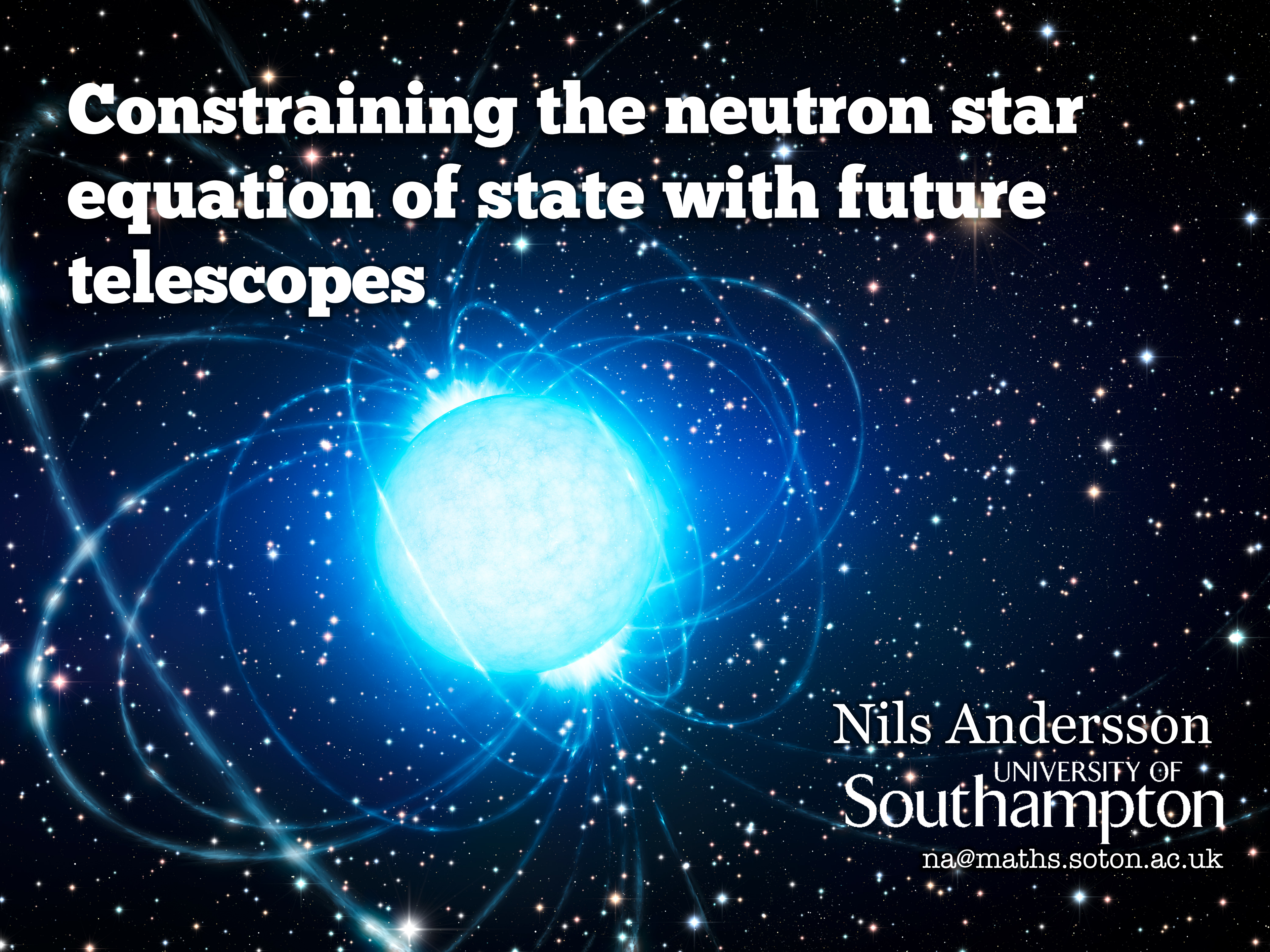

Constraining the neutron star equation of state with future telescopes

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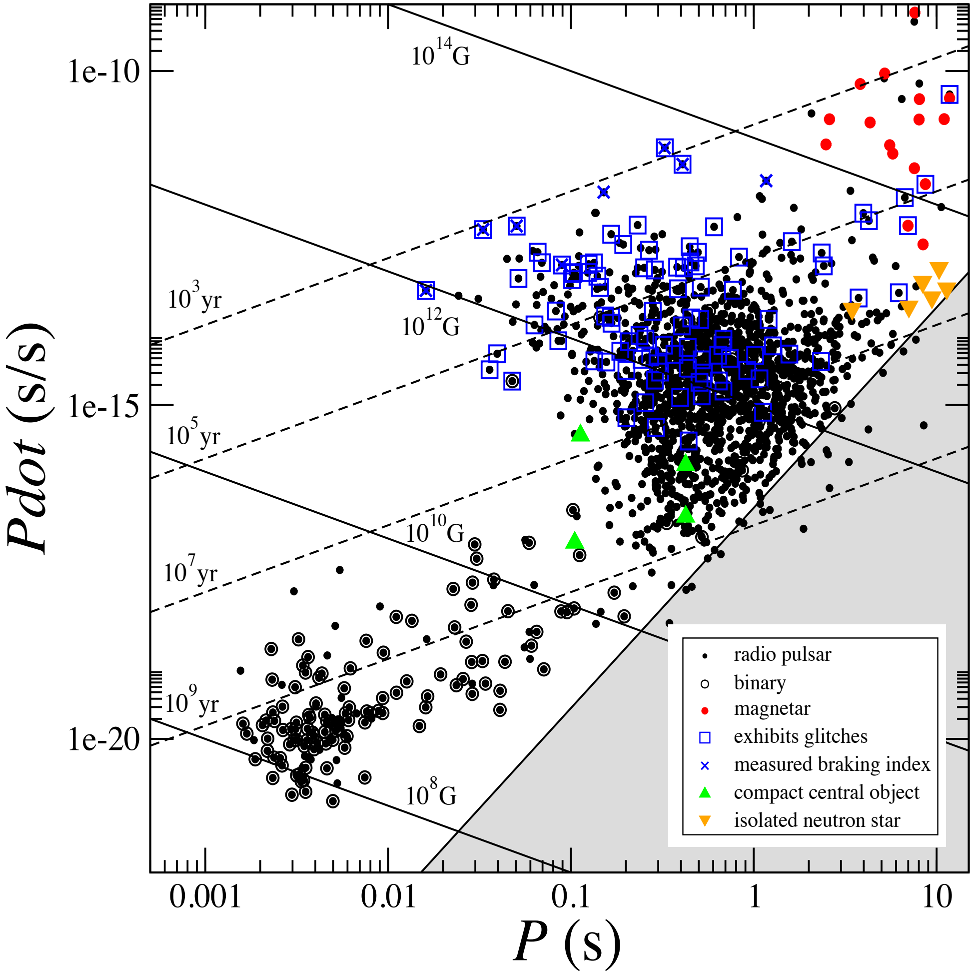
observer's view



We have precise timing data for more than 2,300 (mainly radio) pulsars.

Different classes of neutron stars populate different parts of the P-P-dot diagram.

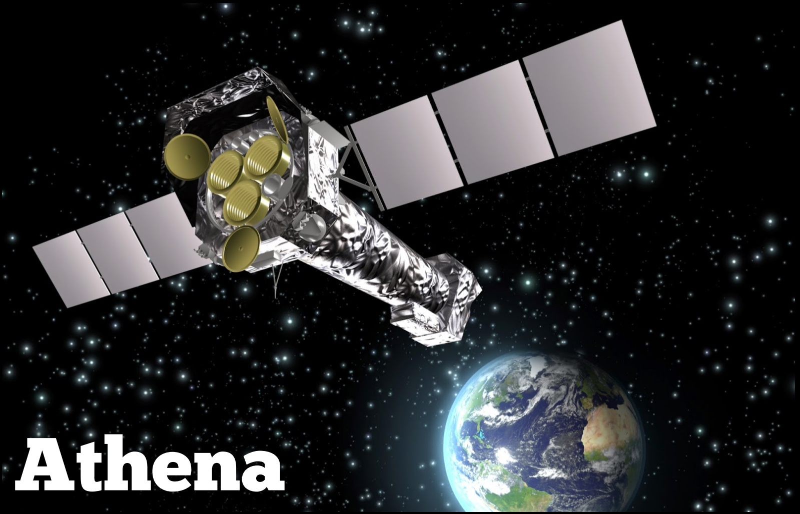
The spin-down rate allows us to infer the star's (exterior) magnetic field.

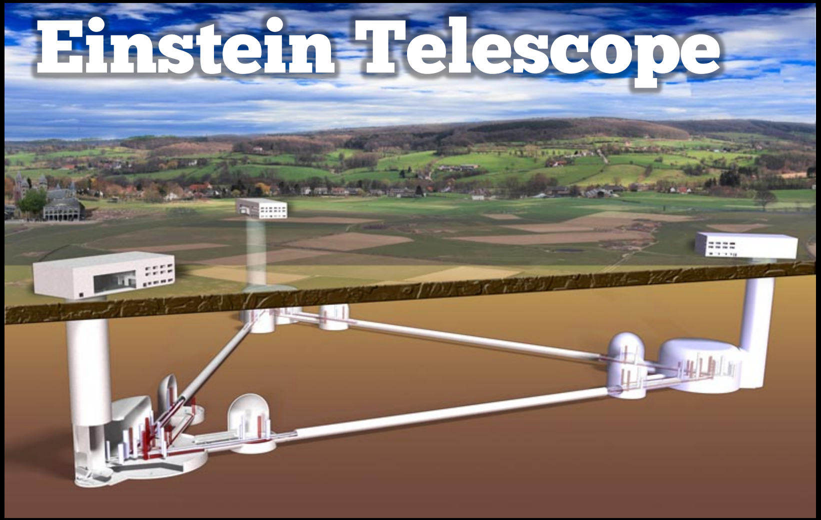
But... as soon as we consider evolutionary aspects (braking index, glitches and so on), we run into difficulties.

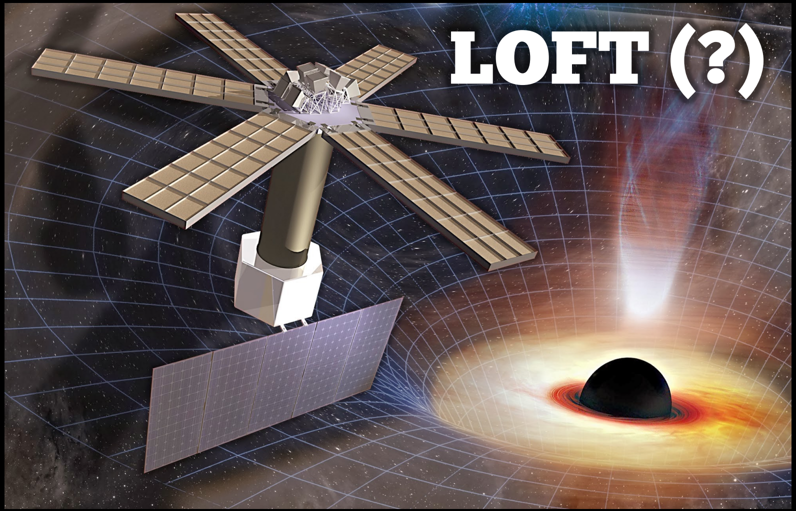
Keep in mind: After more than 35 years we don't know why pulsars pulse!

the future

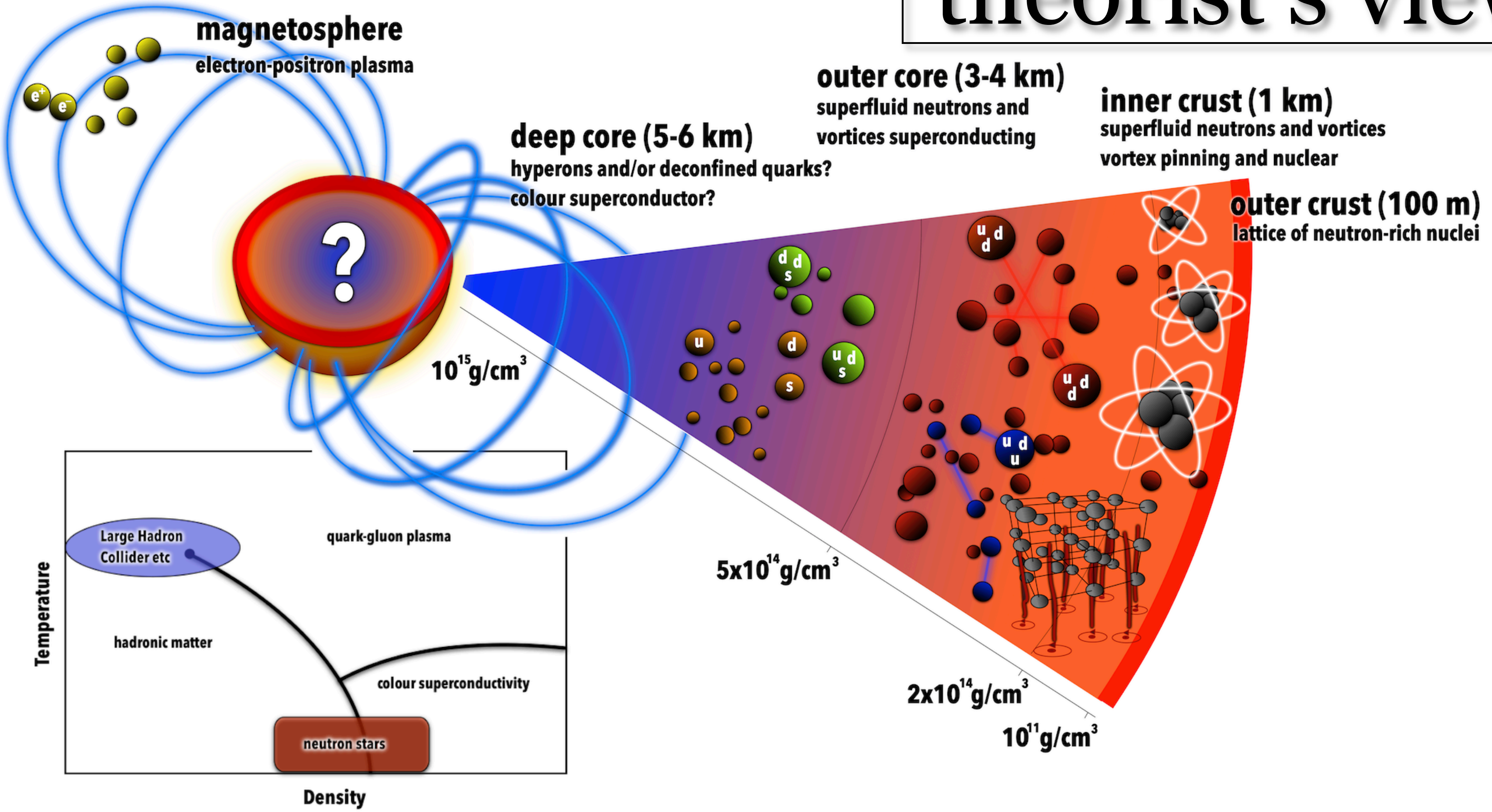
In the next decade(s) a generation of revolutionary telescopes will come on-line, providing high quality information in a range of observing “bands”.







theorist's view



All four fundamental forces at play:

Gravity, holds the star together (gravitational waves?)

Electromagnetism, makes pulsars pulse and magnetars flare (radio/X-rays)

Strong interaction, determines the internal composition

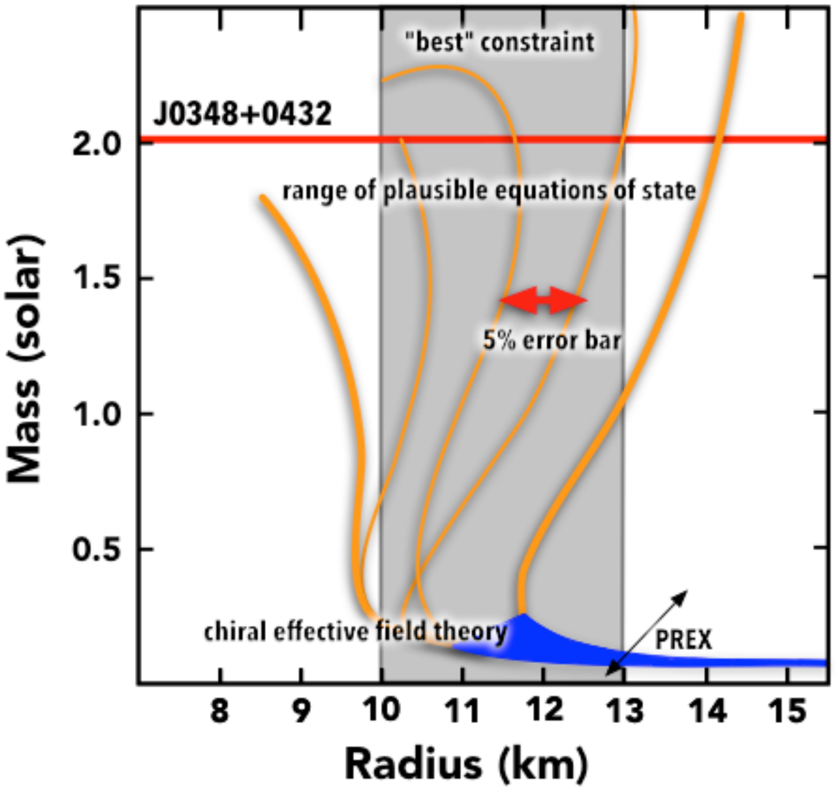
Weak interaction, affects reaction rates - cooling and internal viscosity

fundamental physics

The **equation of state** is the main diagnostic of dense matter interactions.

Each model generates a unique mass-radius relation, predicting a characteristic radius for a range of masses and a maximum mass above which a neutron star collapses to a black hole.

Constrain the physics by combining data from different observational channels.



Orbital data for binaries provide accurate masses; the maximum mass must be above 2 $M_\odot$.

Surface phenomena constrain the radius of a 1.4 $M_\odot$ star to 11-12 km.

The data is beginning to impact on the nuclear physics…

Keep in mind:

- first principles calculations are challenging,
- astrophysics may do better than upcoming nuclear physics experiments (e.g. PREX).

fundamental physics

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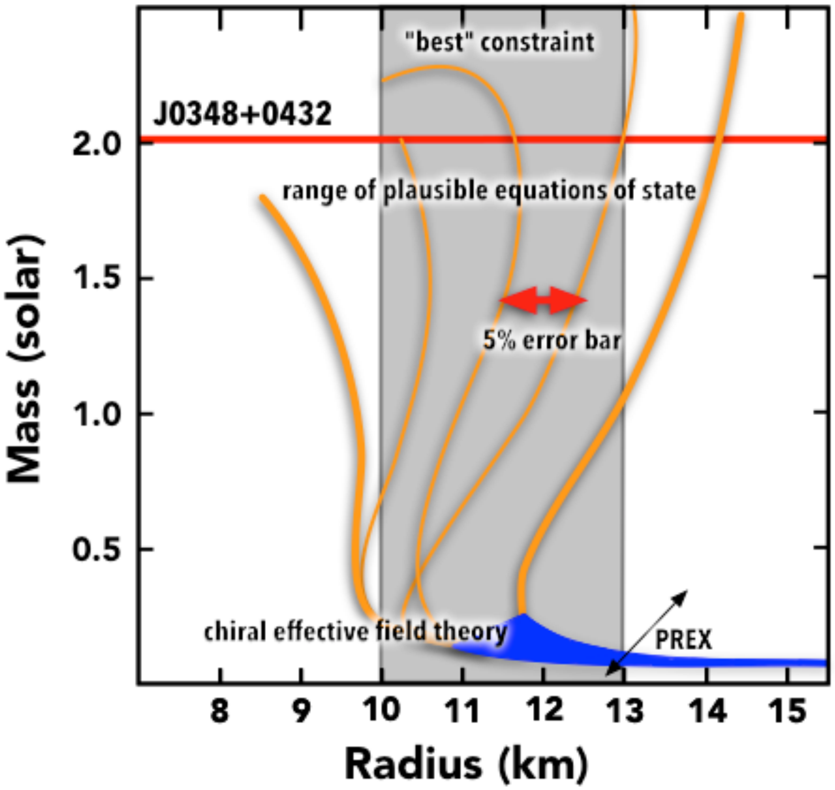


NASAs NICER mission will provide an “accurate” data point.

LIGO (and eventually ET) will (!) detect binaries and infer individual masses (compressibility from Love number?).

SKA will provide a much larger sample of neutron star masses.

Athena will add to the wealth of surface data (Chandra, XMM, NuSTAR).

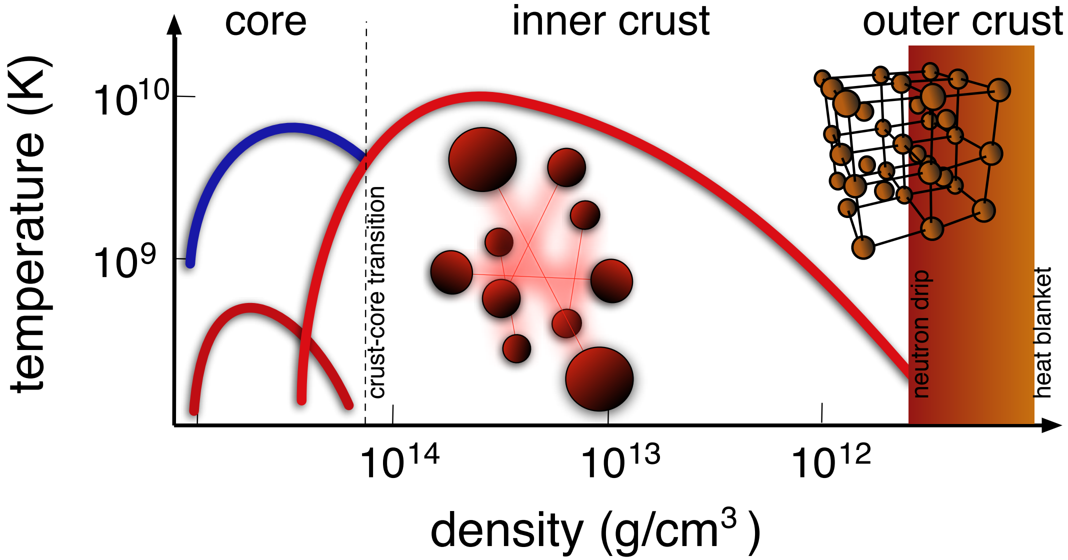
Need a precision X-ray timing mission (like LOFT) to study burst dynamics and magnetar seismology.

state of matter

Now… there is more to neutron stars than the bulk properties.

Need to “dig deeper” to understand the **state** and **composition** of matter.

This is particularly important for **evolutionary** aspects and **dynamics**, as they involve transport coefficients (thermal, viscous, resistive).



Mature neutron stars are “cold” (10^8K<< T_{Fermi}=10^{12}K) so they **should be** either solid or superfluid.

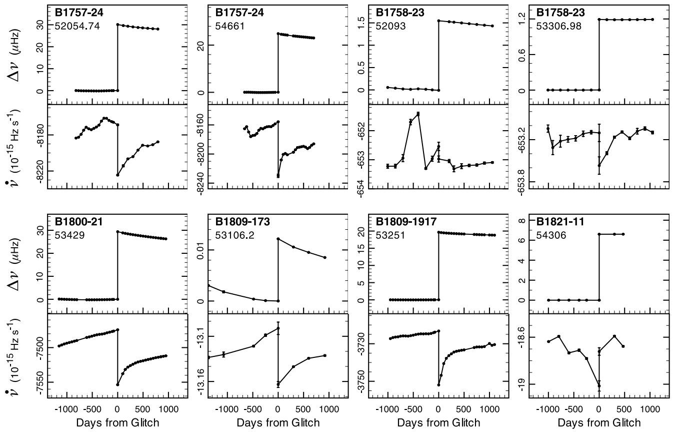
Superfluidity **suppresses** reactions and scattering and **adds** dynamical degrees of freedom.

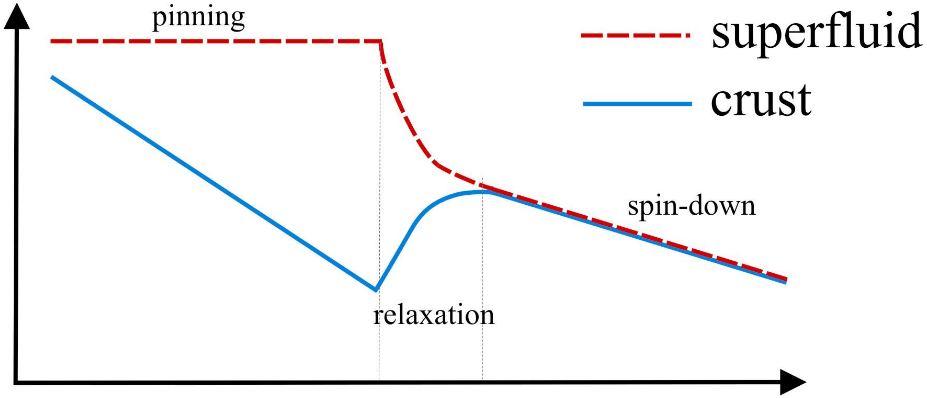
Neutron stars are **multi-fluid systems** (cf. 2-fluid model for Helium).

pulsar glitches

The strongest (current) constraint on superfluid parameters comes from the "real-time" cooling of the Cas A remnant.

We also have "convincing" evidence for the presence of superfluidity from observed pulsar glitches.





Cartoon explanation:

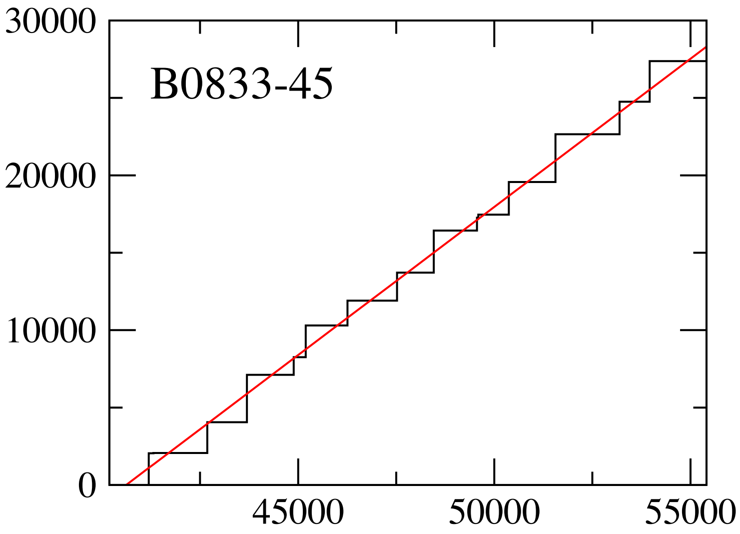
1. the crust slows down due to magnetic braking,
2. the superfluid can only spin down if vortices (by means of which the superfluid rotates) move outwards,
3. if the vortices are pinned (to the crust, say), the superfluid lags behind,
4. at some critical level, a large number of vortices are released. As a result the crust is spun up.

the crust is not enough

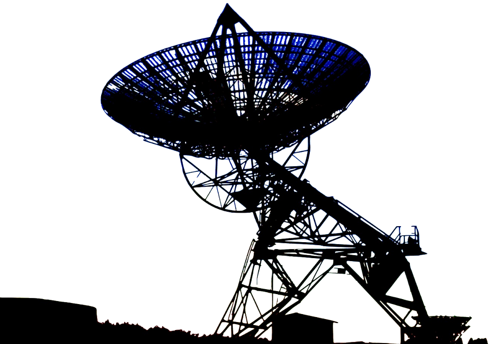
There is no quantitative model for glitch dynamics, but for regular glitchers, one can estimate the superfluid moment of inertia.

Need to involve up to 2% of the total.

The **crust superfluid** can do this; as long as we do not worry about the actual mobility of the superfluid component (entrainment).



The large effective neutron mass in the crust (due to Bragg scattering of neutrons by the nuclear lattice) lowers the effective superfluid moment of inertia by a factor of 5 or so. This is problematic...



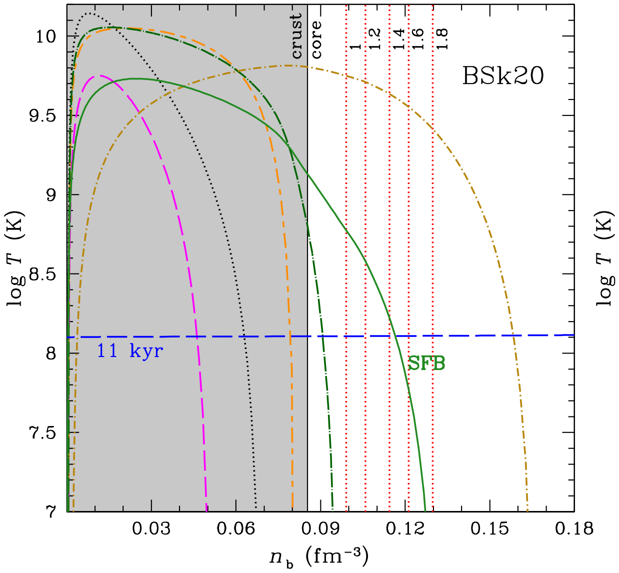
1. A fraction of the **core superfluid** could be involved, but why would the glitches be "the same size"?
2. The (singlet) pairing gap could lead to a superfluid region just large enough to explain the observations.
3. Lack of "precision": Need more accurate parameters.

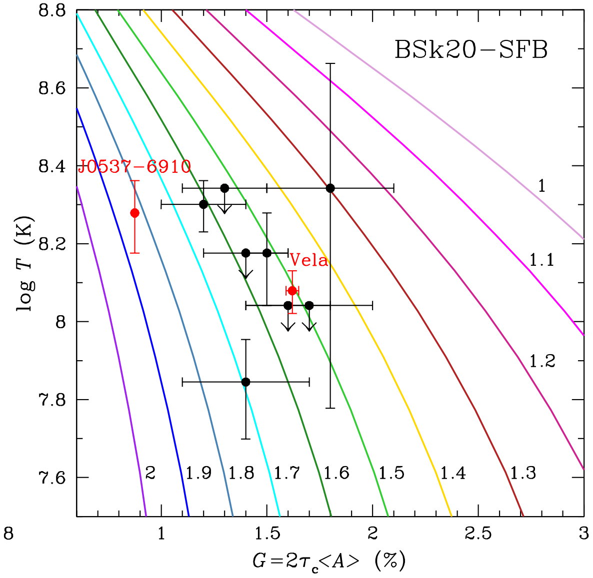
mind the gap

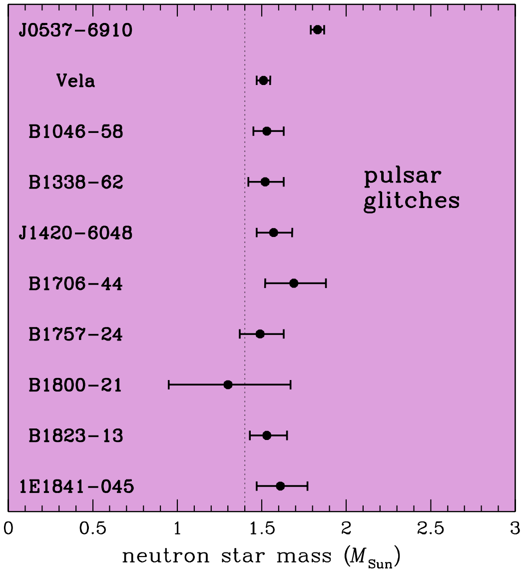
A possible resolution to the problem would be to involve only the singlet superfluid in the crust + outer region of the core.

The data can then be turned into a constraint on the superfluid pairing gap (provided one has some idea of the star’s temperature, and assuming that the angular momentum reservoir is exhausted in each glitch event).

Interestingly, most available gap models **fail** this test.







If we take the pairing gap as given, we can infer the mass of a glitching pulsar.

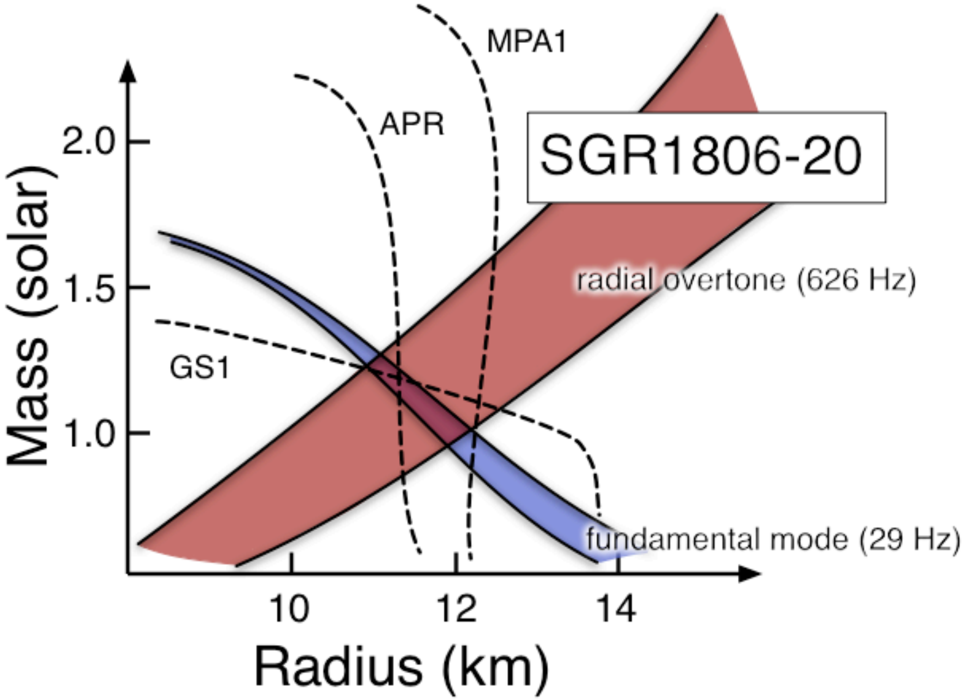
SKA will add significantly to the data (revolve actual glitch rise?), so…

seismology

A neutron star has a rich spectrum of oscillation modes.

Different classes of modes depend (sometimes quite sensitively) on specific physics, making "asteroseismology" a promising strategy for probing the star's interior.

Observed quasi-periodic oscillations in X-ray tail from magnetar giant flares provided the first real opportunity to put this scheme into practice.



If the observed oscillations are associated with the crust then we can constrain **both** mass and radius.

However…

- the magnetic field couples the crust to the core (need field configuration/superconductor?)
- the presence of a superfluid component affects the oscillations (entrainment)

beyond equilibrium

Any state-of-the-art model for neutron star dynamics must account for the fact that these are **multi-component** multi-fluid systems (the composition varies and there are relative flows – heat, charge currents, superfluids).

This requires “**beyond equilibrium**” equation of state information.

As example, consider the pressure perturbation for npe-matter;

$$p = p(n_n, n_p, n_e) \quad \Rightarrow$$

$$\delta p = n_n \delta\mu_n + n_p \delta\mu_p + n_e \delta\mu_e = \qquad \text{[1. definition]}$$

$$= n_n \delta\mu_n + n_p \left(\delta\mu_p + \delta\mu_e\right) \qquad \text{[2. charge neutrality]}$$

$$= n\left(1 - x_p\right)\delta\mu_n + n x_p \left(\delta\mu_p + \delta\mu_e\right) \qquad \text{[3. introduce proton fraction]}$$

$$= n\delta\mu_n + n x_p \left(\delta\mu_p + \delta\mu_e - \delta\mu_n\right) \qquad \text{[4. beta equilibrium]}$$

$$= n\delta\mu_n$$

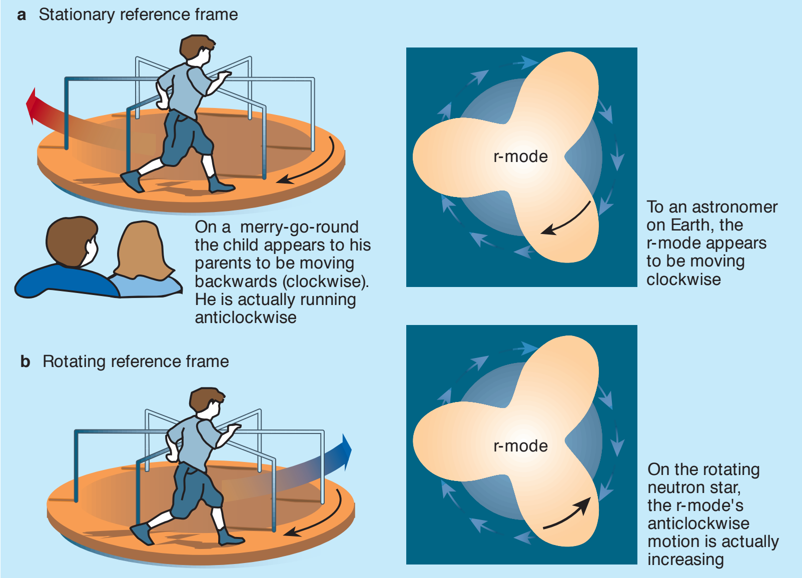
Depending on the state of matter (normal/superfluid) and the regime (fast/slow reactions), one may have to keep track of many thermodynamical derivatives. These can not be (easily) inferred from a tabulated equilibrium equation of state.

CFS instability

In order to be observable, oscillations must be excited to a large amplitude. Instabilities are particularly interesting.

Gravitational waves may drive a secular instability in rotating relativistic stars.

This mechanism may limit the spin of neutron stars at the same time as it generates detectable gravitational waves.



a Stationary reference frame

On a merry-go-round the child appears to his parents to be moving backwards (clockwise). He is actually running anticlockwise

To an astronomer on Earth, the r-mode appears to be moving clockwise

b Rotating reference frame

On the rotating neutron star, the r-mode's anticlockwise motion is actually increasing

Cartoon explanation: A given mode is unstable if the star is losing “negative energy”.

A “neutral” mode of oscillation signals the onset of instability.

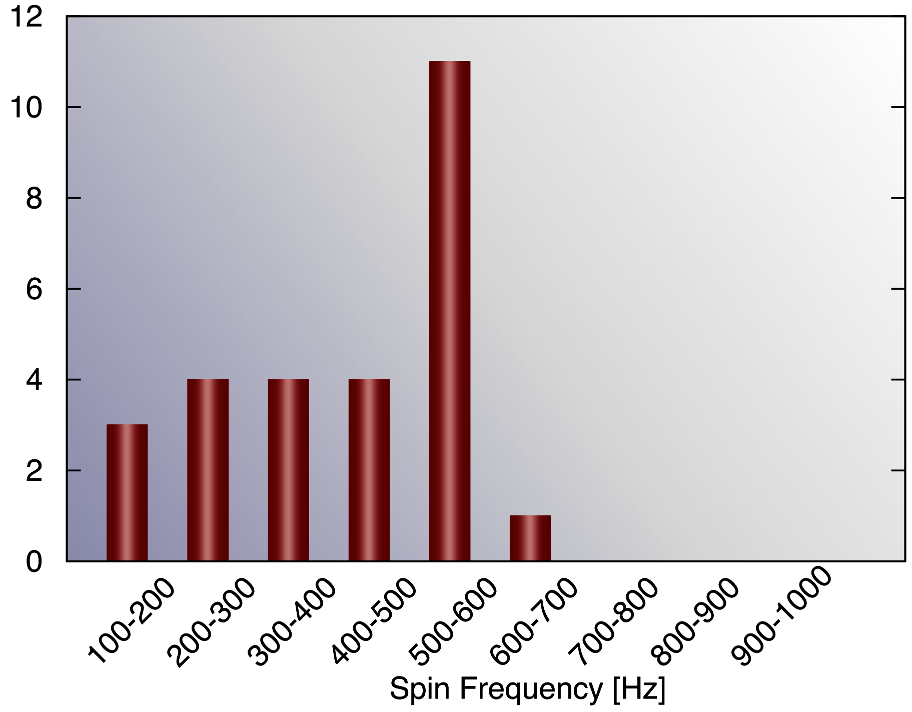
The modes that are thought to be the most important are the “acoustic” f-modes, and the “Coriolis driven” r-modes.

Instability windows depend sensitively on uncertain physics. Simplest models involve shear- and bulk viscosity, but a range of mechanisms have been considered.

LMXBs

Millisecond pulsars form by accreting matter (and angular momentum) from a binary companion.

Some accreting systems are seen as X-ray pulsars, like J1808-3658 which has a spin period of 2.5 ms, for others the spin is inferred from X-ray burst oscillations.



[27 systems from Patruno, unpublished]

All known systems rotate well below the break-up limit.

The fastest rotating LMXB neutron star, 4U 1608, spins at 620 Hz.

Some kind of speed-limit seems to be enforced.

Explanations:

- "refined" accretion torque,
- gravitational wave emission (eg. r-modes).

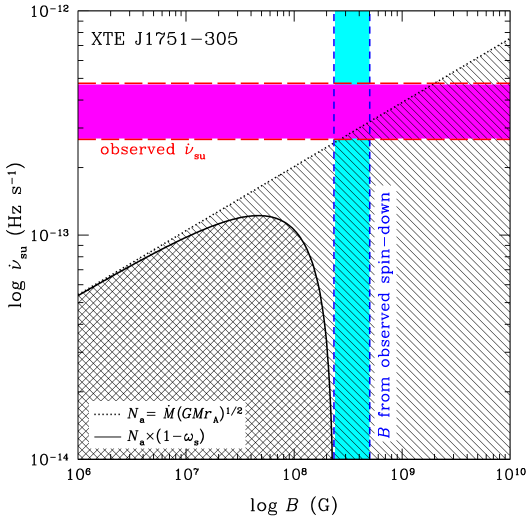
The latter might lead to a pile-up at the highest frequencies (some hint at this in the data?).

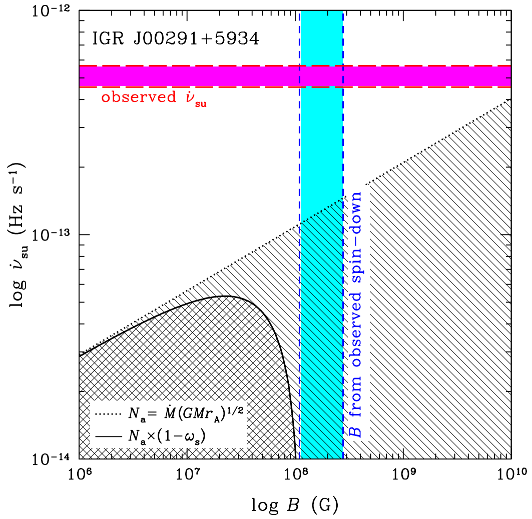
spin evolution

A key question concerns to what extent these systems “require” an additional spindown torque, e.g. gravitational-wave emission.

No compelling evidence for “anomalous” spin behaviour in the observed LMXBs, e.g. the X-ray pulsar J1808-3658.

Evolution seems consistent with accretion spin-up balanced by electromagnetic dipole spin-down.





However, the theory does not work “perfectly”. If it did, the inferred magnetic field from spin-down would be consistent with the spin-up torque. It is not.

Take home message: The accretion torque needs to be better understood.

a "conundrum"

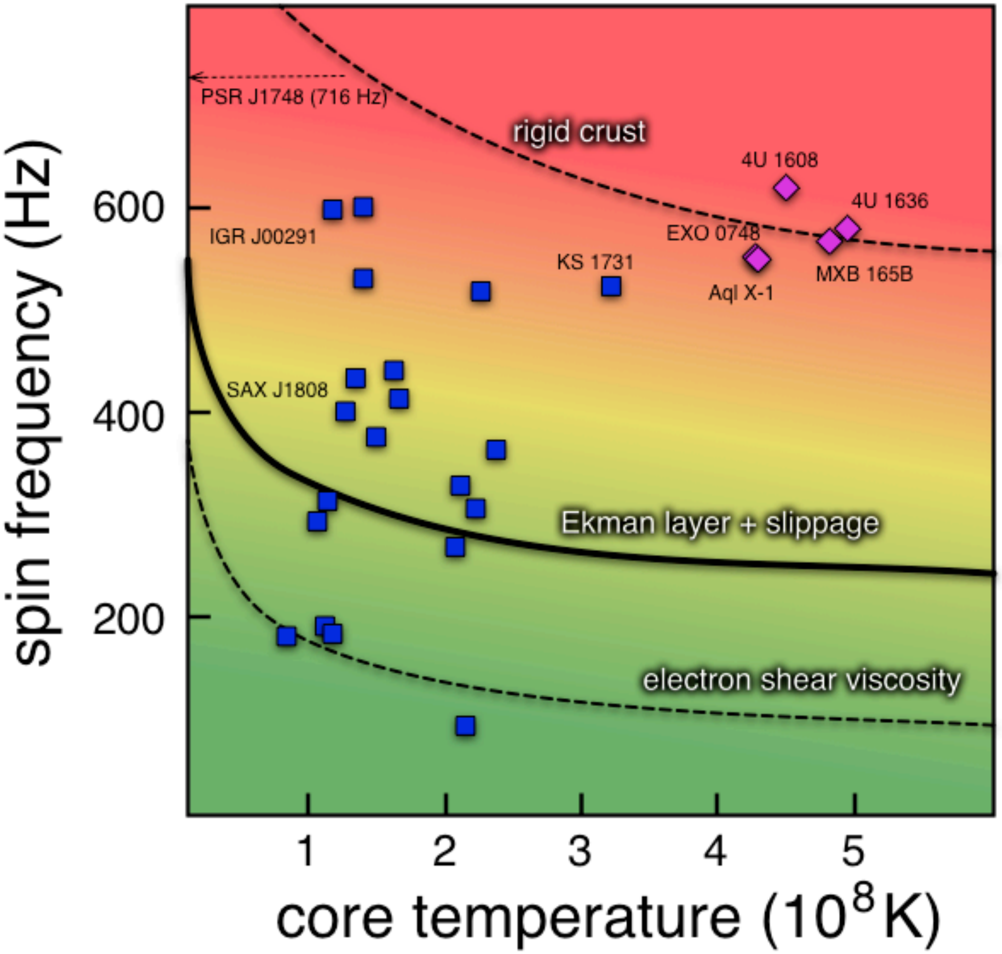
Given the "best estimate" for the main r-mode damping mechanisms, many observed accreting neutron stars in LMXBs **should be** unstable.

Rigid crust with viscous (Ekman) boundary layer would lead to sufficient damping…

… but the crust is more like jelly, so the effect is reduced.

Saturation amplitude due to mode-coupling is too large to allow evolution far into instability region.

Keep in mind: The star's magnetic field may play an important role, even if it is too weak to affect the nature of the r-mode itself.



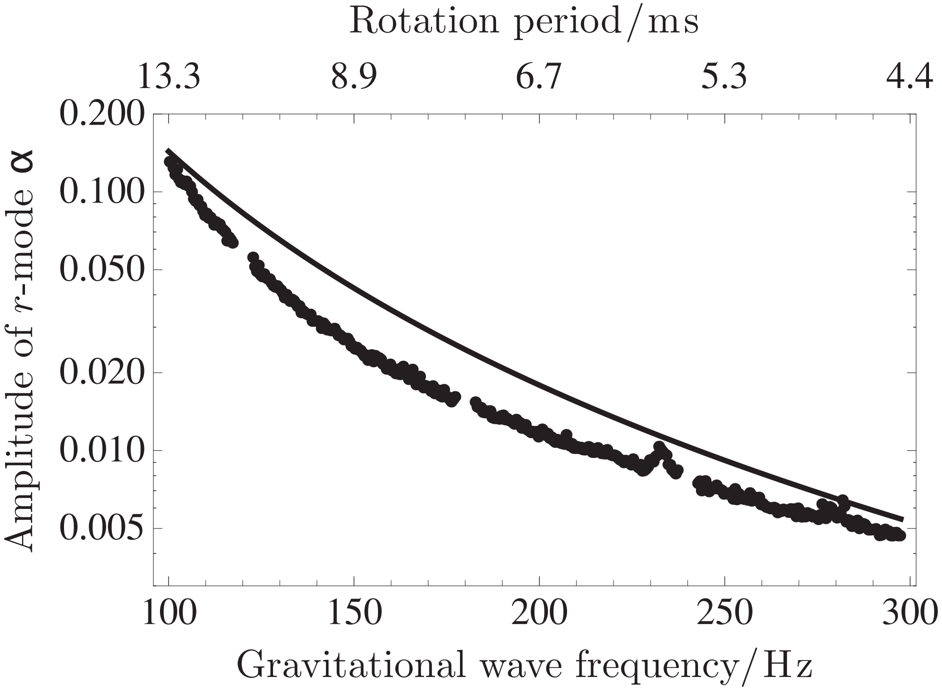
Cassiopeia A

The Cassiopeia A remnant hosts the youngest neutron star in the galaxy (roughly 300 yrs).

LIGO searched 12 days of S5 data for periodic gravitational waves.

Searches are complicated by the fact that the spin rate of the star is unknown.

Still… the results provide the first observational upper limit for the r-mode amplitude.



With the advanced LIGO/Virgo detectors and a 1 year observation these constraints may improve by a factor of 50 or so (ET gives another factor of 10).

Basically: ET may push the limit into the regime where the r-mode is expected to saturate (but will not reach the amplitude required to balance accretion torque).

XTE 1751-305

XTE 1751-305 is an accreting millisecond pulsar spinning at $f_s = 435$ Hz.

Recent work reports evidence for coherent oscillations in RXTE data from the 2002 discovery burst at

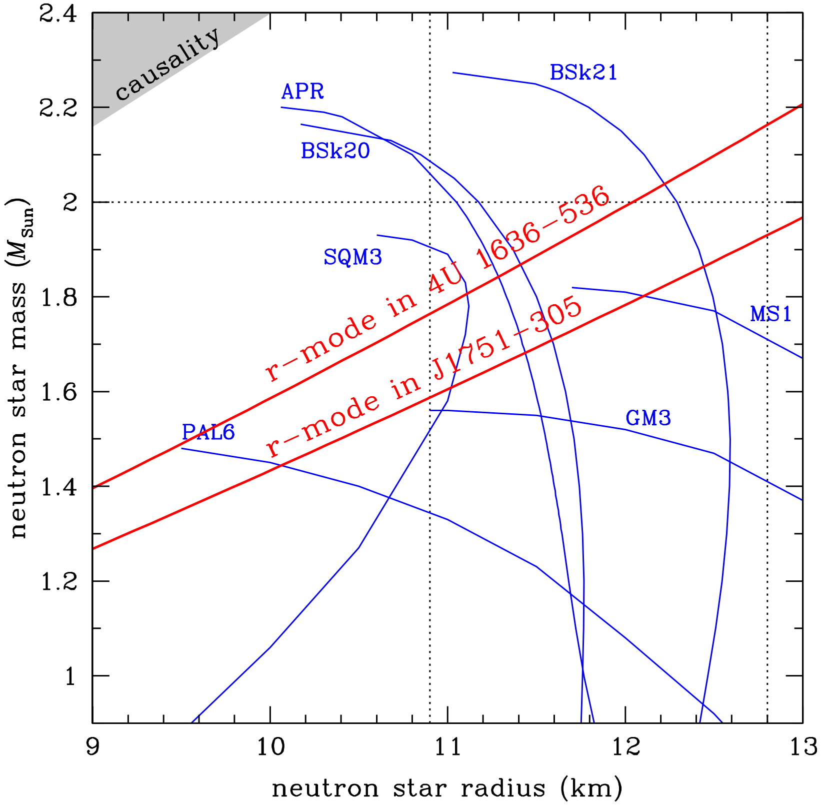
$$\mathbf{f = 0.5727597 \times f_s}$$

This is “consistent” with an r-mode once one accounts for relativistic corrections.

In principle, this would constrain the star’s mass (making use of radius constraints from other X-ray sources).

However, the suggested amplitude is too large to be reconciled with the observed spin-evolution of the system.

There is **always** a spin-down penalty associated with r-mode excitation, even if the mode is stable (unless we are missing something…).



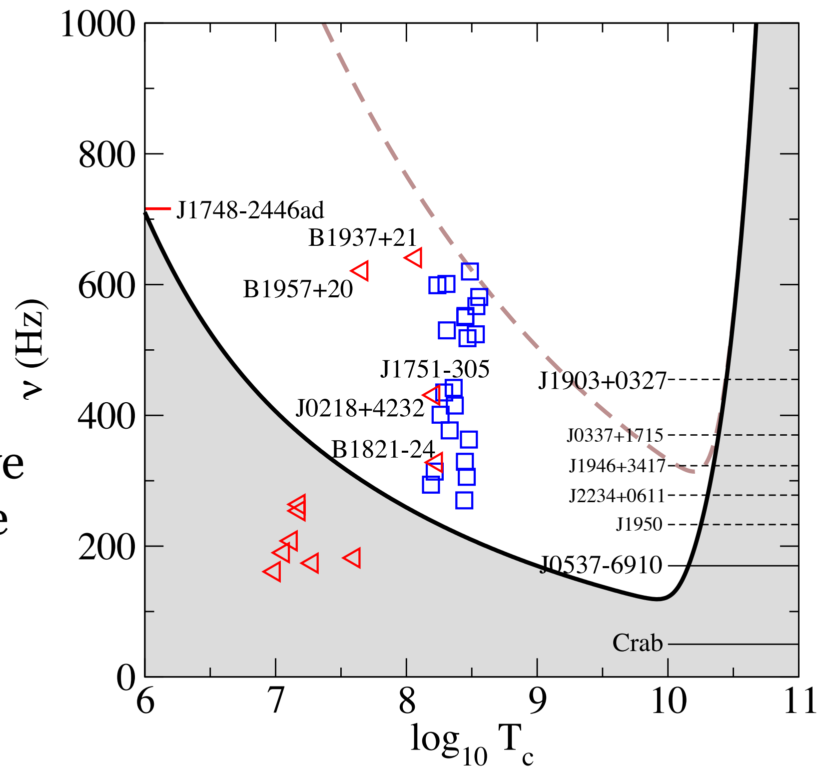
radio constraints

A neutron star is born hot, but it rapidly cools to temperatures where the r-mode instability would act. If the star rotates fast enough, it should spin down as gravitational waves are emitted.

1. Tracing the history of the Crab pulsar (fixed braking index), it would have been "born" at a period of 19 ms.

2. The X-ray pulsar J0537-6910, which currently spins at 16 ms, would have been born with a period in the range 6-9 ms.

3. The most severe constraint comes from observed millisecond pulsars that may have formed through accretion induced collapse of white dwarfs.



The fastest spinning of these systems, J1903+0327, has a period of 2.2 ms and orbital ellipticity e=0.44.

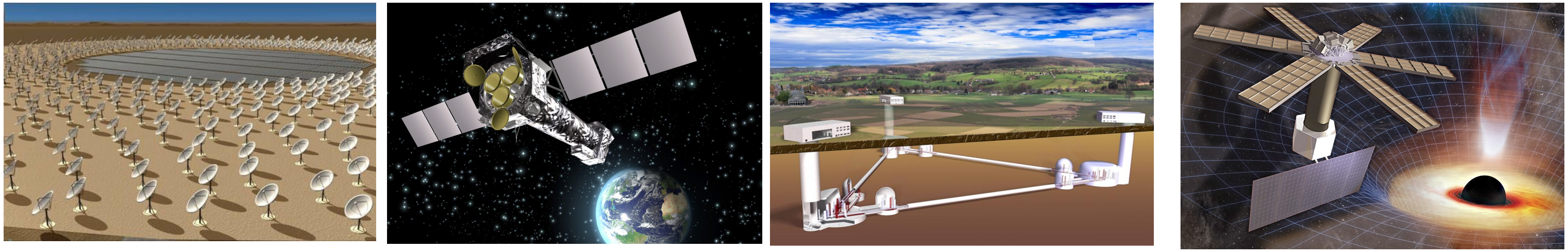
Keep in mind: Any evolutionary scenario must also allow the formation of recycled millisecond pulsars…

final remarks

Neutron stars are Nature's own extreme physics laboratories.

Observations allow us to probe regimes that can never be reached on Earth, complementing information gleaned from colliders like the LHC, RHIC etc.

A new generation of telescopes will ("soon") provide a wealth of relevant data.



However... these are hands-off laboratories.

If we want to move beyond "zoology", and make maximal use of data to constrain fundamental physics, we need to combine information from different "channels".

We need urgent progress on the theory side;

- next generation models should incorporate "all" the expected physics (identify key issues and parameterise ignorance if required),
- need to figure out how to model (nonlinear) systems that evolve on a secular timescale...