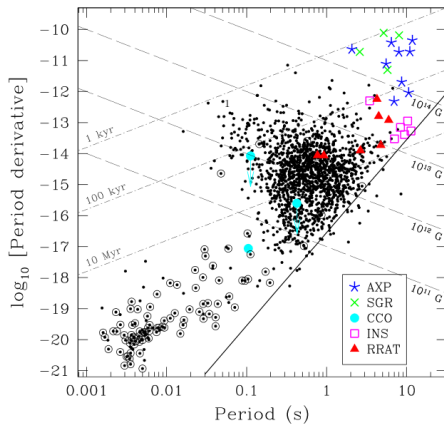


# (Some comments on) Magnetic, thermal, and rotational evolution of neutron stars.

Jose A. Pons  
University of Alicante, Spain



# The observational zoo of Neutron Stars



- Rotation-Powered Pulsars (RPPs), including High-B Pulsars, Radio-Transients (RRATs) and high-energy pulsars (gamma and TeV)
- X-ray Isolated Neutron Stars (XINSs or “the Magnificent Seven”)
- Magnetars (Anomalous X-ray Pulsars and Soft Gamma-ray repeaters):  $\sim 20$
- Central Compact Objects (CCOs):  $\sim 10$
- *Binaries and recycled pulsars (not considered in this talk)*

This is a historical classification not based on physical differences !!

# Probing fundamental physics with multi-band observations of neutron stars.

It has long been hoped that NS internal properties would be deciphered with high quality X-ray observations (Equation of state of dense matter, composition, internal properties).

## Timing analysis.

Accurate values of  $P, \dot{P} \Rightarrow B_d = 6.4 \times 10^{19} G \sqrt{P \dot{P} I_{45} R_{10}^{-3}}, \tau_c = P/2\dot{P}$

WARNING: this is NOT a direct measure ! At most a (very rough/wrong) estimate.

Other torques ? Time variation of B field, moment of inertia, angles ?

## Spectral analysis

Ideally, a combination of accurate spectral modeling and observations can constrain at the same time gravity (atmosphere model) and redshift (lines), thus rendering a measure of  $M$  and  $R$ .

Surprisingly, best spectra of nearby NSs are too close to BBs. In a few cases, spectral lines (or better: deviations from BB) are seen, but interpretation is unclear.

PROBLEM: typical  $kT \approx 0.1$  keV, the interesting part of the spectrum is strongly absorbed.

## NS cooling tracks (focus of this talk).

The long term (1 Myr) cooling history of NSs can also tell us about internal physics. Accurate estimates of temperature/luminosity AND age for a significant number of sources are required.

PROBLEM: B field effects mostly ignored, but they are important. Need better theoretical models.

# Thermally emitting INs: systematic errors

What should we use for cooling curves: bolometric thermal luminosity or temperature ?  
Both are dependent on, and correlated with, the way we model non-thermal contributions (power-law, hard tails) and the other free parameters (ISM absorption, normalization).

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## Factors affecting mainly the uncertainty in temperature

- emission model (BB, atmospheres, condensed surface emission...): color-correction factor 1.5-3 between light element atmospheres and BBs
- anisotropic distribution of the surface temperature in data (value of  $R_{bb}$ ) and models

Pro: insensitive to the distance estimate (flux normalization is related to  $(R_{\infty}/d)^2$ ).

---

## Factors affecting the uncertainty in thermal luminosity

- distance (estimates from SNR associations or absorption/DM)
- absorption model: possible low-temperature contributions not seen due to absorption (very important for absorbed sources  $N_h > 10^{22} \text{ cm}^{-2}$ )

Pros:

- surface-integrated quantity, so slightly dependent on spectral model and anisotropy patterns.
- luminosity spans 5-6 orders of magnitude during the evolution (as opposed to  $\sim 2$  of temperature): errors by factors of a few are acceptable.

# Thermally emitting INSs: the sample

Criteria: clearly detected thermal emission + age estimate

- Good quality spectra
- Thermal component(s) statistically required in the fit
- Distance estimation
- Characteristic and/or kinematic age (e.g., Sedov age of SNRs, proper motion plus association to birth place)

[www.neutronstarcooling.info](http://www.neutronstarcooling.info), online “Coolers catalog”,  
by D. Viganò, N. Rea, J.A. Pons, D. N. Aguilera, D. Page

over 40 sources

- 4 CCOs
- 13 RPPs, incl. 4 high-B PSRs and 1  $\gamma$ -ray radio-quiet PSR
- 7 XINSs
- 17 Magnetars

# Standard 1D cooling theory

[e.g. Dany Page and St. Petersburg group works]

## Energy balance equation

$$c_v \frac{\partial T}{\partial t} + \frac{\partial}{\partial r} \left[ -\kappa \frac{\partial T}{\partial r} \right] = -Q_\nu$$

Ingredients:

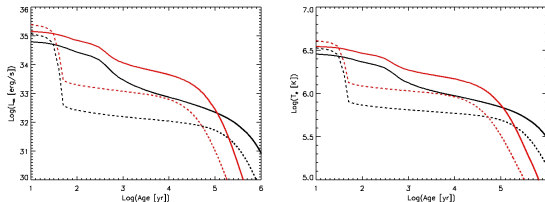
- Neutron star model (structure, EOS)
- Specific heat  $c_v(T, \rho)$ : main contribution by neutrons
- Thermal conductivity  $\hat{\kappa}(T, \rho)$ , very large in the core (rapidly isothermal). Important timescales given by the electron relaxation time in the crust.
- Neutrino emissivities  $Q_\nu(T, \rho)$
- Boundary condition: model of envelope (i.e., liquid outermost  $\sim 100$  m, with strong gradient of temperature), linking internal  $T_b$  to surface  $T_s$ ; emission model (atmosphere, BB...)

## Low field INSs ( $B < 10^{13}$ G)

1D models are reasonably correct (anisotropy, if any, in the envelope)

The influence of magnetic field is not terribly relevant (maybe in older NSs, but this are too cool to be observable).

# The first million years: Standard cooling curves for weak magnetic fields ( $B \lesssim 10^{13}$ G)



EoS: Douchin & Haensel (2001); Baym+ (1971). Fe envelope (black) and accreted envelope (red) (Potekhin+ 2001, 2007; Pons+ 2009).  
For both models, we show  $M = 1.10M_\odot$  (solid) and  $M = 1.76M_\odot$  (dashed).

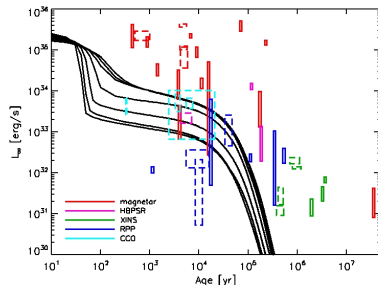
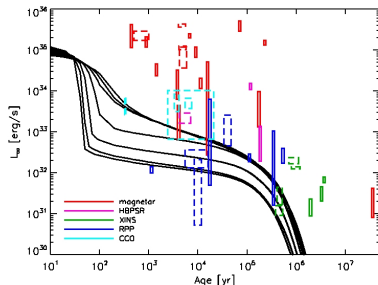
## General trend

- $\approx 1 - 10$  yrs: thermal relaxation of the crust, after that the star is nearly isothermal
- $10^1 - 10^3$  yr: superfluid gaps-dependent activation of CPFBS processes;
- $t < 10^4 - 10^5$  yr: neutrino-cooling era;  $L_\infty \sim t^{-\alpha}$ ,  $\alpha \sim 0.2 - 0.5$
- $t > 10^5 - 10^6$  yr: photon-cooling era;  $L_\infty \sim t^{-\beta}$ ,  $\beta \sim 4.5 - 6$
- Heavier stars (i.e., large central density) are cooler if Direct URCA processes are activated

# Cooling curve, weak magnetic fields ( $B \lesssim 10^{13}$ G)

Different lines: masses from  $1.10 M_{\odot}$  to  $1.76 M_{\odot}$ . Iron (left)/accreted (right) envelope.

**Caveat: solid boxes are UPPER LIMITS** ( $\tau_c < t_{real}$ )



Most of RPPs agree with theoretical predictions.

Vela, B2334 and PSR 1740+1000 need fast cooling.

CCOs are only compatible with light envelope models.

**Most magnetars, high-B PSRs, XINSs (and CCOs with  $Fe$  envelope) are hotter than expected.**

⇒ **Extra energy needed: magnetic field decay**



# Why do we care about magnetic fields ?

## Motivations

What is the NS model that includes the minimum reasonably well known physics and can explain or connect all (as many as possible) different classes, in terms of timing, spectral and bursting properties?

There is one thing we are sure: NSs do have magnetic fields. Despite MF-related issues are usually overlooked –for simplicity –, it is a necessary ingredient in any NS model.

## How thermal evolution is affected by B field ?

- 1 Joule dissipation (source of heat  $Q_j$ , non-isothermal crust!)
- 2 anisotropic thermal conductivity  $\hat{\kappa}$
- 3 neutrino synchrotron process
- 4 quantizing effects (unimportant in the crust)
- 5 magnetized envelope models  $T_s(T_b, \vec{B})$  must also include all these effects

## Final goal

Study the evolution of a NS during its first Myr of life (the surface still hot enough to be seen) considering the feedback between  $T$  and  $B$  evolution in the crust and the CORE (!?)

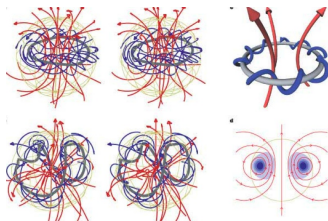
# The major uncertainty: The initial conditions.

A neutron star is born hot and liquid (melting  $T \sim 10^{10}$  K). Let us believe that somehow a strong magnetic field has been created (how ??)

Hydrodynamics is appropriate, and if a strong magnetic field is present we can use MHD.

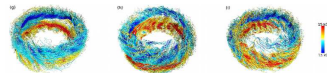
Stable MHD solutions are complex and require a toroidal component.

MHD equilibrium must be established in a few dynamical timescales (seconds), still during the PNS phase, so what is the topology of the B field in MHD equilibrium ?



[Braithwaite & Spruit 2004,2005]

Toroidal field concentrated near the equator, inside closed poloidal field lines  
(see also Lander+ 2009,2012,2014, Ciolfi et al 2009, 2010)

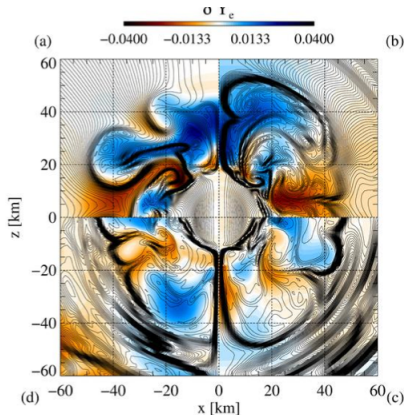


[Nelson+ 2013]

## The initial conditions.

Most detailed up-to-date 2D simulations of proto-neutron star evolution with realistic EoS and neutrino transport show that there is more life beyond the magnetic dipole. A proto-NS is convectively unstable during many seconds !!

QUESTION: does it make sense to assume NSs settle in "perfectly ordered" MHD equilibria ?



[Obergaullinger, Janka, Aloy 2014]

# Magnetic field evolution

## CRUST

A few minutes/hours after birth, a SOLID crust is formed and grows. It can be considered a simplified version of a Hall plasma: ions have very restricted mobility and only electrons can move freely through the lattice, carrying currents and heat: the proper equations are Hall MHD. If ions are strictly fixed in the lattice, the limit is known as EMHD (electron MHD).

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left\{ \eta \vec{\nabla} \times (e^\nu \vec{B}) - \left[ \frac{ce^{-\nu}}{4\pi en_e} \vec{\nabla} \times (e^\nu \vec{B}) \right] \times (e^\nu \vec{B}) \right\}$$

$\eta = \frac{c^2}{4\pi\sigma}$  is the magnetic diffusivity, and the electron fluid velocity is

$$\vec{v}_e = -\frac{\vec{J}}{en_e} = -\frac{ce^{-\nu}}{4\pi en_e} \vec{\nabla} \times (e^\nu \vec{B})$$

In the limit of small deformations, the metric is still spherically symmetric. Relativistic corrections included with the  $e^{-\nu}$  factor.

---

## CORE

Not clear how much flux penetrates into the core, and what is the evolution of a SC fluid (fluxoids drift and interact with vortices? magnetic buoyancy? Does ambipolar diffusion work with superfluid neutrons or SC protons?).

# Ohmic dissipation

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left\{ \eta \vec{\nabla} \times \vec{B} \right\}$$

## Ohmic dissipation timescale

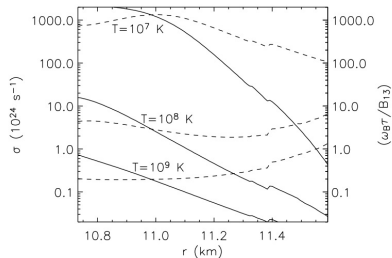
Assume a force-free background field  
 $\vec{\nabla} \times \vec{B} = \mu \vec{B}$  and constant  $\eta$ .

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left\{ \eta \mu \vec{B} \right\}$$

$$\frac{\partial \vec{B}}{\partial t} = -\eta \mu^2 \vec{B}$$

$$\vec{B} = \vec{B}(t=0) e^{-t/\tau_{Ohm}}$$

with  $\tau_{Ohm} = 1/(\eta \mu^2) \approx L^2/\eta$ , where  $L$  is the typical scale of the curvature of the magnetic field



**PROBLEM:** everything varies by several orders of magnitude. Estimates are not very useful, which region in this diagram should we look at ??  
 What is the real scale ( $L$ ) of the B field ??

# The Hall term

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left\{ \eta \left[ \vec{\nabla} \times \vec{B} - \omega_B \tau_e \left( \vec{\nabla} \times \vec{B} \right) \times \vec{b} \right] \right\}$$

Here  $\omega_B \tau_e$  is the “magnetization parameter”, where  $\omega_B$  is the gyro-frequency.

For high temperatures (large resistivity) or weak fields: diffusive regime  $\omega_B \tau_e \ll 1$

For low T ( $T \lesssim 10^8$  K) or strong fields: non-linear hyperbolic regime  $\omega_B \tau_e \gg 1$ , i.e. Hall activity.

## Linear regime: wave modes [Huba 2005]

Background field  $\vec{B} = B_0 \hat{z}$

- constant  $n_e$ , **whistler (or helicon) waves** propagating along field lines  
dispersion relation  $\omega = k^2 B / 4\pi e n_e$   
phase velocity  $\propto k_z \Rightarrow$  restrictive Courant condition
- $n_e = n_e(x)$ , **Hall drift waves** in the  $\vec{B} \times \vec{\nabla} n_e$  direction  
dispersion relation  $\omega = k_y B_0 / [4\pi e (dn_e/dx)]$   
phase velocity  $B_0 / [4\pi e (dn_e/dx)]$

## Hall timescale.

$$\tau_{Hall} = \frac{4\pi e n_e}{cB} L^2$$

$$\omega_B \tau_e = \tau_{Ohm} / \tau_{Hall}$$

# Weak field

# Strong field



# Core field

# Core+crust field

# Multipolar field

# Toroidal field

Initial topology is quickly reorganized: the Hall term removes the freedom to chose arbitrarily large toroidal fields able to inject very large extra energy (Viganò + 2012, 2013)  
The long term structure looks similar for all models (Pons & Geppert 2007, Gourgouliatos+ 2013,2014) Models have more predictive power.

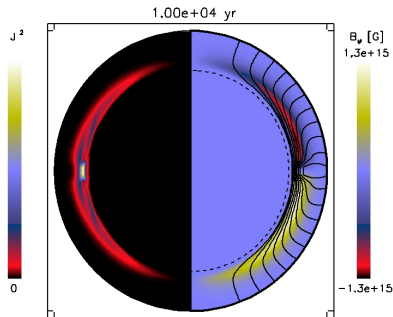
# Evolution of magnetic field

**Magnetic field dissipation makes the spin-down inefficient at late ages**

⇒ **Asymptotic maximum period** [see, e.g., phenomenological model by Colpi+ 2000]

Take results from magneto-thermal evolution.

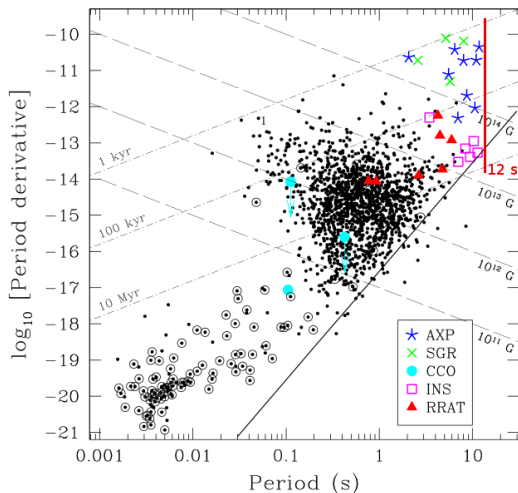
- Large magnetic fields sustained by crustal currents keep the star detectable longer (Myr).
- Magnetic, rotational and thermal evolution agree with timing and spectral observations of magnetised neutron stars.
- The bulk of crustal currents circulate in the dense, inner crust due to the Hall drift.



## Bottomline

IF most currents are in the crust (external layers) at birth, the physics of the INNER CRUST directly regulates the observed TIMING properties!

# Maximum period of isolated neutron stars?



[adapted from Kaspi 2010]

# Selection effects

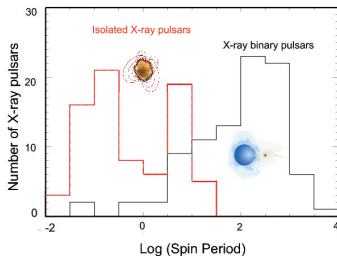
## RADIO

Intrinsic mechanisms of radio emissions, although not understood, loses efficiency for increasing period  $\rightarrow$  narrow beams, low luminosity

Beaming fraction  $f = 9[\log(P/10\text{s})]^2 + 3$  (phenomenological fit, Tauris & Manchester 1998)

## X-RAY

No beaming selection effects against long  $P$ . Hot magnetised stars are more detectable.



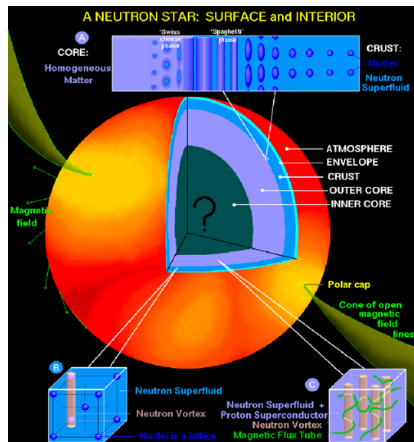
**Evolved magnetars should reach periods of several tens of seconds and still be X-ray detectable.**

Why no evolved magnetars ( $t \gtrsim 10^4 - 10^5$  yr) or XDINS are seen with, e.g.,  $P = 30$  or  $50$  s?  
 Why all the magnetars period (also the oldest ones, like SGR 0418) cluster in the same region?

# The inner crust: nuclear pasta phase

## The NS crust structure

- Density:  $\rho \sim 4 \times 10^{11} - 10^{14} \text{ g cm}^{-3}$
- Composition: relativistic degenerate electrons, free (superfluid) neutrons, lattice made of ions (ground state with large  $A$  and impurities)
- Pasta phase: in the 50-100 m innermost layer of the crust ( $\rho \gtrsim 5 \times 10^{13} \text{ g cm}^{-3}$ ), the large Coulomb energy cost can favour nuclei in pasta shapes (rods, slabs, bubbles, most likely irregular structures of few 100 fm size).



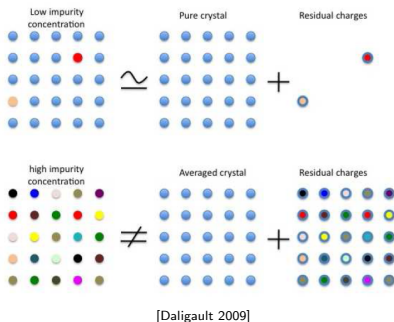
[Page & Reddy 2006]



# Impurity parameter

Lattice properties in inner crust are poorly known

- Structure? Concentration and localization of defects? Effects of magnetic fields?
- Disordered [Jones 2004]? Crystalline [Horowitz group]? Heterogeneous [Magierski & Heenen 2001]?
- Transport properties of pasta phase are largely unexplored.



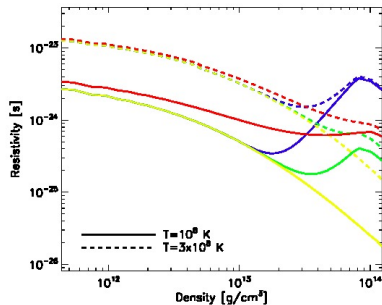
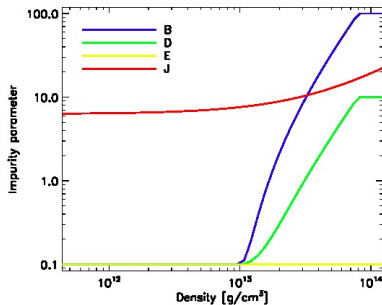
## IMPURITY PARAMETER

$$Q_{imp} = \langle Z^2 \rangle - \langle Z \rangle^2$$

In absence of more detailed calculations,  $Q_{imp}$  parametrizes our ignorance about the crystal structure and charge distribution.

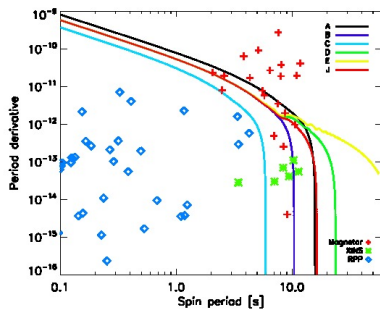
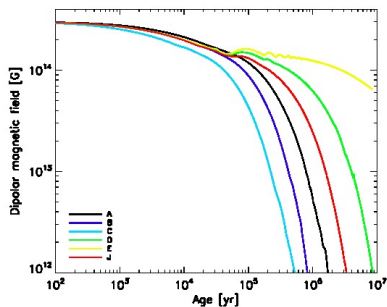
# Conductivity of inner crust

- Crystalline lattice:  $e^-$ -phonon scattering (strongly  $T$ -dependent).
- Disorder resistivity (pasta phase or amorphous crust) is almost independent on  $T$

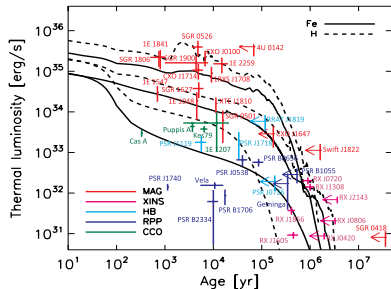
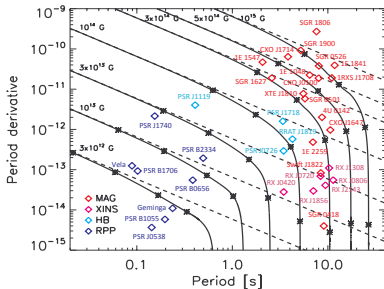


# Timing properties

Initial magnetic field:  $B = 3 \times 10^{14}$  G.  
 A and C models: same as B, with different mass



# Data vs. models



## Constraining models

Data seem to favour a large  $Q_{imp} \gtrsim 10 - 20$  in the inner crust (high magnetic diffusivity), where most of the current is placed.

If  $Q_{imp} \lesssim O(1)$ , there should be evolved magnetars with much longer periods, where are they ?  
Are there other unexplored core/crust interface mechanisms with the same effect ?

# Population synthesis

Statistical Method to average uncertainties in populations with complex evolution.

Monte Carlo simulations of isolated Neutron Stars: birth (initial distributions), evolution (magnetic, thermal, spin evolution models) and detection (observational biases, selection effects).

Comparison between simulated and observed samples in radio AND thermal X-ray bands.

## Main goals

- Constraining the large initial parameter space
- Validating particular models

## References

*Fauchere-Guiguère and Kaspi, 2006.*

*Popov et al., 2010*

*Pierbattista et al. 2012*

*Gullón et al. 2014, 2015*

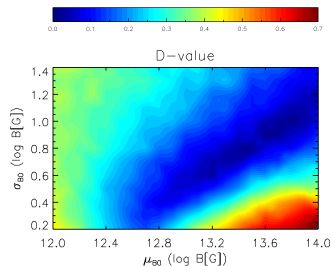
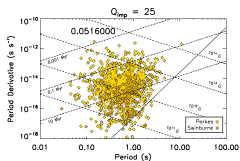
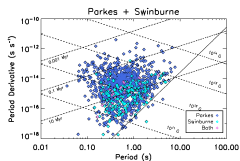


# Radio-pulsar population

Initial magnetic field  $\log(B_0)$  and period  $P_0$  assumed to have gaussian distributions. Different physics in the models: Impurity parameter in the inner-crust  $Q_{imp}$ , envelope composition or additional toroidal component.

$D$ -value accounts for the goodness-of-fit of a 2-dimensional Kolmogorov-Smirnov test

- Most models tested can reproduce the observed radio-pulsar  $P - \dot{P}$  distribution.
- Correlation between the mean  $\mu_{B_0}$  and sigma  $\sigma_{B_0}$  values of the  $B_0$  distribution.



# X-ray thermally emitting NSs

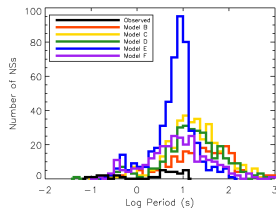
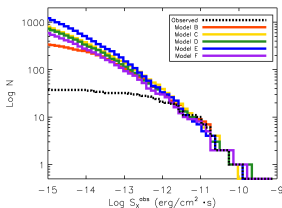
## X-ray detection

Thermal emission coming from residual cooling and magnetic field decay.

Absorption from interstellar media.

Spectra modeled by blackbody + Resonant Compton Scattering (for high B) [Lyutikov & Gavril (2006)].

- Break the degeneracy to fit at the same time logN-logS distributions in X-ray pulsars.
- The observational catalogue seems to be complete for  $S_X > 3 \times 10^{-12}$
- Many NSs with periods longer than 12 s that should have been observed !?

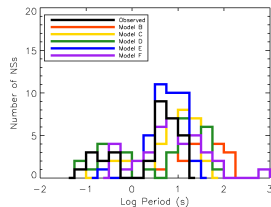
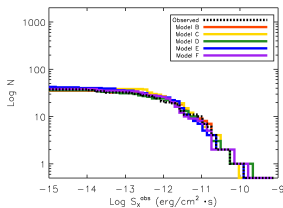


# X-ray thermally emitting NSs

- We propose a simple empirical expression for the *detectability function* depending on the flux.

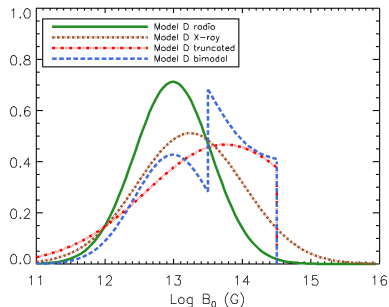
$$p_{\text{Obs}} = \min \{ \eta S_{-11}, 1 \}, \eta \sim 3 - 5$$

- Even correcting for this *selection effect*, still there should be a few  $P > 12$  s X-ray pulsars. Where are they ?





## Beyond gaussian distributions for $B_0$



### Truncated distribution

- Gaussian distribution imposing a cutoff at  $B_0 \sim 5 \times 10^{14}$  G.

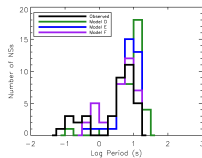
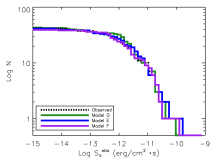
### Bimodal distribution

- Two populations of NSs: radio-pulsars and magnetars with different origins (single vs. binary systems ?)

# Beyond gaussian distributions for $B_0$

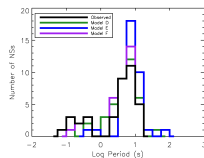
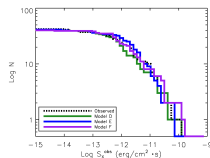
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- Gaussian distribution imposing a cutoff at  $B_0 \sim 5 \times 10^{14}$  G.



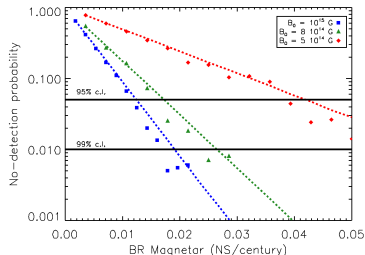
## Bimodal distribution

- Two populations of NSs: radio-pulsars and magnetars with different origins (single vs. binary systems ?)



# Upper limit to the magnetar Birth rate

- What is the maximum number of stars born with fixed  $B_0$  compatible with the lack of observations of X-ray pulsars with  $P > 12$  s?
- For  $B_0 = 10^{15}$  G and a confidence level of 99 %, this implies a maximum birthrate of 0.02 NSs/century! (less than 1% of the NS population).



# Summary

I am likely running out of time, so just one PROVOCATIVE conclusion:

No isolated neutron star is seen (yet) with  $P > 12$  s.

Possibly the first direct evidence of a highly resistive layer in the inner crust, compatible with the existence of the pasta phase. But even including this effect, it is very hard to explain the lack of long period X-ray pulsars ... unless there are no neutron stars with  $B_0 > 10^{15}$  G.