# QCD Thermodynamics from Effective Field Theories

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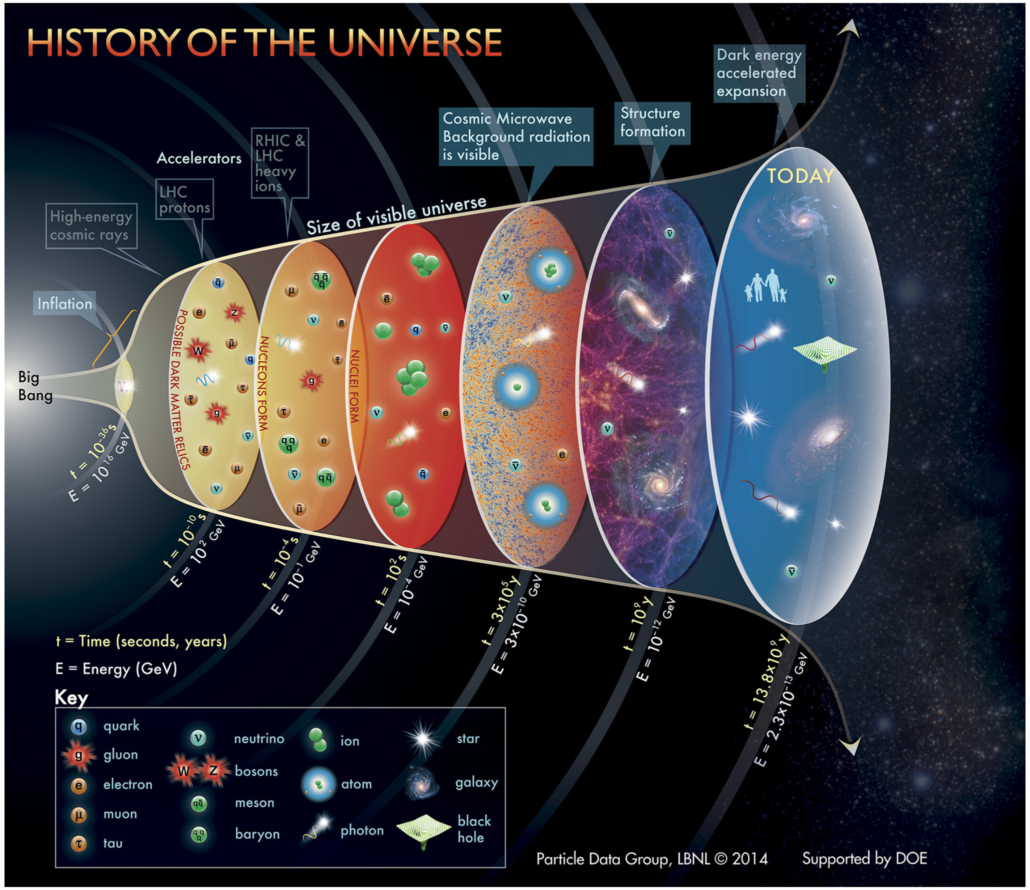
**Selected Issues:**

- interplay between chiral symmetry breaking and confinement
- phase diagram of hot/dense QCD
- hadronic matter near chiral symmetry restoration

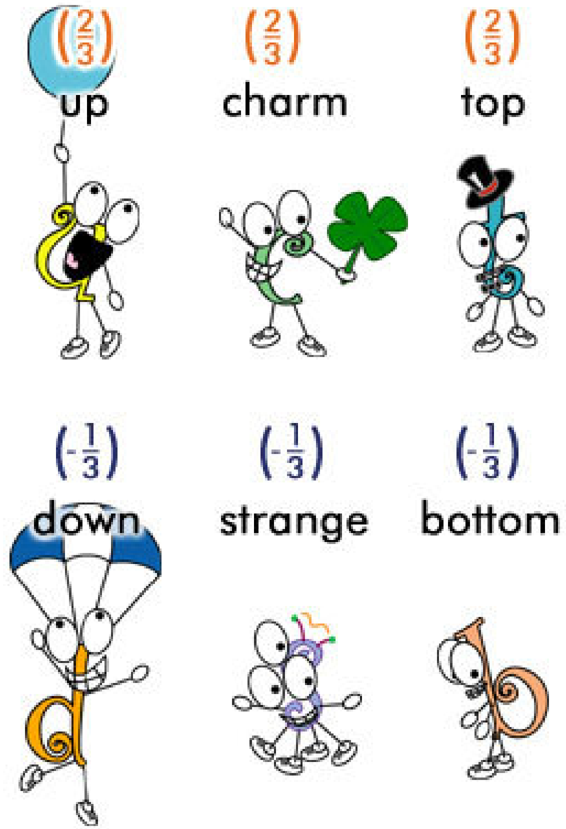
# What is the origin of matter?




- after EW phase transition: Standard Model (EM & Weak plus Strong int.)
- QCD phase transition is the last transition in the evolution of the Universe!

up, down quarks ($\sim 5$ MeV): SM ingredients
Where does the nucleon mass ($\sim 1$ GeV) come from?
10 % from EW/Higgs, and 90 % from QCD!

- scale and confinement? from elementary particle to composite states?
  $\cdots$ fundamental issues in non-abelian gauge theories!




**Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$**
Baryons are fermionic hadrons.
There are a few of the many types of baryons.

| Symbol | Name | Quark content | Electric charge | GeV/c$^2$ | Spin |
|---|---|---|---|---|---|
| p | proton | uud | 1 | 0.938 | 1/2 |
| $\bar{p}$ | antiproton | $\bar{u}\bar{u}\bar{d}$ | –1 | 0.938 | 1/2 |
| n | neutron | udd | 0 | 0.940 | 1/2 |
| Λ | lambda | uds | 0 | 1.116 | 1/2 |
| $\Omega^-$ | omega | sss | –1 | 1.672 | 3/2 |

**Mesons $q\bar{q}$**
Mesons are bosonic hadrons
There are a few of the many types of mesons.

| Symbol | Name | Quark content | Electric charge | GeV/c$^2$ | Spin |
|---|---|---|---|---|---|
| $\pi^+$ | pion | $u\bar{d}$ | +1 | 0.140 | 0 |
| $K^-$ | kaon | $s\bar{u}$ | –1 | 0.494 | 0 |
| $\rho^+$ | rho | $u\bar{d}$ | +1 | 0.770 | 1 |
| $B^0$ | B-zero | $d\bar{b}$ | 0 | 5.279 | 0 |
| $\eta_c$ | eta-c | $c\bar{c}$ | 0 | 2.980 | 0 |

- effective field theories:
  - "particles" contained: either fundamental or composite
  - based on fundamental/emergent symmetries
  - broad spectrum of applications to QCD(-like), SM, BSM

## Objectives: from hadrons to quarks and gluons

*QCD underlies physics of mesons and baryons ··· nonperturbative!*

∗ how are color-singlet states formed? exotic hadrons?

∗ dynamical origin of hadronic interactions and masses?

∗ composition of QCD matter at finite temperature/density?

### Dynamical $\chi$SB & Color Confinement

$$L = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \bar{q}(i\not{D} - m)q$$

color confinement

instantons

trace anomaly

$U(1)_A$ anomaly

dynamical chiral symmetry breaking

pion as NG boson

nuclear force

# I. Confinement vs. Dynamical Chiral symmetry Breaking

# Hot and dense QCD matter

- relativistic heavy-ion collisions (hot, dense), compact stars (dense)
- how to explore the dynamics? effective theories, simulations on a lattice
- toward EFT at high $\rho$: lattice input at $\mu_q \sim 0$ & NM properties at $\rho_0$

## QCD thermodynamics at $\mu_q \simeq 0$ from lattice simulations

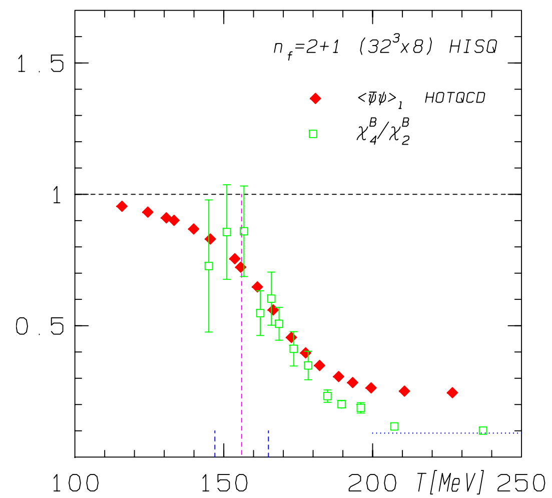
- deconfinement and SU(2) chiral restoration set in at $T_{\rm pc} \sim 155$ MeV.

  $\hookrightarrow$ conserved charges $\hookrightarrow$ melting chiral condensate

- strange (s) and charmed (c) mesons are deconfined together with light (u,d) mesons at $T_{\rm pc}$!

e.g. kurtosis of baryon number fluctuations

$$\kappa = \chi_4^B / \chi_2^B = B^2 \,,$$
$$\chi_n^B = \partial^n (P/T^4) / \partial(\mu_B/T)^n$$



## Confinement vs. dynamical chiral symmetry breaking (D$\chi$SB)

**to which extent does D$\chi$SB contain information on confinement?**

- Banks-Casher relation: low-lying Dirac eigenmodes generate $\langle \bar{q}q \rangle$.

$$\langle \bar{q}q \rangle = - \lim_{m_q \to \infty} \lim_{V \to \infty} \pi \rho(0) \, , \quad \rho(\lambda) = \frac{1}{V} \sum_n \langle \delta(\lambda - \lambda_n) \rangle$$

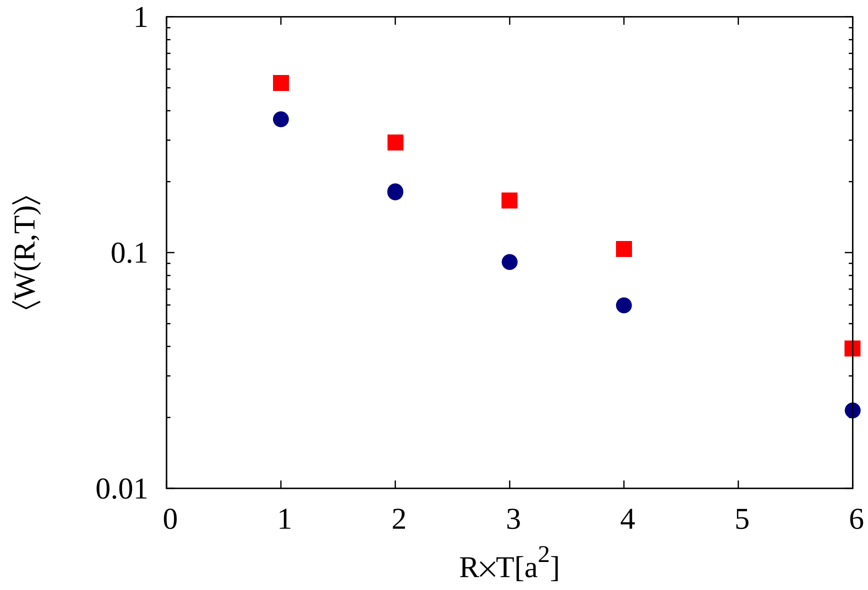
**removal of low-lying Dirac modes $\Rightarrow$ NO D$\chi$SB**
**Q. does confinement disappear simultaneously?**

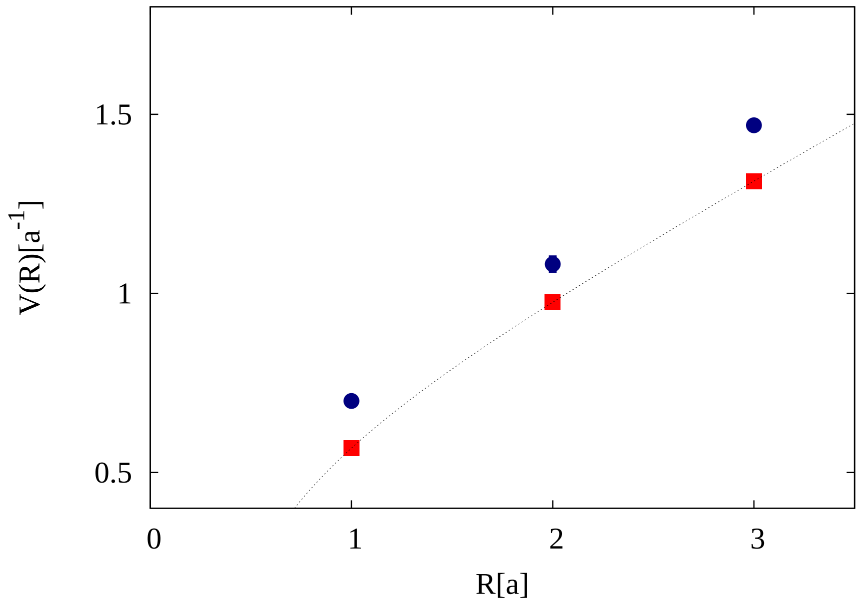
- linking Polyakov loop to spectral function of *lattice* Dirac operator
  [Gattringer ('06); Bruckmann, Gattringer, Hagen ('07); Synatschke, Wipf, Langfeld ('08)]
  manifestly gauge invariant formalism
  [Gongyo, Iritani, Suganuma ('12); Doi, Iritani, Suganuma ('13,14)]

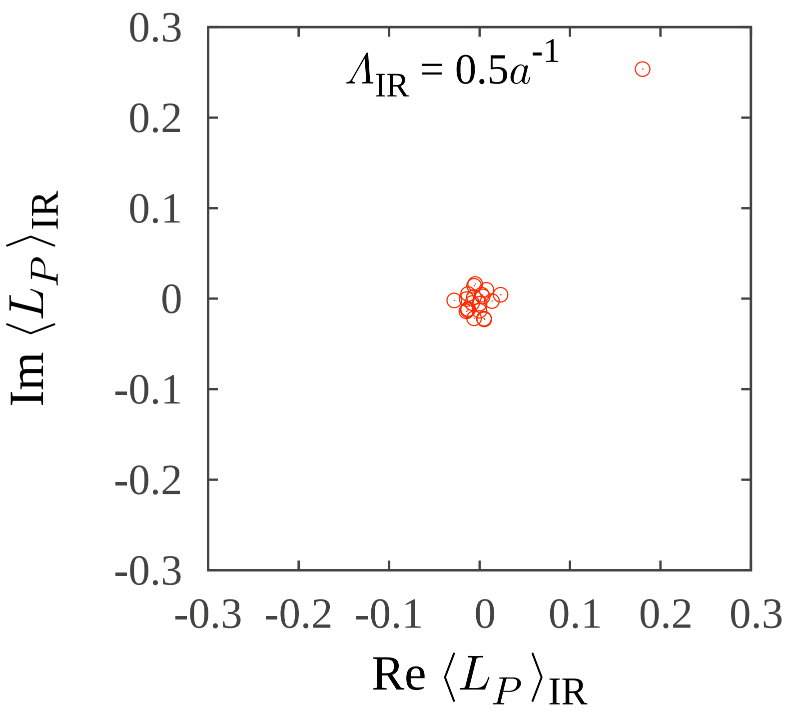
**NO particular Dirac-modes that crucially affect confinement!**

$\Downarrow$

**disappearance of D$\chi$SB DOES NOT mean deconfinement?**




■ full vs. ● w/o low-lying modes

- Wilson loop, quark potential: $\langle W \rangle \propto e^{-\sigma RT}$: slope parameter = string tension ⇒ unchanged!

  [Gongyo, Iritani, Suganuma ('12)]

- Polyakov loop susceptibilities unmodified

  [Doi, Redlich, CS, Suganuma ('15)]

## How to suppress unphysical d.o.f.? [T=0: Benic, Mishustin, CS ('15)]

- IR cutoff $b$: NJL, SD eq., AdS/QCD [Ebert, Feldmann, Reinhardt ('96); $\cdots$]
  $1/b \sim$ typical size of a hadron $\Rightarrow$ modified FD distribution functions

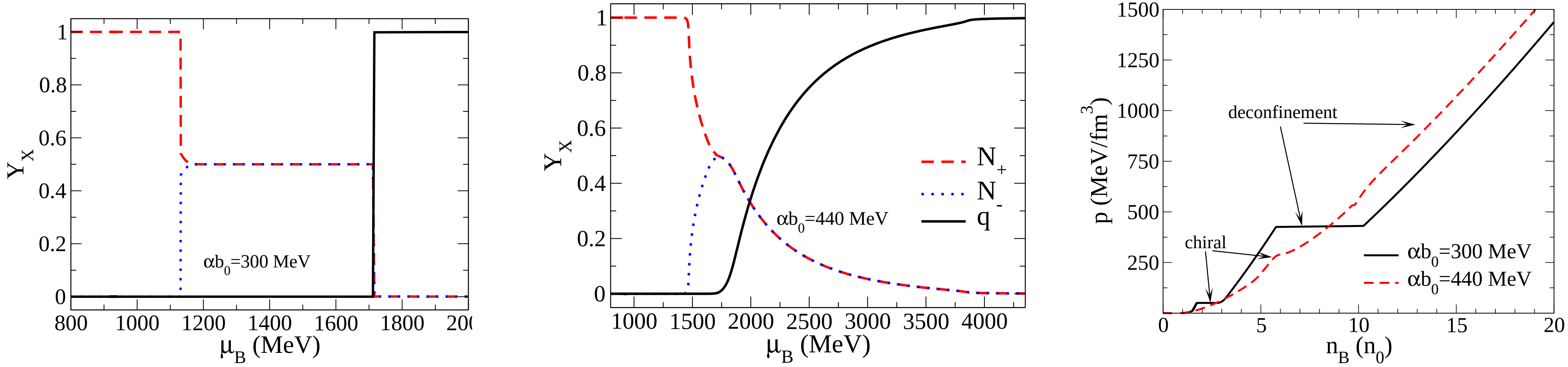
$$n_q = \theta(\vec{p}^2 - b^2) f_q\,, \quad n_{N_\pm} = \theta(\alpha^2 b^2 - \vec{p}^2) f_{N_\pm}\,, \quad f_X = 1/1 + e^{\beta(E_X \mp \mu_X)}\,.$$

- asymptotic behavior at high density:
  *non-int. quark gas, restored chiral symmetry*
  $\Rightarrow$ density dep. IR cutoff, upper limit $\alpha_{\max} = 2^{-1/3} \sim 0.8$
- model setup: $\Omega = \sum_{X=N_\pm,q} \Omega_X + V_\sigma + V_\omega + V_\chi + V_b$

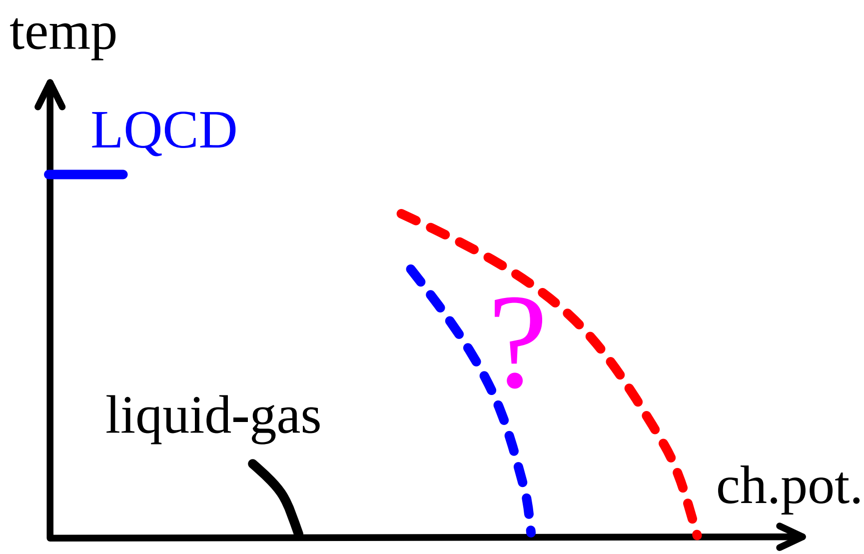
$$V_b = -\frac{\kappa_b^2}{2} b^2 + \frac{\lambda_b}{4} b^4\,.$$

  parameter fixing: $(\kappa_b, \lambda_b) \leftarrow (\epsilon_{\rm vac}, T_{\rm ch})$ from lattice QCD
- deconfinement criteria

$$Y_{N_+} + Y_{N_-} = Y_q\,; \quad Y_{N_\pm} = \frac{\rho_{N_\pm}}{\rho_B}\,, \quad Y_q = \frac{1}{3}\frac{\rho_q}{\rho_B}\,.$$

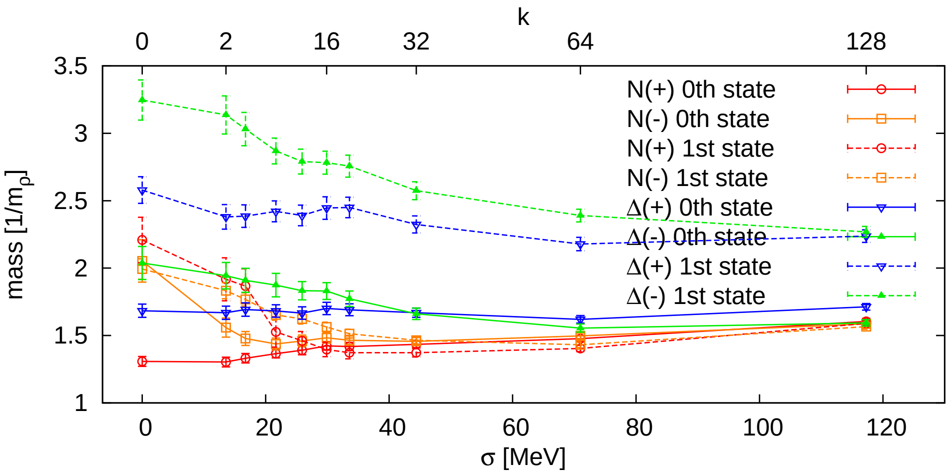


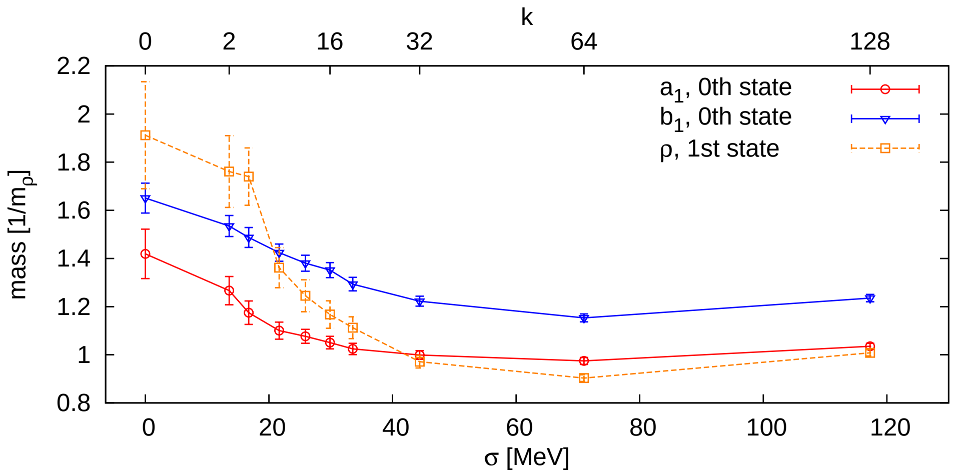

- quark-meson-nucleon hybrid model at $T = 0$ and large $\mu_B$
  $\Rightarrow$ $\rho_{\rm ch}$ separated from $\rho_{\rm dec}$ $\quad$ $\alpha$-dep.: $\Delta\rho_{\rm cr}/\rho_0 \sim$ 4-12
- decreasing $\Delta\rho_{\rm cr}(T)$
  due to bosonic thermal fluctuations
- holographic QCD approach:
  - large $N_c$ limit
  - less parameters



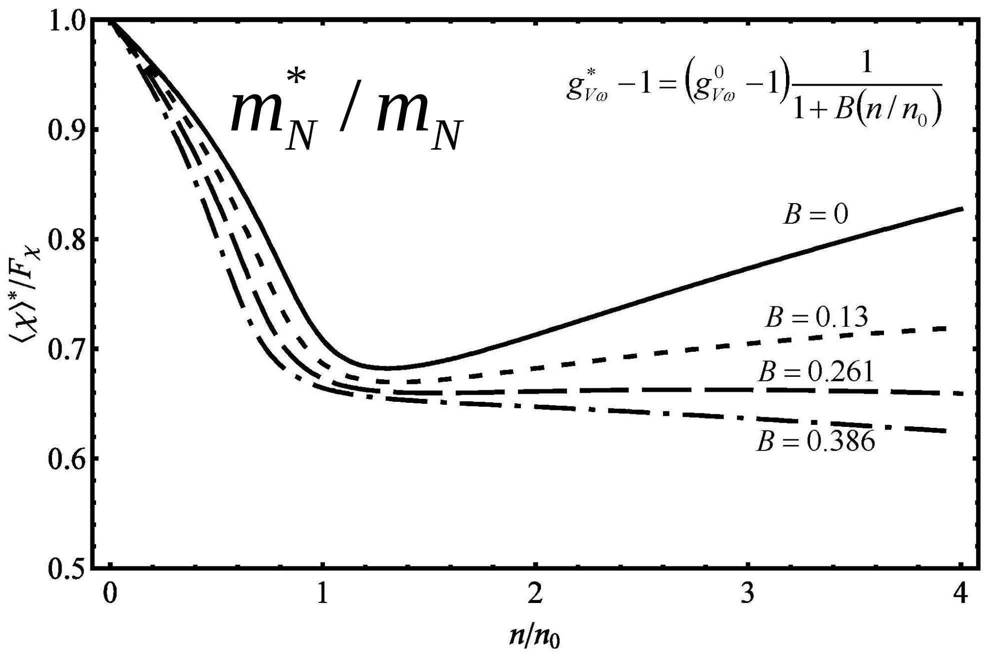
# II. Dense Hadronic Matter near Chiral Restoration

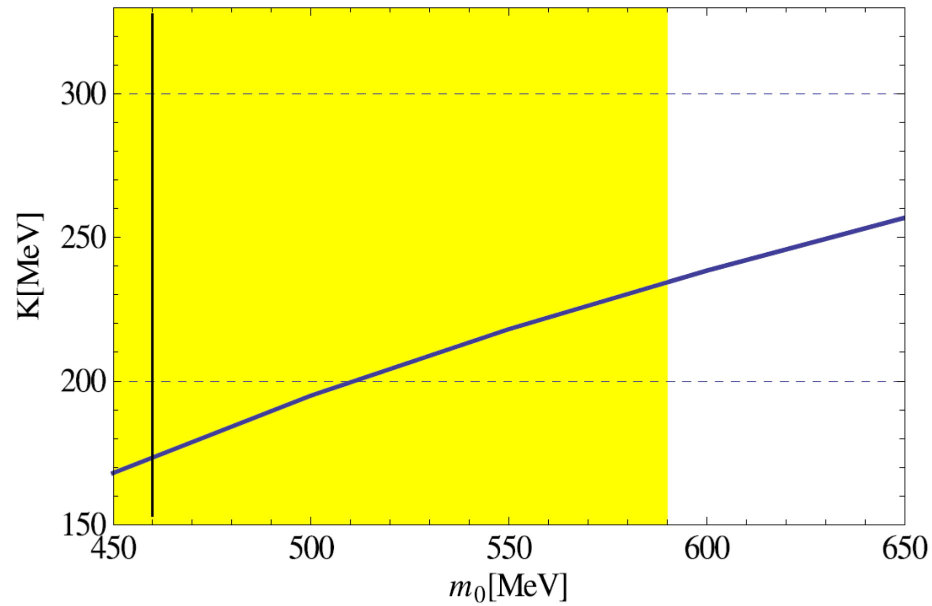
# Fate of hadron masses toward chiral symmetry restoration




- hadron masses vs. truncation level on lattice [Glozman, Lang, Schrock ('12)]
  - removal of lowest Dirac-eigenmodes $\Leftrightarrow$ NO $\langle \bar{q}q \rangle$
    $\Rightarrow$ parity partners degenerate *and* stay quite massive!
  - *no* universal scaling ($m_{\rm meson} \sim 2m$, $m_{\rm baryon} \sim 3m$) found
    $\Rightarrow$ the system remains confined!
- full LQCD: $m_{N^-} \stackrel{T \to T_{\rm ch}}{\longrightarrow} m_{N^+} \sim m_{N^+}^{(\rm vac)}$ [Aarts et al., ('15) ]
- origin of a scale in $\chi$-sym. phase? $\cdots$ trace anomaly $T_\mu^\mu \sim \langle \frac{\alpha_s}{\pi} G^2 \rangle$
  $\Rightarrow$ in-medium gluon condensate: melting toward $T_{\rm ch}$, but non-vanishing




- **embedding gluon condensate:** $U(1)$ vs. $SU(2)$ [Paeng, Lee, Rho, CS ('13)]
  - $U(2)_L \times U(2)_R \sim SU(2)_L \times SU(2)_R \times U(1)_V \longrightarrow SU(2)_V \times U(1)_V$
  - RGE analysis: $\rho NN$-int. "runs" whereas $\omega NN$-int. "walks" with $n_B$!
  - $m_N^* \sim$ const.: emergence of a $\chi$-inv. mass!
- **tetra-quark states in chiral transition:**
  - crucial to nuclear compressibility $K(n_0)$ [Gallas, Giacosa, Pagliara ('11)]
  - new CP? intermediate phase? enlarged symmetry? [Heinz, Struber, Giacosa, Rischke ('08); Harada, CS, Takemoto ('09); Mukherjee, Huang ('13)]

## Chiral mixing in a medium

- chiral (V-A) mixing via $\rho$-$a_1$-$\pi$ interaction
- Weinberg SRs at $T \neq 0$: $f_\pi \Leftrightarrow \rho_{V,A}(s)$ [Weinberg ('67); Kapusta, Shuryak ('94)]
  - smooth reduction of $m_{a_1} \to m_\rho$ and broader width
  - importance of higher-lying states: $\rho', a_1'$ [Hohler, Rapp ('14)]

**dense QCD:**

- no charge-conjugation inv. $\Rightarrow$ $\rho$-$a_1$ mixing at tree, dispersion modified

$$\mathcal{L}_{\rm mix} = 2C\epsilon^{0\nu\lambda\sigma}\mathrm{tr}\left[\partial_\nu V_\lambda \cdot A_\sigma + \partial_\nu A_\lambda \cdot V_\sigma\right]$$

$$\text{transv:} \quad p_0^2 - \vec{p}^2 = \frac{1}{2}\left[m_\rho^2 + m_{a_1}^2 \pm \sqrt{(m_{a_1}^2 - m_\rho^2)^2 + 16C^2\vec{p}^2}\right]$$

- mixing strength: $\chi$EFT [Harada, CS ('09)] vs. AdS/QCD [Domokos, Harvey ('07)]

$C = 0.1$ GeV vs. $1$ GeV at $n_B = n_0$!

$C = 1$ GeV $\Rightarrow$ vector-meson condensation at $n_0$!?

- why $C(n_0) = 1$ GeV in AdS/QCD?

$$C_\omega \ll C = C_\omega + C_{\omega'} + C_{\omega''} + \cdots ?$$

- role of higher KK modes: open moose model [Son, Stephanov ('03)]

$$G_V \overset{Q^2 \to \infty}{\longrightarrow} \frac{1}{8\pi^2} \ln Q^2$$

$\Rightarrow$ correct high-energy behavior for current correlator!

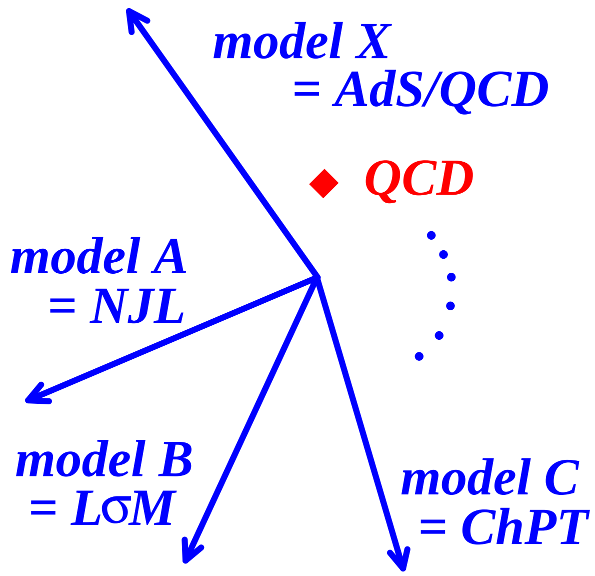
**More and more hadronic states activated toward QCD p.t.!**
**... How to handle them?**

- holographic QCD models: $1/N_c$ corrections?
- 4d effective field theories:
  unknown interactions among higher-lying states?
  if renormalized/integrated out, $T, n_B$-dep. interactions?

$$\mathcal{L}_\infty(\pi, \rho, \omega, \cdots) \sim \mathcal{L}_{\text{low-lying}}(\pi, \rho, \omega, \cdots; g(T, n_B))$$

– might be bridged via DS eq. and FRG?
– fixed along with comparison to lattice observables? [CS ('14)]

## Summary and Remarks




- **EFT approach to hot/dense QCD**
  - approximation of real-life QCD!
  - *multifaceted studies of gauge dynamics* guided by symmetry, topology, ideal limit
  - available data from lat. simulations and HIC
- **Dynamical chiral symmetry breaking vs. confinement**
  - dense matter: 2 separated phase transitions, supported by LQCD w/ Dirac eigenmode expansion
  - one induces another? supported by $\mu = 0$ LQCD
  - cf. adjoint QCD: $T_{\rm dec} \neq T_{\rm ch}$
- **More to come from HICs, LQCD and EFTs**
- **More input from nuclear astrophysics and observations**