A realistic model of a neutron star in a modified theory of gravity

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COST Action MP1304 NewCompStar,

and ***TCPA Foundation***

Plan of the talk:

- General remarks on DM and DE
- Minimal dilatonic gravity (MDG)
- The basic equations of SSSS in MDG
- The boundary conditions for SSSS in MDG
- Neutron SSSS with realistic EOS in MDG

SSSS = Static Spherically Symmetric Stars

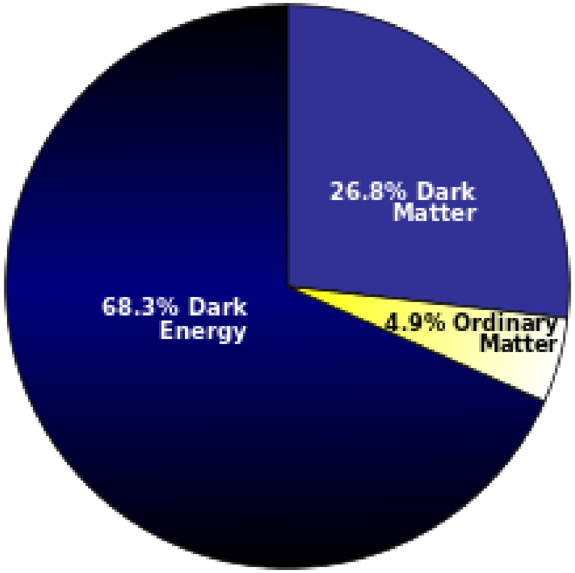
16/06/2015, Budapest

The basic lesson from cosmology:

GR and SPM are not enough !

- One may add some new content: Dark matter (DM), Dark energy (DE)
- One may modify GR: simplest modifications are F(R) and MDG
- Some combination of the above two possibilities may work ?

2015 Planck results:



While the need for **DM** and **DE** is strongly established, their **nature** and their **small-scale distribution** are still largely unknown.

The **only** established part about **DM** is its **gravitational interaction.** We have, in fact, **no evidence that DM has any other interaction but gravity !**

Most probably we need to look

simultaneously and coherently for

a realistic EOS

and for

a realistic modified gravity

which are able to describe a variety of

cosmological, astrophysical, gravitational and star phenomena

at different scales:

Planets, *compact stars,* withe dwarfs, normal stars, stars clusters, dwarf sphericals, galaxies, galaxy clusters, and

the whole Universe

It is not excluded that these objects are related with *different de Sitter vacuums,* suitable for corresponding *different scales* and for *different time epochs.*

Testing General Relativity with Present and Future Astrophysical Observations

TOPICAL REVIEW, arXiv:1501.07274, Emanuele Berti at al. **Class. Quant. Grav. (2015):**

1. The **very existence of compact stars** in f(R) gravity is still a matter of debate.
2. While **it is hard to construct NS equilibrium configurations** in f(R) gravity from a numerical point of view, there is no fundamental obstacle to their existence.
3. **NS** configurations with **realistic values** of the physical parameters have never been constructed in viable f(R) models.
4. **The properties of NS** in the most general scalar-tensor theory with second-order equations of motion (**Horndeski gravity**) have not been explored, even in the static case.
5. **The numerical challenges** they introduce may also serve as a motivation to develop more efficient integration methods.
6. The study of compact objects in f(R) gravity is particularly difficult, especially for realistic configurations.

The new basic results of the talk:

1. The difficulties in numerical investigation of **realistic models** of **NS** in the modified theories of gravity **are surmounted on a general basis**.

2. We present a **realistic model** of static spherically symmetric **NS** with **MPA1 EOS** using **correct boundary conditions**.

3. The critical step is the introduction of a **new field variable** for the scalar degree of freedom which we call "**the dark scalar**".

4. The maximal mass or the **NS** with **MPA1** EOS turns to be around **2.7** solar masses and **depends on the mass of the dark scalar**.

5. We investigate the influence of the **dark scalar** on the gravitational field **inside** the **NS** and its **dark halo outside** the star. The **dark halo** may give some **15 %** of the total mass of the **NS**.

6. The newly introduced pressure and mass density of the **dark matter** and **dark energy** are also discussed.

Minimal dilatonic gravity (MDG)

$$\mathcal{A}_{g,\Phi} = \frac{c}{2\kappa}\int d^4x\sqrt{|g|}\left(\Phi R - 2\Lambda U(\Phi)\right) + \boldsymbol{\mathcal{A}}_{matter}$$

NO Φ enters

In GR with cosmological constant Λ: $\Phi \equiv 1$, $U(\Phi) \equiv 1$.

O'Hanlon: PRL, 1972,

PPF: **Mod. Phys. Lett. A, 15,** 1077 (2000); gr-qc/0202074;

PRD 67, 064016 (2003); **PRD 87**, 0044053 (2013); **PoS** (**FFP14**) 080 (1914);

PPF, K. Marinov: **BAJ**, **23**, 1 (2015)

NEW: variable

$$G(\Phi) = G_N/\Phi \qquad \Lambda U(\Phi)$$

Gravitational factor — Cosmological factor

$\Lambda > 0$
$\Phi > 0$
$U > 0$

Observed value: $\boldsymbol{\Lambda} \approx \mathbf{1.087 \times 10^{-56}}\ cm^{-2}$

Very small

MDG is locally equivalent to **f(R).**

16/06/2015, Budapest

Basic Equations of MDG:

(PPF 2000-2014)

$$\Phi G_{\alpha\beta} + \Lambda U g_{\alpha\beta} + \nabla_\alpha \nabla_\beta \Phi - \Box \Phi g_{\alpha\beta} + 8\pi T_{\alpha\beta} = 0$$

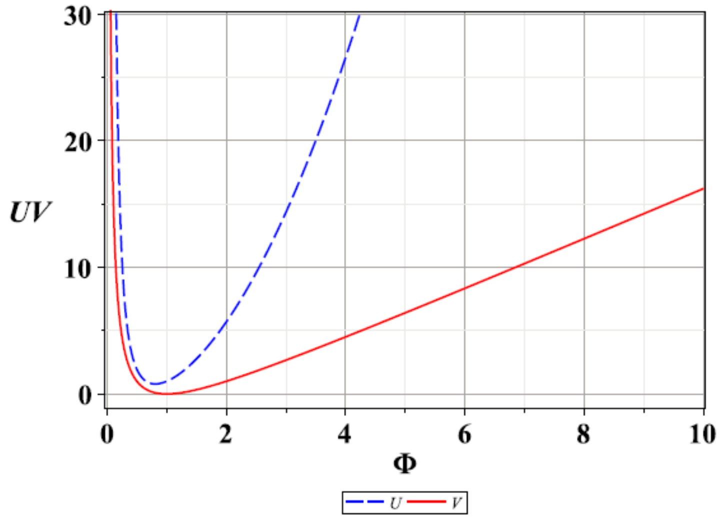
$$\Box \Phi + \Lambda V_{,\Phi}(\Phi) = \frac{8\pi}{3} T$$

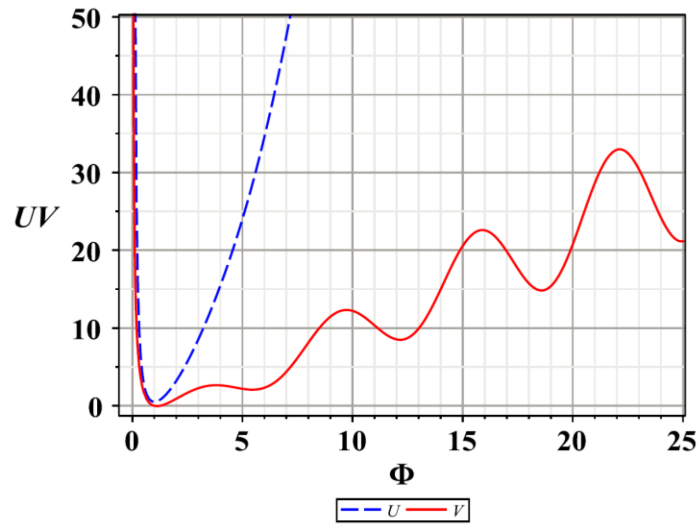
$R = 2\Lambda U_{,\Phi}(\Phi)$ ➡ Cosmological principle respected

$\nabla_\mu T^{\mu\alpha} = 0$ ➡ Energy-momentum conservation respected

Withholding potentials: PPF: MPLA (2000); PRD (2013)

No ghosts! No tachions !



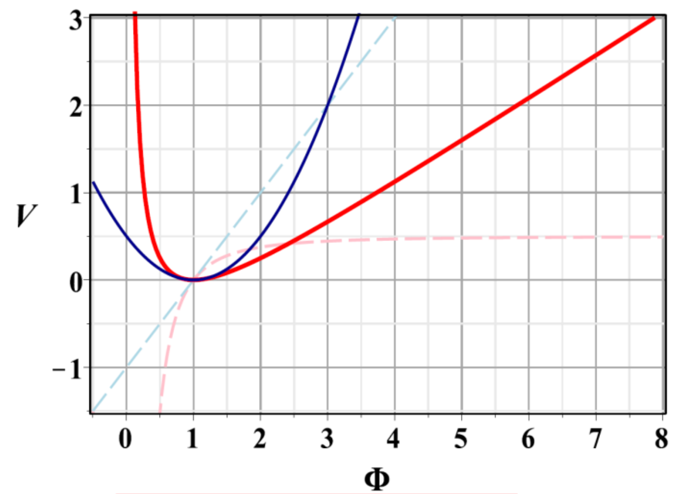


MDG is consistent with Solar system experiments:

$$m_\Phi > 10^{-3}\,\mathrm{eV}/c^2$$

$\lambda_C \sim 10^{-2}$ **cm**

Comparison of the **Starobinsky 1980-2007** potentials V_{St} and dilatonic potential V with identical masses of the scalaron and MDG-dilaton:



$$V_{St}(\Phi) = \frac{\mu^2}{12\Lambda}(\Phi - 1)^2$$

$$m_{scalaron} = m_\Phi \approx 3 \times 10^{-6} M_{Planck}$$

⇒ $\lambda_C \sim 10^{-27}$ **cm**

The basic equations of SSSS in MDG

PF: arXiv:1402.281

Generalized TOV equations:

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\Omega^2$$

Non autonomous:

$$m' = 4\pi r^2 \epsilon_{eff}/\Phi,$$

$$\Phi' = -4\pi r^2 p_{\Phi}/\Delta,$$

$$p'_{\Phi} = -\frac{p_{\Phi}}{r\Delta}\left(3r - 7m - \frac{2}{3}\Lambda r^3 + 4\pi r^3 \frac{\epsilon_{eff}}{\Phi}\right) - \frac{2}{r}\epsilon_{\Phi},$$

$$p' = -\frac{p+\epsilon}{r}\frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_{\Phi}/\Phi},$$

$(\,)' = d/dr$

Decoupled:

$$\nu' = \frac{2}{r}\frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_{\Phi}/\Phi}$$

In GR: 3^{rd} order autonomous ODE

In MDG: 5^{th} order autonomous ODE

$$\epsilon_{eff} = \epsilon + \epsilon_{\Lambda} + \epsilon_{\Phi},$$
$$p_{eff} = p + p_{\Lambda} + p_{\Phi}$$

$$A = 4\pi r^2$$
$$\Delta(r) = r - 2m(r) - \frac{1}{3}\Lambda r^3$$

NOVEL Quantities and EOS:

Dark energy-density and pressure:

$$\epsilon_\Lambda = \frac{\Lambda}{8\pi}\Big(U(\Phi) - \Phi\Big)$$
$$p_\Lambda = -\frac{\Lambda}{8\pi}\Big(U(\Phi) - \frac{1}{3}\Phi\Big)$$

$$-\Lambda U = \left(\epsilon_\Lambda + 3p_\Lambda\right)/2 < 0$$

Dark matter-density and pressure:

$$\epsilon_\Phi = \frac{1}{8\pi}\frac{1}{A}\frac{d}{dl}\left(A\frac{d\Phi}{dl}\right)$$
$$p_\Phi = -\frac{1}{8\pi}\frac{1}{A}\frac{dA}{dl}\frac{d\Phi}{dl}$$

$$ds^2 = \alpha(l)^2 dt^2 - dl^2 - A(l)d^2\Omega$$
$$d^2\Omega = \left(d\theta^2 + \sin^2(\theta)d\phi^2\right)/4\pi$$

Three equations of state:

$$\epsilon_\Lambda = -p_\Lambda - \frac{\Lambda}{12\pi}\Phi\,;$$ ← **DE** EOS

$$\epsilon_\Phi = p - \frac{1}{3}\epsilon + \frac{\Lambda}{8\pi}V'(\Phi) + \frac{p_\Phi}{2}\frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_\Phi/\Phi}\,;$$ ← **DM** EOS

$$\epsilon = \epsilon(p)$$ ← **M** EOS

Schematic procedure for calculations

Center of the star **Fixed singular boundary**		Edge of the star **Moving regular boundary**		Boundary of the Universe **Moving singular boundary**
$r_c = 0$		r_*		r_U
$m_c = 0$	**4 Eqs**	m_*	**3 Eqs**	m_{tot}
$p_c > 0$	→	$p_* = 0$	→	$\Delta = 0$
$\Phi_c > 1$	Non autonomous	$\Phi_* > 1$	Non autonomous	$\Phi_U = 1$
$p_{\Phi c}(p_c, \Phi_c) = \left\vert \frac{2}{3}(\varepsilon/3+p) - \frac{2\Lambda c}{3\varkappa} V_\Phi \right\vert_c$		$p_{\Phi *}$		$p_{\Phi U}$

$$F_\Phi(p_{\Phi c}, p_c, \Phi_c) = 0, \quad F_\Lambda(p_c, \Phi_c) = 0,$$ ← **Two specific MDG relations**

One parametric (p_c) family of SSSS – as in GR and the Newton gravity !

Logarithmic variables

$\mathbf{r} \in [0, r_*]$:

$$\xi = log_{10}(\rho) \quad \leftrightarrow \quad \rho = 10^{\xi}$$
$$\zeta = log_{10}(p) \quad \leftrightarrow \quad p = 10^{\zeta}$$

$\mathbf{r} \in [r_*, r_U]$:

$$x = \ln(r)$$

$$\Phi = \exp(\boldsymbol{a}\exp(\varphi)\text{-}1)$$ – **The dilaton**

$$\varphi = \ln(1 + \boldsymbol{a}^{-1}\ln\Phi)$$ – **The dark scalar**

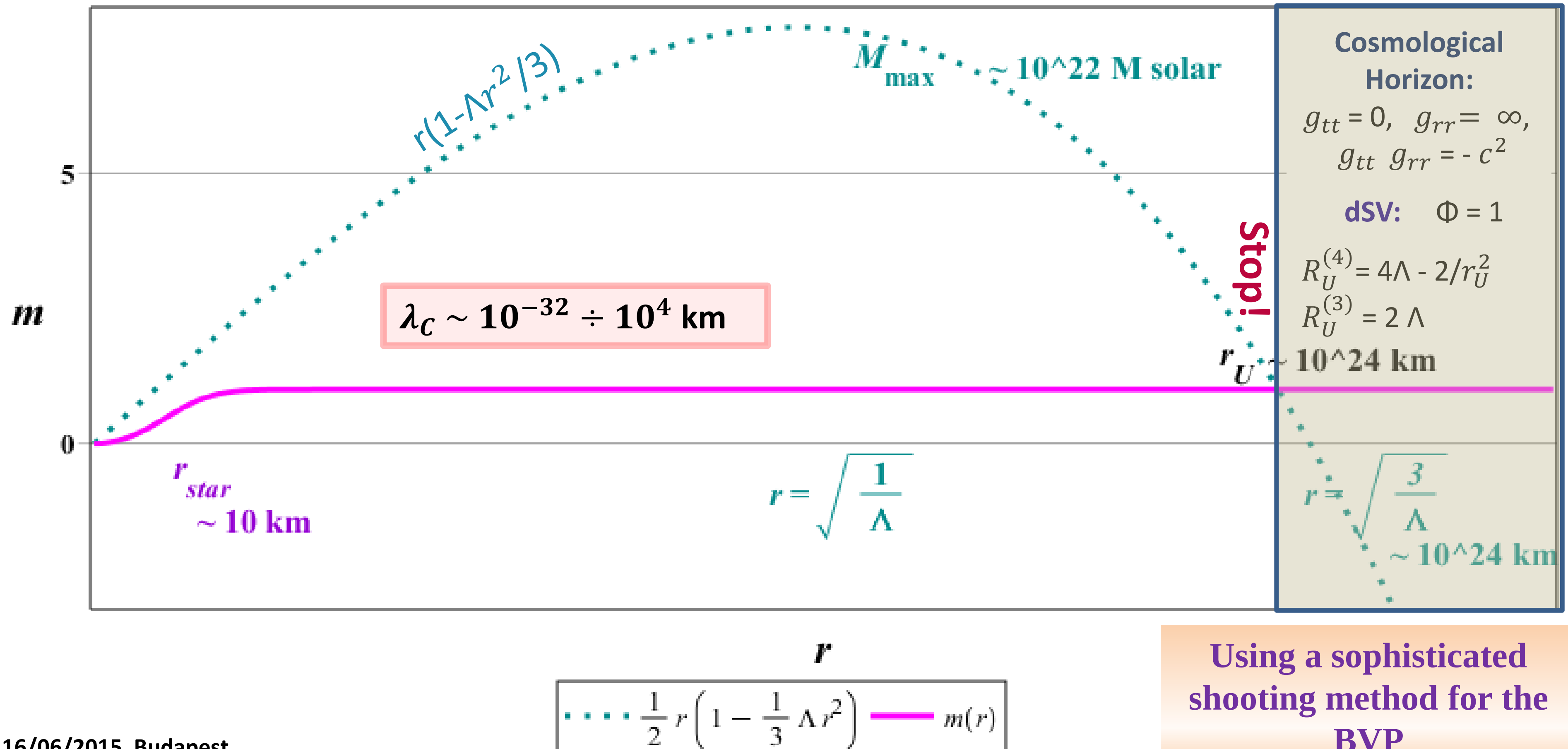
$$\boldsymbol{a} > 0 \quad \rightarrow \quad 0 < \Phi < \infty\,, -\infty < \varphi < \infty \qquad (\boldsymbol{a} = 1)$$

The Border of the MDG-Kottler-Weyl-like Universe

Kottler (1918)-Weyl (1919): $ds^2 = \left(1 - \frac{2m}{r} - \frac{\Lambda}{3} r^2\right) dt^2 - \left(1 - \frac{2m}{r} - \frac{\Lambda}{3} r^2\right)^{-1} dr^2 - r^2 d\Omega^2$

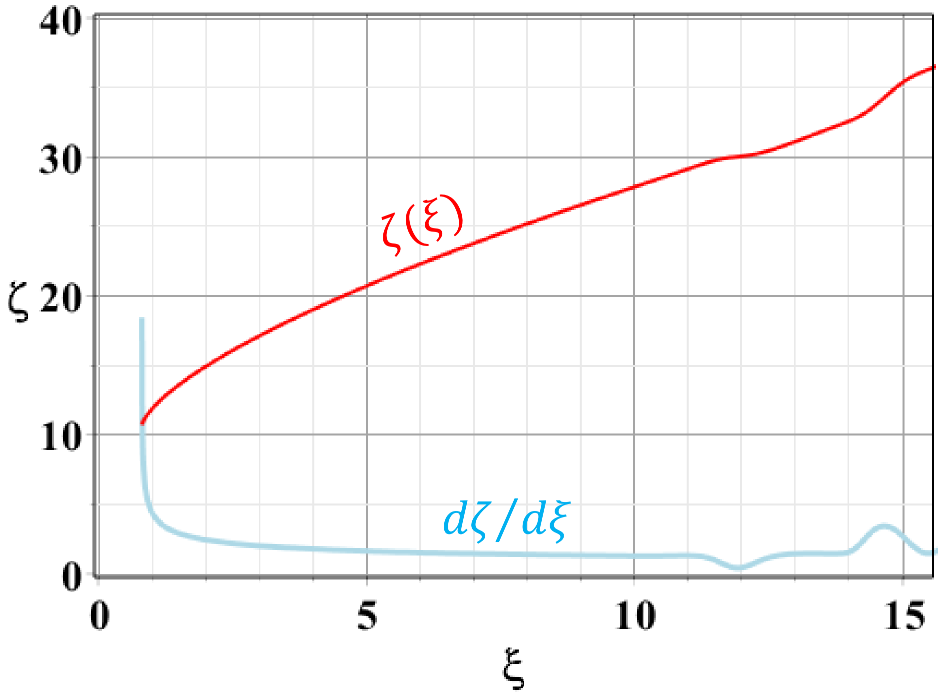
$m = const$ (Schwarzschild-de Sitter Universe)

The MDG-One-Star-Universe: (non-real scales!)



Using a sophisticated shooting method for the BVP

MEOS AMP1:



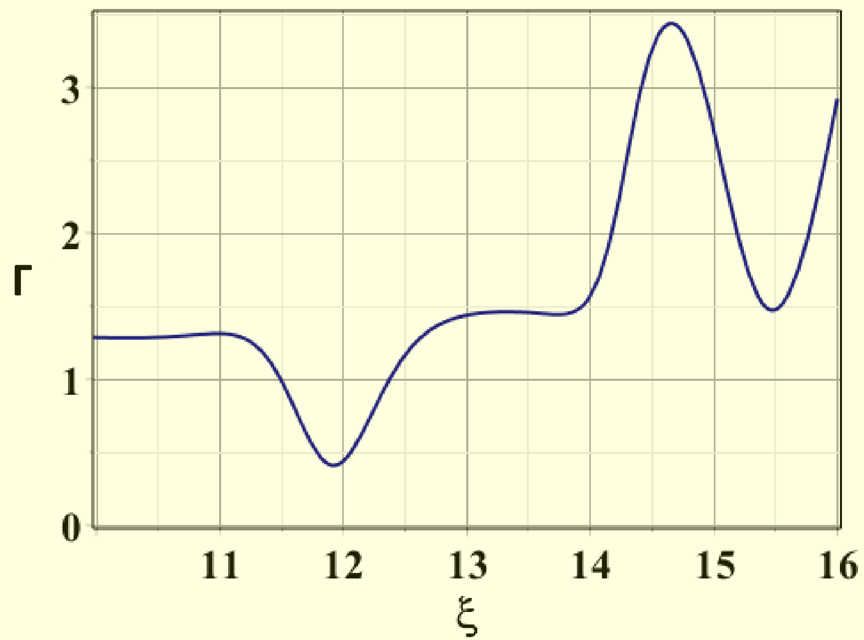
The MEOS **MPA1**

H. MUTHER, M.PRAKASH,T.L. AINSWORTH
Extensoin of the **Brueckner-Hartree-Fock** approach for nuclear matter to dense neutron matter, PHYSICS LETTERS B, **199, (1987)**

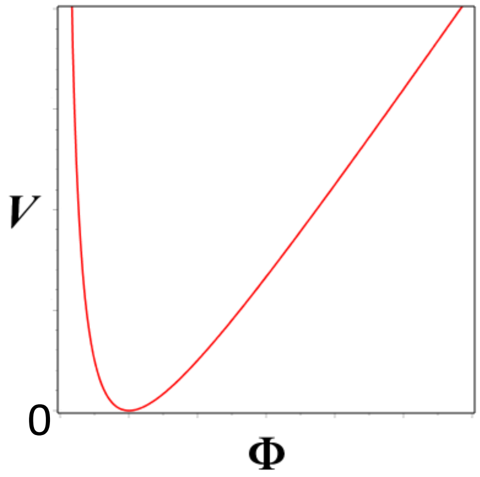
C. Güngör, K. Y. Ekşi, **arXiv:1108.2166**

The adiabatic index

$$\Gamma = \left[1 + \frac{P}{\rho c^2}\right] \frac{\rho}{P} \frac{\mathrm{d}P}{\mathrm{d}\rho}$$



Some new MDG-results for MEOS AMP1:



$$V(\Phi) = \frac{\Phi + \frac{1}{\Phi} - 2}{2\,\Pi^2}$$

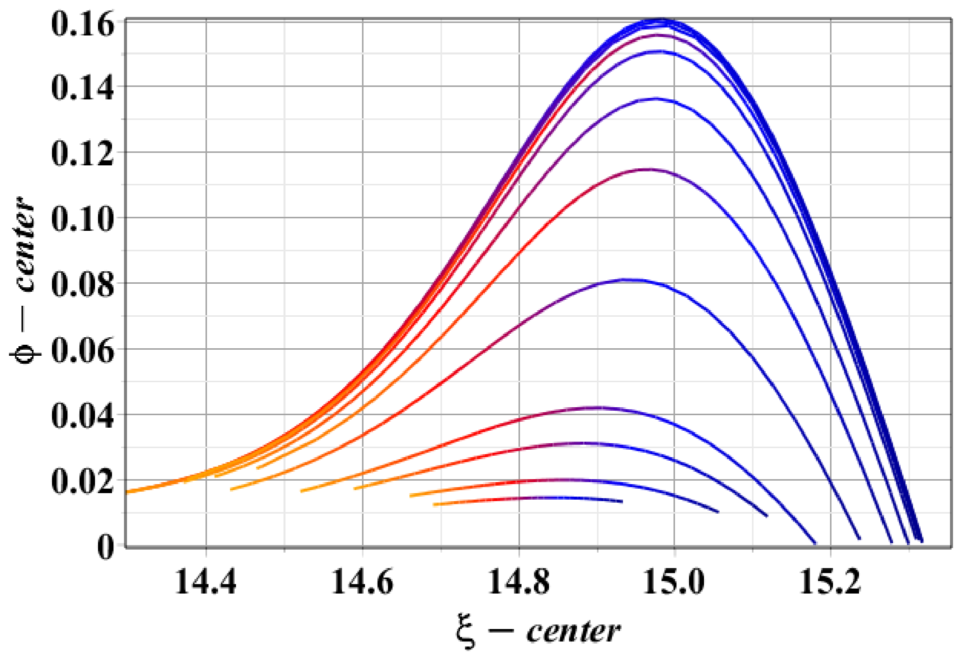
$$\Pi = \lambda_C \sqrt{\Lambda} \sim$$

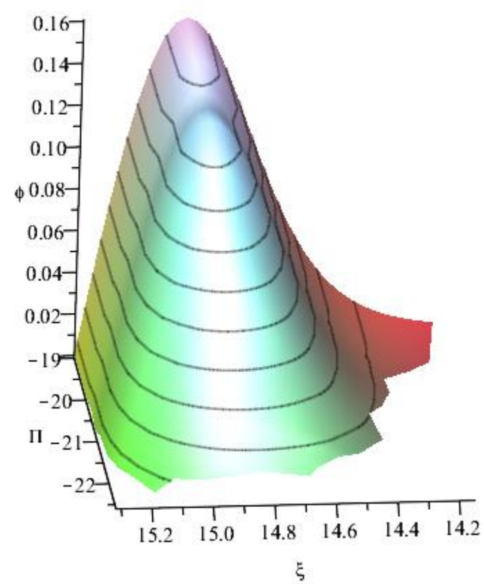
$$\sim 10^{-19} \div 10^{-61}$$

$$\lambda_C = \hbar / m_\Phi c$$

A new phenomenon: Shrinkage of the domain of initial conditions approaching the bifurcation point:

MPA1: xi-center versus phi-center for different masses of dark scalar





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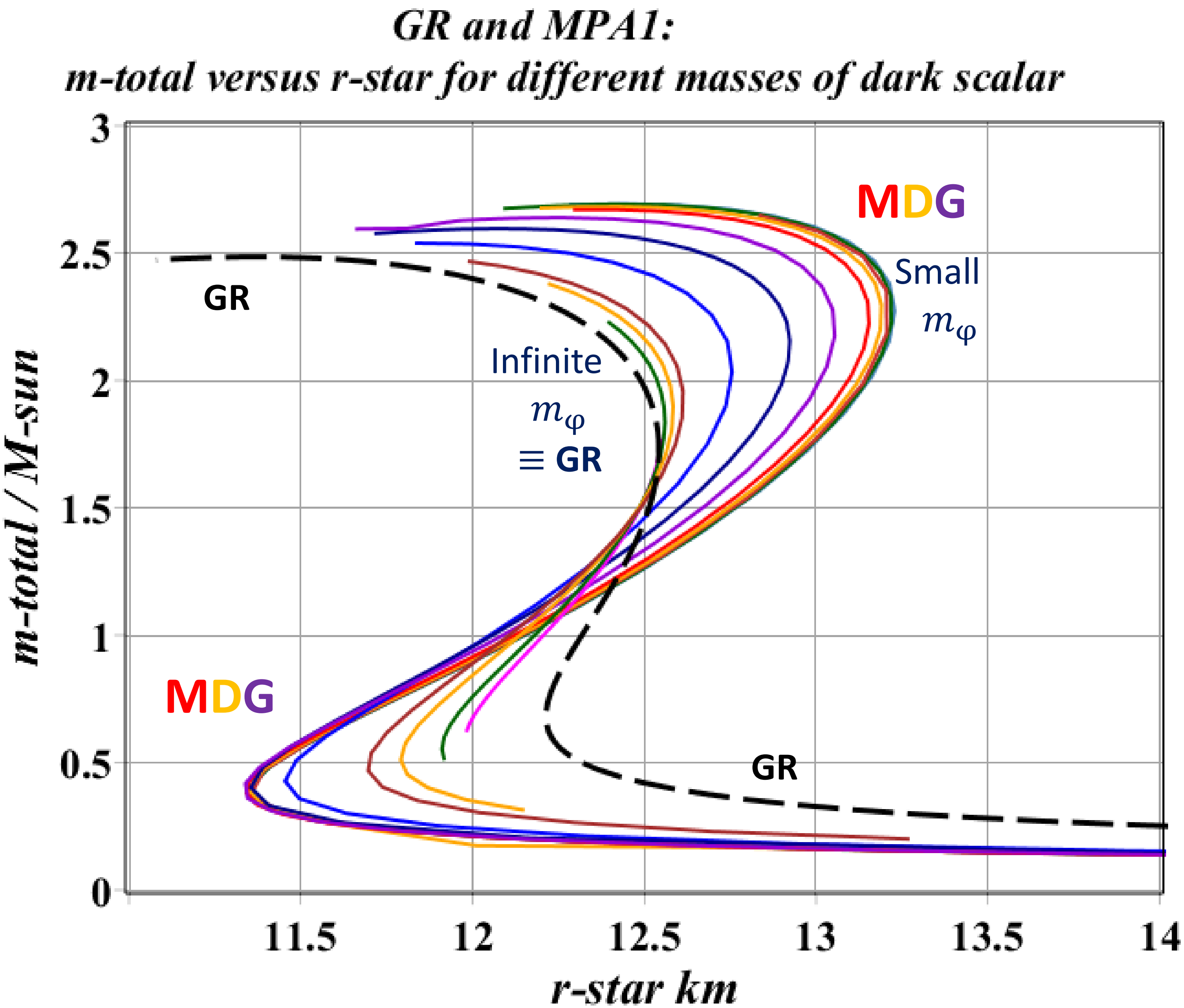
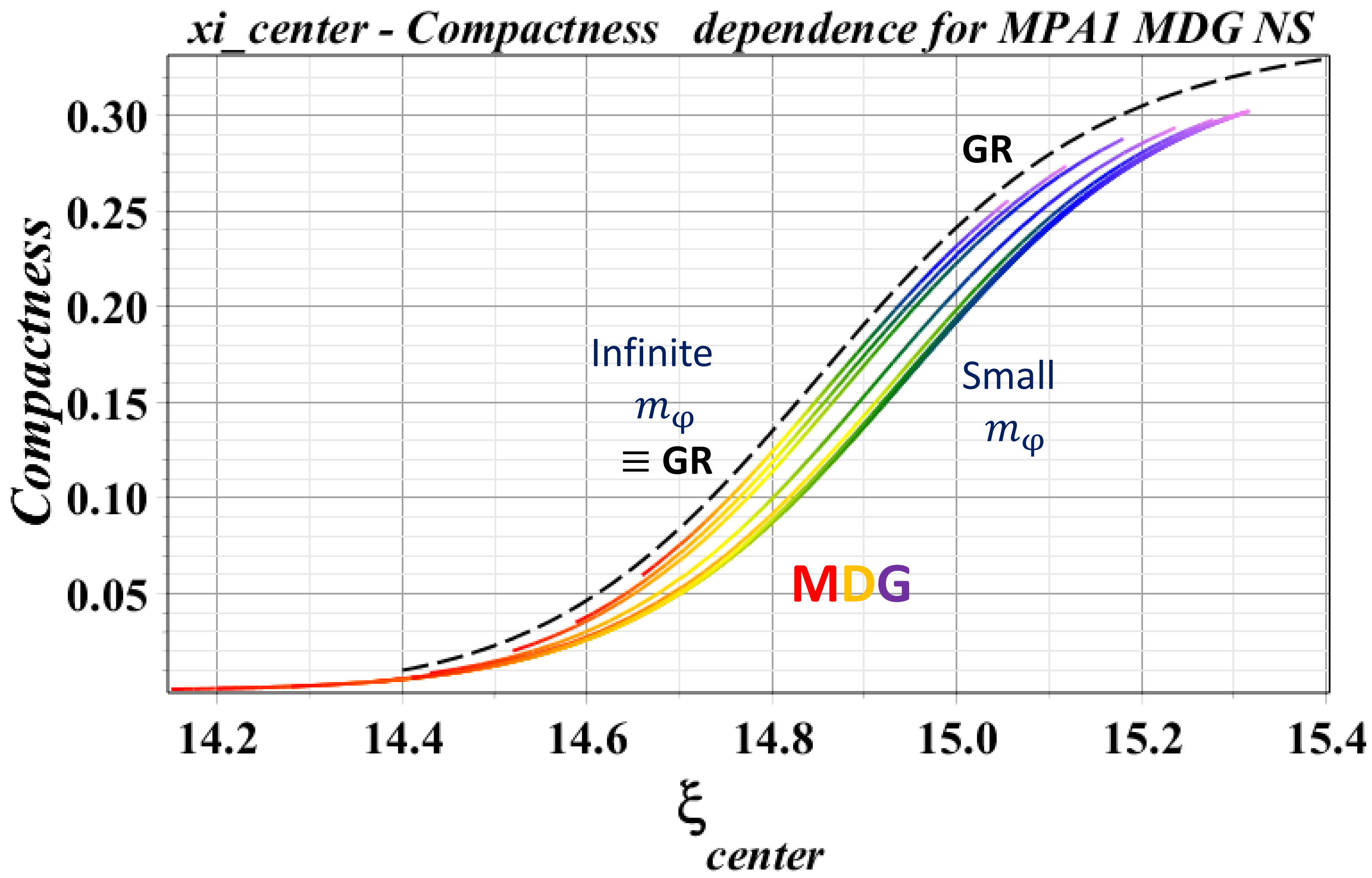
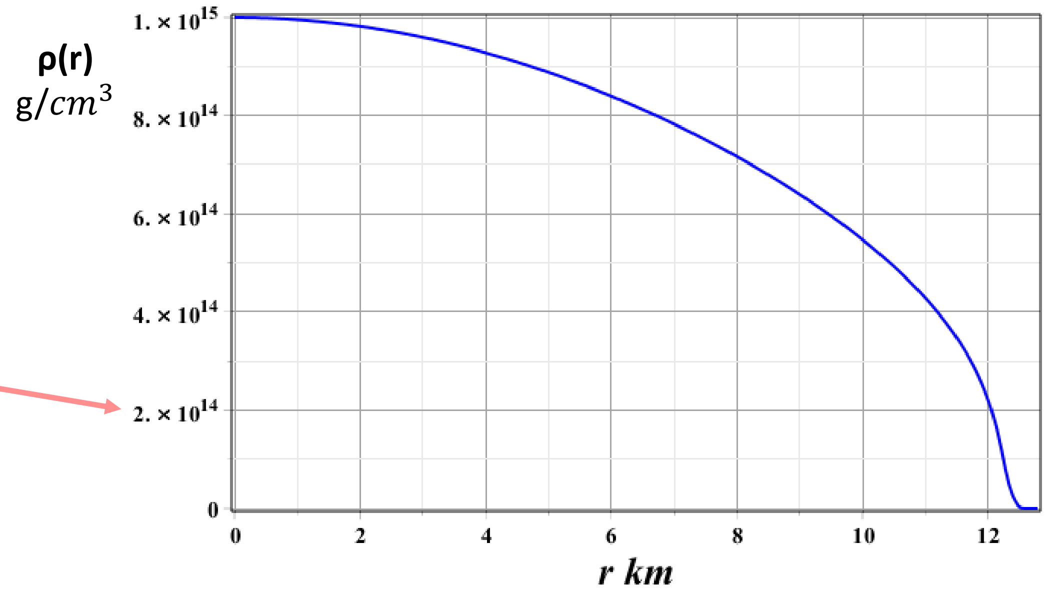


Compactness of MDG-NS for MEOS AMP1 and for different masses of the dark filed:

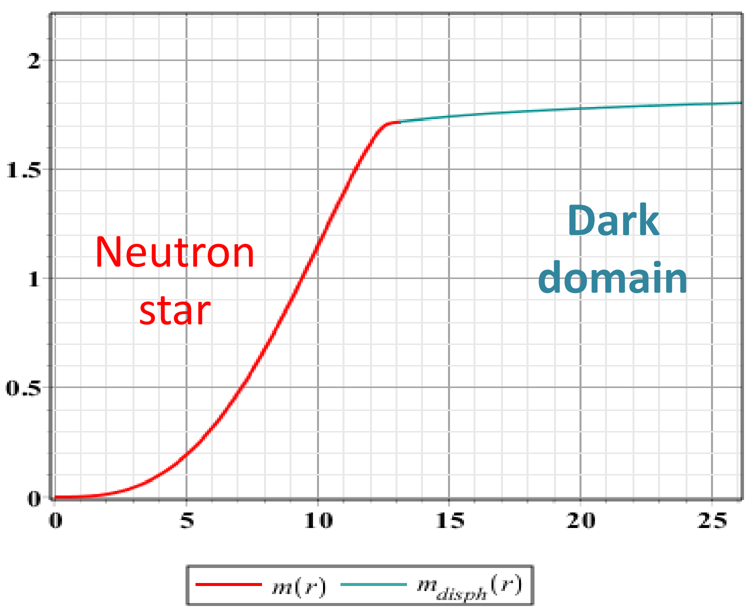


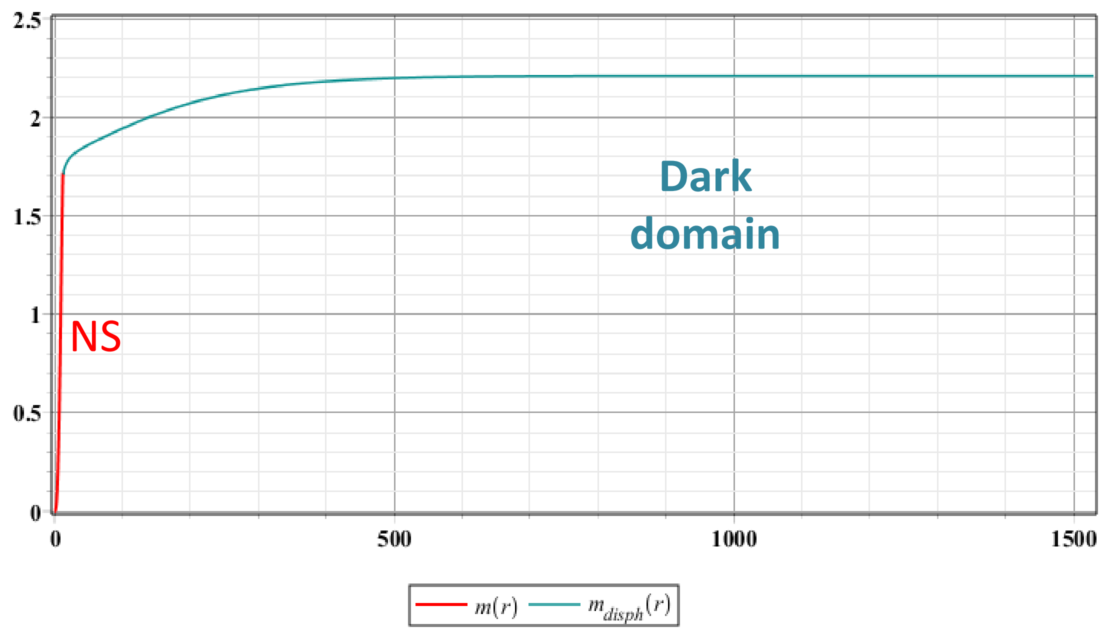
Some new MDG-results for MEOS AMP1:

Nuclear densitiy 2.04×10^{14} g/cm^3

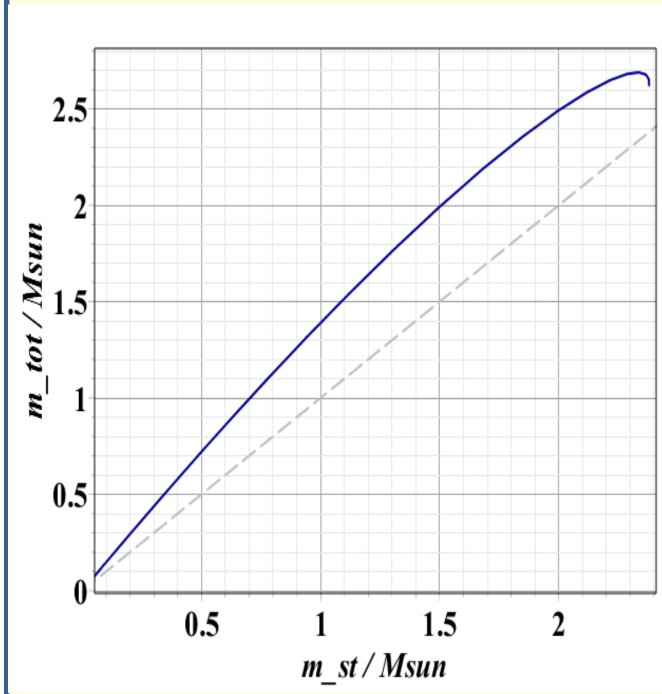


Fe 56 densities 6.49 g/cm^3





Stability of MDG-NS with MEOS AMP1:

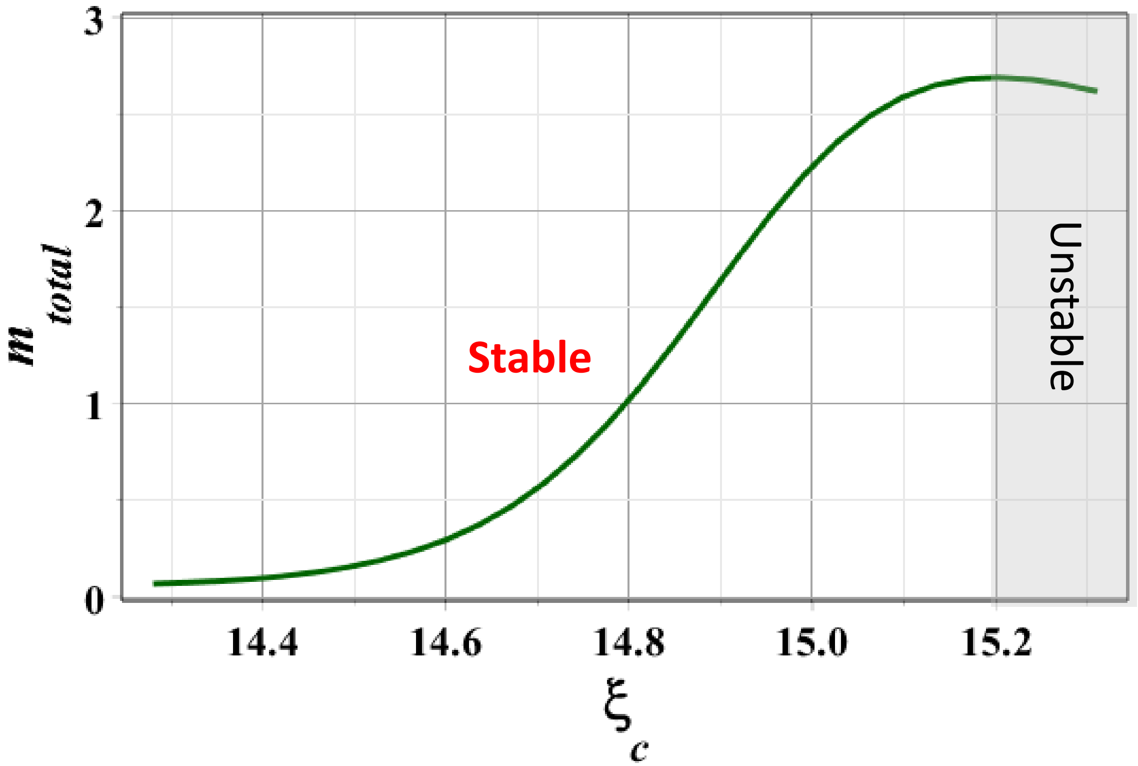


m_{total}(r)
as a function of
m_{stsar}(r)
for r ∈ [0, r_{star}]

m_{total}

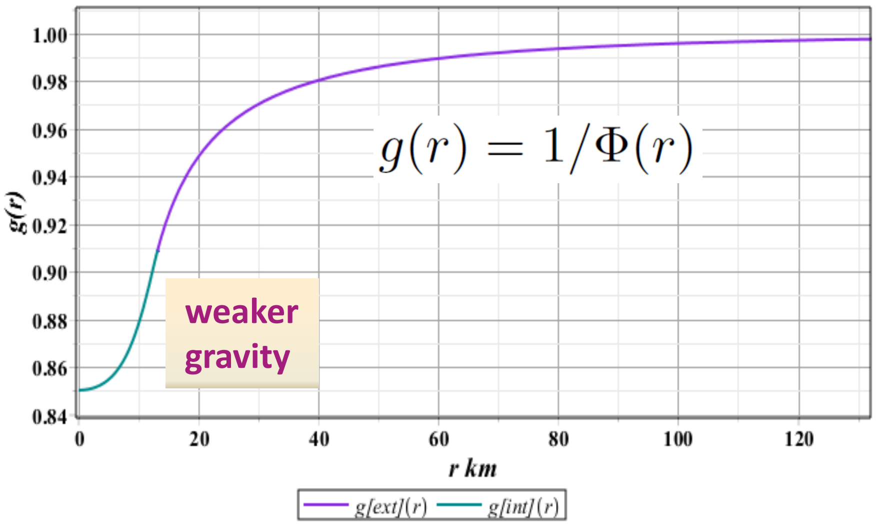
as a function of central density

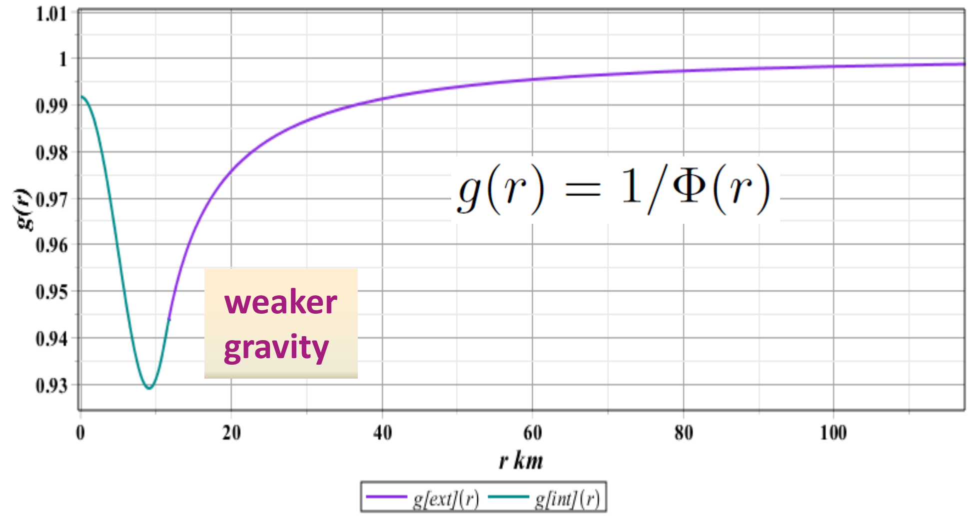
$\xi_c = log_{10} (\rho_c \;\; in \; g/cm^3)$

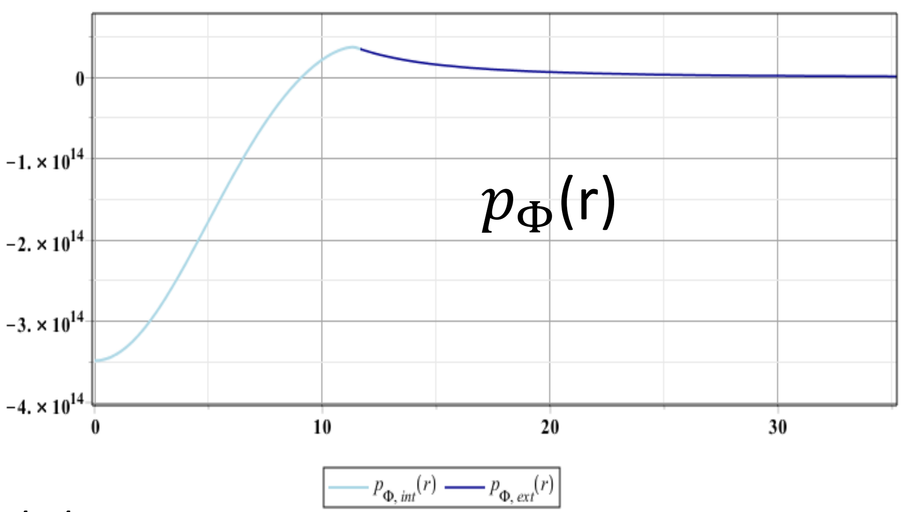


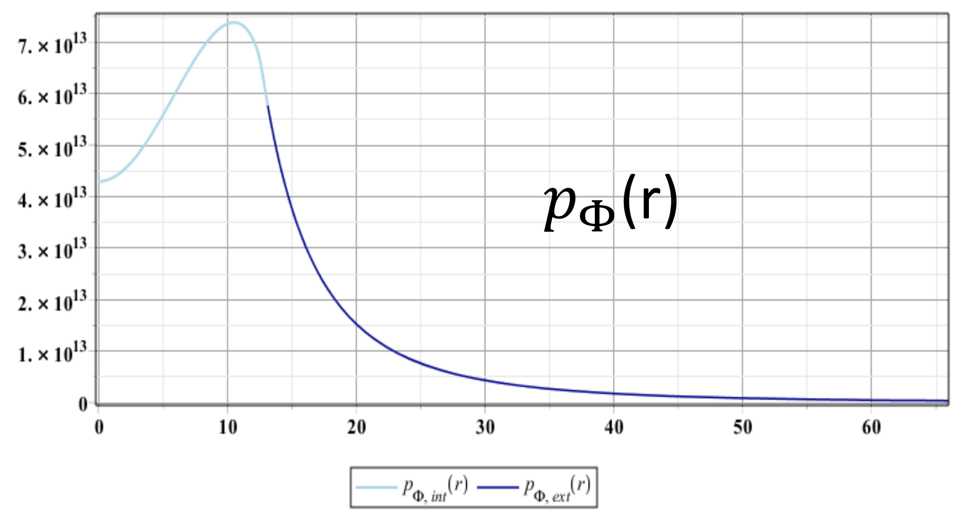
In MDG we have the same stability properties of SSSS as in GR

Some new MDG-results for MEOS AMP1:

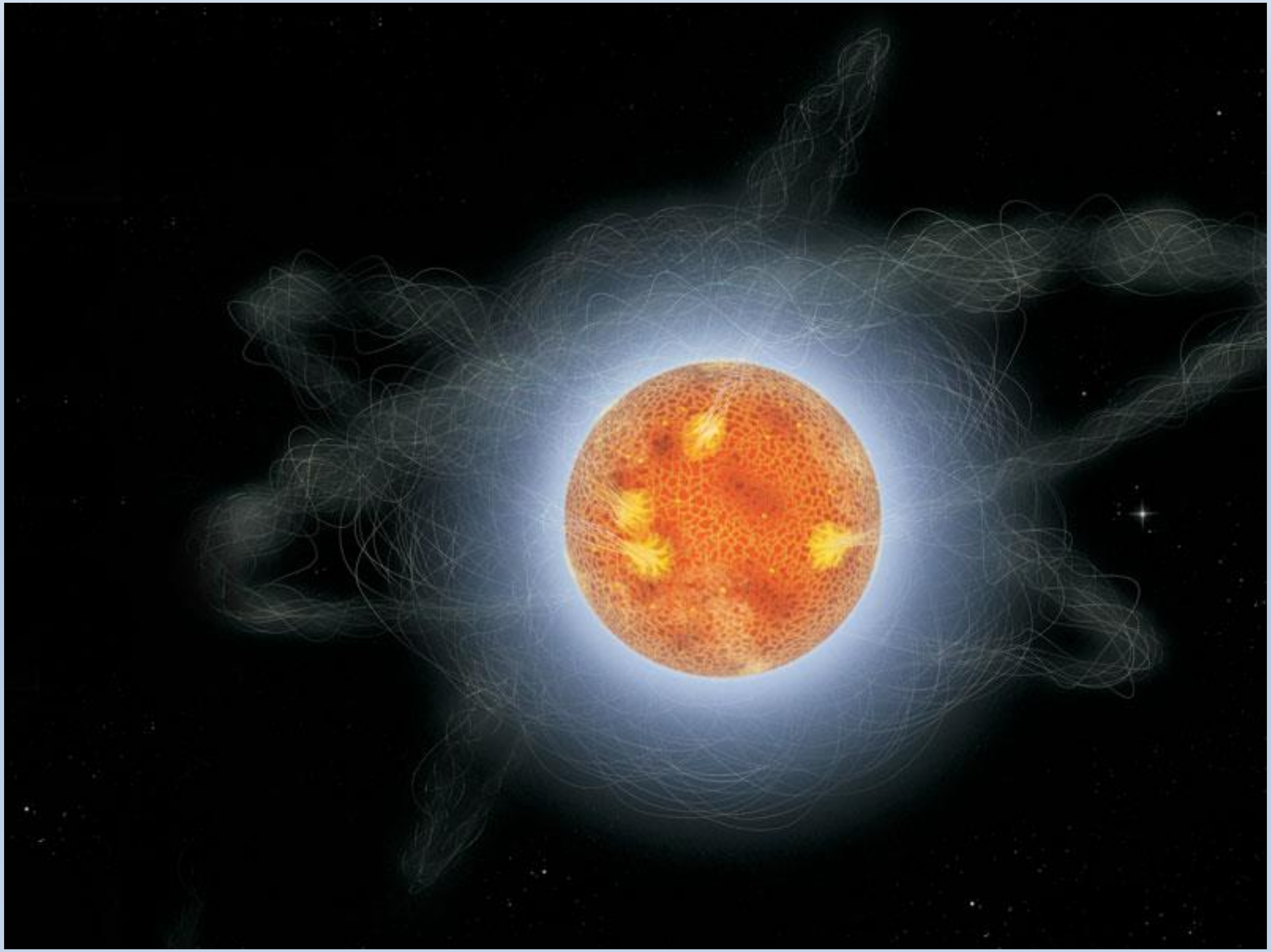








More efforts are needed to know more about the influence of dark matter and dark energy on NS



Thank you!

The boundary conditions for SSSS in MDG

PF: arXiv:1402.281

Assuming:
$r_c = 0$

$$m(0) = m_c = 0, \quad \Phi(0) = \Phi_c, \quad p(0) = p_c,$$
$$p_\Phi(0) = p_{\Phi c} = \frac{2}{3}\left(\frac{\epsilon(p_c)}{3} - p_c\right) - \frac{\Lambda}{12\pi}V'(\Phi_c).$$

SSSS edge:
$P = 0 \Longrightarrow r^*$

$$m^* = m(r^*; p_c, \Phi_c), \quad \Phi^* = \Phi(r^*; p_c, \Phi_c),$$
$$p_\Phi^* = p_\Phi(r^*; p_c, \Phi_c).$$

Cosmological horizon:
r_U

$$r \in [r^*, r_U] \quad p \equiv 0 \text{ and } \epsilon \equiv 0$$
$$r_U: \ \Delta(r_U; p_c, \Phi_c) = 0, r_U \sim 1/\sqrt{\Lambda} \sim 10^{23}\,\text{km}$$

$$\Phi(r_U; p_c, \Phi_c) = 1 \quad \longleftarrow \quad \text{De Sitter vacuum}$$

$$F_\Phi(p_{\Phi c}, p_c, \Phi_c) = 0, \quad F_\Lambda(p_c, \Phi_c) = 0. \quad \longleftarrow \quad \text{Two specific MDG relations}$$

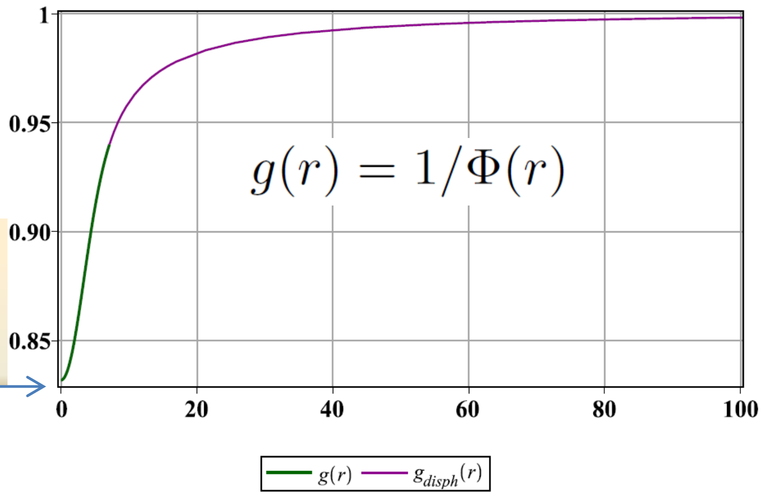
One parametric (p_c) family of SSSS – as in GR and the Newton gravity !

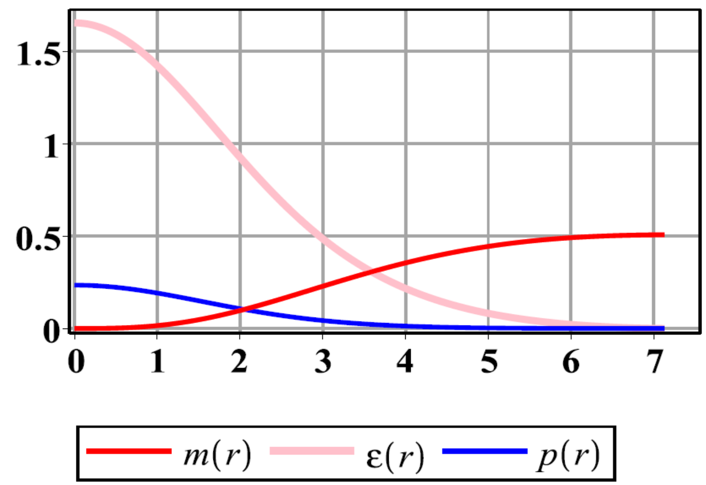
Chandrasechkar (1935), TOV (1939) MEOS in MDG

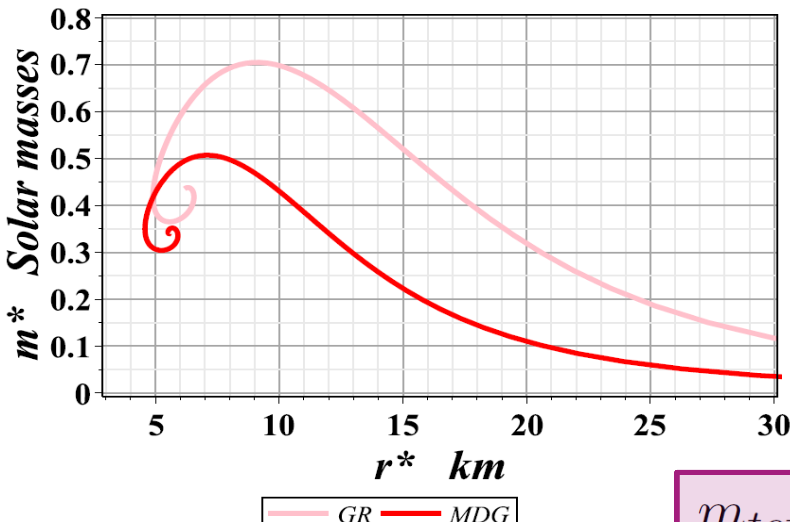
PF: arXiv:1402.281

$$\epsilon = \frac{1}{4\pi}(\sinh t - t), \qquad p = \frac{1}{12\pi}(\sinh t - 8\sinh(t/2) + 3t).$$

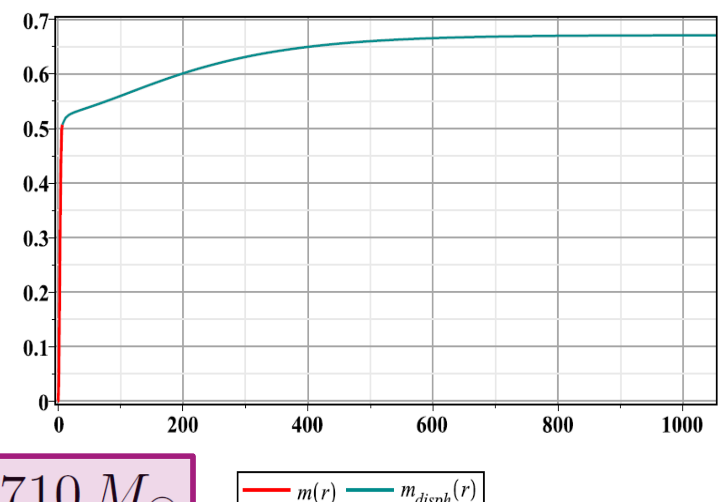
17% weaker gravity

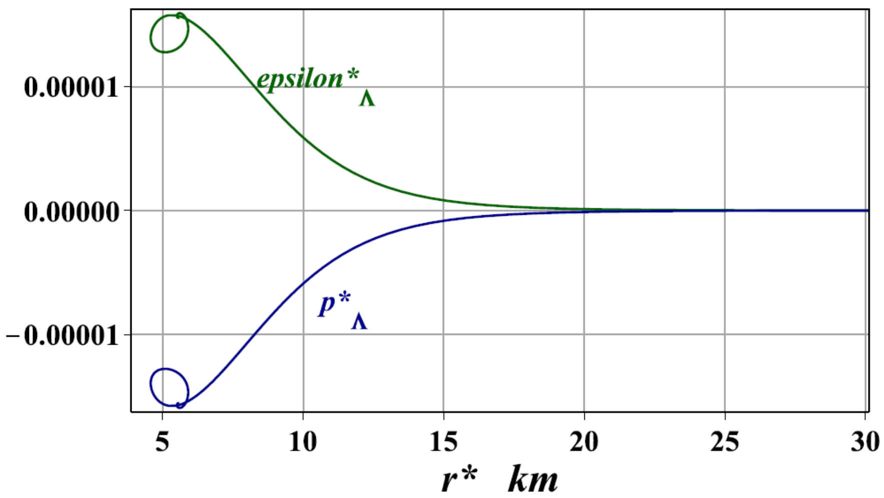


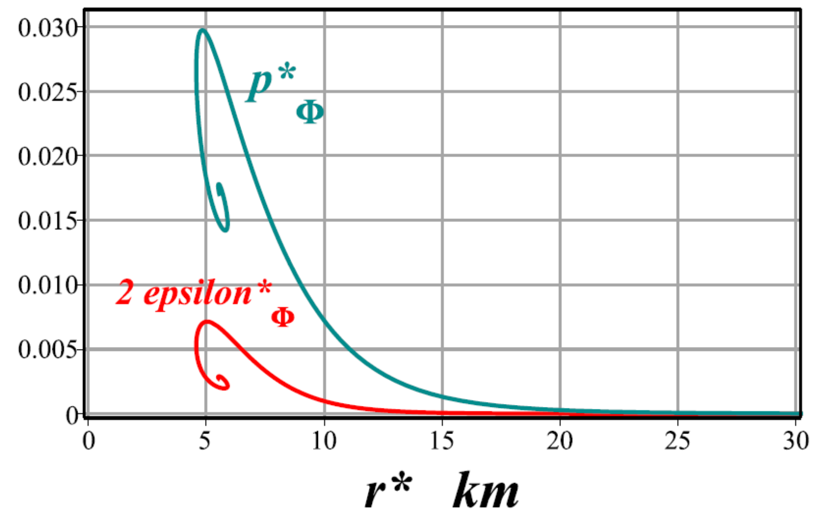


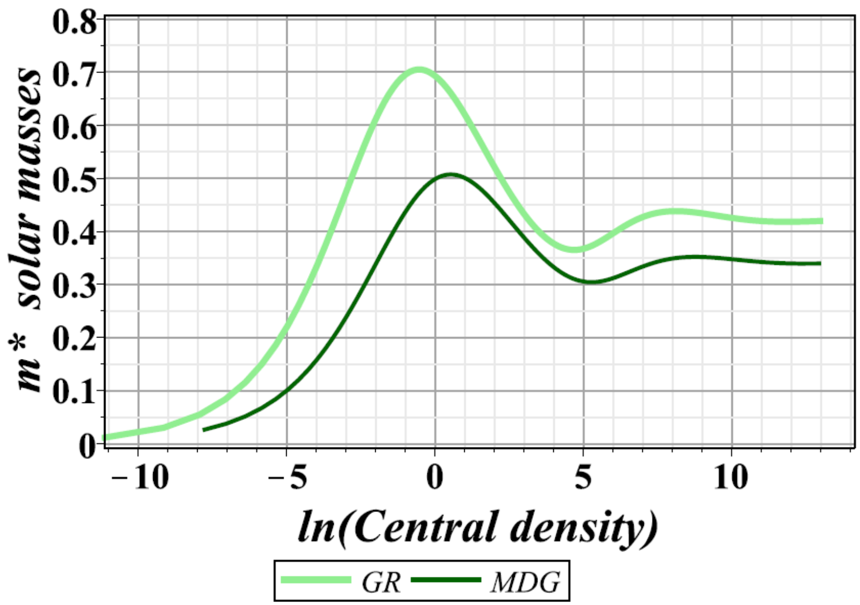


$$m_{total} \approx 0.6710\, M_{\odot}$$









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