Models of differentially rotating Strange Quark Stars in General Relativity

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NATIONAL COHESION STRATEGY
FNP
Foundation for Polish Science
EUROPEAN UNION
EUROPEAN REGIONAL DEVELOPMENT FUND

Equation of State

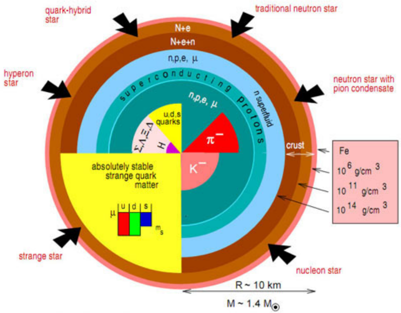

Figure : Weber, J. Phys. 27, 465 (2001))

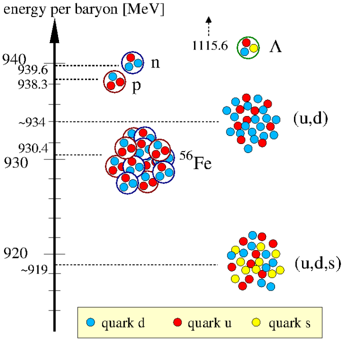

Figure : Energy per baryon for quark matter. (Gourgoulhon)

Strange Quark Stars

The EoS describing strange quark matter can be fairly accurately approximate by a linear dependence $P(\rho)$ even for very complicated cases (Gondek-Rosińska et al., 2000; Zdunik, 2002):

$$P = ac^2(\rho - \rho_0)$$

In general, equation of that type corresponds to a self-bound matter at the density ρ_0 at zero pressure and with a fixed sound velocity ($\sqrt{ac^2}$).
We are considering the simplest MIT Bag model ($m_s = 0$), in which above formula is exact. In the model $a = 1/3$ and $\rho_0 = 4.2785 \times 10^{14}\mathrm{gcm}^{-3}$.

Important steps in Strange Quark Stars modeling

- 1971 - Bodmer - hypothesis that quark matter (u, d, s quarks and e^-) could be a ground state of matter.
- 1984 - Witten - stars build of quark matter could exist (strange stars).
- 1986 - Haensel et al., Alcock et al. - first static models of quark stars.
- 1999 - Gourgoulhon et al., Stergioulas et al. - first fully relativistic calculations of uniformly rotating strange stars described by MIT Bag Model.
- 2000 - Gondek-Rosińska et al., Bombaci et al. - first fully relativistic calculations of uniformly rotating strange stars described by the Dey model (Dey et al., 1998, PhL B438, 123).
- 2015 - Szkudlarek & Gondek-Rosińska - first fully relativistic calculations of differentially rotating strange stars described by MIT Bag model.

Static Stars

- For given EoS the maximum mass of non-rotating star is uniquely determined by the Tolman-Oppenheimer-Volkoff equations

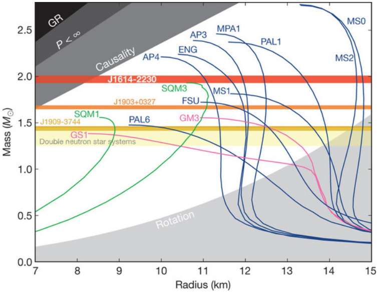

Figure : Mass-radius dependence for different models of neutron and strange stars. (Demorest et al., Nature 467, 1081-1083)

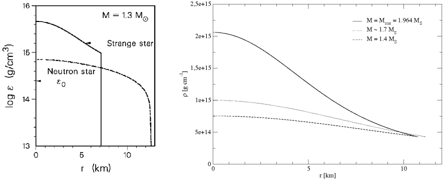

Figure : *Left:* Density profile for neutron and strange quark star with the same masses. (Glendenning, 1997) *Right:* Density profile for strange quark stars with three different masses and surface density $\rho_0 = 4.2785 \times 10^{14} \mathrm{g\ cm}^{-3}$.

Rotating stars - rigid rotation

- For given EoS the maximum mass of non-rotating star is uniquely determined by the Tolman-Oppenheimer-Volkoff equations.
- Rotation can stabilize stars with masses larger than maximum mass of non-rotating star (Baumgarte, Shapiro, Shibata, 2000, ApJ 528, L29).

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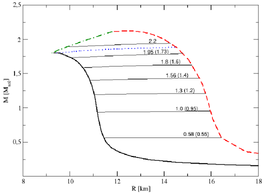
Neutron Stars 17 % - 20 % higher masses than $M_{\mathrm{max,stat}}$ (Cook, Shapiro, Teukolsky, 1994, ApJ 424, 823)
Strange Quark Stars 44 % higher masses than $M_{\mathrm{max,stat}}$ (Gondek-Rosińska et al., 2000, A&A 363, 1005; Gourgoulhon et al., 1999, A&A 349, 851).



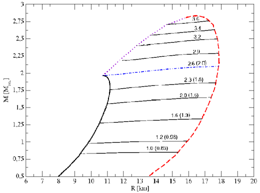


Figure : Mass-radius relation for static and uniformly rotating strange stars (left) and neutron stars (right). (Gourgoulhon, Gondek-Rosińska, Haensel, 2003, A&A 412, 777)

Differential rotation

For axisymmetric simulations of strange stars we use FlatStar code (Ansorg, Gondek-Rosińska, Villain, 2009, MNRAS 396, 4, 2359). To describe the differential rotation we use an astrophysically motivated rotation law first presented by Komatsu et al. in 1989:

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$$F(\Omega) = A^2(\Omega_c - \Omega)$$

- Ω - angular velocity,
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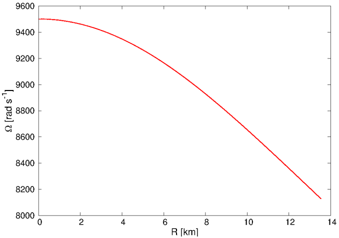

In this rotation law the highest angular velocity is in the center of the star, and it is decreasing with increasing radius. That is why we can spin-up the stars more than in the case of rigid rotation.

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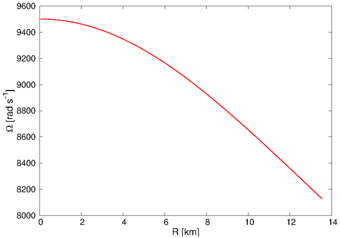

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First we start from spherically symmetric configuration, then we spin-up the star with set maximum density and degree of differential rotation - zero for uniform rotation, higher values for larger Ω_c/Ω_e ratio. According to these two parameters we obtain four types of star configurations.

RESULTS

Code comparison

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- First fully relativistic calculations of rigidly rotating strange quark stars described by MIT Bag model was made with these two codes (LORENE - Gourgoulhon et al., 1999; RNS - Stergioulas, Kluźniak, Bulik, 1999).

In static, non-rotating case we searched for configuration with maximum mass. In case of rigid rotation the determination of maximum mass was more difficult, because of lower accuracy in RNS and different results in FlatStar. To have good comparison of each code we took configuration with maximum mass from Lorene and performed calculations for corresponding ρ_c with RNS and FlatStar.

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	M_{max}^{stat}				
	$M[M_\odot]$	$M_B[M_\odot]$	$\rho_c[10^{15}\mathrm{g\ cm^{-3}}]$	H_c	$R_{circ}[\mathrm{km}]$
FlatStar	1.9637	2.6489	2.059	0.4514	10.71
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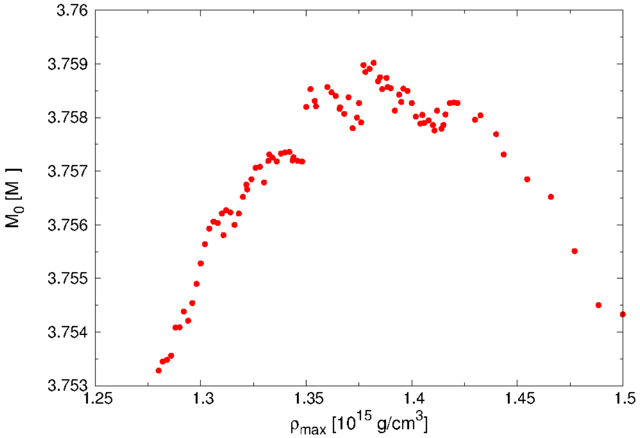
M_{max}^{rig}							
	$M[M_\odot]$	$M_B[M_\odot]$	$\Omega[\text{rad s}^{-1}]$	$\rho_c[10^{15}\text{g cm}^{-3}]$	H_c	$R_{circ}[\text{km}]$	r_{pole}/r_{eq}
FlatStar	2.8188	3.778	9560.19	1.261	0.32	16.463	0.4643
Lorene	2.831	3.751	9547.0	1.261	0.32	16.54	0.4618
RNS	2.8339	3.759	9562.61	1.261	0.32	16.425	0.466

Rigidly rotating stars - RNS vs FlatStar

Maximum mass search with RNS code for rigidly rotating SQS at mass-sheding limit with $\rho_0 = 4.2785 \cdot 10^{14}\mathrm{g/cm^3}$.



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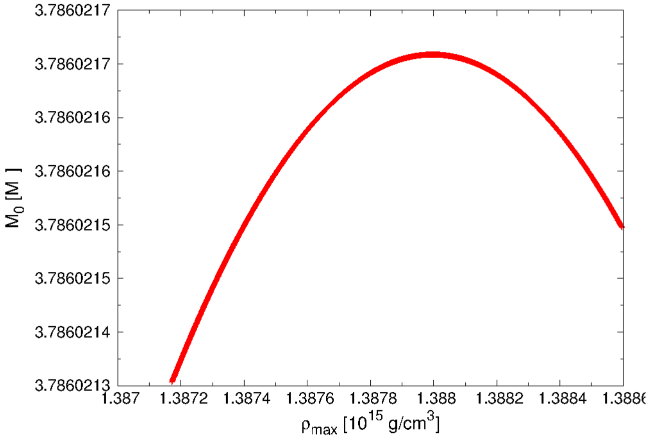
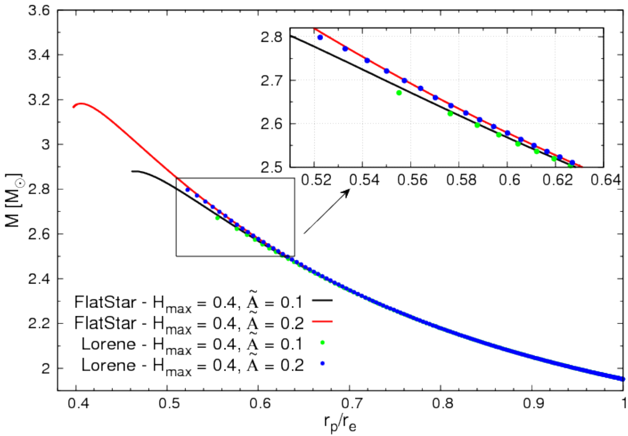
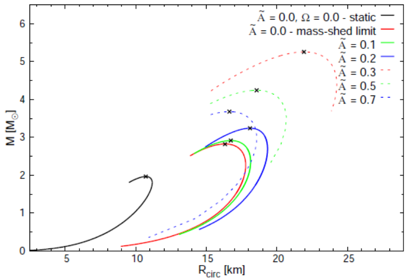
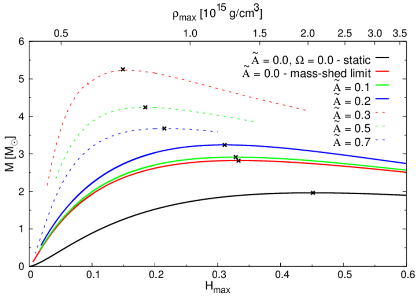


Differentially rotating stars - Lorene vs FlatStar

Differentially rotating Strange Quark Stars made with FlatStar code (solid lines) and Lorene code (dots), for two values of $\tilde{A} = 0.1,\ 0.2$ for fixed $H_{\max} = 0.4(\rho_{max} = 1.7 \cdot 10^{15} g/cm^3)$.







Sequences of configurations of non-rotating, rigidly rotating and differentially rotating SQS - gravitational mass in function of radius and wide range of maximum enthalpy H_{max} 0 - 0.6 (maximum density ρ_{max} $4.2785 \cdot 10^{14} - 3.5 \cdot 10^{15} \mathrm{g/cm}^3$.

For small values of $\tilde{A}$ the maximum mass increases with increasing $\tilde{A}$. At some point the maximum mass reaches the highest value and then decreases with increasing $\tilde{A}$.

New types of solutions

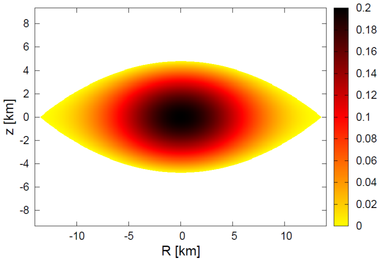


Figure : Crosssection of Strange Quark Star at kepler limit with $\tilde{A} = 0.2$ and $H_{\max} = 0.2$ ($\rho_{\max} = 0.82 \cdot 10^{15} g/cm^3$).

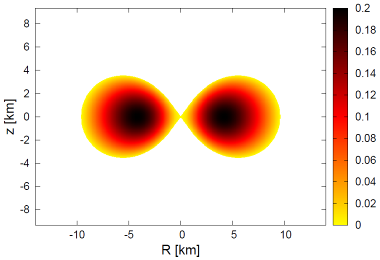
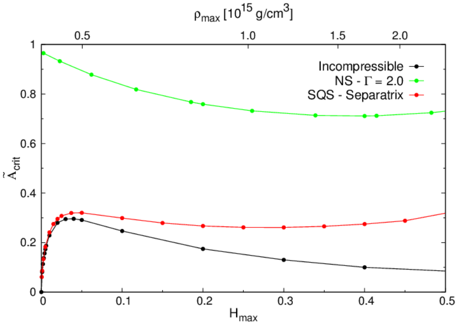


Figure : Crosssection of Strange Quark Star with $r_{\rm pole}/r_{\rm eq} = 0.01$, $\tilde{A} = 0.5$ and $H_{\max} = 0.2$ ($\rho_{\max} = 0.82 \cdot 10^{15} g/cm^3$).

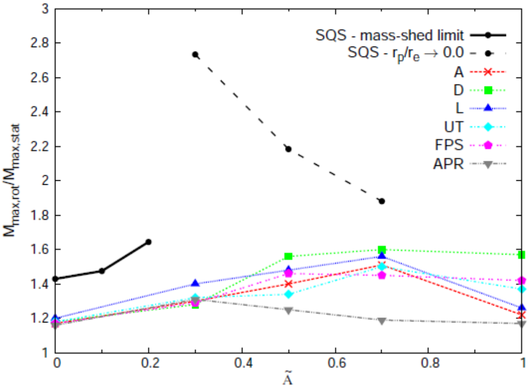
Separatrix

Sepratrix is a critical value of degree of differential rotation ($\tilde{A}_{crit}$) which divides solution space into four parts. Sequences found in each part are described as A, B, C and D types of solutions. (Ansorg, Gondek-Rosińska, Villain, 2009).



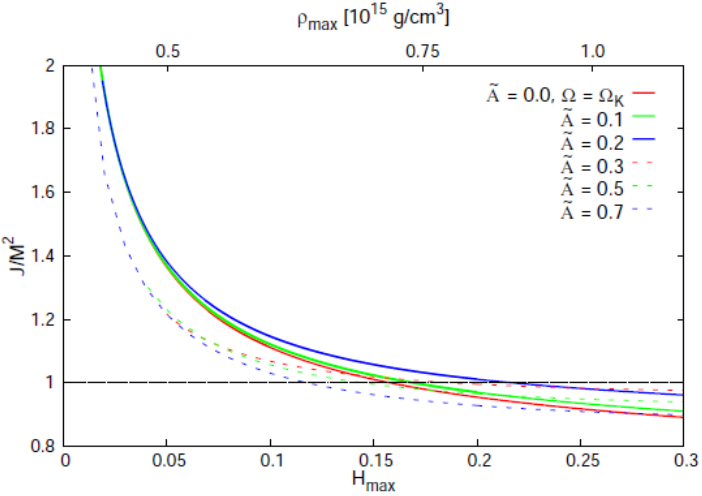
Configurations of Strange Quark Stars from toroidal class appears at much lower values of degree of differential rotation $\tilde{A}$ comparing to Neutron Stars described by polytropic EoS with $\Gamma = 2.0$.

Comparison to Neutron Stars



Maximum achieved baryon mass $M_{0,max}$ as a function of degree of differential rotation $\tilde{A}$ for Starnge Quark Stars, Neutron stars described by politropic EoS with $\Gamma = 2.0$ and Neutron Stars described by realistic EoS (Morrison et al. 2004). The highest increase of mass for fixed $\tilde{A}$ is for SQS.

Kerr parameter



Conclusions

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- Configurations with low maximum density can be stabilize by differential rotation against colapse into black hole.

Thank You For Your Attention