An Advanced Mean-Field Model
for Dense Nuclear Matter

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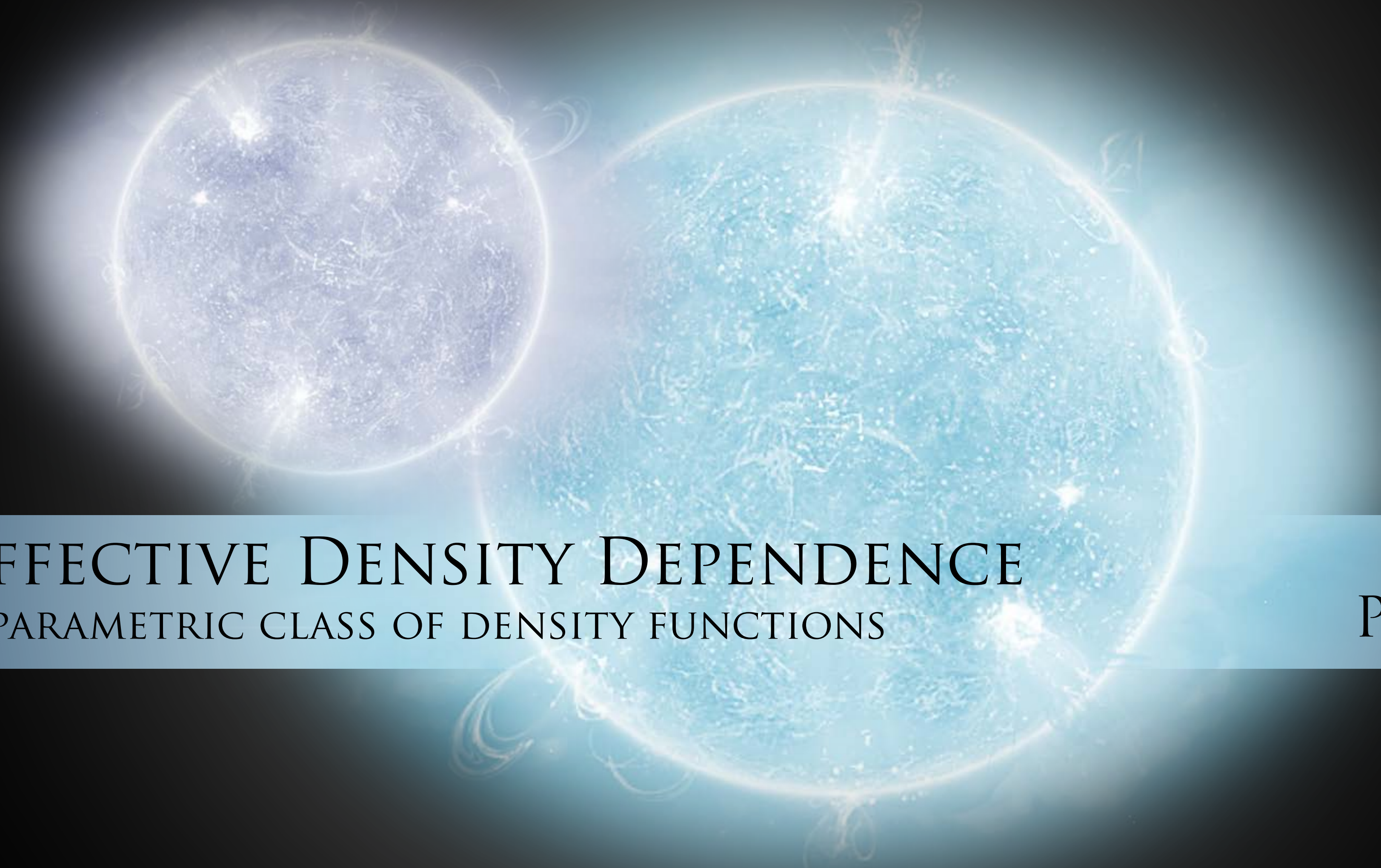


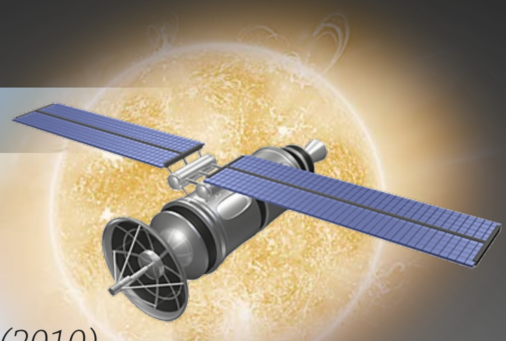
Effective Density Dependence

2-parametric class of density functions

Part 1

NEUTRON STARS

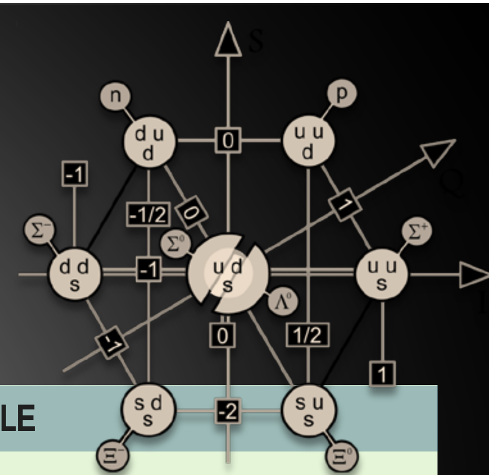
present issues



! MASSIVE PULSARS OBSERVED

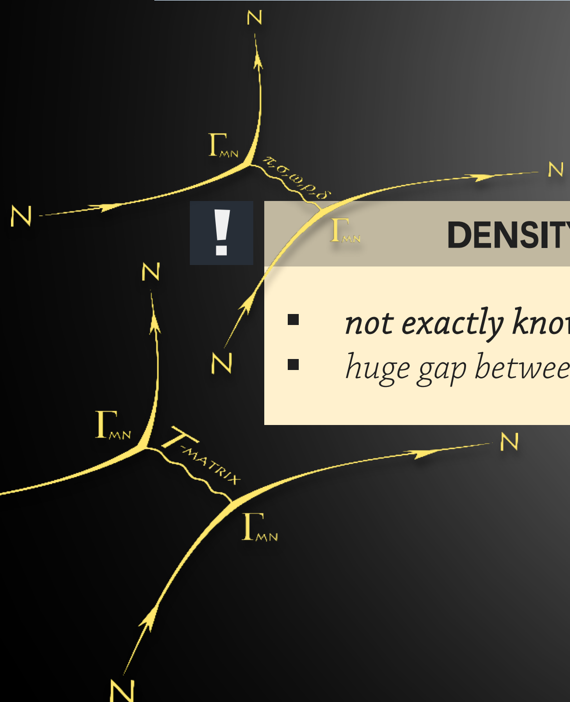
- *masses around* ***2Msol***
- *precise measurements*

P. B. Demorest et al., ***Nature*** *467, 1081 (2010)*
J. Antoniadis et al., ***Science*** *340, 448 (2013)*



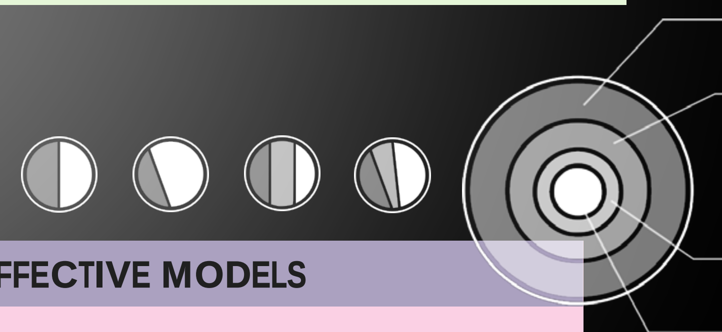
? HYPERON PUZZLE

- ***hyperons*** *generally* ***soften*** *the equation of state*
- ***many models*** *with exotic matter (hyperons, kaon condensates, ...)* ***are ruled out***



! DENSITY DEPENDENCE

- ***not exactly known*** *for a nuclear interaction*
- *huge gap between* ***compact stars*** *and* ***nuclei***



? EFFECTIVE MODELS

- ***simplicity*** *vs* ***fundamentality***
- ***better results*** *with a "****black box****"– like understanding*

MODEL

density dependent theory

Lagrangian density (n, p)

$$
\begin{aligned}
\mathcal{L}_{\text{fermi}} &= \bar{\psi}_N \left[i\gamma^\mu \partial_\mu - m_N \right] \psi_N, \\
\mathcal{L}_{\text{bose}} &= \frac{1}{2}\partial_\mu \hat{\sigma} \partial^\mu \hat{\sigma} - \frac{1}{2} m_\sigma^2 \hat{\sigma}^2 \\
&\quad - \frac{1}{4}\hat{\omega}_{\mu\nu}\hat{\omega}^{\mu\nu} + \frac{1}{2} m_\omega^2 \hat{\omega}_\mu \hat{\omega}^\mu \\
&\quad + \frac{1}{2}\partial_\mu \hat{\boldsymbol{\delta}} \cdot \partial^\mu \hat{\boldsymbol{\delta}} - \frac{1}{2} m_\delta^2 \hat{\delta}^2 \\
&\quad - \frac{1}{4}\hat{\boldsymbol{\rho}}_{\mu\nu} \cdot \hat{\boldsymbol{\rho}}^{\mu\nu} + \frac{1}{2} m_\rho^2 \hat{\boldsymbol{\rho}}_\mu \cdot \hat{\boldsymbol{\rho}}^\mu, \\
\mathcal{L}_{\text{int}} &= \bar{\psi}_N [\hat{\Gamma}_\sigma \hat{\sigma} + \hat{\Gamma}_\delta \boldsymbol{\tau}_N \cdot \hat{\boldsymbol{\delta}} \\
&\qquad - \hat{\Gamma}_\omega \hat{\omega}_\mu \gamma^\mu - \hat{\Gamma}_\rho \boldsymbol{\tau}_N \cdot \hat{\boldsymbol{\rho}}_\mu \gamma^\mu] \psi_N .
\end{aligned}
$$

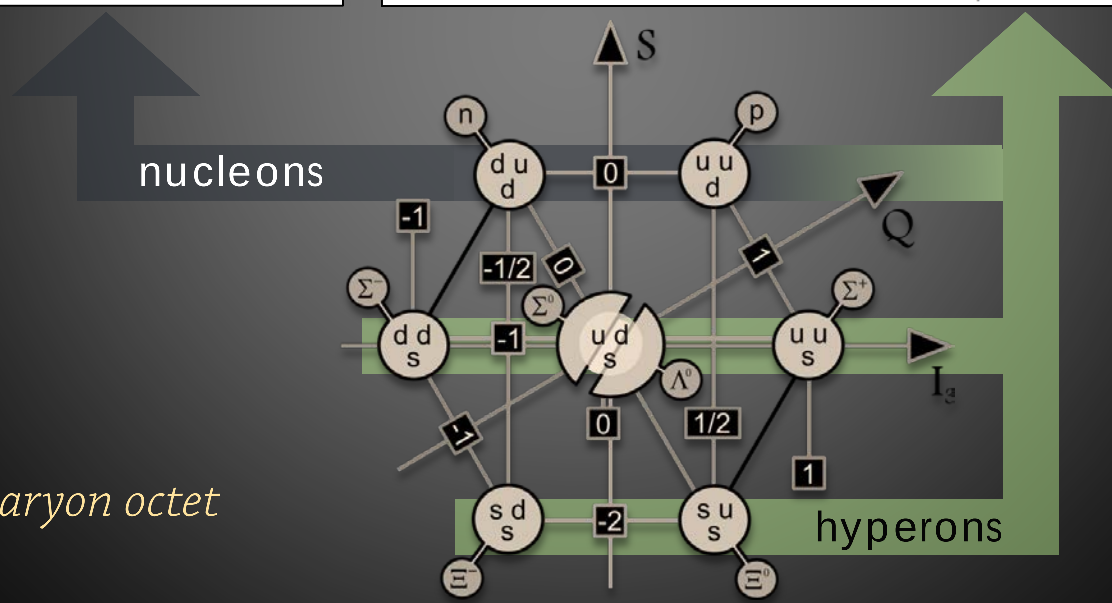
Lagrangian density (n, p, hyperons)

$$
\begin{aligned}
\mathcal{L}^{\text{fermi}} &= \bar{\psi}_B \left[i\gamma^\mu \partial_\mu - m_B \right] \psi_B + \bar{\psi}_{e^-} \left[i\gamma^\mu \partial_\mu - m_{e^-} \right] \psi_{e^-} + \bar{\psi}_{\mu^-} \left[i\gamma^\mu \partial_\mu - m_{\mu^-} \right] \psi_{\mu^-}, \\
\mathcal{L}^{\text{bose}} &= \frac{1}{2}\partial_\mu \hat{\sigma} \partial^\mu \hat{\sigma} - \frac{1}{2} m_\sigma^2 \hat{\sigma}^2 - \frac{1}{4}\hat{\omega}_{\mu\nu}\hat{\omega}^{\mu\nu} + \frac{1}{2} m_\omega^2 \hat{\omega}_\mu \hat{\omega}^\mu \\
&\quad + \frac{1}{2}\partial_\mu \hat{\boldsymbol{\delta}} \partial^\mu \hat{\boldsymbol{\delta}} - \frac{1}{2} m_\delta^2 \hat{\delta}^2 - \frac{1}{4}\hat{\boldsymbol{\rho}}_{\mu\nu} \cdot \hat{\boldsymbol{\rho}}^{\mu\nu} + \frac{1}{2} m_\rho^2 \hat{\boldsymbol{\rho}}_\mu \cdot \hat{\boldsymbol{\rho}}^\mu \\
&\quad + \frac{1}{2}\partial_\mu \hat{\sigma}_s \partial^\mu \hat{\sigma}_s - \frac{1}{2} m_{\sigma_s}^2 \hat{\sigma}_s{}^2 - \frac{1}{4}\hat{\phi}_{\mu\nu}\hat{\phi}^{\mu\nu} + \frac{1}{2} m_\phi^2 \hat{\phi}_\mu \hat{\phi}^\mu \\
&\quad - \frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}, \\
\mathcal{L}^{\text{int}} &= \bar{\psi}_B [\hat{\Gamma}_{\sigma B} \hat{\sigma} + \hat{\Gamma}_{\sigma_s B} \hat{\sigma}_s + \hat{\Gamma}_{\delta B} \boldsymbol{\tau}_B \cdot \hat{\boldsymbol{\delta}} \\
&\quad - \hat{\Gamma}_{\omega B} \hat{\omega}_\mu \gamma^\mu - \hat{\Gamma}_{\phi B} \hat{\phi}_\mu \gamma^\mu - \hat{\Gamma}_{\rho B} \boldsymbol{\tau}_B \cdot \hat{\boldsymbol{\rho}}_\mu \gamma^\mu - e \hat{Q}_B \hat{A}_\mu \gamma^\mu] \psi_B .
\end{aligned}
$$

Dense nuclear matter

symmetric matter
asymmetric matter

simple neutron stars



Dense nuclear matter

matter in β equilibrium

lambda matter
exotic neutron stars

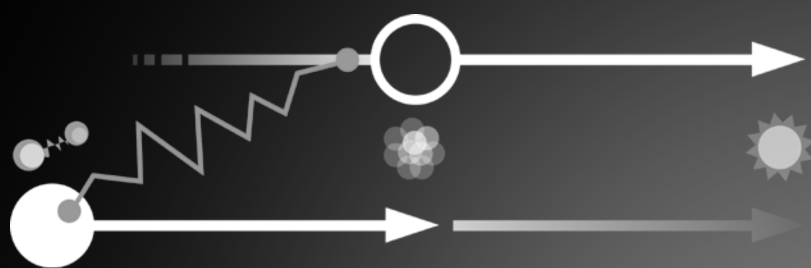
full baryon octet

Protons and neutrons
Λ^0 Σ^- Σ^0 Σ^+ Ξ^- Ξ^0

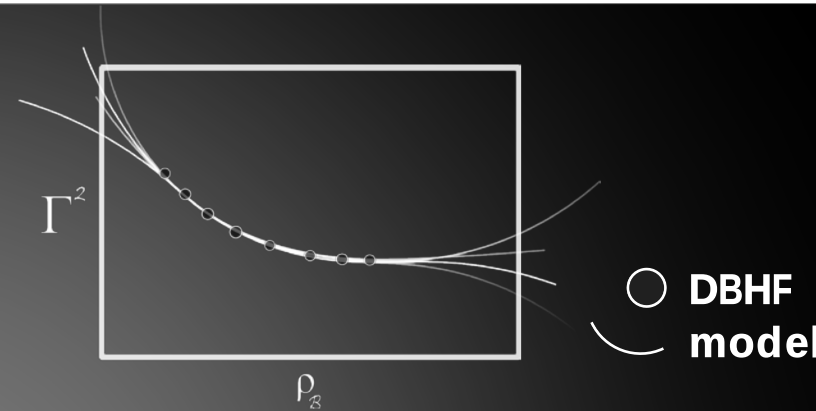
NUCLEAR MATTER

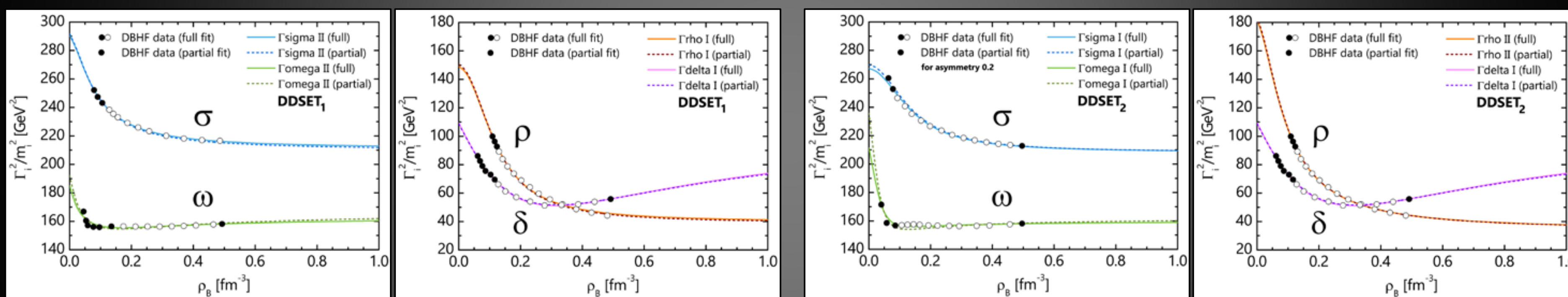
parametrization

effective model



Dirac-Brueckner-Hartree-Fock





DDSET 1

E. N. E. van Dalen, *et al.*, **Eur. Phys. J. A 31**, 29-42 (2007)

DBHF data

DDSET 2

DDSET 1:

$$^{\mathrm{II}}\Gamma_\sigma = a_\sigma\left[\frac{b_\sigma + x^{3/2}}{0.33 + x^{3/2}}\right] \qquad ^{\mathrm{II}}\Gamma_\omega = a_\omega\left[\frac{b_\omega + (x + c_\omega)^{3/2}}{0.5 + x^{3/2}}\right]$$

$$^{\mathrm{I}}\Gamma_\rho = a_\rho\left[\frac{b_\rho + x^2}{0.6 + x^2}\right] \qquad ^{\mathrm{I}}\Gamma_\delta = a_\delta\left[\frac{b_\delta + (x + c_\delta)^2}{3 + x^2}\right]$$

DDSET 2:

$$^{\mathrm{I}}\Gamma_\sigma = a_\sigma\left[\frac{b_\sigma + x^2}{0.6 + x^2}\right] \qquad ^{\mathrm{I}}\Gamma_\omega = a_\omega\left[\frac{b_\omega + (x + c_\omega)^2}{0.05 + x^2}\right]$$

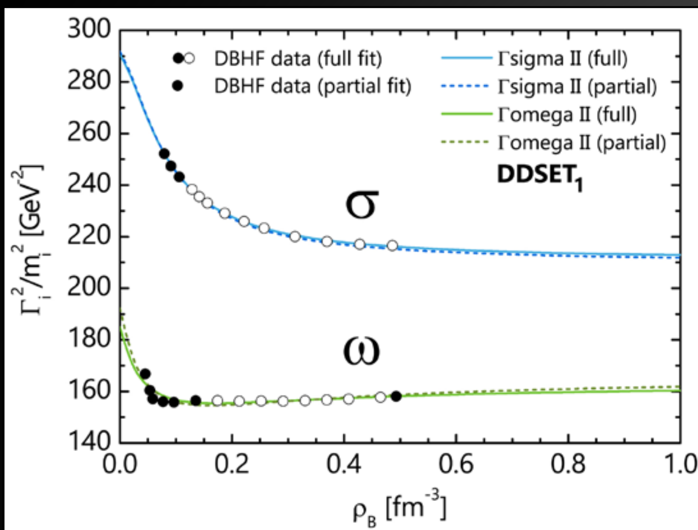
$$^{\mathrm{II}}\Gamma_\rho = a_\rho\left[\frac{b_\rho + x^{3/2}}{0.55 + x^{3/2}}\right] \qquad ^{\mathrm{I}}\Gamma_\delta = a_\delta\left[\frac{b_\delta + (x + c_\delta)^2}{3 + x^2}\right]$$

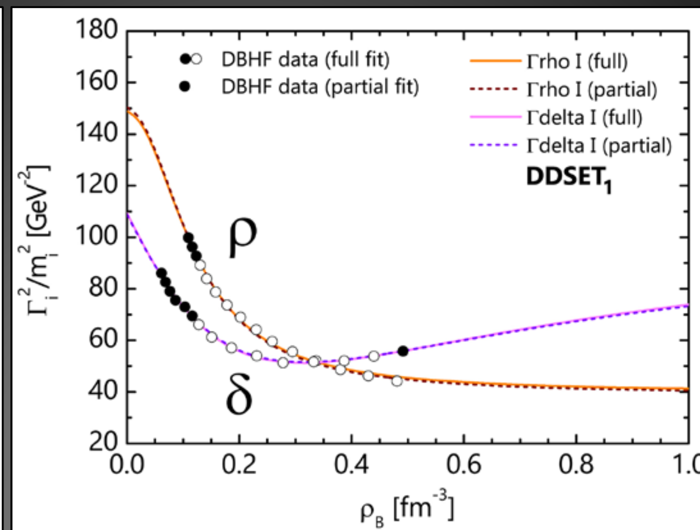
2-parametric functions

K. Petrík and S. Gmuca, **J. Phys. G: Nucl. Part. Phys. 39**, 085113 (2012)
K. Petrík and S. Gmuca, **Astronom. Nachr. 334**, No.9, 1043 (2013)

Nuclear matter

parametrization





$DDSET_1$

$$^{\mathrm{II}}\Gamma_\sigma = a_\sigma\left[\frac{b_\sigma + x^{3/2}}{0.33 + x^{3/2}}\right] \qquad ^{\mathrm{II}}\Gamma_\omega = a_\omega\left[\frac{b_\omega + (x + c_\omega)^{3/2}}{0.5 + x^{3/2}}\right]$$

$$^{\mathrm{I}}\Gamma_\rho = a_\rho\left[\frac{b_\rho + x^2}{0.6 + x^2}\right] \qquad ^{\mathrm{I}}\Gamma_\delta = a_\delta\left[\frac{b_\delta + (x + c_\delta)^2}{3 + x^2}\right]$$

*only **10** free parameters **(total)***

- *stable extrapolations*
- *excellent reproduction of DBHF data*
- *predictability*

2-parametric functions

K. Petrík and S. Gmuca, **J. Phys. G: Nucl. Part. Phys. 39**, 085113 (2012)
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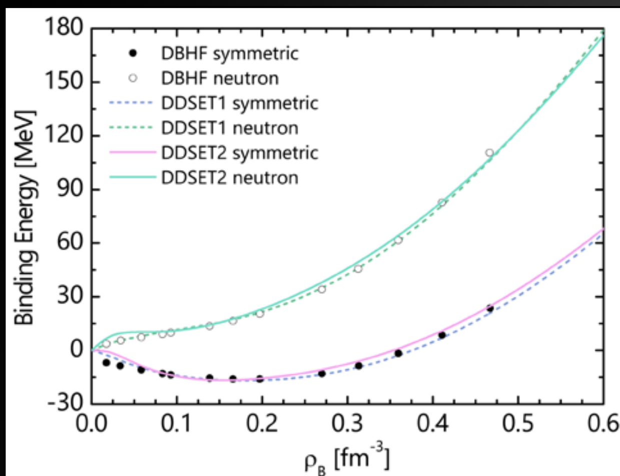
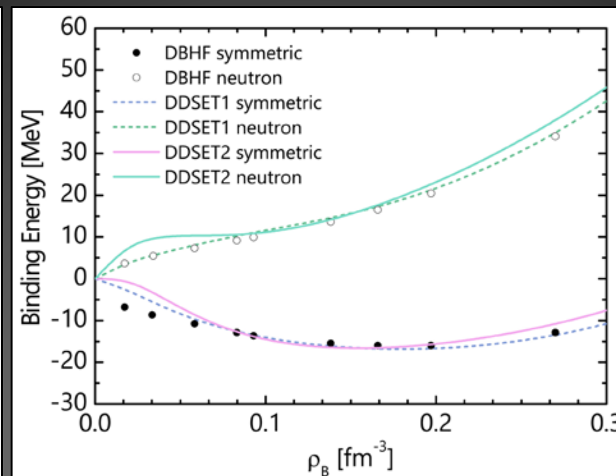
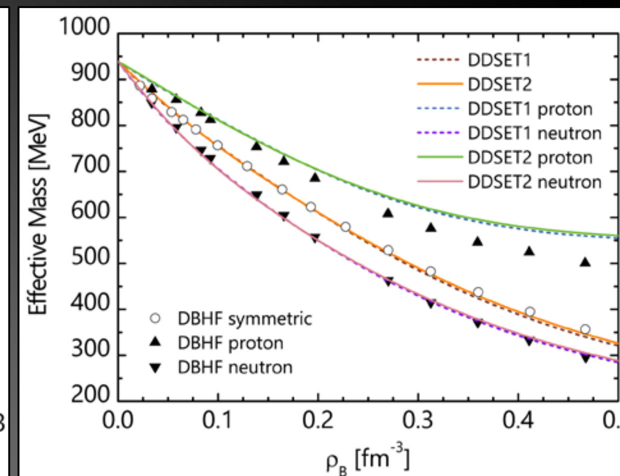
NUCLEAR MATTER

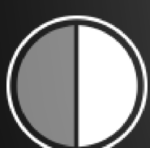
equation of state







Symmetric nuclear matter



saturation density

Fermi momentum

binding energy

compression modulus

symmetry energy

symmetry energy slope

effective mass

	[MeV]	DBHF	$DBHF_A$	$DDSET_1$	$DDSET_2$
ρ_0	$[fm^{-3}]$	0.181	0.1815	0.1801	0.1535
k_F		1.3889	1.39	1.3867	1.3148
E_B		-16.15	-16.62	-16.88	-16.68
K		230	233	235.07	235.85
E_{sym}		34.36	34.8	35.03	31.88
L		-	71.2	67.24	58.24
m^*_{sat}		-	636	637	674

DBHF data comparison

DBHF E. N. E. van Dalen, *et al.*, **Eur. Phys. J. A 31**, 29-42 (2007)

$DBHF_A$ T. Katayama and K. Saito, **Phys. Rev. C 88**, 035805 (2013)

SYMMETRIC MATTER

symmetry energy

Binding energy of nuclear matter

Symmetry energy

$$E_{sym}(\rho_{\mathcal{B}}) = \frac{1}{2}\left[\frac{\partial^2 E(\rho_{\mathcal{B}},\alpha)}{\partial\alpha^2}\right]_{\alpha=0} = E_{sym}(\rho_0) + L\chi + \frac{K_{sym}}{2}\chi^2 + O(\chi^3)$$

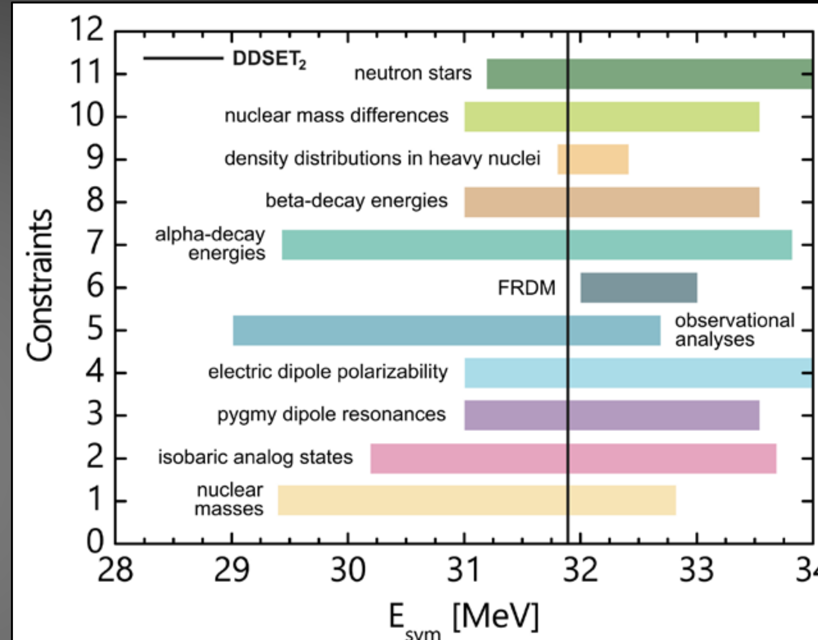
$$E(\rho_{\mathcal{B}},\alpha) \approx E_B + \frac{K}{2}\chi^2 + \left[E_{sym} + L\chi + \frac{K_{sym}}{2}\chi^2\right]\alpha^2$$

Slope of symmetry energy

$$L = 3\rho_0 \frac{\partial E_{sym}(\rho_{\mathcal{B}})}{\partial \rho_{\mathcal{B}}}\bigg|_{\rho_{\mathcal{B}}=\rho_0}$$

$$K_{sym} = 9\rho_0^2 \frac{\partial^2 E_{sym}(\rho_{\mathcal{B}})}{\partial \rho_{\mathcal{B}}^2}\bigg|_{\rho_{\mathcal{B}}=\rho_0}$$

Curvature of symmetry energy



constraints from references

[1] M. Liu, et al., Phys. Rev. C **82**, 064306 (2010)
[2] B. K. Agrawal, J. N. De, and S. K. Samaddar, Phys. Rev. Lett. **109**, 262501 (2012)
[3] M. Warda, et al., Phys. Rev. C **80**, 024316 (2009)
[4] P. Danielewicz, and J. Lee, Nucl. Phys. A **922**, 1 (2014)
[5] A. Carbone, et al., Phys. Rev. C **81**, 041301(R) (2010)
[6] M. B. Tsang, et al., Phys. Rev. Lett. **102**, 122701 (2009)
[7] Z. Zhang, and L. W. Chen, Phys. Rev. C **90**, 064317 (2014)
[8] J. M. Lattimer, Ann. Rev. Nucl. Part. Sci. **62**, 485 (2012)
[9] P. Möller, et al., Phys. Rev. Lett. **108**, 052501 (2012)
[10] J. Dong, W. Zuo, and J. Gu, Phys. Rev. C **87**, 014303 (2013)
[11] J. Dong, et al., Phys. Rev. C **88**, 014302 (2013)
[12] B. K. Agrawal, et al., Phys. Rev. C **87**, 051306(R) (2013)
[13] N. Wang, and T. Li, Phys. Rev. C **88**, 011301(R) (2013)
[14] A. W. Steiner, J. M. Lattimer, and E. F. Brown, Astrophys. J. **722**, 33 (2010)
[15] X. Fan, J. Dong, and W. Zuo, Phys. Rev. C **89**, 017305 (2014)
[16] A. W. Steiner, and S. Gandolfi, Phys. Rev. Lett. **108** 081102 (2012)
[17] L. Trippa, G. Colò, and E. Vigezzi, Phys. Rev. C **77**, 061304(R) (2008)

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Symmetric matter

comparisons

Binding energy of nuclear matter

$$E(\rho_{\mathcal{B}},\alpha) \approx E_B + \frac{K}{2}\chi^2 + \left[E_{sym} + L\chi + \frac{K_{sym}}{2}\chi^2\right]\alpha^2$$

Symmetry energy

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Slope of symmetry energy

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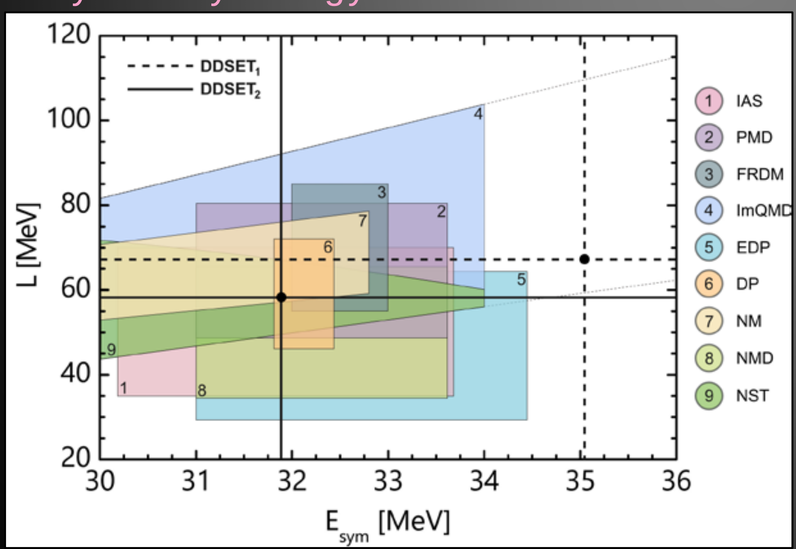
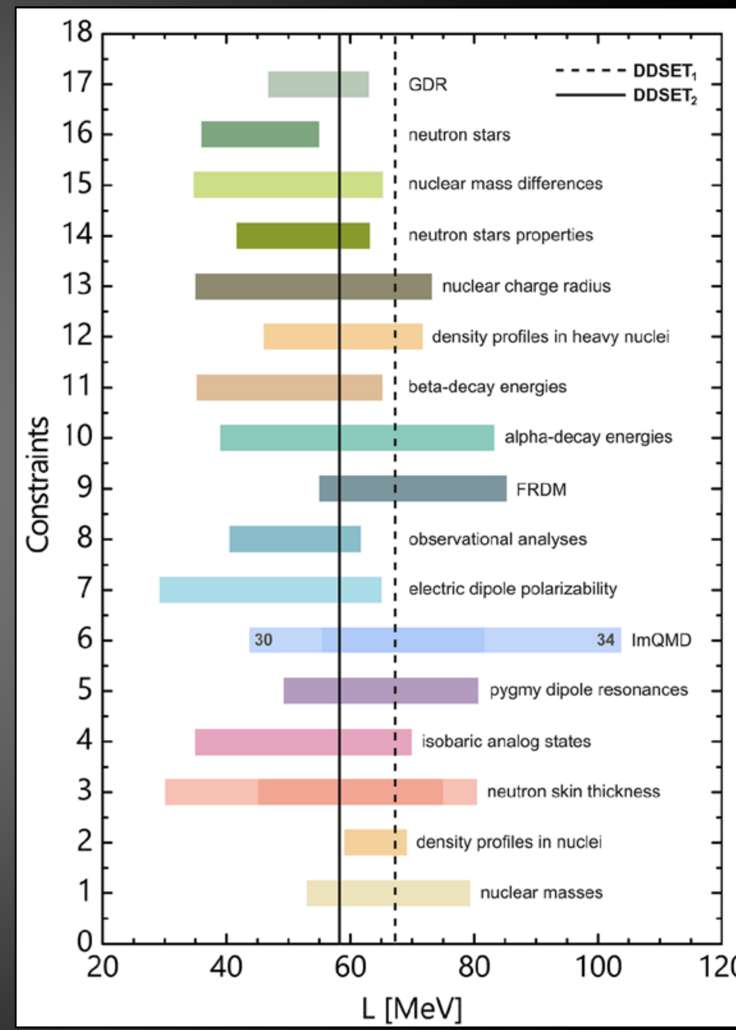
Curvature of symmetry energy



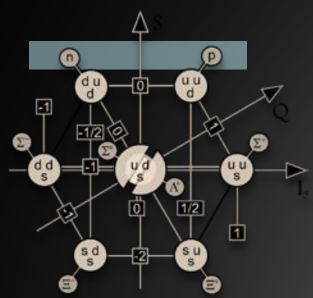


Neutron stars

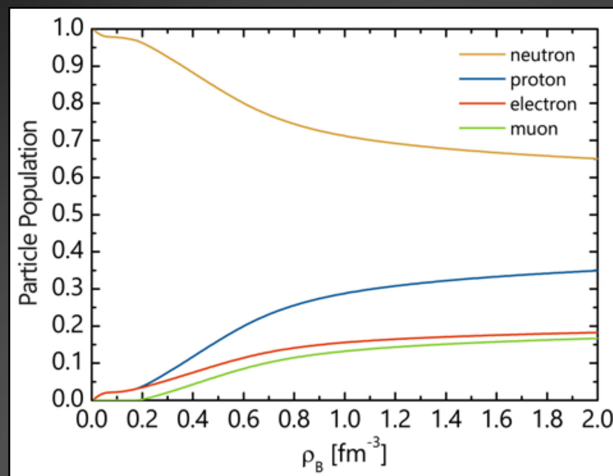
beta equilibrium

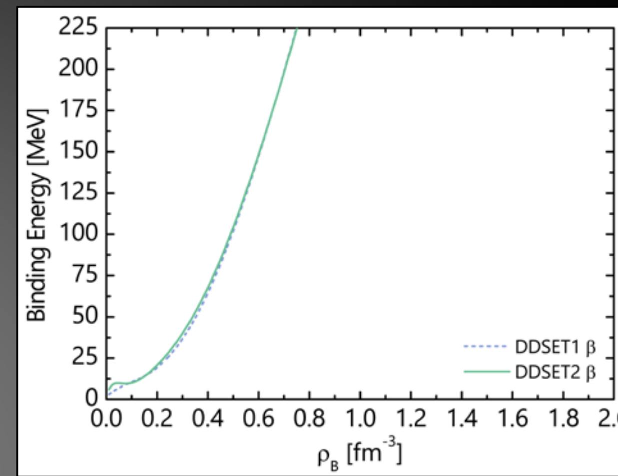
Particle populations

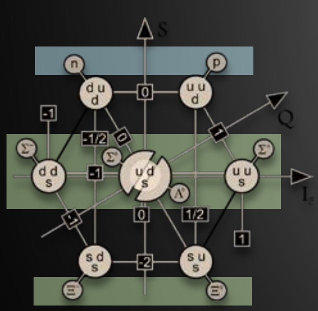
Binding energy



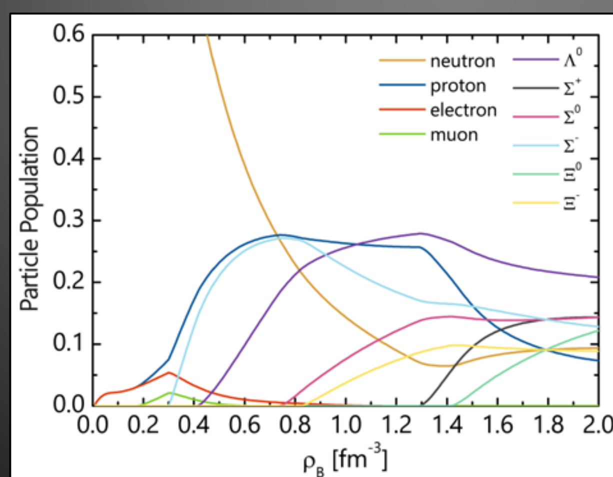
nucleons

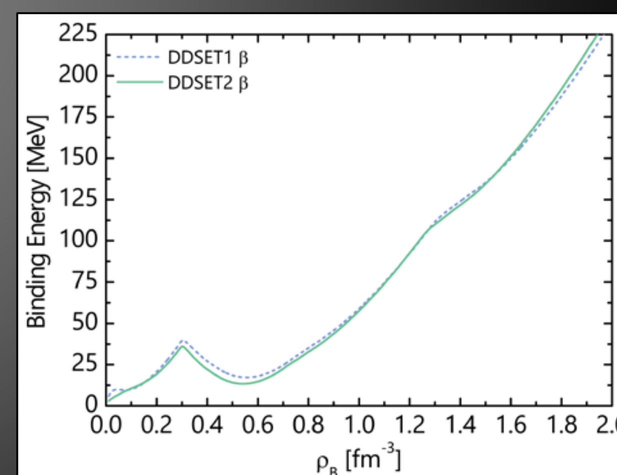






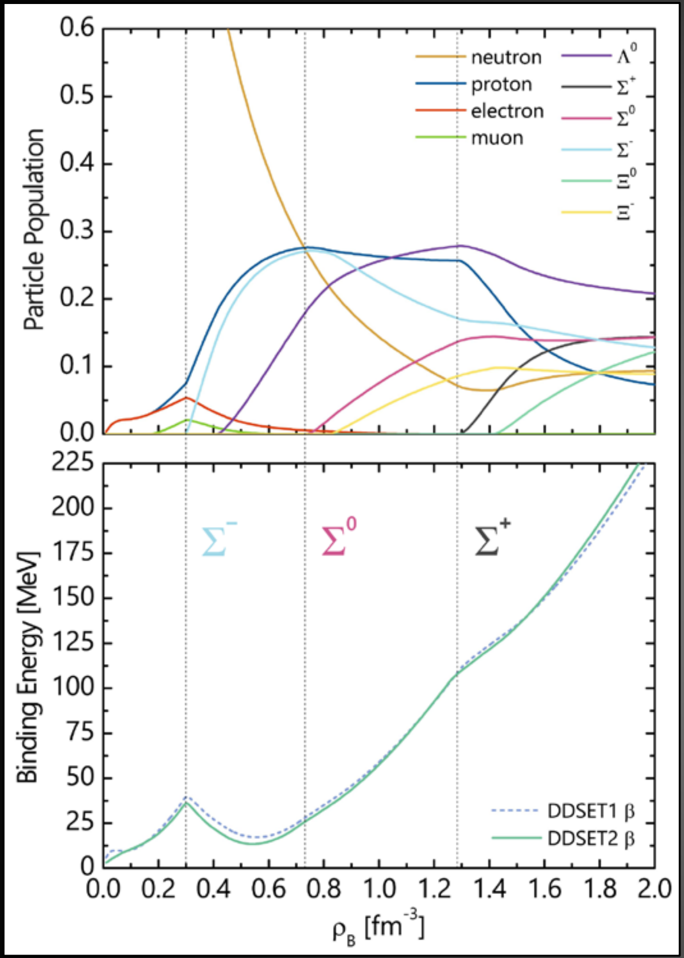
nucleons + hyperons

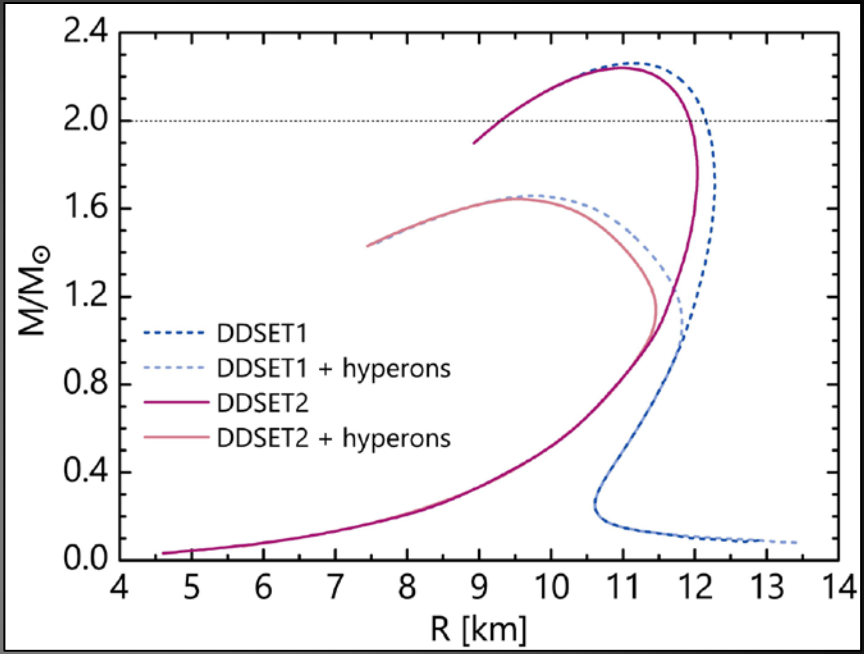




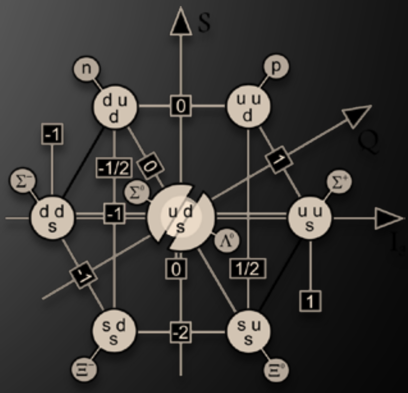
Neutron stars

beta equilibrium

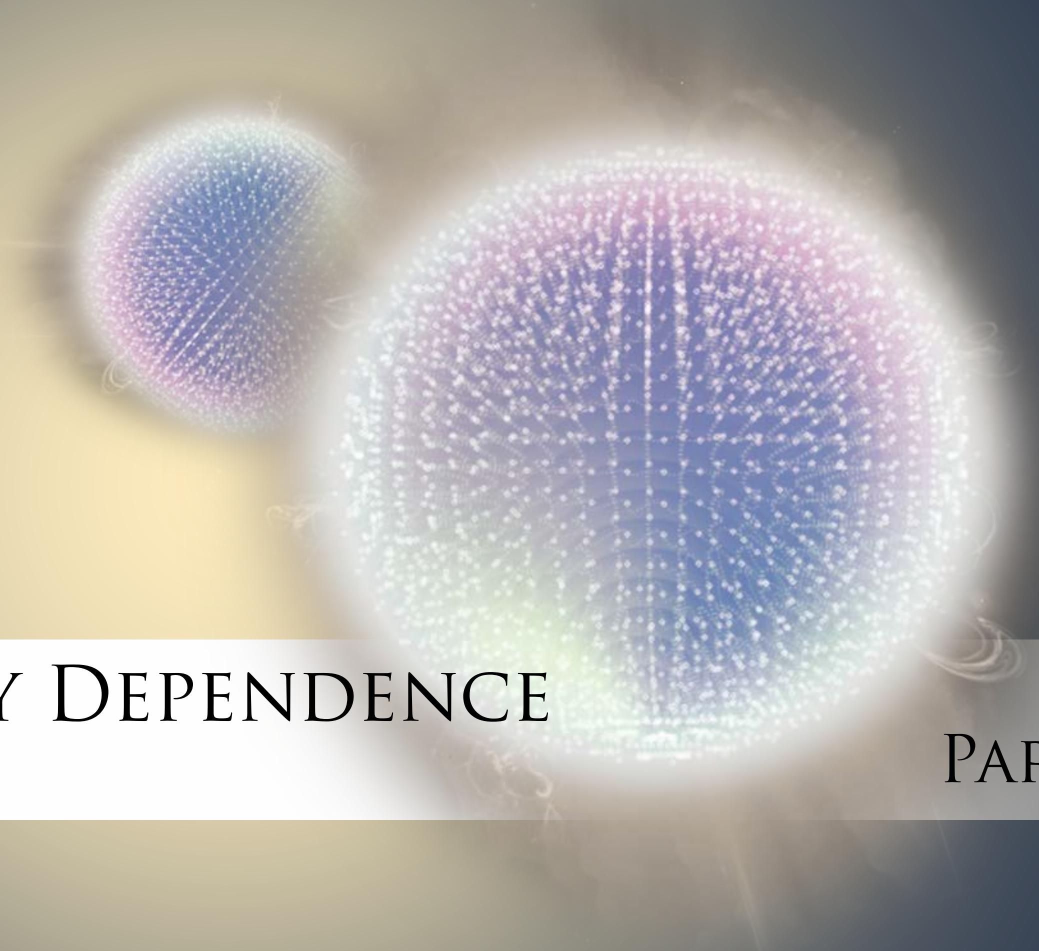




Softening of the Equation of State



nucleons + hyperons



Effective Density Dependence

Fock Correlations

Part 2

DIRAC DENSITY FUNCTIONAL

Dirac-Hartree-Fock Approach

Mean-Filed *vs* **Hartree-Fock**

properties

- *Momentum independent*
- *No exchange (Fock) part in the interaction*
- *Phenomenological density dependence*

properties

- *Momentum dependent*
- *Exchange correlations*
- *Inherent density dependence*

DIRAC-HARTREE-FOCK MODEL

LAGRANGIAN DENSITY

$$\mathcal{L}_{HF} = \mathcal{L}^{\text{fermi}} + \mathcal{L}^{\text{bose}} + \mathcal{L}^{\text{int}}$$

$$\mathcal{L}^{\text{fermi}} = \bar{\psi}[i\gamma^\mu\partial_\mu - m_N]\psi + \bar{\psi}_{e^-}[i\gamma^\mu\partial_\mu - m_{e^-}]\psi_{e^-} + \bar{\psi}_{\mu^-}[i\gamma^\mu\partial_\mu - m_{\mu^-}]\psi_{\mu^-},$$

$$\begin{aligned}\mathcal{L}^{\text{bose}} =& +\frac{1}{2}\partial_\mu\hat{\boldsymbol{\pi}}\cdot\partial^\mu\hat{\boldsymbol{\pi}} - \frac{1}{2}m_\pi^2\hat{\pi}^2 \\ &+\frac{1}{2}\partial_\mu\hat{\sigma}\partial^\mu\hat{\sigma} - \frac{1}{2}m_\sigma^2\hat{\sigma}^2 - \frac{1}{4}\hat{\omega}_{\mu\nu}\hat{\omega}^{\mu\nu} + \frac{1}{2}m_\omega^2\hat{\omega}_\mu\hat{\omega}^\mu \\ &+\frac{1}{2}\partial_\mu\hat{\boldsymbol{\delta}}\cdot\partial^\mu\hat{\boldsymbol{\delta}} - \frac{1}{2}m_\delta^2\hat{\delta}^2 - \frac{1}{4}\hat{\boldsymbol{\rho}}_{\mu\nu}\cdot\hat{\boldsymbol{\rho}}^{\mu\nu} + \frac{1}{2}m_\rho^2\hat{\boldsymbol{\rho}}_\mu\cdot\hat{\boldsymbol{\rho}}^\mu \qquad \text{free meson fields} \\ &-\frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}, \qquad \text{free photon field}\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{\text{int}} =& +\bar{\psi}\left[g_\sigma\hat{\sigma} + g_\delta\boldsymbol{\tau}\cdot\hat{\boldsymbol{\delta}} - g_\omega\hat{\omega}_\mu\gamma^\mu - g_\rho\boldsymbol{\tau}\cdot\hat{\boldsymbol{\rho}}_\mu\gamma^\mu\right]\psi \\ &+\bar{\psi}\left[-\frac{f_\pi}{m_\pi}\gamma^5\gamma^\mu\partial_\nu\boldsymbol{\tau}\cdot\boldsymbol{\pi} + \frac{f_\omega}{2m_N}\sigma_{\mu\nu}\partial^\nu\omega^\mu + \frac{f_\rho}{2m_N}\sigma_{\mu\nu}\partial^\nu\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu\right]\psi \\ &+\bar{\psi}\left[-e\hat{Q}_N\hat{A}_\mu\gamma^\mu\right]\psi \qquad \text{interaction of photons with baryons}\end{aligned}$$

mesons

π σ ω ρ δ

approximations

- *no* ***retardation effects***
- ***zero-range contribution*** *from the pion exchange is subtracted*

sources

- *C. J. Horowitz and B. D. Serot,* ***NPA 399****, 529 (1983)*
- *A. Bouyssy, et al.,* ***PRC 36****, 380 (1987)*

DIRAC DENSITY FUNCTIONAL

DHF Energy Density Functional

idea

*To **map** the Fock exchange correlations onto the Hartree level*

options

- *using the **self-energy** (needs momentum averaging)*
- *using the **energy density***

1 SELF-ENERGY

$$\Sigma(k,k_F)=\Sigma^s(k,k_F)-\gamma^0\Sigma^0(k,k_F)+\boldsymbol{\gamma}\cdot\boldsymbol{k}\Sigma^v(k,k_F)$$

important

- ***scalar** and **vector** parts obtain exchange (Fock) terms*
- ***space-like** vector part is a bit problematic*

2 ENERGY DENSITY FUNCTIONAL

$$\mathcal{E}=\langle\psi_0|\mathcal{H}|\psi_0\rangle=\epsilon_{kin}+\epsilon_D+\epsilon_E$$

HARTREE

$$\epsilon^D=\sum_{\sigma,\omega,\rho,\delta}\epsilon_i^D$$

$$\epsilon_\sigma^D=-\frac{1}{2}\frac{g_\sigma^2}{m_\sigma^2}(\rho_p^s+\rho_n^s)^2$$

$$\epsilon_\omega^D=+\frac{1}{2}\frac{g_\omega^2}{m_\omega^2}(\rho_p^v+\rho_n^v)^2$$

$$\epsilon_\rho^D=+\frac{1}{2}\frac{g_\rho^2}{m_\rho^2}(\rho_p^v-\rho_n^v)^2$$

$$\epsilon_\delta^D=-\frac{1}{2}\frac{g_\delta^2}{m_\delta^2}(\rho_p^s-\rho_n^s)^2$$

FOCK

$$\epsilon^E=\sum_{\mathcal{K}}\epsilon_{\mathcal{K}}^E,\qquad \mathcal{K}=\sigma,\omega,\delta,\rho_V,\rho_T,\rho_{VT},\pi_{PV}$$

$$\epsilon_{\mathcal{K}}^E=\frac{1}{2}\frac{1}{(2\pi)^4}\sum_{i,j}F_{ij}\int_0^{k_{F_i}}kdk\int_0^{k_{F_i}}qdq\Big[A_{\mathcal{K}}(k,q)+\hat{M}(k)\hat{M}(q)B_{\mathcal{K}}(k,q)+\hat{P}(k)\hat{P}(q)C_{\mathcal{K}}(k,q)\Big]$$

$$\hat{M}=\frac{M^*}{E^*},\qquad \hat{P}=\frac{p^*}{E^*}$$

$F^{ij}=\delta^{ij}$ for isoscalar mesons σ,ω

$F^{ij}=2-\delta^{ij}$ for isovector mesons ρ,π,δ

$$E^*=E-\Sigma^0=\sqrt{q^{*2}+M^{*2}},$$

$$M^*=M+\Sigma^s,$$

$$\boldsymbol{q}^*=\boldsymbol{q}+\boldsymbol{q}\Sigma^v,\qquad q^*=|\boldsymbol{q}^*|$$

!

in the following the focus will be put on the approach through the **self-energy**

DIRAC DENSITY FUNCTIONAL

Self-Energy Structure

SELF-ENERGY

SCALAR

TIME-LIKE VECTOR

SPACE-LIKE VECTOR

$$\Sigma(k,k_F) = \Sigma^s(k,k_F) - \gamma^0\Sigma^0(k,k_F) + \boldsymbol{\gamma}\cdot\boldsymbol{k}\,\Sigma^v(k,k_F)$$

$$\Sigma^s(k,k_F) = -\frac{g_\sigma^2}{m_\sigma^2}\rho_s(k_F) + \frac{1}{(4\pi)^2 k}\int_0^{k_F} q\,dq\,\frac{M^*(q)}{E^*(q)}\left[g_\sigma^2\Theta_\sigma(k,q) - 4g_\omega^2\Theta_\omega(k,q) - 3\left(\frac{f_\pi}{m_\pi}\right)^2 m_\pi^2\Theta_\pi(k,q)\right]$$

$$\Sigma^0(k,k_F) = -\frac{g_\omega^2}{m_\omega^2}\rho_B(k_F) - \frac{1}{(4\pi)^2 k}\int_0^{k_F} q\,dq\left[g_\sigma^2\Theta_\sigma(k,q) + 2g_\omega^2\Theta_\omega(k,q) - 3\left(\frac{f_\pi}{m_\pi}\right)^2 m_\pi^2\Theta_\pi(k,q)\right]$$ **!**

$$\Sigma^v(k,k_F) = -\frac{1}{(4\pi)^2 k^2}\int_0^{k_F} q\,dq\,\frac{q^\star}{E^\star(q)}\left[2g_\sigma^2\Phi_\sigma(k,q) + 4g_\omega^2\Phi_\omega(k,q) - 6\left(\frac{f_\pi}{m_\pi}\right)^2\Pi_\pi(k,q)\right]$$

ANGULAR EXCHANGE FACTORS

$$\Theta_i(k,q) = \ln\left[\frac{(k+q)^2+m_i^2}{(k-q)^2+m_i^2}\right],$$

$$\Phi_i(k,q) = \Theta_i(k,q)\left[\frac{k^2+q^2+m_i^2}{4kq}\right] - 1,$$

$$\Pi_i(k,q) = (k^2+q^2)\Phi_i(k,q) - kq\Theta_i(k,q),$$

SOURCE DENSITIES

$$\rho_s(k_F) = \frac{2}{\pi^2}\int_0^{k_F} q^2 dq\,\frac{M^*(q)}{E^*(q)}$$

$$\rho_B(k_F) = \frac{2}{\pi^2}\int_0^{k_F} q^2 dq = \frac{2}{3\pi^2}k_F^3$$

! demonstration for the **vector self-energy**

$$I_{exc}(k,k_F) = \frac{1}{k}\int_0^{k_F} q\,dq\,\ln\left[\frac{(k+q)^2+m^2}{(k-q)^2+m^2}\right]$$

observation

Each contribution depends on the specific exchange integral

goal

An analytic expression of the integral that is easy and convenient to use

Dirac Density Functional

Mapping of Fock Terms

Mean-Filed *vs* **Hartree-Fock**

properties

- *Momentum independent*
- *No exchange (Fock) part in the interaction*
- *Phenomenological density dependence*

properties

- *Momentum dependent*
- *Exchange correlations*
- *Stiff density dependence*

METHOD **demonstration for the vector self-energy**

(a) *evaluate* the exchange integral

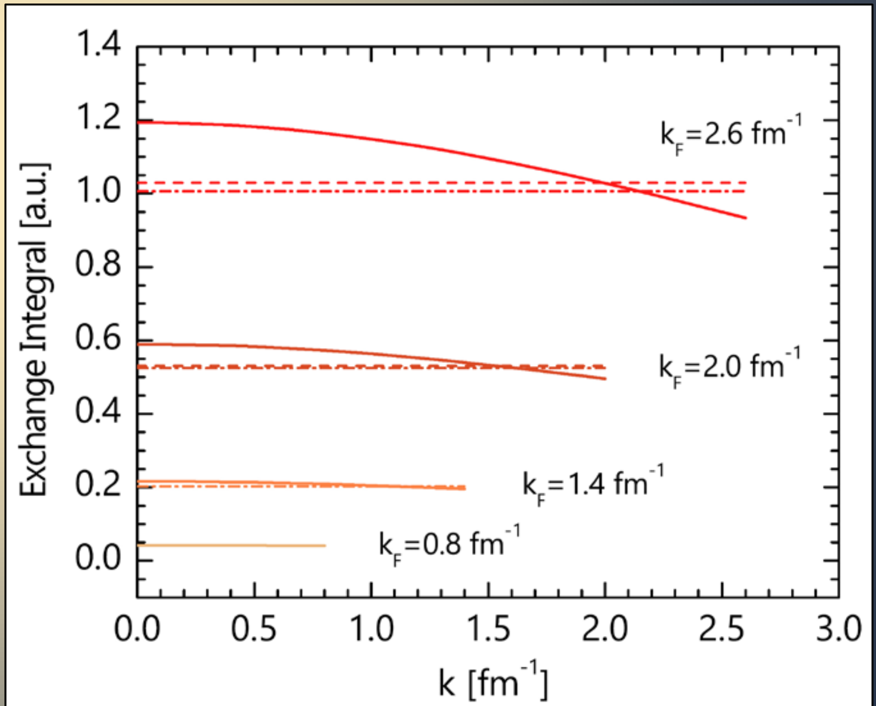
(b) *expand* it in k at k=0 and keep the terms up to k^2, $O(k^4)$

(c) *average* it over the Fermi sphere

$$I_{exc}(k,k_F) = \frac{1}{k}\int_0^{k_F} q dq \ln\left[\frac{(k+q)^2+m^2}{(k-q)^2+m^2}\right]$$

approximate, *k*-averaged expression

$$\bar{I}_{exc}(k_F) = 4\left[k_F - m\arctan\left(\frac{k_F}{m}\right) - \frac{k_F^5}{5(k_F^2+m^2)^2}\right]$$



The difference between the averaged results of the exact integral and the averaged results of approximate expression is less than **2%**

Dirac Density Functional

New Density Dependent Vertices

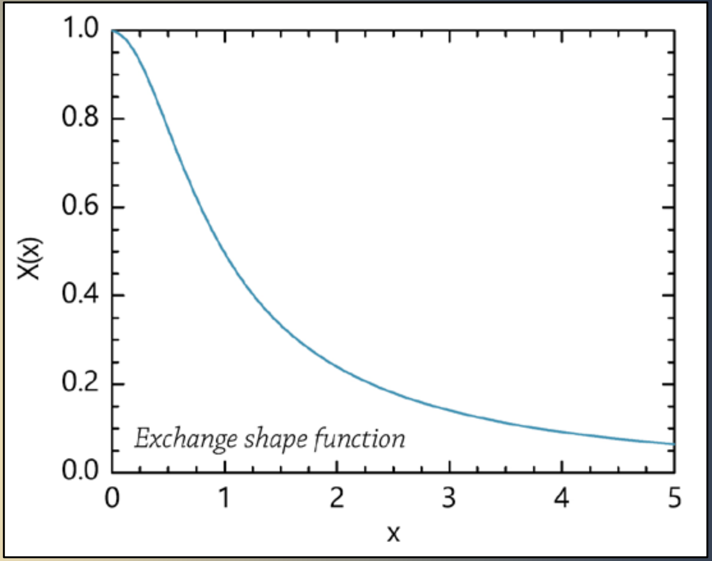
$$\bar{I}_{exc}(k_F) = 4\left[k_F - m\arctan\left(\frac{k_F}{m}\right) - \frac{k_F^5}{5(k_F^2+m^2)^2}\right]$$

approximate Fock integral

$$X_i(x) = 3\left[x - \arctan(x) - \frac{x^5}{5(1+x^2)^2}\right]/x^3$$

normalized as $X_i(0) = 1$, where $x = k_F/m_i$.

auxiliary exchange function



idea

*To **create** a **new type** of **density dependent meson-nucleon vertices**, with the Fock exchange correlations taken into account*

SCALAR CHANNEL

$$\Sigma^s(k_F) = -\frac{\Gamma_\sigma^2(k_F)}{m_\sigma^2}\rho_s(k_F)$$

$$\frac{\Gamma_\sigma^2(k_F)}{m_\sigma^2} = \frac{g_\sigma^2}{m_\sigma^2} - \frac{1}{8}\left[\frac{g_\sigma^2}{m_\sigma^2}X_\sigma - 4\frac{g_\omega^2}{m_\omega^2}X_\omega - 3\frac{f_\pi^2}{m_\pi^2}X_\pi\right]$$

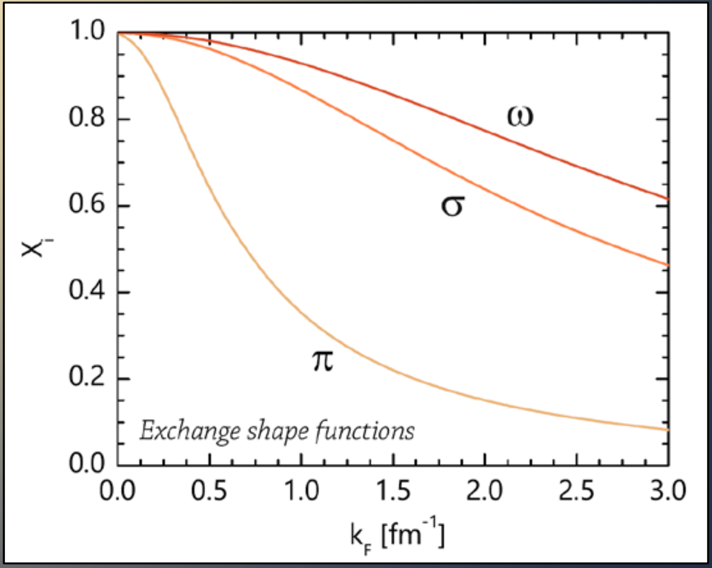
HARTREE
free space couplings

FOCK
meson mixing

VECTOR CHANNEL

$$\Sigma^0(k_F) = -\frac{\Gamma_\omega^2(k_F)}{m_\omega^2}\rho_B(k_F)$$

$$\frac{\Gamma_\omega^2(k_F)}{m_\omega^2} = \frac{g_\omega^2}{m_\omega^2} + \frac{1}{8}\left[\frac{g_\sigma^2}{m_\sigma^2}X_\sigma + 2\frac{g_\omega^2}{m_\omega^2}X_\omega - 3\frac{f_\pi^2}{m_\pi^2}X_\pi\right]$$



Dirac Density Functional

Results

$$\Sigma^s(k_F) = -\frac{\Gamma_\sigma^2(k_F)}{m_\sigma^2}\rho_s(k_F)$$

$$\frac{\Gamma_\sigma^2(k_F)}{m_\sigma^2} = \frac{g_\sigma^2}{m_\sigma^2} - \frac{1}{8}\left[\frac{g_\sigma^2}{m_\sigma^2}X_\sigma - 4\frac{g_\omega^2}{m_\omega^2}X_\omega - 3\frac{f_\pi^2}{m_\pi^2}X_\pi\right]$$

SCALAR CHANNEL

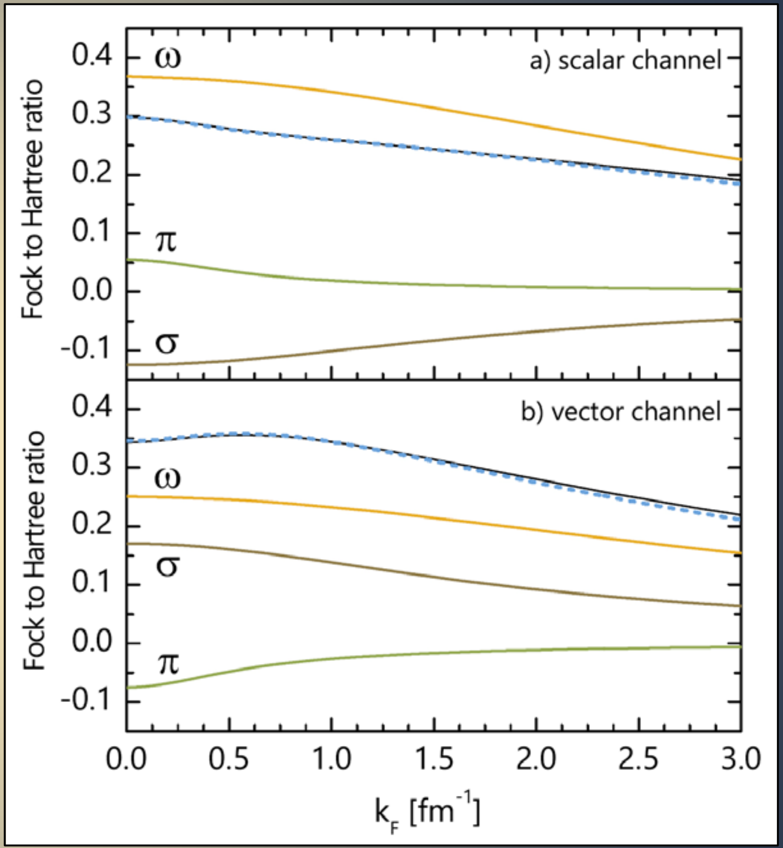
BLACK LINE full exact calculation

BLUE DASHED LINE our approximate calculation

VECTOR CHANNEL

$$\Sigma^0(k_F) = -\frac{\Gamma_\omega^2(k_F)}{m_\omega^2}\rho_B(k_F)$$

$$\frac{\Gamma_\omega^2(k_F)}{m_\omega^2} = \frac{g_\omega^2}{m_\omega^2} + \frac{1}{8}\left[\frac{g_\sigma^2}{m_\sigma^2}X_\sigma + 2\frac{g_\omega^2}{m_\omega^2}X_\omega - 3\frac{f_\pi^2}{m_\pi^2}X_\pi\right]$$



Conclusion

- *All mesons affect all channels – **mixing** is important !*
- *Fock density dependence is **weak***
- *Approximate analytical expression of the Fock integral gives results **very close** to the exact ones*

coupling constants

A. Bouyssy, et al., PRC 36, 380 (1987)

Conclusions

prospects

present

! **2-parametric class of density functions**

- *more **reliable** and **stable** results*
- *significantly **smaller** number of free parameters*
- ***excellent reproduction** of DBHF data*

! **Improvement of saturation properties**

- *DDSET$_2$ improves the **saturation density** and the **symmetry energy** of nuclear matter*
- *still **great description** of DBHF calculations*

future

? **Density dependence**

- ***beyond Hartree** approximation*
- ***inclusion** of the meson **self-interactions***
- ***new** reliable DBHF data*

? **Compact stars**

- ***hyperon** interaction **scaling** ?*
- *other **exotics** ?*
- ***precise** measurements of **neutron star radii** ?*

Thank you !

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Dirac Density Functional

DHF Energy Density Functional

κ	A_κ	B_κ	C_κ
σ	$g_\sigma^2\theta_\sigma$	$g_\sigma^2\theta_\sigma$	$-2g_\sigma^2\theta_\sigma$
δ	$g_\delta^2\theta_\delta$	$g_\delta^2\theta_\delta$	$-2g_\delta^2\theta_\delta$
π	$-\left(\frac{f_\pi}{m_\pi}\right)^2 m_\pi^2\theta_\pi$	$-\left(\frac{f_\pi}{m_\pi}\right)^2 m_\pi^2\theta_\pi$	$2\left(\frac{f_\pi}{m_\pi}\right)^2 [(p^2+p'^2)\phi_\pi - pp'\theta_\pi]$
ω	$2g_\omega^2\theta_\omega$	$-4g_\omega^2\theta_\omega$	$-4g_\omega^2\theta_\omega$
ρ_V	$2g_\rho^2\theta_\rho$	$-4g_\rho^2\theta_\rho$	$-4g_\rho^2\theta_\rho$
ρ_T	$-\left(\frac{f_\rho}{2M}\right)^2 m_\rho^2\theta_\rho$	$-3\left(\frac{f_\rho}{2M}\right)^2 m_\rho^2\theta_\rho$	$4\left(\frac{f_\rho}{2M}\right)^2 [(p^2+p'^2-m_\rho/2)\phi_\rho - pp'\theta_\rho]$
ρ_{VT}	$D = 12\frac{f_\rho g_\rho}{2M}(p\theta_\rho - 2p'\phi_\rho)$		