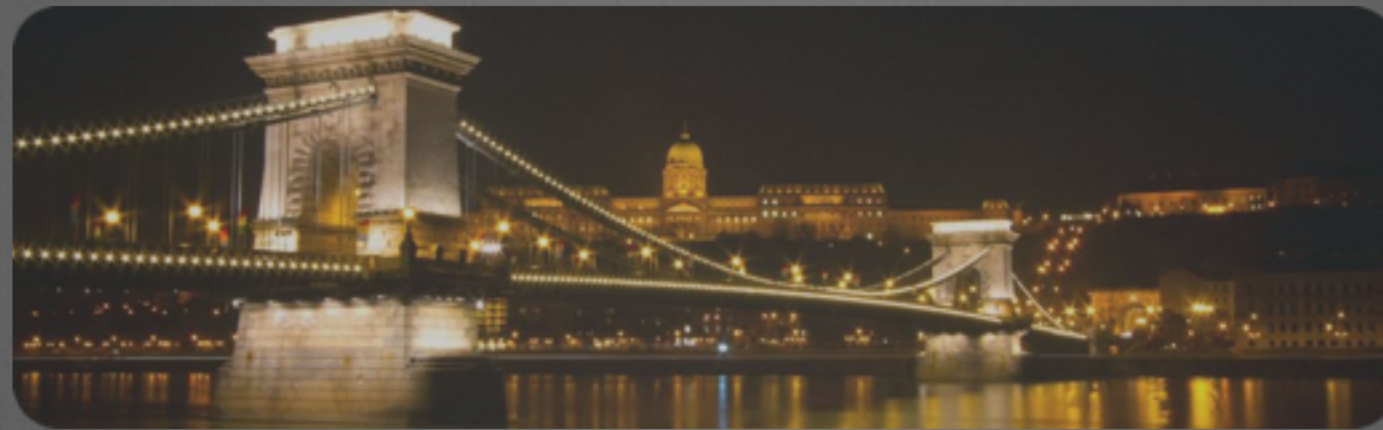

Modelling Strongly Magnetized Neutron Stars in General Relativity

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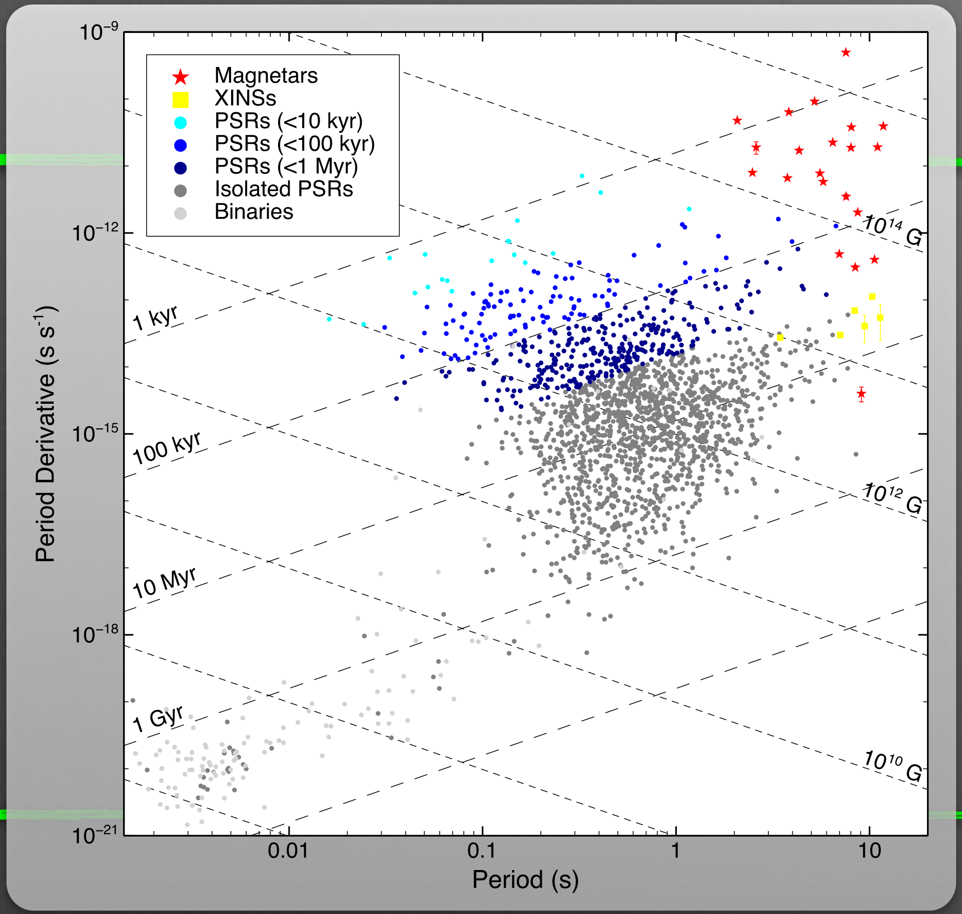
Annual NewCompStar Conference 2015

Budapest, 06-18-2015

The Magnetic Field of NSs



Magnetar

$$\dot{P} \sim 10^{-11}\ \mathrm{s}$$

$$P \sim 2 - 12\ \mathrm{s}$$

$$\tau \sim P/2\dot{P} \sim 10^{4}\ \mathrm{yr}$$

$$B \propto \sqrt{P\dot{P}} \sim 10^{14}\ \mathrm{G}$$

SGR & AXP

Short bursts	$L_X \sim 10^{41}$ erg s^{-1}
Giant Flares	$L_X > 10^{44}$ erg s^{-1}
Persistent Luminosity	$L_X \sim 10^{33}$-10^{36} erg s^{-1}

$$\frac{\mathrm{dE_{rot}}}{\mathrm{dt}} << L_X$$

- Emission is fed by the magnetic energy
- B-field amplified at birth
  - core compression
  - differential rotation
  - dynamo
- The initial magnetic configuration rearranges in ~100s to a metastable configuration
- Twisted-Torus configuration

Gravitational waves

- Fast rotation at birth (~ ms)
- Strong magnetic field induces deformation of the star

→ GWs emission

$$\dot{\omega} = \frac{K_{\rm d}}{2}\omega^3 - \frac{K_{\rm GW}}{4}\omega^5$$

$$K_{\rm d} = (B_{\rm d}^2 R^6)/3Ic^3$$

$$K_{\rm GW} = (2/5)f(\chi)(G/c^5)I\epsilon_B^2$$

$$\epsilon_B \propto B^2$$

Dall'Osso et al. 2009

Efficiency of the emission depends on the geometry of the magnetic field:

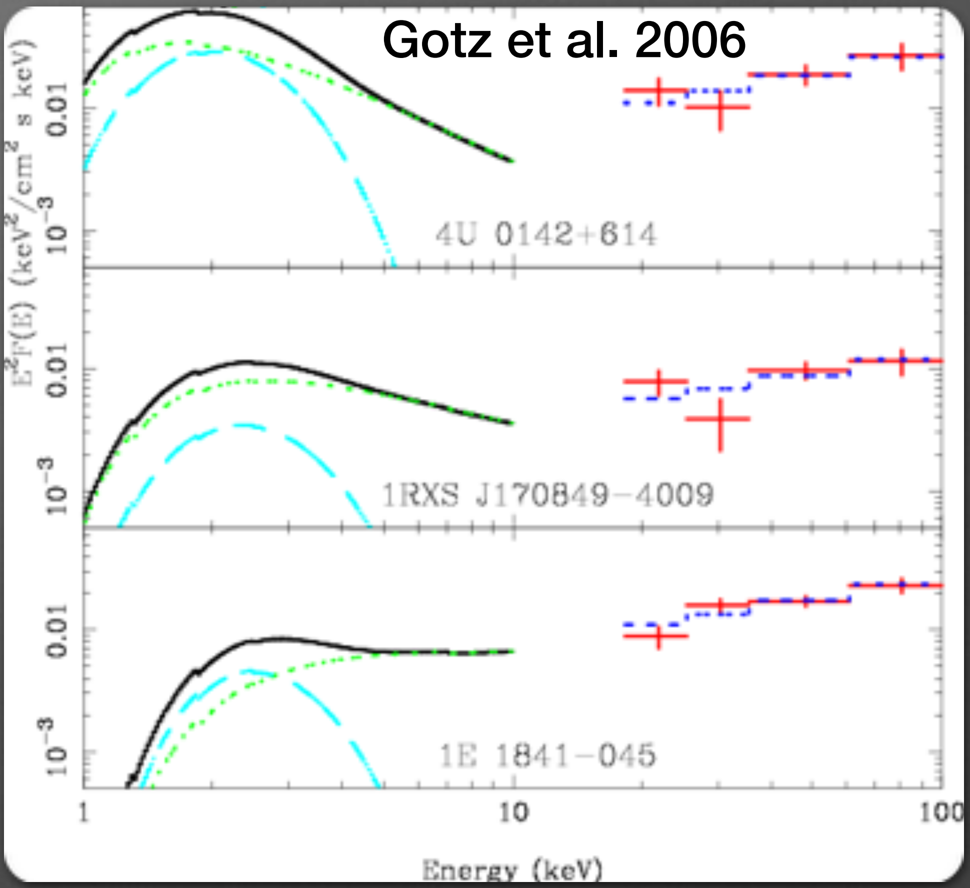
- poloidal magnetic fields makes the star oblate
- toroidal magnetic field makes the star prolate

- Orthogonalization of the rotation of the magnetic axis (Cutler 2002)
- Maximize the efficiency of GW emission

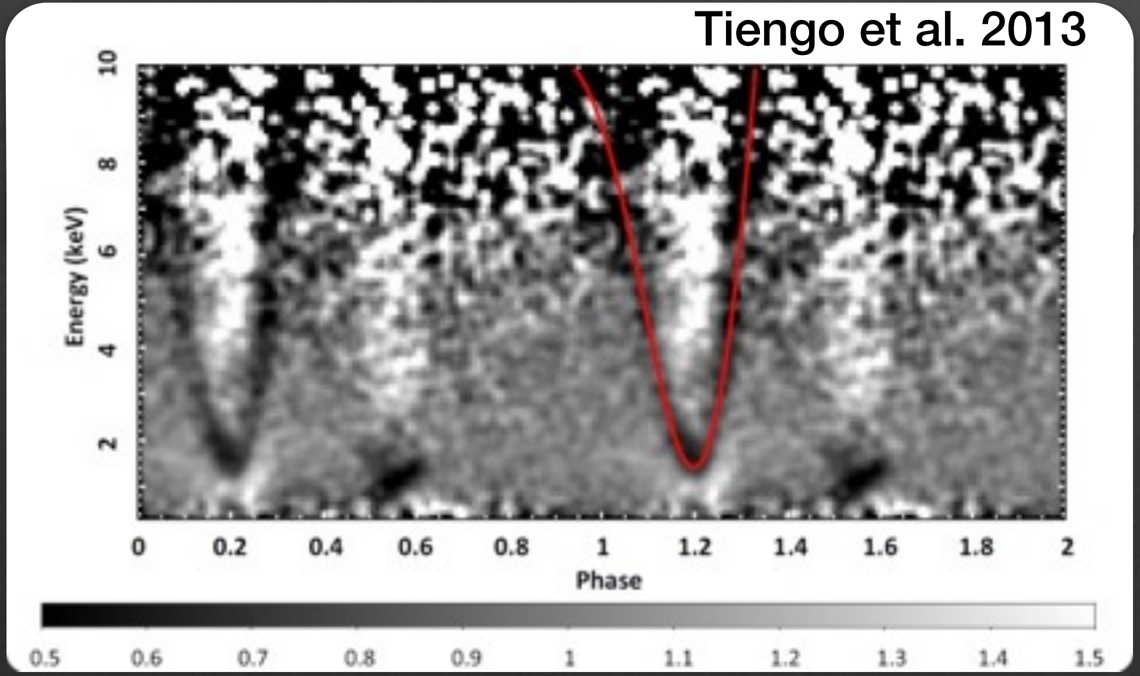
Twisted Magnetosphere

Persistent X-ray spectra:
Black body (@ kT ~ 0.5 keV) + power-law tail from 10keV

Twisted magnetosphere:
Resonant Cyclotron Scattering of thermal photons with electric currents in the magnetosphere (Thompson et al. 2002, Beloborodov & Thompson 2007)



- Low Pdot Magnetar SGR 0418+5729 (B_{dipole} ~ 10^{12}G)
- Proton-Cyclotron Line ➡ Strong but localised magnetic field B~10^{14}G



- Computation of synthetic spectra and light curves (Pavan et al. 2009,Psaltis et al. 2014)

Model the emission on self-consistent GR magnetized NSs

Governing equations

Einstein Equations

- Static and axisymmetric spacetime
- 3+1 formalism of GR → solve only constraint equations
- Conformally flat metric approximation:

$$ds^2 = -\underbrace{\alpha^2(r,\theta)}_{\text{Lapse function}} + \underbrace{\psi^4(r,\theta)}_{\text{Conformal factor}}[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2]$$

- Einstein equations reduce to:
  - $\Delta\psi = -2\pi\hat{E}\psi^{-1}$
  - $\Delta(\alpha\psi) = [2\pi(\hat{E} + 2\hat{S})\psi^{-2}](\alpha\psi)$

$\hat{E} = \psi^6 E$ energy density

$\hat{S} = \psi^6 S$ trace of stress energy tensor

- numerically stable form
- ensure local uniqueness
- standard and accurate numerical technique
- consistency with full GR (10^{-4})

$$\Delta u = s u^q$$

local unicity if $sq > 1$

Governing equations

GRMHD

- Ideal magnetised plasma in static and axisymmetric spacetime
- Barotropic equation of state (polytropic EOS: $P = K\rho^\gamma$ with $K=110$ and $\gamma=2$)
- GRMHD system reduces to Euler equations:

$$\partial_i \ln h + \partial_i \alpha = \frac{L_i}{\rho h}$$

ρ rest mass density
h specific enthalpy

L_i Lorentz force

$$L_i = \epsilon_{ijk} J^j B^k$$

Conduction currents

$$J^i = \alpha^{-1}\epsilon^{ijk}\partial_j(\alpha B_k)$$

- If Lorentz force term is exact $L_i = \rho h \partial_i \mathcal{M}$

➡Bernoulli integral $\ln\left(\frac{h}{h_c}\right) + \ln\left(\frac{\alpha}{\alpha_c}\right) - \mathcal{M} = 0$

Magnetisation function

Purely Toroidal B-Field

$$\partial_i \ln(h) + \partial_i \ln(\alpha) + \frac{\alpha B_\phi \partial_i(\alpha B_\phi)}{G} = 0$$

$$G := \rho h\, \alpha^2 \psi^4 r^2 \sin^2\theta$$

Integrability requires:

$$B_\phi = \alpha^{-1}\mathcal{I}(G), \quad \mathcal{M}(G) = -\int \frac{\mathcal{I}}{G}\frac{\mathrm{d}\mathcal{I}}{\mathrm{d}G} dG$$

"Barotropic magnetization law"

$$\mathcal{I}(G) = K_\mathrm{m} G^m, \quad \mathcal{M}(G) = -\frac{m K_\mathrm{m}^2}{2m-1} G^{2m-1}$$

Governing equations

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Magnetisation function ($\mathcal{M}$)

L_i Lorentz force
$$L_i = \epsilon_{ijk} J^j B^k$$
Conduction currents
$$J^i = \alpha^{-1}\epsilon^{ijk}\partial_j(\alpha B_k)$$

With poloidal B-Field

- Axisymmetry
  ➡Magnetic quantities depend only on the magnetic flux function A_ϕ

$$B^r = \frac{\partial_\theta A_\phi}{\psi^6 r^2 \sin\theta}, \quad B^\theta = -\frac{\partial_r A_\phi}{\psi^6 r^2 \sin\theta}, \quad B^\phi = \frac{\mathcal{I}(A_\phi)}{\alpha\psi^4 r^2 \sin^2\theta}.$$

Grad-Shafranov Equation

$$\tilde{\Delta}_3 \tilde{A}_\phi + \frac{\partial A_\phi \partial \ln(\alpha\psi^{-2})}{r\sin\theta} + \psi^8 r\sin\theta\left(\rho h \frac{d\mathcal{M}}{dA_\phi} + \frac{\mathcal{I}}{\sigma}\frac{d\mathcal{I}}{dA_\phi}\right) = 0$$

The XNS code

Available at:

www.arcetri.astro.it/science/ahead/XNS/

Scheme of iterations:

- starting guess for the metric and energy/matter (TOV @ first step)
- solve equations for the metric
- solve the Grad-Shafranov equation for A_ϕ (if B poloidal)
- solve Bernoulli integral and update fluid quantities

Semi-spectral method

- Use of vector and spherical harmonics
- Elliptic PDEs → system of radial ODEs for each harmonic
- II order radial discretization → direct inversion of tri-diagonal matrices
- Boundary → parity and asymptotic properties of the multipole
- No compactified domains

Typical Run

N_r=500, N_θ=250

N harmonics = 20 ÷ 60

Computational time:
few minutes

$$\Delta q = H(q)$$

$$q(r,\theta) = \sum_{l=0}^{\infty} A_l Y_l(\theta)$$

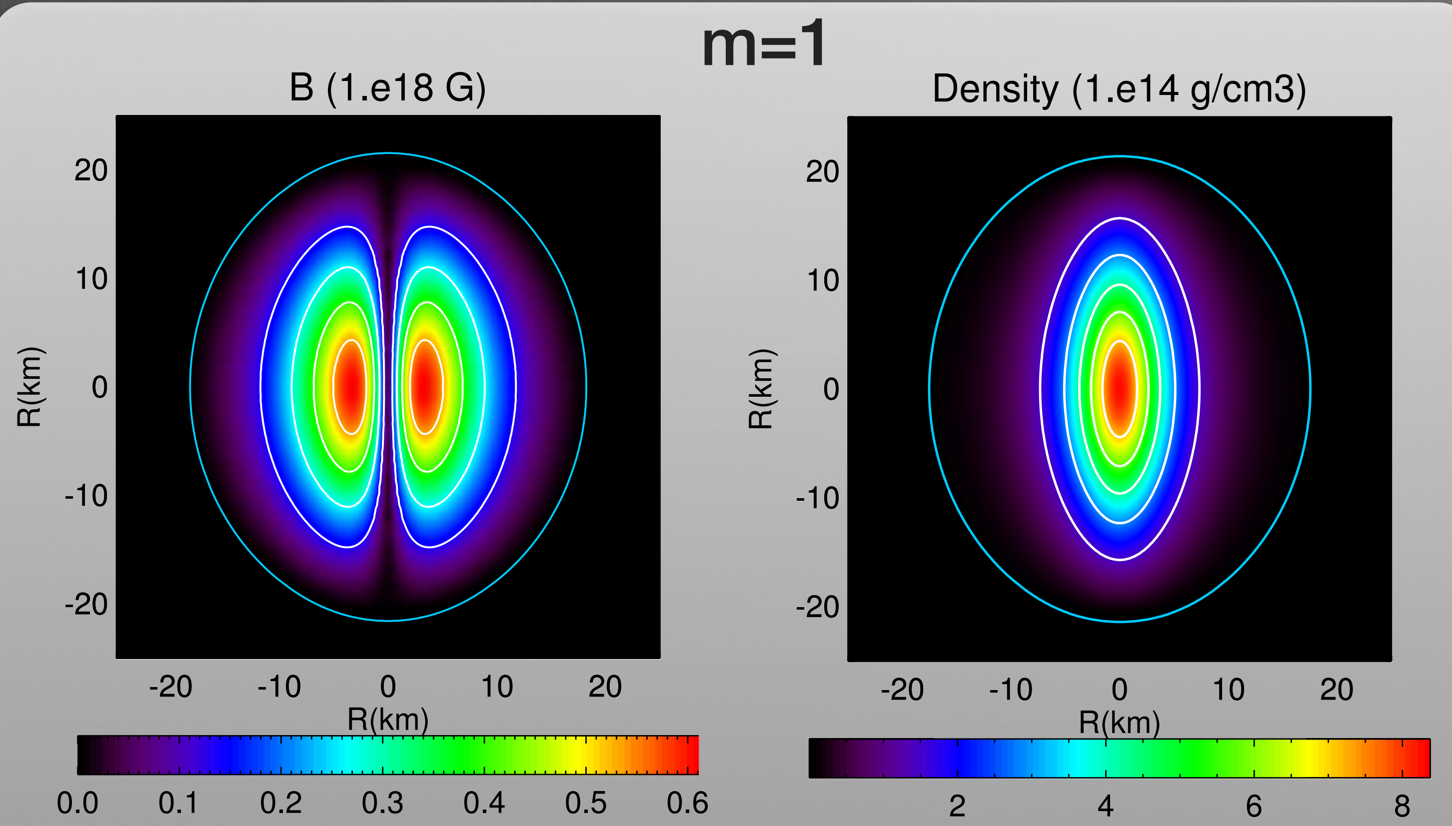
$$\frac{d^2 A_l}{dr^2} + \frac{2}{r}\frac{dA_l}{dr} - \frac{l(l+1)}{r^2} A_l = H_l$$

$$H_l(r) := \int d\Omega\, H(r,\theta) Y_l(\theta)$$

Analogously for the GS with vector harmonics

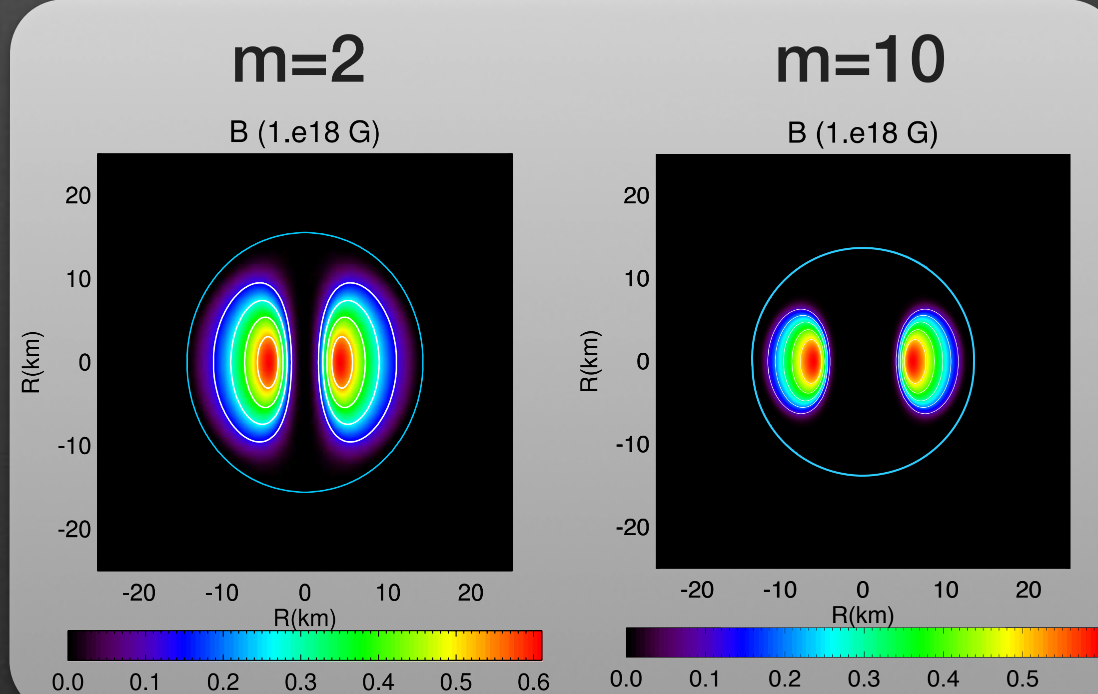
Purely toroidal models

$$\mathcal{M}(G) = -\frac{m K_{\mathrm{m}}^2}{2m-1} G^{2m-1} \qquad G := \rho h\, \alpha^2 \psi^4 r^2 \sin^2\theta$$



Pili et al. 2014

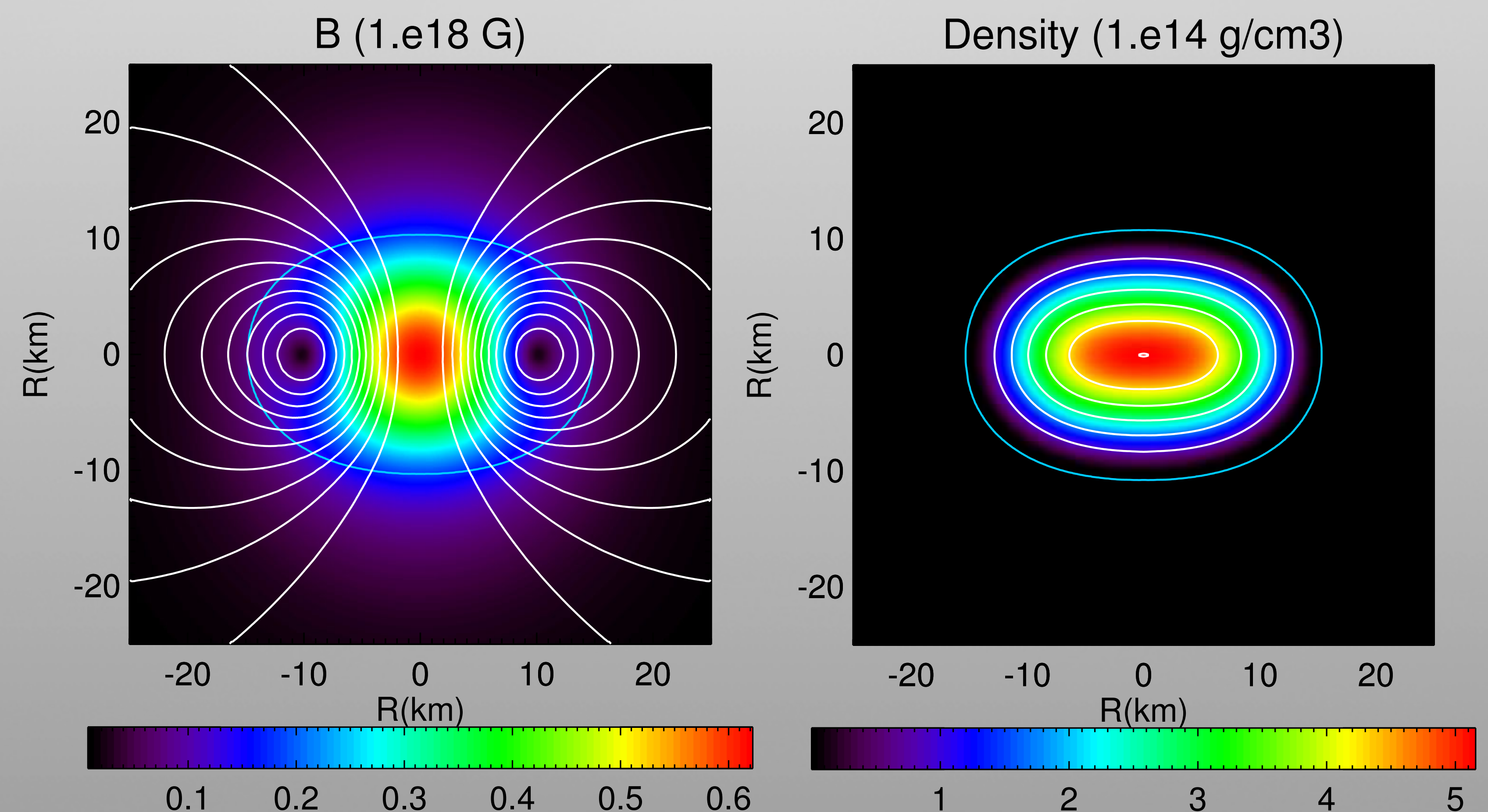
Prolate deformation:
- axial compression
- inflation of low-density outer layers



Trends with increasing m:
- B peaks at higher radius
- reduced effect on the stellar structure
- Magnetic tension B^2/R_{line}

Purely poloidal models

$$\mathcal{M}(A_\phi) = k_{\mathrm{pol}}\, A_\phi$$



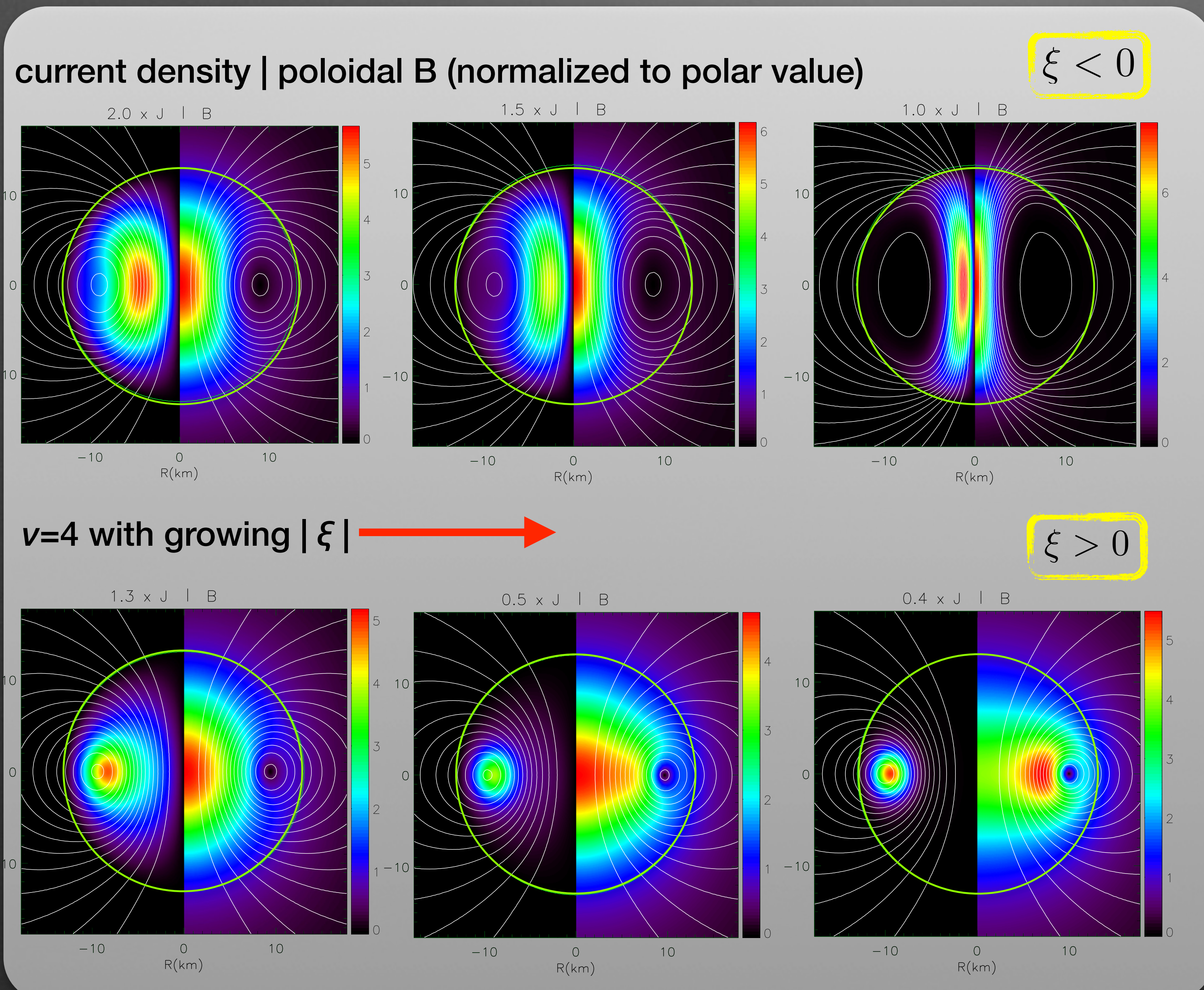
Pili et al. 2014

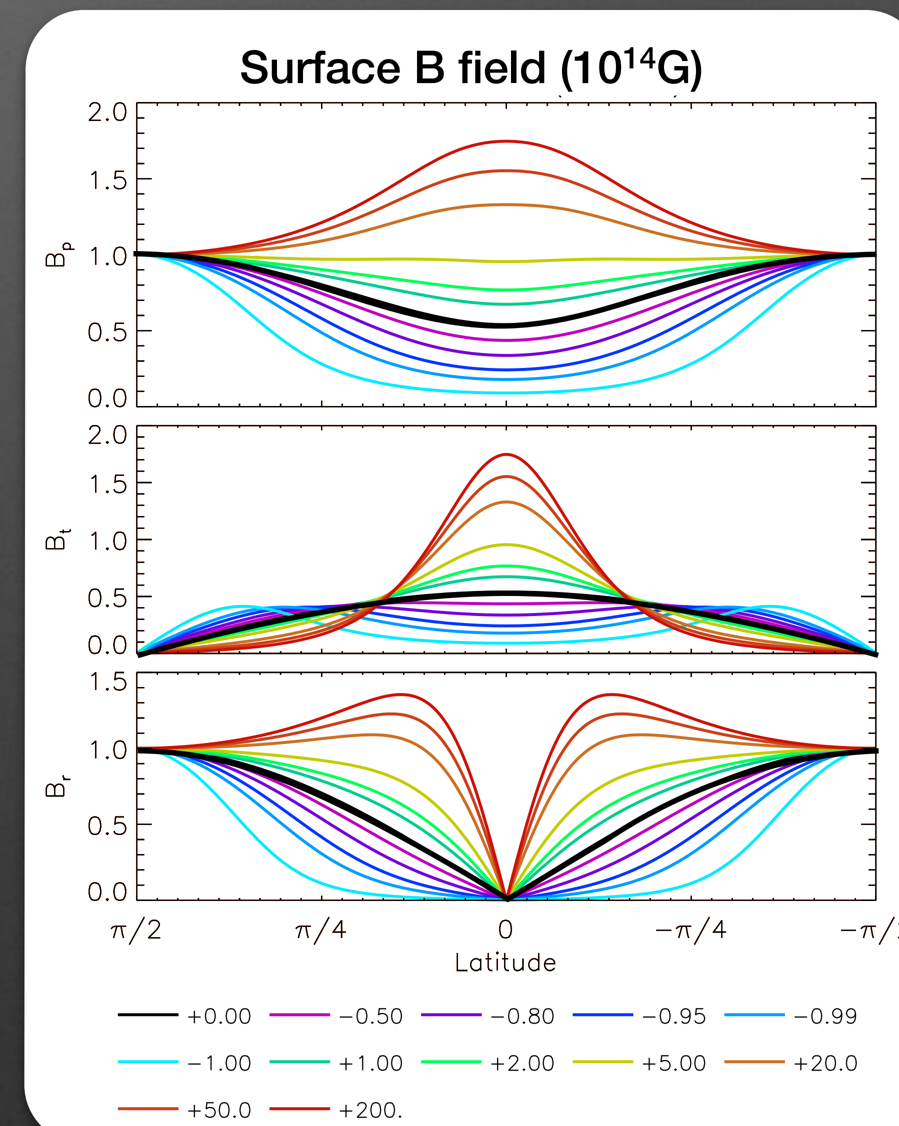
- Oblate deformation
- Flatter density profile perpendicularly to the magnetic axis

Poloidal Magnetic Field: role of current terms

$$\mathcal{M}(A_\phi) = k_{\rm pol} A_\phi \left[1 + \frac{\xi}{\nu+1}\left(\frac{A_\phi}{A_\phi^{\rm max}}\right)^{\nu}\right]$$

- k_{pol} mag. constant
- ν mag. index
- A_ϕ^{max} maximum of the potential





Bucciantini et al. 2015

Poloidal Magnetic Field: role of current terms

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- k_{pol} mag. constant
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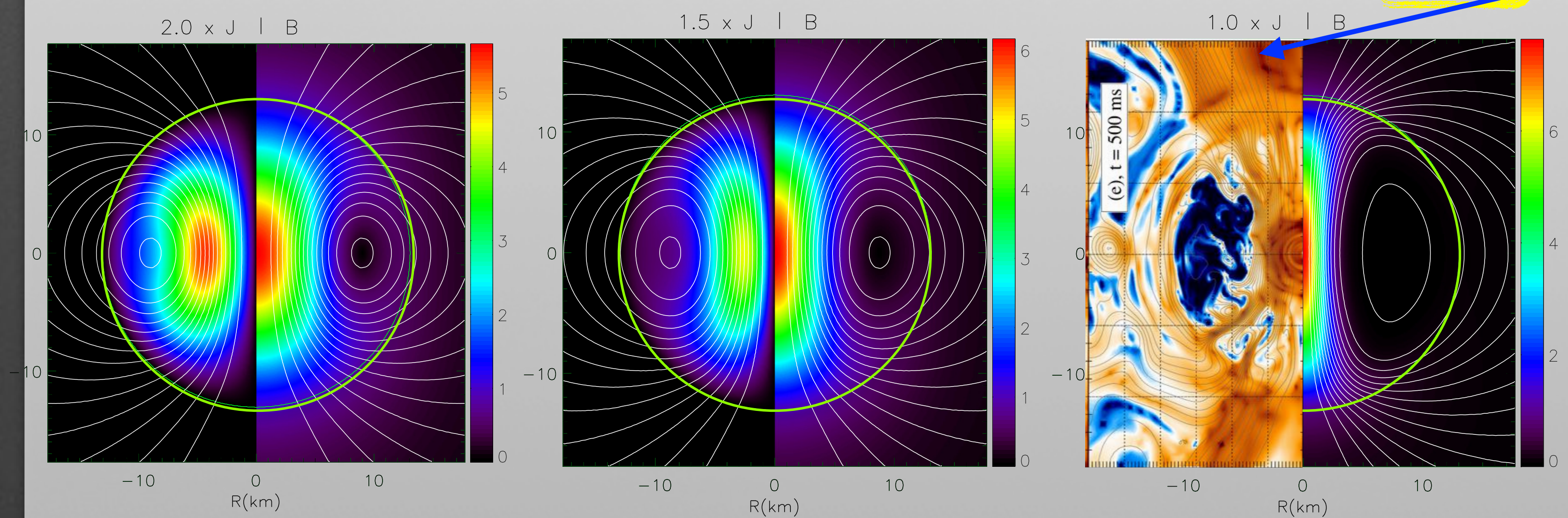
current density | poloidal B (normalized to polar value)

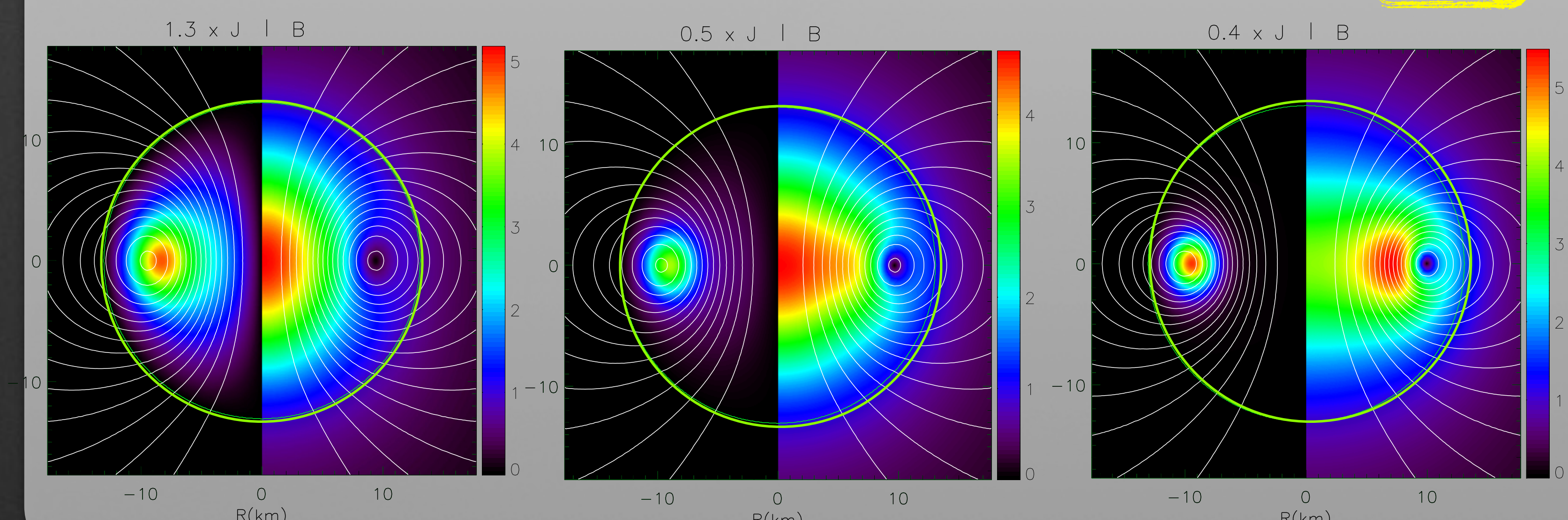
$\xi < 0$

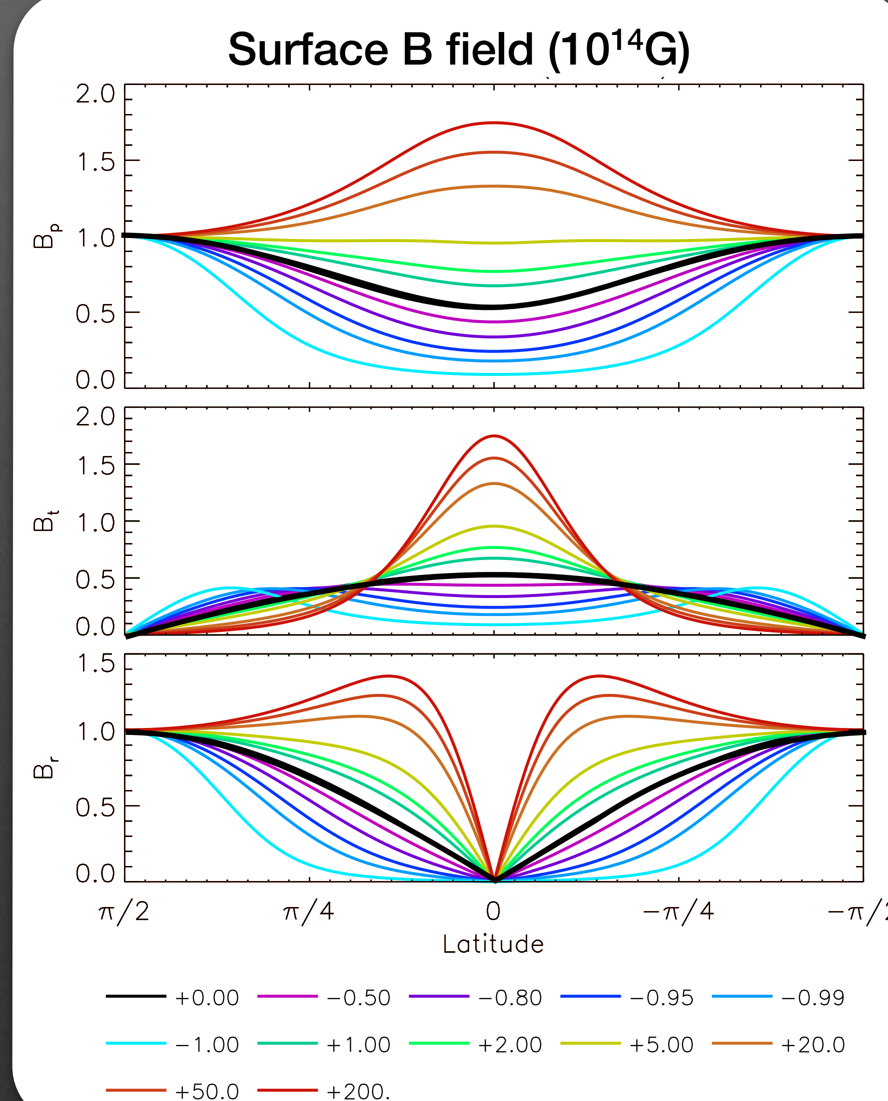
Obergaulinger et al 2014



ν=4 with growing | ξ |

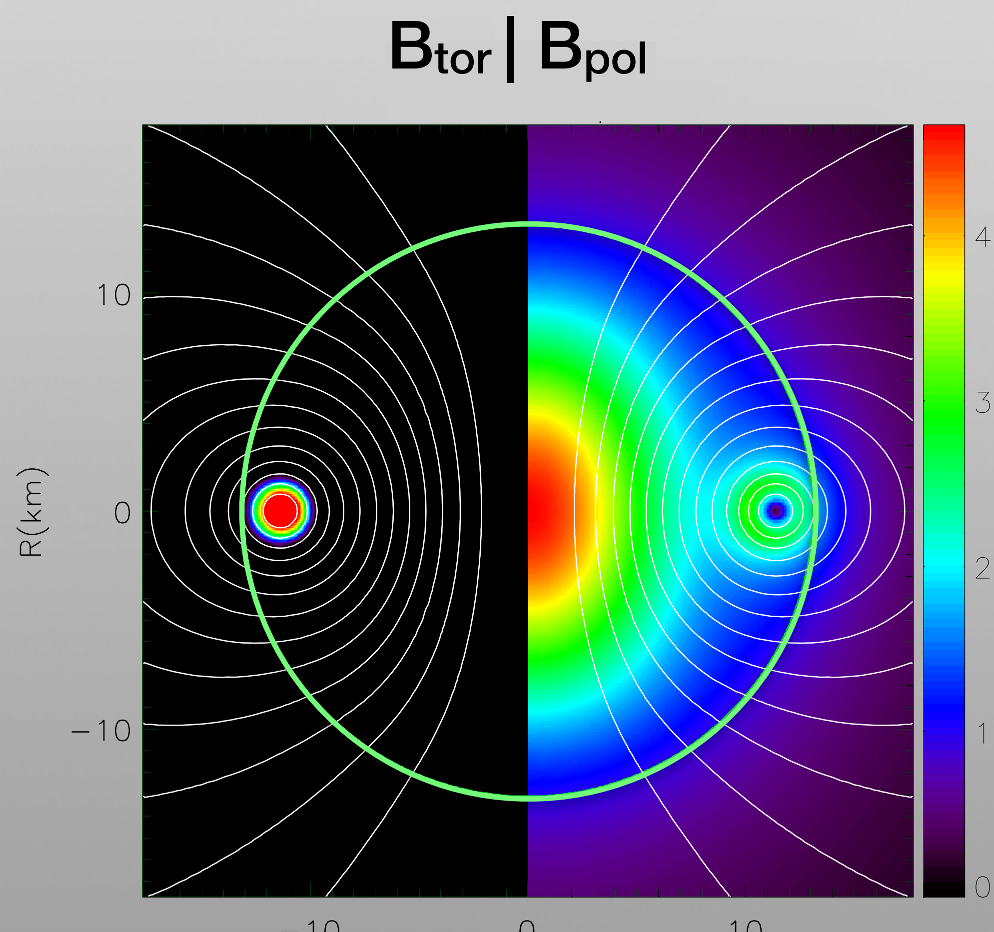
$\xi > 0$





Bucciantini et al. 2015

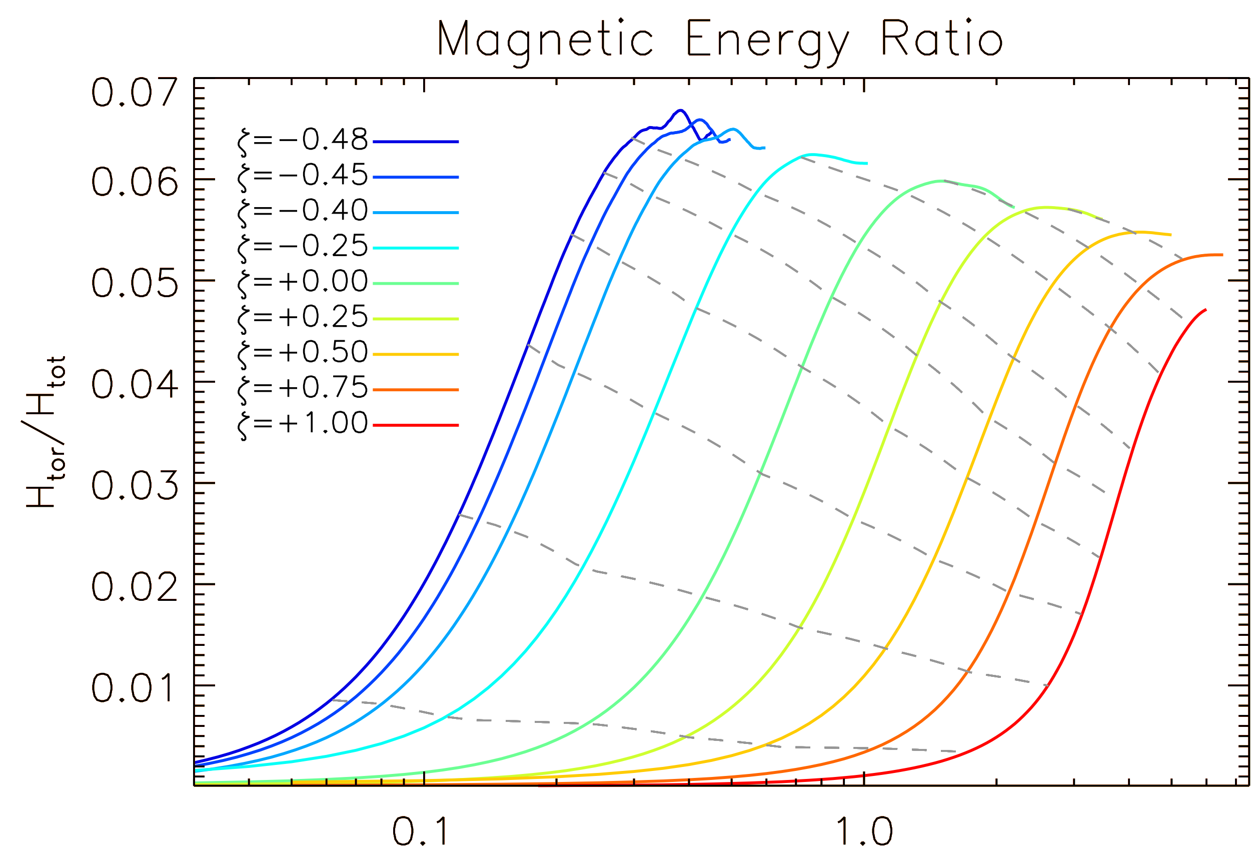
Twisted Torus Configurations



Pili et al. 2014

$$\mathcal{M}(A_\phi) = k_{\rm pol} A_\phi$$

$$\mathcal{I}(A_\phi) = \frac{a}{\zeta+1}\Theta[A_\phi - A_\phi^{\rm sur}]\frac{(A_\phi - A_\phi^{\rm sur})^{\zeta+1}}{(A_\phi^{\rm sur})^{\zeta}}$$



Bucciantini et al. 2015

Stability criterion for twisted torus magnetic field:

$$0.2 \leq \frac{\mathcal{H}_t}{\mathcal{H}} \lesssim 0.99$$

(Braithwaite et al. 2009, Duez et al. 2010)

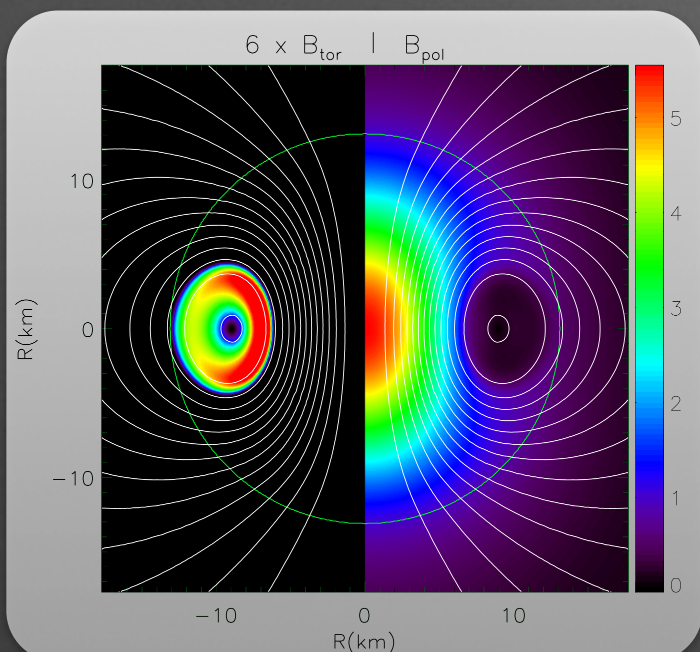
The introduction oppositely flowing currents might allow toroidal dominated configurations

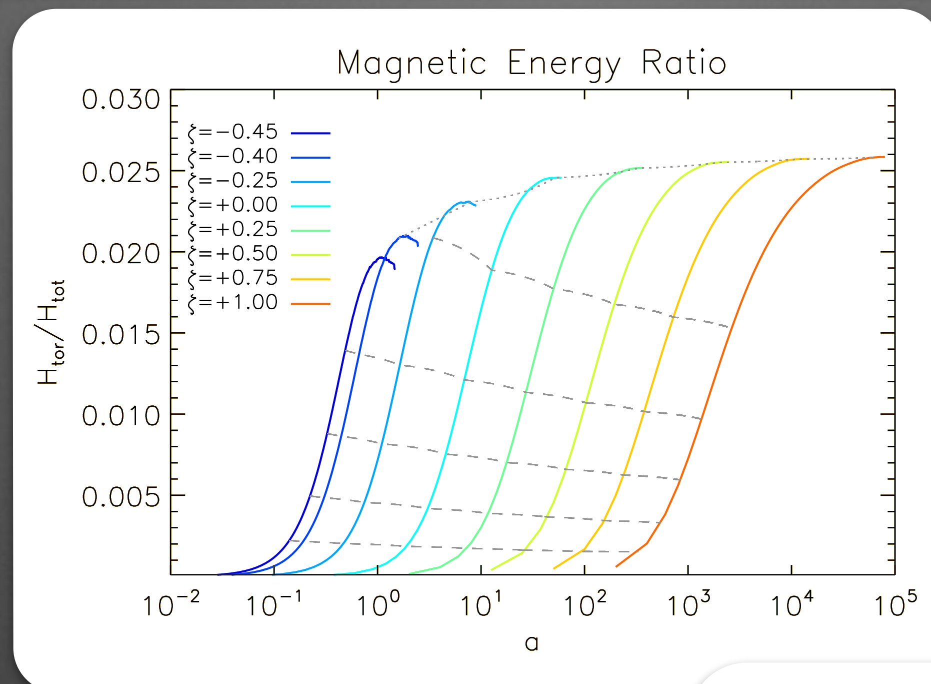
(Ciolfi et al. 2013, Fujisawa et al. 2015)

Models with subtractive currents

Twisted Ring

$$\mathcal{M}(A_\phi) = k_{\rm pol} A_\phi \qquad \mathcal{I}(A_\phi) = \frac{a}{\zeta+1}\Theta[A_\phi - A_\phi^{\rm sur}]\frac{(A_\phi - A_\phi^{\rm sur})^{\zeta+1}(A_\phi^{\rm max} - A_\phi)^{\zeta+1}}{(A_\phi^{\rm sur}A_\phi^{\rm max})^{\zeta+1/2}}$$

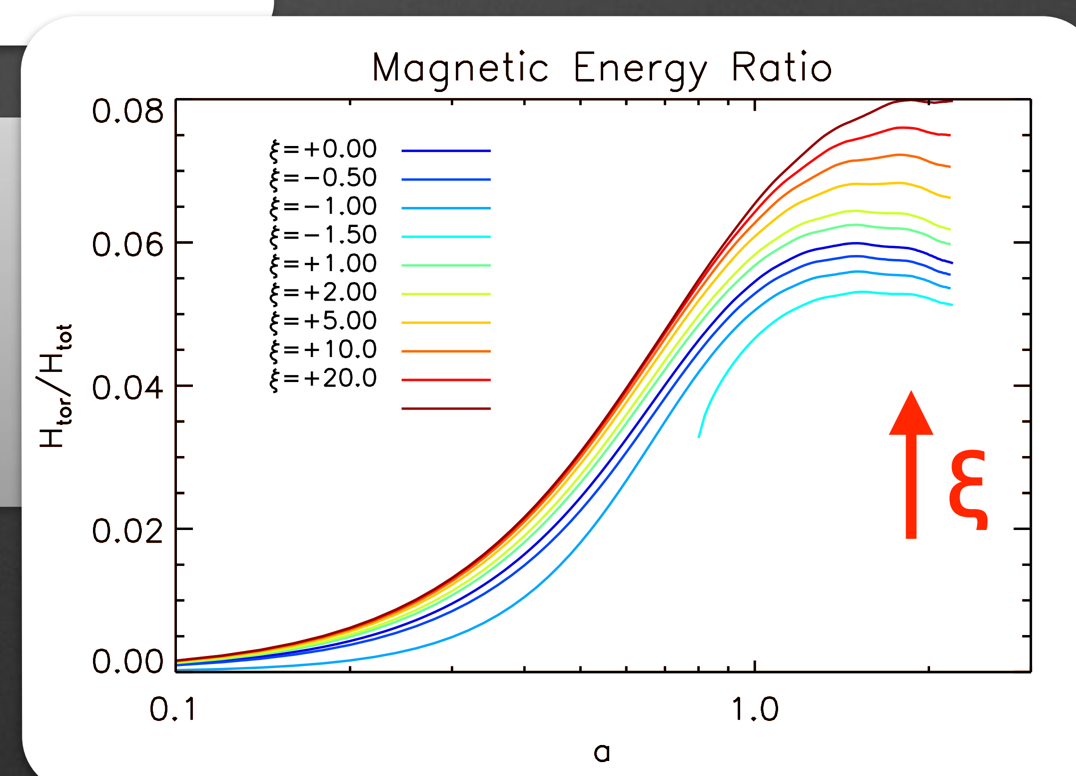




- Current changes sign within the torus like region
- The toroidal field vanishes on the neutral line
- Poloidal dominated configurations

Mixed current configurations

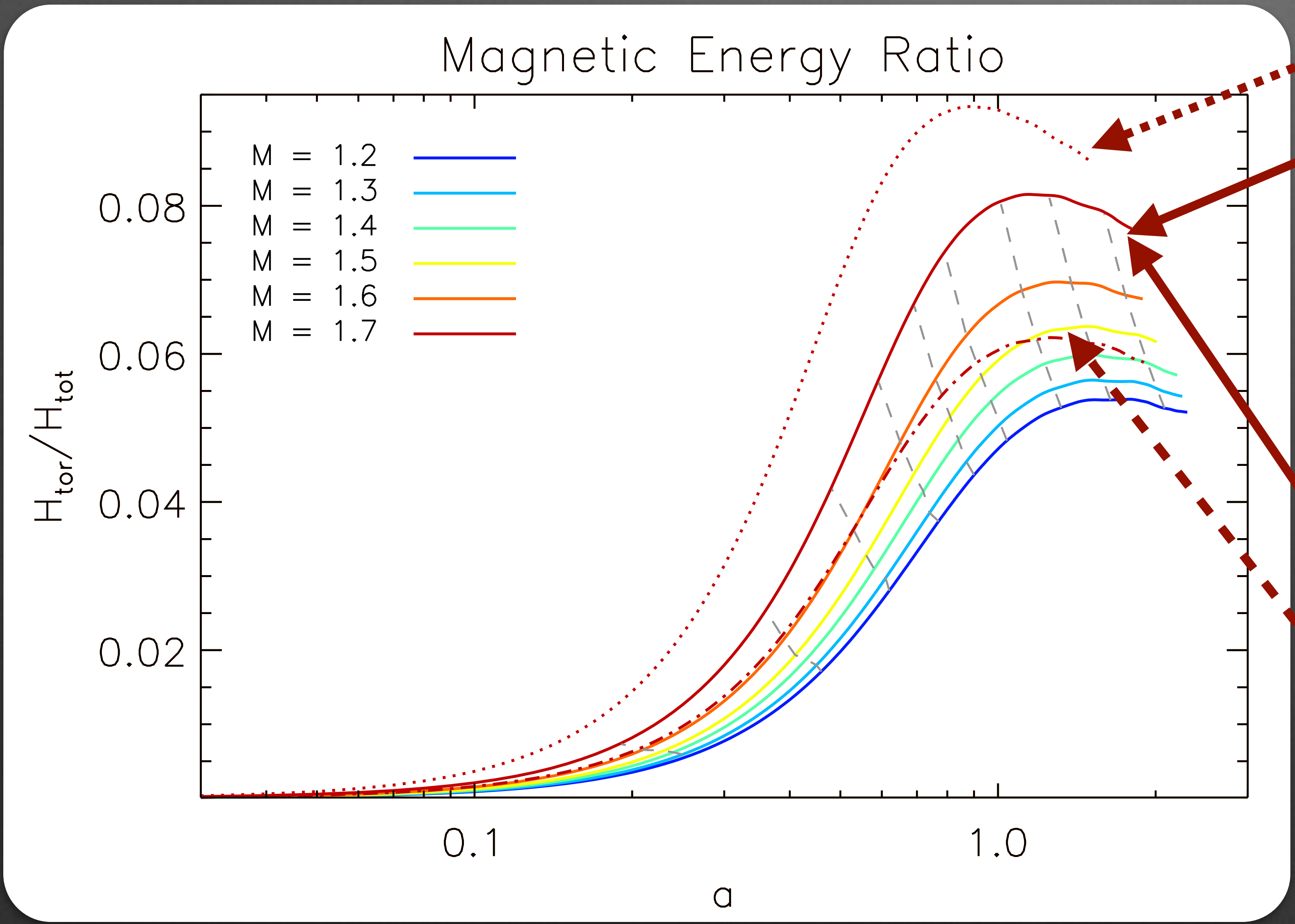
TT with $\mathcal{M}(A_\phi) = k_{\rm pol} A_\phi \left[1 + \frac{\xi}{\nu+1}\left(\frac{A_\phi}{A_\phi^{\rm max}}\right)^\nu\right]$



Magnetic energy ratio grows with ξ

Bucciantini et al. 2015

Dependence on the stellar model



Bucciantini et al. 2015

M=2.0 $M_\odot$

M=1.7 $M_\odot$

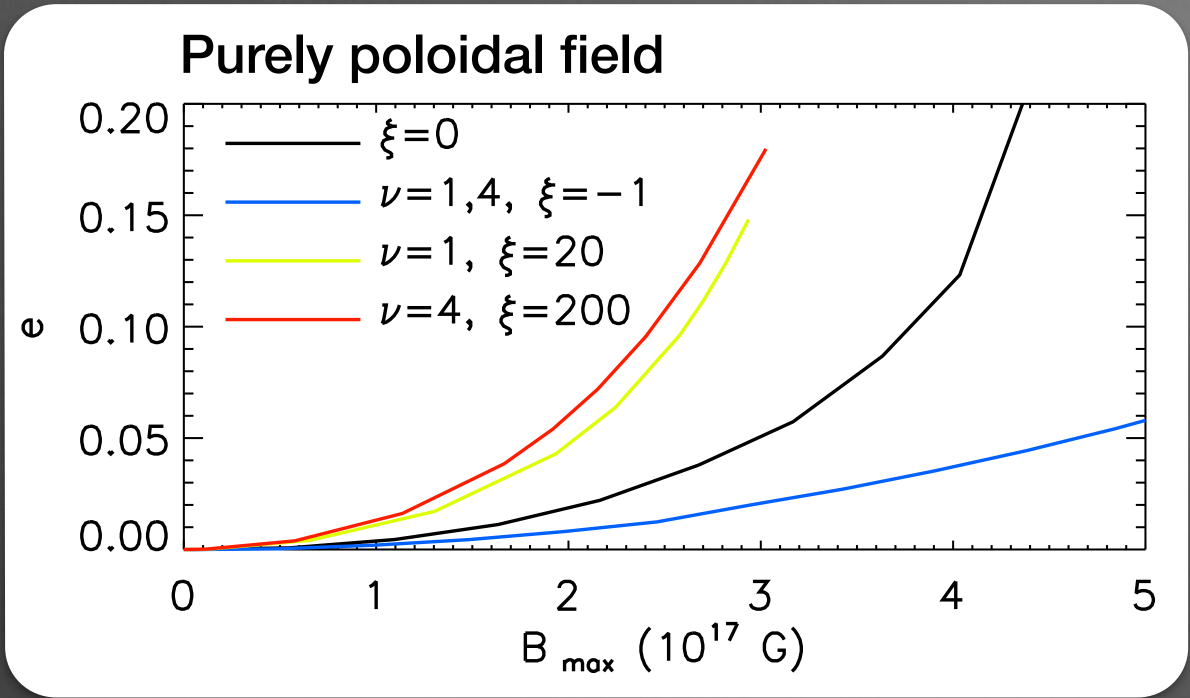
Same central density but different mass

Same mass (1.7 $M_\odot$) but different central density

Higher density

Lower density

"Comparing" deformations



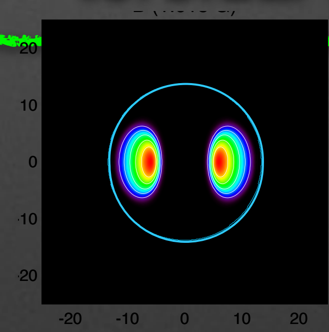
$$\bar{e} := \frac{I_{zz} - I_{xx}}{I_{zz}} \qquad I_{zz} := \int e r^4 \sin^3\theta \, dr \, d\theta \, d\phi \qquad I_{xx} := \frac{1}{2}\int e r^4 \sin\theta (1+\cos^2\theta) \, dr \, d\theta \, d\phi .$$

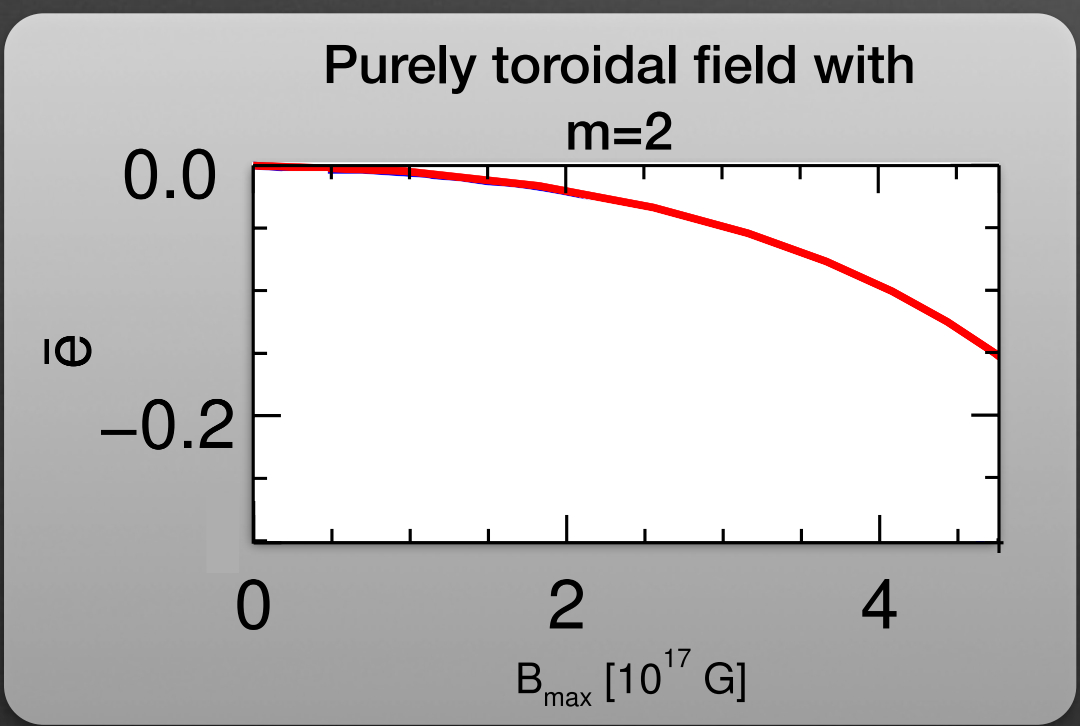
$$\epsilon_B = -\frac{3}{2}\frac{\mathcal{I}_{zz}}{I} \sim 0.4\bar{e}$$

$$\bar{e} \sim 5 \times 10^{-5} B_{16}^2$$

@ 4×10^{17}G:

- poloidal field ⟶ $\bar{e} \sim 0.12$
- toroidal field (m=2) ⟶ $\bar{e} \sim -0.1$
- toroidal field (m=10) ⟶ $\bar{e} \sim -0.02$





Oblate deformation ➜ GW emission is quenched
➜ GRB-like events
(Bucciantini et al. 2009, 2012
Metzger et al 2011)

Twisted Magnetosphere

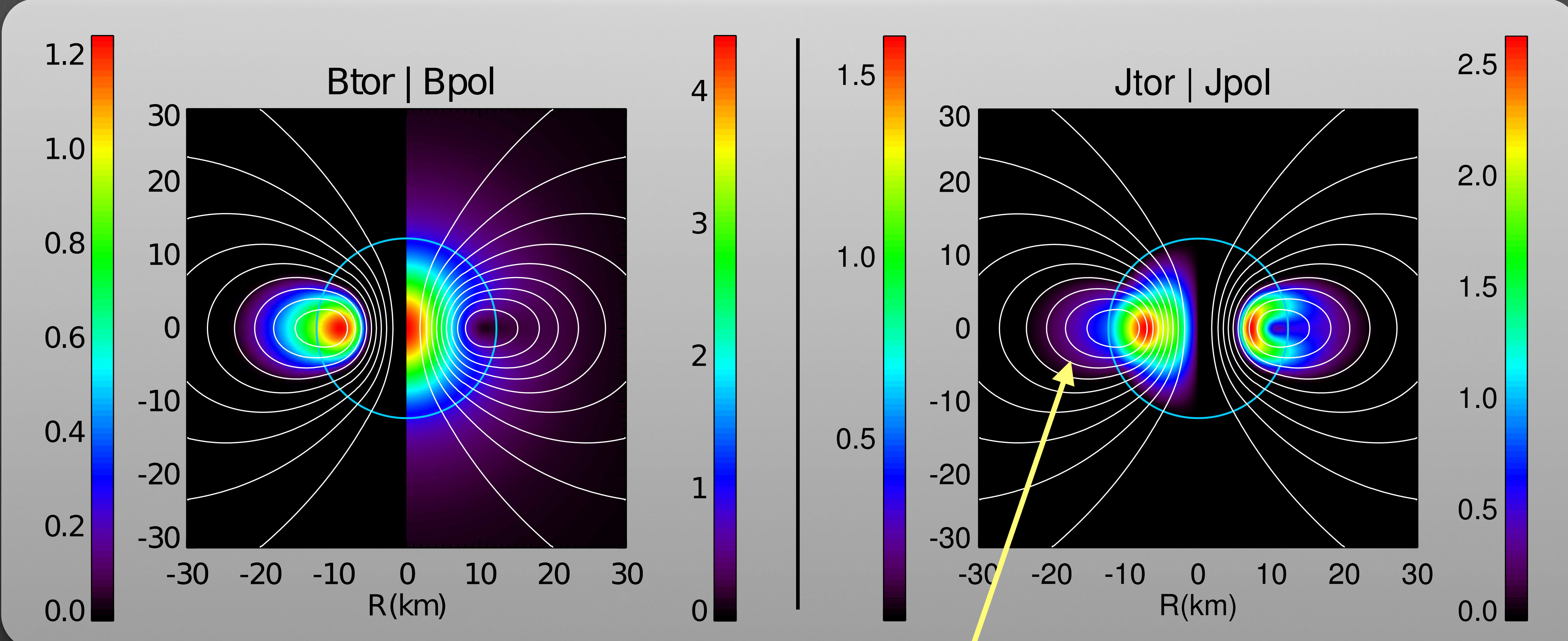
$$\mathcal{M}(A_\phi) = k_{\rm pol} A_\phi$$

$$\mathcal{I}(A_\phi) = \frac{a}{\zeta + 1} \Theta \left[A_\phi - A_\phi^{\rm ext}\right] \frac{\left(A_\phi - A_\phi^{\rm ext}\right)^{\zeta+1}}{\left(A_\phi^{\rm max}\right)^{\zeta+1/2}}$$

$$A_\phi^{\rm ext} = A_\phi(\lambda R_{\rm NS}, \pi/2)$$

λ controls the size of the twisted magnetosphere

Glampedakis et al. 2014
Pili et al. 2015
Fujisawa et al. 2015

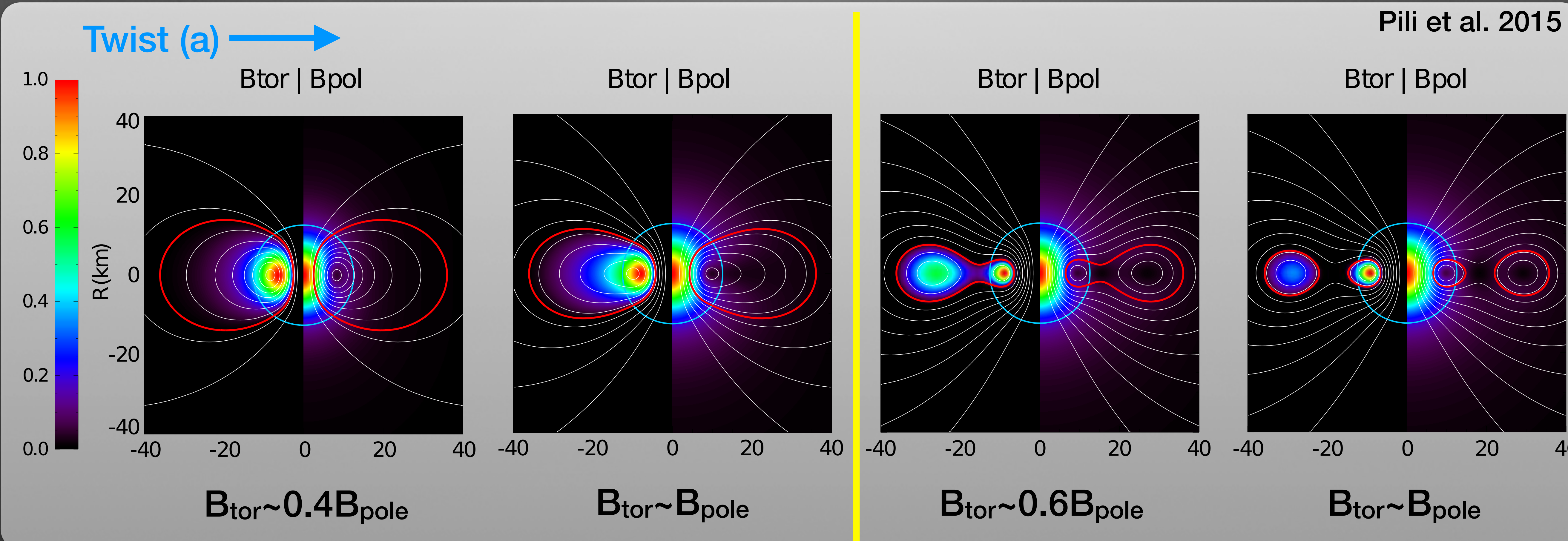


- Model with λ=2
- Highest value of H_{tor}/H=0.11

$$J^\phi = \frac{a^2}{(\zeta + 1)\varpi^2} \Theta \left[A_\phi - A_\phi^{\rm ext}\right] \frac{\left(A_\phi - A_\phi^{\rm ext}\right)^{2\zeta+1}}{\left(A_\phi^{\rm max}\right)^{2\zeta+1}} + \rho h\, k_{\rm pol}$$

Twisted Magnetosphere

In the magnetosphere energy of toroidal field ~ 20% poloidal B-field

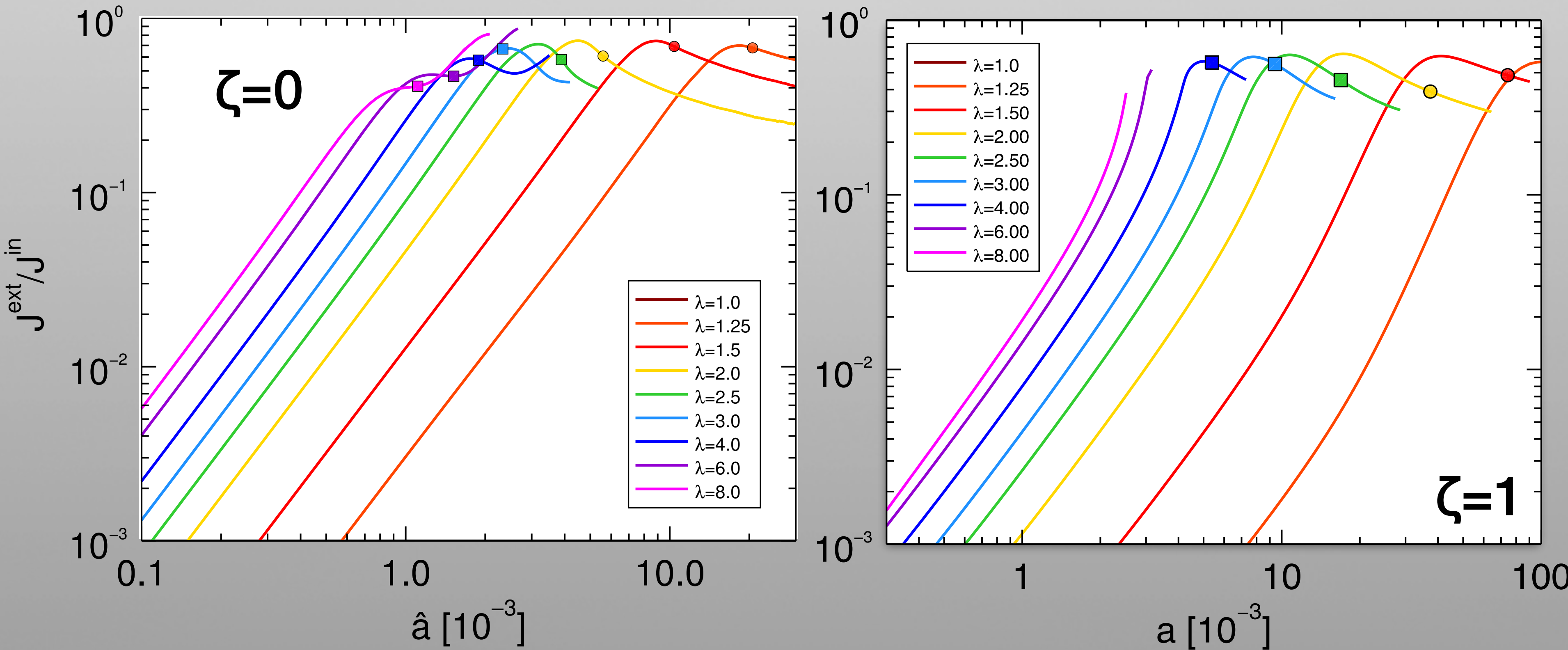


- Azimuthal displacement of footpoints does not exceed 2 rad
- Below the critical value 3.65 rad (Parfrey et al. 2013)

- Onset of X-points and detached flux ropes
  - Kruskal-Shafranov condition (kink instability)
  - Safety factor ≲1
- Finally plasmoid-like solutions

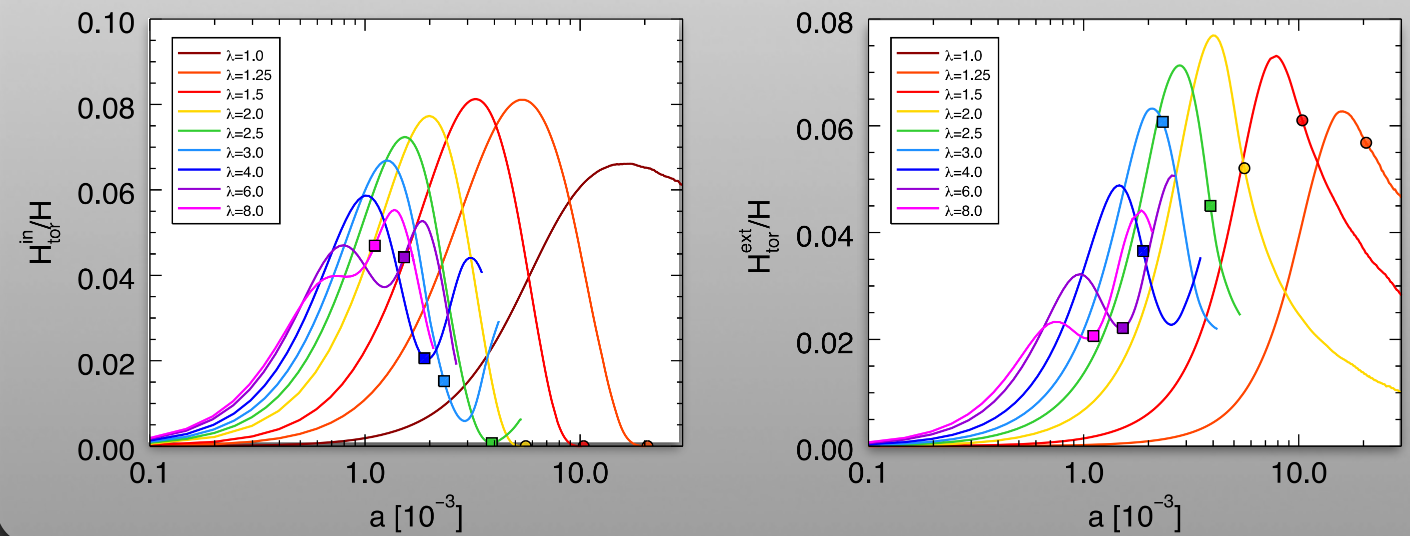
Twisted Magnetosphere

Exterior ϕ-current density / Interior ϕ-current density

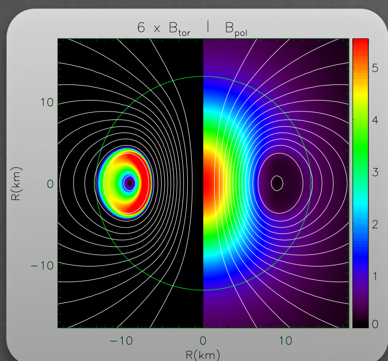


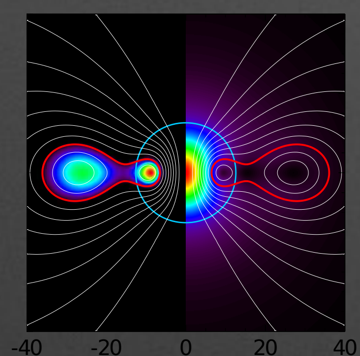
Pili et al. 2015

- External toroidal currents can not exceed the internal ones
- The toroidal energy density in the exterior is comparable with that in the interior

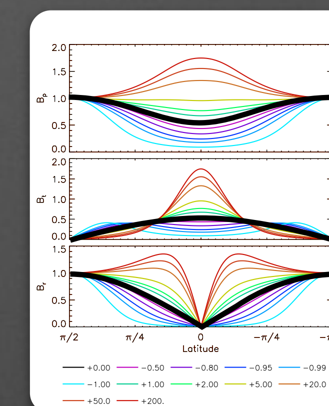


Conclusions





- Models with subtractive currents are poloidal dominated (we can not reach inversion currents)
- The surface B-field is strongly influenced by the location and the distribution of currents
- Limit on the magnetospheric twist: self-regulating mechanism between internal and external currents



Next

Rotating magnetized models
Emission models in GR

Thank you!

A special thanks go to **NewCompStar COST action MP1304** for the travel and local support!!