On the tidal Love numbers of rotating Neutron Stars

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Mostly based on:

P. Pani, L.G., A. Maselli, V. Ferrari, arXiv:1503.07365, submitted to Phys. Rev. D

P. Pani, L.G., A. Maselli, V. Ferrari, in preparation

Tidal deformability: a powerful tool to study the NS EoS

NS-NS coalescing binaries are probably the most promising sources for ground-based GW detectors: Advanced LIGO/Virgo is expecting to detect from ~few to ~hundreds of them per year.

The gravitational signal emitted by these processes contains valuable information on the behaviour of matter at supranuclear densities, i.e., on the EoS of matter in the inner core of NSs.

Where can we find the imprint of the NS EoS on the GW signal from coalescing binaries?

- In the merger signal, when the binary forms (under appropriate conditions) a metastable hypermassive NS, oscillating at characteristic frequencies which depends on the NS radius (*Shibata, PRL '05; Oechslin & Janka, PRL '07; Stergioulas et al., MNRAS '11; Bauswein et al., PRL '12, PRD '12, 14; Takami at al., PRL '14, PRD '15*)
- In the late inspiral signal, when the tidal deformation of the NSs is large enough to affect the gravitational waveform (*Flanagan & Hinderer, PRD '08; Hinderer et al, PRD '10; Baiotti et al. PRL '10, PRD '11; Lackey et al., PRD '12, PRD '14; Bini et al., PRD '12, Damour et al. PRD '12, Maselli et al. PRD '13, Favata PRL '14; Yagi & Yunes PRD '14, Bernuzzi et al. '15*)

Tidal deformability: a powerful tool to study the NS EoS

In recent years, a relativistic theory of tidal deformations has been developed

(*Flanagan & Hinderer PRD '08; Hinderer ApJ '08; Binnington & Poisson PRD '09; Damour & Nagar PRD '09*)

Main idea: in the timescale of orbital motion, << proper oscillations (*but see Maselli et al., PRD '12*), tidal deformations can be treated as stationary and multipole moments are linear in the moments of the exterior tidal field.

The proportionality constants are the Love numbers, which characterize the deformatility properties of the star and strongly depend on the NS EoS.

Most important is the quadrupolar tidal deformation:

$$Q_{ij} = -\frac{2}{3}k_2 R^5 \mathcal{E}_{ij} = -\lambda_2 \mathcal{E}_{ij}$$

Quadrupole tensor

l=2 electric tidal
Love number

Tidal deformability

Tidal tensor

Tidal deformability: a powerful tool to study the NS EoS

Tidal deformation affects the PN waveform of late inspiral through λ_2

$$h_{PN}(x) = \mathcal{A}(x)e^{\mathrm{i}[\Psi_{PP}(x)+\Psi_T(x)]}$$

$$x = (\pi m f)^{2/3} \qquad m = m_1 + m_2 \qquad \nu = m_1 m_2/m^2 \qquad \mathcal{M} = m\nu^{3/5}$$

$$\mathcal{A}(x) = \sqrt{\frac{5}{24}}\frac{\mathcal{M}^{5/6}}{\pi^{2/3}d}f^{-7/6}\left[1+\beta_1 x+\beta_2 x^2+\dots\right]$$

$$\Psi_{PP}(x) = 2\pi f t_c - \phi_c + \frac{3}{128\nu x^{5/2}}\left[1+\alpha_2 x+\alpha_3 x^{3/2}+\alpha_4 x^2+\dots\right]$$

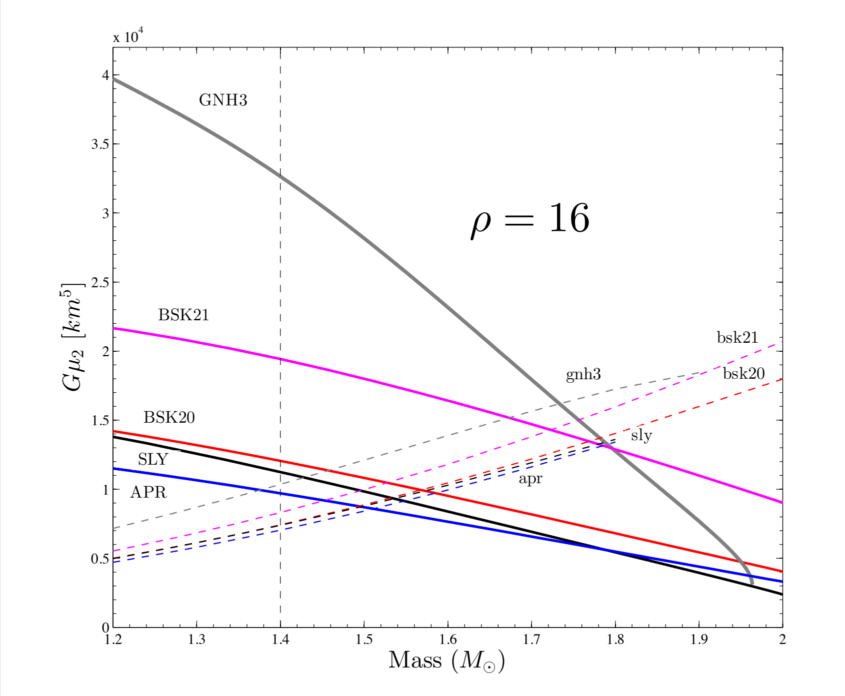
$$\Psi_T(x) = -\frac{117\tilde{\lambda}}{8\nu m^5}x^{5/2}\left[1+\tilde{\alpha}_2 x+\tilde{\alpha}_3 x^{3/2}+\dots\right] \qquad \tilde{\lambda} = \frac{m_1+12m_2}{26m_2}\lambda_2$$

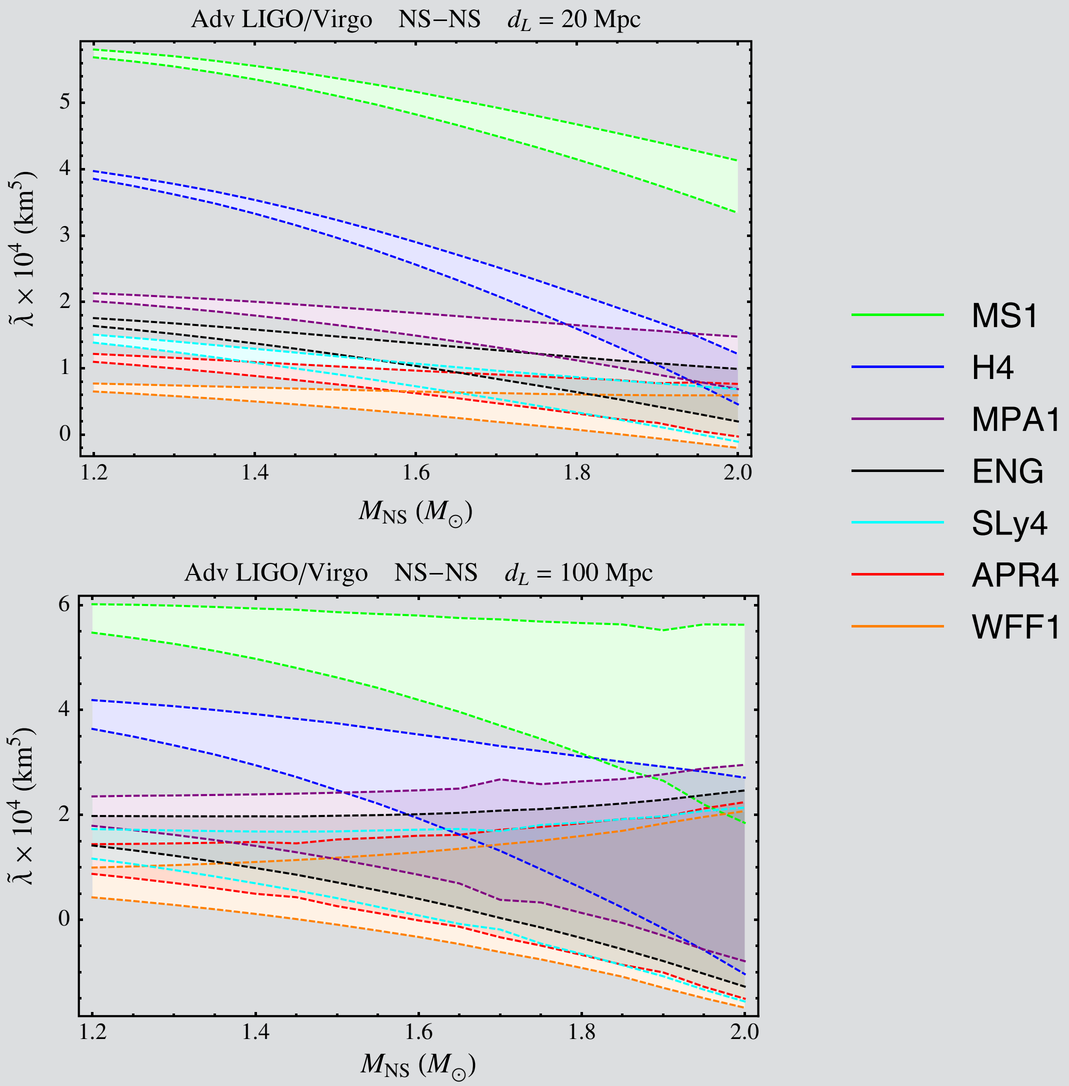
(formally 5PN, but $\lambda \sim R^5$ => $\lambda/m^5 \sim (R/m)^5 >> 1$)

Second generation GW detectors (Adv LIGO/Virgo) may detect λ with enough accuracy to constrain the NS EoS.
Third generation detectors (i.e. ET) will be even more sensitive.
(*Damour et al. PRD '12; Del Pozzo et al. PRL '13; Maselli et al. PRD '13*)

Tidal deformability: a powerful tool to study the NS EoS

Tidal deformation affects the PN waveform of late inspiral through λ





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Tidal deformability: a powerful tool to study the NS EoS

However, these computations are still preliminary.
We need to better understand tidal deformation.

When we compute Love numbers,
we consider a single,static, isolated star
deformed by a generic tidal field.

The perturbation of the spherically symmetric background has two components:
the external tidal field, and the deformation induced by this field.

$$g_{tt} = -1 + \frac{2m}{r} + \frac{3Q_{ij}}{r^3}n^i n^j - \mathcal{E}_{ij} r^2 n^i n^j + \dots$$

The former is described by the tidal tensor, and grows with r^l;
the latter is described by the quadrupole moments, and decrease with r^{-l+1}.

Solving Einstein's equations for
static perturbations, one finds
the coefficients relating tidal tensor
and quadrupole tensor: the Love numbers.

l=2: $Q_{ij}n^i n^j = -\lambda_2 \mathcal{E}_{ij} n^i n^j$

$$\begin{aligned} ds^2 &= -e^{2\Phi(r)}\left[1 + H(r)Y_{20}(\theta,\varphi)\right]dt^2 \\ &\quad + e^{2\Lambda(r)}\left[1 - H(r)Y_{20}(\theta,\varphi)\right]dr^2 \\ &\quad + r^2\left[1 - K(r)Y_{20}(\theta,\varphi)\right]\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \end{aligned}$$

Tidal deformability: a powerful tool to study the NS EoS

The l=2 electric-type tidal Love number
(associated with the quadrupolar tidal deformability λ_2)
is just one of the Love numbers
characterising the deformability properties of the NS.

More generally: M_l, S_l mass and current multipoles ($M_0=m=M$, $S_1=J$, $M_2=Q$, …),
$\mathcal{E}^{(l)}$, $\mathcal{B}^{(l)}$ electric- and magnetic-type components of the tidal field ($\sim R_{0i0j}$)

$$g_{00} = -1 + \frac{2M_0}{r} + \frac{(l=0)}{r^3} + 2P_2(\theta)\left[\frac{M_2}{r^3} - \mathcal{E}^{(2)}r^2 + (l<2)\right] + 2P_4(\theta)\left[\frac{M_4}{r^5} - \mathcal{E}^{(4)}r^4 + (l<4)\right] + \ldots$$

$$g_{03} = -2\left[\frac{S_1}{r} + \mathcal{B}^{(1)}r\right] + \ldots$$

…

$$M_l = \lambda_l^e \mathcal{E}^{(l)}$$

$$S_l = \lambda_l^m \mathcal{B}^{(l)}$$

$\{\lambda^e_l, \lambda^m_l\}$: electric-type and Love numbers

However, this construction only concerns non-rotating NSs.
Since NSs do rotate, it is natural to ask:
which are the Love numbers of rotating NSs?
Do we need to include rotation in our analysis,
in order to extract the EoS information from GW data?

Tidal deformations of a rotating compact object

A first step toward the determination of Love numbers of rotating NSs:

II order in rotation, axisymmetric, stationary tidal fields

(*P. Pani, L.G., A. Maselli, V. Ferrari, arXiv:1503.07365*; *P. Landry, E. Poisson, arXiv:1503.07366*)

I order in rotation, non-axisymmetric, quasi-stationary tidal fields

Our assumptions:

- central object slowly spinning: χ=J/M^2<<1 (neglect O(χ^3))
- weak tidal field: tidal sources at large distance r>>M
- ~stationary tidal field (and stationary distorted star)
- axisymmetry m=0 (if not, no stationary solution due to precession)

Two-parameter expansion, in the NS spin and in the tidal field

Tidal deformations of a rotating compact object

(P. Pani, L.G., A. Maselli, V. Ferrari, arXiv:1503.07365)

$$
\begin{aligned}
g_{tt} &= -e^{\nu}\left[1+2\epsilon_a^2\left(j_0+j_2P_2-\frac{r^2e^{-\nu}}{2}(\Omega-\bar{\omega})^2\right)\right.\\
&\quad +\epsilon_a^2\delta H_0^{(0)}Y^{00}+\left(H_0^{(2)}+\epsilon_a^2\delta H_0^{(2)}\right)Y^{20}\\
&\quad \left.+\epsilon_a^2\delta H_0^{(4)}Y^{40}\right],\\
g_{t\varphi} &= -\epsilon_a r^2(\Omega-\bar{\omega})\sin^2\vartheta\\
&\quad +\epsilon_a\sin\vartheta\left(h_0^{(1)}Y^{10}_{,\vartheta}+h_0^{(3)}Y^{30}_{,\vartheta}\right),
\end{aligned}
$$

Slowly rotating background

$$
\begin{aligned}
g_{rr} &= \left[1-\frac{2\mathcal{M}}{r}\right]^{-1}\times\left[1+2\epsilon_a^2\frac{m_0+m_2P_2}{r-2\mathcal{M}}\right.\\
&\quad +\epsilon_a^2\delta H_2^{(0)}Y^{00}+\left(H_2^{(2)}+\epsilon_a^2\delta H_2^{(2)}\right)Y^{20}\\
&\quad \left.+\epsilon_a^2\delta H_2^{(4)}Y^{40}\right],\\
g_{\vartheta\vartheta} &= r^2\left[1+2\epsilon_a^2(v_2-j_2)P_2\right.\\
&\quad +\epsilon_a^2\delta K^{(0)}Y^{00}+\left(K^{(2)}+\epsilon_a^2\delta K^{(2)}\right)Y^{20}\\
&\quad \left.+\epsilon_a^2\delta K^{(4)}Y^{40}\right],\\
g_{\varphi\varphi} &= \sin^2\vartheta g_{\vartheta\vartheta},
\end{aligned}
$$

Perturbations induced by tidal source are expanded in tensor spherical harmonics.
Einstein's equations are solved at 2nd order in spin and 1st order in tidal field.

Coupled system of ODEs among polar and axial parity components with different harmonic indexes

Schematic structure of the equations:

$$
\begin{aligned}
0 &= \mathcal{A}_\ell + \epsilon_a^2\hat{\mathcal{A}}_\ell\\
&\quad + \epsilon_a(\mathcal{Q}_\ell\tilde{\mathcal{P}}_{\ell-1}+\mathcal{Q}_{\ell+1}\tilde{\mathcal{P}}_{\ell+1})\\
&\quad + \epsilon_a^2\left[\mathcal{Q}_{\ell-1}\mathcal{Q}_\ell\breve{\mathcal{A}}_{\ell-2}+\mathcal{Q}_{\ell+2}\mathcal{Q}_{\ell+1}\breve{\mathcal{A}}_{\ell+2}\right]+\mathcal{O}(\epsilon_a^3)\\
0 &= \mathcal{P}_\ell + \epsilon_a^2\hat{\mathcal{P}}_\ell\\
&\quad + \epsilon_a(\mathcal{Q}_\ell\tilde{\mathcal{A}}_{\ell-1}+\mathcal{Q}_{\ell+1}\tilde{\mathcal{A}}_{\ell+1})\\
&\quad + \epsilon_a^2\left[\mathcal{Q}_{\ell-1}\mathcal{Q}_\ell\breve{\mathcal{P}}_{\ell-2}+\mathcal{Q}_{\ell+2}\mathcal{Q}_{\ell+1}\breve{\mathcal{P}}_{\ell+2}\right]+\mathcal{O}(\epsilon_a^3),
\end{aligned}
$$

Tidal deformations of a rotating compact object

(P. Pani, L.G., A. Maselli, V. Ferrari, arXiv:1503.07365)

We considered l=2 m=0 polar-parity tidal field,
exciting a set of (polar l=2)-led perturbations,
and - to begin with - solved the equations in the exterior of the NS.

General solution:

$$\delta g_{\mu\nu} = \alpha \delta g_{\mu\nu}^{(\alpha)} + \gamma \delta g_{\mu\nu}^{(\gamma)} + \sum_{l=0}^{4} \gamma_l \delta g_{\mu\nu}^{(\gamma_l)}$$

Response of the compact object to the external field; vanishes at infinity

$\alpha = \mathcal{E}^{(2)} M^2$

External tidal field, divergent at infinity ~r^2

Modification of multipole moments

Imposing the boundary conditions
(for BHs: regularity at the horizon; for NSs: matching with interior solution)
one finds the γs in terms of the αs, and then the Love numbers

$$\lambda_l^e = \frac{\partial M_l}{\partial \mathcal{E}^{(2)}} \qquad \lambda_l^m = \frac{\partial S_l}{\partial \mathcal{E}^{(2)}}$$

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General solution:

$$\delta g_{\mu\nu} = \cancel{\alpha\delta g^{(\alpha)}_{\mu\nu}} + \gamma\delta g^{(\gamma)}_{\mu\nu} + \sum_{l=0}^{4} \gamma_l \delta g^{(\gamma_l)}_{\mu\nu}$$

Strictly speaking, multipole moments can only be defined
in asymptotically flat spacetimes!

Then, one considers the solution without the external tidal field
and works out the multipoles M_l, S_l in terms of the γ_l,
and then compute the Love numbers

$$\lambda_l^e = \frac{\partial M_l}{\partial \mathcal{E}^{(2)}} \qquad \lambda_l^m = \frac{\partial S_l}{\partial \mathcal{E}^{(2)}}$$

Tidal deformations of a rotating compact object

(P. Pani, L.G., A. Maselli, V. Ferrari, arXiv:1503.07365)

Problem: it is not obvious how to distinguish between the external tidal field and the response of the compact object.

This introduces some arbitrariness in the definition of Love numbers.

This problem also occurs in the static, spherically symmetric case, but it is less severe than in the static, non-rotating case, since in that case the powers of r in the two solutions do not overlap:

$$\delta g_{tt} = \alpha\left(\frac{r}{M}\right)\left(1-\frac{2M}{r}\right)^2 + \gamma\left(\frac{8}{5}\frac{M^3}{r^3}+\frac{8}{5}\frac{M^4}{r^4}+\frac{64}{35}\frac{M^5}{r^4}+\dots\right)$$

In the rotating case, the α part has also terms as $1/r^3$, $1/r^4$, …
We made the simplest choice, corresponding to a well-behaved BH solution, extended to the case of NSs.
But we should keep in mind that these issues need to be clarified, even in the non-rotating case, to be sure that the Love numbers defined in this way are really those appearing in the GW waveform.

Tidal deformations of a rotating compact object

(P. Pani, L.G., A. Maselli, V. Ferrari, arXiv:1503.07365)

In the case of BHs we explicitly found the full solution, finding that the Love numbers of a rotating BH are vanishing (extending results of *Binnington & Poisson PRD '09*) at least up to 2nd order in spin

Multipole moments of a slowly rotating BH are not affected by an external, weak, axisymmetric tidal field (no-hair apply to tidally deformed BHs)

$$M_l + \mathrm{i}S_l = M^{l+1}(\mathrm{i}\chi)^l \quad (\chi = a/M)$$

I will not say more on this BH solution, since we are interested on NSs…

(P. Pani, L.G., A. Maselli, V. Ferrari, in preparation)

The case of NSs is more complex, since we have to solve the equations inside the star. Using the same prescription to define Love numbers, we solved the equations up to first order in the spin parameter χ. We generalized the tidal source to both electric and magnetic terms, and l=2,3 (but still m=0).

Tidal deformations of a rotating neutron stars

(P. Pani, L.G., A. Maselli, V. Ferrari, in preparation)

Rotation couples contributions with different parities and different values of l.

$$\mathcal{P}_2 + \epsilon_a(\mathcal{Q}_1\mathcal{A}_1 + \mathcal{Q}_3\mathcal{A}_3) = 0$$

Polar l=2 tidal source $\longrightarrow$ polar l=2 Love number: O(χ^2) — Our computation at O(χ) can not see it!

Axial l=1,3 tidal source $\longrightarrow$ polar l=2 Love number: O(χ) — Our computation at O(χ) can see it!

Rotational corrections to Love numbers: $\delta\lambda^e_{ll'} = \dfrac{\partial\delta M_l}{\partial\mathcal{B}^{(l')}}$ $\quad\delta\lambda^m_{ll'} = \dfrac{\partial\delta S_l}{\partial\mathcal{E}^{(l')}}$

We can compute $\delta\lambda^e_{23}$, not $\delta\lambda^e_{22}$.
One would expect the contribution of $\delta\lambda^e_{23}$ to be irrelevant,
because for typical sources the axial, "magnetic" component is negligible ($\mathcal{B} \ll \mathcal{E}$)
but it turns out that $\delta\lambda^e_{23}$ can be very large: $\delta\bar{\lambda}^e_{23} \sim 300\bar{\lambda}^e_2$ $\quad\left(\bar{\lambda}^e_2 = \dfrac{\lambda^e_2}{M^3}\,,\ \delta\bar{\lambda}^e_{23} = \dfrac{\delta\lambda^e_{23}}{M^4}\right)$

Tidal deformations of a rotating neutron stars

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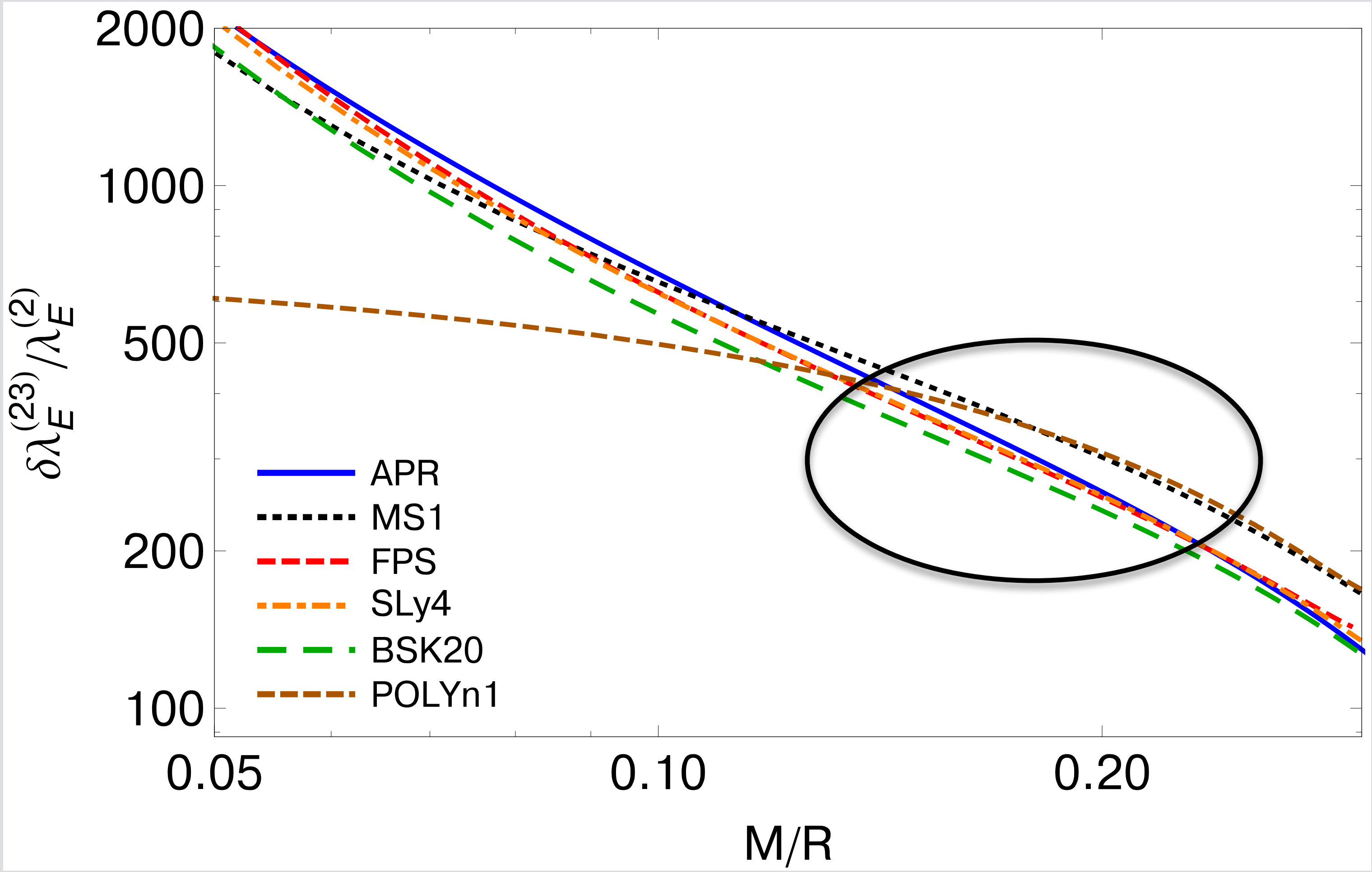

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Tidal deformations of a rotating neutron stars

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Let us consider a concrete example.

A NS-NS coalescing binary, with $m_1=m_2=M$, at distance r.
One NS is the source of the tidal field acting on the other. Spin ~ orbital rotation.
Of course, it is very rough to treat the deformed star as stationary:
it is just an order-of-magnitude estimate of the relevance of these corrections.

The l=2, m=0 component of the tidal field is

$$\mathcal{E}^{(2)} \sim \frac{M}{r^3} \quad \mathcal{B}^{(3)} \sim \frac{M^{3/2}}{r^{9/2}}$$ thus

$$\frac{M_2}{M^3} \sim (\bar{\lambda}_2^e + \chi^2 \delta\bar{\lambda}_{22}^e) M^2 \mathcal{E}^{(2)} + \chi \delta\bar{\lambda}_{23}^e M^4 \mathcal{B}^{(3)}$$

$$\sim \frac{M^3}{r^3} \left(\bar{\lambda}_2^e + \chi \left(\frac{M}{r} \right)^{3/2} \delta\bar{\lambda}_{23}^e + \chi^2 \delta\bar{\lambda}_{22}^e \right)$$

and the error in neglecting rotational contribution can be

$$\frac{\delta\lambda_2^e}{\lambda_2^e} \sim \chi \left(\frac{M}{r} \right)^{3/2} \frac{\delta\bar{\lambda}_{22}^e}{\bar{\lambda}_2^e}$$

if r=5R, R=6M, up to ~10-20%

Conclusions

- Tidal deformations of NSs can be extremely important to extract information on the NS EoS from the gravitational waveform emitted by coalescing NS-NS
- They are characterized by Love numbers, relating multipole moments of the tidally deformed NS with those of the tidal field. They appear in the waveform, and are sensitive of the EoS, but if we want to use it as a main tool we should get a better theoretical understanding.
- We studied tidal deformations of a rotating NS.
  Exterior solution solved up to $O(\chi^2)$, full solution up to $O(\chi)$,
  under simplifying assumption (axisymmetric, stationary source).
- This source is a bad model for interaction in coalescing binary, but still an order-of-magnitude computation reveals that rotation, coupling l=3 to l=2, could affect quadrupolar Love number in late inspiral up to ~10-20%.
- Even a smaller effect would significantly affect the validity of I-Love-Q
- There is a potential ambiguity in the definition of Love numbers. It mainly affects rotating NSs; the non-rotating case should be ok, but we'll be 100% sure only with explicit computation of the 5PN tidal contribution to the waveform