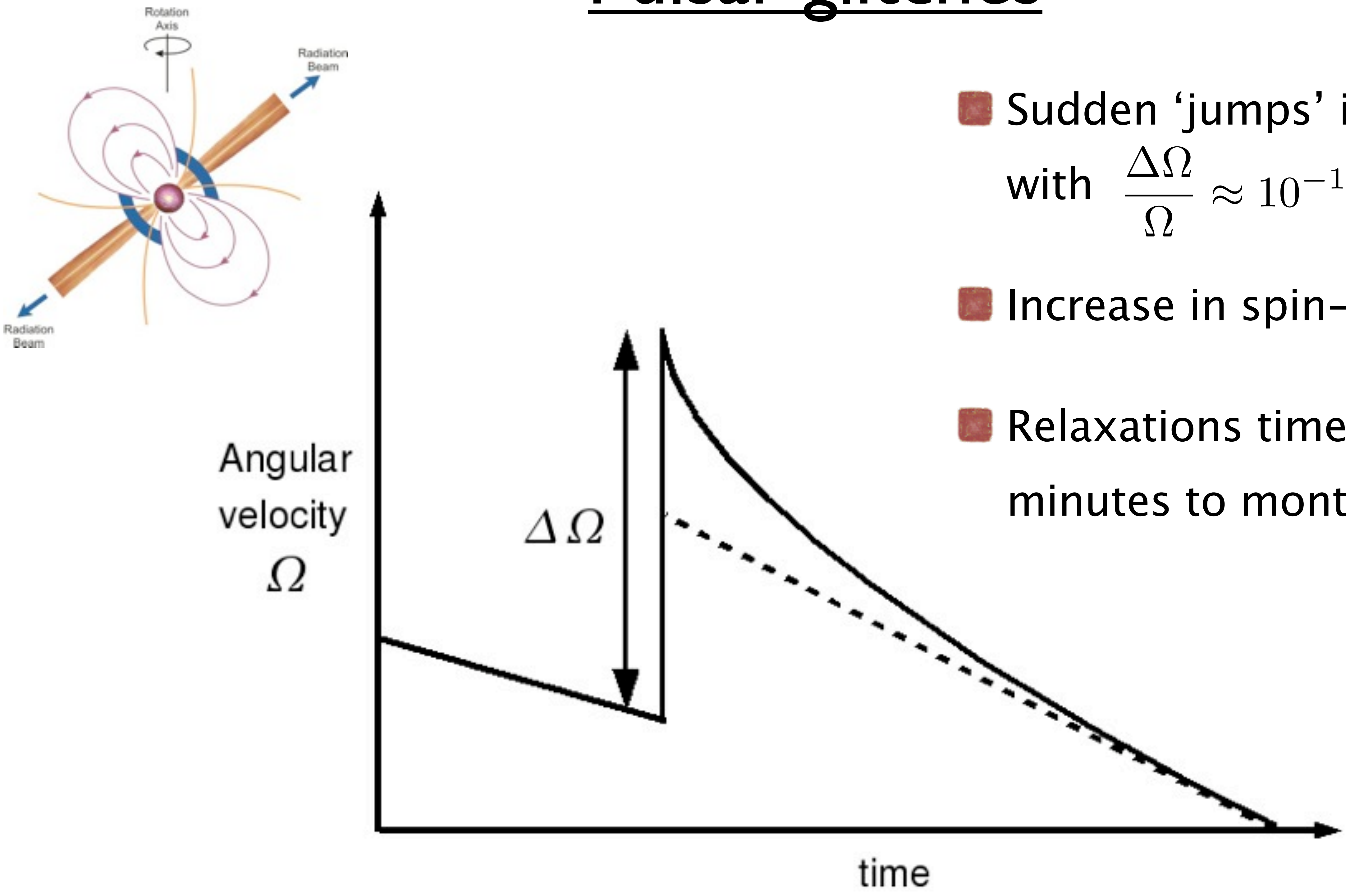
Vortex motion in NS crusts and Pulsar Glitches

Brynmor Haskell

compstar

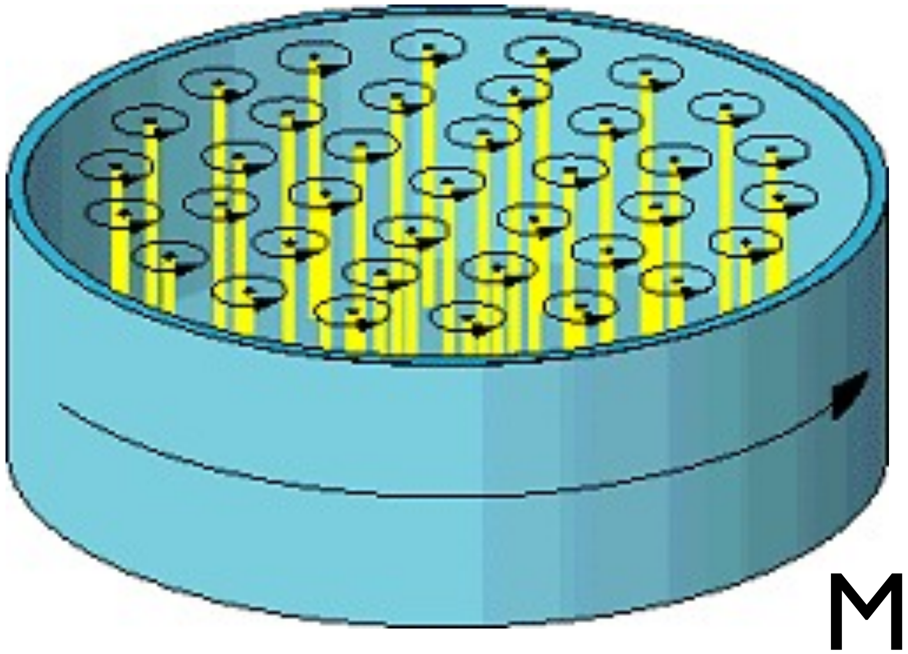
THE UNIVERSITY OF MELBOURNE

Pulsar glitches



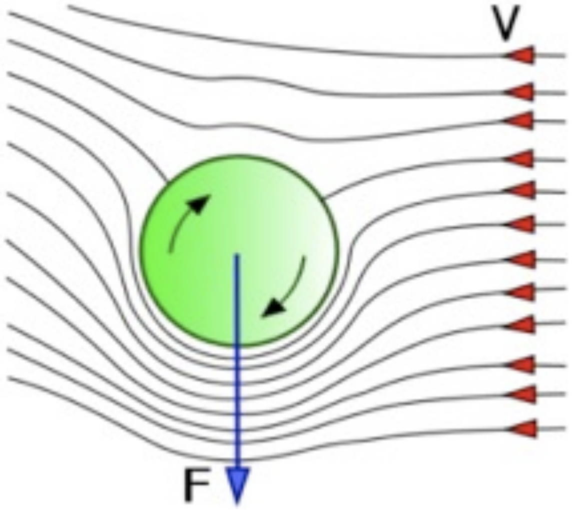
- Sudden ‘jumps’ in frequency, with $\frac{\Delta\Omega}{\Omega} \approx 10^{-10} - 10^{-5}$
- Increase in spin-down rate
- Relaxations timescales from minutes to months

Vortex dynamics



- Superfluids rotate by forming quantised vortices
- Vortex density determines spin :

  vortices must move out to spin down the fluid!

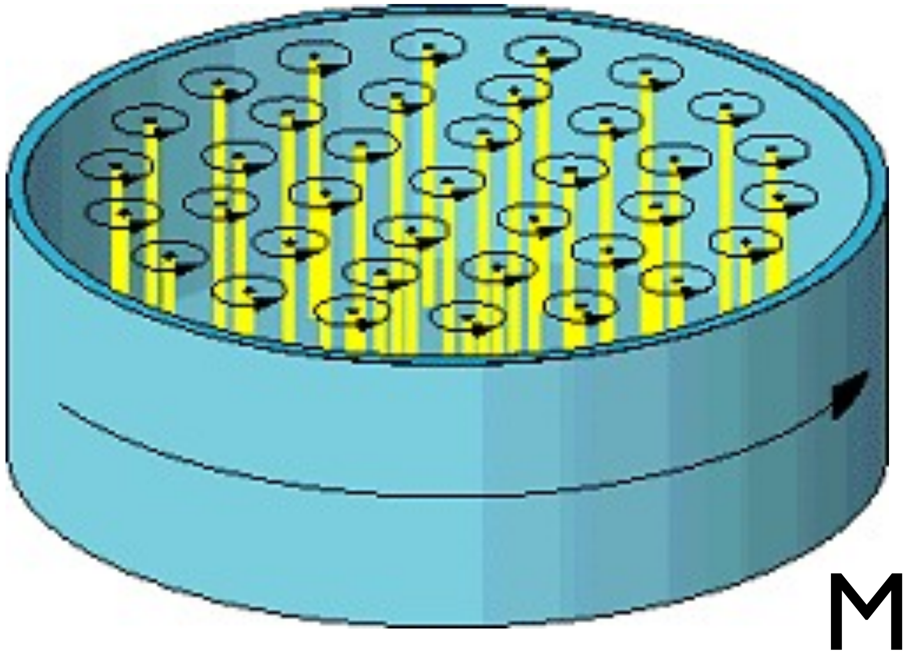


Magnus Force

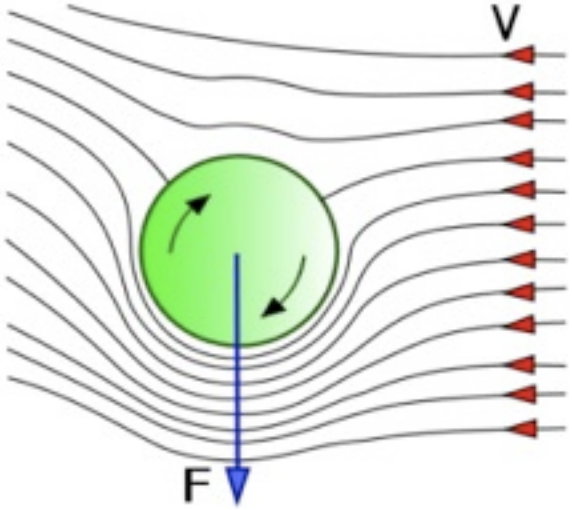
$$\text{FREE :}\quad \boxed{\epsilon^{ijk}\hat{k}_j(v_k^{\mathrm{v}} - v_k^{\mathrm{n}})} + \mathcal{R}(v_{\mathrm{c}}^i - v_{\mathrm{v}}^i) = 0$$

$$\text{PINNED :}\quad \epsilon^{ijk}\hat{k}_j(v_k^{\mathrm{v}} - v_k^{\mathrm{n}}) + F_{\mathrm{p}}^i = 0$$

Vortex dynamics



If vortices cannot move out the superfluid cannot spin down.



Magnus Force

$$\text{FREE}: \quad \boxed{\epsilon^{ijk}\hat{k}_j(v^{\rm v}_k - v^{\rm n}_k)} + \mathcal{R}(v^i_{\rm c} - v^i_{\rm v}) = 0$$

$$\text{PINNED}: \quad \epsilon^{ijk}\hat{k}_j(v^{\rm v}_k - v^{\rm n}_k) + F^i_{\rm p} = 0$$

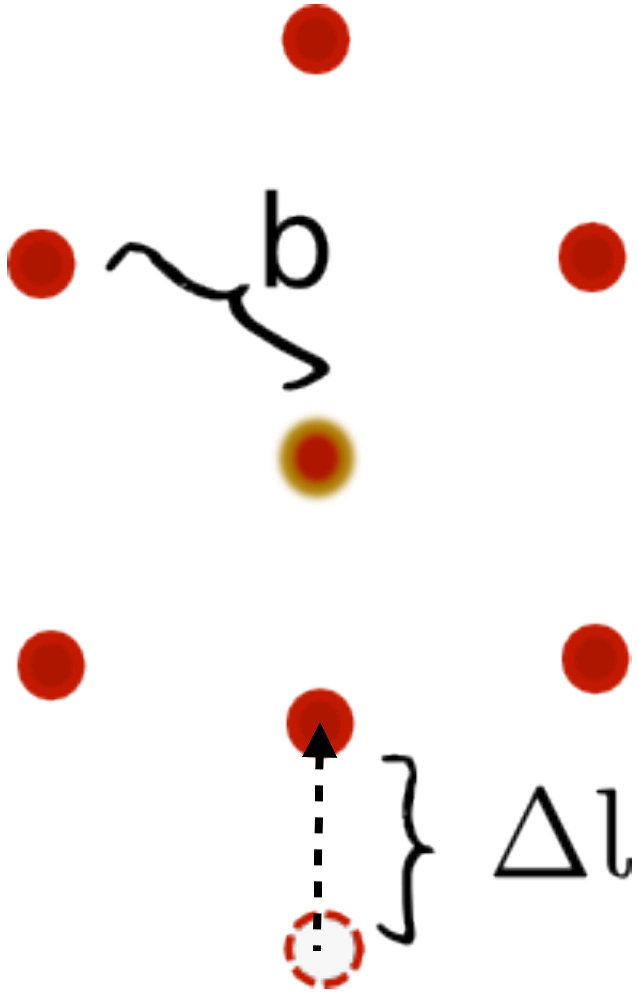
What triggers a glitch?

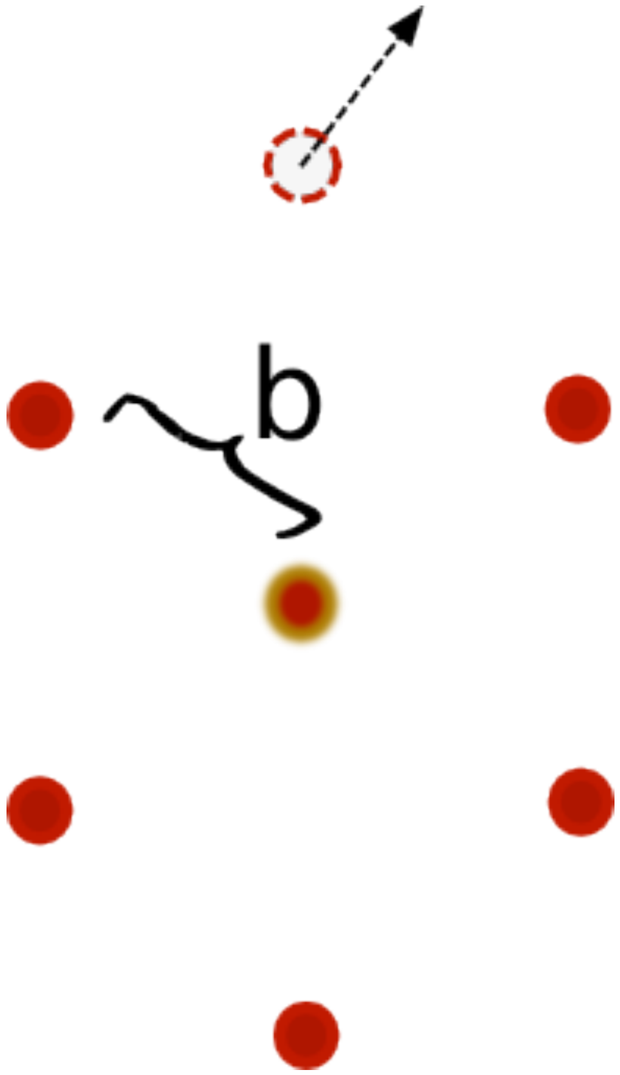
- Starquakes [Ruderman 69, 76]
- Hydrodynamical instabilities

  [Andersson et al. 2003, Glampedakis & Andersson 2009]
- Vortex avalanches [Cheng et al. 88, Alpar et al. 96]

See Haskell & Melatos 2015 for a review

vortex-vortex interactions



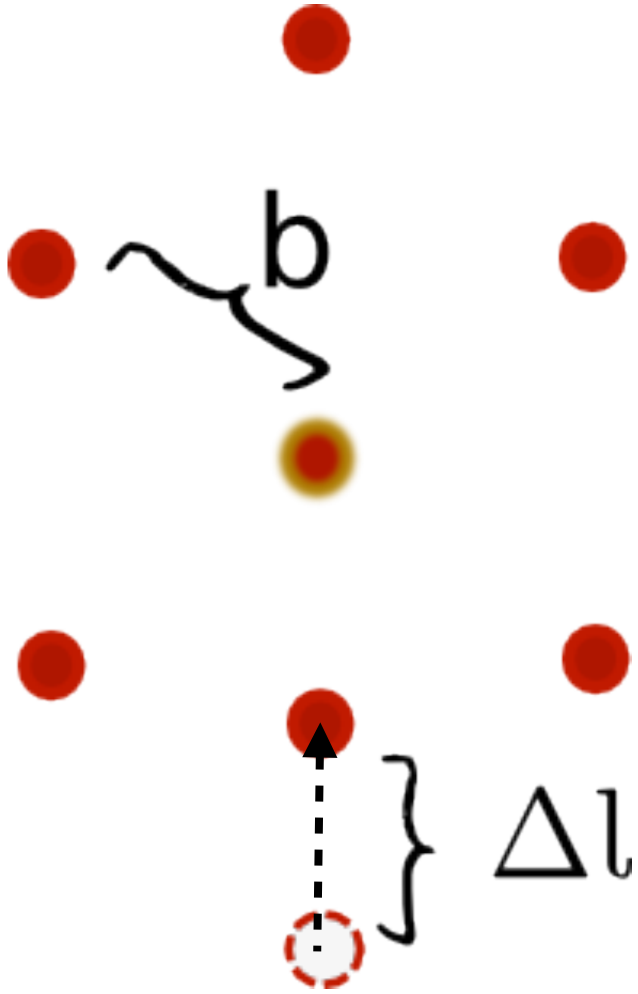


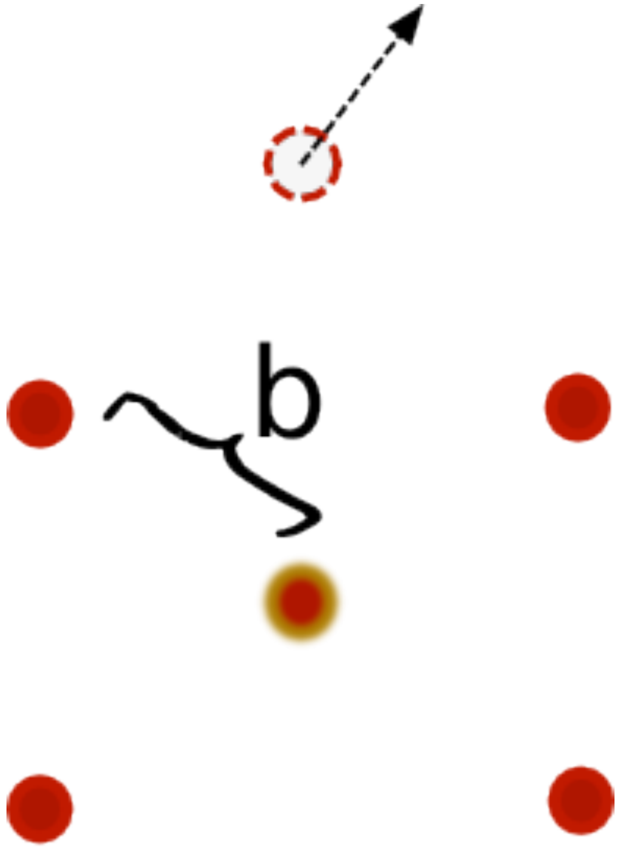
$$|\Delta v| \approx 0.1 \left(\frac{\nu}{10\mathrm{hz}}\right)^{-1/2} \left(\frac{\Delta l}{a}\right) \mathrm{cm/s}$$

$$V_{cr} \approx 10^2 - 10^4 \mathrm{cm/s}$$

[Seveso, Pizzochero, Grill,BH 2015]

vortex-vortex interactions





Avalanche if

$$\Delta l \approx b$$

$$(V_{cr} - V)/(V_{cr}) \approx 0.001 - 0.1$$

$$|\Delta v| \approx 0.1 \left(\frac{\nu}{10\mathrm{hz}}\right)^{-1/2} \left(\frac{\Delta l}{a}\right) \mathrm{cm/s}$$

$$V_{cr} \approx 10^2 - 10^4 \mathrm{cm/s}$$

[Seveso, Pizzochero, Grill,BH 2015]

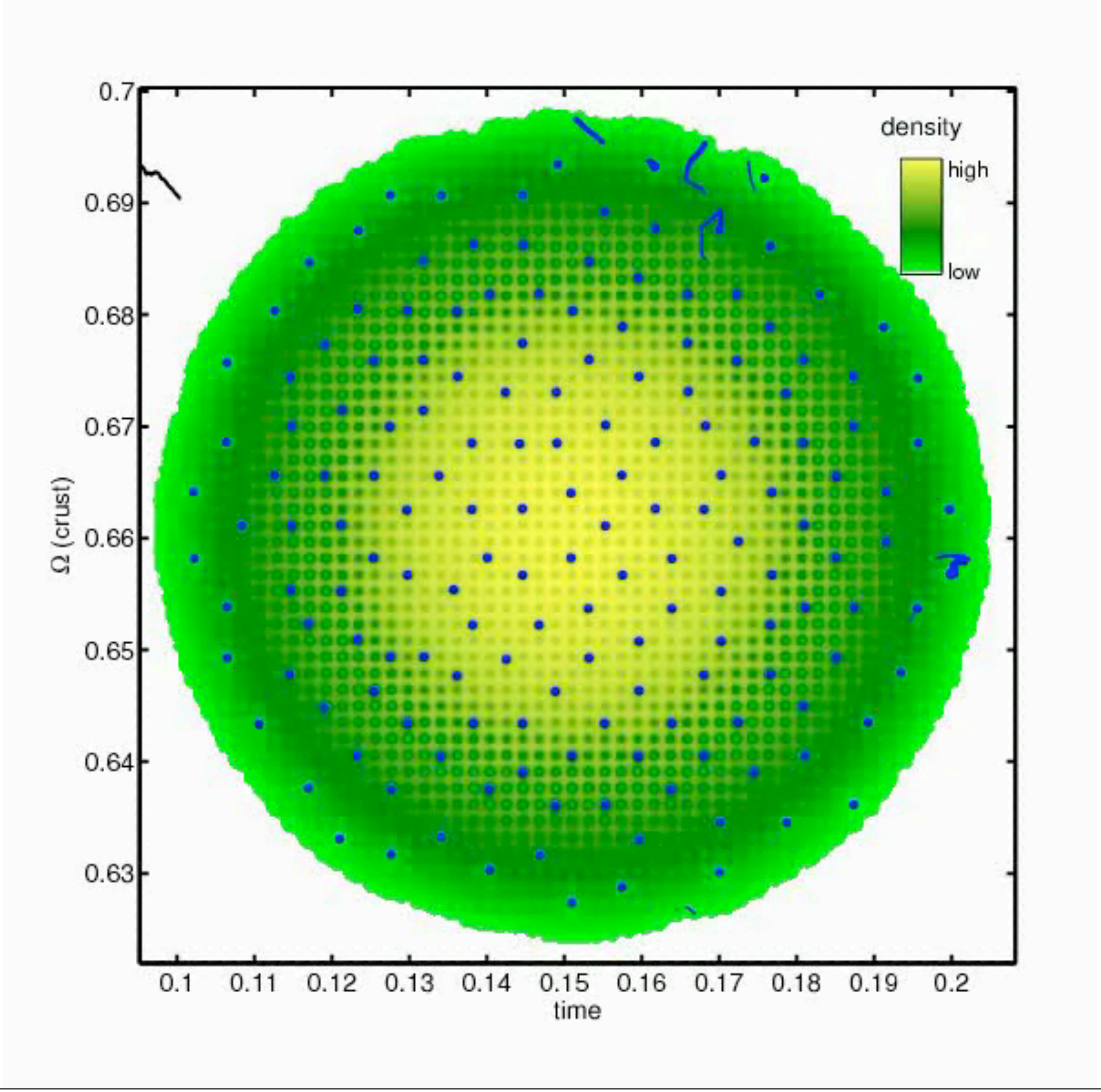
Gross Pitaevskii simulations:

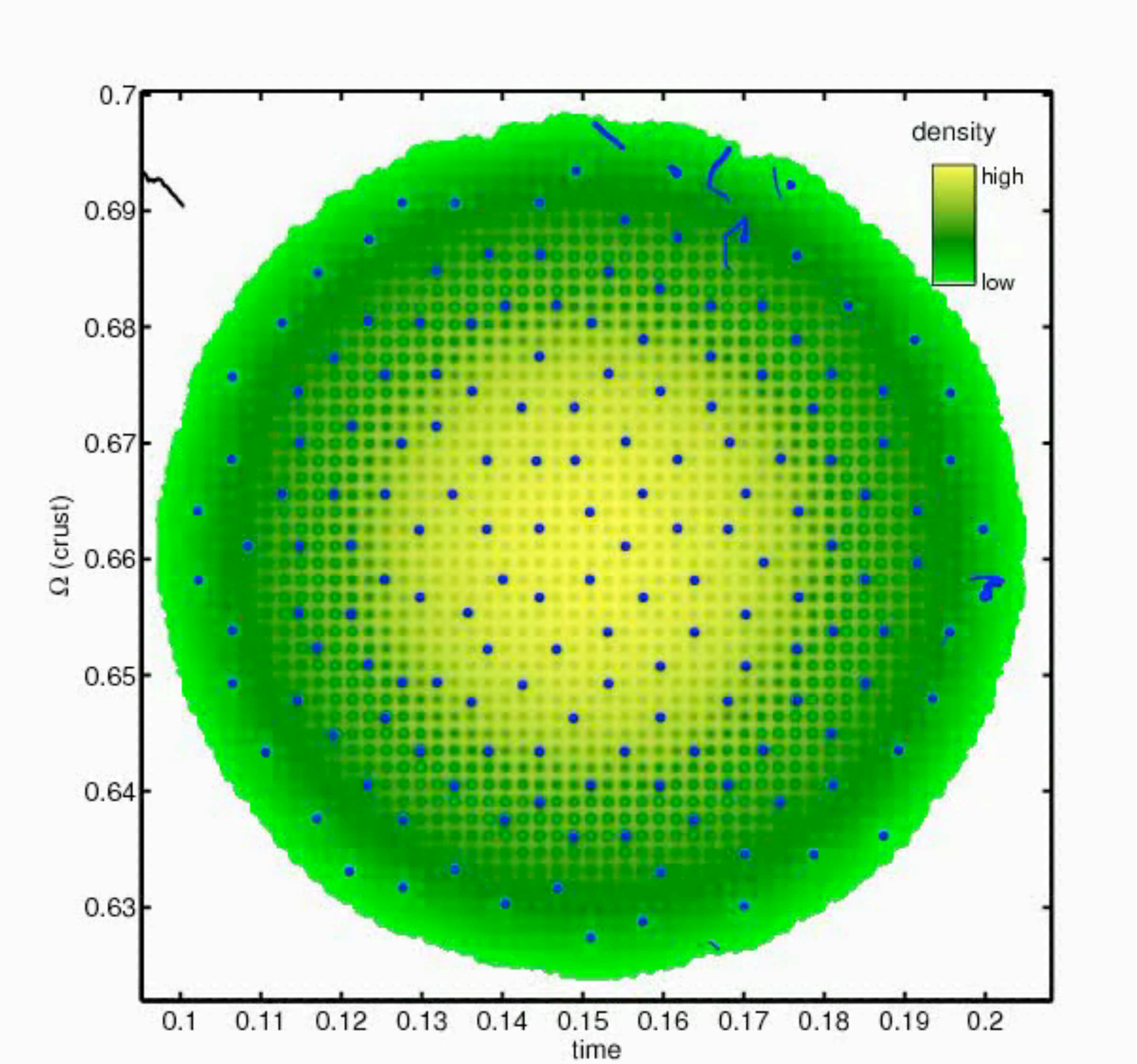
$$(i-\gamma)\bar{h}\frac{\partial\psi}{\partial t} = -\frac{\bar{h}^2}{2m}\nabla^2\psi - (\mu - V - g|\psi|^2)\psi - \Omega\hat{L}_z\psi,$$

$$I_c\frac{d\Omega}{dt} = -\frac{d<\hat{L}_z>}{dt} + N_{EM}, \qquad V = V_{\rm trap} + \sum V_i[1+\tanh(\Theta(r-R_i)]$$

- Good description of BEC dynamics in which interactions are weak
- Can it describe vortex dynamics in NS, in which the interactions are strong and long-range?

[Warszawski & Melatos, 2008]

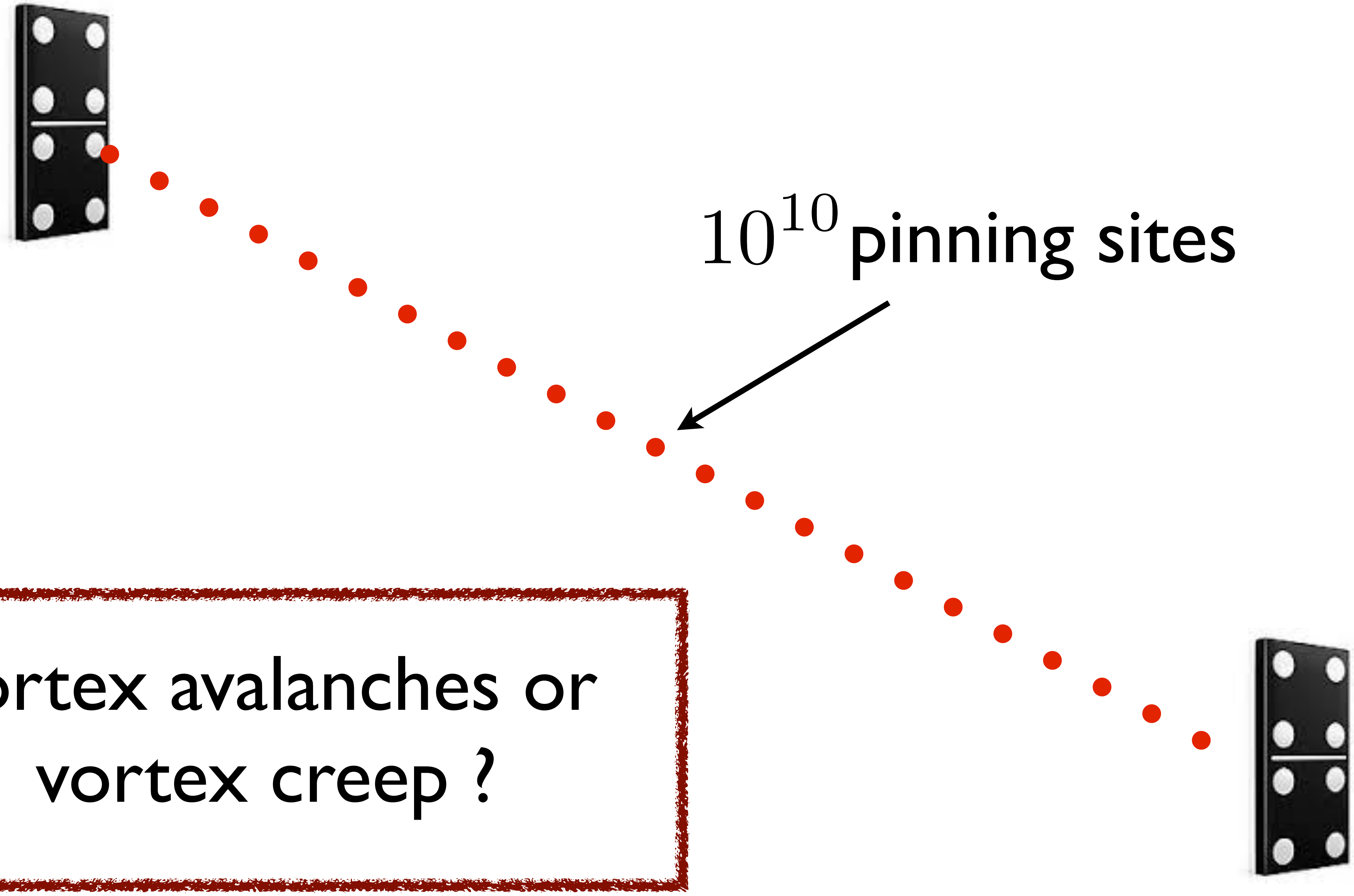

density
high
low
0.7
0.69
0.68
0.67
0.66
0.65
0.64
0.63
Ω (crust)
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.2
time



(Courtesy of James Douglass)

Can an avalanche propagate in a NS?

Can an avalanche propagate in a NS?



10^{10} pinning sites

vortex avalanches or
vortex creep ?

Vortex Motion:

Magnus

Pinning

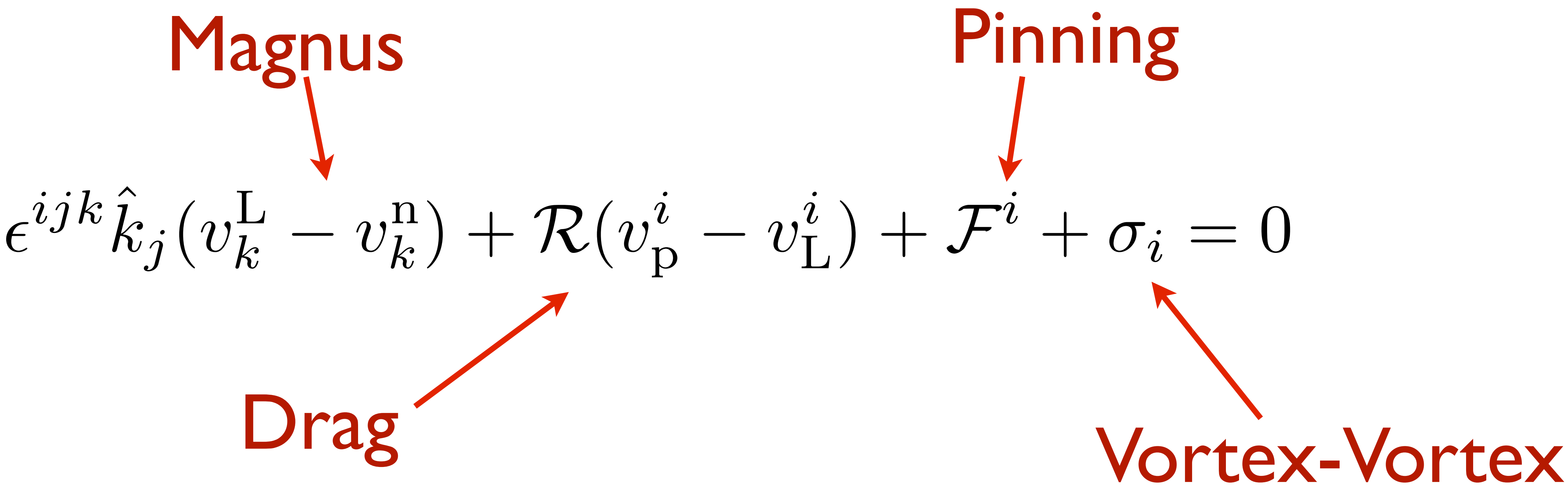
$$\epsilon^{ijk}\hat{k}_j(v^{\mathrm{L}}_k - v^{\mathrm{n}}_k) + \mathcal{R}(v^i_{\mathrm{p}} - v^i_{\mathrm{L}}) + \mathcal{F}^i + \sigma_i = 0$$

Drag

Vortex-Vortex

$$\mathcal{R} \approx 10^{-4}$$

Vortex Motion:



Magnus

Pinning

$$\epsilon^{ijk}\hat{k}_j(v_k^{\rm L} - v_k^{\rm n}) + \mathcal{R}(v_{\rm p}^i - v_{\rm L}^i) + \mathcal{F}^i + \sigma_i = 0$$

Drag

Vortex-Vortex

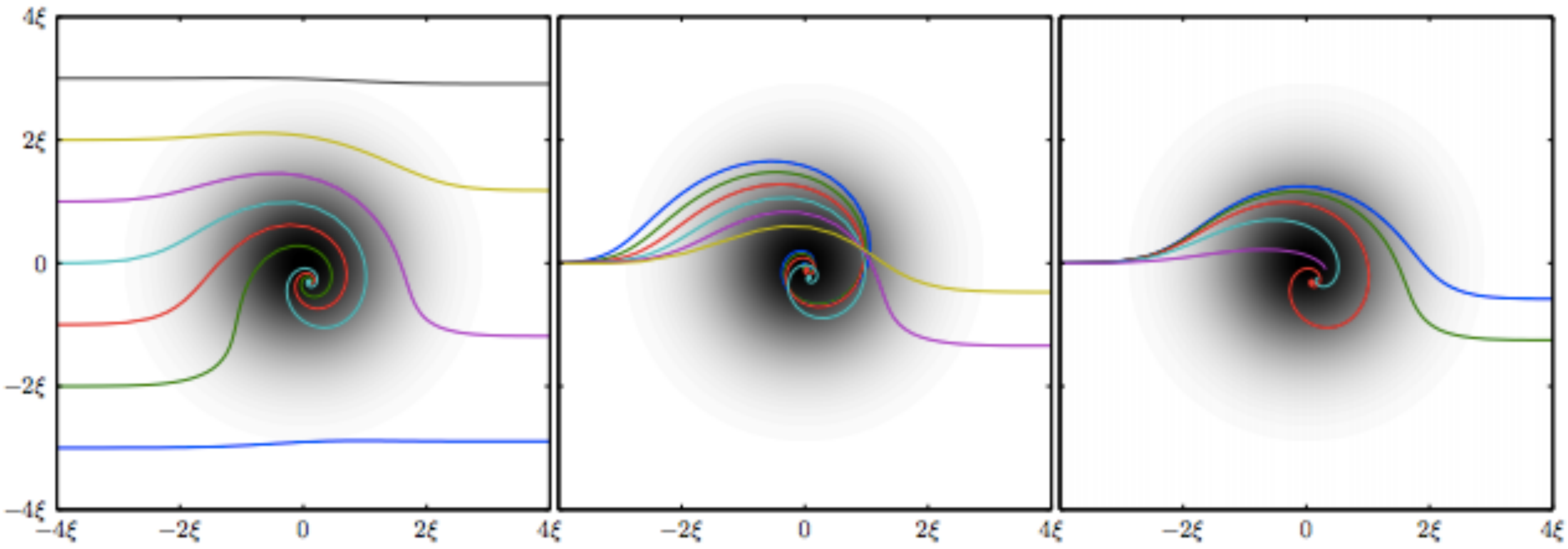
$$\mathcal{F}^i = -\nabla^i V$$

$\mathcal{R} << 1$ Phonons in the crust

$$\mathcal{R} \approx 10^{-4}$$

in the core

$\mathcal{R} \approx 1$ Kelvons in the crust

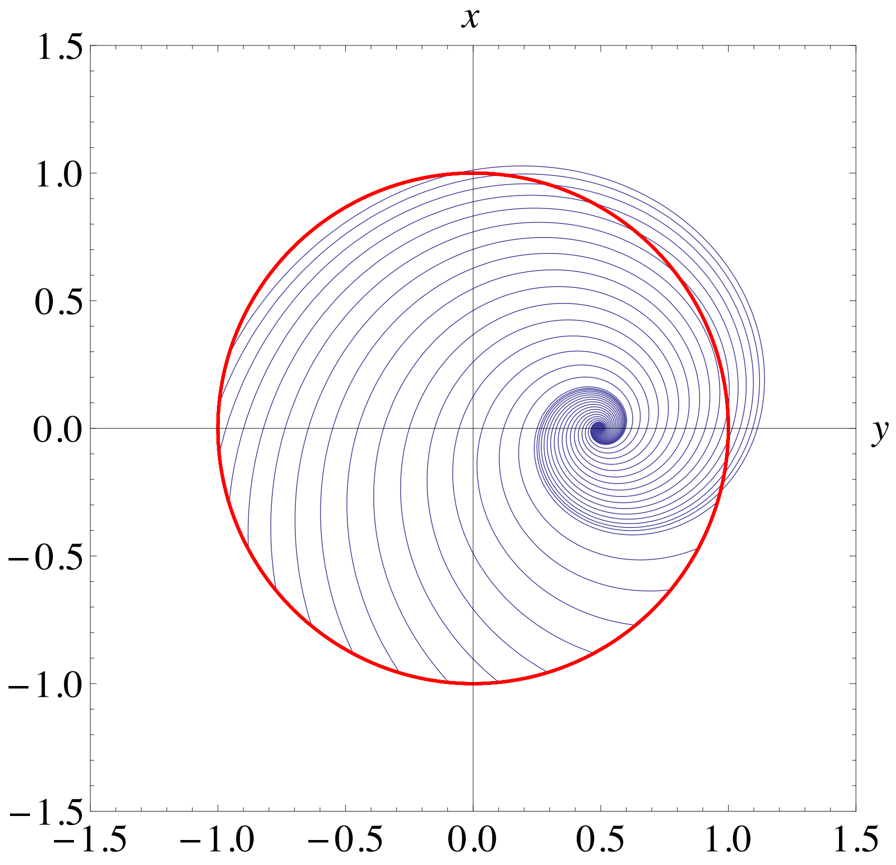


- N-body (Barnes-Hut) code [Melatos, Douglass & BH, in preparation]

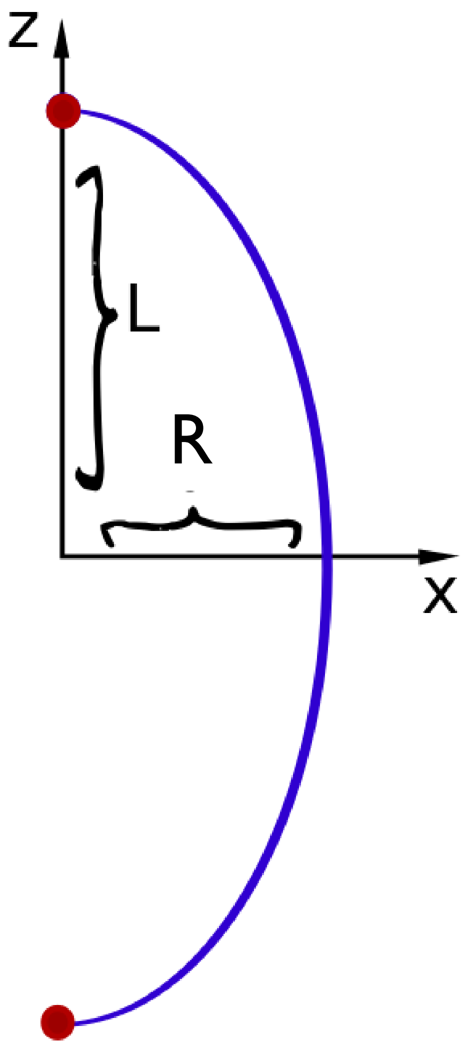
$$V(r) = -E_p \exp\left(-\frac{|r-r_p|^2}{2\xi^2}\right)$$

- Analytic cross section [Sedrakian 95, BH & Melatos, in preparation]

$$V(r) = \frac{E_p}{2}\left(r-r_p\right)^2 \quad \text{for} \quad |r-r_p| < R_r$$



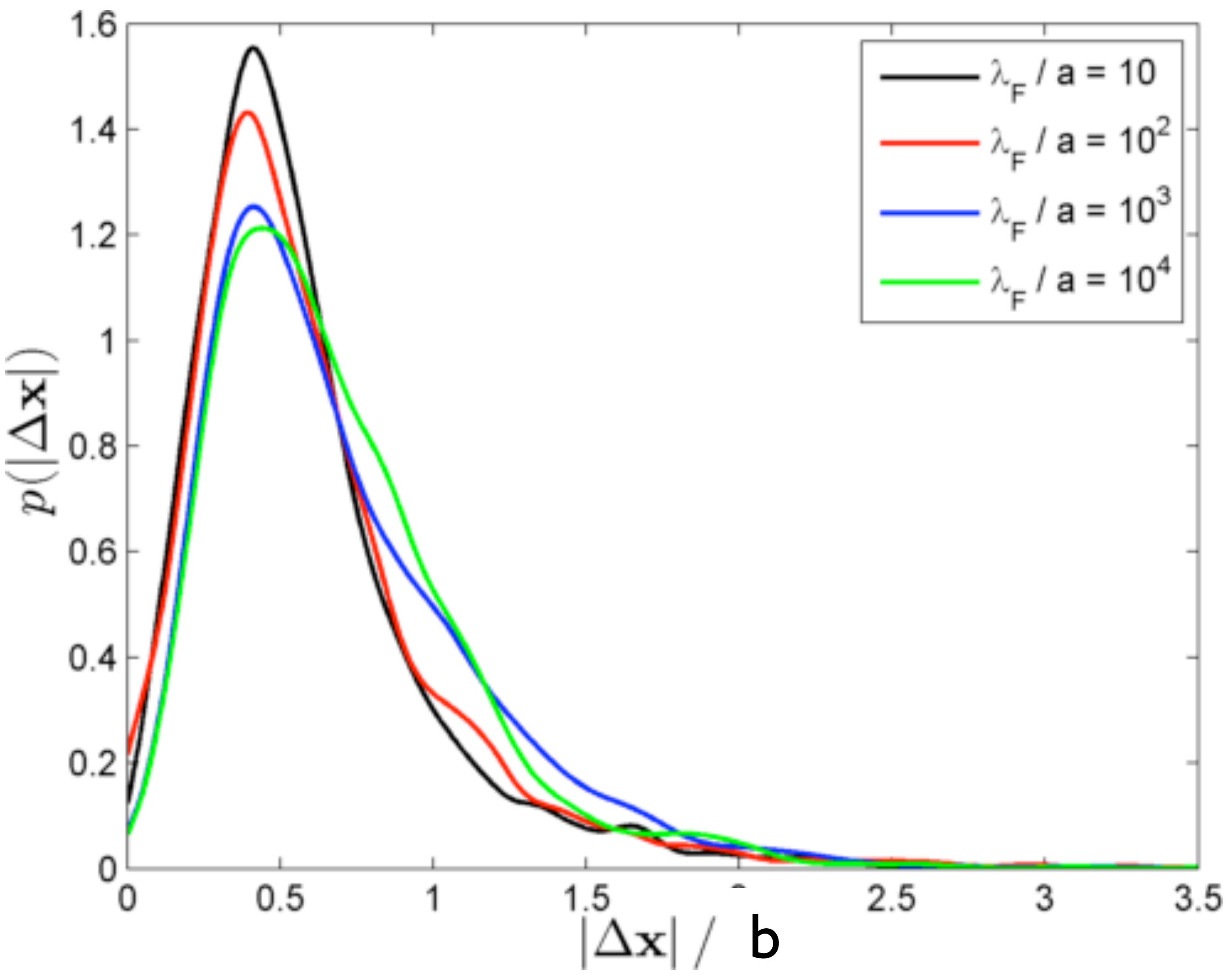
Tension:

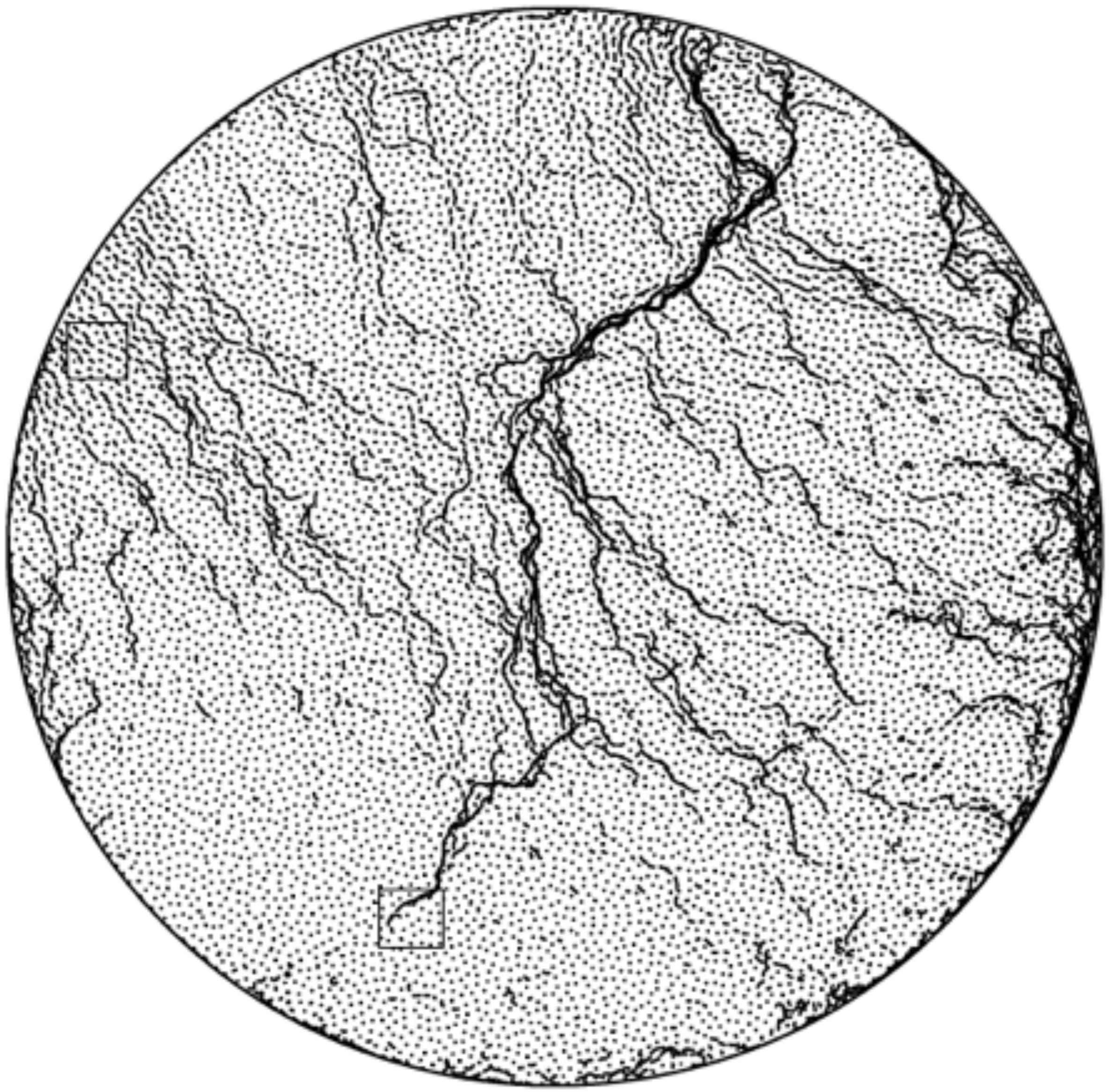


$$L \approx \frac{T R_{\mathrm{WS}}^2}{|E_p|} \approx 1000 \quad R_{\mathrm{WS}}$$

- Tension decreases the pinning force for large L [Seveso, Pizzochero, Grill, BH 2015]
- Oscillations can be excited leading to damping [Link 2009]

N-body results

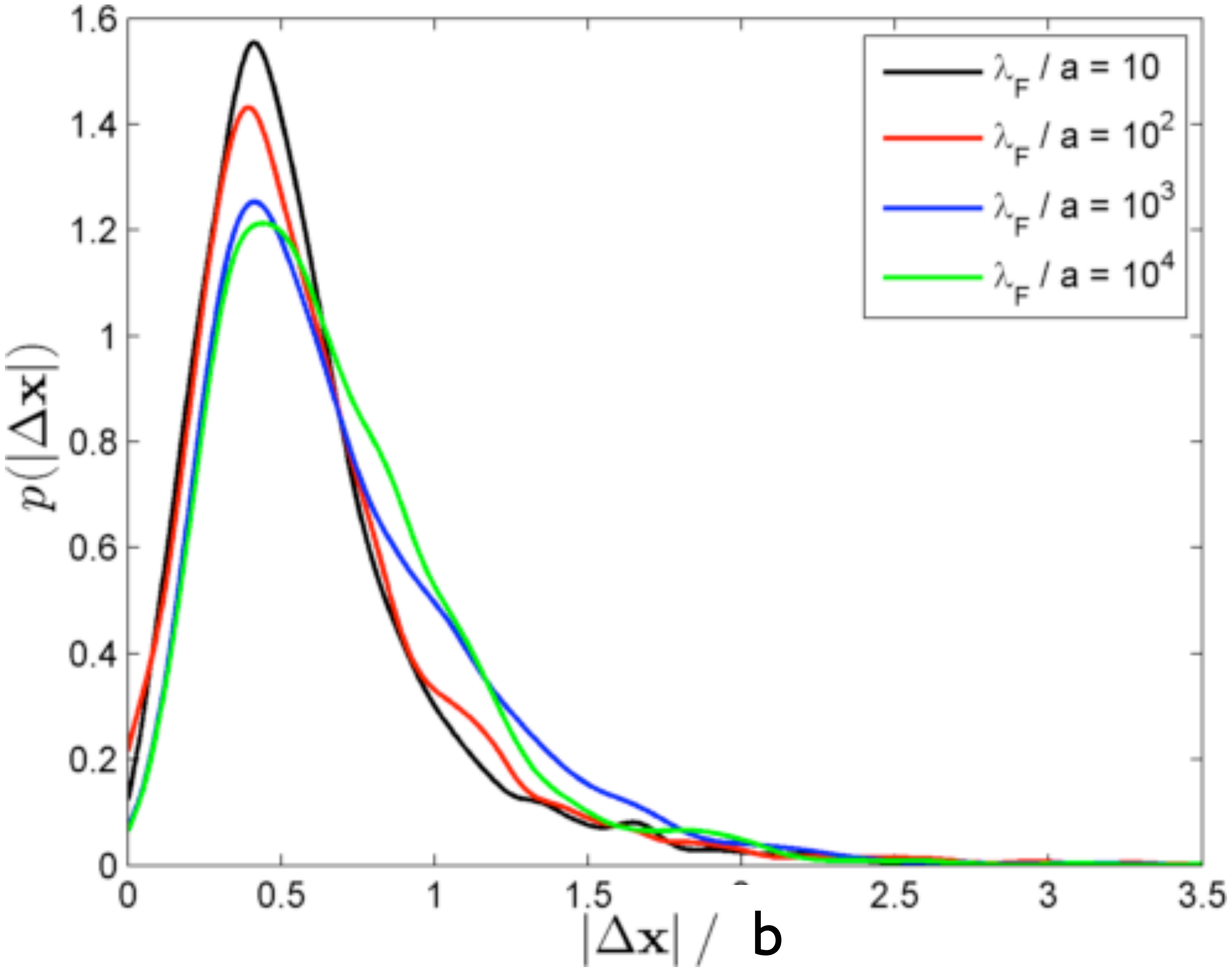




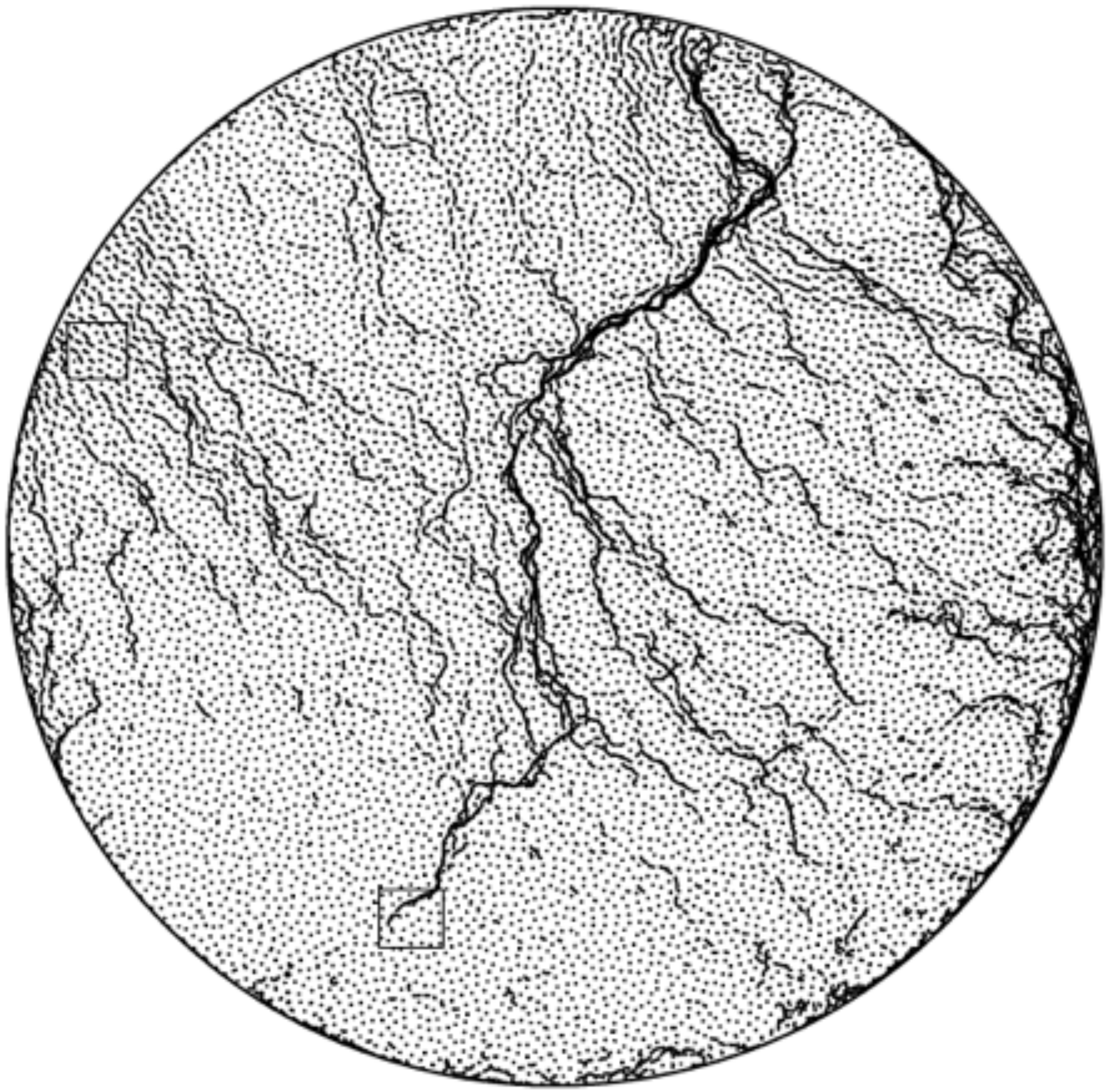
10^4 vortices

$$\mathcal{R} \approx 0.3$$

N-body results



$$\mathcal{R} \approx 0.3$$



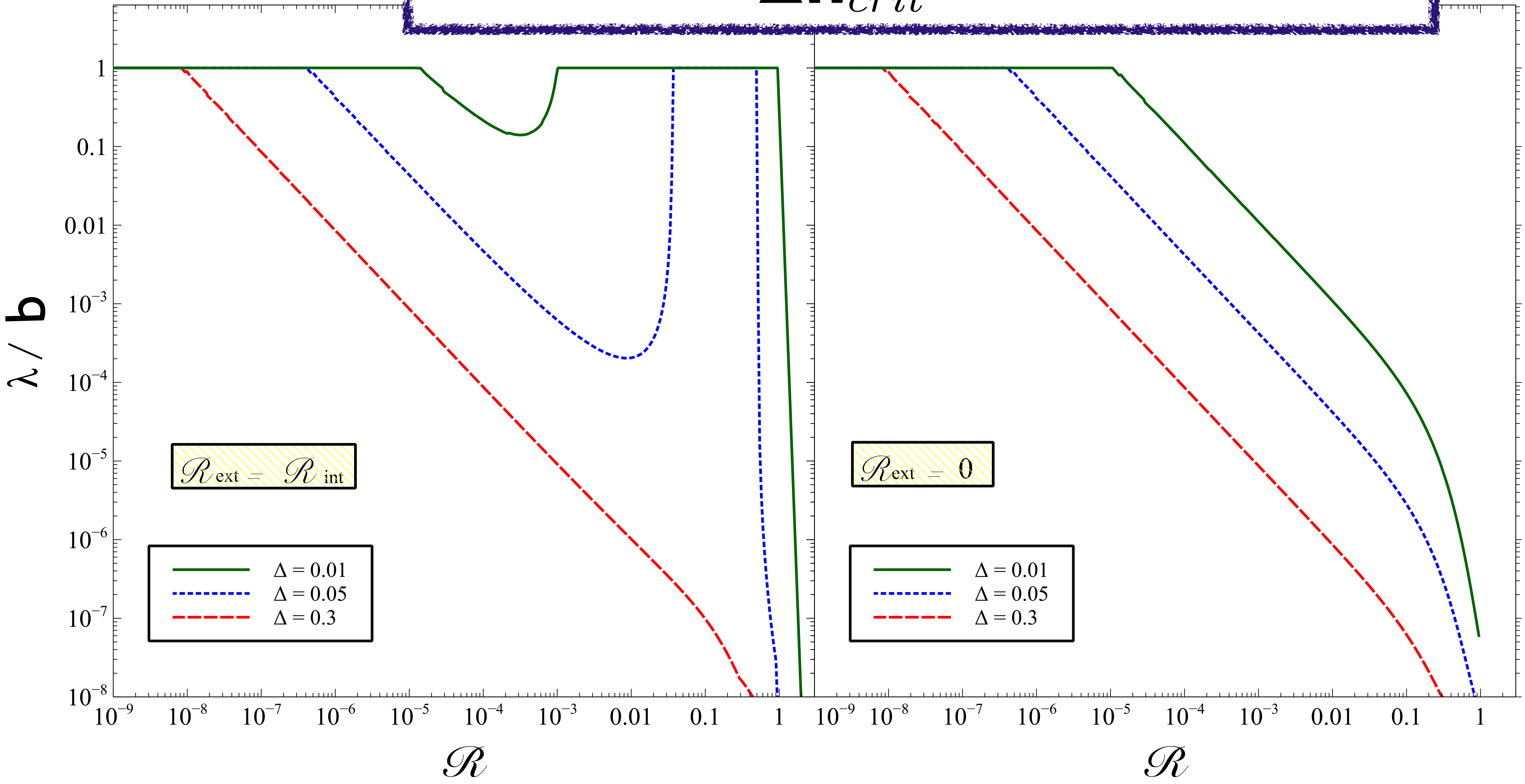
10^4 vortices

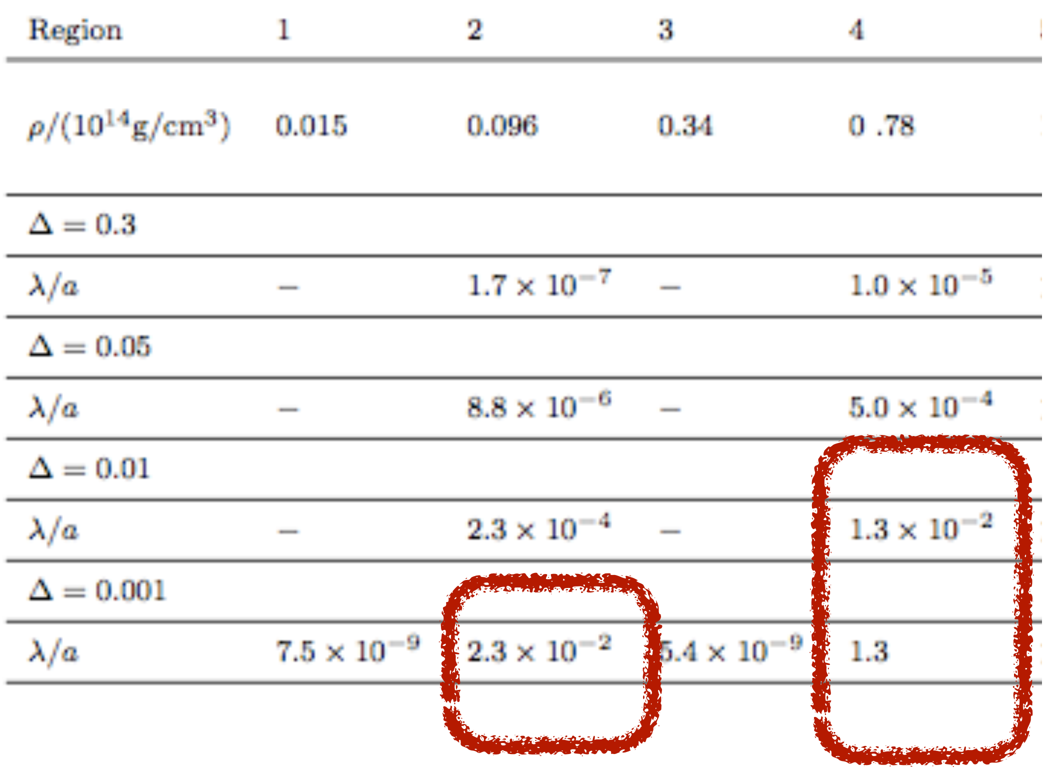
(Melatos, Douglass & BH in preparation)

Analytic Results:

[BH & Melatos, in preparation]

$$\Delta = \frac{\Delta\Omega_{crit} - \Delta\Omega}{\Delta\Omega_{crit}} \approx 0.001 - 0.1$$



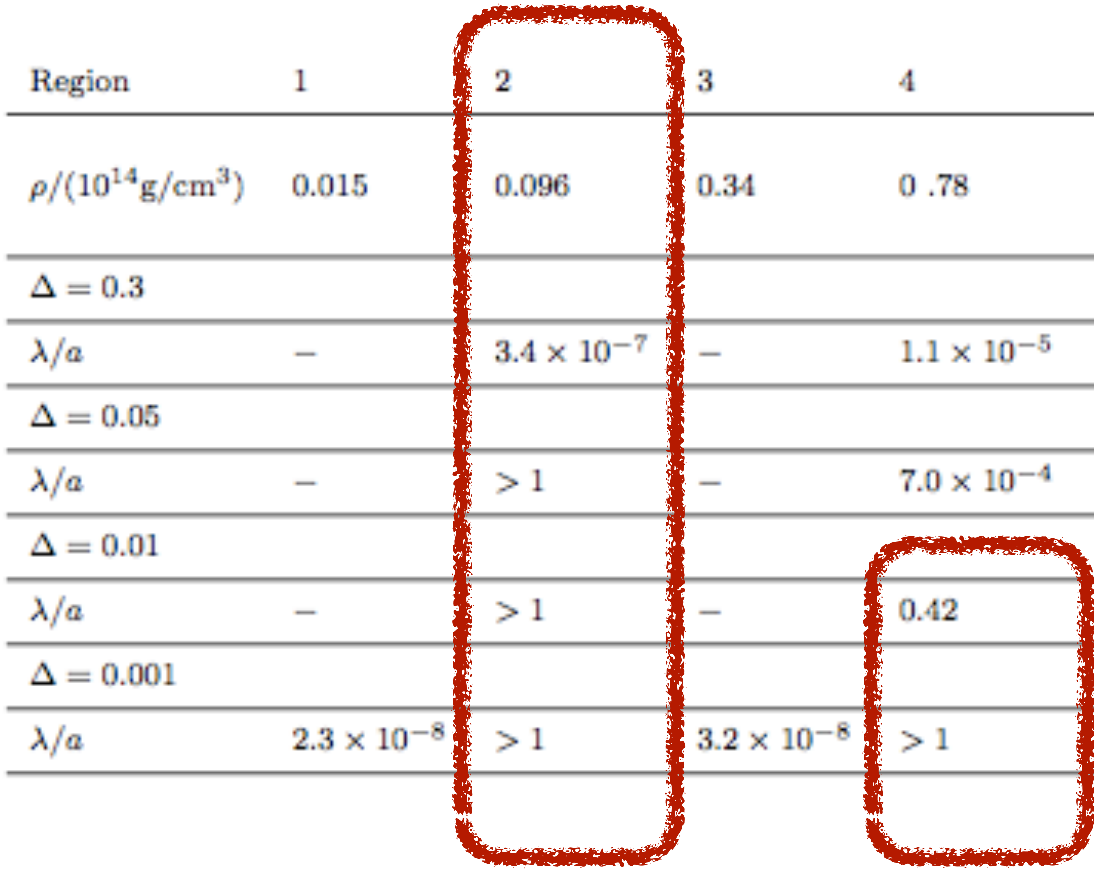


Region	1	2	3	4
$\rho/(10^{14}\mathrm{g/cm^3})$	0.015	0.096	0.34	0 .78
$\Delta = 0.3$				
λ/a	–	1.7×10^{-7}	–	1.0×10^{-5}
$\Delta = 0.05$				
λ/a	–	8.8×10^{-6}	–	5.0×10^{-4}
$\Delta = 0.01$				
λ/a	–	2.3×10^{-4}	–	1.3×10^{-2}
$\Delta = 0.001$				
λ/a	7.5×10^{-9}	2.3×10^{-2}	5.4×10^{-9}	1.3

$$\mathcal{R}_{ext} = 0$$

[BH & Melatos, in preparation]

$$\mathcal{R}_{ext} = \mathcal{R}_{in}$$



Region	1	2	3	4
$\rho/(10^{14}\mathrm{g/cm^3})$	0.015	0.096	0.34	0 .78
$\Delta = 0.3$				
λ/a	–	3.4×10^{-7}	–	1.1×10^{-5}
$\Delta = 0.05$				
λ/a	–	> 1	–	7.0×10^{-4}
$\Delta = 0.01$				
λ/a	–	> 1	–	0.42
$\Delta = 0.001$				
λ/a	2.3×10^{-8}	> 1	3.2×10^{-8}	> 1

Conclusions - Part I

- Vortex avalanches **can** propagate in NS interiors

(need better constraints on superfluid drag and the role of tension)

Equations of motion

$$\begin{aligned}\dot{\Omega}_{\rm n}(\tilde{r}) &= \kappa n_v \frac{\mathcal{B}(\Omega_{\rm p}-\Omega_{\rm n})}{(1-\varepsilon_{\rm n}-\varepsilon_{\rm p})} - f(\varepsilon_{\rm p})\mathcal{A}\Omega_{\rm p}^3 \\ \dot{\Omega}_{\rm p}(\tilde{r}) &= -\kappa n_v \frac{\rho_{\rm n}}{\rho_{\rm p}} \frac{\mathcal{B}(\Omega_{\rm p}-\Omega_{\rm n})}{(1-\varepsilon_{\rm n}-\varepsilon_{\rm p})} - \mathcal{A}\Omega_{\rm p}^3\end{aligned}$$

$$\kappa n_v = f(T, \Omega_p - \Omega_n)$$

$$\mathcal{B} = \frac{\mathcal{R}}{1+\mathcal{R}^2}$$

Equations of motion

$$\begin{aligned}
\dot{\Omega}_{\mathrm{n}}(\tilde{r}) &= \kappa n_v \frac{\mathcal{B}(\Omega_{\mathrm{p}}-\Omega_{\mathrm{n}})}{(1-\varepsilon_{\mathrm{n}}-\varepsilon_{\mathrm{p}})} - f(\varepsilon_{\mathrm{p}})\mathcal{A}\Omega_{\mathrm{p}}^3 \\
\dot{\Omega}_{\mathrm{p}}(\tilde{r}) &= -\kappa n_v \frac{\rho_{\mathrm{n}}}{\rho_{\mathrm{p}}} \frac{\mathcal{B}(\Omega_{\mathrm{p}}-\Omega_{\mathrm{n}})}{(1-\varepsilon_{\mathrm{n}}-\varepsilon_{\mathrm{p}})} - \mathcal{A}\Omega_{\mathrm{p}}^3
\end{aligned}$$

$$\kappa n_v = f(T, \Omega_p - \Omega_n)$$

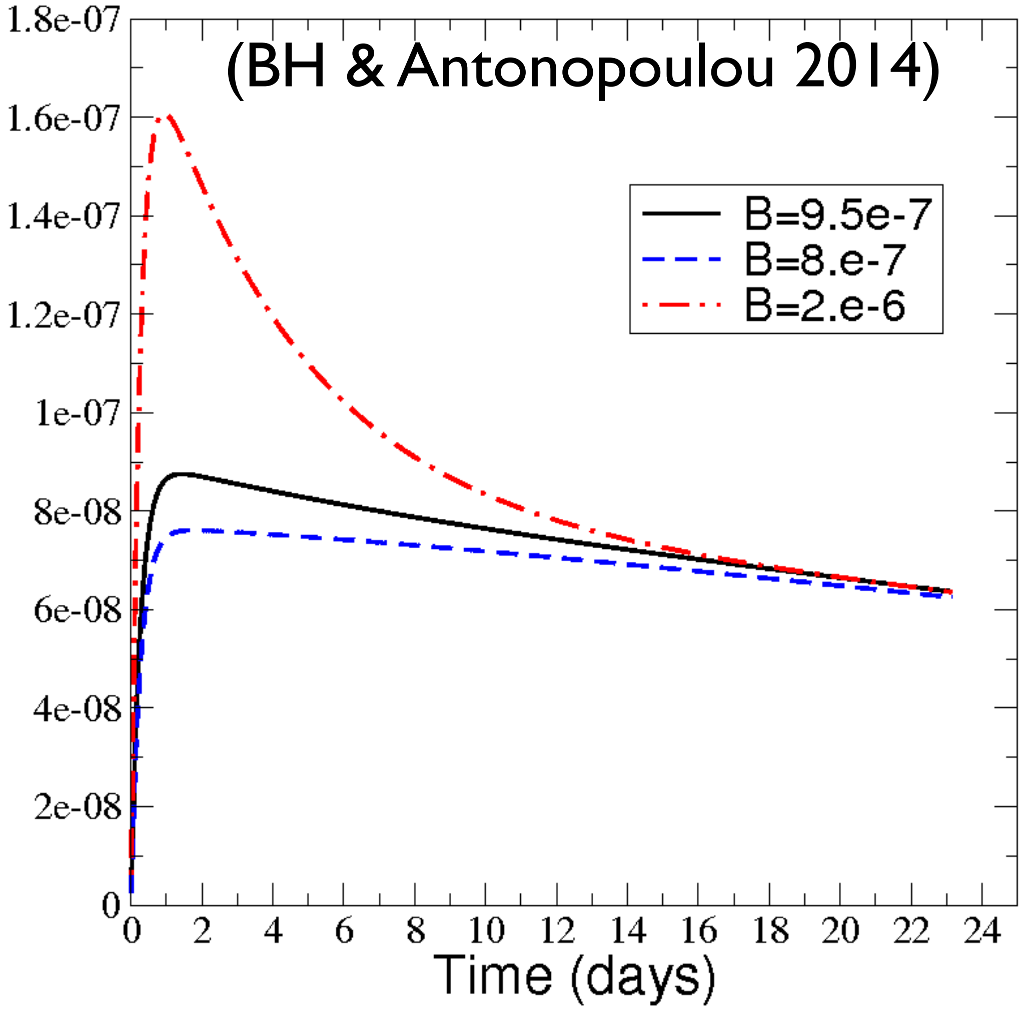
$$\mathcal{B} = \frac{\mathcal{R}}{1+\mathcal{R}^2}$$

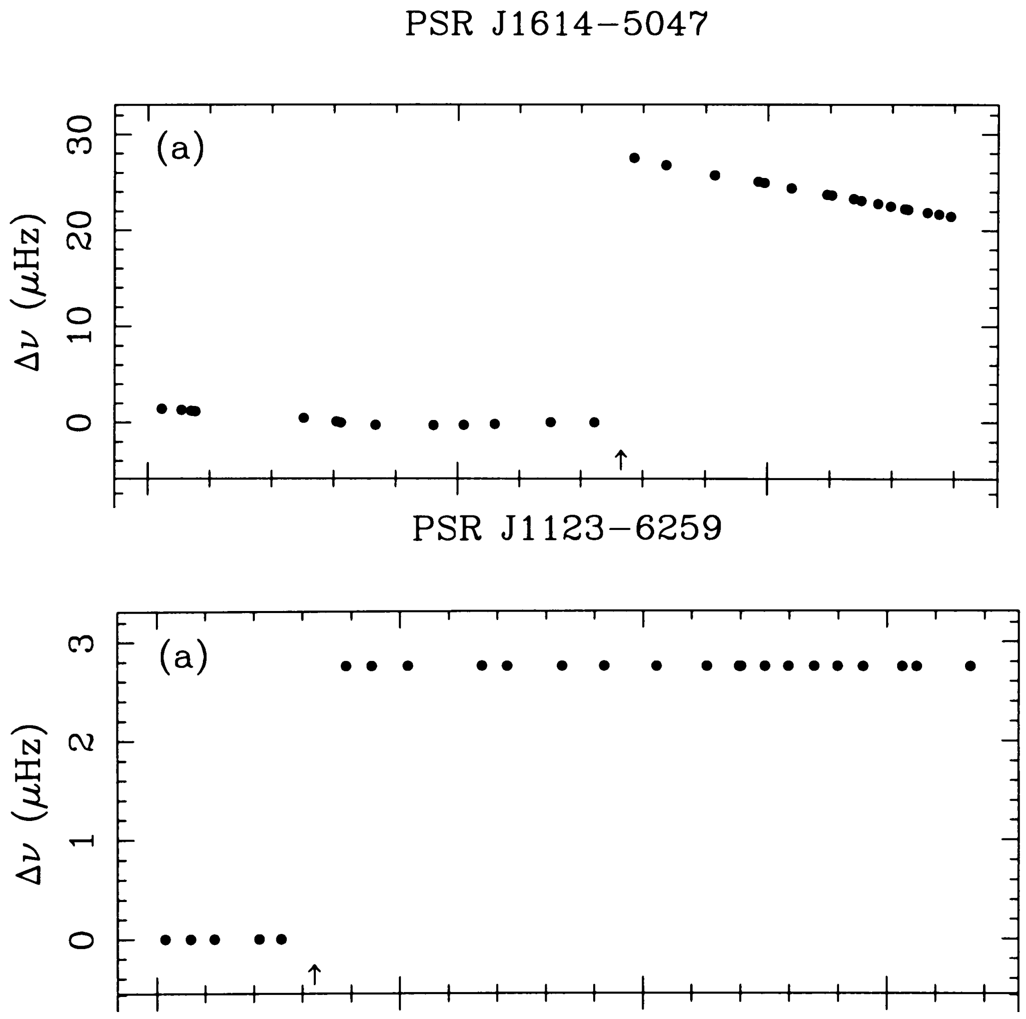
Important in the crust!

(Chamel 2012, Andersson et al. 2012)

- Bragg scattering reduces neutron mobility (Chamel et al. 2012)
- The crust is not enough (Andersson et al. 2012, Chamel 2013)

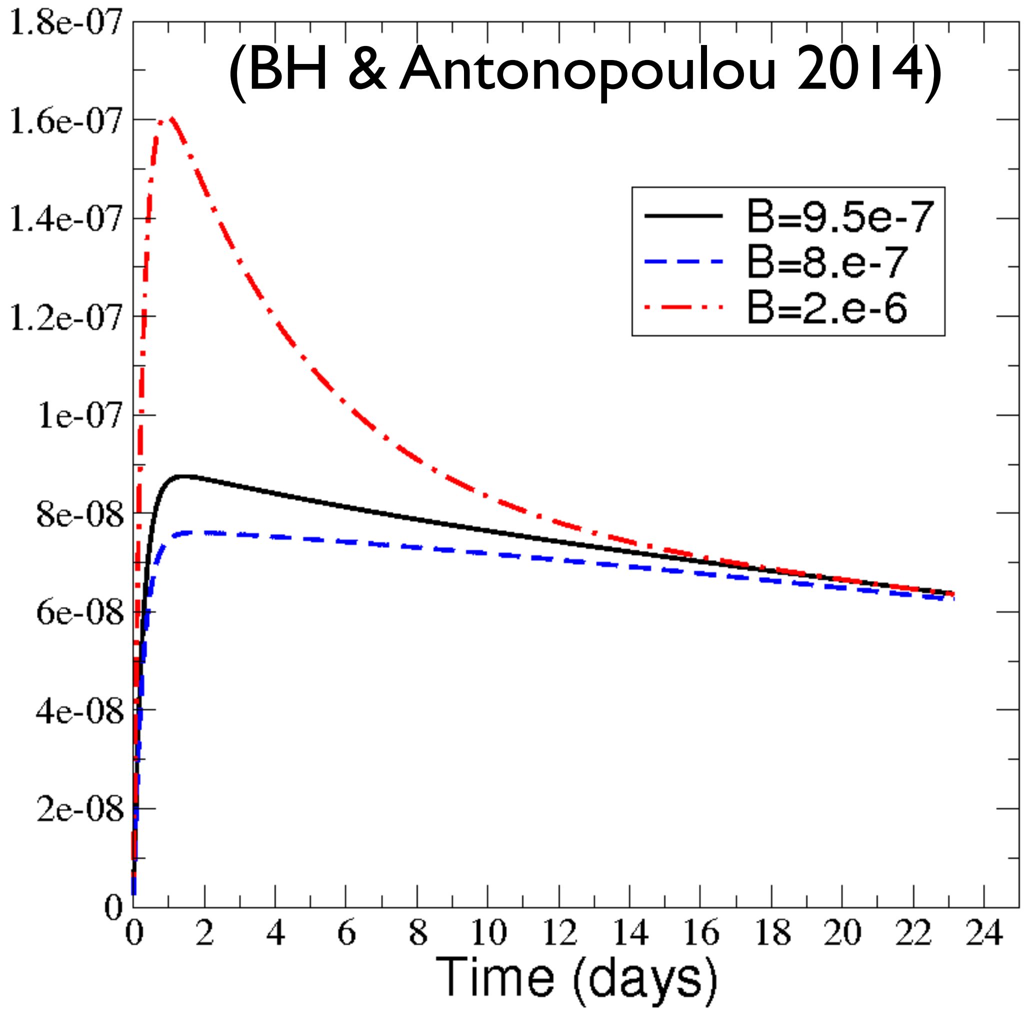
- Response of the star can lead to different relaxation behaviours



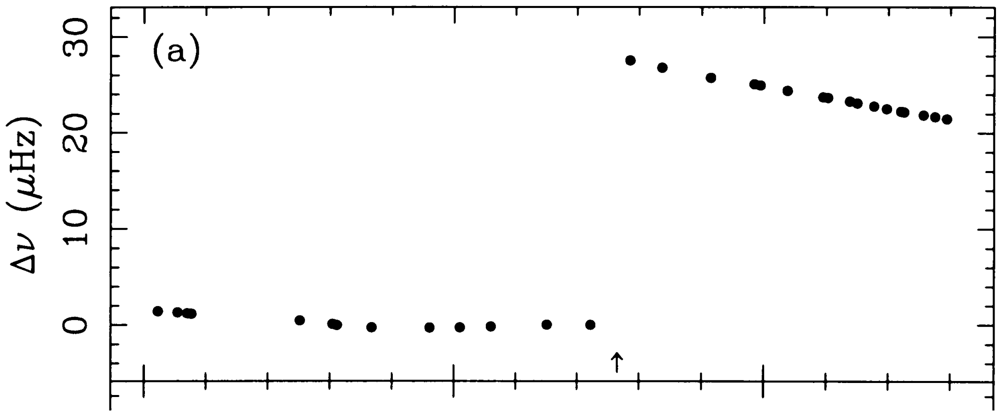


- Bragg scattering reduces neutron mobility (Chamel et al. 2012)
- The crust is not enough (Andersson et al. 2012, Chamel 2013)
- Also true for realistic crust-core coupling (Newton, Berger, BH 2015)

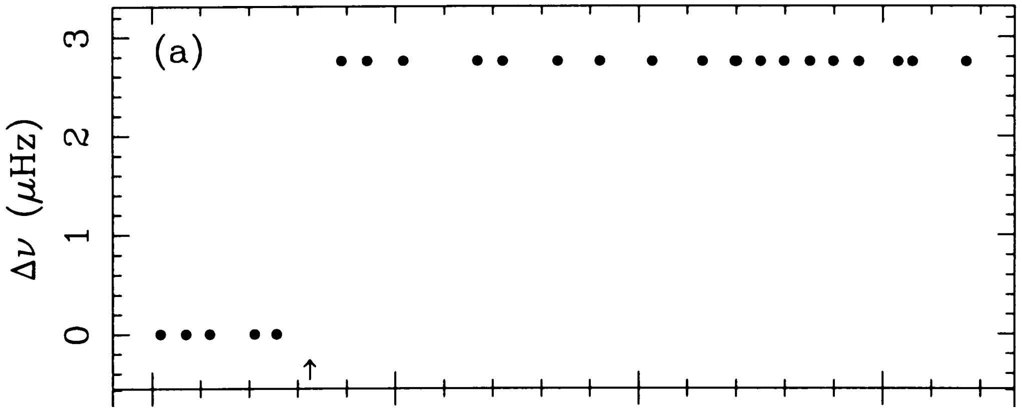
- Response of the star can lead to different relaxation behaviours



PSR J1614−5047



PSR J1123−6259



Conclusions

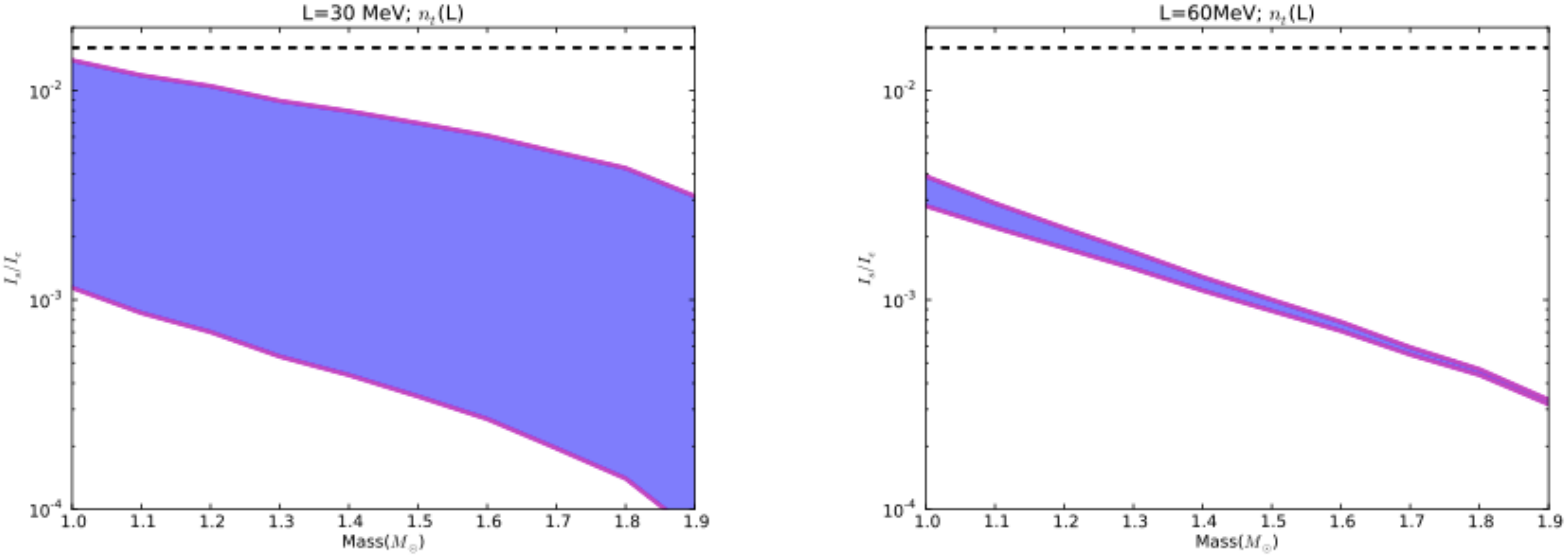
- Vortex avalanches **can** propagate in NS interiors

  (need better constraints on superfluid drag and the role of tension)

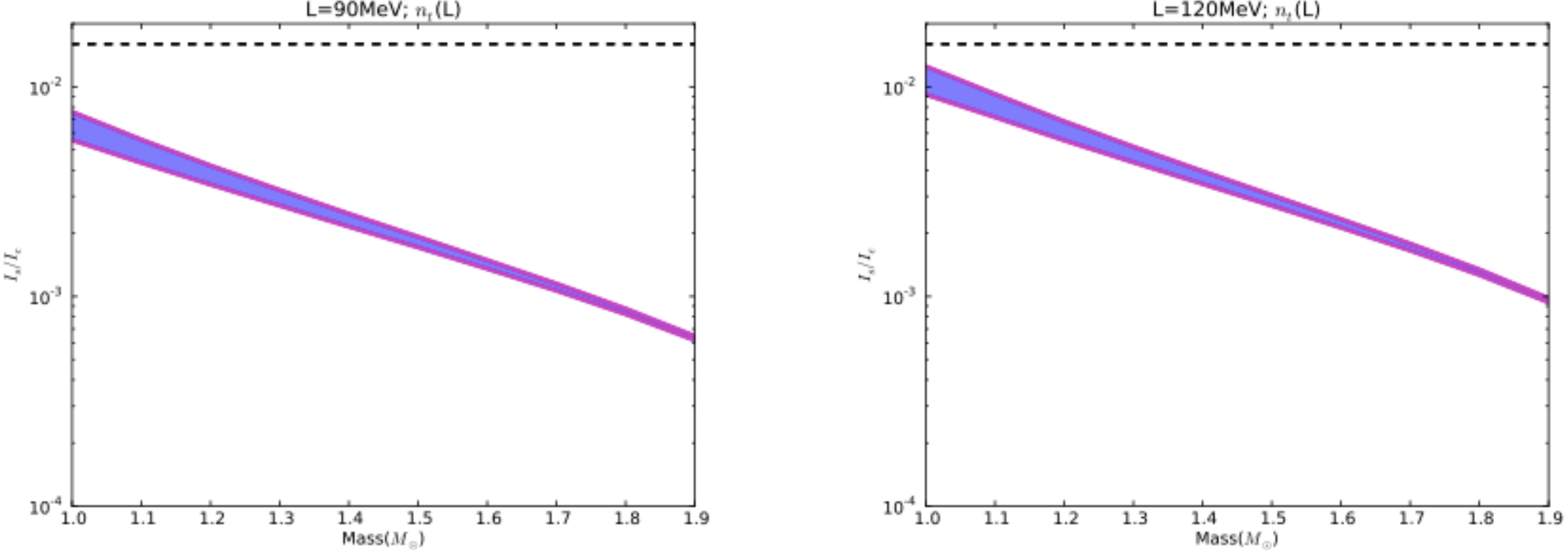
- Coupling of the fluid to vortex motion is crucial

  (additional physics in the core must be involved)

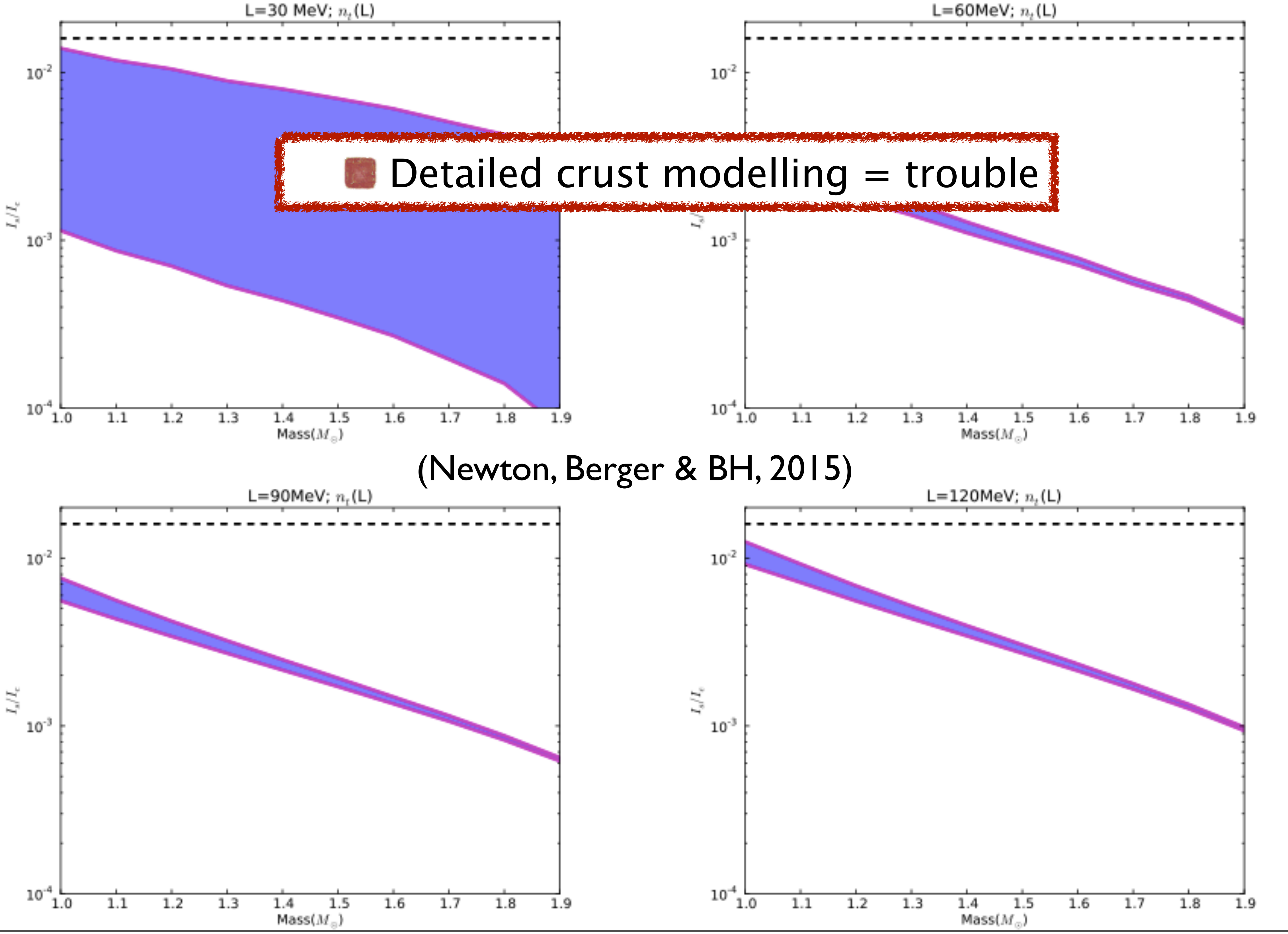
IUFSU parametrisation (Fattoyev et al. 2012, Hooker et al. 2015)



(Newton, Berger & BH, 2015)



IUFSU parametrisation (Fattoyev et al. 2012, Hooker et al. 2015)



- Detailed crust modelling = trouble

(Newton, Berger & BH, 2015)