



# Neutron Stars:

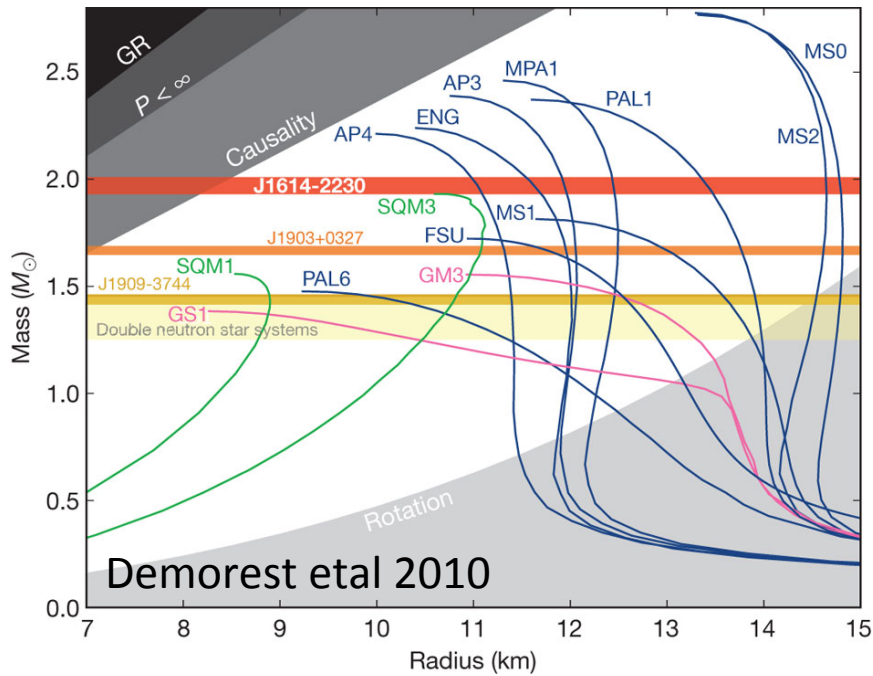
## Oscillations, Instabilities and Gravitational Waves

**Kostas Kokkotas**

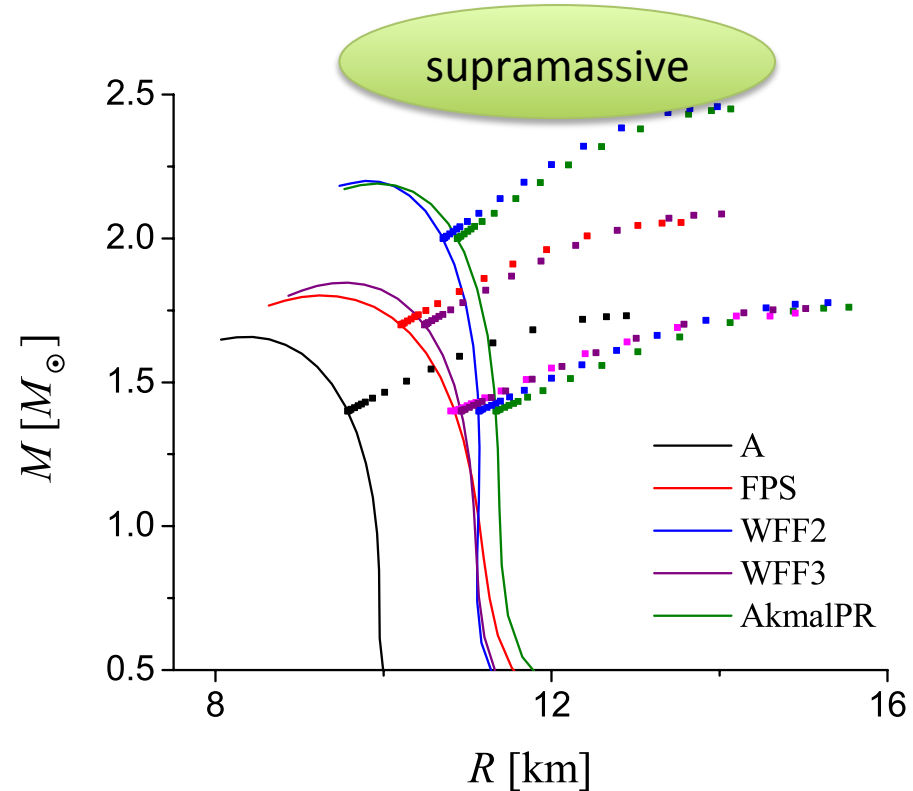
Theoretical Astrophysics  
Eberhard Karls University of Tübingen

# Neutron Stars: Mass vs Radius

Static Models



Rotating Models



# Neutron Stars & “universal relations”

Need for relations between the “**observables**” and the “**fundamentals**” of NS physics

Average Density

$$\bar{\rho} \sim M / R^3$$

Compactness

$$z \sim M / R \quad \eta = \sqrt{M^3 / I}$$

Moment of Inertia

$$I \sim MR^2 \quad I \sim J / \Omega$$

Quadrupole Moment

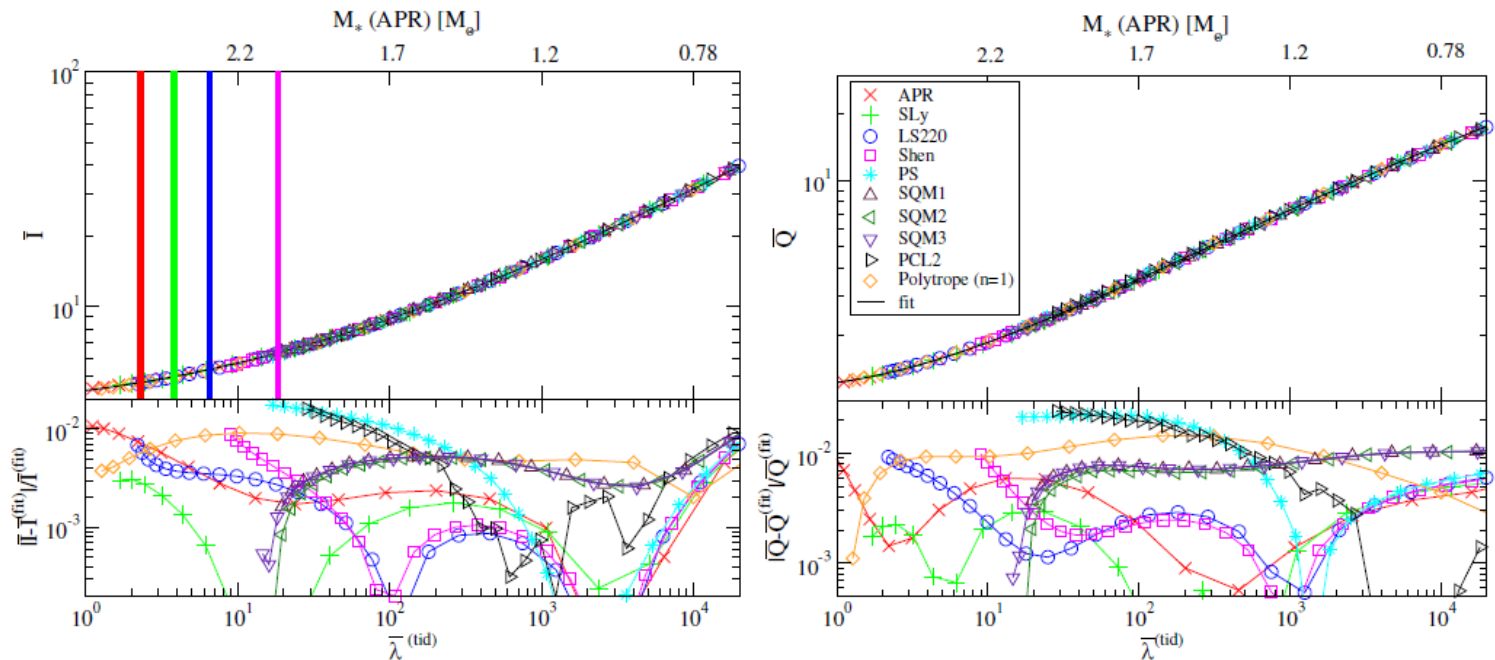
$$Q \sim R^5 \Omega^2$$

Tidal Love Numbers

$$\lambda \sim I^2 Q$$

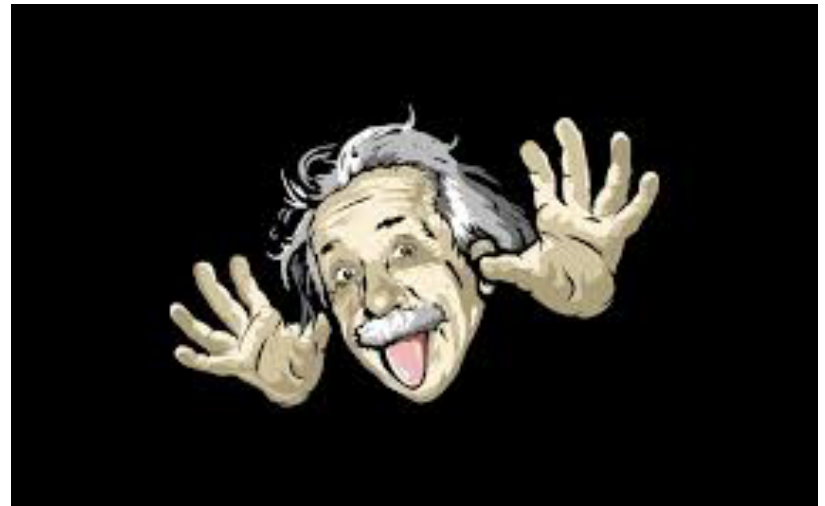
# I-Love-Q relations

EOS independent relations were derived by Yagi & Yunes(2013) for non-magnetized stars in the slow-rotation and small tidal deformation approximations.



... the relations proved to be valid (*with appropriate normalizations*) even for *fast rotating and magnetized stars*

# NEUTRON STARS & ALTERNATIVE THEORIES OF GRAVITY



# STT of gravity - Motivation

- The **Scalar Tensor Theory (STT)** is one of the most natural generalizations of the **Einstein's Theory of Gravity (ETG)**
- Their essence is in one or several scalar fields that are mediators of the gravitational interaction in addition to the spacetime metric of classical ETG
- Scalar fields appear in the reduction of the Kaluza-Klein theories to 4 dimensions, in string theory and in higher dimensional gravity but STT can be defined completely independently
- STT can be considered as an ETG with variable gravitational constant
- They fit to the observational data very well
- They are also an essential part of dark energy and dark matter models
- The  $f(R)$  theories are mathematically equivalent to the STT

# STT of gravity – Action

- **Physical (Jordan) frame action:**

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-\tilde{g}} [F(\Phi)\tilde{R} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi)] + S_m[\Psi_m; \tilde{g}_{\mu\nu}]$$

- **Einstein frame action (much simpler):**

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g} (R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)) + S_m[\Psi_m; \mathcal{A}^2(\varphi)g_{\mu\nu}]$$

– **Coupling function**

$$k(\varphi) = \frac{d \ln(A(\varphi))}{d\varphi}$$

$$A(\varphi) = e^{\frac{1}{2}\beta\varphi^2}$$

$$k(\varphi) = \beta\varphi$$

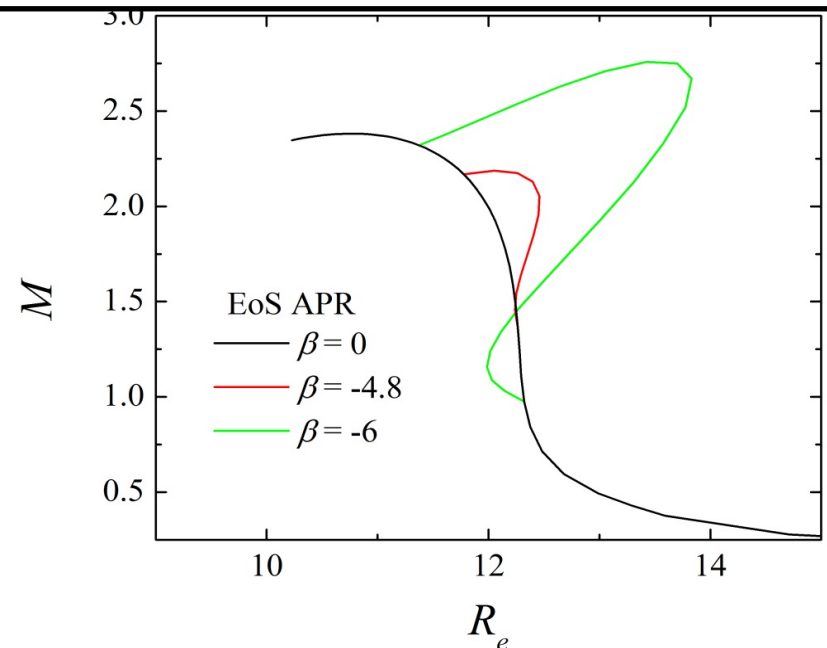
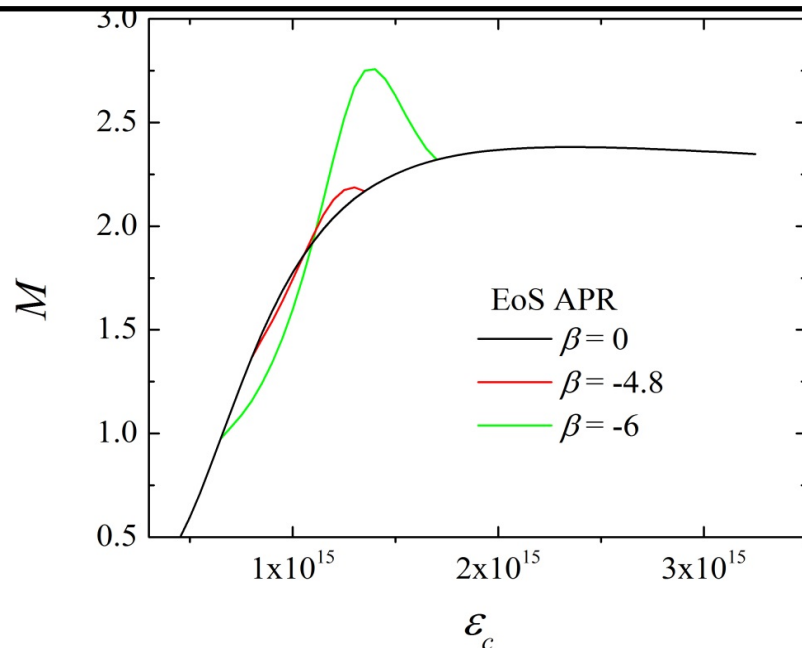
– **We set the potential to zero**

$$V(\varphi) = 0$$

# STT of gravity – Neutron Stars

**Spontaneous Scalarization** is possible for  $\beta < -4.35$   
(Damour+Esposito-Farese 1993)

## Properties of the **static** scalarized neutron stars



The solutions with nontrivial scalar field are *energetically more favorable* than their GR counterpart (Harada 1997, Harada 1998, Sotani+KK 2004).



# STT of gravity - Observations

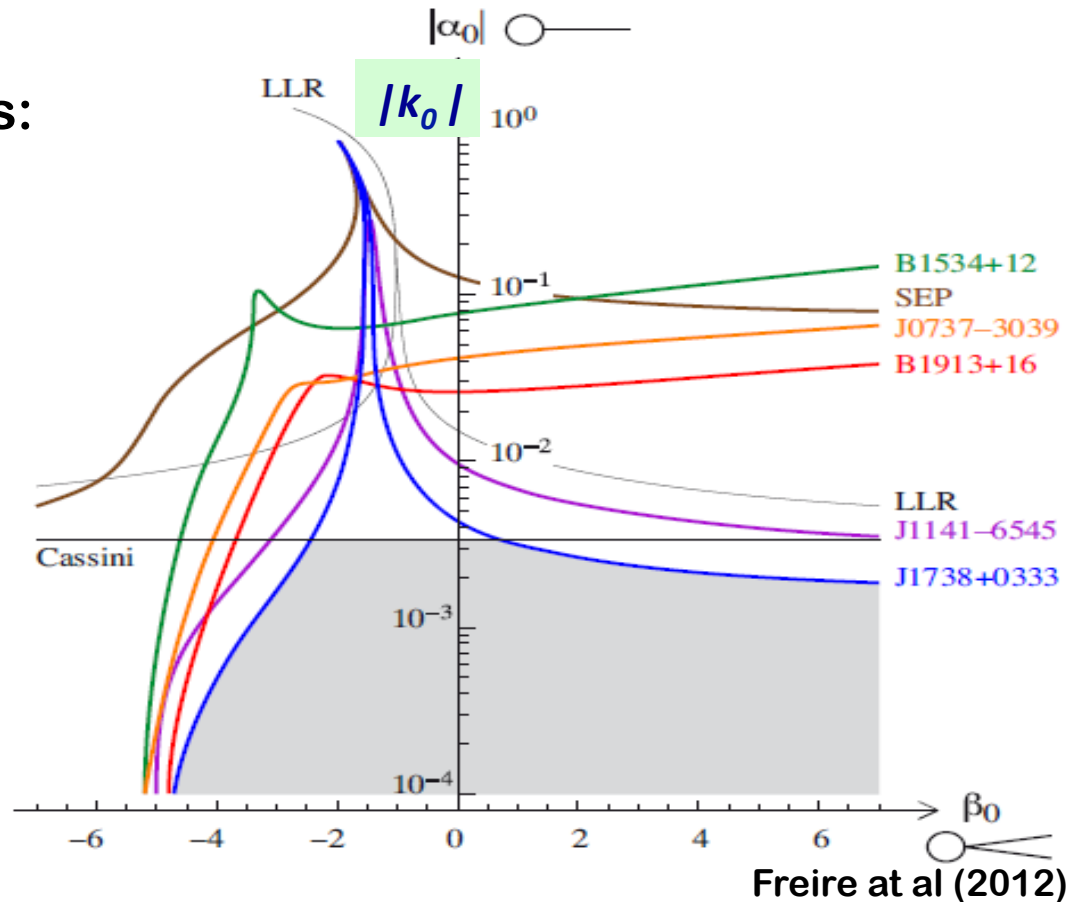
Observational constraints:

$$k_0 < 0.004$$

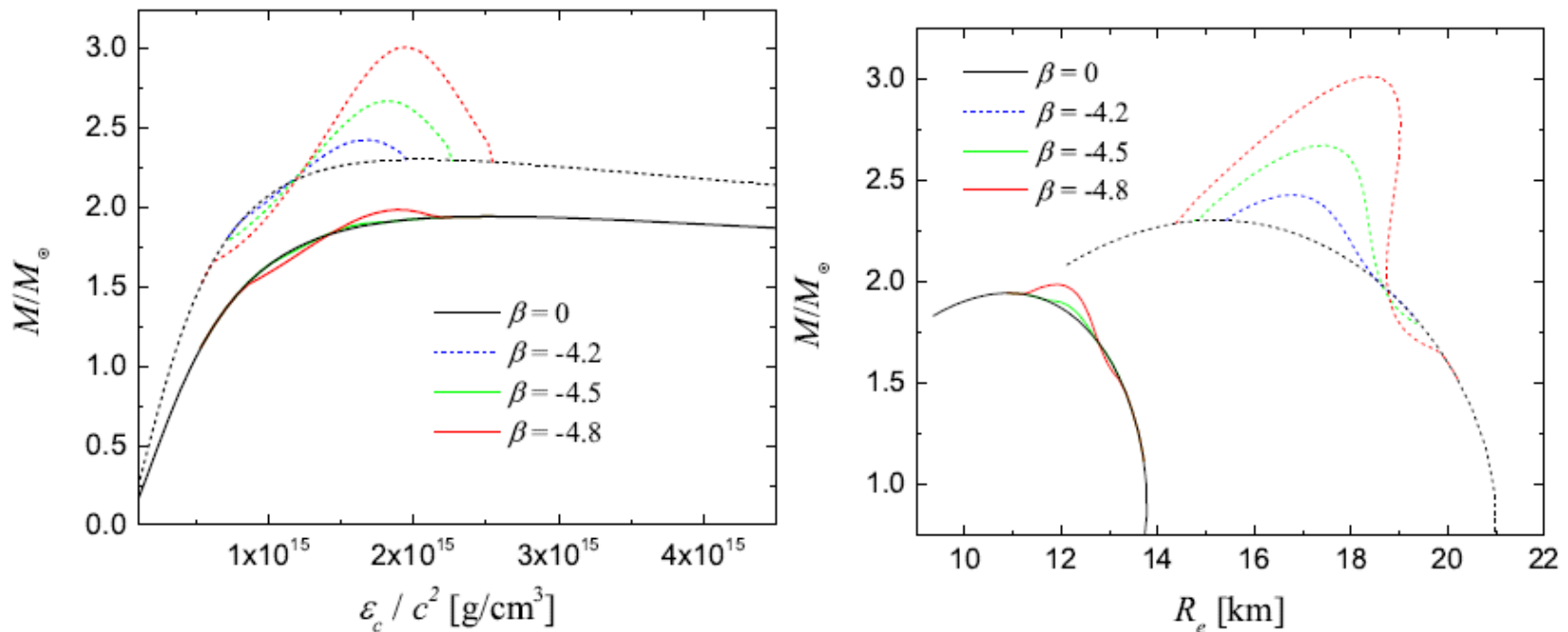
$$\beta > -4.8 \text{ (-4.5)}$$

Damour & Esposito-Farese (1996, 98)

Will (2006), Freire et al (2012)



# STT of gravity – Fast Rotating Stars



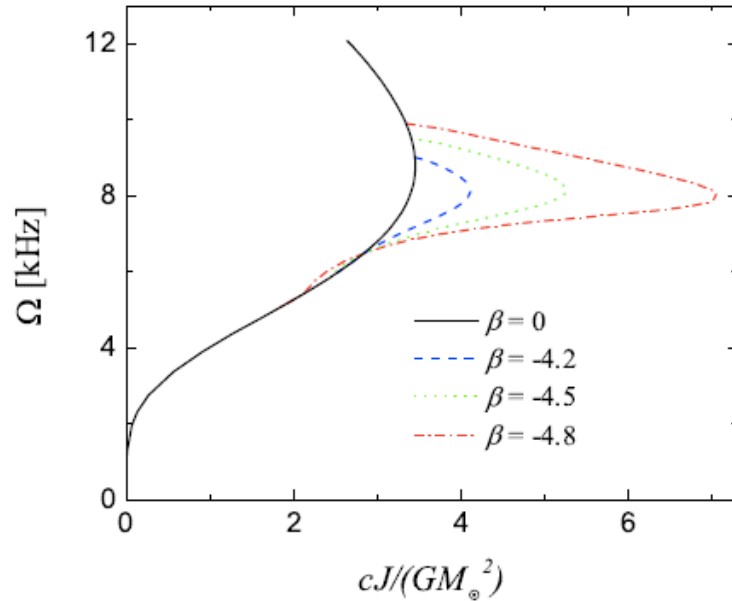
- The effect of scalarization is *much stronger* for fast rotation.
- Scalarized solutions exist for a *much larger range of parameters* than in the static case

Doneva, Yazadjiev, Stergioulas, Kokkotas 2013

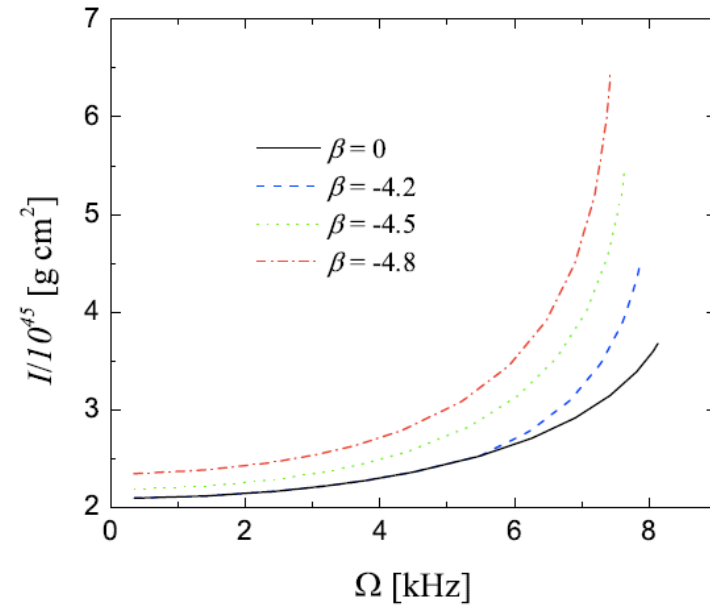
# STT of gravity

## Angular Momentum & Moment of Inertia

Sequences of models rotating at the Kepler limit



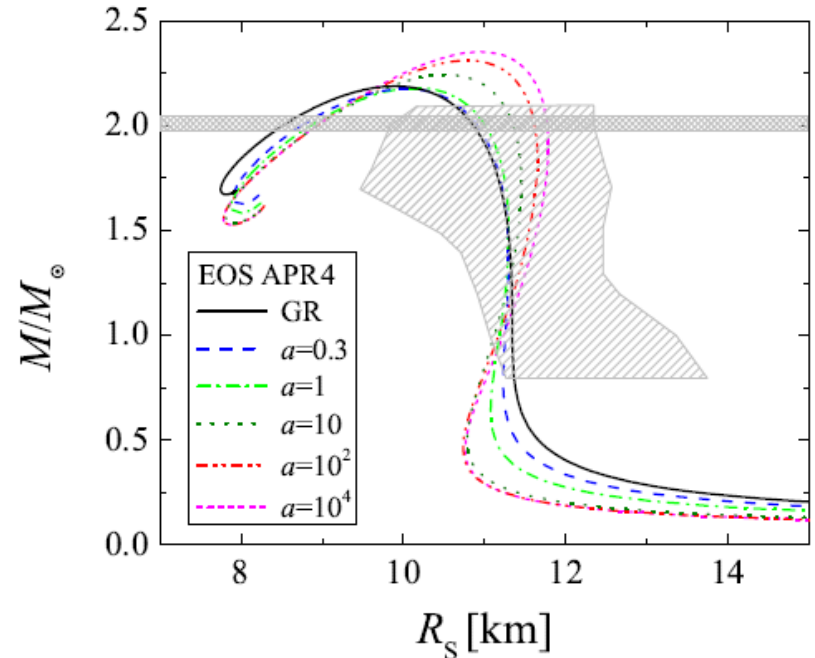
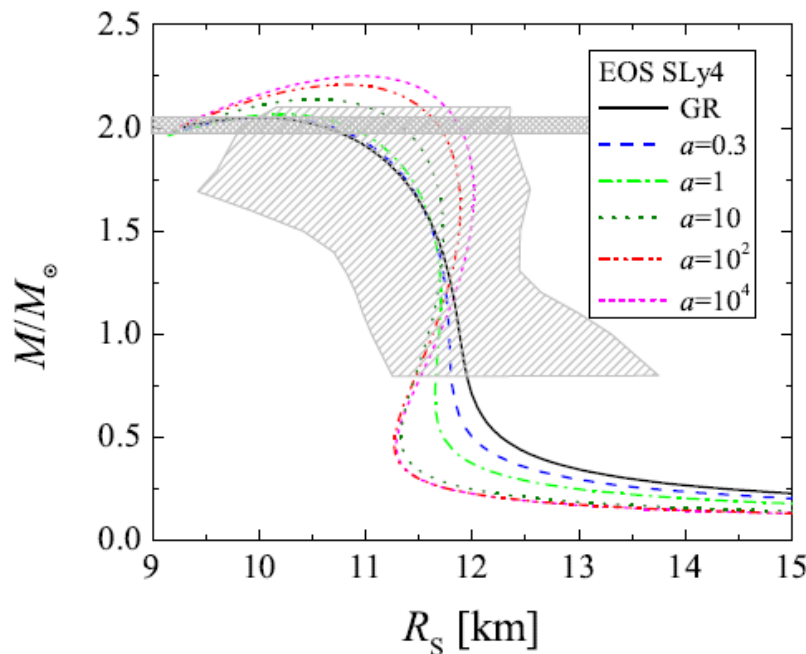
Models with constant central energy density



Not surprising that both **angular momentum** and **moment of inertia** could *differ twice* for scalarized solutions

# NSs in $f(R)$ -gravity: **Static Models**

$$f(R) = R + aR^2$$



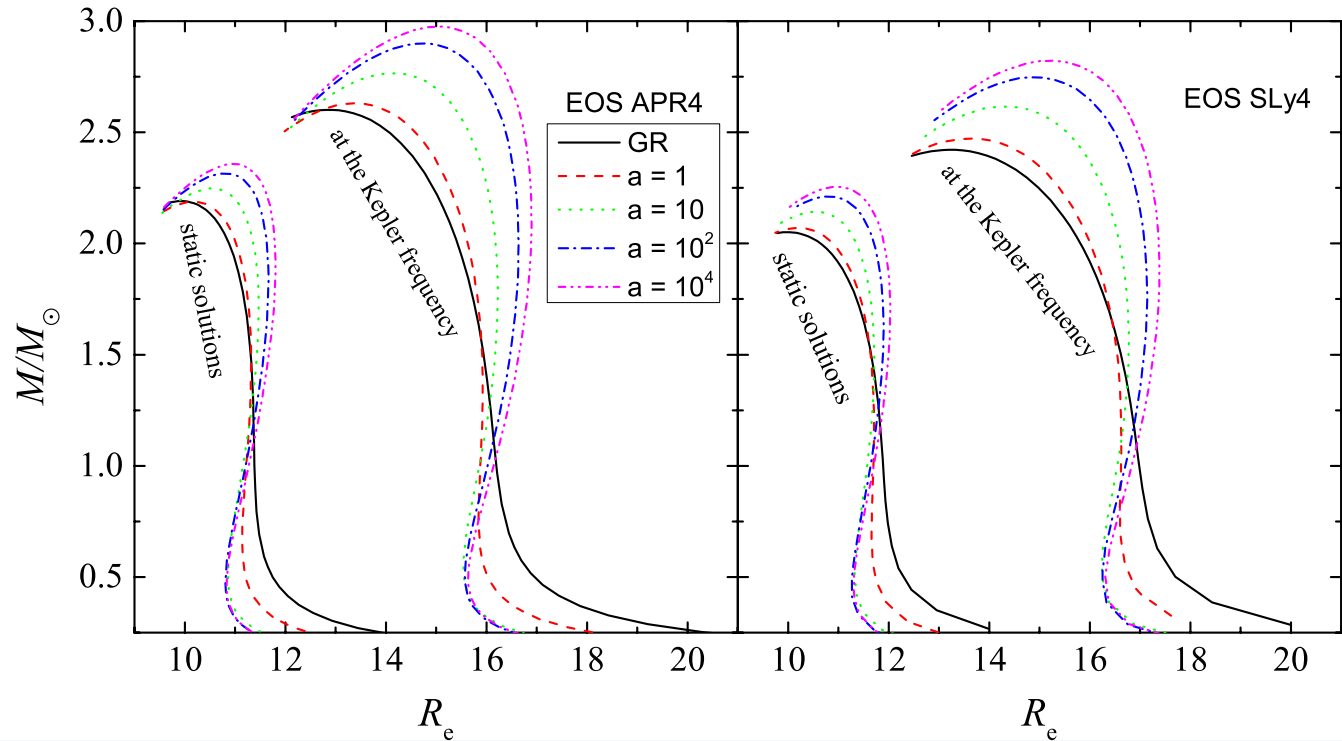
- The differences between the  $R^2$  and GR are comparable with the uncertainties in the nuclear matter equations of state.
- The current observations of the NS masses and radii alone can not put constraints on the value of the parameters  $a$ , **unless the EoS is better constrained in the future.**

Yazadjiev, Doneva, Kokkotas, Staykov (2014)

# NSs in $f(R)$ -gravity: Fast Rotation

$$f(R) = R + aR^2$$

Mass of radius diagrams for two realistic EOS



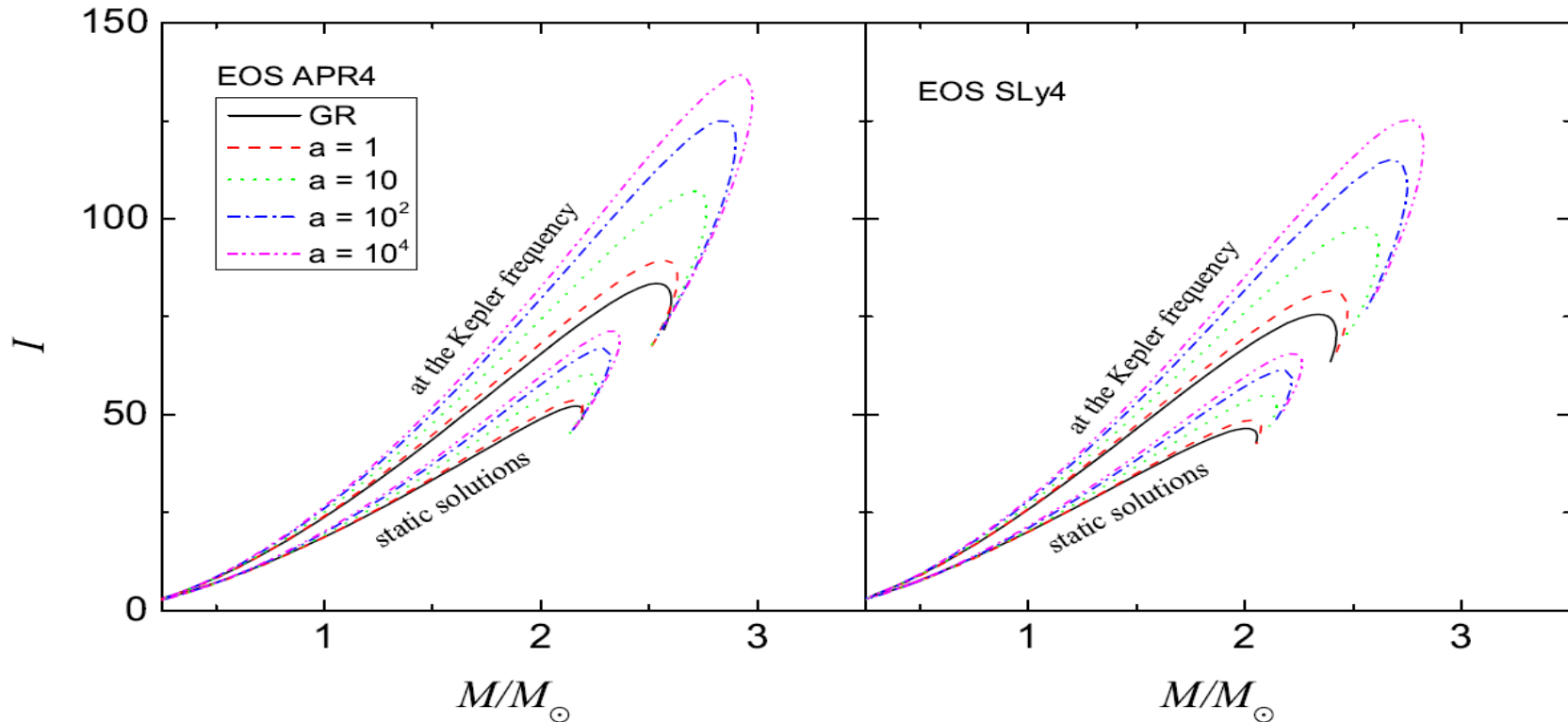
Difficult to set constraints on the  $f(R)$  theories using measurement of the neutron star  $M$  and  $R$  alone, until the EOS can be determined with smaller uncertainty.

Yazadjiev, Doneva, Kokkotas, (2015)

# NSs in $f(R)$ -gravity: Fast Rotation

$$f(R) = R + aR^2$$

Yazadjiev, Doneva, Kokkotas (2015) <sup>4</sup>



- ✓ The differences in the neutron star moment of inertia on the other hand can be much more dramatic.
- ✓ Large deviations can be potentially measured by the forthcoming observations of the NS moment of inertia [Lattimer-Schutz 2005, Kramer-Wex 2009] that can lead to a direct test of the  $R^2$  gravity.

# NSs in f(R)-gravity: I-Q relations / Fast Rotation

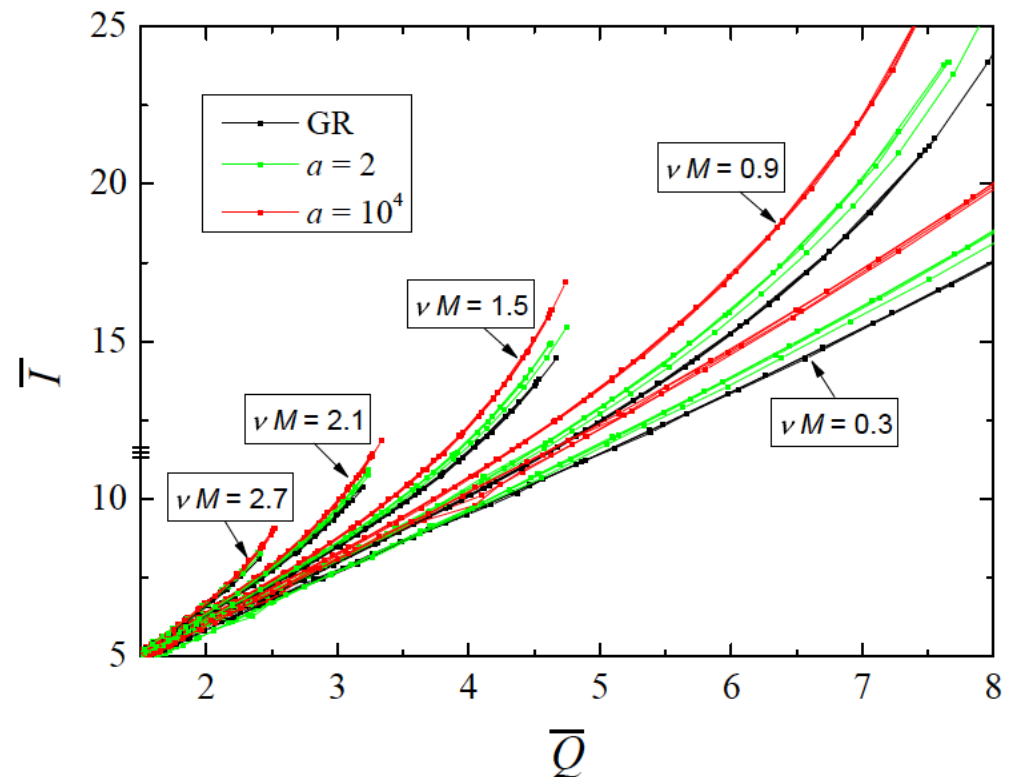
$$f(R) = R + aR^2$$

$$\bar{I} \equiv I / M^3$$

$$\bar{Q} \equiv Q / (M^3 \chi^2)$$

$$\chi \equiv J / M^2$$

$\nu$  : rot. frequency (Hz)



Doneva, Yazadjiev, Kokkotas (2015)

- The results show that the I-Q relation remain **nearly EoS independent** for fixed values of the normalized rotational parameter
- The differences with the pure Einstein's theory can be large reaching **above 20%** for **lower masses** and **slow rotation**.

# Neutron Star “ringing”

**p-modes:** main restoring force is the pressure (**f-mode**) ( $>1.5$  kHz)

$$\sigma \approx \sqrt{\frac{GM}{R^3}}$$

**Inertial modes: (r-modes)** main restoring force is the Coriolis force

$$\sigma \approx \Omega$$

**w-modes:** pure space-time modes (only in GR) ( $>5$  kHz)

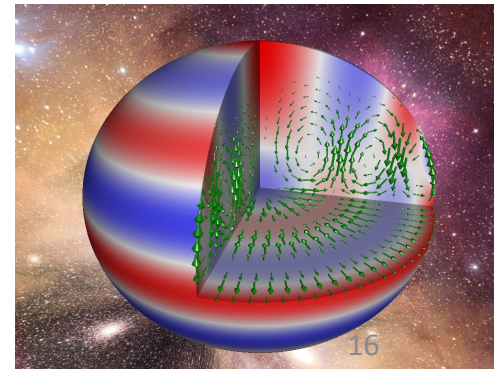
$$\sigma \approx \frac{1}{R} \left( \frac{GM}{Rc^2} \right)$$

**Torsional modes (t-modes)** ( $>20$  Hz) shear deformations. Restoring force, the weak Coulomb force of the crystal ions.

$$\sigma \approx \frac{v_s}{R} \sim 16 \ell \text{ Hz}$$

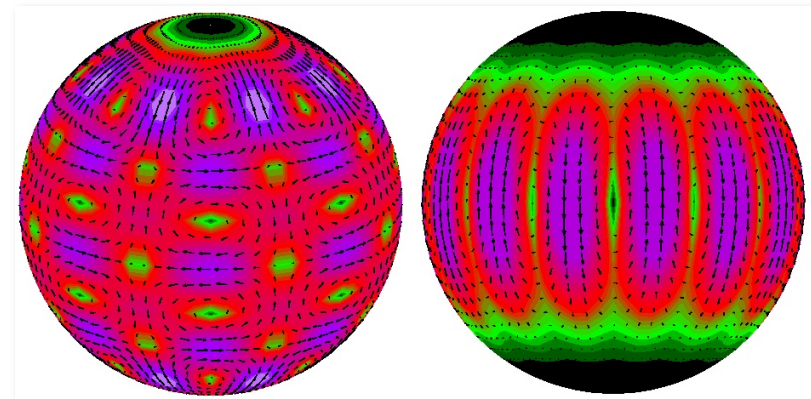
... and many more

shear, g-, Alfven, interface, ... modes



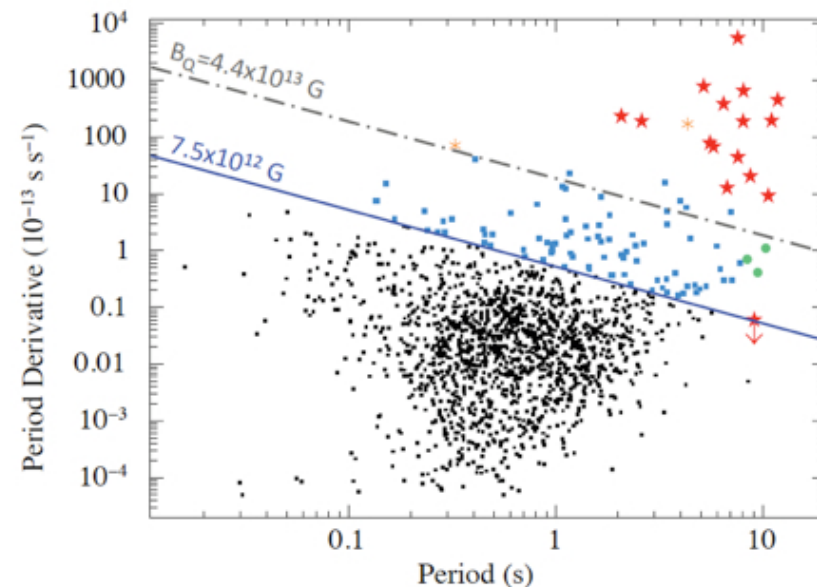


# MAGNETARS: A PROMISING CASE FOR ASTEROSEISMOLOGY



# Magnetars

- Young, slowly spinning ( $P \sim 10\text{s}$ ) systems (**20+**)
- Exhibit regular  $\gamma$ -ray flares
  - Believed to be powered by magnetic field
  - Either trigger or are preceded by starquakes
  - Some linked to glitches or **anti-glitches**
- Three giant flares observed with peak luminosities  $\sim 10^{47}$  erg/s
  - March 5, 1979 : SGR 0526-66
  - August 27, 1998 : SGR 1900+14
  - December 27, 2004: SGR 1806-20
  - Recently few medium ones
- Giant flares
  - QPOs – 10's -100's of Hz
  - Magnetic field reconstruction



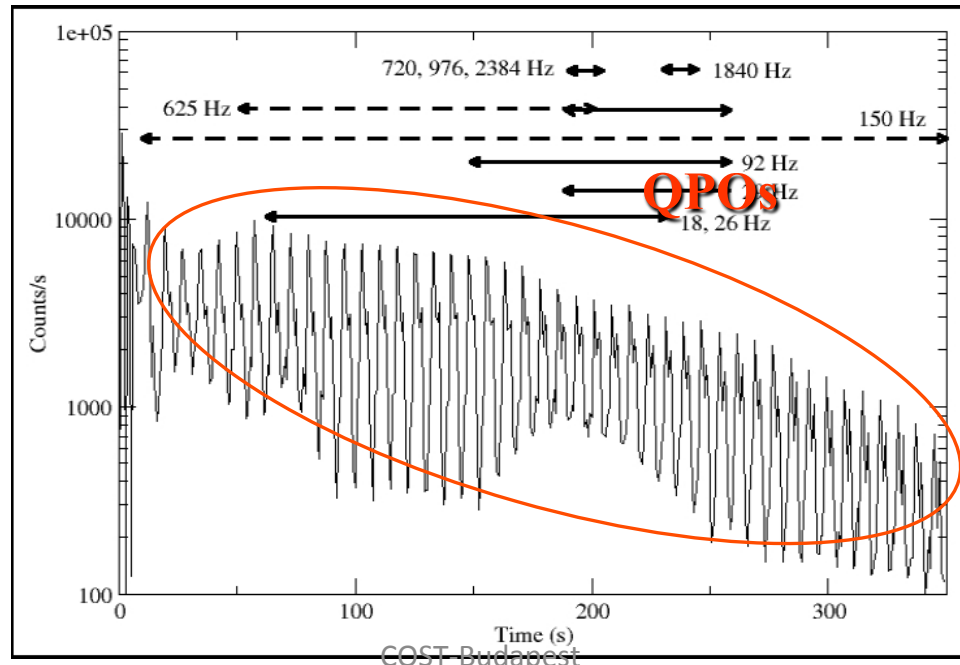
# Magnetars: Quasi-Periodic Oscillations

## ✓ Giant flares in SGRs

- A decaying tail for several hundred seconds follows the flare.

## ✓ QPOs in decaying tail (Israel *et al.* 2005; Watts & Strohmayer 2005, 2006)

- **SGR 1900+14** : 28, 54, 84, and 155 Hz
- **SGR 1806-20** : **18, 26**, 29, 92.5, 150, 626.5, 720, 976, 1837, 2384 Hz
- **SGR 1806-20** : Additional frequencies **22, 16, 116 Hz**, also **720 & 2384 Hz**;  
(Hambaryan, Neuhaeuser, Kokkotas 2011)



# Alfven Continuum and/or Discrete oscillations

## Only Crust Oscillations

- Sotani, Kokkotas, Stergioulas 2007,2008
- Samuelsson, Andersson 2007
- Sotani, Colaiuda, Kokkotas 2008
- Steiner, Watts 2009
- ...
- Sotani etal 2012-15

## Without Crust

- Levin 2007
- Sotani, Kokkotas, Stergioulas 2008
- Colaiuda, Beyer, Kokkotas 2009
- Cerda-Duran, Stergioulas, Font 2009

## Fluid + Crust

- Van Hoven, Levin 2011, 2012
- Cerda-Duran, Stergioulas, Font 2011
- Colaiuda, Kokkotas 2011
- Gabler etal 2012
- Gabler etal 2013

## Superfluidity

- Passamonti, Lander 2012
- Sotani etal 2013
- Gabler etal 2013

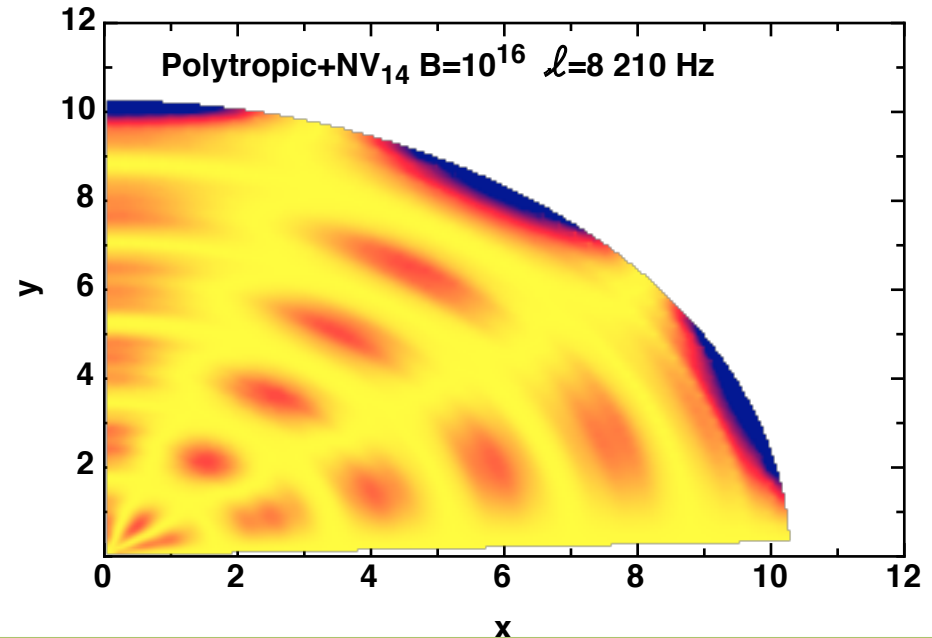
## Mixed axial-polar

- Colaiuda, Kokkotas 2012
- Lee, Yoshida 2015

## Non-axisymmetric

- Sotani, Kokkotas 2012

15.06.2015



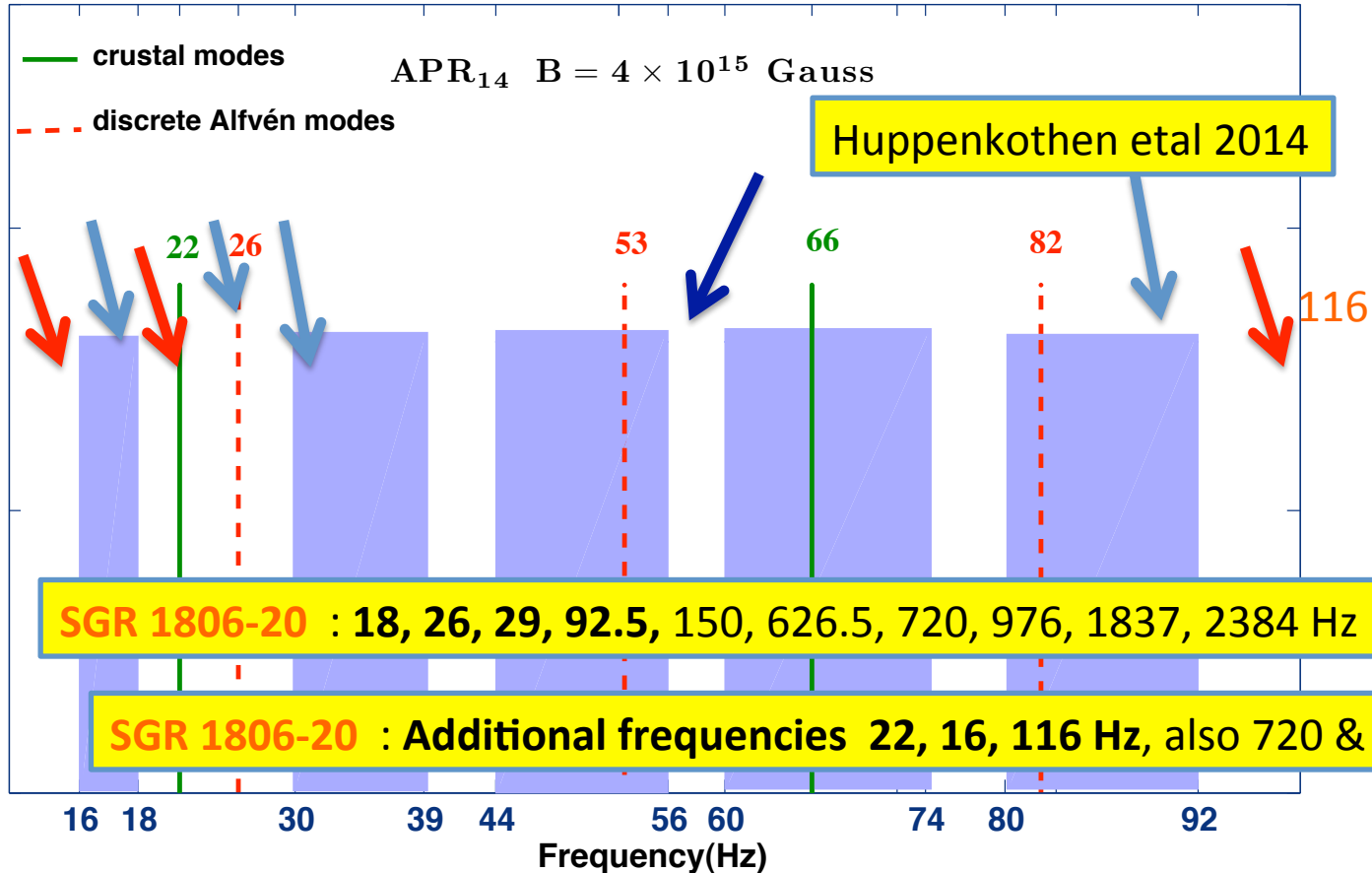
✓ The combination of **poloidal + toroidal** magnetic fields **+crust** leads to **PURE discrete** spectrum

✓ The main results of the magnetar seismology remain unchanged !

(Colaiuda-KK 2012)

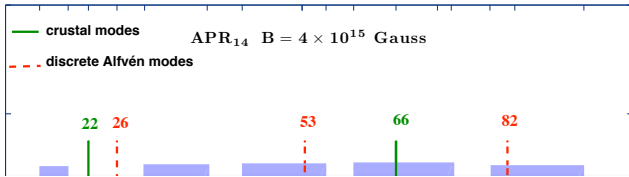
# SGR 1806-20

Colaiuda, KK (2011-12)



# SGR 1806-20

Colaiuda, KK (2011-12)



Do we understand how the QPOs are excited?

The answer is **NO! (?)**

Great progress in the last 7-8 years

A new event of the type of SGR 1806-20, might be catalytic for understanding:

- The mechanism that triggers the hyperflares
- The QPOs in the decaying tail
- The EOS, the Mass, Radius, B-field of magnetars

...and **predicts new that found** after careful analysis of data

Hambaryan, Kokkotas, Neuhauser (2012)

**EoS :** APR (NV)  
**Mass:**  $M = 1.4M_{\odot}$   
**Radius:** 11.57 km  
**B-field :**  $2 \times 10^{15}$  Gauss  
**Crust :** 0.099 R

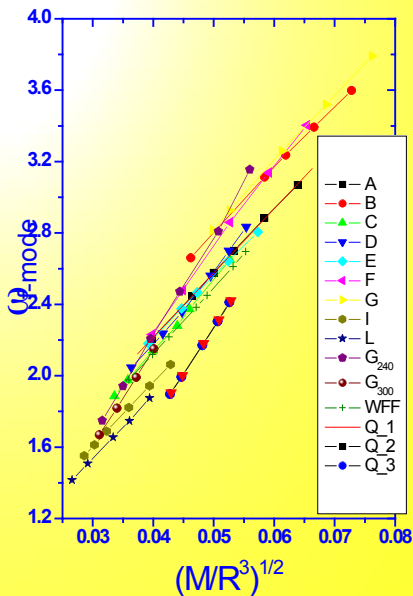
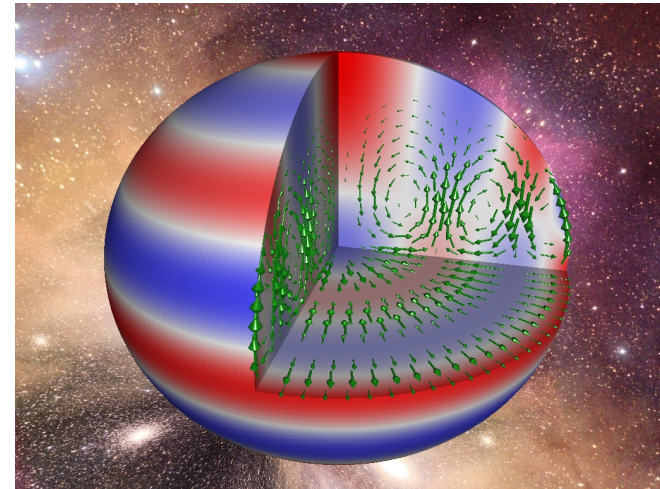
oscillations both poloidal & toroidal B-fields **are unstable!**

The observed QPOs or some of them can be due to magnetospheric phenomena (breathing of the fireball)

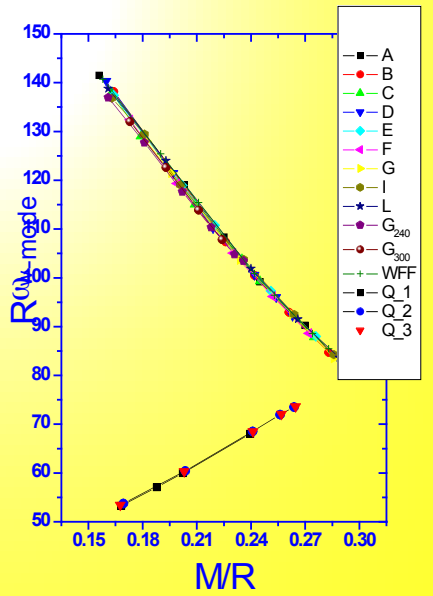
# Gravitational Wave Asteroseismology

Oscillation patterns can reveal the internal structure of neutron stars :

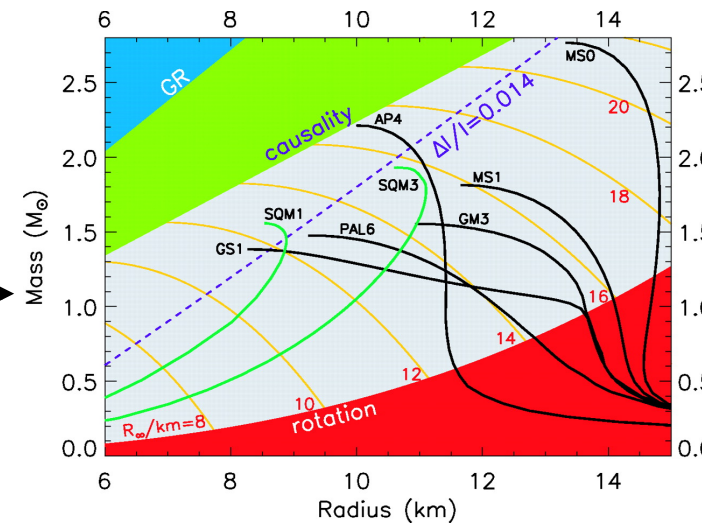
- ✓ mass,
- ✓ radius,
- ✓ EoS,
- ✓ rotation,
- ✓ B-field,
- ✓ crust,...



+



Andersson, Kokkotas 1996, 1998, 2001



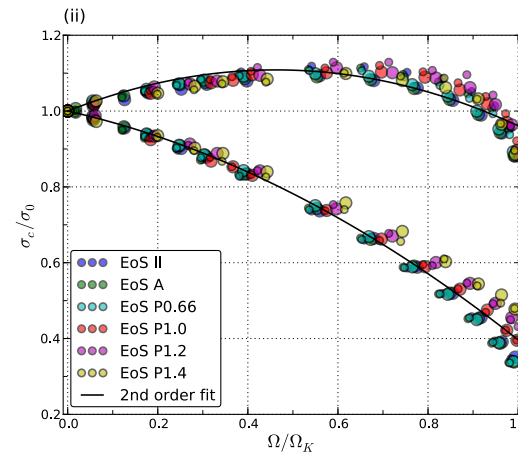
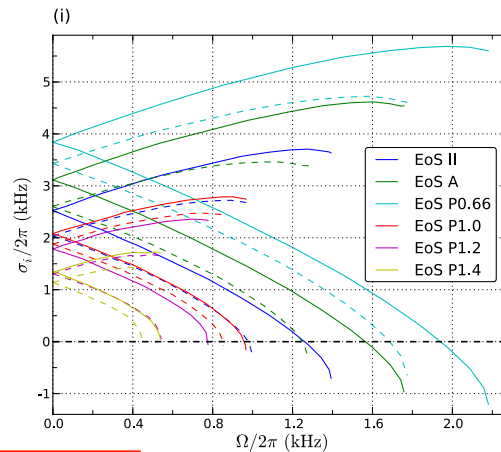
Lattimer+Prakash 2007

# f-modes: Asteroseismology

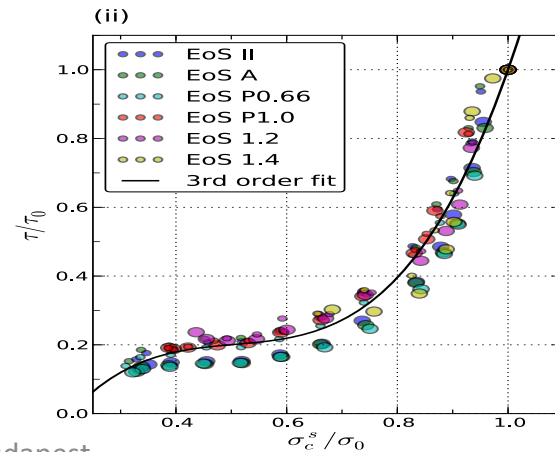
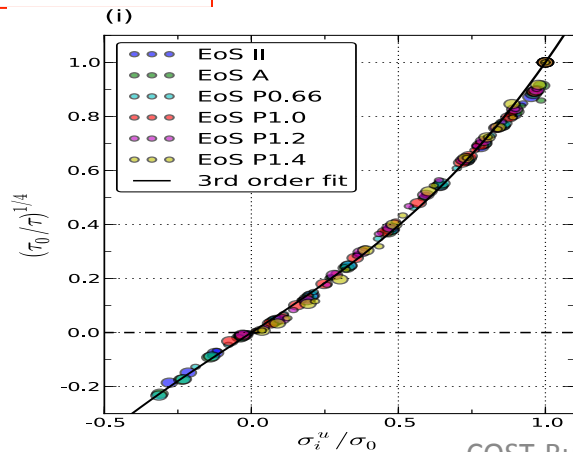
We can produce **empirical relation** relating the parameters of the *rotating neutron stars* to the observed frequencies.

Gaertig-Kokkotas 2008, 2010, 2011

Frequency



Damping/Growth time



Cowling Approximation



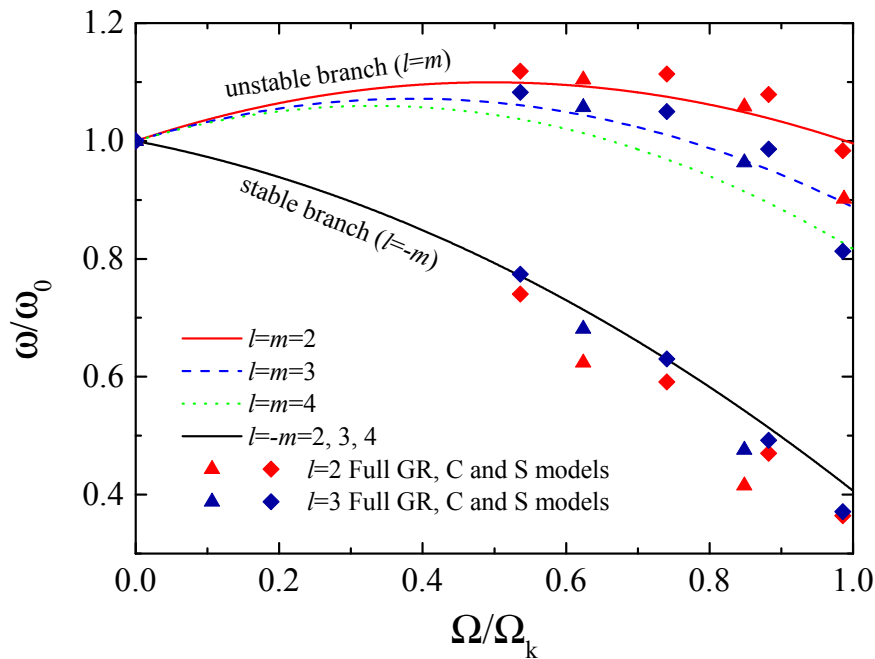
# Asteroseismology: Realistic EoS

Doneva, Gaertig, KK, Krüger (2013)

$$\left(\frac{\omega_c}{\omega_0}\right)_{\ell=2,3,4} \approx f\left(\frac{\Omega}{\Omega_K}\right)$$

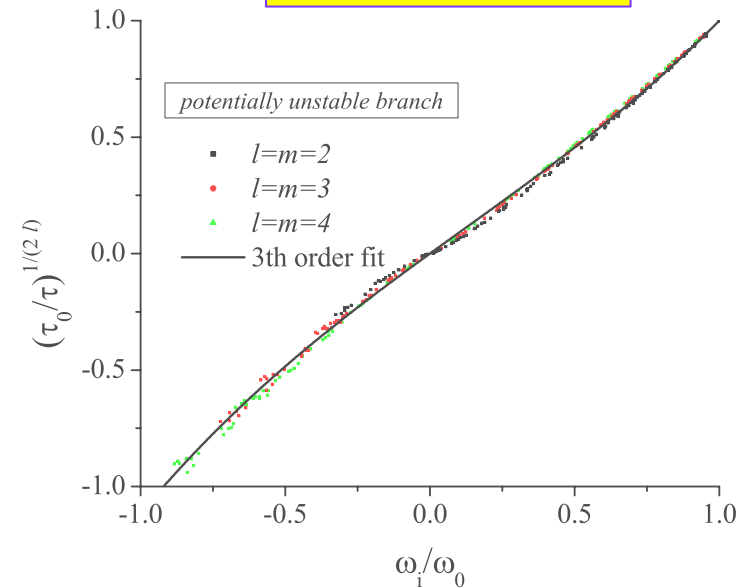
Nearly “universal” fitting formulae for :

- the frequencies
- the damping times
- Independent of GR or Cowling



Oscillation frequencies

$$\left(\frac{\tau_0}{\tau}\right)^{1/2\ell} \approx f\left(\frac{\omega_i}{\omega_0}\right)$$



Damping/Growth Times

# Asteroseismology

Stable Branch

$$\frac{\omega_c^s}{\omega_0} = 1 - 0.235 \left( \frac{\Omega}{\Omega_K} \right) - 0.358 \left( \frac{\Omega}{\Omega_K} \right)^2$$

Unstable Branch

$$\frac{\omega_{c\ l=2}^u}{\omega_0} = 1 + 0.402 \left( \frac{\Omega}{\Omega_K} \right) - 0.406 \left( \frac{\Omega}{\Omega_K} \right)^2$$

$$\frac{\omega_{c\ l=3}^u}{\omega_0} = 1 + 0.373 \left( \frac{\Omega}{\Omega_K} \right) - 0.485 \left( \frac{\Omega}{\Omega_K} \right)^2$$

$$\frac{\omega_{c\ l=4}^u}{\omega_0} = 1 + 0.360 \left( \frac{\Omega}{\Omega_K} \right) - 0.543 \left( \frac{\Omega}{\Omega_K} \right)^2$$

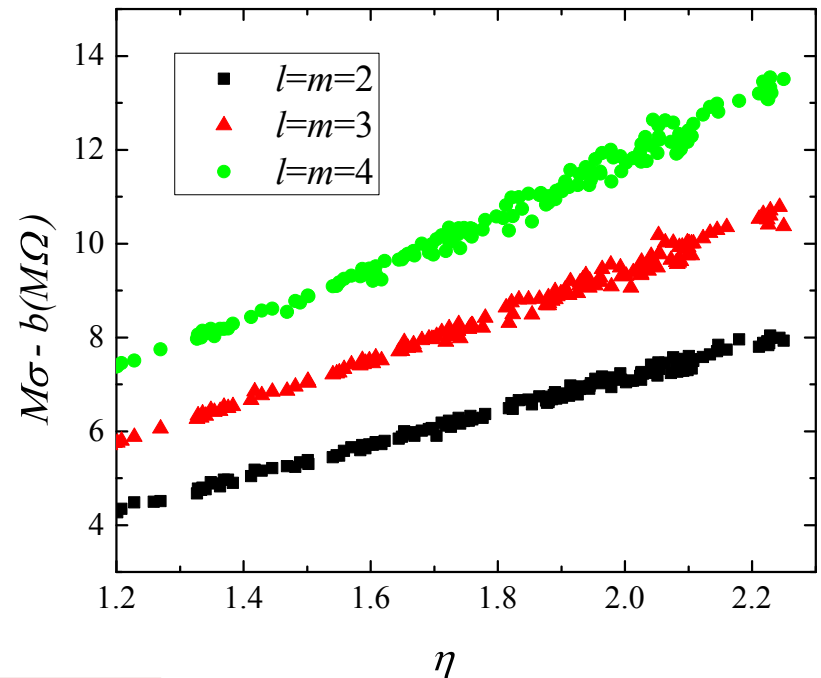
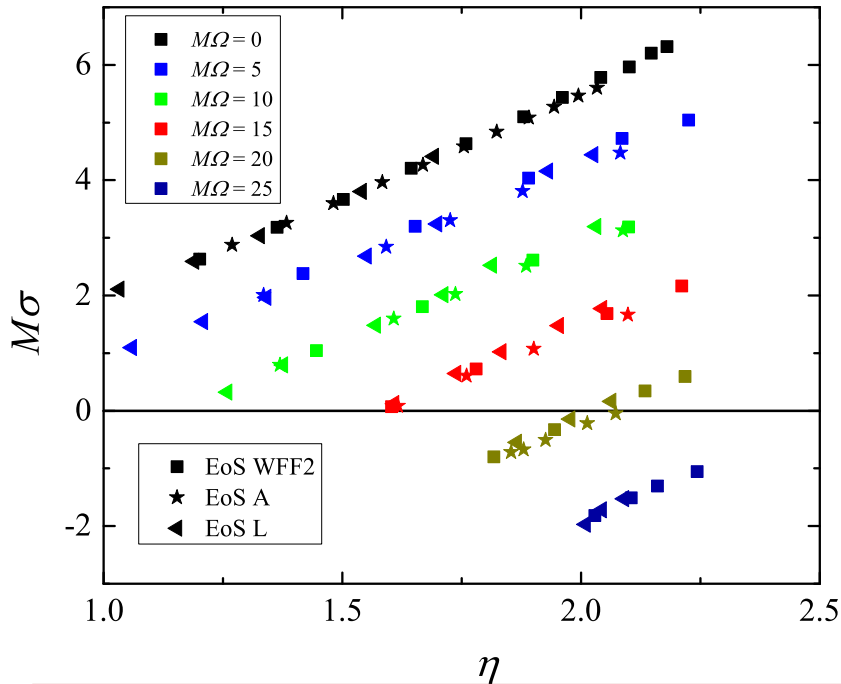
Unstable Branch

$$\frac{\tau_0}{\tau} = \text{sgn}(\omega_i^u) \left( 0.900 \left( \frac{\omega_i^u}{\omega_0} \right) - 0.057 \left( \frac{\omega_i^u}{\omega_0} \right)^2 + 0.157 \left( \frac{\omega_i^u}{\omega_0} \right)^3 \right)^{2l}$$

Doneva, Gaertig, KK, Krüger (2013)

# Asteroseismology: alternative scalings

$$M\sigma_i^{unst} = [(0.56 - 0.94\ell) + (0.08 - 0.19\ell)M\Omega + 1.2(\ell + 1)\eta]$$

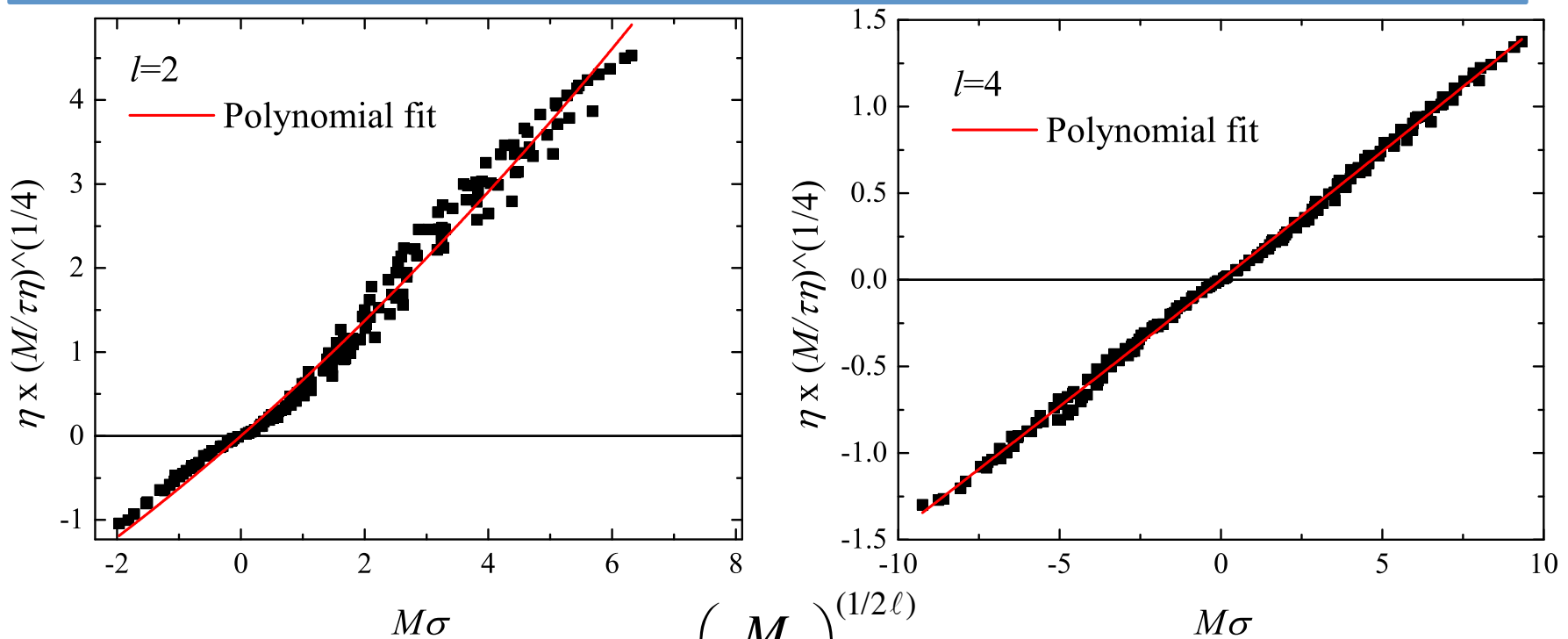


The  $l = 2$  f-mode oscillation frequencies as functions of the parameter  $\eta$

$$\eta = \sqrt{M^3 / I}$$

Doneva-KK 2015

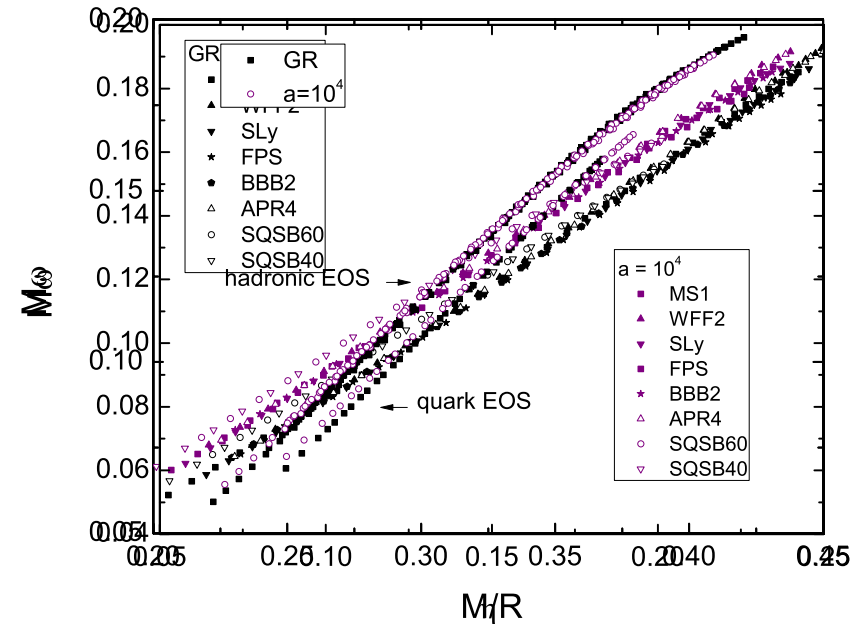
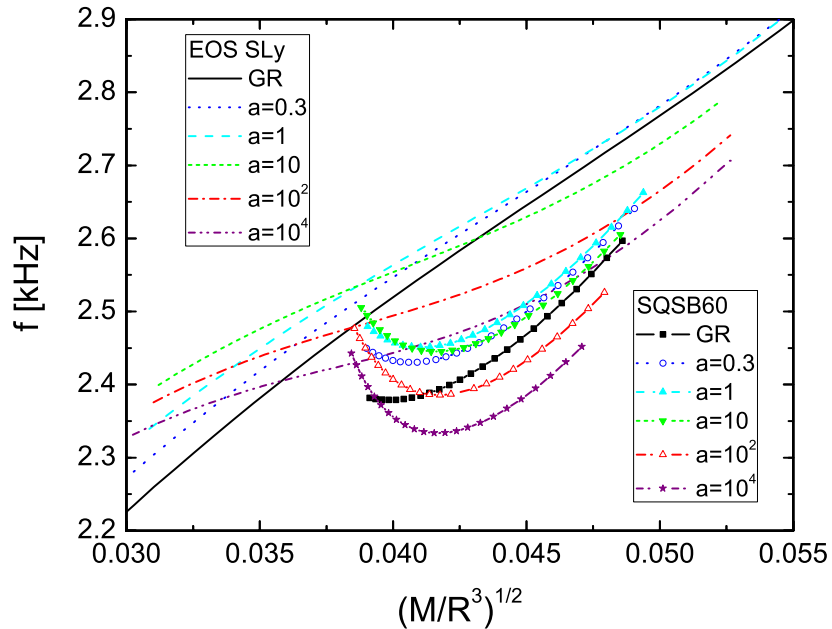
# Asteroseismology: alternative scalings



The normalized damping time  $\eta \left( \frac{M}{\tau\eta^2} \right)^{(1/2\ell)}$  as a function of the normalized oscillation frequency  $M\sigma$  for  $l = m = 2$  &  $l = m = 4$  f-modes.

$\eta = \sqrt{M^3 / I}$

# Asteroseismology: Alternative Theories of Gravity



- The maximum deviation between the f-mode frequencies in GR and  $R^2$  gravity is up to **10%** and depends on the value of the  $R^2$  gravity parameter  $a$ .
- Alternative normalizations show nicer relations

$$\eta = \sqrt{M^3 / I}$$

# The CFS instability

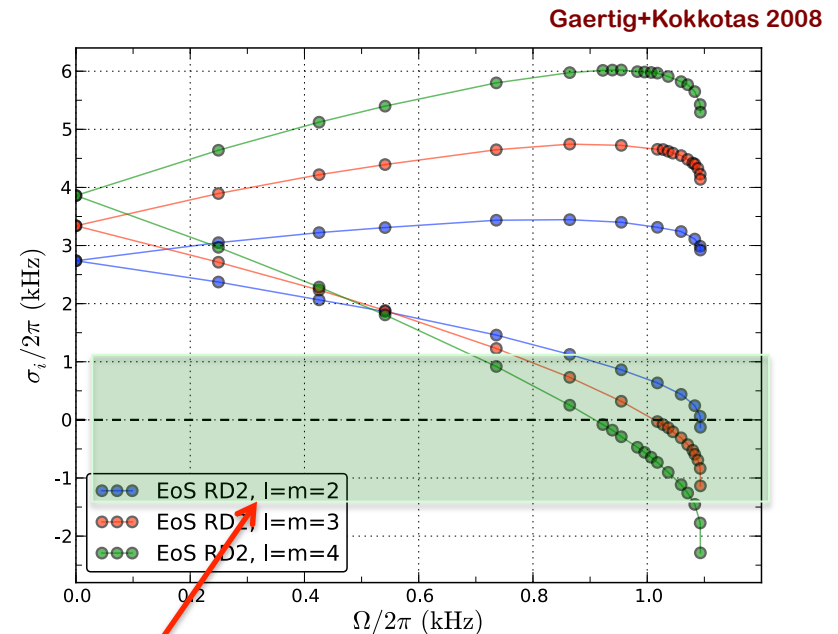
**Chandrasekhar 1970:** Gravitational waves lead to a secular instability

**Friedman & Schutz 1978:** The instability is generic, modes with sufficiently large  $m$  are unstable.

**A neutral mode of oscillation signals the onset of CFS instability**

- ✓ Radiation drives a mode unstable if the mode pattern moves backwards according to an observer on the star ( $J_{rot} < 0$ ), but forwards according to someone far away ( $J_{rot} > 0$ ).
- ✓ They radiate positive angular momentum, thus in the rotating frame the angular momentum of the mode increases leading to an increase in mode's amplitude.

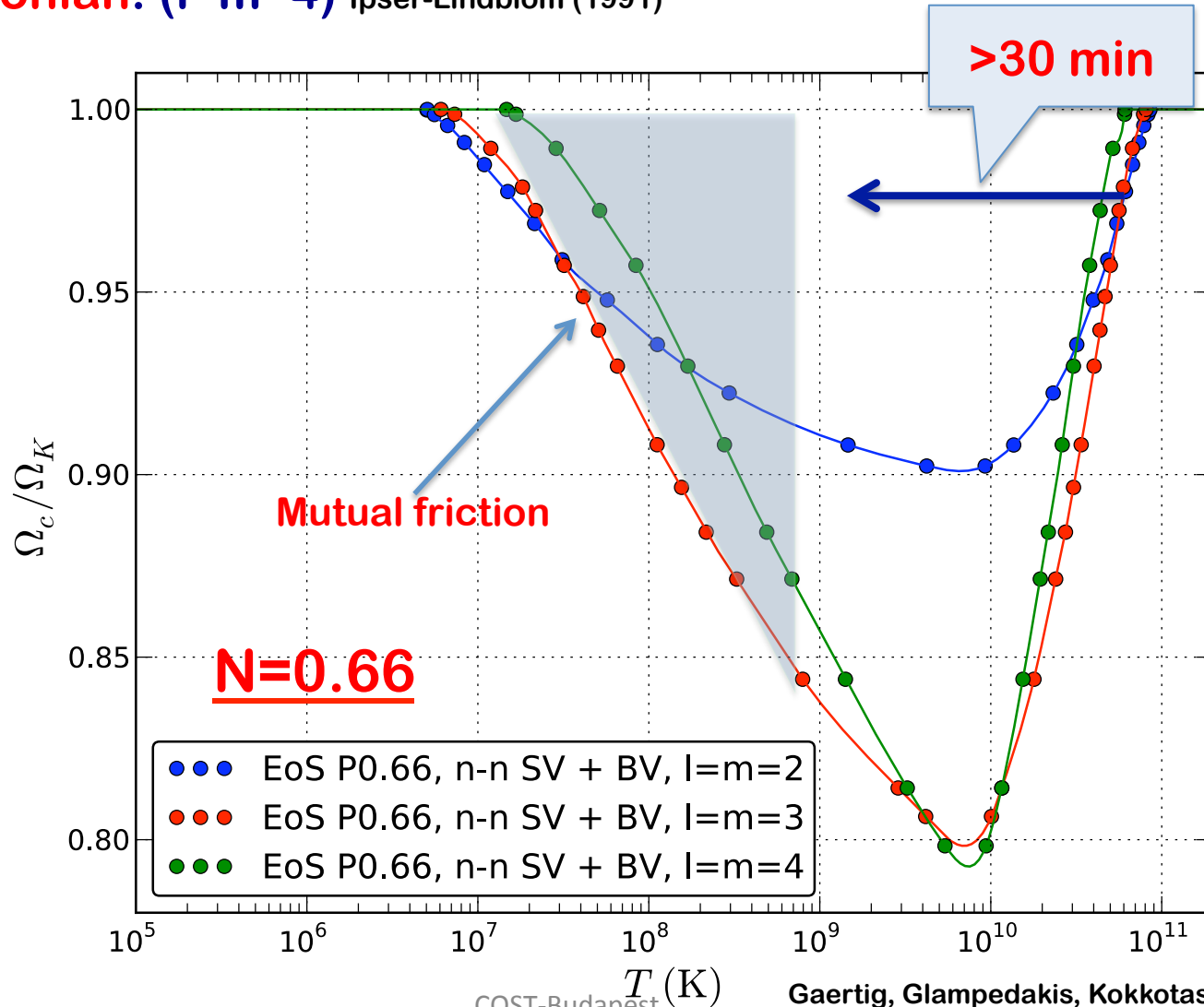
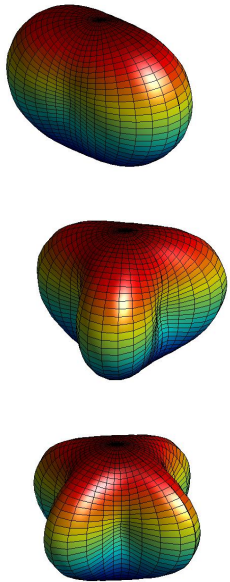
$$\frac{\omega_{in}}{m} = -\frac{\omega_{rot}}{m} + \Omega$$



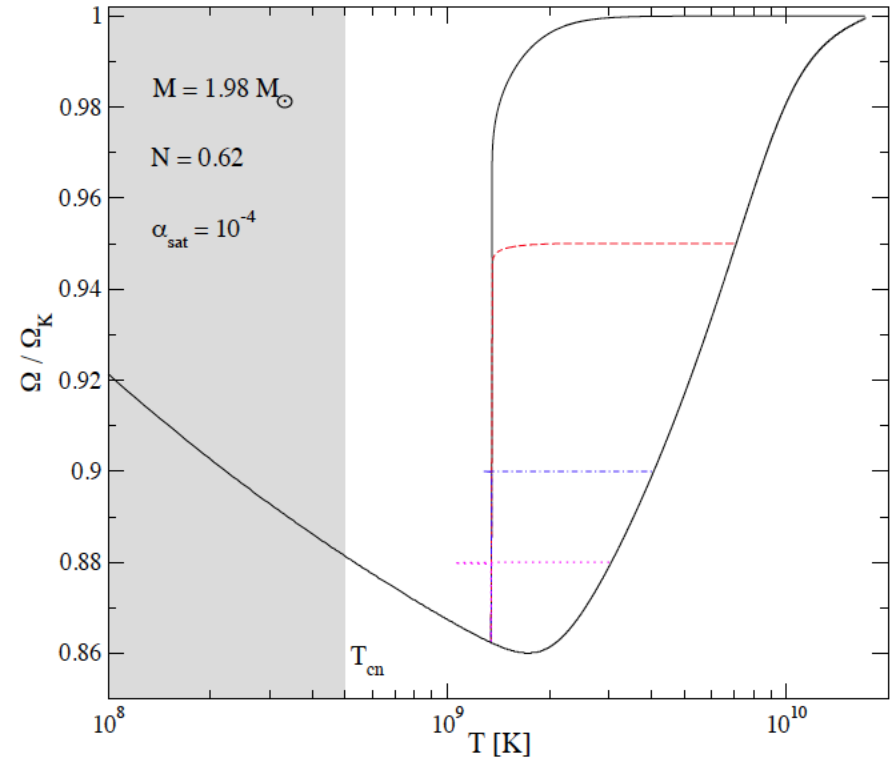
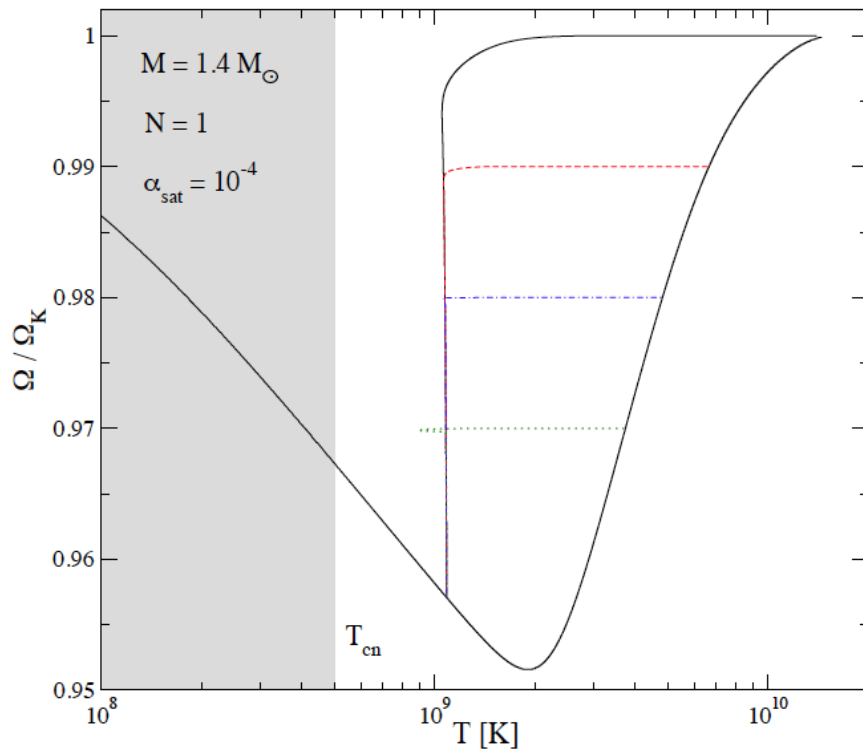
**LIGO/Virgo/GEO/KAGRA/ET band**

# Instability Window

- ✓ For the **first time** we have the window of f-mode instability in **GR**
- ✓ **Newtonian:** ( $l=m=4$ ) Ipser-Lindblom (1991)



# Evolution of a nascent (unstable) NS



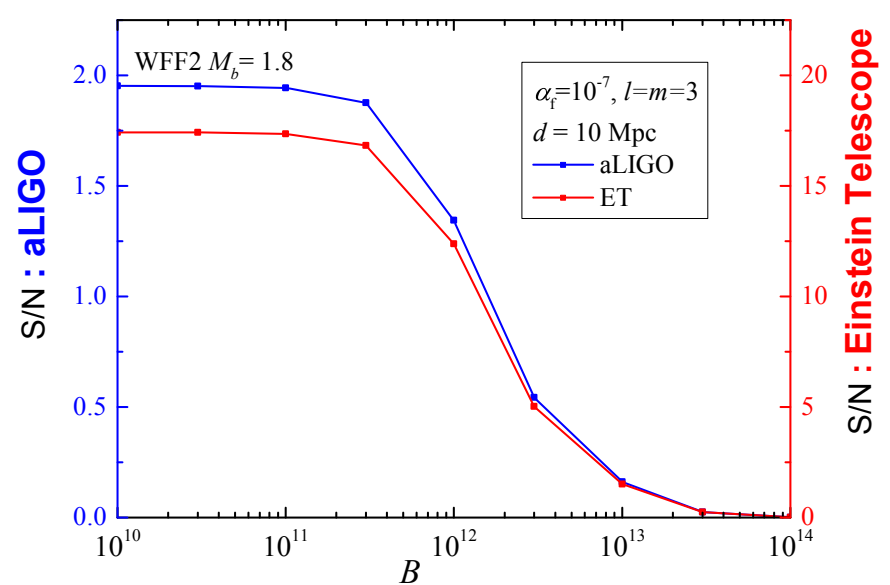
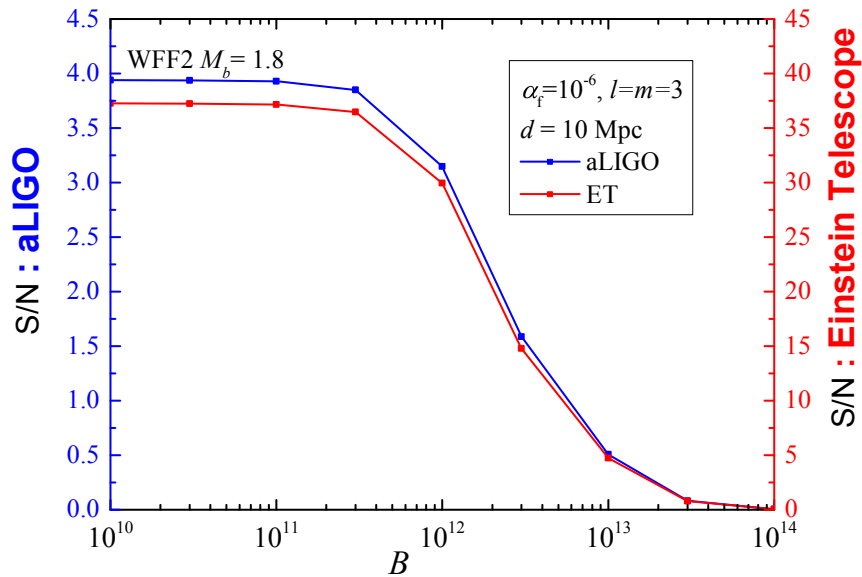
**Mutual Friction plays NO ROLE for the f-mode instability**

Procedure as described in Owen et al 1998 & Anderson, Jones, KK 2002

Passamonti-Gaertig-KK-Doneva (2013)



# Evolution of a nascent (unstable) NS



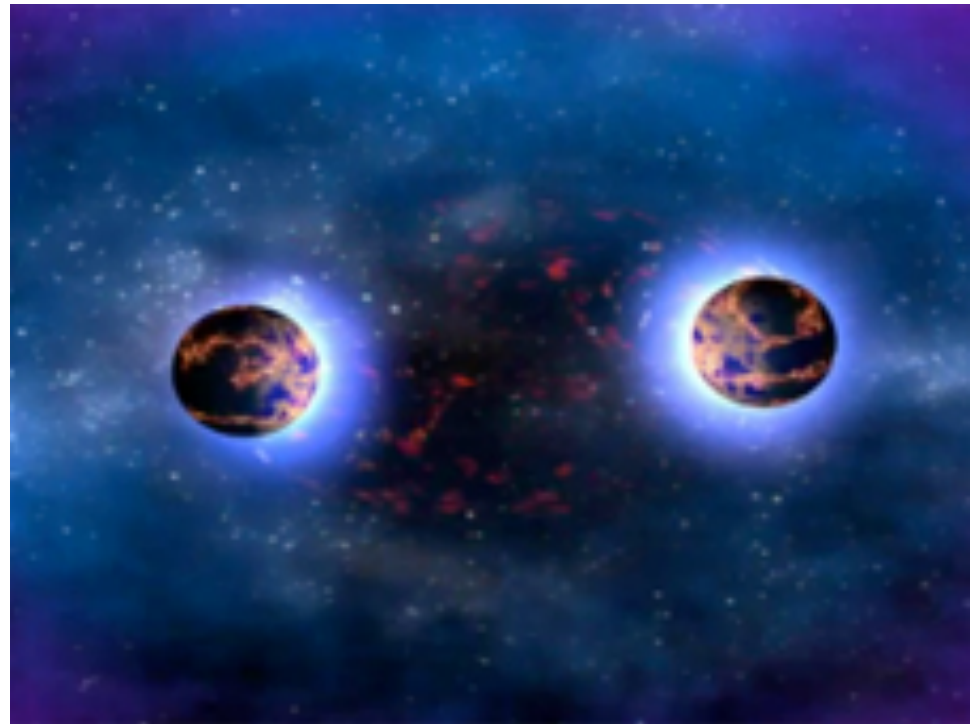
The instability can be potentially observed by events in Virgo cluster

**BUT**

- Event rate is unknown
- Saturation amplitude is **varying during the process**

Passamonti-Gaertig-Kokkotas-Doneva (2013)

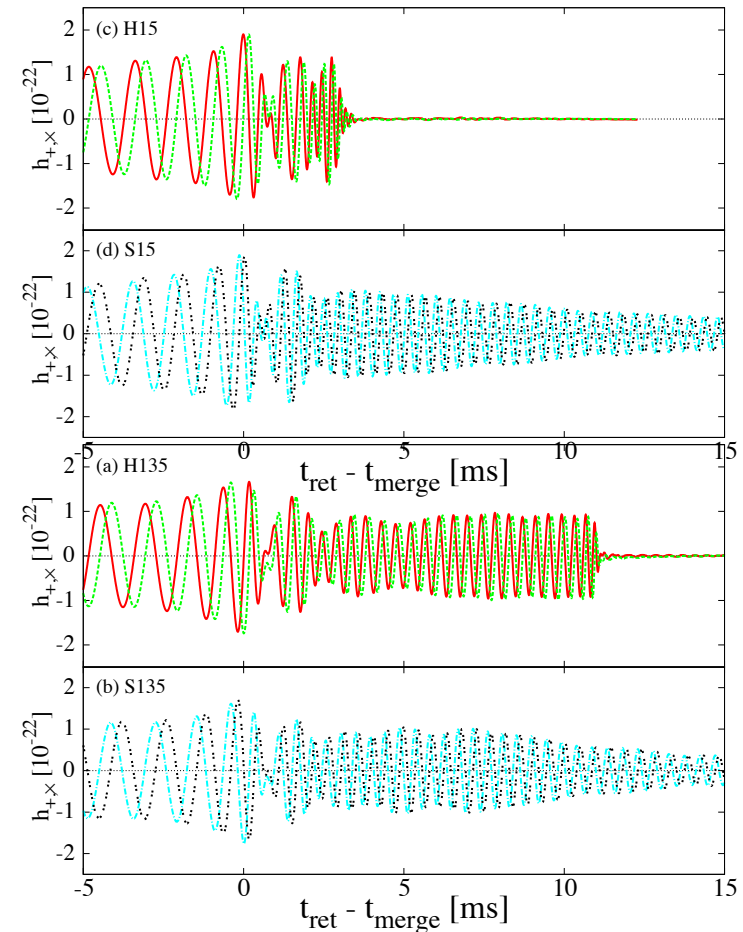
# A GRAVITATIONAL WAVE **AFTERGLOW** IN BINARY NEUTRON STAR MERGERS



# Binary Neutron Star Mergers

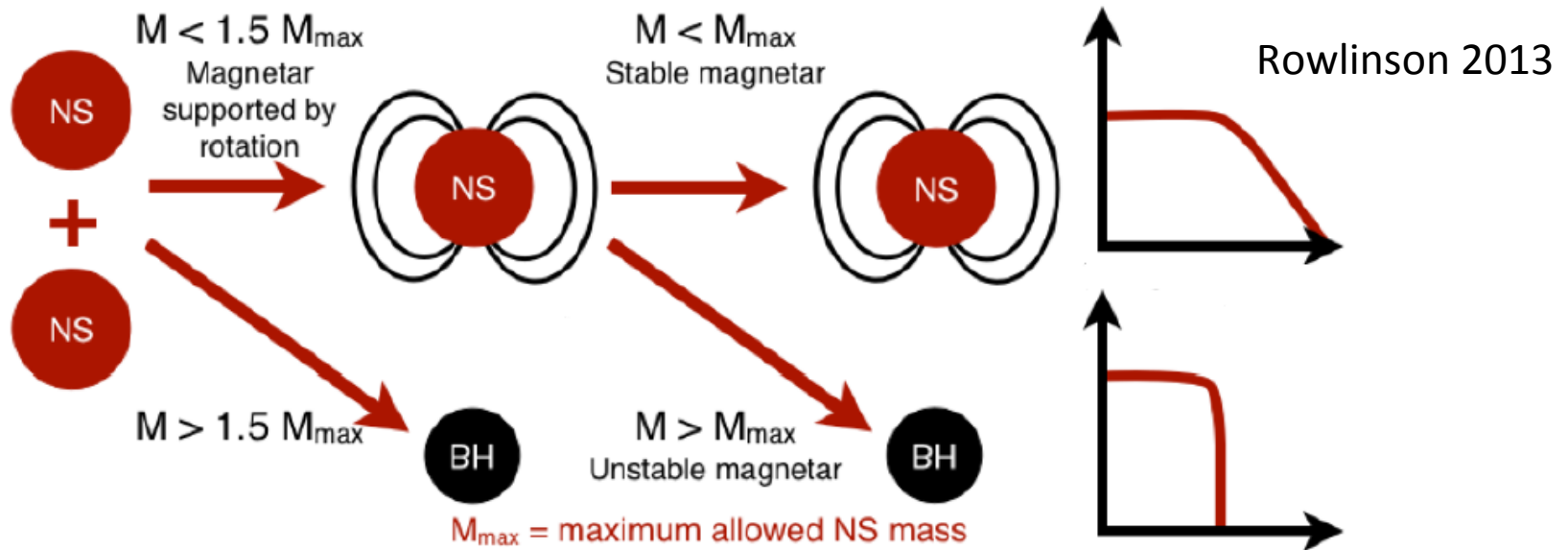
## the standard scenario

- I. After the merging the final body most probably will be a **supramassive NS** ( $2.5-3 M_{\odot}$ )
- II. The body will be **differentially rotating**
- III. The “averaged” **magnetic field** will amplified due to MRI (up to **3-4 orders of magnitude**)
- IV. The strong **magnetic field** and the **emission of GWs** will **drain rotational energy**
- V. This phase **will last only a few tenths of msecs** and can potentially provide information for the Equation of State (EOS)



# Post-Merger Scenario

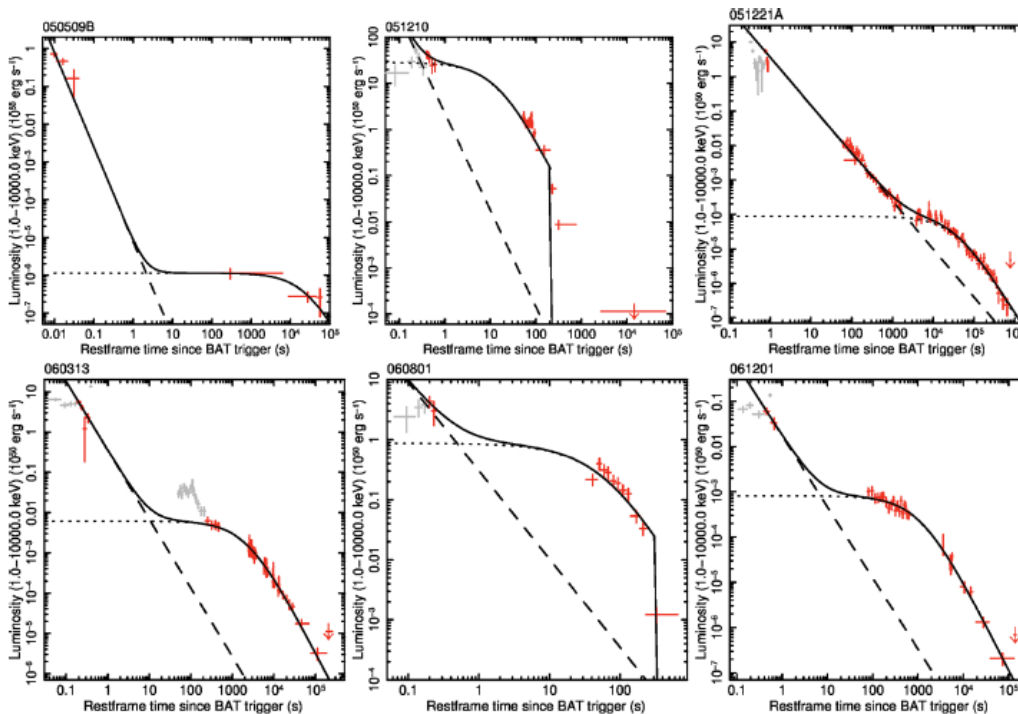
Three different outcomes of the merger of a BNS merger



- ✓ The outcome is dependent upon the mass ( $M$ ) of the central object formed and the maximum possible mass of a neutron star ( $M_{\max}$ ).
- ✓ On the right are sketches of the expected light-curves if a stable (top) or an unstable magnetar (bottom) is formed.

# Short $\gamma$ -ray light curves

- The favored progenitor model for SGRBs is the merger of two NSs that triggers an explosion with a **burst of collimated  $\gamma$ -rays**.
- Following the initial prompt emission, **some SGRBs exhibit a plateau phase** in their X-ray light curves that indicates **additional energy injection from a central engine**, believed to be a **rapidly rotating, highly magnetized neutron star**.
- The collapse of this “protomagnetar” to a black hole is likely to be responsible for a **steep decay in X-ray flux** observed at the end of the plateau.

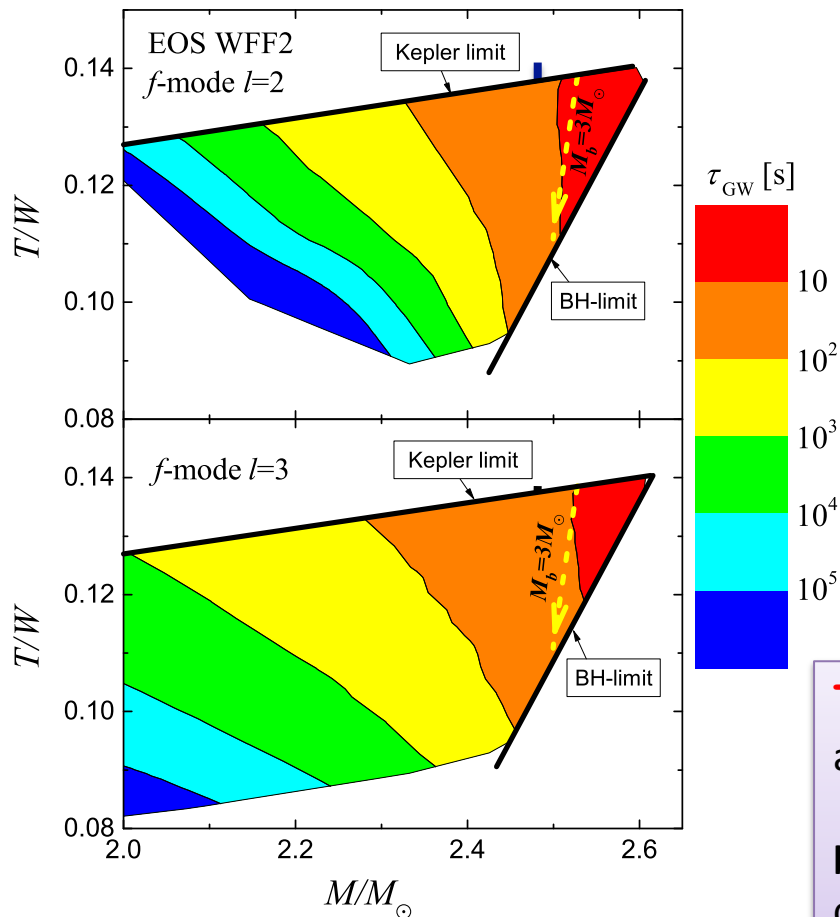


Rowlinson, O’Brien, Metzger, Tanvir, Levan 2013

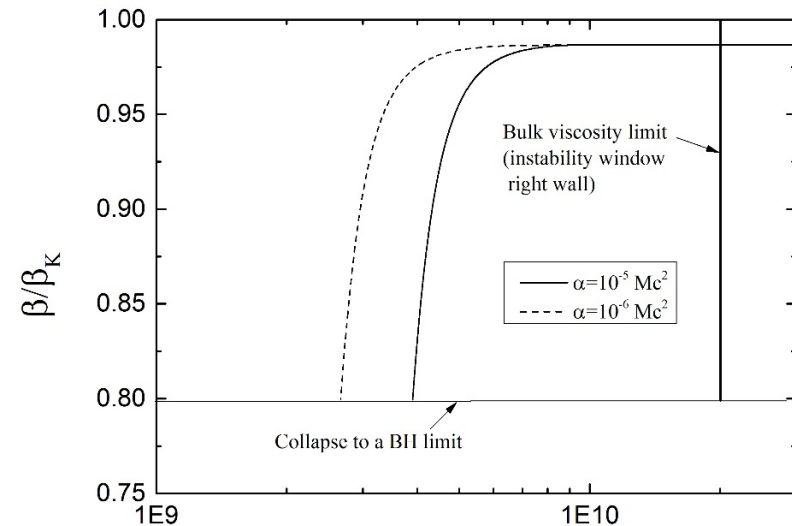
# Post-Merger NS: secular instability

Doneva-KK-Pnigouras 2014

The post-merger object **is still stable** and rotates at nearly Kepler **periods < 1ms**



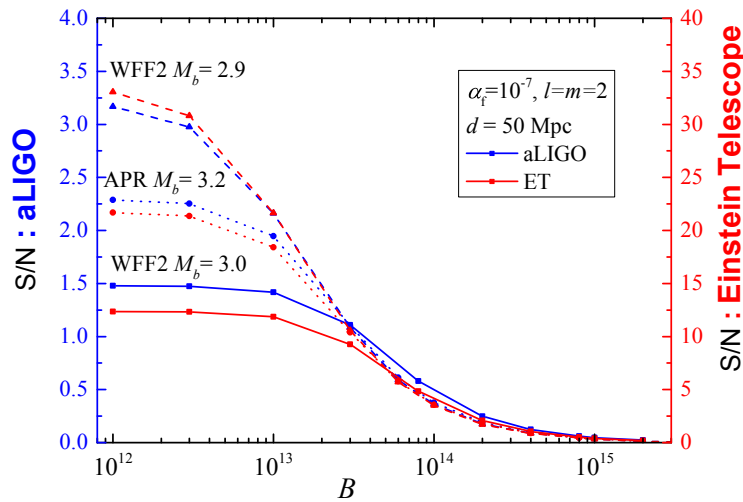
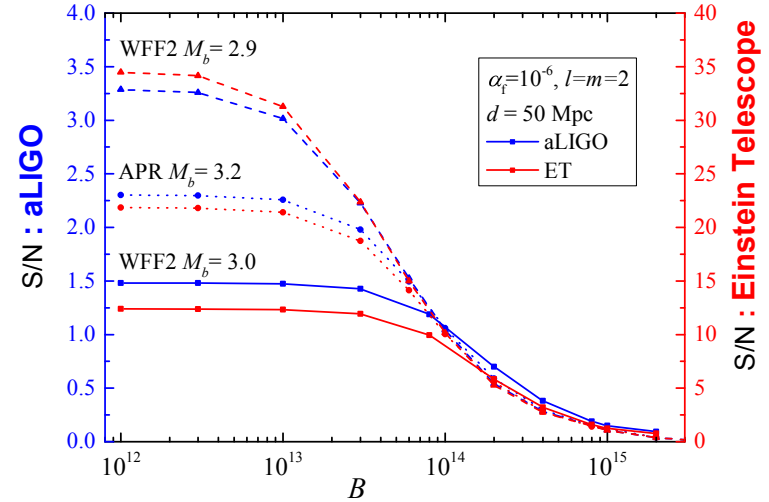
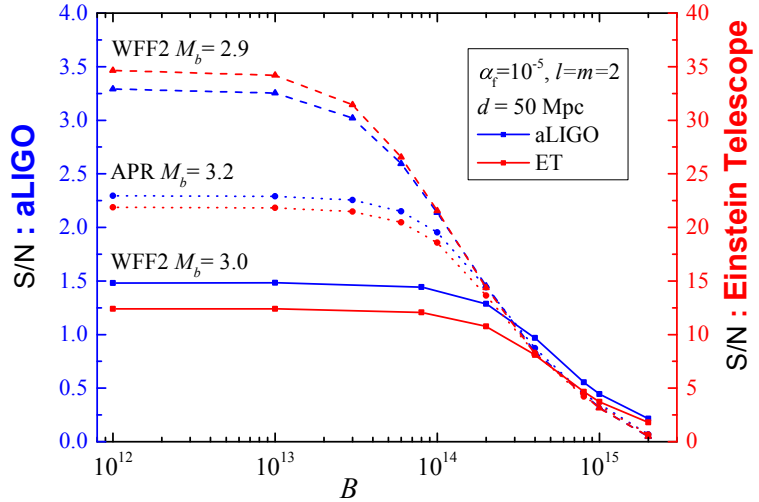
## The evolution into the instability window



## The detailed evolution depends:

- Strength of the **magnetic field** (averaged may reach  $10^{15-16} \text{ G}$  !)
- Equation of state** of the post-merger neutron star
- Fine details of the **non-linear dynamics** (three mode coupling, shock waves, wave breaking)

# Post-Merger NS: secular instability



Competition between the B-field and the secular instability

**GW frequencies:**

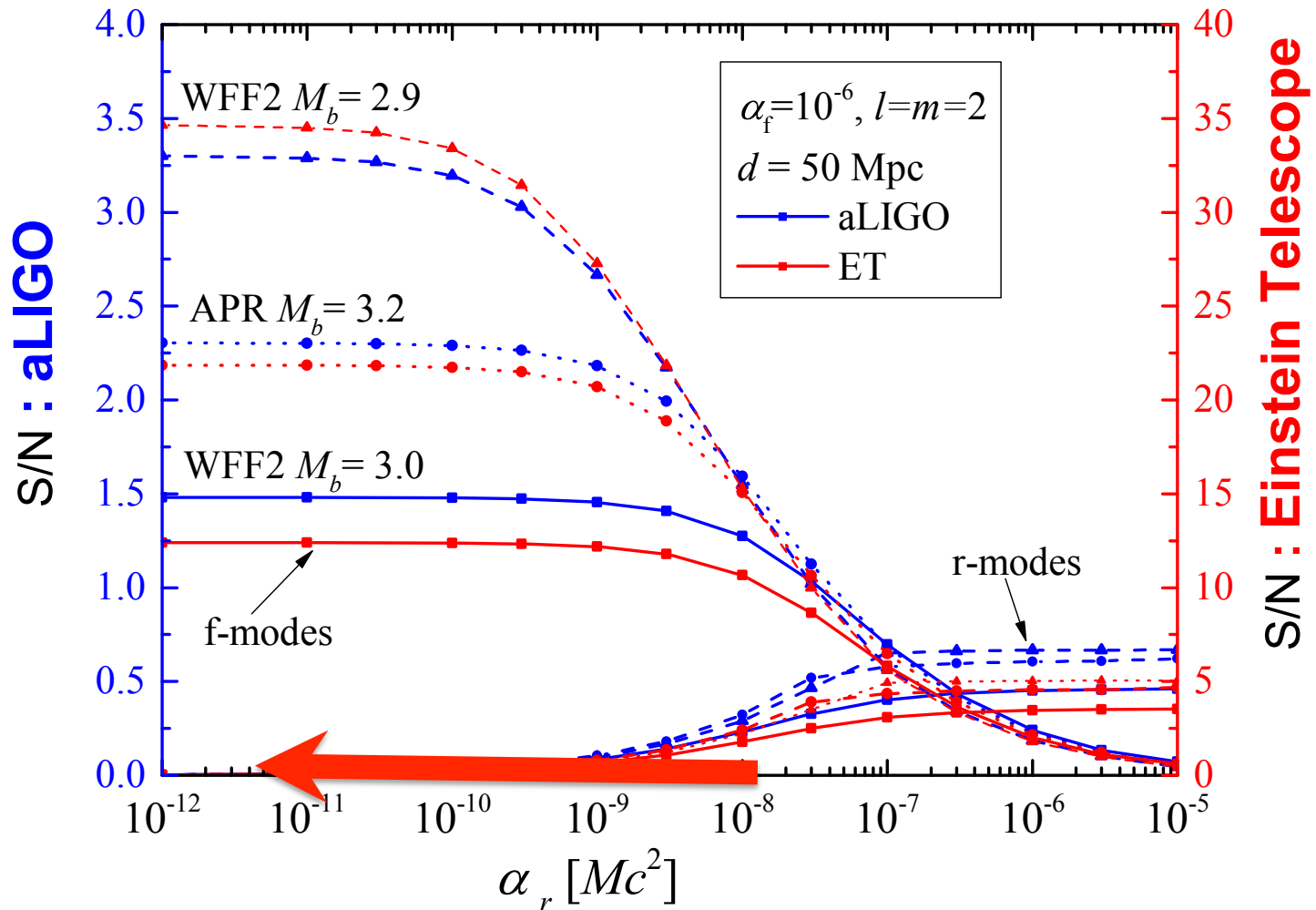
**WW2a: 920-1000 Hz**

**APR: 370-810 Hz**

**WWF2b: 600-780 Hz**

Doneva-KK-Pnigouras 2014

# f vs r-mode





# Conclusions

- ✓ The influence of the scalar field is much more pronounced for fast rotation.
- ✓ This is true also for  $f(R)$  gravity
- ✓ **Asteroseismology for fast rotating stars is possible**
- ✓ **Asteroseismology for magnetars is possible**
- ✓ f-mode instability can be **potentially** a good source for GWs for supramassive NS
- ✓ **Saturation amplitude** and **strength of B-field** are the key factors