

Thermodynamical behavior of the vector meson extended Polyakov quark meson model

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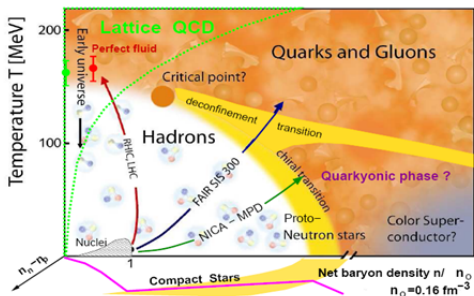
Collaborators: Zsolt Szép , György Wolf

Overview

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 - Vector meson extended PQM model – Polyakov loop
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QCD phase diagram

Phase diagram in the $T - \mu_B - \mu_I$ space



- At $\mu_B = 0$
 $T_C = 151(3) \text{ MeV}$
Y. Aoki, et al., PLB 643, 46 (2006)
- Is there a CEP?
- At $T = 0$ in μ_B where is the phase boundary?
- Behavior as a function of μ_I/μ_S ?

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)

Chiral symmetry, chiral models

If the quark masses are zero (chiral limit) \implies QCD invariant under the following global transformation (**chiral symmetry**):

$$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A = SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

$U(1)_V$ term \longrightarrow baryon number conservation

$U(1)_A$ term \longrightarrow broken through axial anomaly

$SU(3)_A$ term \longrightarrow broken down by any quark mass

$SU(3)_V$ term \longrightarrow broken down to $SU(2)_V$ if $m_u = m_d \neq m_s$

\longrightarrow totally broken if $m_u \neq m_d \neq m_s$ (**realized in nature**)

Since QCD is very hard to solve \longrightarrow **low energy effective models** \longrightarrow

reflecting the global symmetries of QCD \longrightarrow **degrees of freedom:**

observable particles instead of quarks and gluons

Linear realization of the symmetry \longrightarrow linear sigma model

Lagrangian (2/1)

$$\begin{aligned}
\mathcal{L} = & \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\
& + c_1 (\det \Phi + \det \Phi^\dagger) + \text{Tr}[H(\Phi + \Phi^\dagger)] - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\
& + \text{Tr} \left[\left(\frac{m_1^2}{2} \mathbb{1} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + i \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\
& + \bar{\Psi} i \gamma_\mu D^\mu \Psi - g_F \bar{\Psi} (\Phi_S + i \gamma_5 \Phi_{PS}) \Psi,
\end{aligned}$$

$$D^\mu \Phi = \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu) - ie A_e^\mu [T_3, \Phi],$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ie A_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ie A_e^\nu [T_3, L^\mu]\},$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ie A_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ie A_e^\nu [T_3, R^\mu]\},$$

$$D^\mu \Psi = \partial^\mu \Psi - i G^\mu \Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a.$$

+ Polyakov loop potential

Lagrangian (2/2)

the **matter** and **external** fields are

$$\Phi = \sum_{i=0}^8 (\sigma_i + i\pi_i) T_i, \quad H = \sum_{i=0}^8 h_i T_i \quad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^8 (\rho_i^\mu - b_i^\mu) T_i, \quad L^\mu = \sum_{i=0}^8 (\rho_i^\mu + b_i^\mu) T_i, \quad \Delta = \sum_{i=0}^8 \delta_i T_i$$

$$\Psi = (u, d, s)^T$$

non strange – strange base:

$$\xi_N = \sqrt{2/3}\xi_0 + \sqrt{1/3}\xi_8,$$

$$\xi_S = \sqrt{1/3}\xi_0 - \sqrt{2/3}\xi_8, \quad \xi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle \sigma_{N/S} \rangle \equiv \bar{\sigma}_{N/S}$

Included fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^8 \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & K^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_S = \sum_{i=0}^8 \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & K_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: π (138), K (495), η (548), η' (958)

Scalars: a_0 (980 or 1450), K_S (800 or 1430),

2 of f_0 (500, 980, 1370, 1500, 1710)

Structure of scalar mesons

	Mass (MeV)	width (MeV)	decays
$a_0(980)$	980 ± 20	50 – 100	$\pi\pi$ dominant
$a_0(1450)$	1474 ± 19	265 ± 13	$\pi\eta, \pi\eta', K\bar{K}$
$K_s(800) = \kappa$	682 ± 29	547 ± 24	$K\pi$
$K_s(1430)$	1425 ± 50	270 ± 80	$K\pi$ dominant
$f_0(500) = \sigma$	400–550	400 – 700	$\pi\pi$ dominant
$f_0(980)$	980 ± 20	40 – 100	$\pi\pi$ dominant
$f_0(1370)$	1200–1500	200 – 500	$\pi\pi \approx 250, K\bar{K} \approx 150$
$f_0(1500)$	1505 ± 6	109 ± 7	$\pi\pi \approx 38, K\bar{K} \approx 9.4$
$f_0(1710)$	1722 ± 6	135 ± 7	$\pi\pi \approx 30, K\bar{K} \approx 71$

Possible scalar states: $\bar{q}q$, tetraquarks, glueballs

scalar $\bar{q}q$ nonet content: 1 a_0 , 1 K_s , and 2 f_0

tetraquarks: $f_0(500)$, $f_0(980)$, $a_0(980)$, $K_s(800)$???

glueballs: $f_0(1500)$, $f_0(1710)$???

Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^8 \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & K^{*0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^8 b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & K_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770)$, $K^*(894)$, $\omega_N = \omega(782)$, $\omega_S = \phi(1020)$

Axial vectors: $a_1(1230)$, $K_1(1270)$, $f_{1N}(1280)$, $f_{1S}(1426)$

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra is not
 → SSB:

$$\sigma_{N/S} \rightarrow \sigma_{N/S} + \bar{\sigma}_{N/S} \quad \bar{\sigma}_{N/S} \equiv \langle \sigma_{N/S} \rangle$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)]$:

$$\begin{aligned}
 \eta_N - f_{1N}^\mu &: -g_1 \bar{\sigma}_N f_{1N}^\mu \partial_\mu \eta_N, \\
 \pi - a_1^\mu &: -g_1 \bar{\sigma}_N (a_1^{\mu+} \partial_\mu \pi^- + a_1^{\mu 0} \partial_\mu \pi^0) + \text{h.c.}, \\
 \eta_S - f_{1S}^\mu &: -\sqrt{2} g_1 \bar{\sigma}_S f_{1S}^\mu \partial_\mu \eta_S, \\
 K_S - K_\mu^* &: \frac{ig_1}{2} (\sqrt{2} \bar{\sigma}_S - \bar{\sigma}_N) (\bar{K}_\mu^{*0} \partial^\mu K_S^0 + K_\mu^{*-} \partial^\mu K_S^+) + \text{h.c.}, \\
 K - K_1^\mu &: -\frac{g_1}{2} (\bar{\sigma}_N + \sqrt{2} \bar{\sigma}_S) (K_1^{\mu 0} \partial_\mu \bar{K}^0 + K_1^{\mu+} \partial_\mu K^-) + \text{h.c.}
 \end{aligned} \tag{1}$$

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\text{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\text{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp \left[i \int_0^\beta d\tau G_4(\vec{x}, \tau) \right]$$

↔ signals center symmetry (\mathbb{Z}_3) breaking at the deconfinement transition

low T : confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$

high T : deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

- **Polyakov gauge:** $G_4(\vec{x}, \tau) = G_4(\vec{x})$, plus gauge rotation to diagonal form in color space
- further simplification: \vec{x} -independence

$$\hookrightarrow L = e^{i\beta G_4} = \text{diag}(a, b, c) \left(\overset{!}{\in} SU(N_c) \right); \quad a, b, c \in \mathbb{Z}$$

↔ use this to calculate partition function of free quarks on constant gluon background

Effects of Polyakov loops on FD statistics

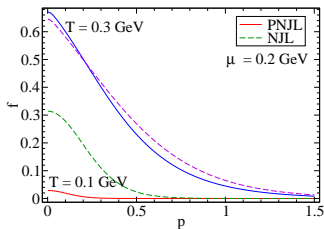
Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p - \mu_q) \longrightarrow f_{\Phi}^+(E_p) = \frac{(\bar{\Phi} + 2\Phi e^{-\beta(E_p - \mu_q)}) e^{-\beta(E_p - \mu_q)} + e^{-3\beta(E_p - \mu_q)}}{1 + 3(\bar{\Phi} + \Phi e^{-\beta(E_p - \mu_q)}) e^{-\beta(E_p - \mu_q)} + e^{-3\beta(E_p - \mu_q)}}$$

$$f(E_p + \mu_q) \longrightarrow f_{\Phi}^-(E_p) = \frac{(\Phi + 2\bar{\Phi} e^{-\beta(E_p + \mu_q)}) e^{-\beta(E_p + \mu_q)} + e^{-3\beta(E_p + \mu_q)}}{1 + 3(\Phi + \bar{\Phi} e^{-\beta(E_p + \mu_q)}) e^{-\beta(E_p + \mu_q)} + e^{-3\beta(E_p + \mu_q)}}$$

$$\Phi, \bar{\Phi} \rightarrow 0 \implies f_{\Phi}^{\pm}(E_p) \rightarrow f(3(E_p \pm \mu_q)) \quad \Phi, \bar{\Phi} \rightarrow 1 \implies f_{\Phi}^{\pm}(E_p) \rightarrow f(E_p \pm \mu_q)$$

three-particle state appears: mimics confinement of quarks within baryons



the effect of the Polyakov loop
is more relevant for $T < T_c$

at $T = 0$ there is no difference between
models with and without Polyakov loop:

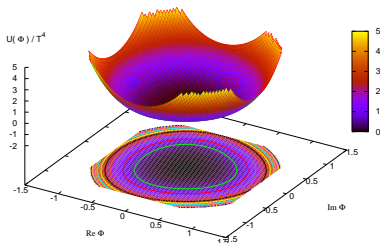
$$\Theta(3(\mu_q - E_p)) \equiv \Theta((\mu_q - E_p))$$

H. Hansen et al., PRD75, 065004

Polyakov loop potential

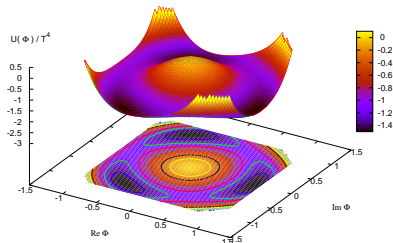
“Color confinement”

$\langle \Phi \rangle = 0 \rightarrow$ no breaking of \mathbb{Z}_3
one minimum



“Color deconfinement”

$\langle \Phi \rangle \neq 0 \rightarrow$ spontaneous breaking of \mathbb{Z}_3
minima at $0, 2\pi/3, -2\pi/3$
one of them spontaneously selected



from H. Hansen et al., PRD75, 065004 (2007)

Form of the potential

I.) Simple **polynomial potential** invariant under \mathbb{Z}_3 and charge

conjugation: R.D.Pisarski, PRD 62, 111501

$$\frac{\mathcal{U}_{\text{poly}}^{\text{YM}}(\Phi, \bar{\Phi})}{T^4} = -\frac{b_2(T)}{2} \bar{\Phi} \Phi - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\bar{\Phi} \Phi)^2$$

with
$$b_2(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \frac{T_0^2}{T^2} + a_3 \frac{T_0^3}{T^3}$$

II.) **Logarithmic potential** coming from the $SU(3)$ Haar measure of group

integration K. Fukushima, Phys. Lett. **B591**, 277 (2004)

$$\frac{\mathcal{U}_{\text{log}}^{\text{YM}}(\Phi, \bar{\Phi})}{T^4} = -\frac{1}{2} a(T) \Phi \bar{\Phi} + b(T) \ln \left[1 - 6 \Phi \bar{\Phi} + 4 (\Phi^3 + \bar{\Phi}^3) - 3 (\Phi \bar{\Phi})^2 \right]$$

with
$$a(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \frac{T_0^2}{T^2}, \quad b(T) = b_3 \frac{T_0^3}{T^3}$$

$\mathcal{U}^{\text{YM}}(\Phi, \bar{\Phi})$ models the free energy of a pure gauge theory

→ the parameters are fitted to the pure gauge lattice data

Improved Polyakov loop potential

Previous potentials describe successfully the first order phase transition of the pure $SU(3)$ Yang–Mills

↔ taking into account the gluon dynamics (quark polarization of gluon propagator) → QCD **glue potential**

↔ can be implemented by changing the reduced temperature

$$t_{\text{glue}} \equiv \frac{T - T_c^{\text{glue}}}{T_c^{\text{glue}}}, \quad t_{\text{YM}} \equiv \frac{T^{\text{YM}} - T_c^{\text{YM}}}{T_c^{\text{YM}}}$$

$$t_{\text{YM}}(t_{\text{glue}}) \approx 0.57 t_{\text{glue}}$$

$$\frac{\mathcal{U}^{\text{glue}}}{T^4}(\Phi, \bar{\Phi}, t_{\text{glue}}) = \frac{\mathcal{U}^{\text{YM}}}{(T^{\text{YM}})^4}(\Phi, \bar{\Phi}, t_{\text{YM}}(t_{\text{glue}}))$$

Field equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$

Field equations for the condensates ($\phi_{N/S}$)

Hybrid approach: fermions at one-loop, mesons at tree-level

→ calculate Ω the **grand canonical potential**

$$\Omega(T, \mu_q) = U_{\text{meson}}^{\text{tree}}(\langle\phi\rangle) + \Omega_{\bar{q}q}^{\text{vac}} + \Omega_{\bar{q}q}^T(T, \mu_q) + \mathcal{U}^{\text{glue}}(\Phi, \bar{\Phi}, t_{\text{glue}}(T))$$

$$i.) \quad \frac{\partial \Omega}{\partial \bar{\sigma}_N} = \frac{\partial \Omega}{\partial \bar{\sigma}_S} \Big|_{\bar{\sigma}_N = \phi_N, \bar{\sigma}_S = \phi_S} = 0$$

$$m_0^2 \phi_N + \left(\lambda_1 + \frac{1}{2} \lambda_2 \right) \phi_N^3 + \lambda_1 \phi_N \phi_S^2 - h_N + \frac{g_F}{2} N_c (\langle u\bar{u} \rangle_T + \langle d\bar{d} \rangle_T) = 0$$

$$m_0^2 \phi_S + (\lambda_1 + \lambda_2) \phi_S^3 + \lambda_1 \phi_N^2 \phi_S - h_S + \frac{g_F}{\sqrt{2}} N_c \langle s\bar{s} \rangle_T = 0$$

$$\langle q\bar{q} \rangle_T = -4m_q \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{2E_q(p)} (1 - f_{\Phi}^{-}(E_q(p)) - f_{\Phi}^{+}(E_q(p)))$$

Field equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$

Field equations for the Polyakov-loop variables

$$n.) \quad \frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \bar{\Phi}} \Big|_{\bar{\sigma}_N = \phi_N, \bar{\sigma}_S = \phi_S} = 0,$$

$$- \frac{d}{d\Phi} \left(\frac{U(\Phi, \bar{\Phi})}{T^4} \right) + \frac{2N_c}{T^3} \sum_{q=u,d,s} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left(\frac{e^{-\beta E_q^-(p)}}{g_q^-(p)} + \frac{e^{-2\beta E_q^+(p)}}{g_q^+(p)} \right) = 0$$

$$- \frac{d}{d\bar{\Phi}} \left(\frac{U(\Phi, \bar{\Phi})}{T^4} \right) + \frac{2N_c}{T^3} \sum_{q=u,d,s} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left(\frac{e^{-\beta E_q^+(p)}}{g_q^+(p)} + \frac{e^{-2\beta E_q^-(p)}}{g_q^-(p)} \right) = 0$$

$$g_q^+(p) = 1 + 3 \left(\bar{\Phi} + \Phi e^{-\beta E_q^+(p)} \right) e^{-\beta E_q^+(p)} + e^{-3\beta E_q^+(p)}$$

$$g_q^-(p) = 1 + 3 \left(\Phi + \bar{\Phi} e^{-\beta E_q^-(p)} \right) e^{-\beta E_q^-(p)} + e^{-3\beta E_q^-(p)}$$

$$E_q^\pm(p) = E_q(p) \mp \mu_B/3, \quad E_{u/d}(p) = \sqrt{p^2 + m_{u/d}^2}, \quad E_s(p) = \sqrt{p^2 + m_s^2}$$

Curvature masses

$$\mathcal{M}_{i,ab}^2 = \left. \frac{\partial^2 \Omega(T, \mu_f)}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} = m_{i,ab}^2 + \Delta_0 m_{i,ab}^2 + \Delta_T m_{i,ab}^2,$$

$m_{i,ab}^2 \longrightarrow$ tree-level mass matrix,

$\Delta_{0/T} m_{i,ab}^2 \longrightarrow$ fermion vacuum/thermal fluctuation,

$$\Delta_0 m_{i,ab}^2 = \left. \frac{\partial^2 \Omega_{q\bar{q}}^{\text{vac}}}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} = -\frac{3}{8\pi^2} \sum_{f=u,d,s} \left[\left(\frac{3}{2} + \log \frac{m_f^2}{M^2} \right) m_{f,a}^{2(i)} m_{f,b}^{2(i)} + m_f^2 \left(\frac{1}{2} + \log \frac{m_f^2}{M^2} \right) m_{f,ab}^{2(i)} \right],$$

$$\Delta_T m_{i,ab}^2 = \left. \frac{\partial^2 \Omega_{q\bar{q}}^{\text{th}}}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \right|_{\min} = 6 \sum_{f=u,d,s} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_f(p)} \left[(f_f^+(p) + f_f^-(p)) \left(m_{f,ab}^{2(i)} - \frac{m_{f,a}^{2(i)} m_{f,b}^{2(i)}}{2E_f^2(p)} \right) \right. \\ \left. + (B_f^+(p) + B_f^-(p)) \frac{m_{f,a}^{2(i)} m_{f,b}^{2(i)}}{2TE_f(p)} \right],$$

where $m_{f,a}^{2(i)} \equiv \partial m_f^2 / \partial \varphi_{i,a}$, $m_{f,ab}^{2(i)} \equiv \partial^2 m_f^2 / \partial \varphi_{i,a} \partial \varphi_{i,b}$

Curvature masses

Table : The first and second derivatives of the squared quark mass with respect to the scalar (S) and pseudoscalar (P) meson fields evaluated at the minimum in the $N - S$ basis. In the first two columns a summation over $\ell \in \{u, d\}$ is understood.

i	ab	$m_{\ell,a}^{2(i)} m_{\ell,b}^{2(i)} / g^4$	$m_{\ell,ab}^{2(i)} / g^2$	$m_{s,a}^{2(i)} m_{s,b}^{2(i)} / g^4$	$m_{s,ab}^{2(i)} / g^2$
S	11	$\frac{1}{2} \phi_N^2$	1	0	0
S	44	0	$\frac{Z_K^2 \phi_N}{\phi_N - \sqrt{2} \phi_S}$	0	$\frac{-\sqrt{2} Z_K^2 \phi_S}{\phi_N - \sqrt{2} \phi_S}$
S	NN	$\frac{1}{2} \phi_N^2$	1	0	0
S	SS	0	0	ϕ_S^2	1
P	11	0	Z_π^2	0	0
P	44	0	$\frac{Z_K^2 \phi_N}{\phi_N + \sqrt{2} \phi_S}$	0	$\frac{\sqrt{2} Z_K^2 \phi_S}{\phi_N + \sqrt{2} \phi_S}$
P	NN	0	$Z_{\eta N}^2$	0	0
P	SS	0	0	0	$Z_{\eta S}^2$

Determination of the parameters

14 unknown parameters ($m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, \mathbf{gF}$) \longrightarrow determined by the **min. of χ^2** :

$$\chi^2(x_1, \dots, x_N) = \sum_{i=1}^M \left[\frac{Q_i(x_1, \dots, x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2,$$

$(x_1, \dots, x_N) = (m_0, \lambda_1, \lambda_2, \dots)$, $Q_i(x_1, \dots, x_N) \longrightarrow$ from the model, $Q_i^{\text{exp}} \longrightarrow$ PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$

multiparametric minimalization \longrightarrow **MINUIT**

- PCAC \rightarrow 2 physical quantities: f_π, f_K

- Curvature masses \rightarrow 16 physical quantities:

$m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$

- Decay widths \rightarrow 12 physical quantities:

$\Gamma_{\rho \rightarrow \pi\pi}, \Gamma_{\Phi \rightarrow KK}, \Gamma_{K^* \rightarrow K\pi}, \Gamma_{a_1 \rightarrow \pi\gamma}, \Gamma_{a_1 \rightarrow \rho\pi}, \Gamma_{f_1 \rightarrow KK^*}, \Gamma_{a_0}, \Gamma_{K_S \rightarrow K\pi}, \Gamma_{f_0^L \rightarrow \pi\pi}, \Gamma_{f_0^L \rightarrow KK}, \Gamma_{f_0^H \rightarrow \pi\pi}, \Gamma_{f_0^H \rightarrow KK}$

Features of our approach

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables with \mathcal{U}^{YM} or $\mathcal{U}^{\text{glue}}$
- constituent quarks
- Four order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}) \longrightarrow$
four coupled T/μ_B -dependent equations
- Fermion **vacuum** fluctuations
- Fermion **thermal** fluctuations
- Fermion contributions to the tree-level meson masses \longrightarrow
curvature masses
- + **Thermal pion fluctuations for the pressure and other thermodynamical quantities**

Consequence of scalar mesons sector

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$a_0(980)$	980 ± 20	$50 - 100$	$\pi\pi$ dominant
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$f_0(1500)$	1505 ± 6	109 ± 7	$\pi\pi \approx 38, K\bar{K} \approx 9.4$
$f_0(1710)$	1722 ± 6	135 ± 7	$\pi\pi \approx 30, K\bar{K} \approx 71$

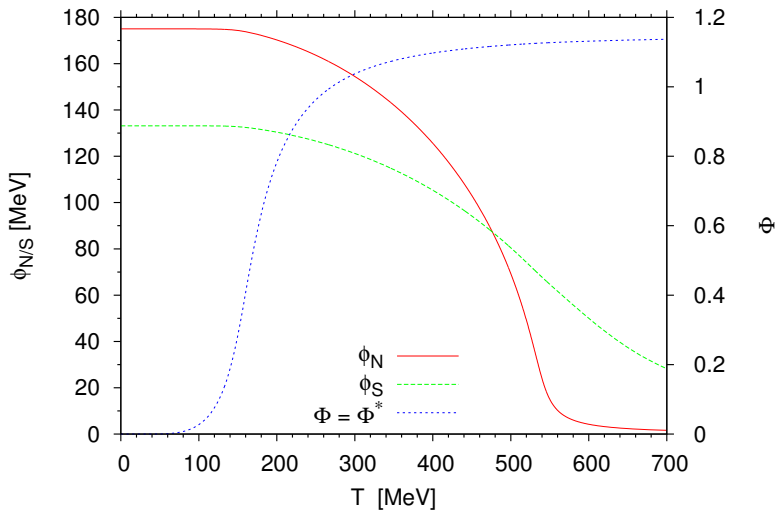
↔ We have 40 assignment possibilities!

Different parameterizations can give different thermodynamical behavior

T dependence of the order parameters

With high mass scalars, $m_{f_0^L} = 1326$ MeV

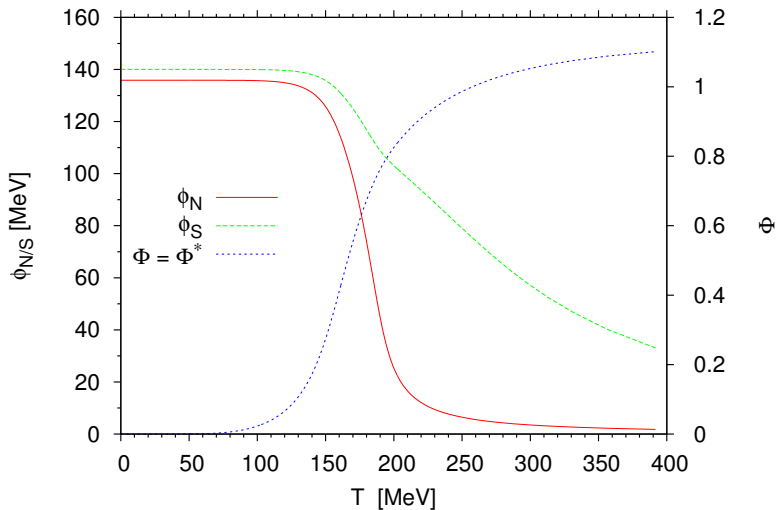
Condensates and Polyakov loop variables with vacuum fluctuations



T dependence of the order parameters

With low mass scalars, $m_{f_0^L} = 402$ MeV

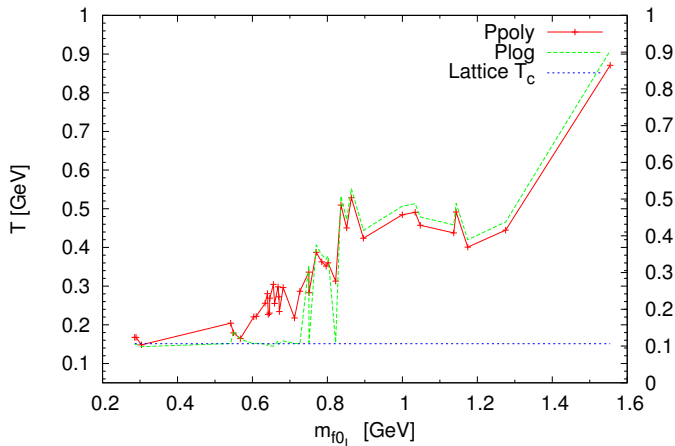
Condensates and Polyakov loop variables with vacuum fluctuations



T dependence of the order parameters

T_c at $\mu_B = 0$ for various parameterizations

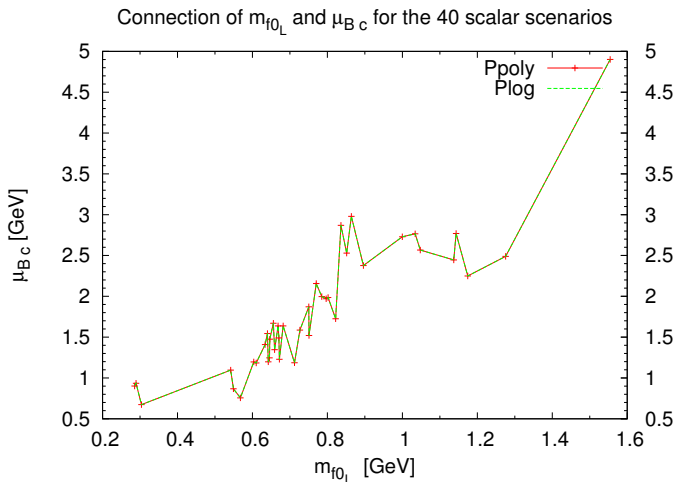
Connection of $m_{f_{0L}}$ and T_c for the 40 scalar scenarios



40 parameterizations \rightarrow all the possible combinations for the masses of the scalar sector

T dependence of the order parameters

μ_{B_c} at $T = 0$ for various parameterizations

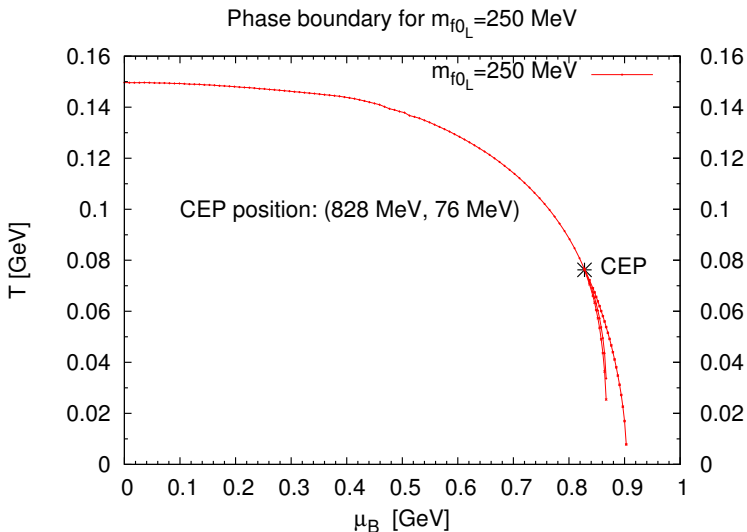


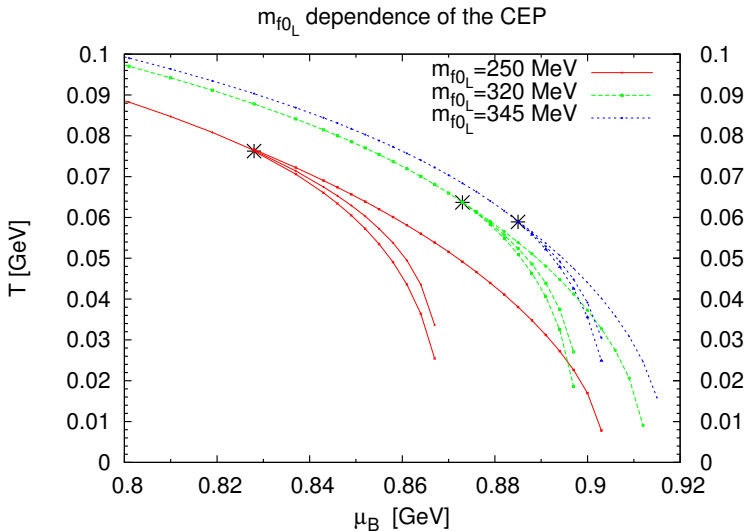
40 parameterizations \rightarrow all the possible combinations for the masses of the scalar sector

Remarks

- In all 40 cases the best χ^2 solution was chosen
- Only parameterizations, which produced $m_{f_0^L} \lesssim 800$ MeV can have $T_c \approx 151$ MeV (lattice data)
- Only parameterizations, which produced $m_{f_0^L} \lesssim 400$ MeV can have 1st order transition in $\mu_B \implies$ there is CEP
- If $T_c \approx 150$ MeV and the CEP exists $\implies m_{a_0}$ and m_{K_S} are also below 1 GeV
- Important note: **scale dependence** \longrightarrow can **change the numbers**

The phase boundary



CEP for different f_0^L masses

Subtracted condensate, pressure, energy density, etc...

Calculation of thermodynamical quantities

$$\text{pressure: } p = \frac{\partial(T \ln Z)}{\partial V} = -\Omega$$

$$\text{entropy density: } s = \frac{1}{V} \frac{\partial(T \ln Z)}{\partial T} = \frac{\partial p}{\partial T}$$

$$\text{energy density } (\mu_B = 0): \epsilon = -p + Ts$$

mesonic thermal 1-loop contribution to the pressure:

$$p_{\text{meson}} = -\Omega_{\text{meson}}^{1\text{-loop}, T} = -NT \int \frac{d^3 p}{(2\pi)^3} \ln \left(1 - e^{-\beta \omega(p)} \right)$$

$$\text{where, } \omega(p) = \sqrt{p^2 + m^2}$$

to compare with the lattice \rightarrow

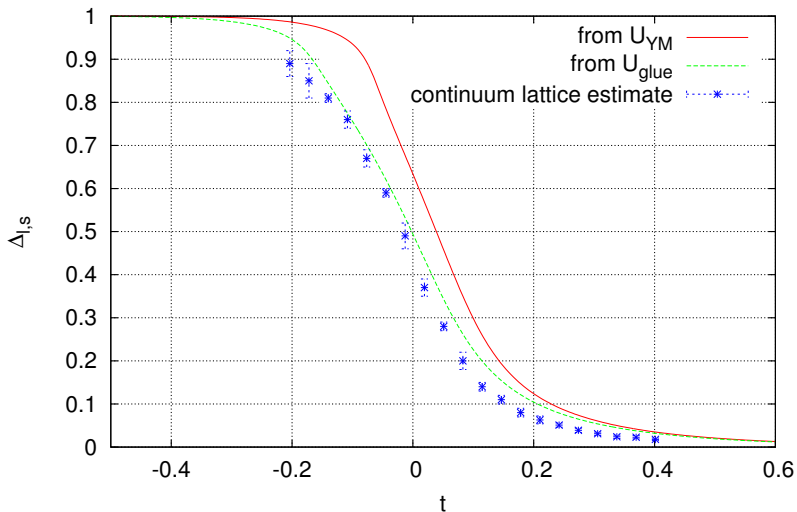
$$\text{subtracted condensate: } \Delta_{l,s} = \frac{\Phi_N - \frac{\hbar_N}{\hbar_S} \cdot \Phi_S|_T}{\Phi_N - \frac{\hbar_N}{\hbar_S} \cdot \Phi_S|_{T=0}}$$

$$\text{interaction measure: } I/T^4 = (\epsilon - 3p)/T^4$$

Subtracted condensate, pressure, energy density, etc...

The subtracted condensate

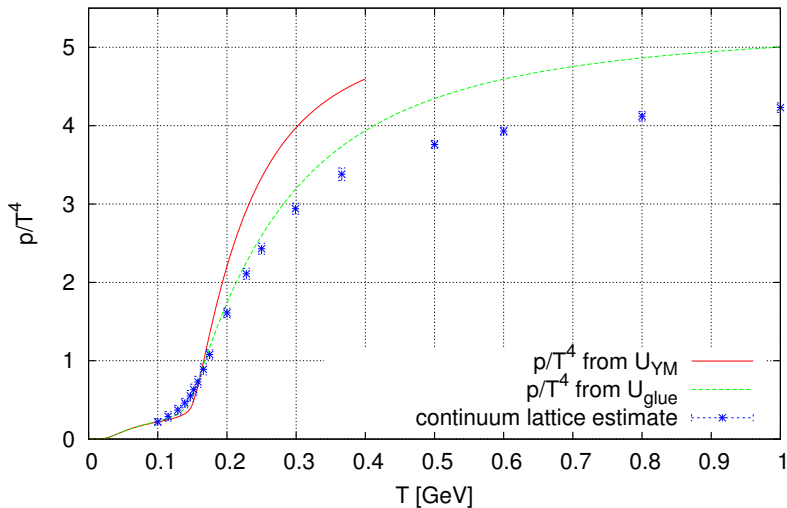
t dependence of subtracted condensate



Subtracted condensate, pressure, energy density, etc...

The normalized pressure

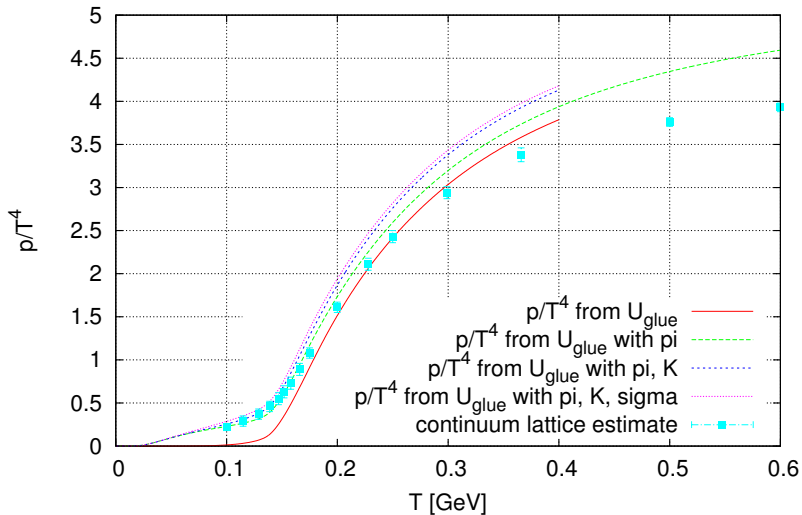
T dependence of pressure with thermal pions



Subtracted condensate, pressure, energy density, etc...

Different mesonic thermal fluctuations to the pressure

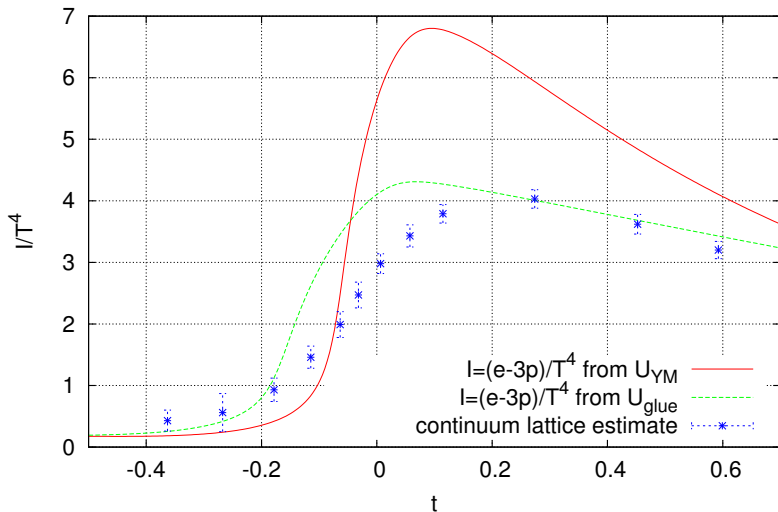
Influence of the mesonic thermal fluctuations on $p(T)$



Subtracted condensate, pressure, energy density, etc...

Interaction measure or trace anomaly

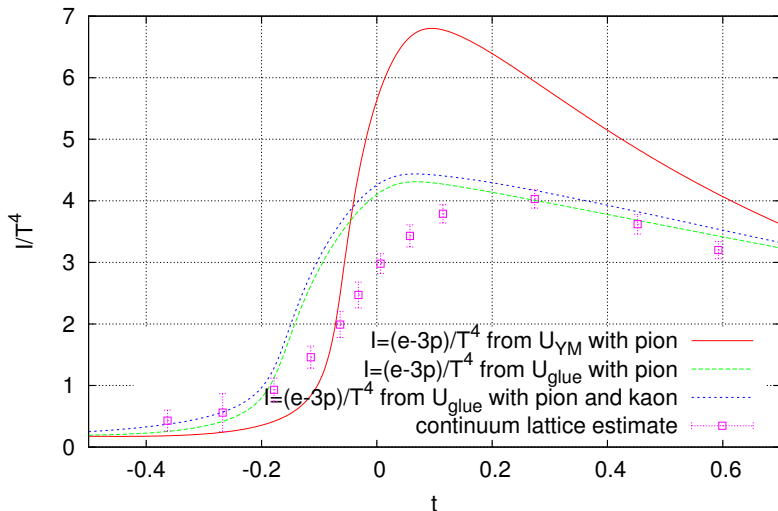
t dependence of interaction measure with thermal pions



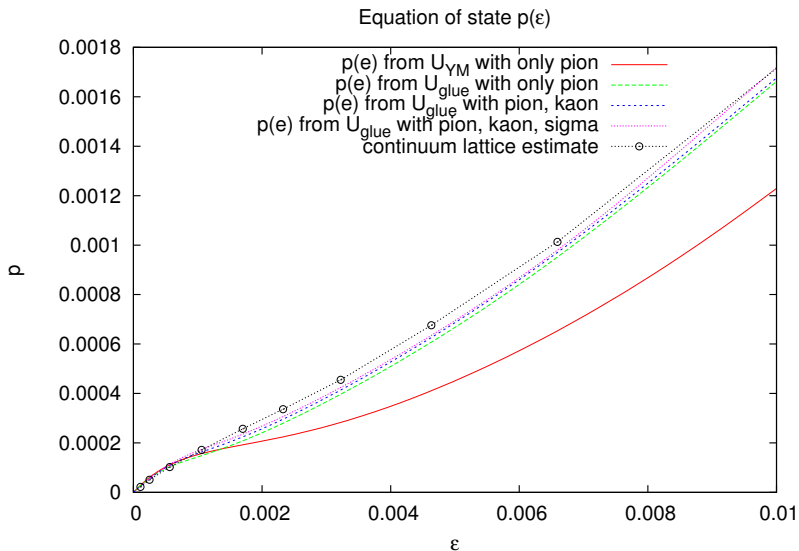
Subtracted condensate, pressure, energy density, etc...

Influence of thermal mesons on the interaction measure

Influence of the mesonic thermal fluctuations on the trace anomaly



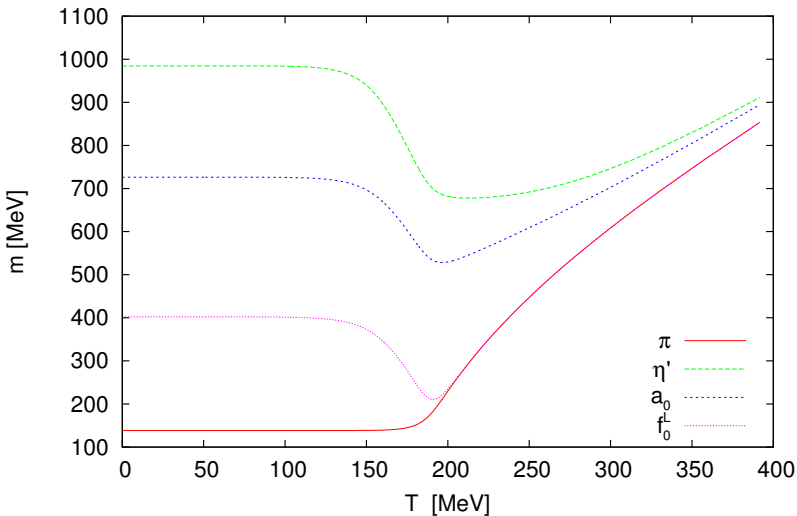
Subtracted condensate, pressure, energy density, etc...

Equation of state at $\mu_B = 0$ 

T dependence of the (pseudo)scalar masses

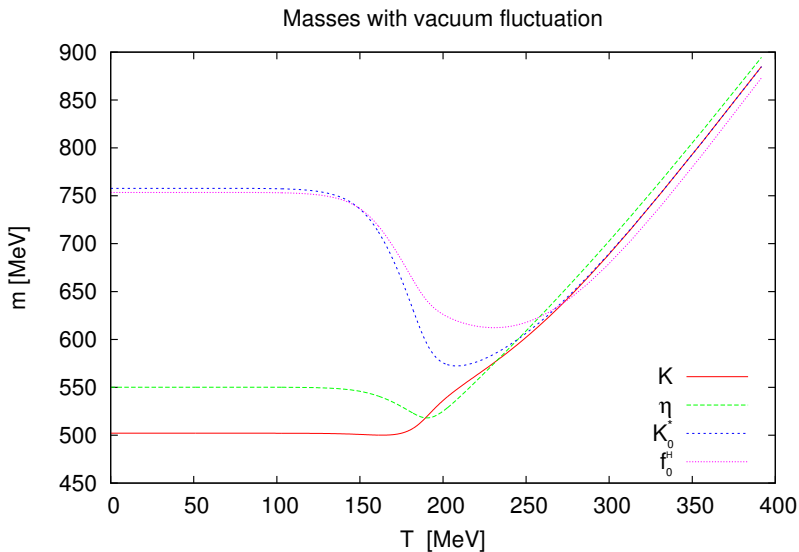
π, η', a_0, f_0^L masses

Masses with vacuum fluctuation



T dependence of the (pseudo)scalar masses

K, η, K^*, f_0^H masses



Summary

- Thermodynamics of a vector meson extended Polyakov quark meson was investigated
- We used a hybrid approach: fermion vacuum/thermal fluctuations (has the largest contribution), bosons at tree-level (except in pressure and such)
- We investigated the 40 possible scalar parameterization scenarios
- At finite T/μ_B there was 4 coupled equations for the 4 order parameters
- Various thermodynamical quantities were calculated, which show quiet good agreement with lattice results, if we use the improved Polyakov potential

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- At finite T/μ_B there was 4 coupled equations for the 4 order parameters
- Various thermodynamical quantities were calculated, which show quiet good agreement with lattice results, if we use the improved Polyakov potential
- Model can be used to give predictions at finite densities (masses, decay width, all the thermodynamical quantities)

Thank you for your attention!

μ_B dependence of the π, η, η', K masses

Masses with vacuum fluctuation

