Post-TOV formalism for relativistic stars

(based on arXiv:1504.02455)

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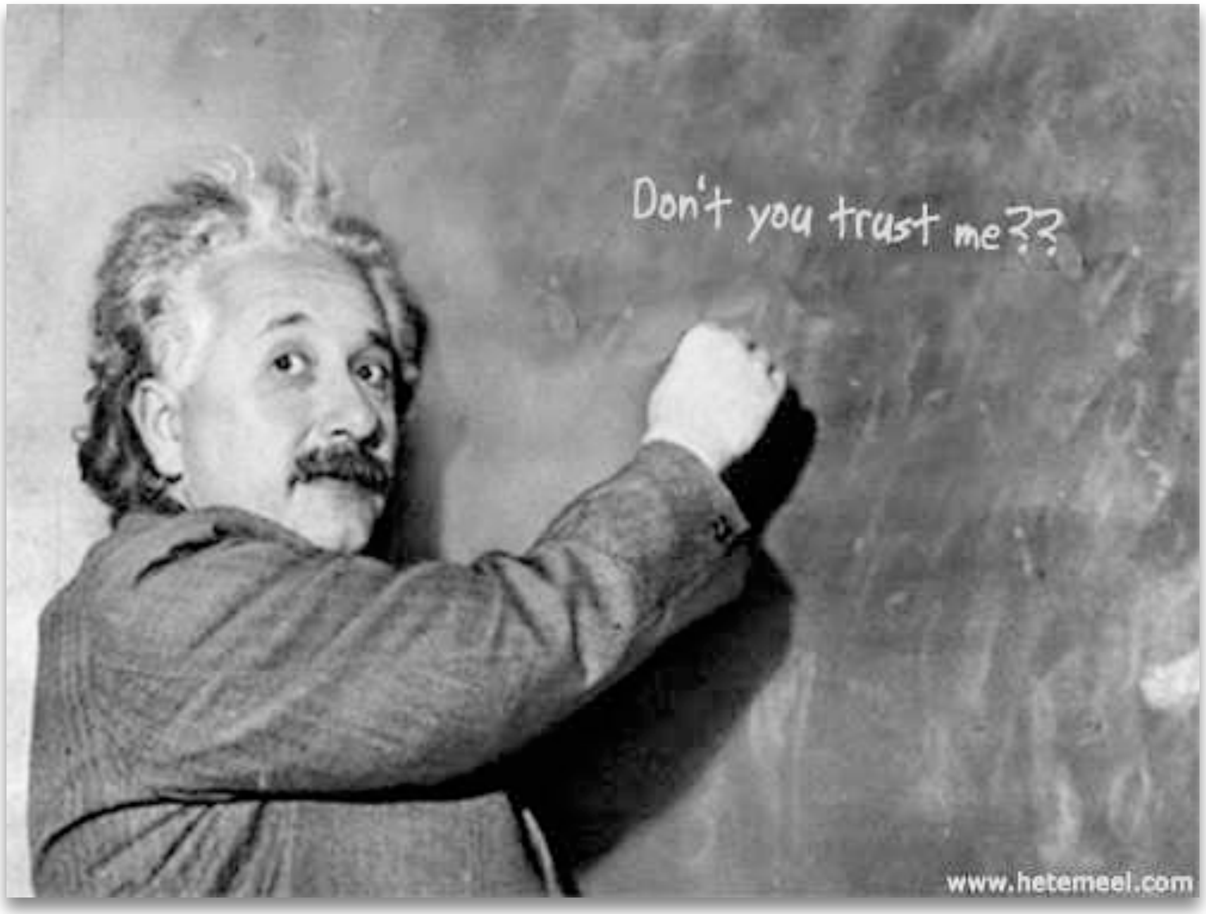
Budapest, June 2015

Context

- General Relativity is arguably the most elegant theory invented so far and is, most likely, *The* theory of gravity.
- Good science: test all theories, no matter how elegant!
- Build up phenomenology --> how "special" is GR?
- A host of modified theories of gravity exists on the market.

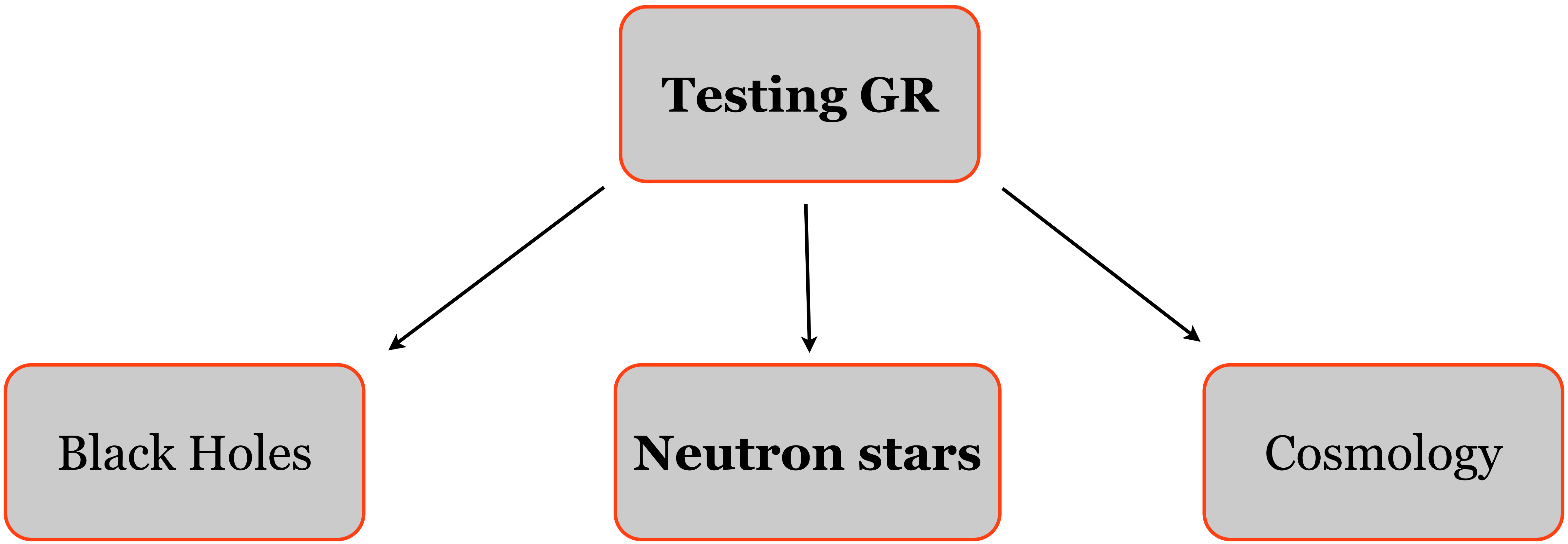


The zoo of gravity theories

Theory	Field content	Strong EP	Massless graviton	Lorentz symmetry	Linear $T_{\mu\nu}$	Weak EP	Well-posed?	Weak-field constraints
Extra scalar field								
Scalar-tensor	S	✗	✓	✓	✓	✓	✓ [30]	[31–33]
Multiscalar	S	✗	✓	✓	✓	✓	✓?	[34]
Metric $f(R)$	S	✗	✓	✓	✓	✓	✓ [35, 36]	[37]
Quadratic gravity								
Gauss-Bonnet	S	✗	✓	✓	✓	✓	✓?	[38]
Chern-Simons	P	✗	✓	✓	✓	✓	✗✓? [39]	[40]
Generic	S/P	✗	✓	✓	✓	✓	?	
Horndeski	S	✗	✓	✓	✓	✓	✓?	
Lorentz-violating								
Æ-gravity	SV	✗	✓	✗	✓	✓	✓?	[41–44]
Khronometric/ Hořava-Lifshitz	S	✗	✓	✗	✓	✓	✓?	[43–46]
n-DBI	S	✗	✓	✗	✓	✓	?	none ([47])
Massive gravity								
dRGT/Bimetric	SVT	✗	✗	✓	✓	✓	?	[16]
Galileon	S	✗	✓	✓	✓	✓	✓?	[16, 48]
Nondynamical fields								
Palatini $f(R)$	–	✓	✓	✓	✗	✓	✓	none
Eddington-Born-Infeld	–	✓	✓	✓	✗	✓	?	none
Others, not covered here								
TeVeS	SVT	✗	✓	✓	✓	✓	?	[33]
$f(R)\mathcal{L}_m$	?	?	✓	✓	✓	✗	?	
$f(T)$	?	✗	✓	✗	✓	✓	?	[49]

[Berti et al. 2015]

Testing strong gravity



Probably the "cleanest" probes of gravity, although the Kerr metric is not unique to GR.

Many observational "handles" but intrinsically plagued by the matter-gravity degeneracy.

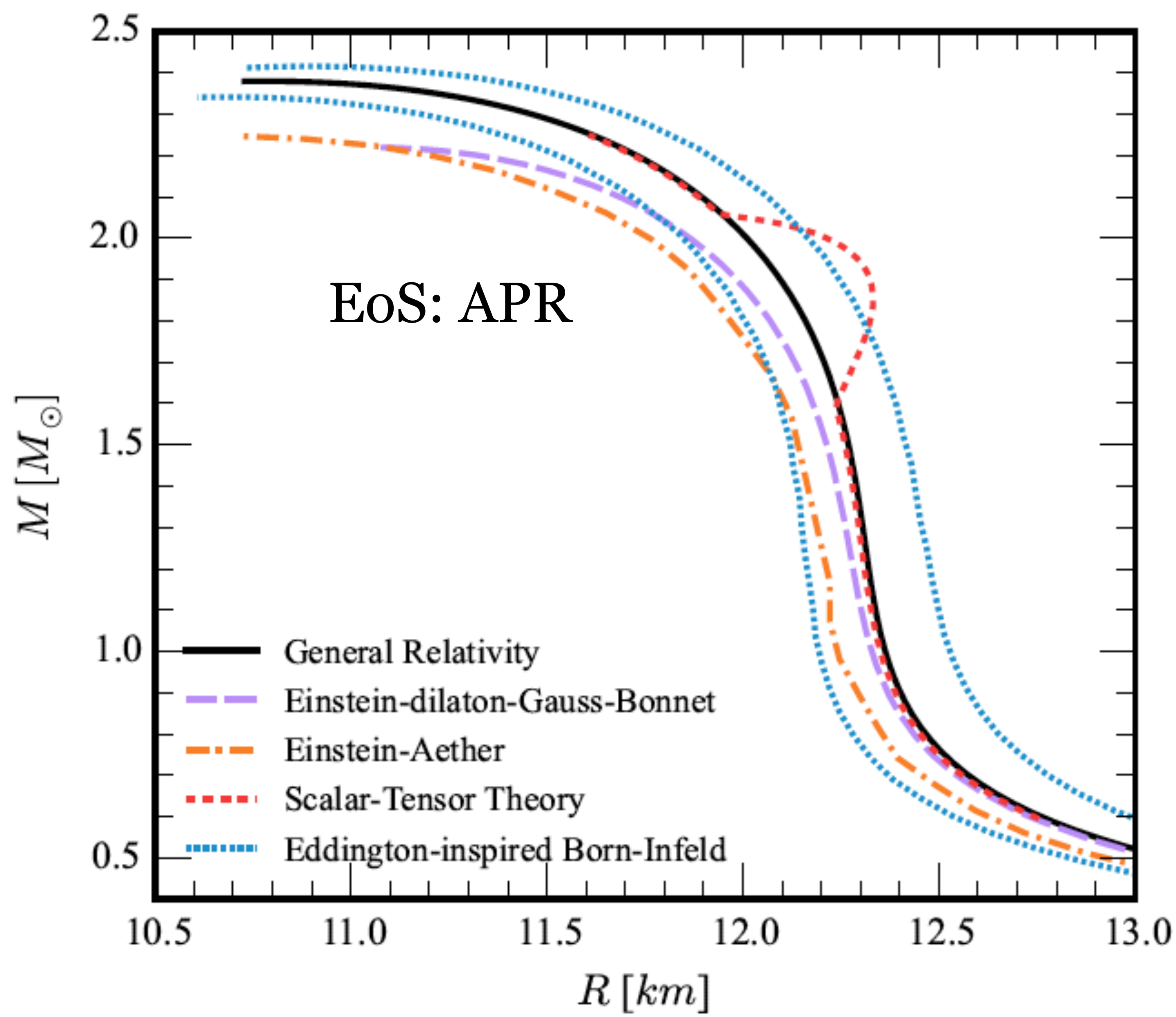
Need to go beyond the scale factor evolution $\alpha(t)$, typically this can be "reverse-engineered" by the extra degrees of freedom.

The “gravity theory degeneracy”

- Two types of Mass-Radius degeneracy in neutron stars:

✓ *Matter-gravity* (changes in EoS can mimic departures from GR).

✓ *Gravity theory*: for the same EoS, different theories of gravity can cause similar departures from GR.



An alternative approach

- *Do not commit to any particular "pet" theory of gravity.*

- Parametrize deviations from GR in a PPN theory-manner.

- At the same time allow for strong gravity, by going beyond "simple" PN expansions.

- Examples:

  "Bumpy" or "quasi-Kerr" BH metrics,

  "post-Friedmannian" cosmology,

  "post-Einsteinian" GW waveforms.

Recipe for a "post-TOV" formalism

- This is a formalism for *relativistic stars in spherical symmetry* (i.e. no rotation), not a full-fledged theory of gravity.

- **Main idea:**
  augment the General Relativistic TOV stellar structure equations by adding 1PN and 2PN corrections with *arbitrary coefficients*.
  These terms are to be built out of the available parameters:

$$p,\ \ \rho,\ \ \Pi,\ \ m,\ \ r \qquad \text{matter energy density:} \quad \epsilon = \rho(1+\Pi)$$

- The hydrostatic equations for the pressure $p(r)$ and mass function $m(r)$ take the symbolic form:

$$\frac{dp}{dr} = \left(\frac{dp}{dr}\right)_{\rm GR} + \{\ \text{PN corrections}\ \}, \quad \frac{dm}{dr} = \left(\frac{dm}{dr}\right)_{\rm GR} + \ \{\ \text{PN corrections}\ \}$$

Looking for post-TOV corrections

- **1PN order**: these can be extracted from the existing PPN theory:

$$\Lambda_1 \sim \Pi, \ \frac{m}{r}, \ \frac{r^3 p}{m}$$

- **2PN** : use dimensional analysis (excluding the presence of dimensional coupling constants):

General 2PN term (dimensionless): $\Lambda_2 \sim \Pi^\theta (r^2 p)^\alpha (r^2 \rho)^\beta \left(\frac{m}{r}\right)^{2-2\alpha-\beta-\theta}$

- Limits on { α, β, θ }: (i) avoid divergence at the stellar center/surface and (ii) assume field equations with a linear dependence on the stress-energy tensor:

$$\{\,\text{geometry}\,\} \sim 8\pi T^{\mu\nu}$$

$$\{\,\text{geometry}\,\} \sim (\epsilon + \tau p)^n$$

$$\tau, n = \mathcal{O}(1)$$

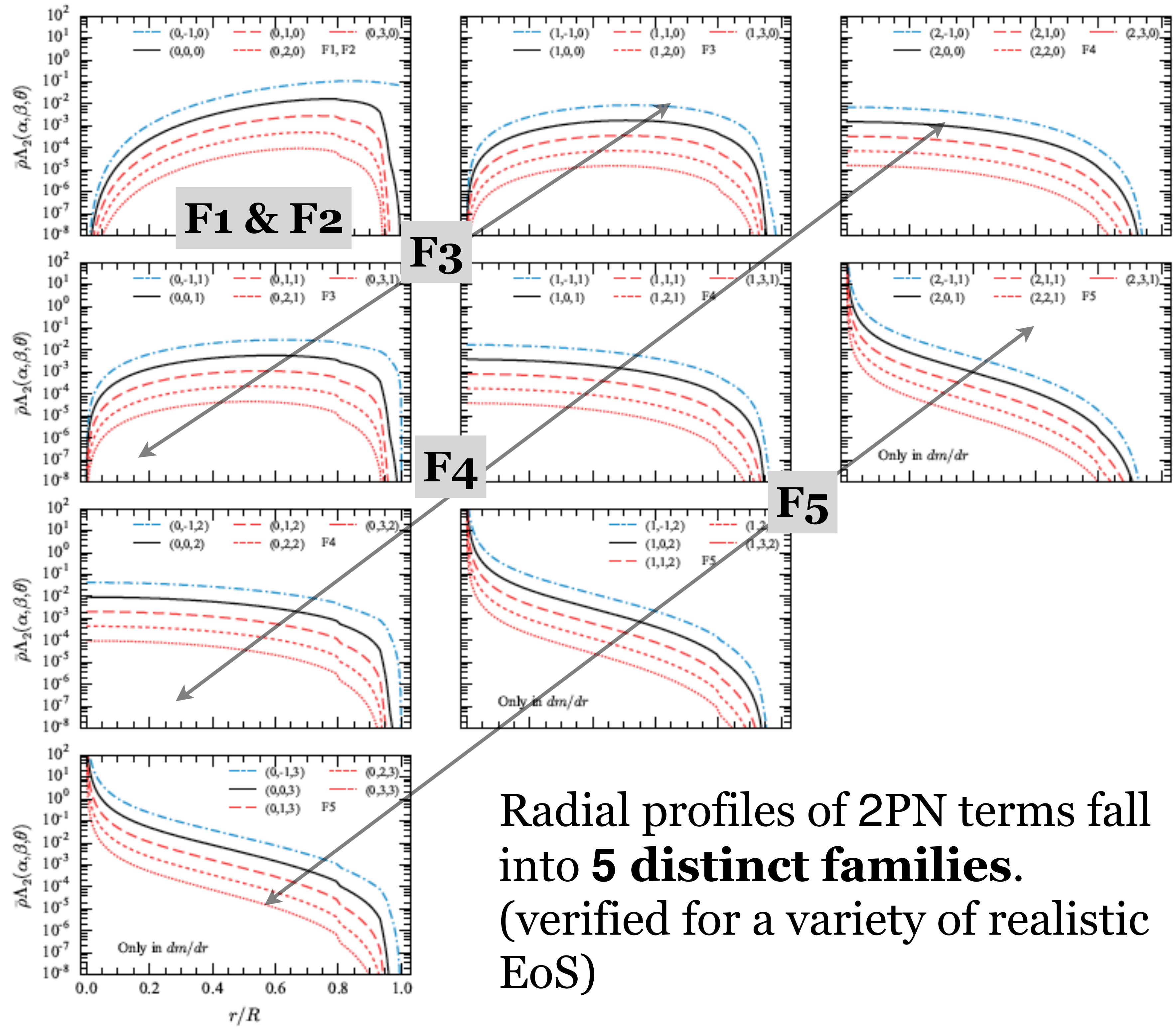
$$\beta \geq -1$$

$$0 \leq \theta \leq 2 \text{ or } 3$$

$$0 \leq \alpha \leq 2-\theta \text{ or } 3-\theta$$

It's all about Families (of 2PN terms)!

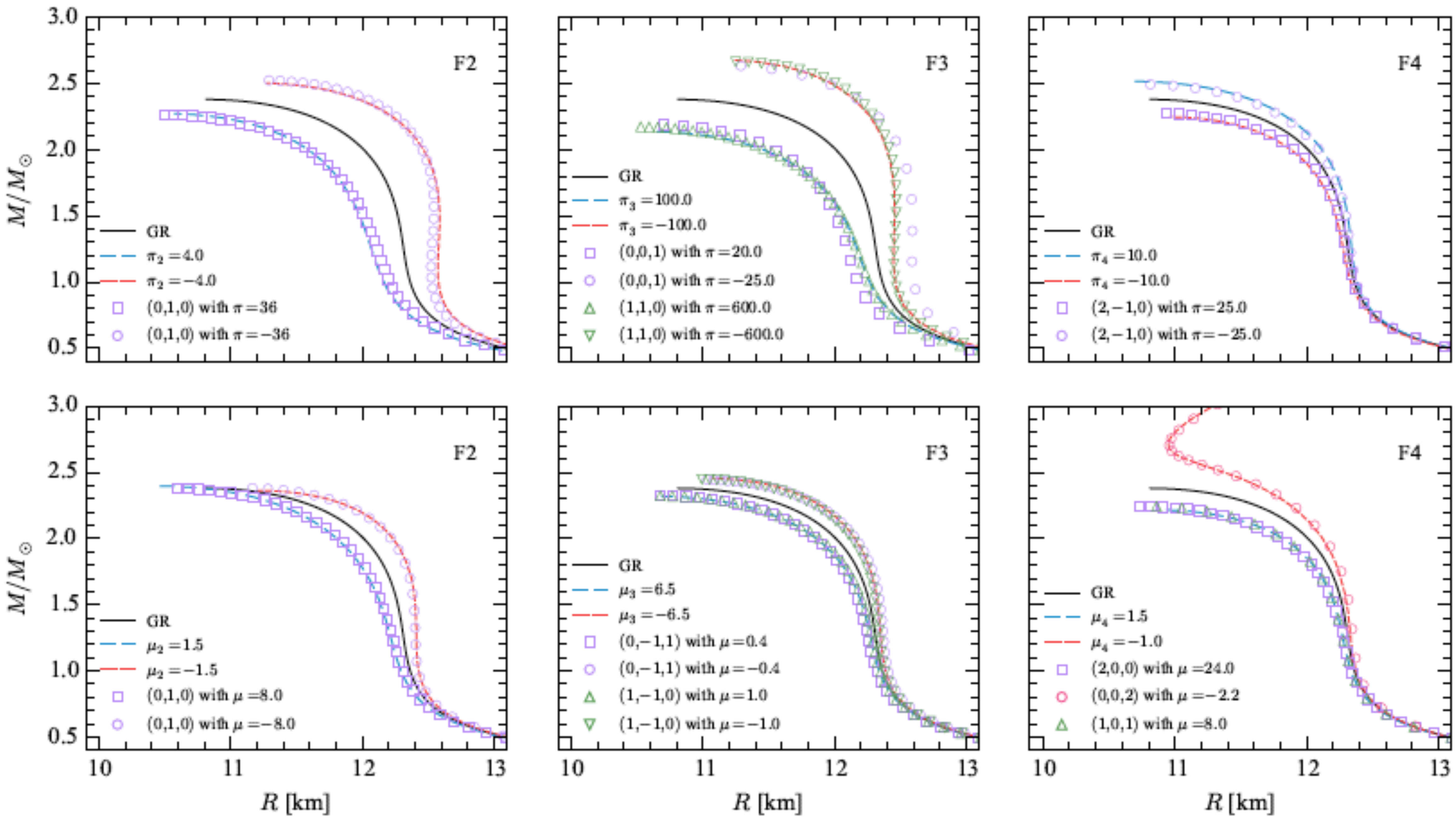
Family	2PN term	(α, β, θ)
F1	$m^3/(r^5\rho)$	$(0, -1, 0)$
F2	$(m/r)^2$	$(0, 0, 0)$
F2	$rm\rho$	$(0, 1, 0)$
F3	$mp/(r\rho)$	$(1, -1, 0)$
F3	r^2p	$(1, 0, 0)$
F3	$\Pi m^2/(r^4\rho)$	$(0, -1, 1)$
F3	$\Pi m/r$	$(0, 0, 1)$
F3	$r^2\Pi\rho$	$(0, 1, 1)$
F4	$r^3p^2/(\rho m)$	$(2, -1, 0)$
F4	$r^6p^2/(m^2)$	$(2, 0, 0)$
F4	$\Pi p/\rho$	$(1, -1, 1)$
F4	$\Pi r^3p/m$	$(1, 0, 1)$
F4	$\Pi^2 m/(r^3\rho)$	$(0, -1, 2)$
F4	Π^2	$(0, 0, 2)$
F5	$\Pi r^4p^2/(\rho m^2)$	$(2, -1, 1)$
F5	$\Pi r^7p^2/m^3$	$(2, 0, 1)$
F5	$\Pi^2 rp/m\rho$	$(1, -1, 2)$
F5	$\Pi^2 r^4p/m^2$	$(1, 0, 2)$
F5	$\Pi^3/(r^2\rho)$	$(0, -1, 3)$
F5	$\Pi^3 r/m$	$(0, 0, 3)$



Radial profiles of 2PN terms fall into **5 distinct families**. (verified for a variety of realistic EoS)

Families: *M*-*R* self-similarity

Remarkably, the 2PN terms of the same family lead to self-similar M-R curves when added as post-TOV corrections (results shown assume APR but have been verified for other EoS too).



Post-TOV structure equations (I)

$$\frac{dp}{dr} = \left(\frac{dp}{dr}\right)_{\rm GR} - \frac{\rho m}{r^2}\left(\mathcal{P}_1 + \mathcal{P}_2\right)$$

$$\frac{dm}{dr} = \left(\frac{dm}{dr}\right)_{\rm GR} + 4\pi r^2 \rho \left(\mathcal{M}_1 + \mathcal{M}_2\right)$$

- 1PN-order corrections:

$$\mathcal{P}_1 = \delta_1 \frac{m}{r} + 4\pi \delta_2 \frac{r^3 p}{m}$$

$$\mathcal{M}_1 = \delta_3 \frac{m}{r} + \delta_4 \Pi$$

- Current PPN limits: $|\delta_i| \ll 1 \quad \rightarrow \quad |\mathcal{P}_1|, \ |\mathcal{M}_1| \ll 1$

these terms can be ignored

Post-TOV structure equations (II)

$$\frac{dp}{dr} \approx \left(\frac{dp}{dr}\right)_{\rm GR} - \frac{\rho m}{r^2}\,\mathcal{P}_2$$

$$\frac{dm}{dr} \approx \left(\frac{dm}{dr}\right)_{\rm GR} + 4\pi r^2 \rho\,\mathcal{M}_2$$

- 2PN-order corrections: use *just one representative term per family*, then self-similarity implies that all other terms are accounted for by a simple variation of the corresponding coefficient:

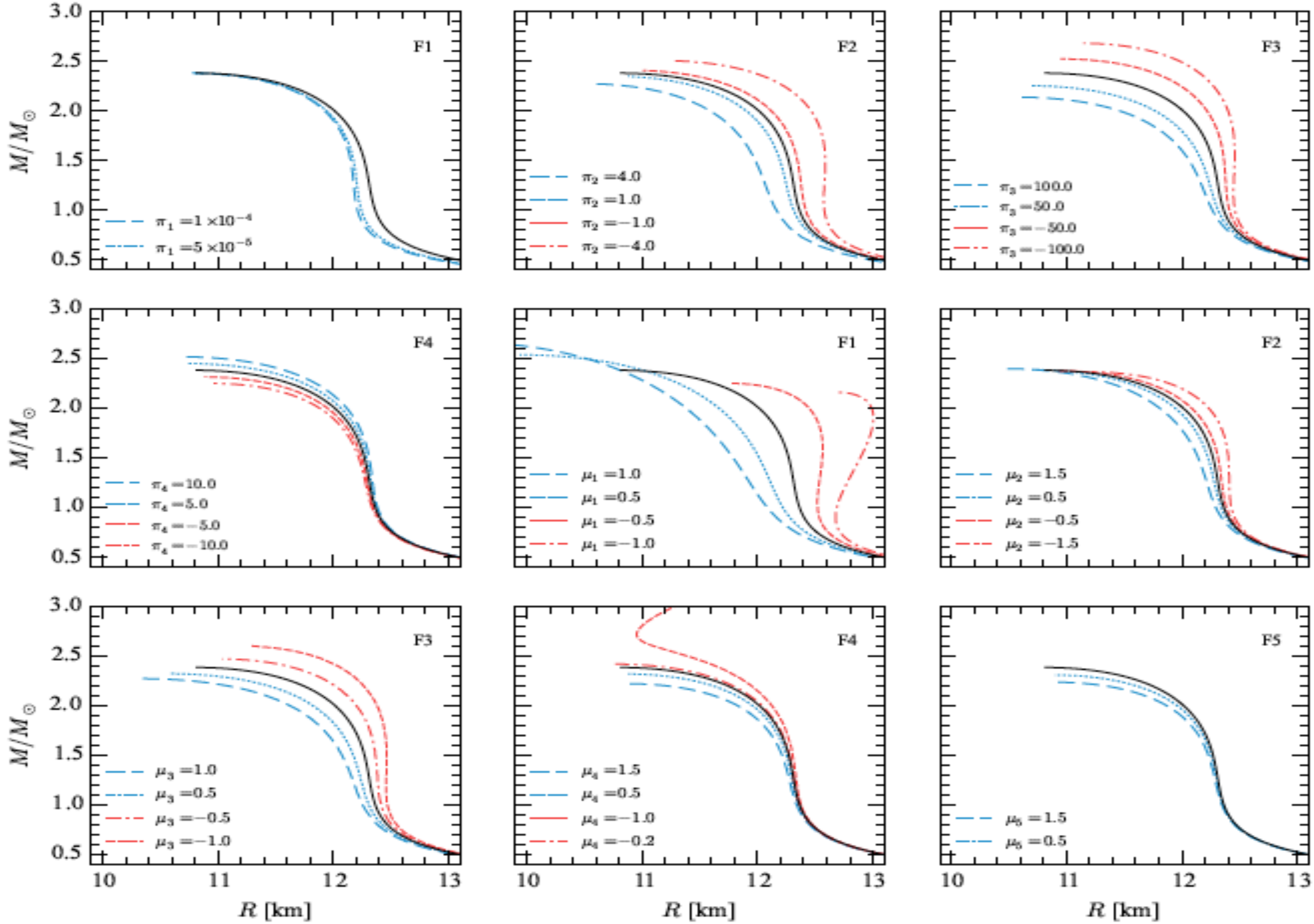
$$\{\pi_i, \mu_i\} \leftrightarrow F_i$$

family

$$\mathcal{P}_2 = \pi_1 \frac{m^3}{r^5 \rho} + \pi_2 \frac{m^2}{r^2} + \pi_3\, r^2 p + \pi_4 \frac{\Pi p}{\rho}$$

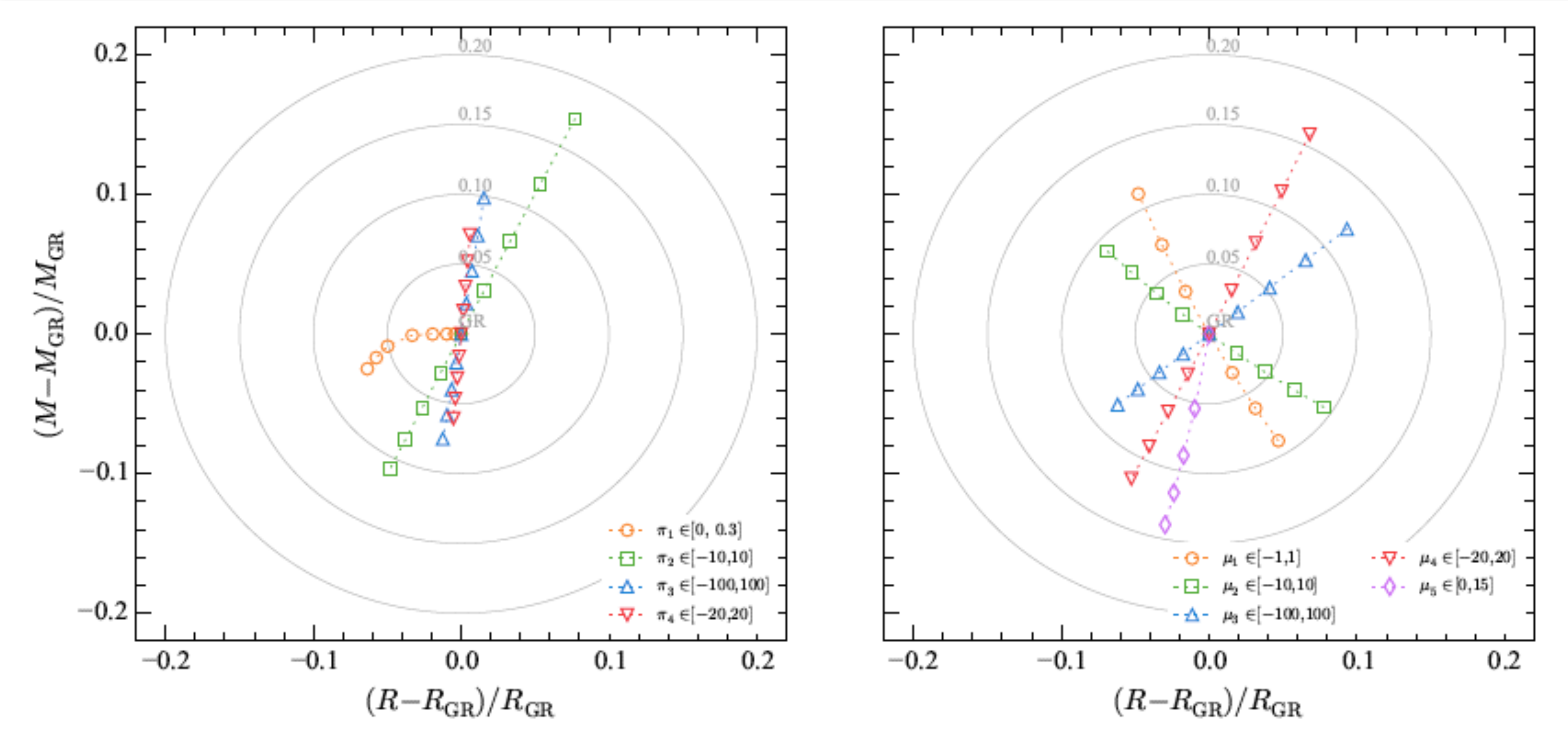
$$\mathcal{M}_2 = \mu_1 \frac{m^3}{r^5 \rho} + \mu_2 \frac{m^2}{r^2} + \mu_3\, r^2 p + \mu_4 \frac{\Pi p}{\rho} + \mu_5\, \Pi^3 \frac{r}{m}$$

post-TOV: sample of *M*-*R* curves

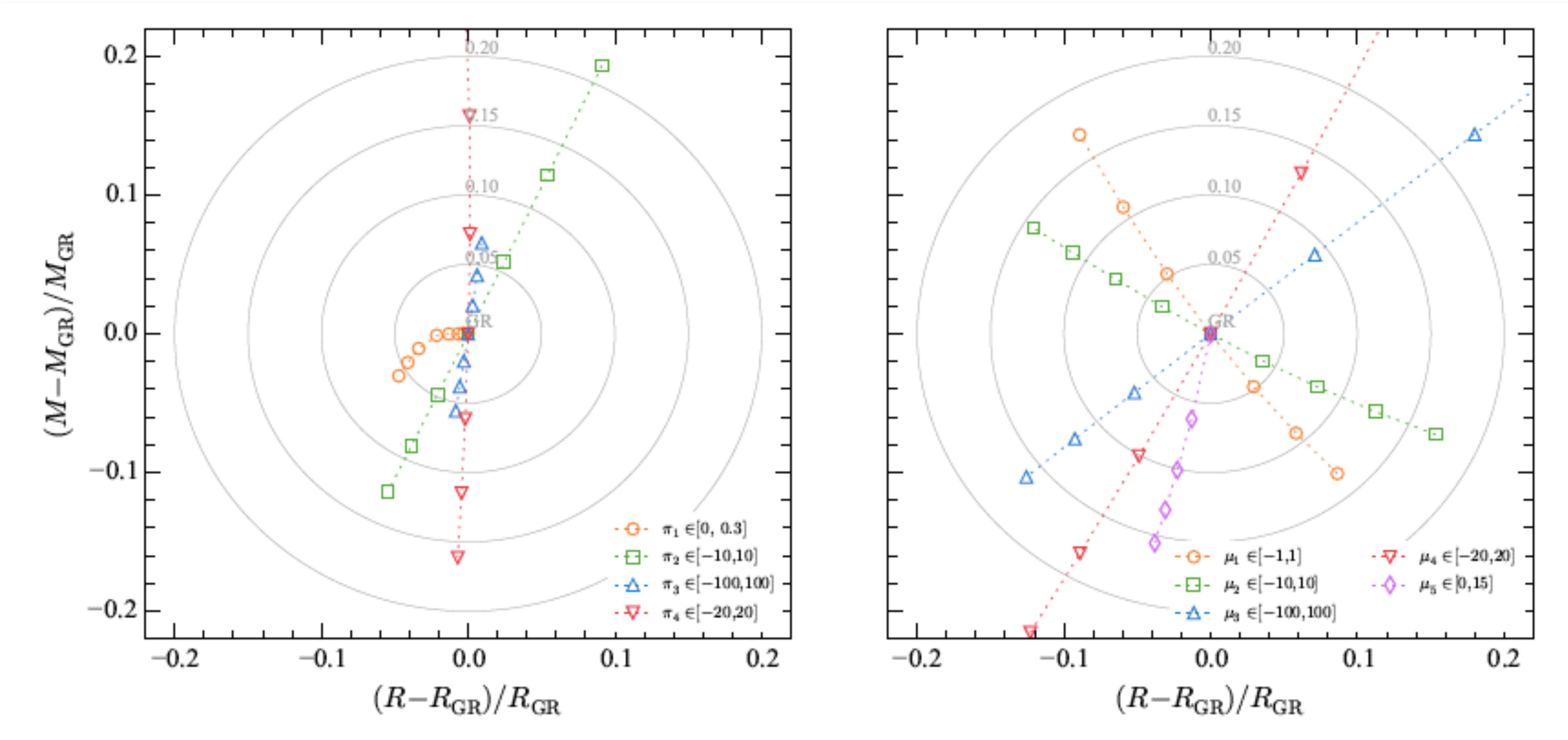


EoS: APR

post-TOV: M-R "directionality" (I)



post-TOV: M-R “directionality” (II)



Post-TOV as "effective GR"

- The post-TOV equations can also be mapped onto an "effective" GR formulation:

$$\nabla_\nu T^{\mu\nu}_{\rm eff} = 0, \qquad T^{\mu\nu}_{\rm eff} = (\epsilon_{\rm eff} + p) u^\mu u^\nu + p g^{\mu\nu}$$

$$\frac{dp}{dr} = -\frac{1}{2}\left(\epsilon_{\rm eff} + p\right)\frac{d\nu}{dr} \qquad \frac{dm}{dr} = 4\pi r^2 \epsilon_{\rm eff}$$

- *Gravity-shifted effective* EoS: $p = p\big(\epsilon_{\rm eff}\big), \quad \epsilon_{\rm eff} = \epsilon + \rho \mathcal{M}_2$

- Effective *interior* metric:

$$g_{\mu\nu} = {\rm diag}[\, e^{\nu(r)}, (1 - 2m(r)/r)^{-1}, r^2, r^2 \sin^2\theta\,]$$

post-TOV: how "general" is it?

- Modified theories of gravity typically include one (less frequently more) additional dynamical degrees of freedom (e.g. a scalar field)

- Our post-TOV formalism, featuring only two equations for $\mathrm{d}p/\mathrm{d}r$, $\mathrm{d}m/\mathrm{d}r$, implies that the extra d.o.f. ψ can be expressed in terms of the matter variables, i.e. $\psi=\psi(p, \rho, \Pi)$.

- This is the case for scalar-tensor theory and the same is likely true for other theories with a single extra d.o.f.

- By construction, the post-TOV scheme assumes "small" departures from GR, so it may *not* capture non-perturbative effects like "spontaneous scalarization".

Outlook

- The post-TOV formalism is a toolkit for building relativistic stellar models with small/moderate departures from GR.

- Its parametrized form should encompass a large class of modified theories of gravity.

- **Post-TOV neutron star astrophysics (ongoing work):**

✓ M-R curves.

✓ Redshift (requires exterior metric), cooling, surface emission from bursting systems.

✓ Add slow rotation, study I-Q "universal" relations (and perhaps lift matter-gravity degeneracy...)