Constraining neutron stars EoS with gravitational wave observations

Valeria Ferrari

SAPIENZA
UNIVERSITÀ DI ROMA

INFN
Istituto Nazionale di Fisica Nucleare

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Neutron stars provide an ideal laboratory to study matter under extreme conditions of temperature, density, and spacetime curvature.

- Which is the equation of state (EoS) of matter in such extreme conditions?

- What is its composition and how does it depend on temperature?

- What are the transport properties, thermal conductivity, shear and bulk viscosities, electric conductivity, neutrino mean free path, etc. which so strongly influence the stellar structure and dynamics?

- How can one put to test the theoretical approaches developed to describe neutron star properties?

Gravitational waves emitted by neutron stars at different stages of their evolution, will hopefully provide a tool to investigate these issues.

Gravitational wave interferometric detectors: first generation

GEO600 (British-German)
Hannover, Germany

LIGO- I (USA)
Hanford, WA

LIGO-II (USA)
Livingston, LA

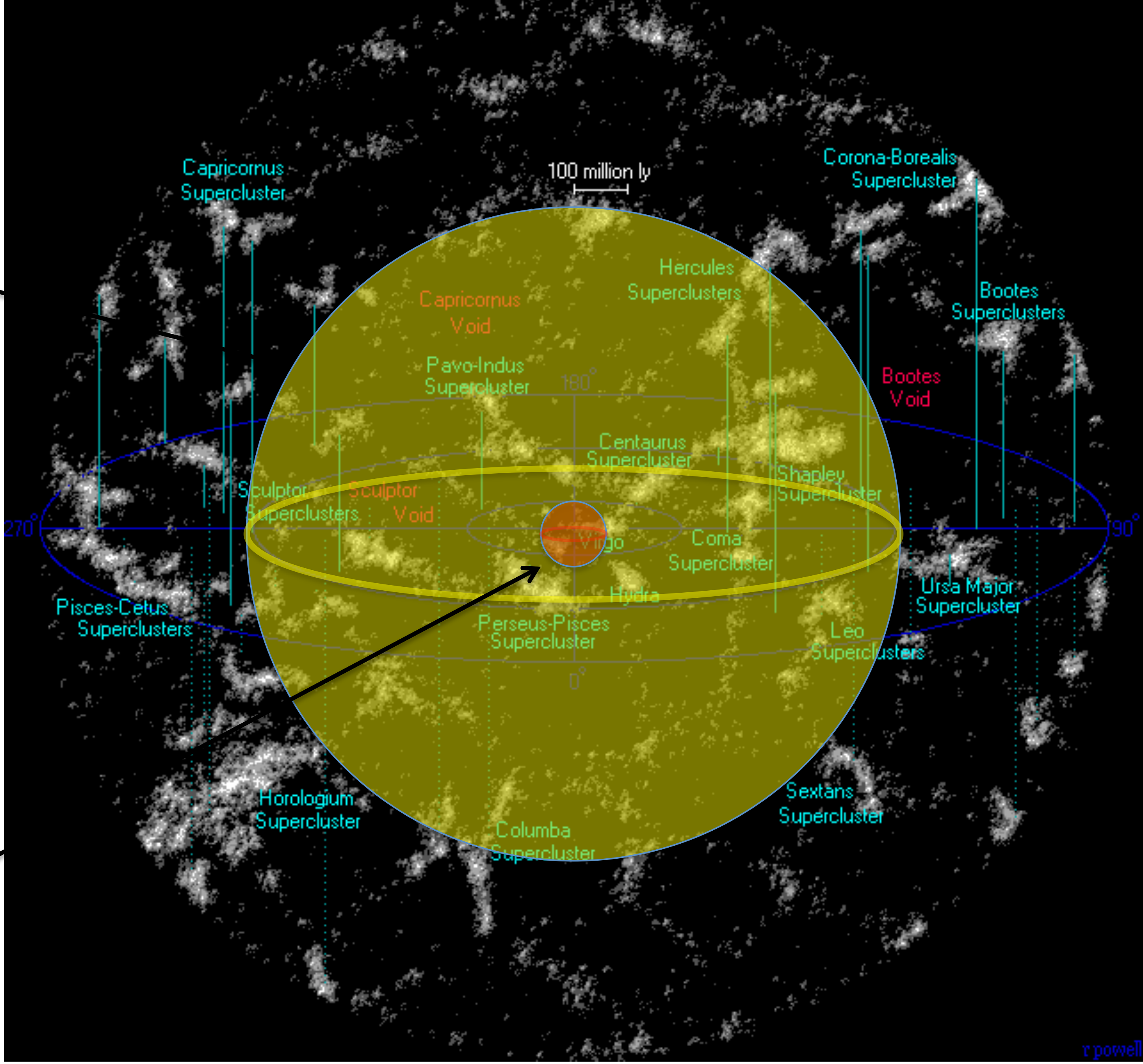
TAMA300 (Japan) (decommissioned)
first detector to take data

Next generation will improve sensitivity by an order of magnitude

Virgo interferometer (Cascina, Italy)

Advanced detectors

LIGO: Fall 2015
Virgo: next year



Initial detectors

One of the main target is the detection of GW signals emitted by coalescing compact binaries

When operating at design sensitivity , ADVANCED LIGO/Virgo are expected to detect

NS-NS coalescence seen up to ~ 300 Mpc

NS-BH (1.4-10) Msun ~ 650 Mpc

ASTROPHYSICAL OBSERVATIONS: FROM OBSERVED SAMPLE OF BINARY PULSARS:

$\sim [10^{-8} – 8x10^{-6}]\ Mpc^{-3}\ yr^{-1}$

RATES FROM POPULATION SYNTHESIS STUDIES in $Mpc^{-3}\ yr^{-1}$

$[10^{-8} - 10^{-5}]$	NS-NS	
$[6x10^{-10} - 10^{-6}]$	NS-BH	*LIGO-Virgo coll. CQG **27**, 2010*

even in the most pessimistic case we can reasonably say that within a decade we should be able to detect several events

will we be able to set constraints on the NS EoS using these observations?

The way in which the EoS enters the GW signal from binary coalescence is mainly through tidal deformation.

This deformation of the neutron star has an effect on the orbital motion , and hence on the gravitational waveform; in particular it enters, as we shall later see in the signal phase

LOVE NUMBERS AND TIDAL DEFORMABILITY

The Newtonian Theory of Tides:

The Love numbers were introduced by August E. H. Love in 1911: they are a set of dimensionless parameters which measure the rigidity of a planetary body and show how its shape changes in response to an external tidal potential.

These numbers can be generalized to stars in General Relativity.
We are interested in particular in one of these numbers, which connects the tidal field with the quadrupolar deformation of the star.

Tidal-induced Quadrupole Moment and tidal deformability of a Neutron Star

If a static, spherically symmetric star of mass M is placed in a static, external, quadrupolar tidal field C_{ij}, it develops a quadrupole moment Q_{ij}

To linear order in the tidal field C_{ij}, the "tidal - induced" quadrupole moment Q_{ij} can be written as

$$Q_{ij} = \lambda C_{ij}$$

where λ is the tidal deformability,

λ is related to the l=2 tidal Love number k_2 (or apsidal constant) by the equation

$$k_2 = \frac{3G}{2R^5}\lambda$$

How do we find λ ?

Tidal-induced Quadrupole Moment and tidal deformability of a Neutron Star

In the star's local asymptotic frame (asymptotically mass-centered Cartesian coordinates) at large distance r, the metric coefficient g_{tt} can be written as (Thorne 1998)

$$\frac{1-g_{tt}}{2} = -\frac{M}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + \mathcal{O}\left(\frac{1}{r^3}\right) + \frac{1}{2}C_{ij}x^i x^j + \mathcal{O}\left(r^3\right)$$

where $n^i = x^i/r$.

For instance in Newtonian theory

$$C_{ij} = \frac{\partial^2 \Phi_{ext}}{\partial x^i \partial x^j}$$

In GR the tidal field C_{ij} is found by projecting the Riemann tensor associated to the external field which produces the star deformation, onto a parallely transported tetrad attached to the deformed star

$$C_{ij} = e^{\alpha}_{(0)} e^{\beta}_{(i)} e^{\gamma}_{(0)} e^{\delta}_{(j)} R_{\alpha\beta\gamma\delta}$$

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where $n^i = x^i/r$.

Since we have assumed $Q_{ij} = \lambda C_{ij}$

$$\frac{1-g_{tt}}{2} = -\frac{M}{r} - \frac{3\,\boldsymbol{\lambda}\,C_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + \mathcal{O}\left(\frac{1}{r^3}\right) + \frac{1}{2}C_{ij}x^i x^j + \mathcal{O}\left(r^3\right)$$

Given the tidal field C_{ij}, to find λ we need to determine the asymptotic behaviour of g_{tt}

Under the action of the external, quadrupolar tidal field the star's metric is perturbed

$$g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h_{\alpha\beta}$$

$g^{(0)}_{\alpha\beta}$ describes the geometry of the static star, $h_{\alpha\beta}$ is the metric perturbation.

Expanding $h_{\alpha\beta}$ in spherical harmonics (in the Regge-Wheeler gauge) allows to write $h_{\alpha\beta}$ as

$$h_{\alpha\beta} = diag\left[-e^{\nu(r)}H_0(r), e^{\lambda(r)}H_2(r), r^2K(r), r^2\sin\theta K(r)\right] Y_{2m}(\theta,\phi)$$

Einstein's equations linearized about $g^{(0)}_{\alpha\beta}$ give:

ν(r) and λ (r) are the metric funtions of the unperturbed star

$$H_0 = -H_2 = H$$

and, by suitably combining the various components, a relation between H(r) and K(r), and a second order differential equation for H(r):

$$H'' + H'\left\{\frac{2}{r} + e^{\lambda}\left[\frac{2m(r)}{r^2} + 4\pi r(p-\rho)\right]\right\} + H\left[-\frac{6e^{\lambda}}{r^2} + 4\pi e^{\lambda}\left(5\rho + 9p + \frac{\rho+p}{dp/d\rho}\right) - (\nu')^2\right] = 0$$

T. Hinderer, ApJ 677,2008

The function H is the only function we need to find the perturbed metric

$$H'' + H'\left\{\frac{2}{r} + e^{\lambda}\left[\frac{2m(r)}{r^2} + 4\pi r(p-\rho)\right]\right\} + H\left[-\frac{6e^{\lambda}}{r^2} + 4\pi e^{\lambda}\left(5\rho + 9p + \frac{\rho+p}{dp/d\rho}\right) - (\nu')^2\right] = 0$$

This equation is solved numerically by imposing regularity at r=0, and continuous matching at r=R with the exterior solution which is known analitically. The metric is thus completely determined, and in particular

$$g_{tt} = g_{tt}^0 + h_{tt} = -e^{\nu}[1 + H(r)]$$

Expanding this quantity at radial infinity and comparing with Thorne's asymptotic expansion

$$\frac{1 - g_{tt}}{2} = -\frac{M}{r} - \frac{3\,\boldsymbol{\lambda}\,C_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + \mathcal{O}\left(\frac{1}{r^3}\right) + \frac{1}{2}C_{ij}x^i x^j + \mathcal{O}\left(r^3\right)$$

the tidal deformability λ and the Love number k_2 can be found

T. Hinderer, ApJ 677,2008

Love number

$$k_2 = \frac{8\mathcal{C}^5}{5}(1-2\mathcal{C})^2\left[2+2\mathcal{C}(y-1)-y\right]\Big\{2\mathcal{C}\left[6-3y\ + \right.$$
$$+3\mathcal{C}(5y-8)] + 4\mathcal{C}^3\left[13-11y+\mathcal{C}(3y-2)\ + \right.$$
$$\left.+2\mathcal{C}^2(1+y)\right] + 3(1-2\mathcal{C})^2\left[2-y+2\mathcal{C}(y-1)\ln(1-2\mathcal{C})\right]\Big\}^{-1},$$

T. Hinderer, ApJ 677,2008

where $\mathcal{C} = M/R$ is the star compactness, and $y = H'(R)R/H(R)$

$$\boxed{\lambda = \frac{2R^5}{3G}k_2}$$

tidal deformability

NOTE THAT:

the Love number k_2 and the tidal deformability λ depend only on the stellar compactness C, **a quantity which depends on the equation of state,** and on the value of H and H' at the surface, which again depends on the EoS through the pressure and density profiles in the unperturbed star

I-Love-Q relations

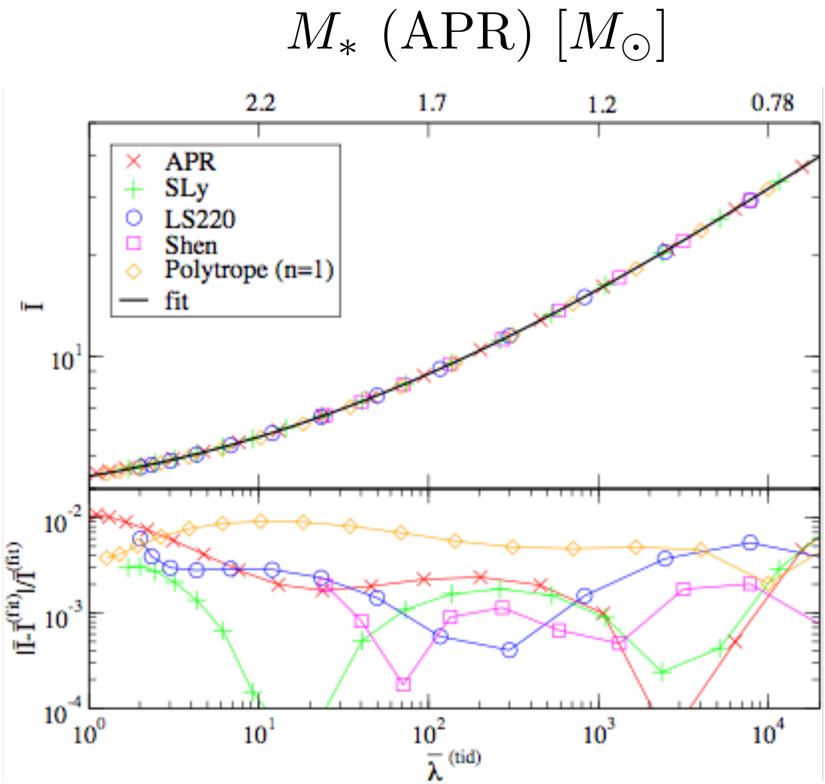
In 2013 Yagi and Yunes discovered new relations between the NS moment of inertia I, the Love numbers and the spin-induced quadrupole moment Q that are essentially EoS independent for slowly- rotating NSs

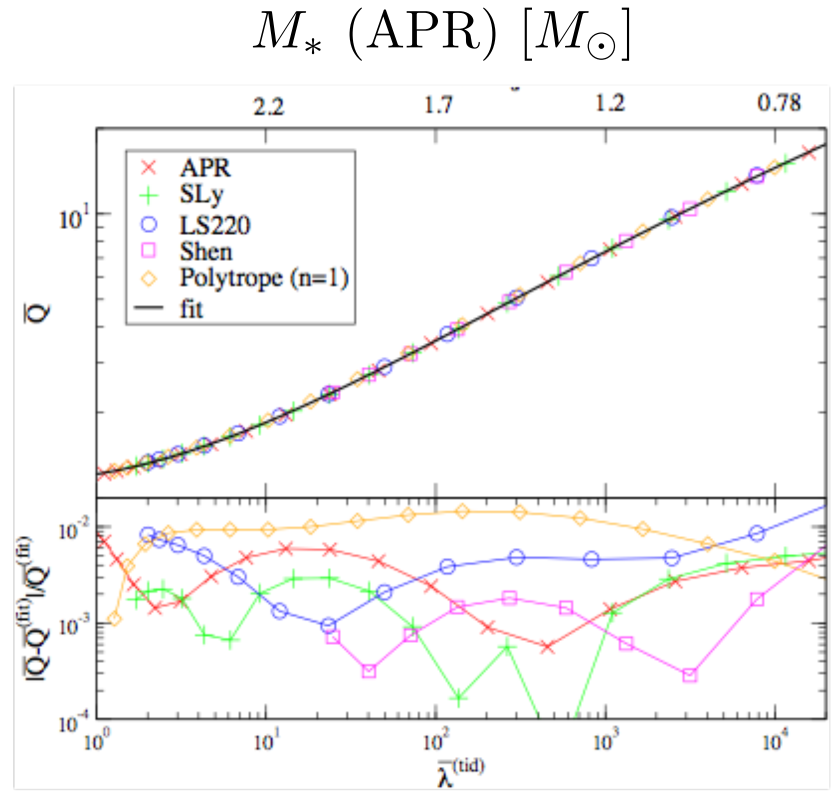
These three quantities give interesting physical information:

- the moment of inertia quantifies how fast a NS can spin assumining a fixed angular momentum
- the spin-induced quadrupole moment describes the NS shape deviates from sphericity due to rotation
- the Love number quantifies how much deformable the NS is.

K. Yagi, N. Yunes, Science 241, 2013; PRD 88, 2013

Each of these quantities depend on the EoS,
the relations between them do not!





K. Yagi, N. Yunes PRD 88, 2013

$\bar{I} = I/M_*^3$ $\quad\quad$ $\bar{\lambda}^{(tid)} = \lambda/M_*^5$ $\quad\quad$ $\bar{Q} = Q^{(rot)}/[M_*^3(S/M_*^2)^2]$

$$\ln y_i = a_i + b_i \ln x_i + c_i(\ln x_i)^2 + d_i(\ln x_i)^3 + e_i(\ln x_i)^4$$

y_i	x_i	a_i	b_i	c_i	d_i	e_i
$\bar{I}$	$\bar{\lambda}^{(tid)}$	1.47	0.0817	0.0149	2.87×10^{-4}	-3.64×10^{-5}
$\bar{I}$	$\bar{Q}$	1.35	0.697	-0.143	9.94×10^{-2}	-1.24×10^{-2}
$\bar{Q}$	$\bar{\lambda}^{(tid)}$	0.194	0.0936	0.0474	-4.21×10^{-3}	1.23×10^{-4}

Tidal effects in the gravitational wave signal emitted in NS-NS binary coalescence

$$h(f) = \mathcal{A}(f) e^{i\psi(f)} \qquad \psi(f) = \psi_{PP} + \psi_{\bar{Q}} + \psi_{\bar{\lambda}}$$

$x = (m\pi f)^{5/3}$ PN expansion parameter

point-particle contribution

$$\psi_{PP}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{1 + \left(\frac{3715}{756} + \frac{55}{9}\eta\right)x - (16\pi - 4\beta)x^{3/2} + \left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2 - 10\sigma\right)x^2 + \mathcal{O}(x^{5/2})\right\}$$

- σ contains spin-spin and spin-orbit terms. Note that it appears in the 2-PN term (x^2)

Quadrupole contribution:

$$\psi_{\bar{Q}} = \frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{-50\left[\left(\frac{m_1^2}{m^2}\chi_1^2 + \frac{m_2^2}{m^2}\chi_2^2\right)(Q_S - 1) + \left(\frac{m_1^2}{m^2}\chi_1^2 - \frac{m_2^2}{m^2}\chi_2^2\right)Q_a\right]x^2\right\}$$

$$Q_S = \frac{\bar{Q}_1 + \bar{Q}_2}{2}, \quad Q_a = \frac{\bar{Q}_1 - \bar{Q}_2}{2}$$

both the quadrupole moments and the spin terms appear at the 2-PN order and cannot be measured independently : in this sense we say that there is complete degeracy

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$$Q_S = \frac{\bar{Q}_1 + \bar{Q}_2}{2}, \quad Q_a = \frac{\bar{Q}_1 - \bar{Q}_2}{2}$$

Tidal contribution: $\psi_{\bar{\lambda}} = -\frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{24[(1 + 7\eta - 31\eta^2)\lambda_S + (1 + 9\eta - 11\eta^2)\lambda_a \delta m]x^5 + \right\} + \mathcal{O}($

$$\lambda_S = \frac{\bar{\lambda}_1 + \bar{\lambda}_2}{2}, \quad \lambda_a = \frac{\bar{\lambda}_1 - \bar{\lambda}_2}{2} \qquad \delta m = \frac{m_1 - m_2}{m}$$

tidal effects are measured from the high frequency part of the wave

degeracy can be removed by expressing the Q's in terms of λ using the universal relations
NOTE THAT: λ is independent of the spins

Although tidal effects enter the phase at high post-Newtonian order (5PN), the tidal deformability which appear in the formula is normalized as

$$\bar{\lambda}(m) = \lambda(m)/m^5 \propto (R/m)^5 \sim 10^2 - 10^5$$

so that it may be observable with advanced detectors

Will it be possible possible to measure the tidal deformability with the accuracy needed to gain information on the NS EoS ?

two possible approaches: Fisher-matrix and Bayesian analysis

Fisher matrix in two words: given a signal $h(t,\vec{\theta})$

where $\vec{\theta} = (\mathcal{M}, t_c, \phi_c, \lambda, \dots)$ is a set of parameters, and given the detector output

$$d(t) = h(t,\vec{\theta}) + n(t)$$

where n(t) is the detector noise which is assumed to be stationary and Gaussian, the estimated values of the source parameters are those which maximize the probability distribution

$$P(\vec{\theta}|d) \propto P^{(0)} exp\left[-\frac{1}{2}\left(h(f,\vec{\theta}) - d(f)\middle|h(f,\vec{\theta}) - d(f)\right)\right]$$

$p^{(0)}$ prior probability on the physical parameters set (tipically uniform in suitable ranges)

where the inner product (..|..) is defined as

$$(a|b) = 2\int_{-\infty}^{+\infty} \frac{a(f)b^*(f) + a^*(f)b(f)}{S_n(f)} df$$

The uncertainties on the parameters are $\sigma_{\theta_i} = \sqrt{\Sigma^{ii}}$, where $\Sigma^{ij} = (\Gamma^{-1})^{ij}$ and

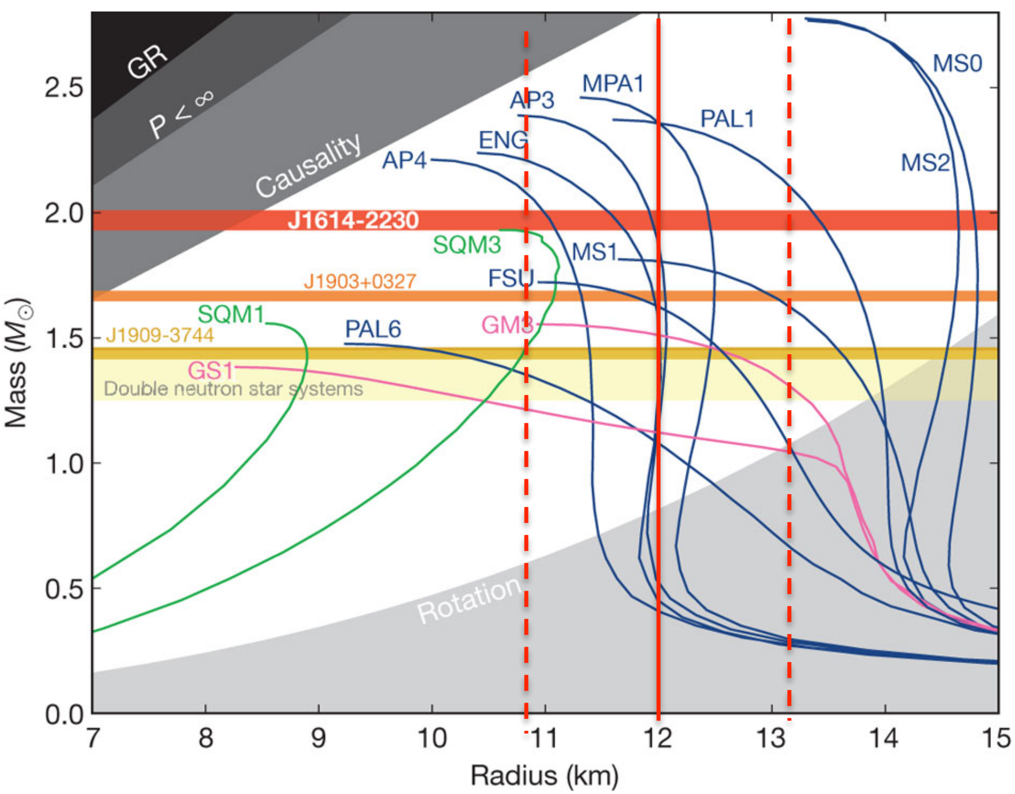
$$\Gamma^{ij} = \left(\frac{\partial h}{\partial \theta_i}\middle|\frac{\partial h}{\partial \theta_j}\right)$$

is the Fisher matrix

RESULTS WITH THE FISHER MATRIX APPROACH

*Read, Markakis, Shibata, Uryu, Creighton, PRD**79**, 2009*

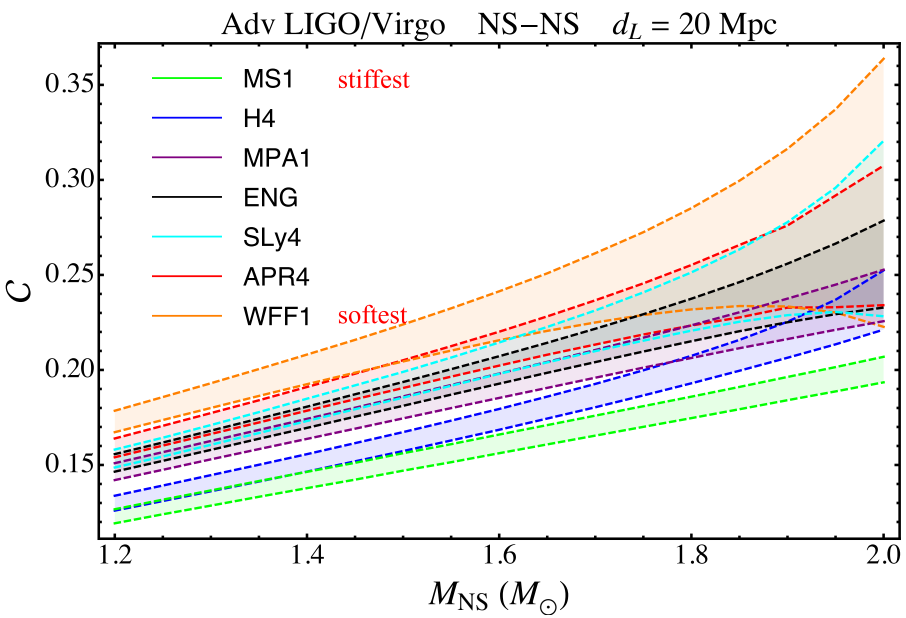
using hybrid waveforms (Post-Newtonian+ full GR over the last 3 orbits)
show that if the EoS is sufficiently stiff, with a single close-by source (at a distance of 100 Mpc) neutron star radius could be constrained to ~10%

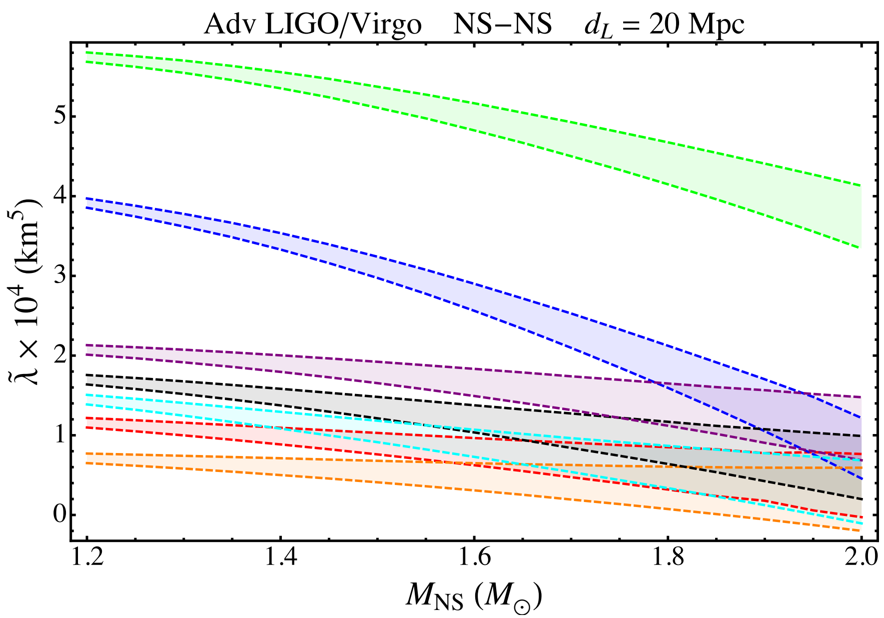


RESULTS WITH THE FISHER MATRIX APPROACH

Is the tidal deformability λ the "right" parameter to discriminate among different EoS?

*Maselli, Gualtieri, Ferrari PRD**88**, 2013*
Using a set of EoS and PN templates including tidal corrections, we evaluated how well Advanced LIGO/Virgo and ET could measure λ or the stellar compactness C=M/R assuming the source at D=20/100 Mpc (LIGO/Virgo) or at D=100 Mpc /2 Gpc (ET)

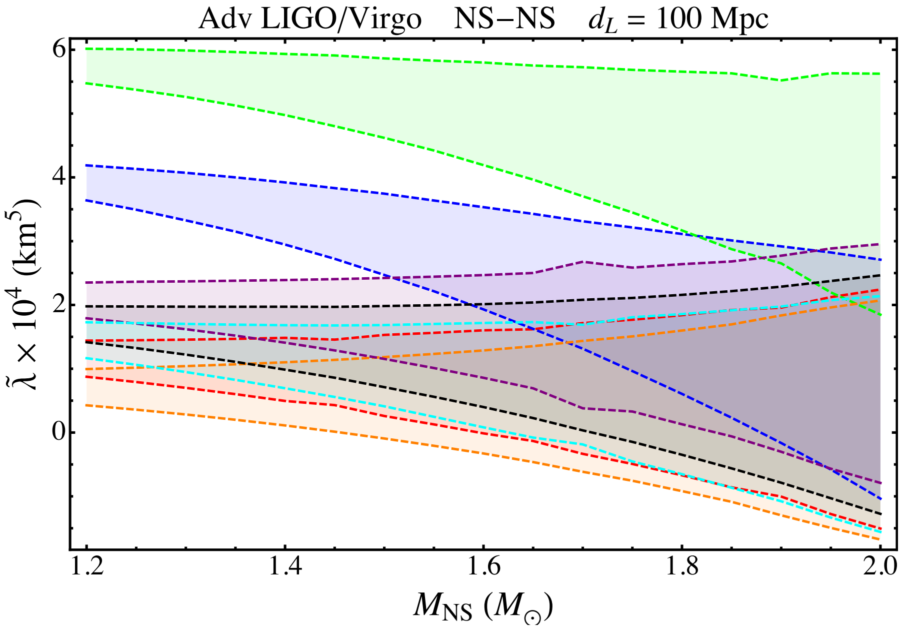


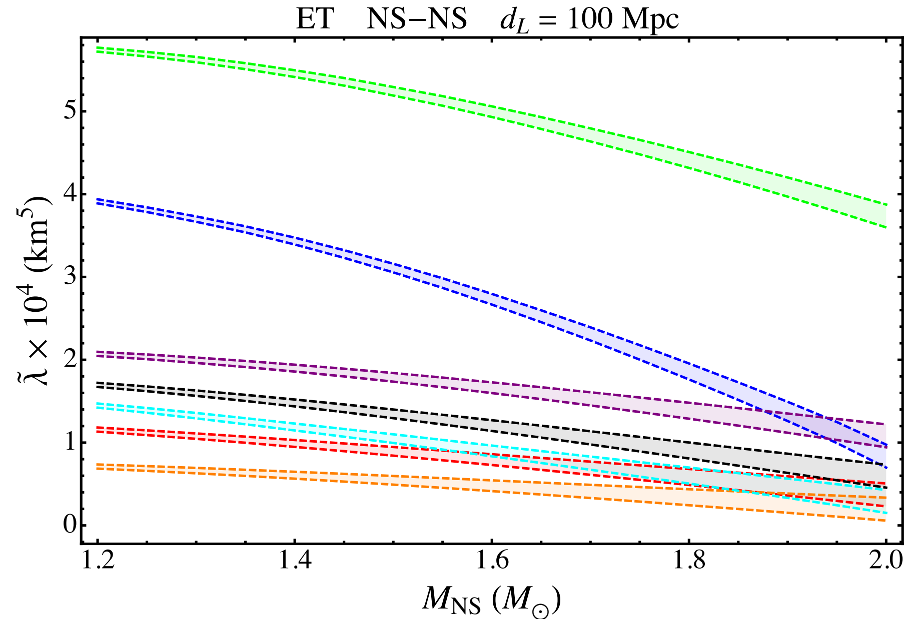


EoS indistinguishable if we use C

stiffest EoSs can be distinguished if we use λ for masses smaller than ~ 1.8 $M_\odot$

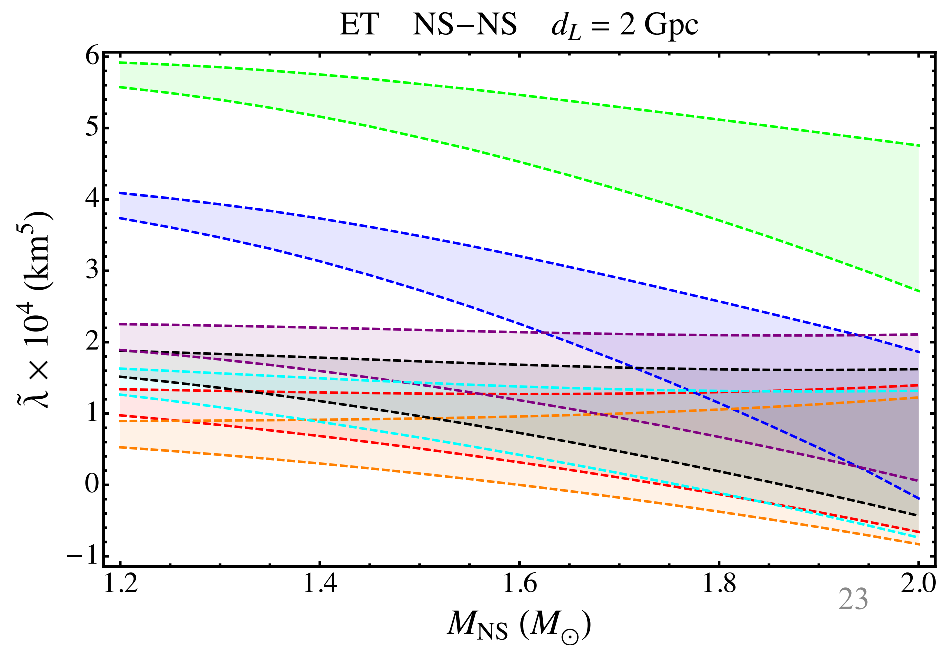
RESULTS WITH THE FISHER MATRIX APPROACH





stiffest EoSs can be distinguished if we use λ for masses smaller than ~ 1.5 $M_\odot$

*Maselli, Gualtieri, Ferrari PRD**88**, 2013*



Similar results were obtained in

Damour, Nagar, Villain PRD85, 2012

they use waveforms constructed with the EOB formalism extrapolated up to the touching of the two NSs.

They show that: if a NS-NS coalescence signal will be detected with a signal-to-noise ratio ≥ 16 by advanced detectors, than the tidal deformability can be measured at 95% confidence level for a large set of EoS and for mass ratio ranging from 0.7 to 1

For BH-NS system the dephasing turns out to be smaller by ≈ 2 orders of magnitude

Similar results also in *Lackey, Kiutoku, Shibata, Brady, Friedman, PRD85, 2012* who used hybrid waveforms for NS-BH coalescence (num rel+ one body approximants)

These results are based on the assumption of a single detection and on the Fisher matrix approach. What if we combine results of multiple detections?

Bayesian analisys for inferring the NS EoS using more sources

Bayesian analysis in three words: suppose that we are looking for a signal $h(t,\vec{\theta})$ where

$\vec{\theta} = (\mathcal{M}, t_c, \phi_c, \lambda, ...)$ is a set of parameters, and the detector output is $d(t)$

be **I** any information that we may have on the signal, for instance the range in which parameters may vary and their distribution functions. We construct the *likelihood function*

$$p(d|h,\vec{\theta},I) = \mathcal{N} exp\left[-2\int_{f_{low}}^{f_{cut}} df \frac{|d(f) - h(f,\vec{\theta})|^2}{S_n(f)}\right]$$

$d(f)$ = Fourier transform of the data stream

$\mathcal{N}$ = normalization factor

then we compute the *evidence* by integrating the likelihood on the parameter space

$$P(d|h,I) = \int d\vec{\theta}\ P(\vec{\theta}|I)\ p(d|h,I)$$

and finally the *posterior probability* that $h(t,\vec{\theta})$ is present in the data, given the collected data stream d(f) and the prior hypothesis **I**

$$P(h|d,I) \propto P(d|h,I)P(h|I)$$

Let us now assume that the signal we are looking for may come from two different EoS , and let us call them h_1 and h_2

Bayesian analisys for inferring the NS EoS using more sources

If the signal we are looking for may come from two different EoS , say h_1 and h_2 ,

For each template we compute the *likelihood function*

$$p(d|h_i,\vec{\theta},I) = \mathcal{N} exp\left[-2\int_{f_{low}}^{f_{cut}} df\ \frac{|d(f)-h_i(f,\vec{\theta})|^2}{S_n(f)}\right]$$

i=1,2

the *evidence* by integrating the likelihood on the parameter space

$$P(d|h_i,I) = \int d\vec{\theta}\ P(\vec{\theta}|I)\ p(d|h_i,I),\quad i=1,2$$

and finally the *posterior probability* that $h_i(f,\vec{\theta})$ is present in the data, given the collected data stream d(f) and the prior hypothesis I

$$P(h_i|d,I) \propto P(d|h_i,I)\ P(h_i,I)$$

If the *odds ratio* O^i_j is greater than 1, this means that the data favour the signal h_i over h_j i.e., for instance the EoS i with respect to the EoS J

$$O^i_j = \frac{P(h_i|d,I)}{P(h_j|d,I)}$$

This analysis can be extended to the case of multiple detections d_1, d_2, …d_N

$$^{(N)}O^i_j = \frac{P(h_i|I)}{P(h_j|I)}\prod_{n=1}^{N}\frac{P(d_n|h_i,I)}{P(d_n|h_j,I)}$$

using this formula we can rank the EoS , i.e. we can establish which is the EoS which is more compatible with the detected data

Using Bayesian analisys for inferring the NS EoS using more sources

Del Pozzo, Agathos, Meidam, Li, Tompitak, Veitch, Vitale,, Van Den Broek, PRL ***111****, 2013*

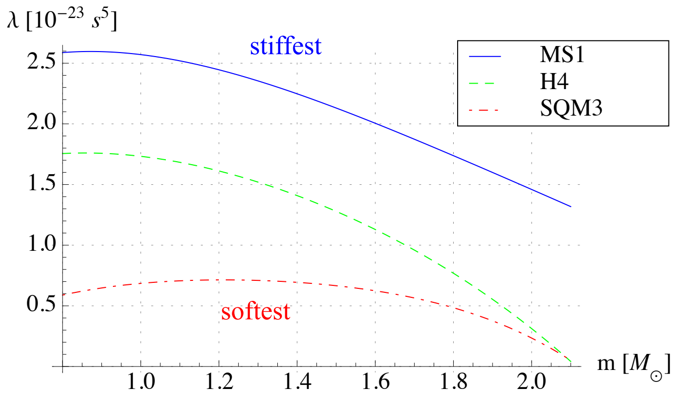


FIG. 1: The tidal deformability parameter $\lambda(m)$ as a function of neutron star mass for three different EOS: a soft one (SQM3), a moderate one (H4), and a hard one (MS1). Adapted from [18].

NS-NS signals are injected into simulated data of Advance LIGO/Virgo noise, assuming design sensitivity.

They consider 3 different EoS , MS1, H4, SM3, which are stiff, moderatly stiff and soft, respectively.

The tidal deformability λ (m) is given in the picture

masses in the range [1,2] $M_\odot$

PN waveforms are truncated at the ISCO or at physical contact

The detectors noise (the three LIGO+Virgo) is taken to be stationary and Gaussian

Injections and Priors

signals are injected assuming sources uniformely distributed in the comoving volume in the distance range D = [100, 250] Mpc

the set of parameters $\vec{\theta}$ includes:
masses, spin, sky position, orientation, distance, time and phase at coalescence.

sky location and orientation are assumed to be uniformly distributed on a sphere
phase at coalescence is uniform in $[0, 2\pi]$

- ❖ spins are set to zero both in the injected signals and in the priors,
- ❖ all injected signals belong to the stiffest EoS, MS1

- ❖ mass density distribution of the injected signals is
  1) flat distribution with masses in the range $[1,2]\ M_\odot$
  2) Gaussian, mean value $1.35\ M_\odot$, spread $0.05 M_\odot$

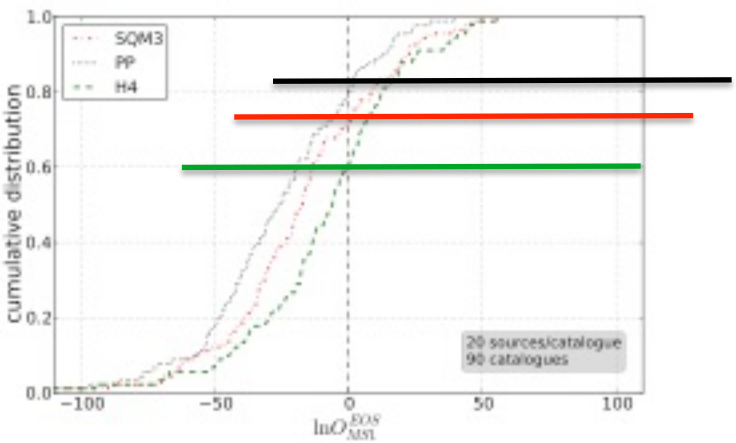
- ❖ prior for the mass density distribution uniform in the range $[1,2]\ M_\odot$

In a further analysis they also include spin effects

1) 20 injected signals -- all injected signals belong to the stiffest EoS, MS1

flat distribution with masses in the range [1,2] $M_\odot$

prior for the mass density distribution assumed to be flat in the range [1,2] $M_\odot$



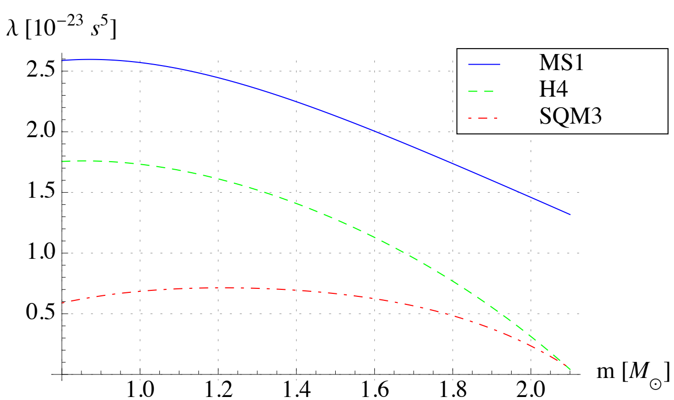
If the *odds ratio* O^i_j is > 1, this means that the data favour the signal h_i over h_j i.e., for instance the EoS i with respect to the EoS J

$$O^i_j = \frac{P(h_i|d,I)}{P(h_j|d,I)}$$

If we assume that h_i belongs to the EoS **H4** **the odds ratios** are larger than 1 in 40% of the data

If we assume that h_i belongs to the waveform with no tidal effects (**PP**), the odds ratio are larger than 1 in 30% of the data

If we assume that h_i belongs to the EoS **SQM3** are larger than 1 in 20% of the data



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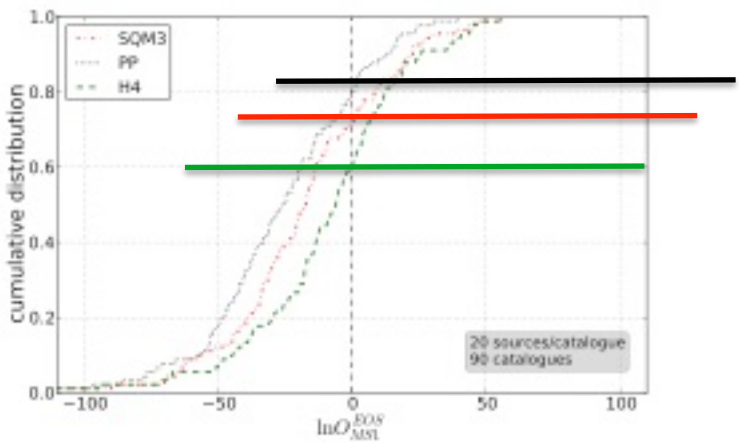
2) injected signals -- all injected signals belong to the stiffest EoS, MS

mass distribution Gaussian, mean value 1.35 $M_\odot$, spread $0.05M_\odot$

prior for the mass density distribution uniform in the range [1,2] $M_\odot$



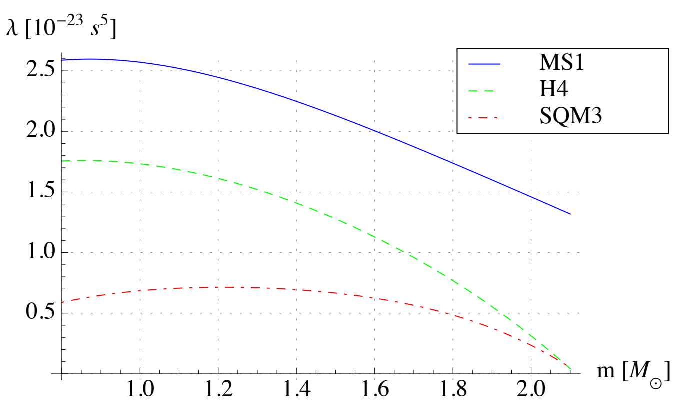
If the *odds ratio* O^i_j is > 1, this means that the data favour the signal h_i over h_j i.e., for instance the EoS i with respect to the EoS J

$$O^i_j = \frac{P(h_i|d,I)}{P(h_j|d,I)}$$

H4 is the EoS with the higher probability to correspond to the *true* signals

SQM3 and the point particle signal (PP) have a lower probability

This ranking is correct, because the EoS H4 is closer to MS1 than SQM3 and PP



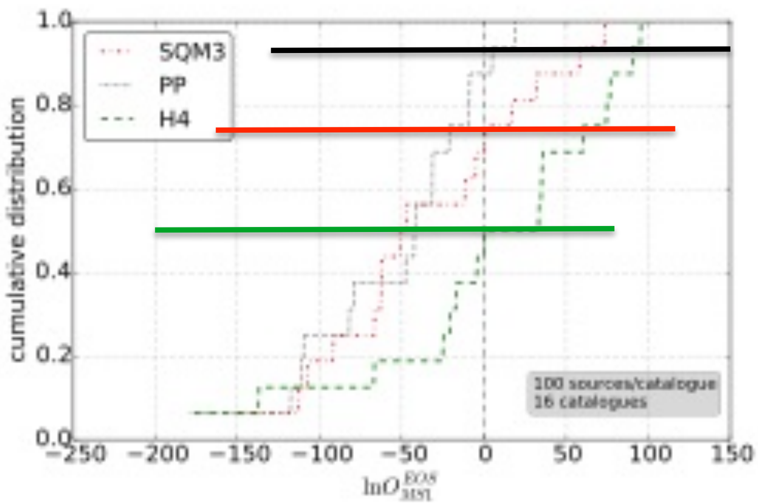
Let us now see how this picture changes if we inject signals with a mass distribution different from the one which is use as a prior

FIG. 1: The tidal deformability parameter $\lambda(m)$ as a function of neutron star mass for three different EOS: a soft one (SQM3), a moderate one (H4), and a hard one (MS1).

2) injected signals -- all injected signals belong to the stiffest EoS, MS

mass distribution Gaussian, mean value 1.35 $M_\odot$, spread 0.05$M_\odot$

prior for the mass density distribution uniform in the range [1,2] $M_\odot$



If the *odds ratio* O^i_j is > 1, this means that the data favour the signal h_i over h_j i.e., for instance the EoS i with respect to the EoS J

$$O^i_j = \frac{P(h_i|d,I)}{P(h_j|d,I)}$$

The ranking of the EoS is again correct:

H4 is the EoS with the higher probability to correspond to the *true* signals

SQM3 and PP have a lower probability

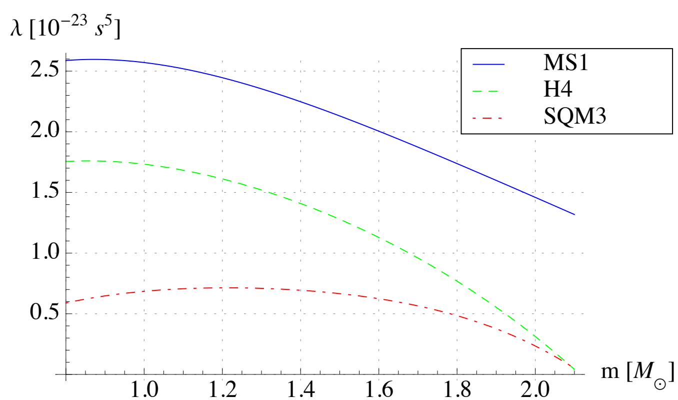


FIG. 1: The tidal deformability parameter $\lambda(m)$ as a function of neutron star mass for three different EOS: a soft one (SQM3), a moderate one (H4), and a hard one (MS1).

But this time to reach a confidence level comparable to the previous one they need to use more than **100 detected sources** (against 20 they used before) !!

If spins are included in the analysis results are similar:

The ranking of the EoS as they come out from the analysis is correct only when the injected signals belong to EoS which are very stiff , i.e. matter is very deformable and the tidal deformability is large, provided a high number of detection will be available.

Concluding Remarks

The identification of the *level of stiffness* of NS EoS using detection of gravitational wave signals from coalescing NS-NS binaries appears to be possible only for very stiff EoSs,

It would be difficult to distinguish moderately stiff EoSs.

If the EoS is very soft, we would only be able to exclude very stiff EoSs.

In any event: a large number of detections is needed.

This mainly arises from our ignorance on the true distribution of masses (and spins) in NS-NS binaries

However there are theoretical uncertainties which may further bias the presented analysis

Theoretical issues

- When computing the tidal deformability, there is some degree of arbitrariness in separating the solution describing the external tidal field from that describing the response of the system, even at the linearized level and for non rotating stars.

Even if the full perturbed solution behaves – by definition – linearly, the multipole moments of the spacetime might be mixed among the two solutions.
In other words, the multipole moments of the central object might in principle be contaminated by the external solution.

An example: Weyl's solution
$ds^2 = -e^{2U}dt^2+e^{2(k-U)}(d\rho^2+dz^2)+W^2e^{-2U}d\varphi^2$, where U, k and W depend only on ρ and z

- if the star rotates multipoles couple and it is not clear how they will appear in the gravitational waveforms which, so far, have been computed using the non-rotating tidal deformability

- due to these couplings the I-Love-Q relations which have been used in the previous analyses may not hold anymore

Yesterday's talk by L. Gualtieri
Pani, Gualtieri, Maselli, Ferrari, 2015, PRD to appear, arXiv150307365P

Much more work has to be done to clarify these important issues