A new view on differentially rotating neutron stars in general relativity

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Differential rotation

Astrophysical arguments

Differentiall rotation plays important role in :

- collapse of massive stars (and observed supernova),
- NS binary merger.

- the dynamics of compact astrophysical objects is a rich and complex topic involving nuclear matter at very high density and gravitation in the strong field regim
- even in stationary situations, differential rotation can make the study of their structure and life much more complicated than in Newtonian physics
- realistic and precise numerical simulations are needed and should enable to learn a lot on both astrophysics and fundamental physics
- problems are challenging not only from the physical points of view, but also from the mathematical and numerical ones

Equilibrium models of differentially rotating NS

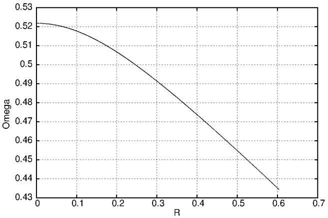
Astrophysically motivated (consistent with the core collapse results) a simple differential rotation law proposed by Komatsu, Eriguchi, Hachisu [1989]:

$$F(\Omega) = \mathbf{R}_0^2(\Omega_c - \Omega)$$

R_0 lenght describing the degree of differential rotation ($r = R_0$: $\Omega = \Omega_c/2$, Ω_c is the 'angular velocity on the axis'. We use $A^{-1} = \tilde{A} = R_e/R_0$ (R_e - equatorial radius, for **uniform rotation** $R_0 -> \infty$, $\tilde{A} -> 0$).

A polytropic EOS: $P = K\rho^\Gamma$ with a polytropic index $\Gamma = 2$

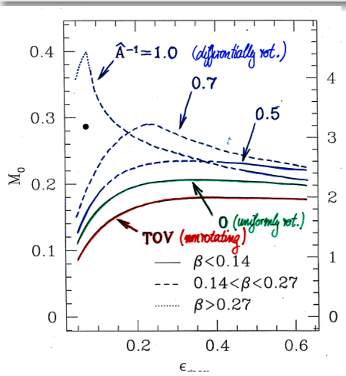
Relativistic multidomain spectral code Flatstar–> **axisymmetric and stationary** solutions for broad ranges of stellar densities and degree of differential rotation $0.01 < A^{-1} < 1.5$



The effect of the rotation on the maximum mass of NS

The effect of **rigid** rotation on the M_{max}:
NS up to 20% (e.g. Cook et al. 1994)
SS $\sim$ 44% (Gourgoulhon et al. 1999, Gondek-Rosinska et al. 2000)

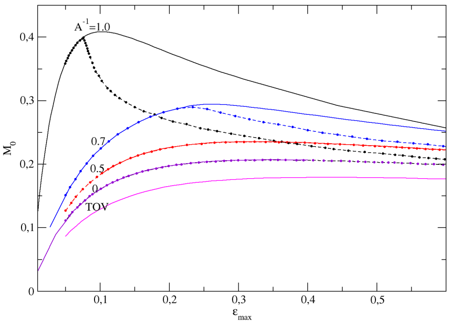
The effect of **differential** rotation on the M_{max} (Baumgarte et al. 2000, Morrison et al. 2004) depends on the degree of differential rotation A^{-1}



Differential rotation significantly increases the maximum allowed mass of NS and may temporarily stabilize the remnant of BNS merger- delayed collapse. The outcomes - no, delayed, or prompt collapse depend on EOS, the maximum density and A^{-1}. GW observations of coalescing BNS or a core collapse may be able to distinguish these outcomes.

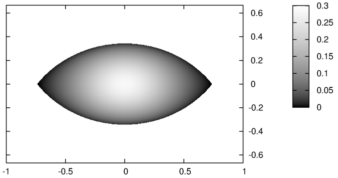
Differentially rotating NS - comparison

Highly accurate and stable code (Ansorg, Gondek-Rosińska, Villain, 2009) allows to construct relativistic models of differentially rotating NS for whole parameter space (broad ranges of degrees of differential rotation, maximal densities and $r_{\rm ratio}$ from 1 to zero.

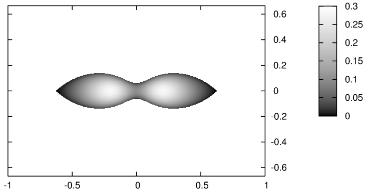


Solid lines - our results (Gondek-Rosinska et al. [2015], in preparation)
dotted lines- Baumgarte, Shapiro, Shibata [2000]
Good agreement for modest degree of differential rotation and/or small densities.

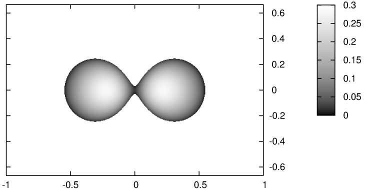
Types of differentially rotating neutron stars



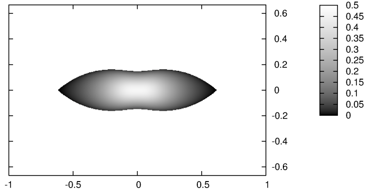
Type A, $\tilde{A} = 0.3$ at mass-shed limit



Type B, $\tilde{A} = 0.3$ at mass-shed limit



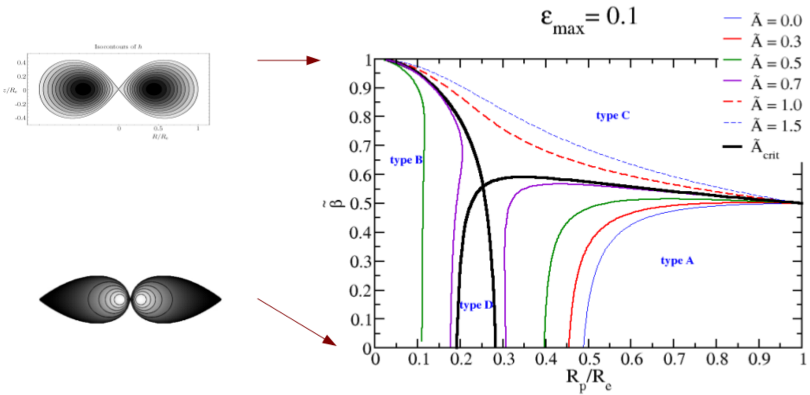
Type C, $\tilde{A} = 1.0$ and $r_{ratio} \sim 0$



Type D, $\tilde{A} = 0.5$ at mass-shed limit

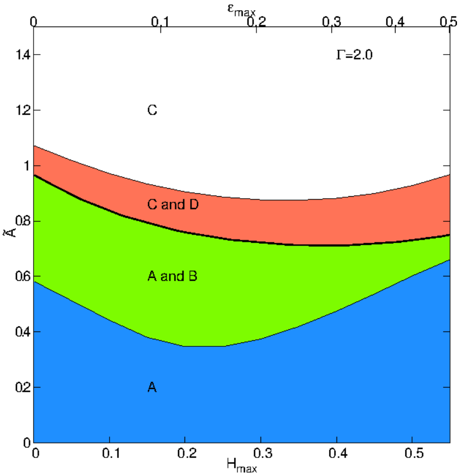
Studzinska, Gondek-Rosinska, Kucaba, Ansorg, Villain, 2015, in preparation

Four types of differentially rotating neutron stars



$\tilde{\beta}$ - the shedding parameter, $\tilde{\beta} = 0$ for mass-sheding limit; $\tilde{\beta} = 0.5$ for non-rotating NS and 1 for toroidal models. A solid line corresponds to $\tilde{A}_{\rm crit}$, the separatrix. Results for fixed maximum density-four types of solution: A, B, C and D

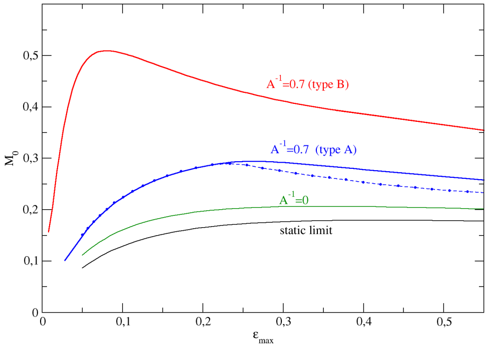
Types of differentially rotating neutron stars



- **Type A** exist for $\tilde{A} < \tilde{A}_{crit}$, start at static configuration and ends at mass-shed limit,
- **Type C** exist for $\tilde{A} > \tilde{A}_{crit}$, start at static case and terminates for $R_p/R_e = 0$,
- **Type B** also exist for $\tilde{A}_B < \tilde{A} < \tilde{A}_{crit}$
- **Type D** exist for $\tilde{A} > \tilde{A}_{crit}$, for narrow range of $\tilde{A}$.

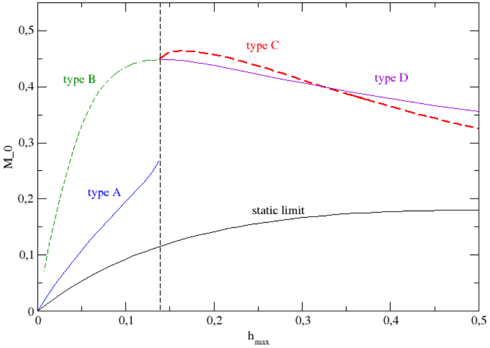
Gondek-Rosinska, Kowalska, Ansorg, Villain, 2015, in prep

The maximum mass for $\tilde{A} = 0.7$, types A and B



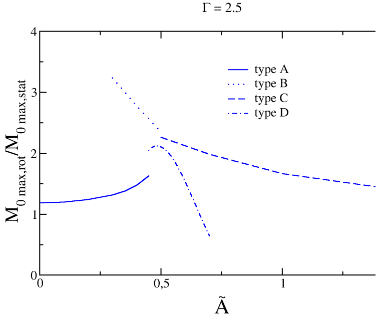
Maximum mass of new type B is much higher than maximum mass for type A with the same degree of differential rotation (eg. for polytrope with $\Gamma = 2.0$).

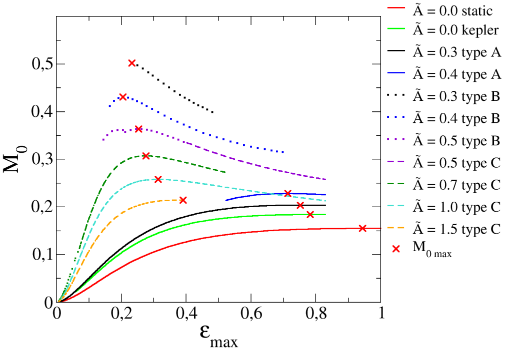
The maximum mass for $\tilde{A} = 0.8$, types A, B,C, D



Maximum mass strongly depend on the type of sollution. For type B is much higher than maximum mass for type A with the same degree of differential rotation. Type C and D have comparable masses (eg. for polytrope with $\Gamma = 2.0$)

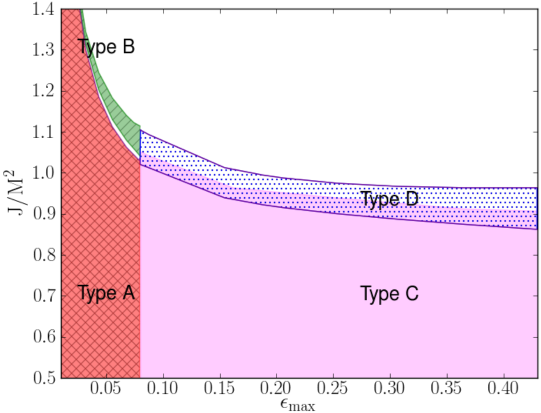
The maximum mass of differentially rotating NS





For type A the maximum mass is increasing function of the degree of differential rotation while for type B and C decreasing. The maximum mass is obtained for type B, the new type of configurations.

Are they stable?



Areas of Kerr parameter for $\Gamma = 2.0$ and $\tilde{A} = 0.8$. For $J/M^2 > 1$ NS can be temporarily stabilized by differential rotation (Baumgarte et al. 2000, Giacomazzo et al. 2011).

Summary

Using highly accurate code based on spectral method (Ansorg, Gondek-Rosińska, Villain [2009]) we have calculated relativistic models of axisymmetric rotating NS for broad ranges of degree of differential rotation

- We have found new types of solutions(existing for modest degree of differential rotation and $r_p/r_e \lesssim 0.3$), which were not considered in previous works based on other algorithms, due to complexity of the problem and numerical limitations
- The maximum mass of differentially rotating NS for given EOS depends on the degree of differential rotation and a type of solution (classified as A,B,C,D)
- Differential rotation significantly increases up to 400% the maximum allowed mass of NS and may temporarily stabilize a new born protoneutron star or a remnant of binary NS merger.
- The highest increase of mass is obtained for the newly found types of differentially rotating NS
- Gravitational waves observations of coalescing binary NS may be able to distinguish the outcomes; prompt collapse, delayed collapse, a stable NS

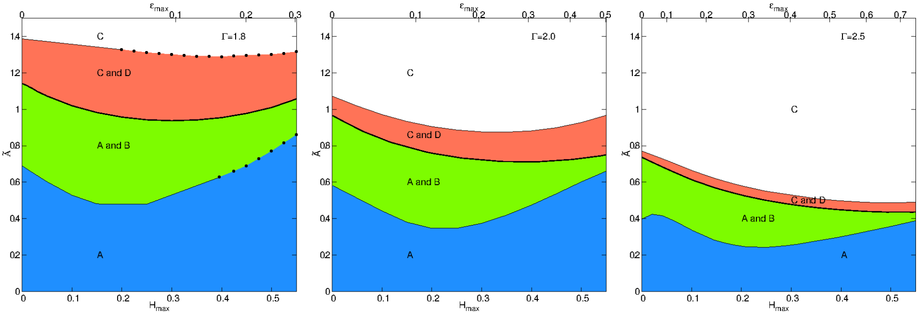
Work in progress

- Calculations for realistic finite temperature EOS
- Initial data for stability analysis (2D, 3D)
- Consider as sources of gravitational waves
- Testing different rotation law

Our results for differentially rotating NS (type A and C)

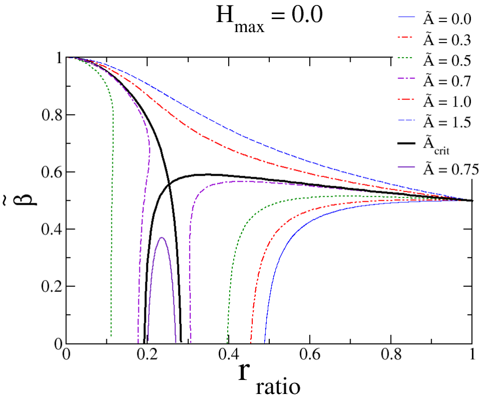
To construct a single NS for given EOS we need 3 parameters e.g. ($\rho_{\max}$, $r_{\rm ratio}$, A) (for rigid rotation only 2 e.g. (ρ_c,Ω) or (ρ_c,$r_{\rm ratio}$) Depending on the value of A^{-1} the maximum mass is obtained close to the mass-shed limit –> **type A** (solid lines) or for toroidal configurations ($R_p/R_e = 0$) –> **type C** (dashed lines). Existence of **separatrix** $A_{\rm crit}$

The effects of EOS on types for diffentially rotating NS



The stiffer EOS is the larger region of type C configurations and narrower for types A,B and D

Clasifications of differentially rotating neutron stars



$\tilde{\beta}$ is 0 for mass-sheding limit; 0.5 for spheroidal and 1 for toroidal models. A solid line corresponds to the separatrix, type A - below separatrix, right corner; type B below separatrix left side; type C - above separatrix

Effects of differential rotation and EOS on the maximum mass of NS

Depending on EOS and degree of differential rotation we can obtain very high increase of maximum mass.