The gravitational wave signal from ensemble of the neutron stars

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Parameters of a neutron star

Population model

In our calculations we use a realistic model of population that sets **the supernova birth every 100yr, the initial values of the kick velocities, the rotation periods (10ms)**. Then it is evolved in time. Position and velocity of each pulsar is calculated from gravitational potential of the Galaxy. Objects further than 30kpc from the center of the Galaxy are excluded from the model. Finally we obtain population of, for example, **6.5M stars with age below 1Gyr**.

The Galaxy

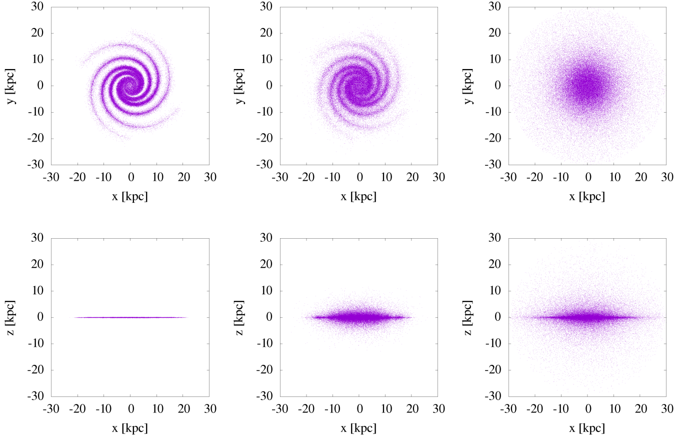


Figure : From left to right examples of 10^5 objects: initial positions, their positions after 10^7 years (center panels) and after 10^9 years (right panels).

Parameters of a neutron star

When a neutron star is rotating and has some distortion it generates gravitational waves. The model parameters are:
P (initial rotation period $P_0 = 10ms$, evolved assuming dipolar emission),
r (distance),
B (magnetic field, constant $\log(B) = 12.5\mathrm{G}$),
I (moment of inertia, constant - $10^{38} kg\, m^2$),
ϵ (distortion value, constant - 10^{-6}),
α (angle between rotation axis and distortion axis, $acos(0.0 - 1.0)$),
i (inclination from $acos(0.0 - 1.0)$).
We may determine some characteristic value for any pulsar:

$$h_0 = -\frac{16\pi^2 G}{c^4} \frac{I\epsilon}{P^2 r}$$

Neutron stars may have ellipsoidal distortion due to magnetic field or some instabilities. The shorter rotation period the stronger emission is. When distortion axis is aligned with rotation axis there is no emission.

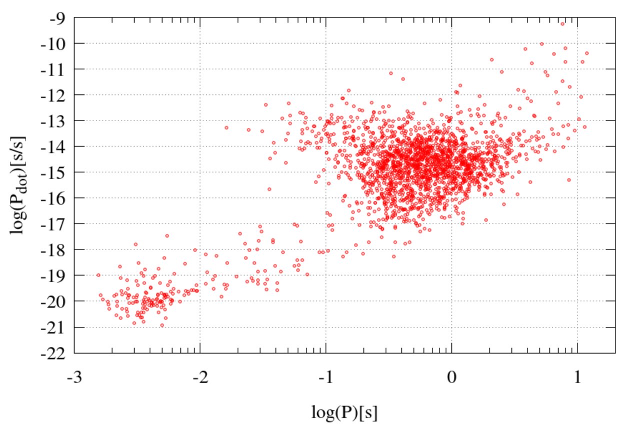
Signal from a single neutron star

We use the equations from Bonazzola, Gourgouhlon (1996):

$$\begin{aligned} h_+ = h_0 \sin(\alpha)[\frac{1}{2}\cos(\alpha)\sin(i)\cos(i)\cos(\Omega(t-t_0)) \\ -\sin(\alpha)\frac{1+\cos^2(i)}{2}\cos(2\Omega(t-t_0))] \end{aligned} \quad (1)$$

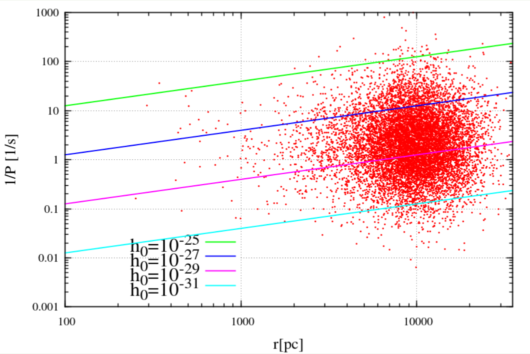
$$\begin{aligned} h_\times = h_0 \sin(\alpha)[\frac{1}{2}\cos(\alpha)\sin(i)\sin(\Omega(t-t_0)) \\ -\sin(\alpha)\cos(i)\sin(2\Omega(t-t_0))] \end{aligned} \quad (2)$$

Observed neutron stars



Rotation period versus derivative of the rotation period of observed pulsars, ATNF data (http://www.atnf.csiro.au/).

Population of neutron stars



Frequency versus distance of an object for a population of 10000 neutron stars. Lines with constant values of h_0.

Signal from an ensemble of the neutrons stars

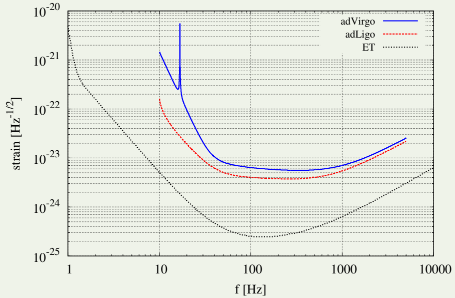
Assumptions

signal **not coherent** so we take a sum of $h^2(t)$ instead of $h(t)$
signal calculated on a **time intervals** $P \ll \tau \ll day$
Earth rotation changes relative position of the object and the detector, described by functions $F(t)$

Average squared signal calculated by the formula:

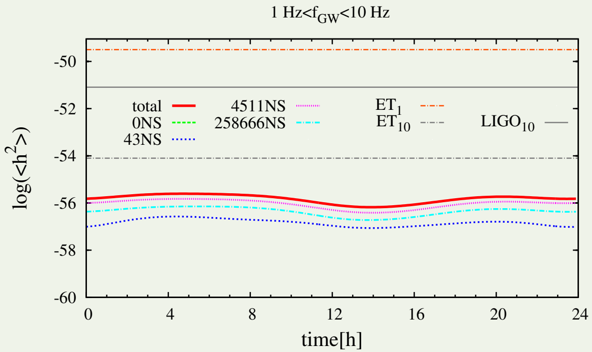
$$< h^2(t) >= \sum_{i=1}^{N} \frac{[F_+^i(t)]^2}{\tau} \left(\int_t^{t+\tau} [h_{1+}^i(t')]^2 dt' + \int_t^{t+\tau} [h_{2+}^i(t')]^2 dt' \right) + \sum_{i=1}^{N} \frac{[F_\times^i(t)]^2}{\tau} \left(\int_t^{t+\tau} [h_{1\times}^i(t')]^2 dt + \int_t^{t+\tau} [h_{2\times}^i(t')]^2 dt' \right) \quad (3)$$

Detectors



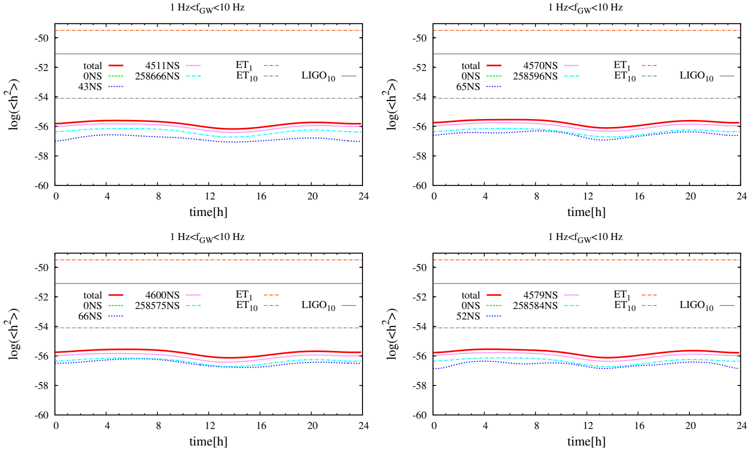
Single rotating neutron stars may be observed by AdvancedLigo, AdvancedVirgo and Einstein Telescope. For AdvancedLigo and AdvancedVirgo observation frequency range is above 10Hz, for Einstein Telescope above 1Hz.

Frequency range 1-10Hz

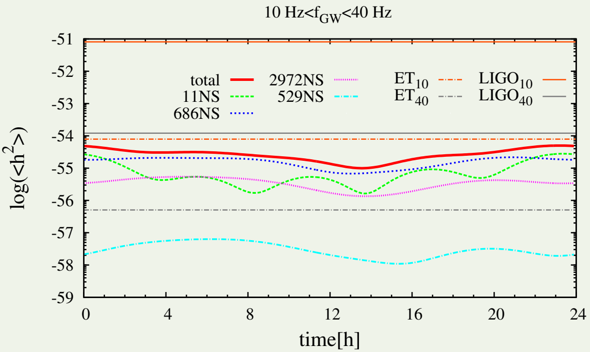


Squared signal for population of neutron stars with gravitational wave frequency 1-10Hz. There is total signal and components for different h_0 ranges.

Frequency range 1-10Hz

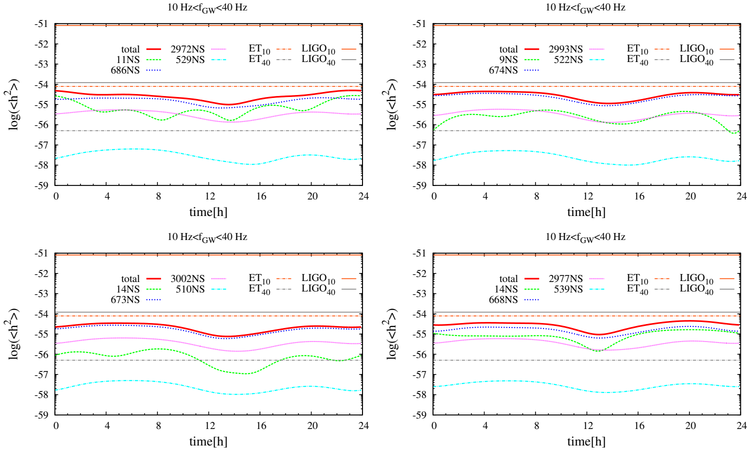


Frequency range 10-40Hz

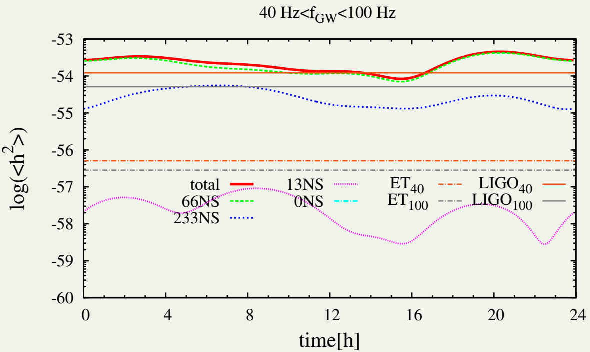


Squared signal for a population of 6.5M neutron stars with gravitational wave frequency 10-40Hz. There is total signal and components for different h_0 ranges.

Frequency range 10-40Hz

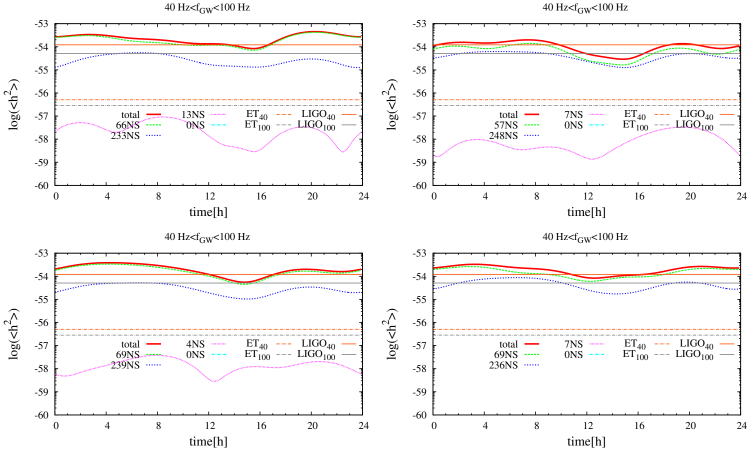


Frequency range 40-100Hz

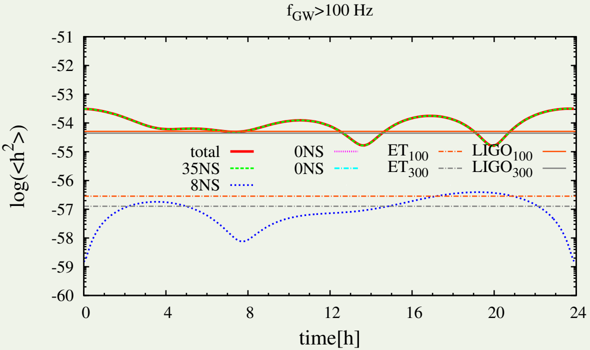


Squared signal for a population of 6.5M neutron stars with gravitational wave frequency 40-100Hz.

Frequency range 40-100Hz

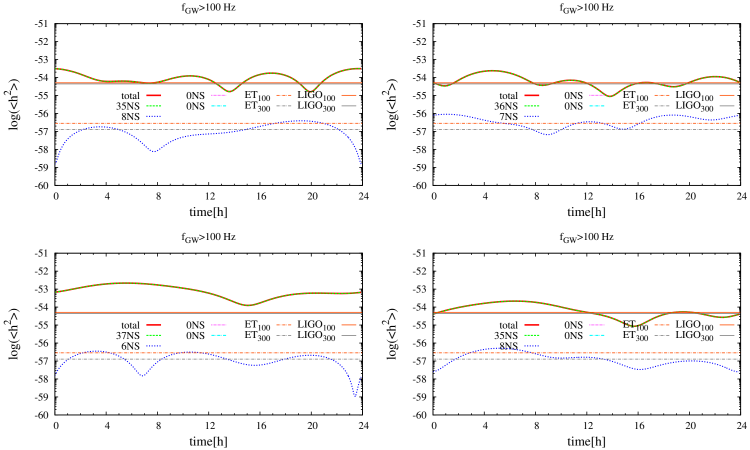


Frequency range above 100Hz



Squared signal for a population of 6.5M neutron stars with gravitational wave frequency above 100Hz.

Frequency range above 100Hz



Conclusions

- close objects or/and with high frequency, in most cases dominate the total signal,
- the signal below 10Hz would be difficult to observe,
- the signal could be detectable for advLigo above 40Hz, and for ET above around 20Hz,
- ranges with small number of objects shows different profile for different realisation of the model, for more numerous ranges the profile is very stable,