R-mode excitation in accreting neutron stars

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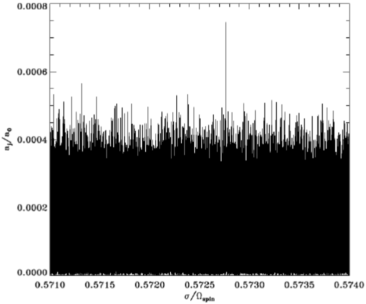
Based on:
Andersson, DIJ & Ho (2014)
DIJ, Santiago-Prieto, Heng & Clarke (in prep.)

The Strohmayer & Mahmoodifar observation

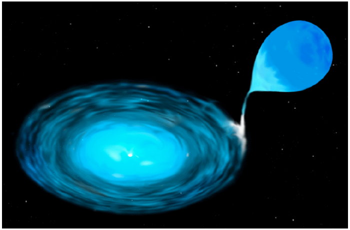
- Strohmayer & Mahmoodifar (2014) observed a coherent oscillation in the X-ray data for XTE J1751-305, $f_{\rm spin} = 435$ Hz.
- They searched over a range of frequecnies, motivated by the possible existence of
  - g-mode oscillations (buoyancy-type modes)
  - r-modes (inertial-type modes)
- Will explore the r-mode intepretation of the data, and ask:

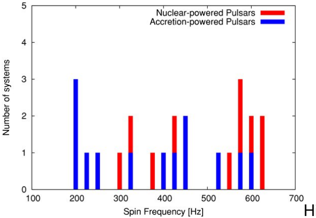
If this is an r-mode, what can we learn about neutron stars and the equation of state?



Why are r-modes potentially relevant?

Gravitational wave emission *may* determine their maximum spin frequency, and r-mode excitation *may* be the relevant mechanism.





Histogram: Patruno (2010)

The Chandrasekhar-Friedman-Schutz (CFS) instability

- Amplitude of a normal mode evolves as $\alpha(t) \sim e^{-t/\tau}$, where

$$\frac{1}{\tau} = \frac{1}{\tau_{\rm GRR}} + \frac{1}{\tau_{\rm damp}}.$$

- R-mode gravitational radiation reaction timescale scales sharply with $f_{\rm spin}$:

$$\tau_{\rm GRR} \approx -1 \text{ hour} \left(\frac{435 \text{ Hz}}{f_{\rm spin}}\right)^6 \frac{1}{M_{1.4} R_{11.7}^4},$$

  for an $n = 1$ polytrope (Owen et al 1998); weak dependence on equation of state.

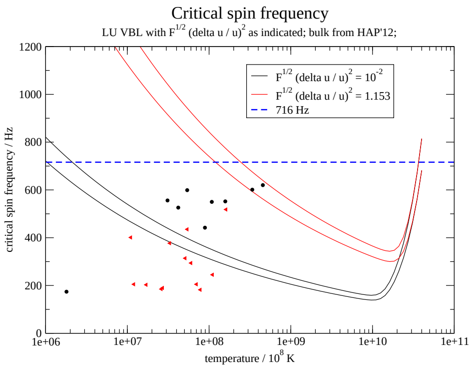
- Viscous damping timescale is temperature and spin frequency dependent:

$$\tau_{\rm damp} = \tau_{\rm damp}(f_{\rm spin}, T) > 0,$$

  strongly depenent on details of the equation of state.

The instability 'band'

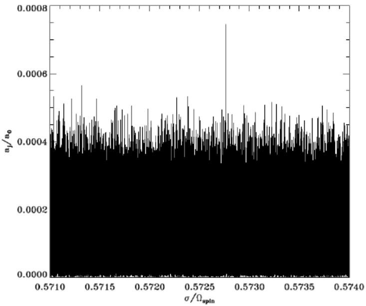
- At relevant temperatures, viscous boundary layer dissipation believed to dominate (Levin & Ushomirsky 2001).
- But instability region too large (Ho et al 2011); can appeal to other physics or simply adjust damping rate 'by hand' to stabalize the observed systems:



- If star lies just *below* curve, impulsive input of energy into r-mode could lead to interesting level of gravitational wave emission (Santiago-Prieto, DIJ, Heng & Clarke, in prep.)

The Strohmayer & Mahmoodifar observation

- Strohmayer & Mahmoodifar (2014) identified *possible* r-mode oscillation from 6 days of X-ray data for XTE J1751-305, $f_{\rm spin} = 435$ Hz.
- Three things that need to be understood/explained:
  1. The *frequency* of the oscillation $f_{\rm mode} = 249$ Hz, $f_{\rm mode}/f_{\rm spin} = 0.57$.
  2. The *evolution* in spin frequency.
  3. The *duration* of oscillation.



Issue 1: The frequency of the oscillation

- For r-mode, simplest model has $f_{\mathrm{mode}}/f_{\mathrm{spin}} = 2/3$, regardless of the perfect fluid EoS.
- This is larger than observed value, but is subject to many corrections, including:
  - Relativistic (GR) effects
  - Rapid rotation
  - Crustal resonances
  - Magnetic fields
  - ...
- Idrisy, Owen & DIJ (2015) estimate relative importance; GR probably largest single correction, away from crustal resonance frequencies at least.
- R-mode frequencies in GR computed by Lockitch, Friedman & Andersson (2003) for polytropic stars; found that full GR result differed significantly from leading-order post-Newtonian approximation.
- Results extended by Idrisy et al. (2015) to realistic EoS: found weak EoS dependence.

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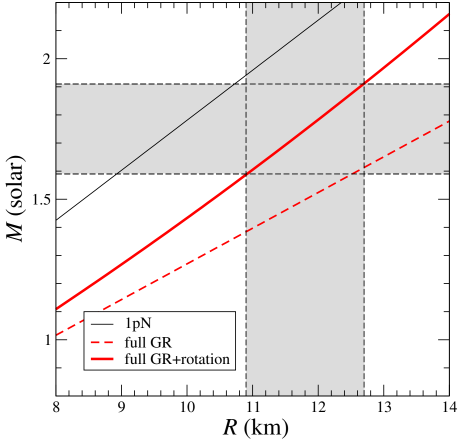
Issue 1: The frequency of the oscillation cont . . .

- Can use departure of observed frequncy from $f_{\mathrm{mode}}/f_{\mathrm{spin}} = 2/3$ to learn about stellar structure.
- Andersson, DIJ & Ho (2014) combined GR & rotational corrections to give curve in *M*–*R* plane:



Issue 2: The evolution in spin frequency

- For r-mode would expect star to *spin-down* at a rate:

$$\dot{f}_{\rm spin} \sim -2 \times 10^{-8}~{\rm Hz\,s^{-1}} \left(\frac{\alpha}{10^{-3}} \right)^2 . \quad (1)$$

- But in fact a *spin-up* was observed, $\dot{f}_{\rm spin} \approx +4 \times 10^{-13}$ Hz s^{-1}.
- Modelling of realistic multi-component star by Lee (2014) suggests that actual r-mode amplitude may be several orders of magnitude *less* than surface emission.
- This partially resolves contradiction; perhaps accretion torque overwhelms r-mode spin-down torque.

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Issue 3: The duration of the observation

- The S & M observation used 6 days of X-ray data.
- This indicates (but does not absolutely prove) that the excitation lasts for at least this long, $\tau \gtrsim 6$ days.
- This is much longer than the gravitational radiation reaction timescale $|\tau_{\rm GRR}| \approx 1$ hour.

$\Rightarrow$ *Near-cancellation between pumping and damping;*
system lies just below its instability curve.

- To connect with ($f_{\rm crit}$–T) instability curve, suppose that dissipation dominated by a single mechanism with power-law scaling with $\tau_{\rm v}^{-1} \sim f^n$.
- Can then show that critical frequency *slightly* exceeds spin frequency:

$$f_{\rm crit} \approx 435\,{\rm Hz}\left[1 + 7 \times 10^{-3}\left(\frac{1}{6-n}\right)\left(\frac{|\tau_{\rm GRR}|}{1\,{\rm hour}}\right)\left(\frac{6\,{\rm days}}{\tau}\right)\right]$$

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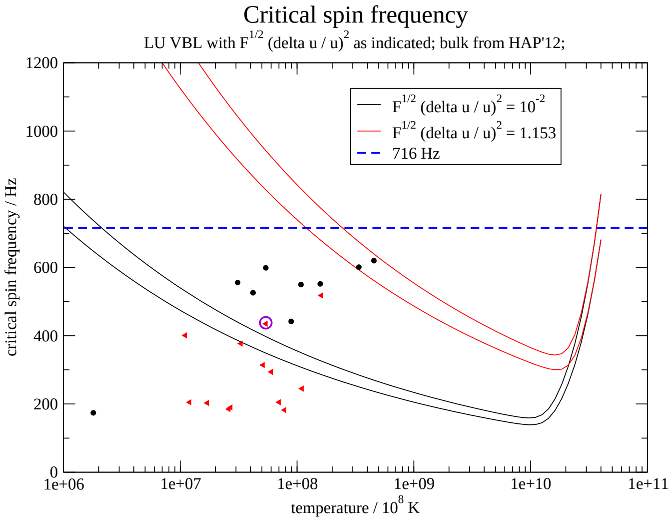
Issue 3: The duration of the observation cont . . .

- This allow us to draw a point in the instability-phase diagram.
- For $f_{\rm crit} = 438$ Hz, $T \lesssim 5.4 \times 10^7$ K.



Issue 3: The duration of the observation cont . . .

- Clearly, cannot simultaneously adjust the viscous boundary layer instability band such that:

  1. XTE J1751 lies just above band **AND . . .**
  2. . . . the faster/hotter LMXBs stabalized

- Two possible resolutions:

  1. The curve has a highly non-monotonic character, caused possibly by resonances involving crust (e.g. Levin & Ushomirsky 2001) or the superfluid (e.g. Gusokov, Chugunov & Kantor 2014).
  2. Instability band is rather wide.

- With regard to second possibility, strong *mutual friction* between superfluid neutron and superconducting protons may be relevant.

- Scaling of mutual friction damping rate with spin frequency is:

$$\frac{1}{\tau_{\rm MF}} \sim f_{\rm spin}^5.$$

- Similarity of scaling $\tau_{\rm GRR}^{-1} \sim f_{\rm spin}^6$ would naturally lead to a relatively wide instability band.

Implications for gravitational wave detection

- Impulsive excitation of r-modes in stable stars generally undetectable.
- If star happens to be close to instability curve, long decay time leads to more easily detectable signal.
- If $\alpha \sim 10^{-3}$, the S & M signal would have been borderline detectable by the LIGO S5 science run . . .
- . . . but event preceded first science run, S1, by a few months.
- Nevertheless, impulsive excitation of r-mode in stars close to instability curve potentially detectable; detailed study in progress (Santiago-Prieto, in preparation).

Summary

- LMXBs/AMXPs might be interesting in terms of *burst-like* GW sources.
- Recent Strohmayer & Mahmoodifar observation could be sign of such behaviour.
- Key issue with r-mode interpretation:
    - Mode frequency corresponds to sensible mass-radius constraint.
    - Observed small spin-up problematic, but not necessarily fatal.
    - Duration of oscillation gives a constraint on location of the instability band.
    - R-mode intepretation implies that instability band is highly non-monotonic, or else is rather wide.
- Further observation of this/similar sources would be useful.