Thermodynamics of friction

Noa Mitsui

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   - tectonics & earthquakes
   - theory of earthquakes
   - relation between earthquakes and friction
2. Purpose: thermodynamic modeling of rock friction
3. Research
   - introduction: rock experiments
   - thermodynamic modeling
4. Summary

Tectonics & earthquakes in the world

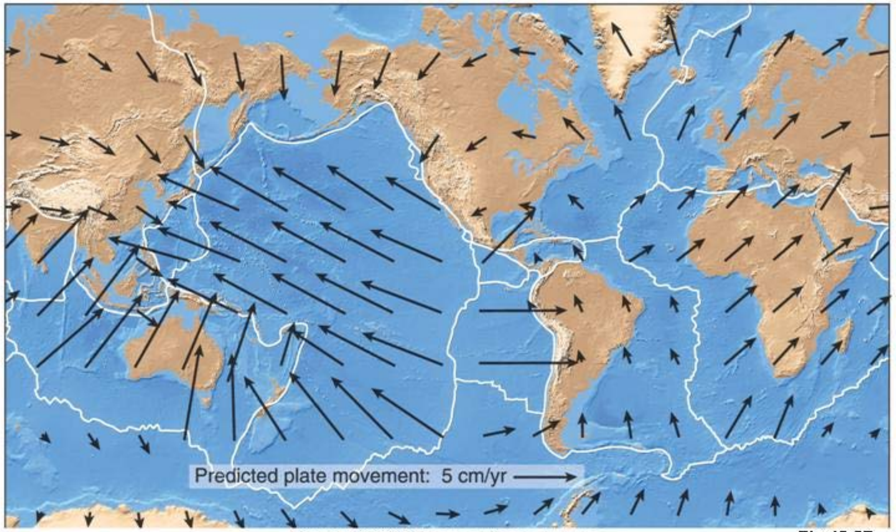
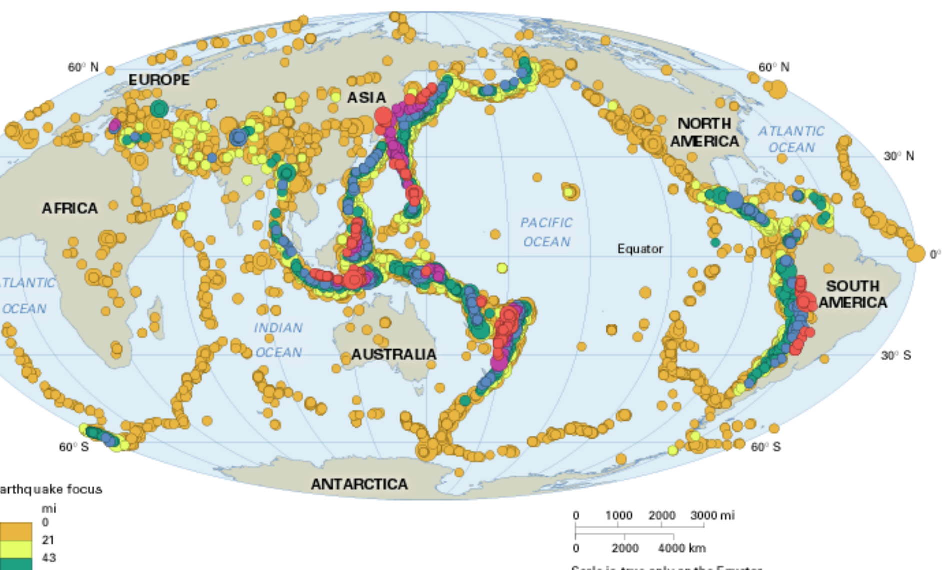


 Fig 12.37

Plate movements and their boundaries

- Plates move relatively at several cm/yr
- Japan locates on some boundaries

GI
earthquakes of magnitude 5.5 and greater

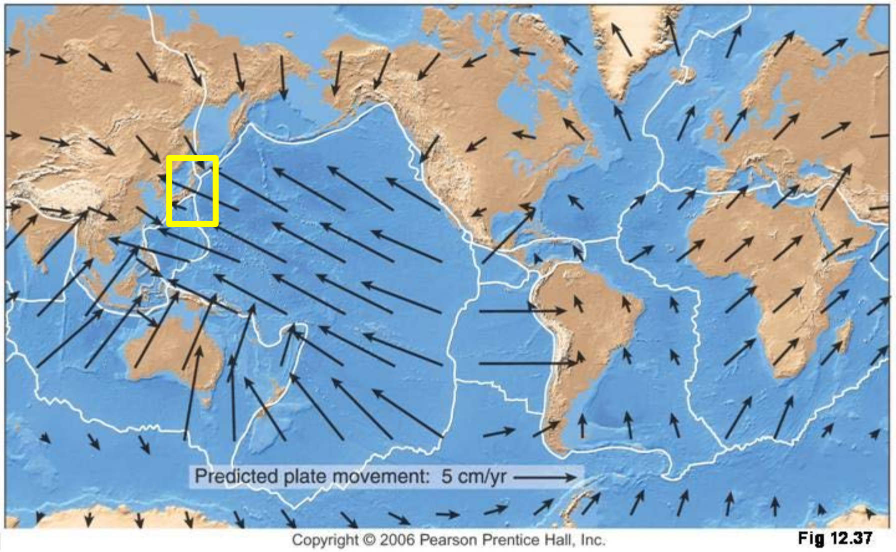


Earthquake epicenter (M5.5<)

- Many earthquakes occur around Japan

$M = -3.2 + 0.67\log_{10}E$ (E: energy)

Tectonics & earthquakes in the world



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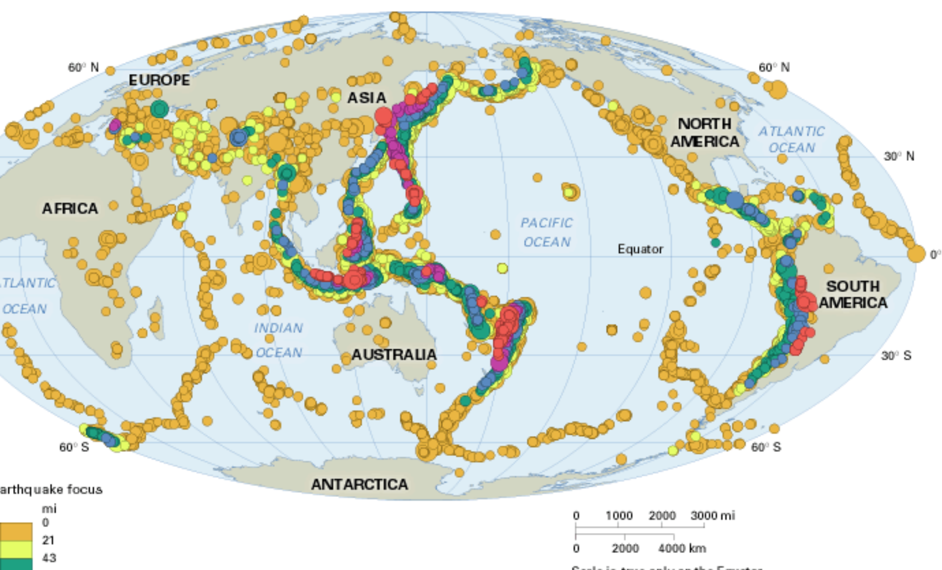

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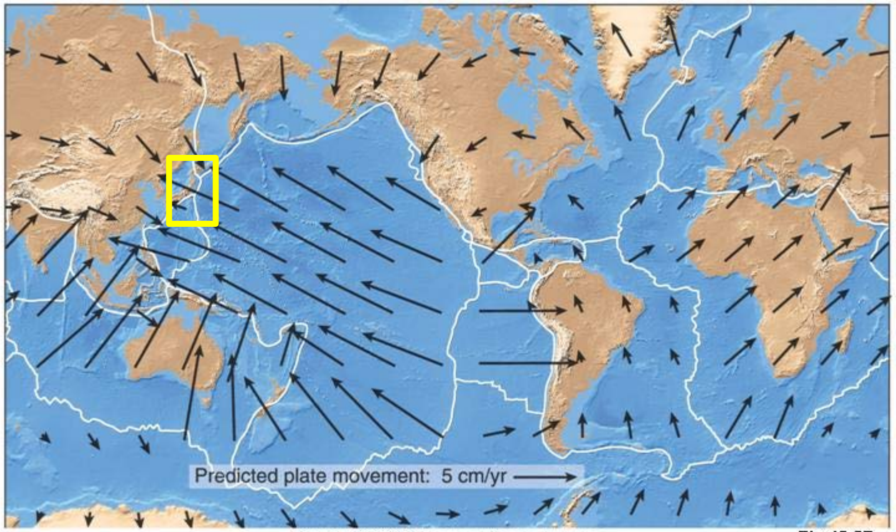

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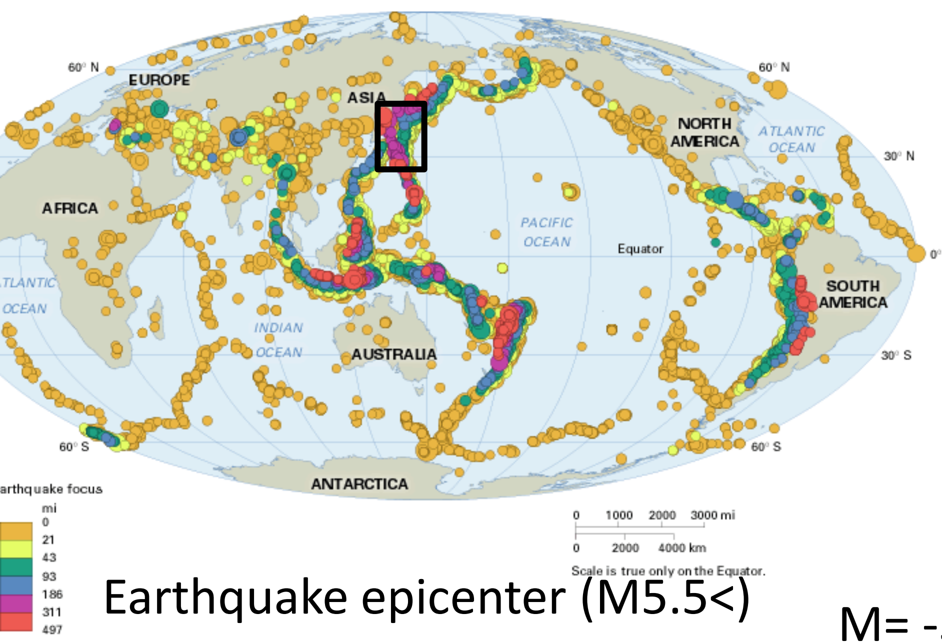


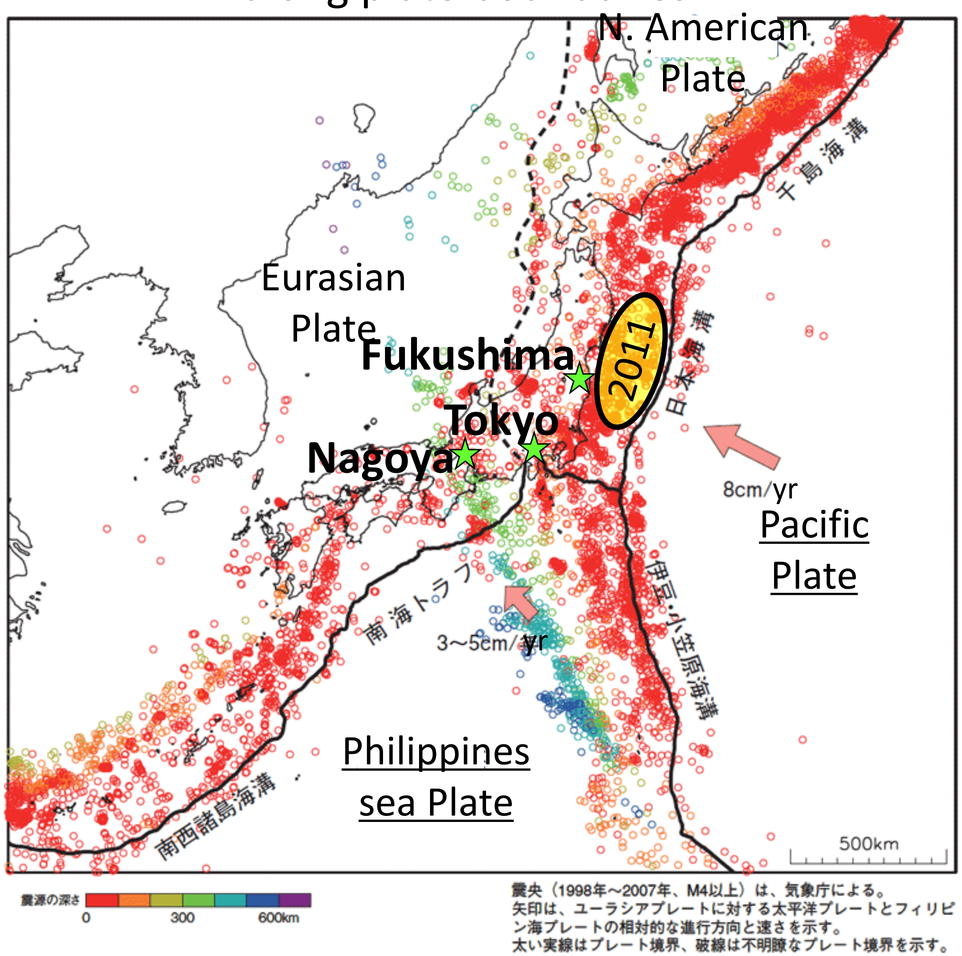
Earthquake epicenter (M5.5<)

- Many earthquakes occur around Japan

$M = -3.2 + 0.67\log_{10}E$ (E: energy)

Difficulty in understanding of earthquake

earthquakes distribution (seismicity)
along plate boundaries



(Source area= ruptured area
estimated by detected seismic waves)

- People want to know "WHEN & WHERE" earthquakes will happen

Difficulty of the forecast
‖
Incompleteness of theory

Difficulty in understanding of earthquake

earthquakes distribution (seismicity)
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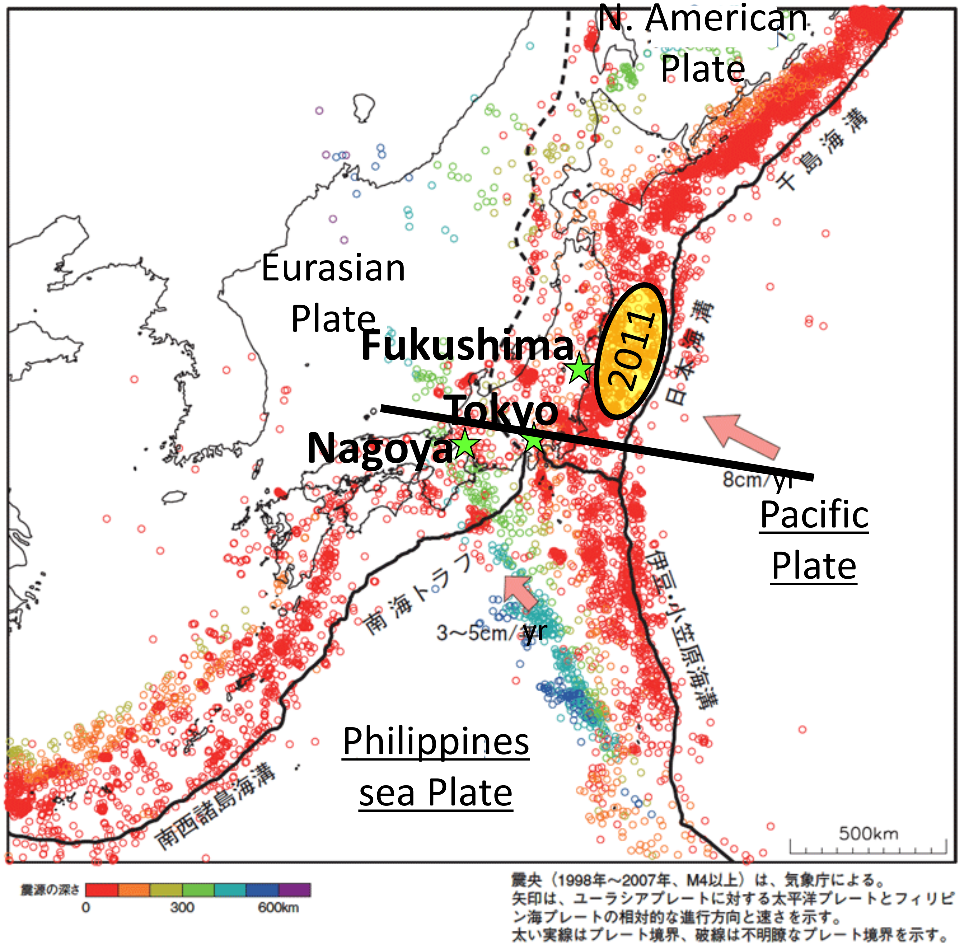

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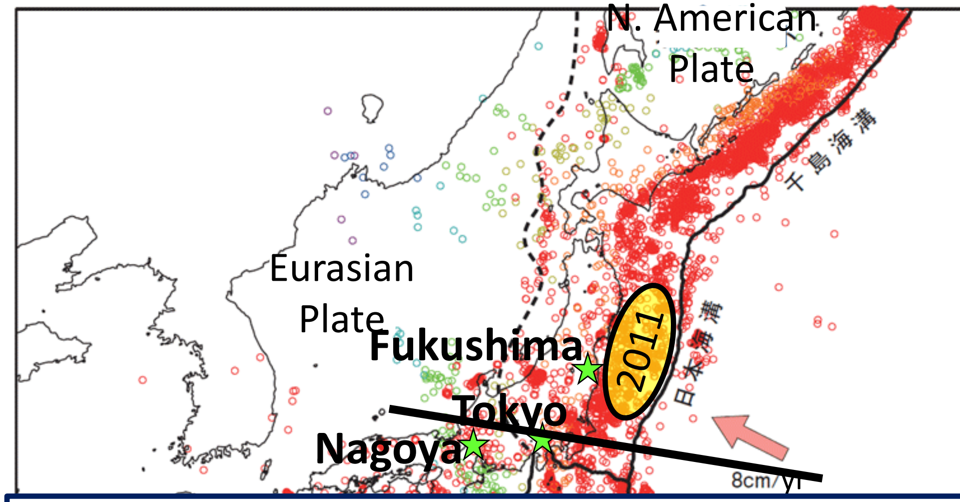
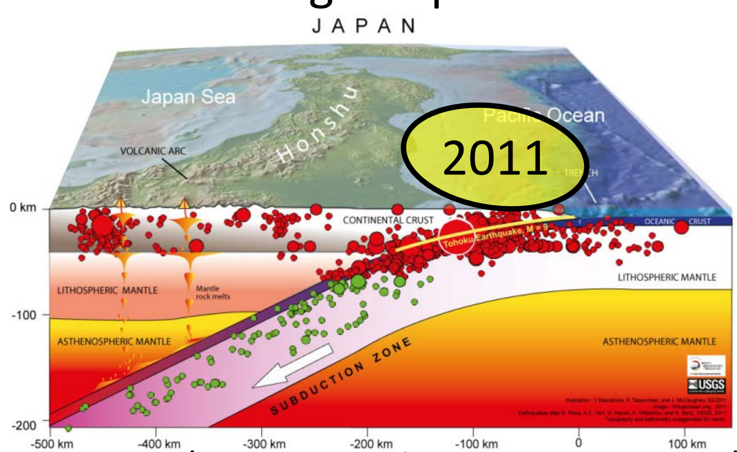


Cross section along the plate movement



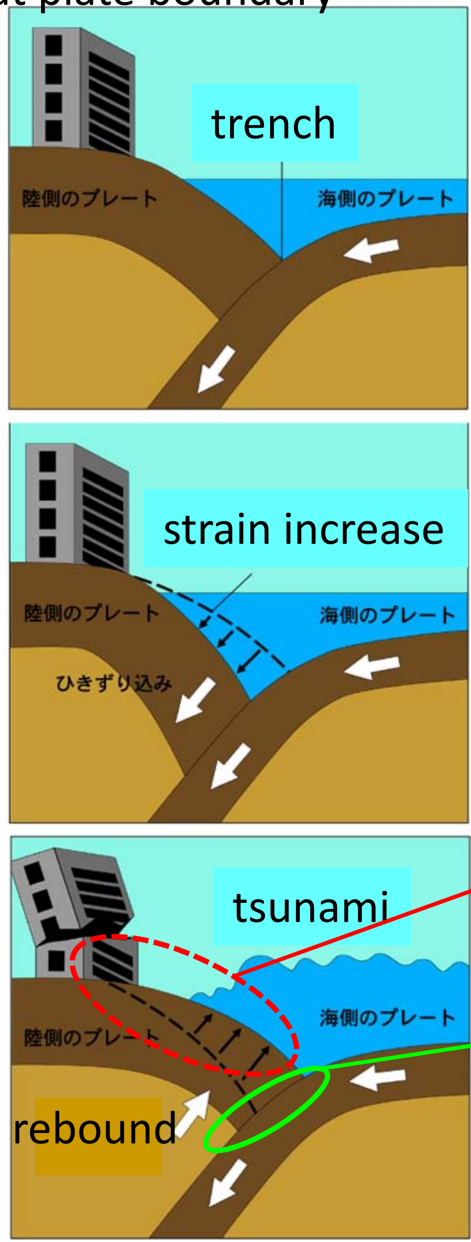
(1964-2007 & 2011, M5<, USGS)

- People want to know "WHEN & WHERE" earthquakes will happen

Difficulty of the forecast
‖
Incompleteness of theory

Incomplete theory = elasticity

Schematic diagram of deformation
at plate boundary



All strain released

Slip only at boundary

Only in theory

- current bases of seismology:
  All strain released by slip
  at <u>contact surface</u>
  ||
  plate boundary

It cannot explain
small earthquakes (=seismicity)
occurring in the plate

↓

<u>realistic deformation theory</u>
<u>is necessary (= motive)</u>

Main theme of today's talk

☆ How we can obtain deformation theory of medium?

(bases of the earthquakes occurrences)

fracture and deformation in the earth's crust

Research object: friction law in rock experiments

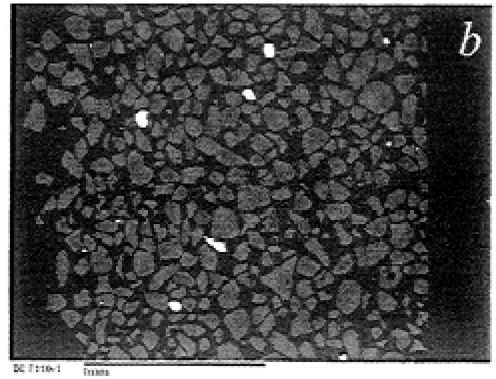
- earthquakes are compared to frictional slips along the tectonic plate boundary
- easiness to understand: rock experiment > earthquakes

I will focus on the rate- and state-dependent friction law

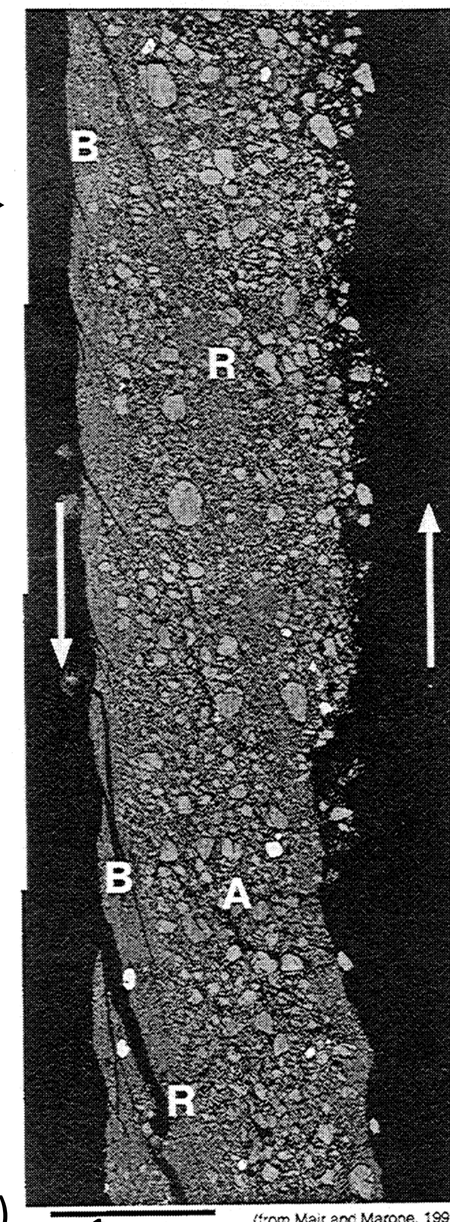
(Dieterich, 1979)

Sand particles in frictional layer between rocks

Before dislocation



After dislocation

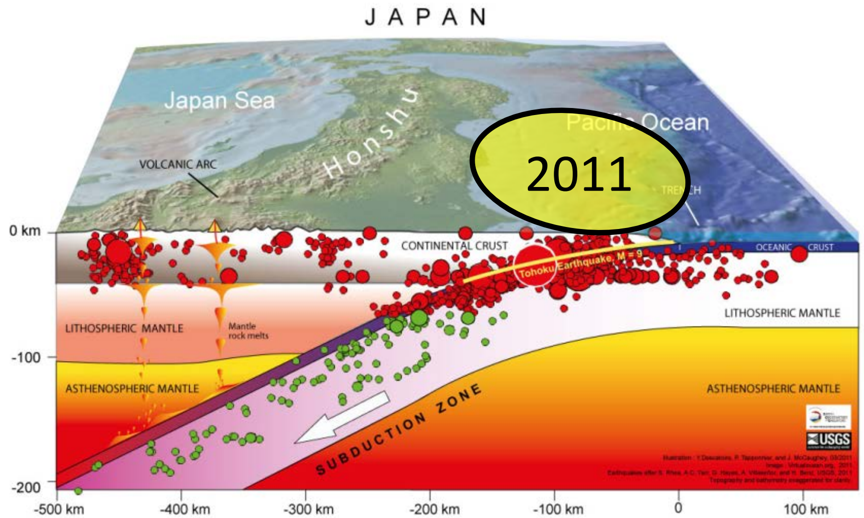


1mm

- Even in the small frictional layer, sand particles are deformed, crushed, compacted.
- These phenomena probably causes the complexity of friction
- The mechanism of friction can be related to that of earthquakes

(Mair & Marone,1999)

Sand particles in frictional layer between rocks



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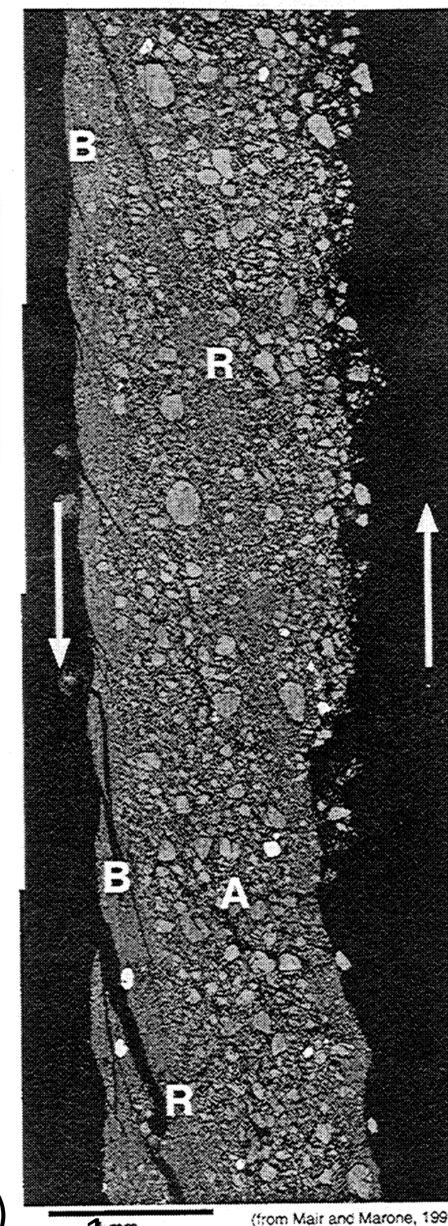

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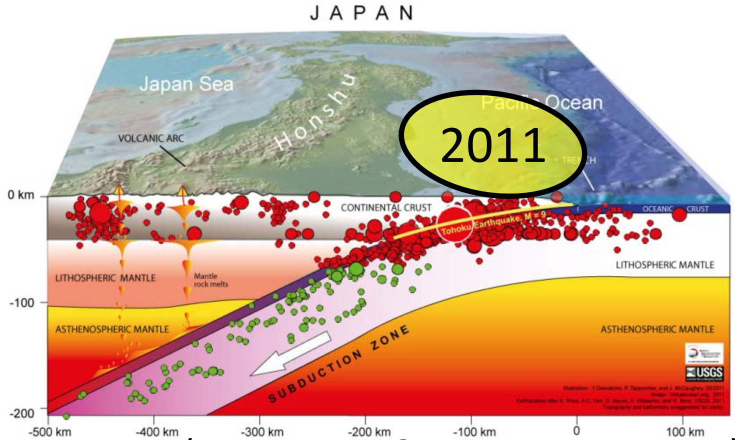
(Mair & Marone,1999)

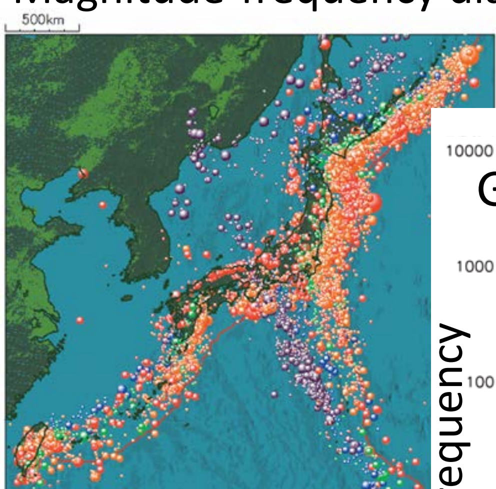
Wideness of earthquakes

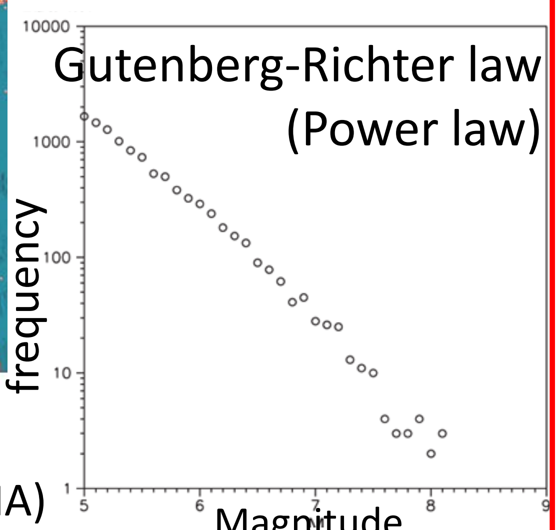
Cross section along the plate movement



(1964-2007 & 2011, M5<, USGS)

Magnitude-frequency diagram





(1924-2007, M5<, JMA)

- wide distribution along the plate boundary
- Magnitude: M9.0-M2(detectable limit)

The mechanism can be related to them

← Another model of G-R law called as "cumulative Gompertz distribution" can have the same entropy formula as that in heavy ion physics (Biro et al, 2015)

Main theme of today's talk

☆ How we can obtain <u>deformation theory</u> of medium?

(It causes the <u>earthquakes</u> occurrences)

fracture and deformation in the earth's crust

Research object: <u>friction law</u> in rock experiments

- earthquakes are compared to frictional slips along the tectonic plate boundary
- easiness to understand: rock experiment > earthquakes

I will focus on the <u>rate- and state-dependent</u> friction law

(Dieterich, 1979)

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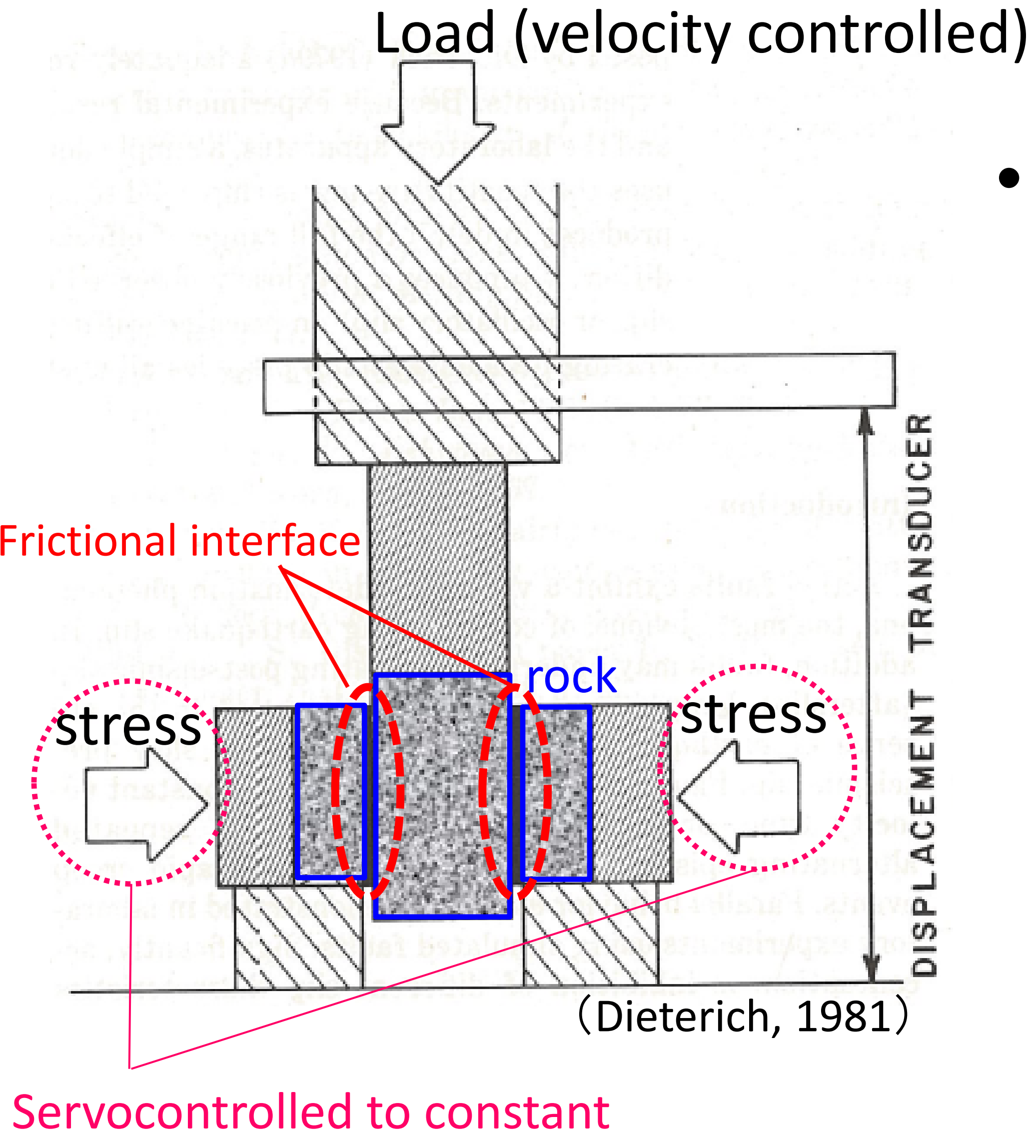
Outline of rock experiments

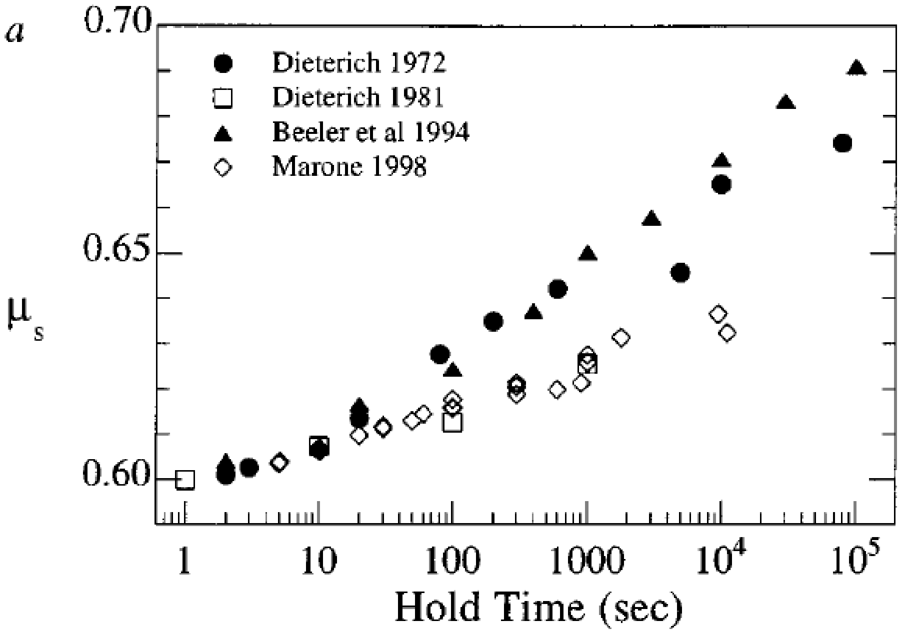


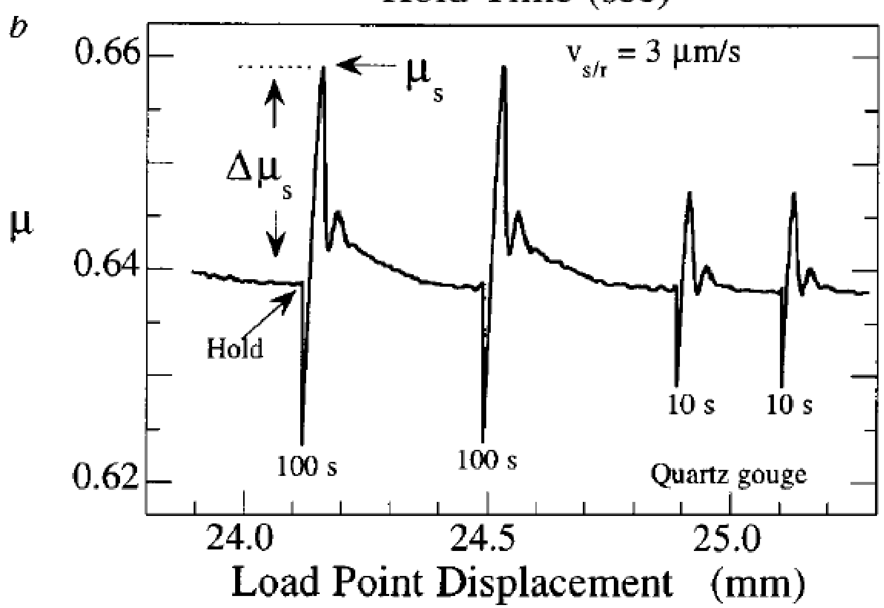
(Dieterich, 1981)

- Static & dynamic friction are tested with various condition:

  (rock type, temperature, humidity, normal stress with/without sand at the interface)

Static friction increases with time

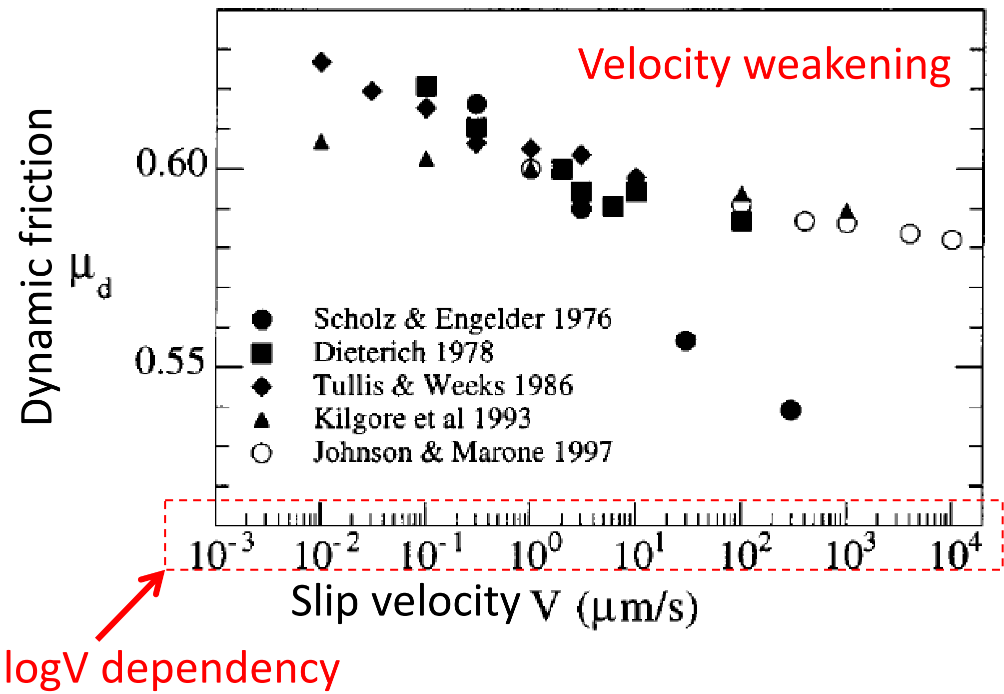




(Marone, 1998)

- Static friction ∝ log (time)
- Friction decreases to dynamic one with displacement

Feature1/3: Stable dynamic friction ∝ log(slip velocity)

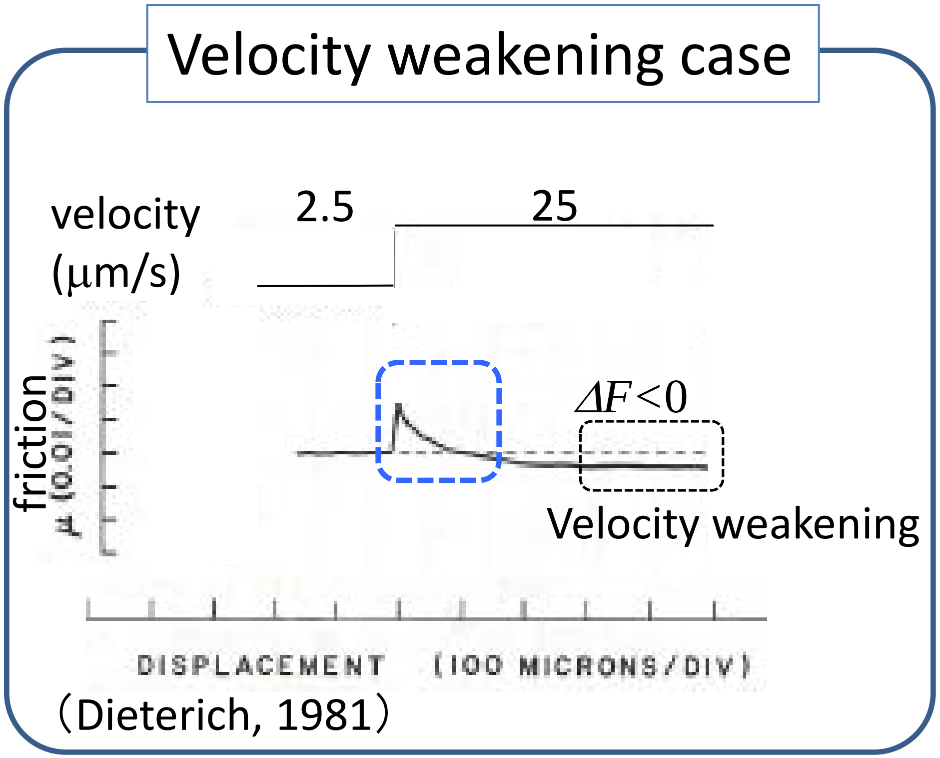


(Marone, 1998)

- Rock samples loaded with constant velocity
- Friction weakens or strengthen with velocity depending on the tested condition

  ex. Interface sand
  without sand: weaken
  with sand: strengthen

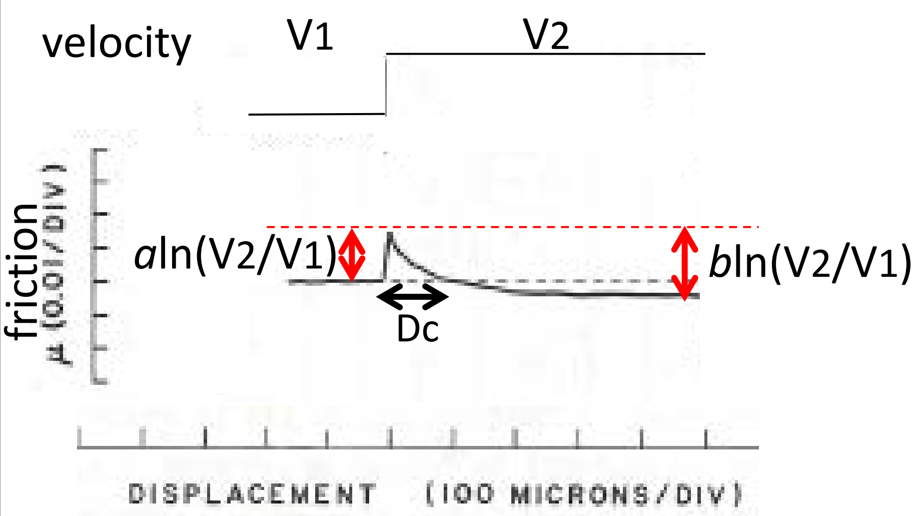
Feature2/3: frictional jump ∝ log(velocity step)



- Loading velocity step causes instantaneous jump of dynamic friction
- Following the jump, the friction transitions to the stable one
- Both jump and relaxation depend on ln(velocity step)

Feature2/3: frictional jump ∝ log(velocity step)

Velocity weakening case



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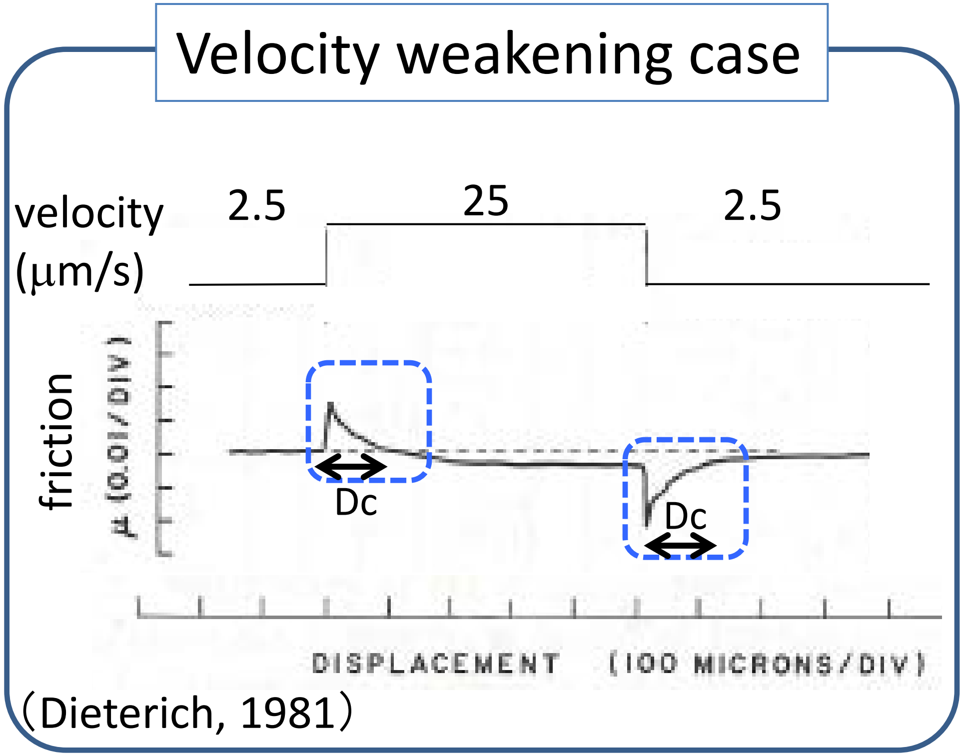
Feature3/3: symmetric relaxation with constant displacement



- Both case of velocity increase and decrease has frictional jump and relaxation with constant displacement (Dc)

vel. increase: jump up

vel. decrease: jump down

Empirical equation of the friction law

Empirical equations of
rate- and state-dependent friction law

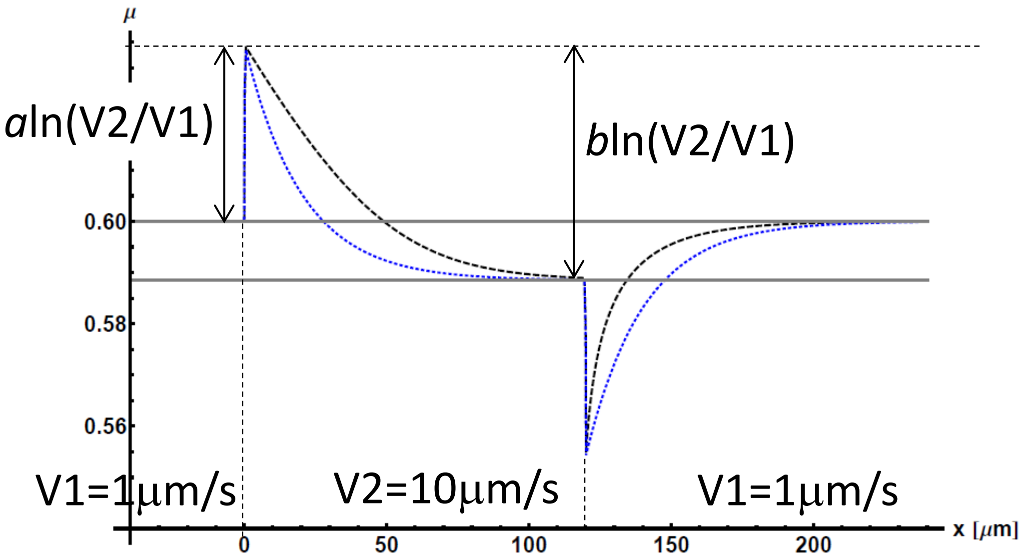
$$\begin{cases} ① \ \mu = \mu_o + a\ ln\left(\frac{V}{V_o}\right) + b\ ln\left(\frac{V_o\theta}{D_c}\right) \\ ② \ \frac{d\theta}{dt} = 1 - \frac{V\theta}{D_c} \quad \text{or} \quad \frac{d\theta}{dt} = -\frac{V\theta}{L}\ln\left(\frac{V\theta}{L}\right) \end{cases}$$

(Dieterich, 1979) (Ruina, 1983)

☺: Static friction Dynamic friction

μ: shear stress t: time V: slip velocity
μ_0:reference friction for V_0
D_c: characteristic slip distance
θ: state variable a,b: positive parameters

Simulated frictional change
with displacement



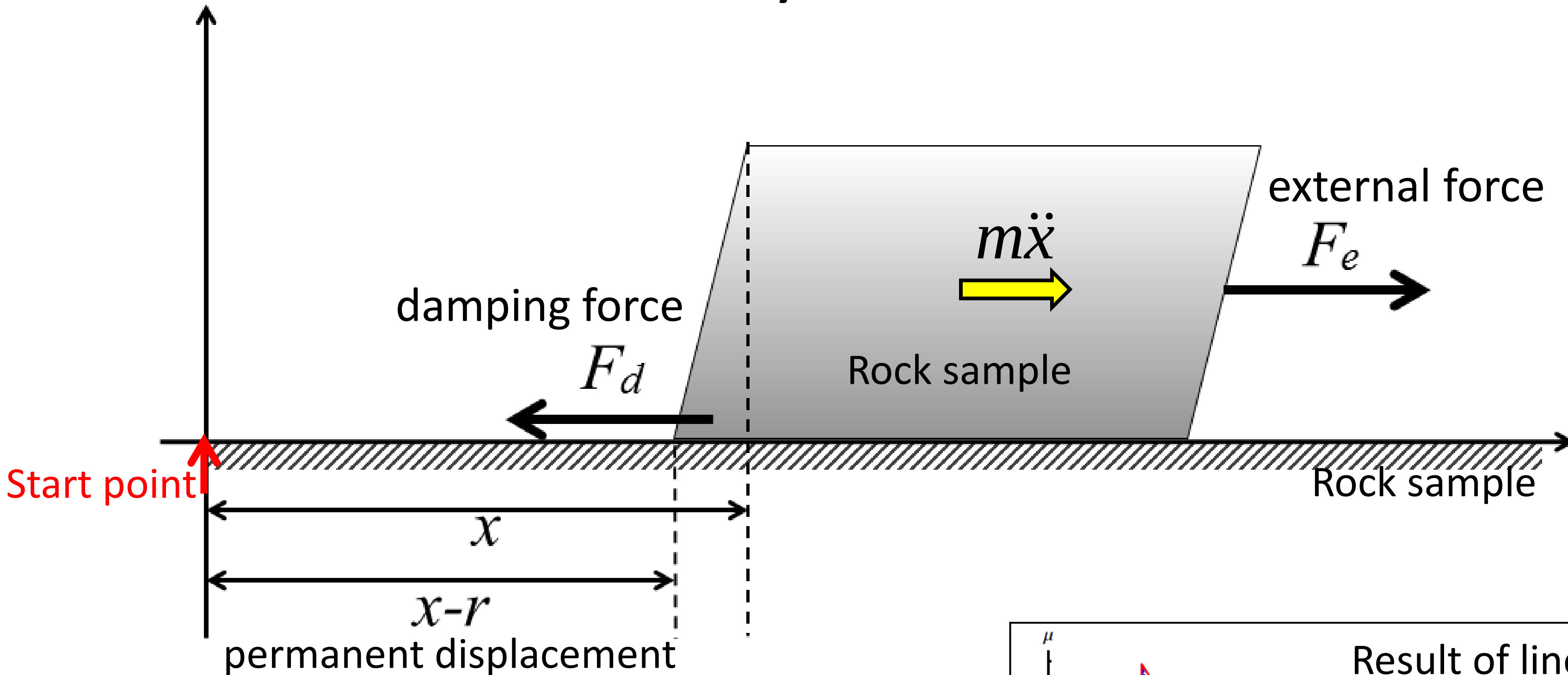
- Neither of them can reproduce both (static & dynamic) friction.

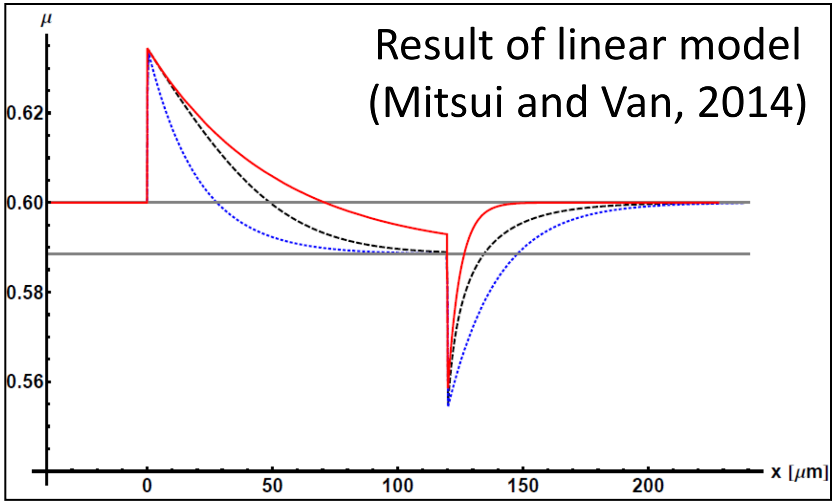
↓

- We will reproduce the experimental result with thermodynamics.

friction = dissipative event -> thermodynamic issue

Previous thermodynamic model (Mitsui and Van, 2014)





equation of motion : $m\ddot{x} = F_e - F_d$

total energy : $\dot{E} = F_e \dot{x} = F_e V$

entropy : $S\left(U = E - mV^2/2 - kr^2/2 \right)$

U ↑ internal energy; k ↑ elastic modulus

entropy balance : $T\dot{S}(U) = F_d V - kr\dot{r} \geq 0 \quad \because$ 2nd law of thermodynamics

Improvement of thermodynamic model

- Important feature

1. instantaneous jump

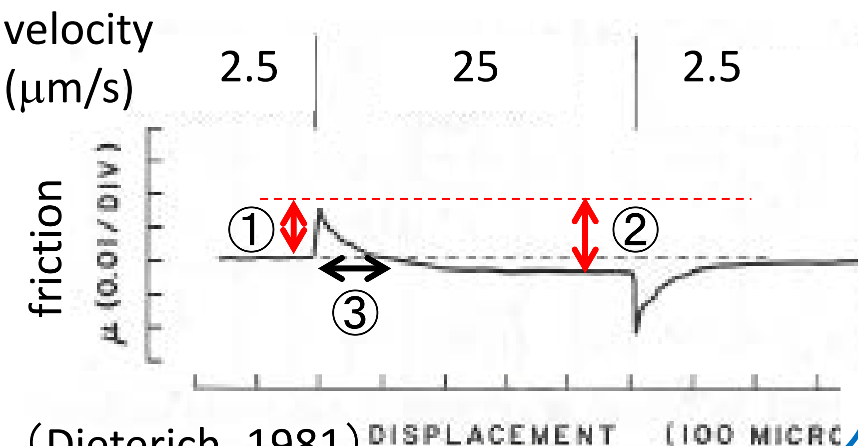
   $a\ln(V_2/V_1)$

2. following relaxation

   $b\ln(V_2/V_1)$

3. symmetric relaxation with disp.

   Dc



(Dieterich, 1981)

(Van, Mitsui, Hatano, 2015, arXiv:1501.04608v1)

- introduced elements

1. partial steady state

$$U = E - mV^2/2 - kr^2/2$$

$$\downarrow$$

$$U = E_0 - m(V - V_0)^2/2 - kr^2/2$$

2. logarithmic increment

   based on thermal activation theory
   (chemical reaction: Eyring, 1936;
   rock friction: Nakatani, 2001)

$$\Delta\mu(V - V_0) \rightarrow \Delta\mu \ln(V/V_0)$$

3. factor of slip reduction

   in elastic displacement

$$\frac{dr}{dt} \rightarrow \frac{dr}{d(x - \alpha r)} = \frac{\dot{r}}{V - \alpha \dot{r}}$$

Improved constitutive equations

entropy balance: $$\frac{T\dot{S}(U)}{\sigma_n A} = \mu V - \kappa r \dot{r} \geq 0$$

$\downarrow$

$$\frac{T\dot{S}(U)}{\sigma_n A} = \mu_0 V_0 + \underbrace{\Delta\mu}_{1} \underbrace{\ln(V/V_0)}_{2} - \kappa r \underbrace{\frac{\dot{r}}{V - \alpha \dot{r}}}_{3} \geq 0$$

cf. Ruina law (1983)

$$\frac{dr}{dt} \to \frac{dr}{dx} = \frac{\dot{r}}{V}$$

Improvements

1. partial steady state
2. logarithmic increment
3. factor of slip reduction

$\downarrow$

constitutive equations:
$$\begin{cases} \Delta\mu = l_1 \ln(V/V_0) - l_{12}\kappa r \\ \dot{r} = V \dfrac{l_{21}\ln(V/V_0) - l_2 \kappa r}{1 + \alpha\left(l_{21}\ln(V/V_0) - l_2 \kappa r\right)} \end{cases}$$

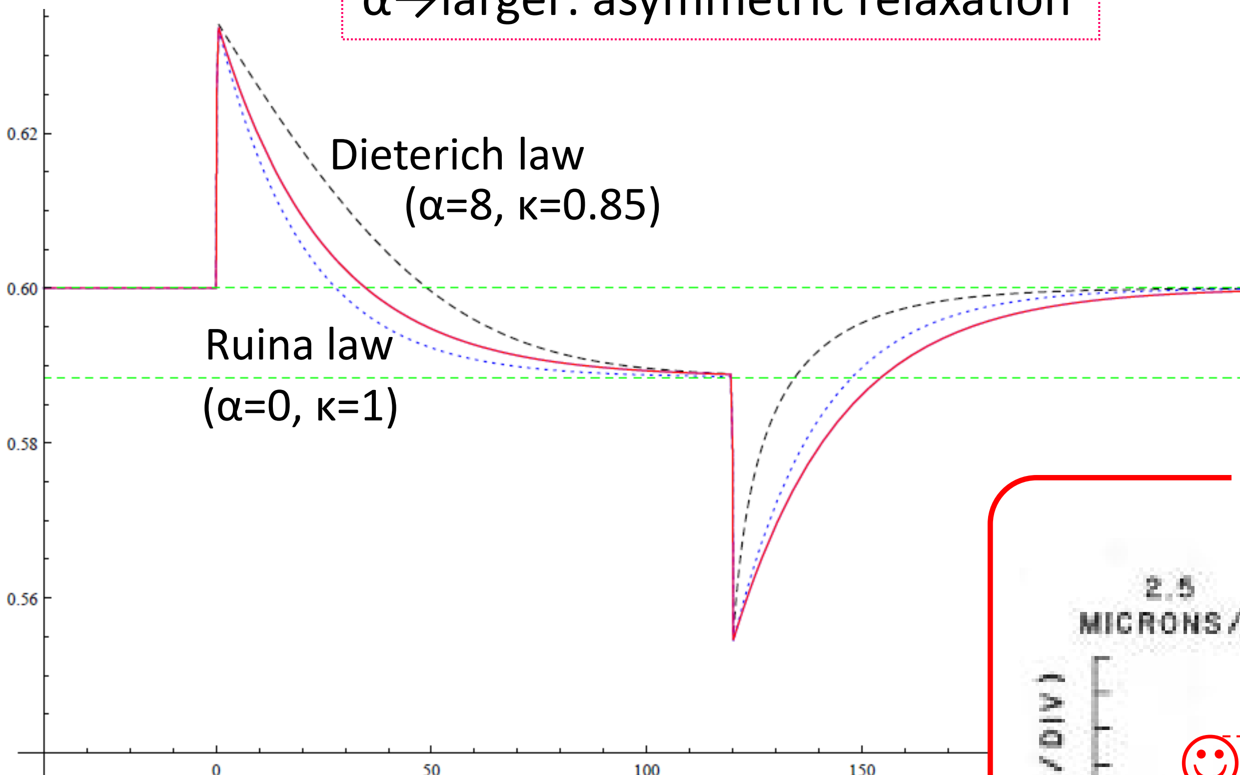
(Van, Mitsui, Hatano, 2015, arXiv:1501.04608v1)

Results

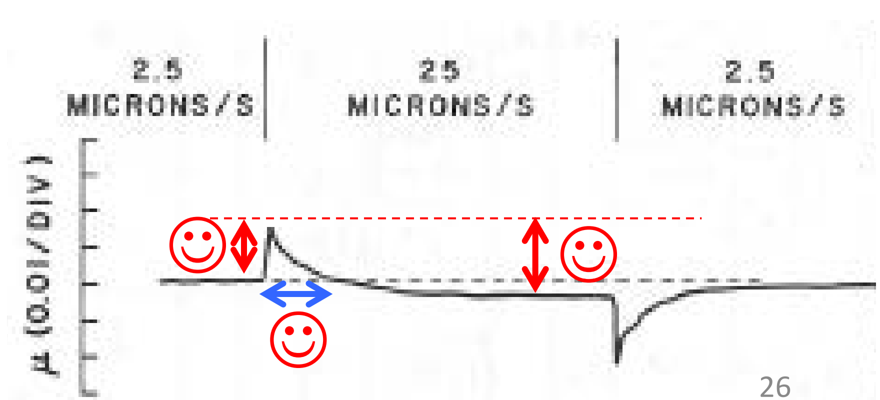
$$\begin{cases} \Delta\mu = l_1 \ln(V/V_0) - l_{12}\kappa r \\ \dot{r} = V \dfrac{l_{21}\ln(V/V_0) - l_2 \kappa r}{1 + \boxed{\alpha}\left(l_{21}\ln(V/V_0) - l_2 \kappa r\right)} \end{cases}$$

$$\left(\begin{array}{l} \alpha = 1, \kappa = 0.8 \\ l_1 = a = 0.015, l_{12} = b = 0.02 \\ l_{21} = l_2 = 1/L = 20\mu m \end{array} \right)$$

α→larger: asymmetric relaxation

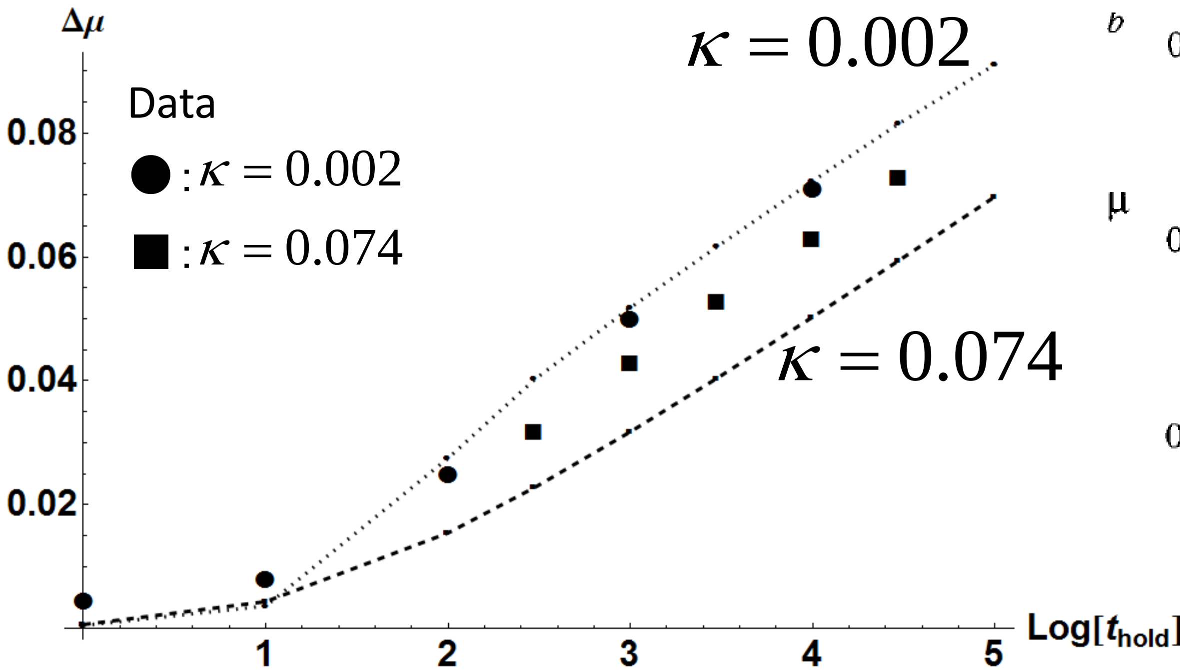


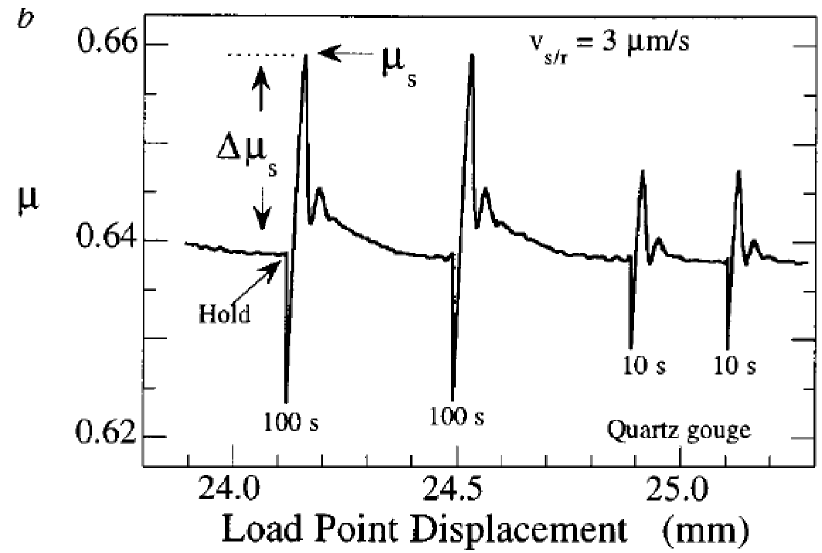
Data comparison



(Van, Mitsui, Hatano, 2015, arXiv:1501.04608v1)

Static friction





(Marone, 1998)

- log(t)-like increase in static friction $\Delta\mu$ is reproduced
- Both dynamic & static friction are reproduced with one set of constitutive equations

(Van, Mitsui, Hatano, 2015, arXiv:1501.04608v1)

Summary

- First motive: to understand mechanism of earthquakes

  ↓

- Current motive: deformation theory of medium under stress

object: static & dynamic friction of rocks

(quasi-thermostatic condition)

Results:

1. friction at reference velocity can be explained by using partial steady-state condition of internal energy
2. logV dependencies of friction are reproduced by the logarithmic increments of entropy production
3. symmetric relaxation can be explained by introducing a factor of slip reduction in elastic evolution
4. Above three elements reproduce log-t increase of static friction

Advantage:

simple model can be applied to geophysics with upscaling

Future plan: physical bases of assumptions in this model, etc.