The crust and the neutron star radius

Constança Providência

CFisUC, Universidade de Coimbra, Portugal

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Motivation

- How does the crust-core EOS matching affects the NS radius and crust thickness?

  collaboration with

  Morgane Fortin, Michael Bejger, Pawel Haensel, Leszek Zdunik (CAMK, Poland)

- What is the effect of including light clusters on the description of the non-homogeous phase of PNS or CCSN matter?

  collaboration with

  Sidney Avancini (U Santa Catarina, Brasil), **Helena Pais, Marcio Ferreira** (U Coimbra)

Outline

Crust-core matching

Unified EOS

Fit of unified EOS

Light clusters in a pasta calculation

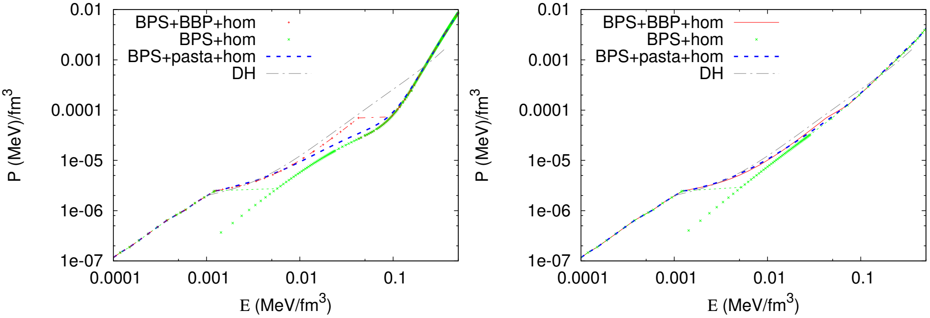
Crust-core matching

- Crust-core matching influences crust thickness and star radius
    - Crust size affects properties such as crustal torsional vibrations or the crustal moment of inertia necessary to explain pulsar glitches.
- How is crust-core matching done?
    - Outer crust: densities below neutron drip $\rho \lesssim 10^{-4}$ fm^{-3}
        - Baym-Pethick-Sutherland (BPS) (1971); Haensel & Pichon (1994); Ruester, Hempel, Schaffner-Biellich (2006)
    - Inner crust: from neutron drip to crust-core transition
        - Baym-Bethe-Pethick (CLDM) 1971; Negele-Vautherin (HF) 1973; Douchin & Haensel (CLDM) 2001
    - Core:
        - β-equilibrium EOS for homogeneous matter

Crust-core matching

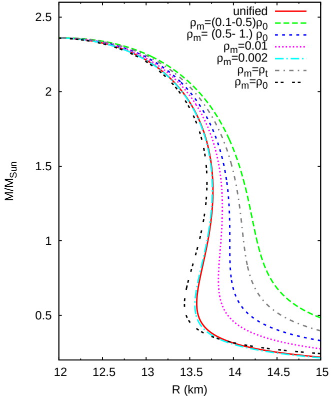
Matching NL3 and NL3$\omega\rho$ with DH

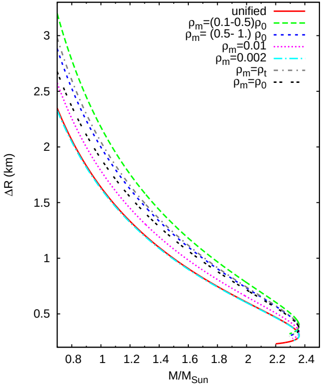
NL3($L = 118$MeV) , NL3$\omega\rho$ ($L = 55$ MeV), DH ($L = 46$ MeV)



Radius and crust-core matching

- Matching with Douchin-Haensel EOS





- Matching at diferent energy densities with causal EOS or linear interpolation
- Causal EOS: $P(\epsilon) = P_m + (\epsilon - \epsilon_m)c^2$

Radius and crust-core matching

Matching with Douchin-Haensel EOS

- Radius and crust thickness uncertainties

	$1\,M_\odot$ R (km)	$1.4\,M_\odot$ R (km)	$1M_\odot$ ΔR (km)	$1.4M_\odot$ ΔR (km)	L
NL3	14.54	14.63	1.91	1.30	118
Largest variation	1.09	0.71	0.55	0.35	
NL3$\omega\rho$	13.42	13.75	2.02	1.43	55
Largest variation	0.15	0.10	0.16	0.10	
GM1	13.71	13.76	1.62	1.09	94
Largest variation	0.66	0.42	0.56	0.35	

Crust-core matching conditions

P, ρ, n matching

- Thermodynamical consistency:
  - P, ϵ, n should match
- Conditions to be fulfilled:
  1. n is increasing function of P

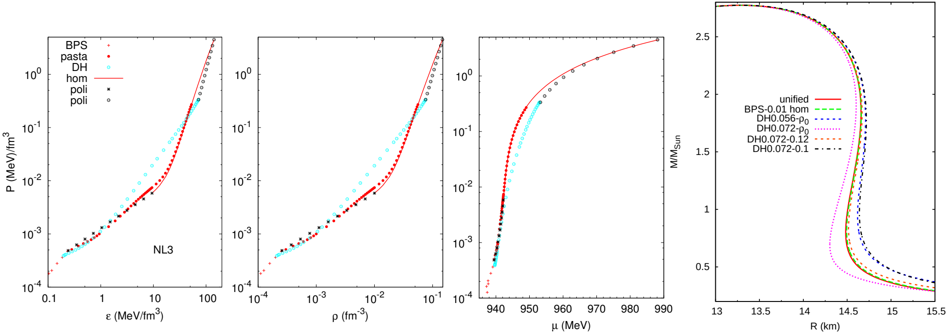
$$n = \frac{\mathrm{d}P}{\mathrm{d}\mu} \rightarrow P(\mu) \text{ is increasing and convex}$$

  2. causality:

$$\frac{\mathrm{d}P}{\mathrm{d}\epsilon} < c^2$$

Matching the crust-core EOS

Matching with polytropes: Douchin-Haensel and NL3



- EOS close to the unified EOS: match neutron drip to 0.01 fm^{-3}
- DH crust considered realistic: match $\rho_t - \rho_0$

Crust-core matching

Conclusions

- Large uncertainty if inner curst EOS and core EOS have very different *L*
- Thermodynamical consistency conditions: causality and P is a convex function of μ
- Results close to unified EOS: use inner crust EOS with similar *L*.

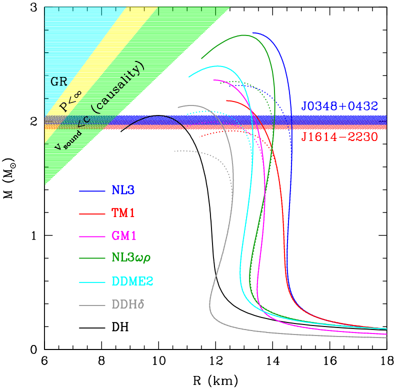
Unified EOS

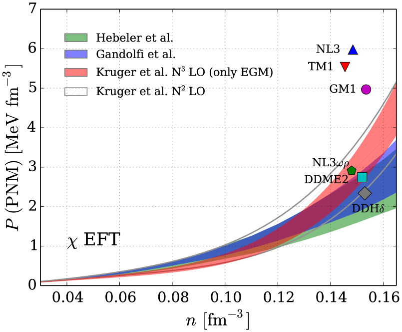
Set of EOSs that predict $2M_\odot$

- Outer crust: densities below neutron drip $\rho \lesssim 10^{-4}$ fm^{-3}
  - Haensel & Pichon (1994)
- Inner crust: from neutron drip to crust-core transition
  - Inner crust EOS within Thomas Fermi calculation of pasta (Grill et al 2014)

    NL3, TM1, GM1, DDME2,DDHδ,NL3$\omega\rho$
- core EOS:
  - nucleonic or hyperonic EOS for homogeneous matter in β-equilibrium
- unified EOS of Douchin-Haensel (Sly4)

Unified EOS

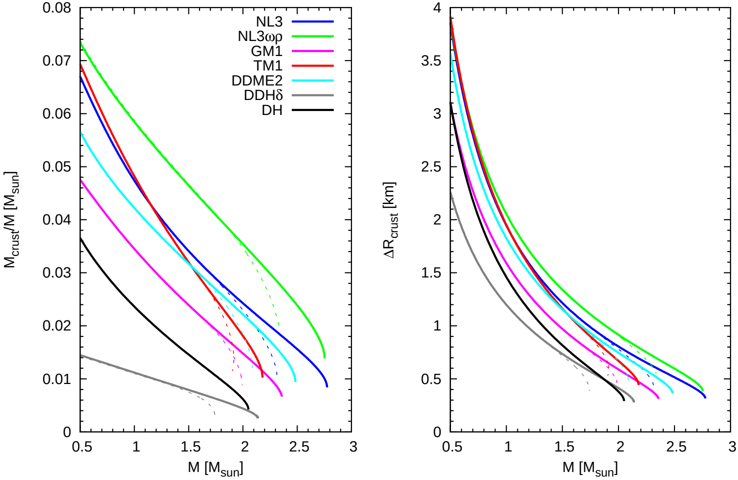




- $M - R$ relations nucleonic (solid) and hyperonic (dotted) EOS
- Pressure at n_0 for pure neutron matter and χ EFT constraints [Hebeler2013, Gandolfi2013 and Krüger2013]

Unified EOS

Crust mass and thickness



Solid lines for nucleonic models and dashed for hyperonic ones.

Unified EOS

Fits by piecewise polytorpes

Following Read2009

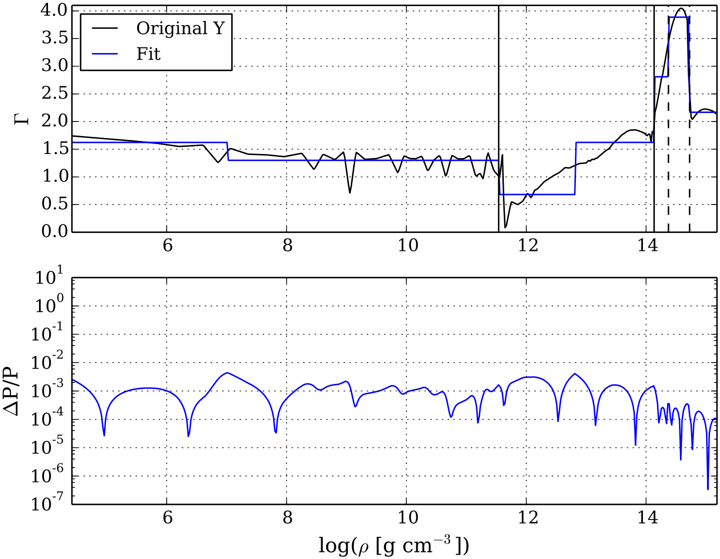
- Unified EoSs are fitted by 7 polytropes:
- $n_1 > n_2 > n_3, \ldots (n_0 = 0)$
- EOS is paramerized by

$$\begin{aligned} P(n) &= \kappa_i n^{\Gamma_i} \qquad \text{for } n_{i-1} < n < n_i \\ \epsilon(n) &= (1 - a_i)n + \frac{\kappa_i}{\Gamma_i - 1} n^{\Gamma_i} \end{aligned}$$

- $a_i \rightarrow \epsilon$ continuous at interface between polytropes
- Dividing densities
  - 4 for the crust and 3 for the core
  - Two dividing densities are fixed to physical values (neutron-drip and core-crust transition densities)
- Relative error of fit (R.E) $X = |X^{\text{fit}} - X^{\text{exact}}|/X^{\text{exact}}|$

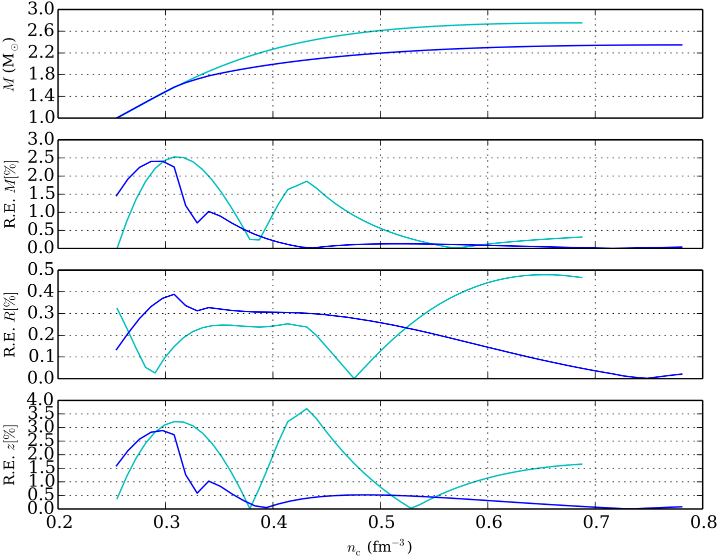
Fit by piecewise polytropes

Polytropic index Γ as a function of density and relative error in the calculation of the pressure for hyperonic EOS



Fit by piecewise polytropes

NL3$\omega\rho$ EOS: Relative error for mass M, radius R, surface redshift z



Light clusters in a pasta calculation

- Light clusters influence supernova matter properties (Arcones et al PRC78 (2008), Heckel et al PRC80 2009)
- Cluster formation has been measured in heavy ion collisions (Qin et al PRL 108 (2012), Hagel et al PRL108,062702): equilibrium constants, Mott points and medium cluster binding energies
- Supernova EOS constrained with equilibrium constants from HIC (Hempel et al PRC91, 045805 (2015))
- At low densities, it is expected a competition between light cluster and heavy cluster formation:
  We will consider a Thomas Fermi calculation including light clusters

Pasta phase EOS with light clusters

- non-homgeneous matter within a Thomas-Fermi calculation
- a preferred single geometry is assumed (least free energy) for a given T, ρ and y_p
- only five possible shapes are considered: droplets, rods, slabs, tubes and bubbles
- β-equilibrium: y_P is very small and only three shapes are energetically favorable: droplets, rods and slabs.
- a regular lattice in the Wigner-Seitz approximation is considered, the WS cell having the shape of the clusters
- a fixed A number at a given density determines the WS volume (minimizes the free energy), β-equilibrium condition or Y_p determines Z, N

EOS

RMF Lagrangian for stellar matter

- Lagrangian density: $\mathcal{L} = \mathcal{L}_N + \mathcal{L}_c + \mathcal{L}_m$
  - nucleons, tritium and helion

$$\mathcal{L}_j = \bar{\psi}_j \left[\gamma_\mu i D_j^\mu - M_j^*\right] \psi_j \qquad i = p,\, n,\, t,\, 3he$$

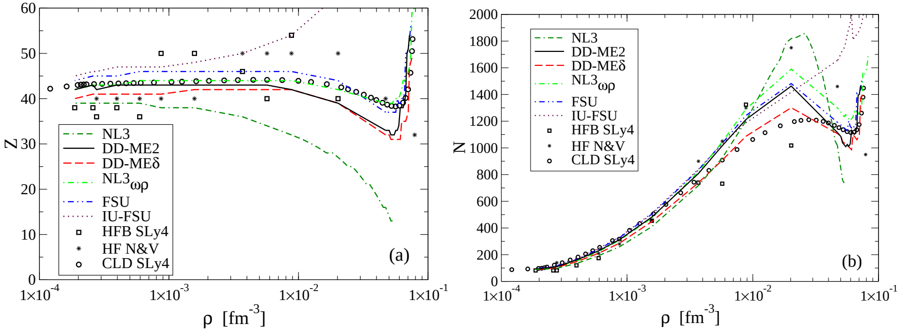
  - alphas, deuterons

$$\begin{aligned}
\mathcal{L}_\alpha &= \frac{1}{2}(iD^\mu_\alpha \phi_\alpha)^*(iD_{\mu\alpha}\phi_\alpha) - \frac{1}{2}\phi^*_\alpha M^2_\alpha \phi_\alpha, \\
\mathcal{L}_d &= \frac{1}{4}(iD^\mu_d \phi^\nu_d - iD^\nu_d \phi^\mu_d)^*(iD_{d\mu}\phi_{d\nu} - iD_{d\nu}\phi_{d\mu}) - \frac{1}{2}\phi^{\mu *}_d M^2_d \phi_{d\mu},
\end{aligned}$$

$$\begin{aligned}
iD^\mu_j &= i\partial^\mu - g^j_v \omega^\mu - \frac{g^j_\rho}{2}\boldsymbol{\tau}\cdot\mathbf{b}^\mu, \\
M^*_j &= M_j - g^j_s \phi, \quad j = p, n, t, h, d, \alpha
\end{aligned}$$

- couplings: constrained by HIC data ou first principle calculations

Z and N in clusters

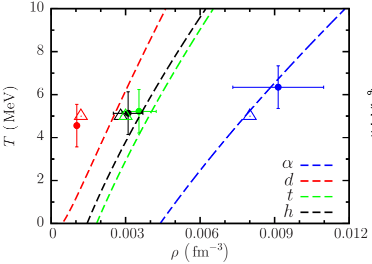


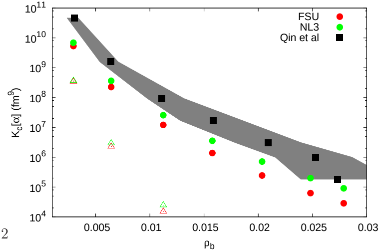
Only spherical: HFB with Sly4 (Grill PRC84 065801,2011); HF (Negele & Vautherin NPA207, 1973), CDM with Sly4 (Douchin & Haensel, A& A380,2001)

droplets, rods, slabs: TF and RMF (Grill PRC85 055808, 2012)

Light clusters: fixing the parameters

- Comparing with experimental Mott points from Hagel et al PRL108,062702
- binding energy of cluster in medium $B_i = A_i M^* - M_i^*$
- $B_i = 0 \rightarrow$ Mott point
- Equilibrium constant: $K_c[i] = \rho_i / (\rho_p^{Z_i} \rho_n^{N_i})$

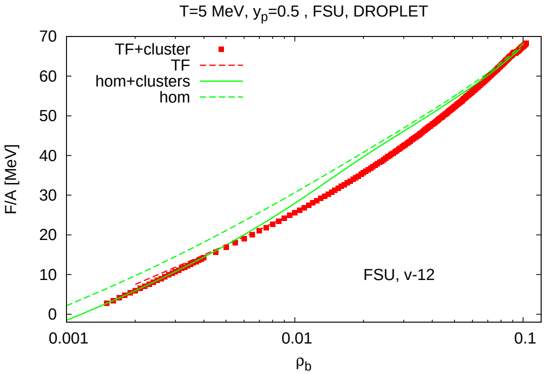




- The Mott densities: dashed lines from Typel *et al.* PRC81 (2010), experimental prediction by Hagel *et al.* (full dots with errorbars

Pasta phases with light clusters

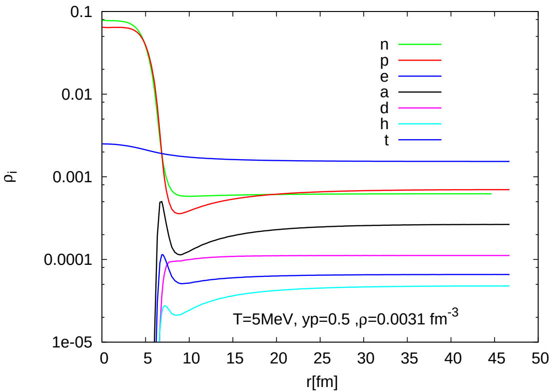
FSU, $T = 5$ MeV and $Y_p = 0.5$.



- light clusters: main contribution at the lower border of the inner crust
- smooth matching with homogeneous EOS

Density profiles

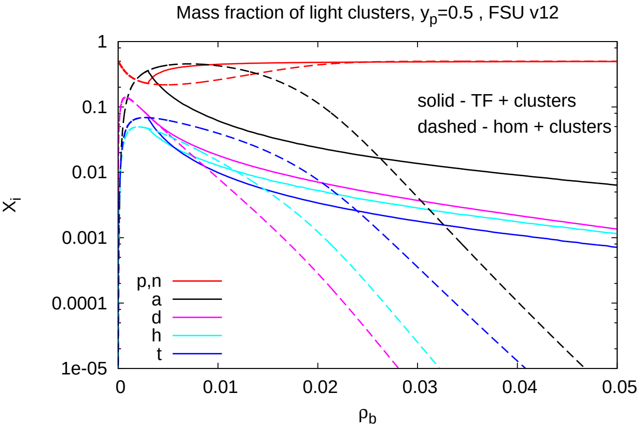
FSU, $T = 5$ MeV and $Y_p = 0.5$.



- light clusters: concentration at the surface of the pasta clusters, and presence in the neutron drip gas

Mass fraction of light clusters

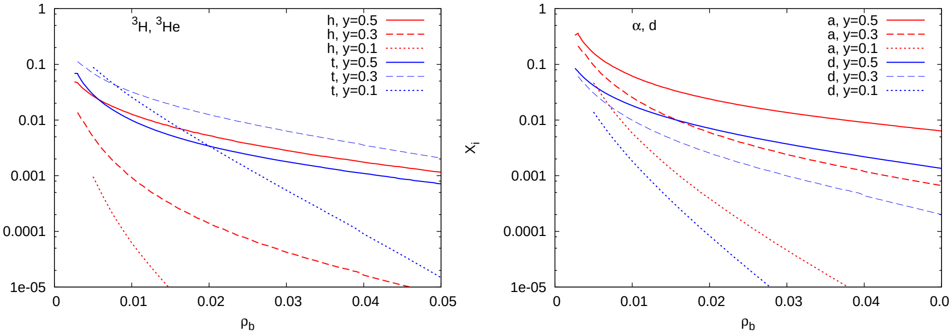
FSU, $T = 5$ MeV and $Y_p = 0.5$.



- light clusters with pasta:
  - the presence of pasta clusters reduce the contribution of light clusters at for densities 0.01-0.03 fm^{-3}
  - for intermediate densities ($\rho \gtrsim 0.03$ fm^{-3}: larger fractions of light clusters in pasta matter, smoother reduction

Mass fraction of light clusters

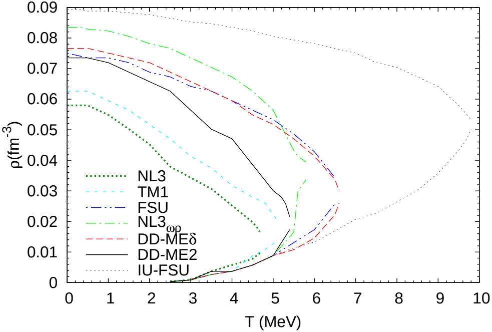
Comparing light clusters in matter with different global Y_p



- Decreasing Y_p:
  - fraction of light clusters α, d and ^{3}He decrease
  - fraction of ^{3}H may increase (at least at low densities)

Effect of T on pasta extension

β-equilibrium matter, no light clusters



- The critical temperature T_{crit} is model dependent
- T_{crit} depends on density dependence of ϵ_{sym} and K:
  - a smaller value of L and a smaller K_0 favor the existence of clusterization at large temperatures
  - inclusion of light clusters will affect the low density border

Conclusions

Pasta calculation with ligh clusters

- **Light clusters**:

  have a non negligible effect at low densities in a self-consistent calculation with heavy clusters

- **Determination of couplings**: experimental data or first principles calculations are required. More work needs to be done

- **Inclusion of light clusters in pasta calculation**:
  - at low densities reduces the amount of light clusters with respect to homogeneous matter
  - at densities above Mott densities: fraction of light clusters is non-negligible

Thank you !