v2 and Jet Fragmentation @LHC & LEP

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Outline

- ***Motivation***

  Multiplicity Fluctuations & Power-law Spectra

- ***Statistical Jet Fragmentation @ LEP & LHC***

  - Longitudinal and Transverse Structure of Jets,

  - Application in Parton Model Calculations

  - Scale evolution, Branching, LPHD?

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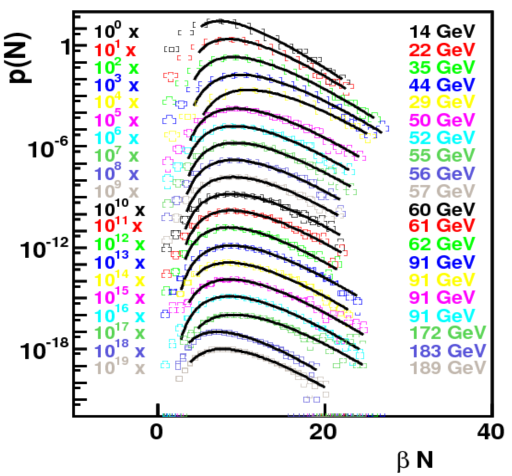
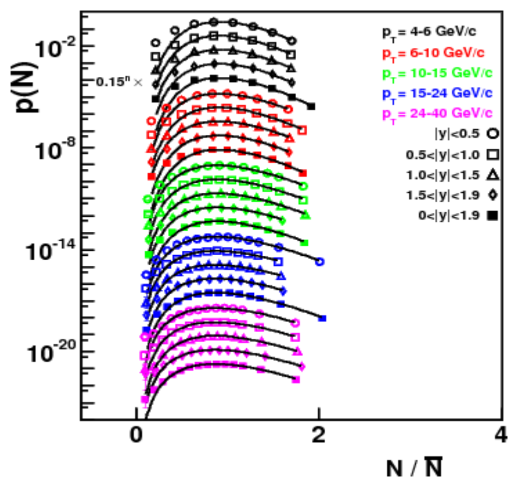
Motivation

Particle Multiplicity fluctuates according to the Gamma-distribution

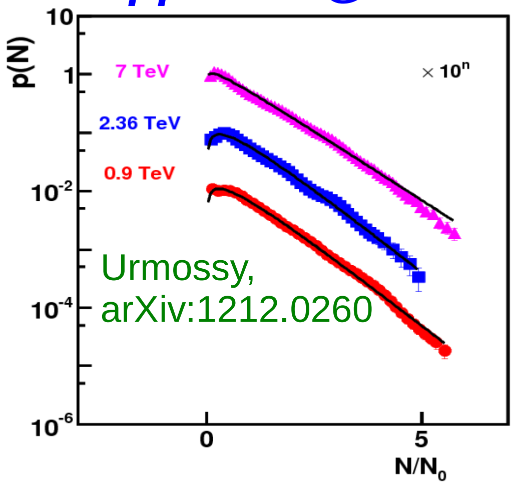
$e^-e^+ \rightarrow h^\pm$



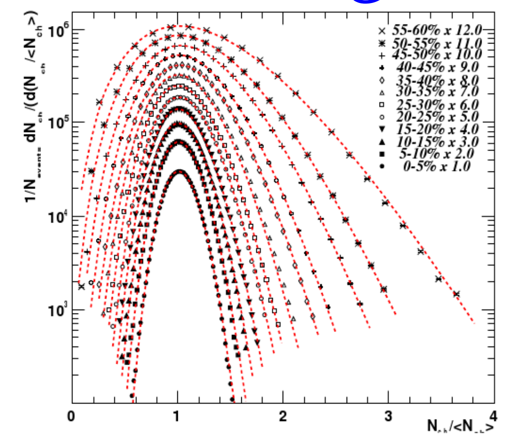
pp → jets @ 7 TeV



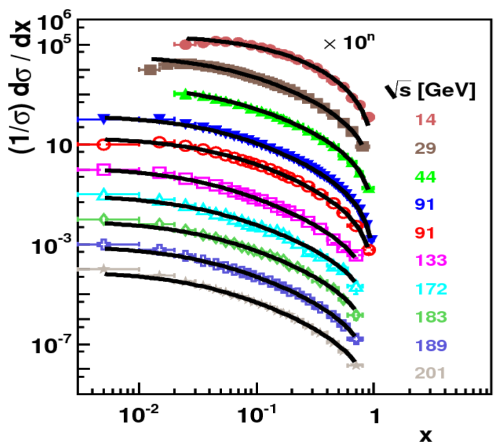
***pp → $h^\pm$* @ LHC**



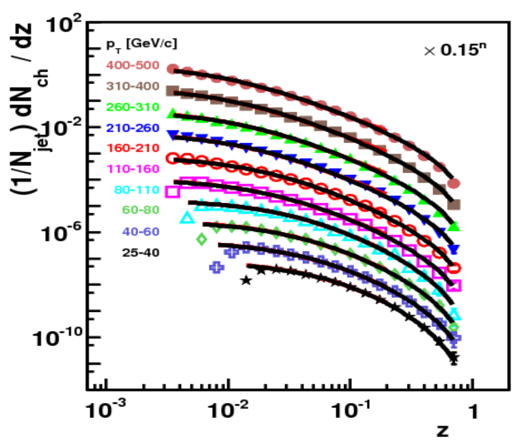
***AuAu → $h^\pm$* @RHIC**



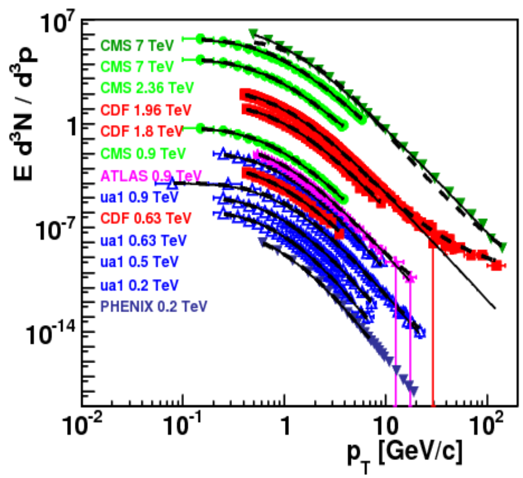
Power-law hadron spectra



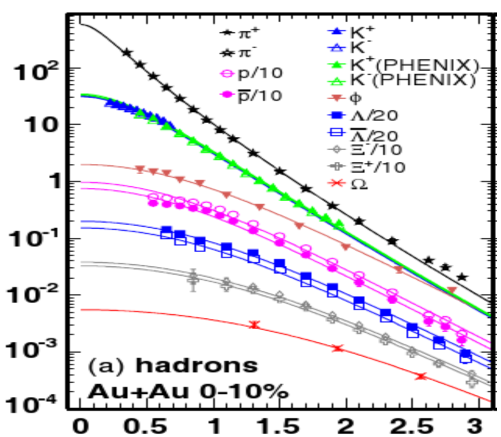
Urmossy et.al.,*PLB*, **701**: 111-116 (2011)



Urmossy et. al., *PLB*, **718**, 125-129, (2012)



Barnaföldi etal, *J. Phys.: Conf. Ser.*, **270,** 012008 (2011)



J. Phys. G: Nucl. Part. Phys. **37** 085104 (2010),

Statistical description & *n* fluctuations

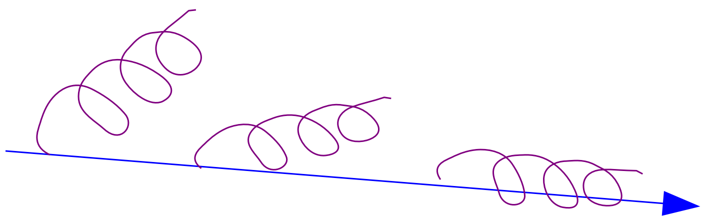
$$p^0 \frac{d\sigma}{d^3p}^{n=fix} \propto (1-x)^{n-3}, \quad x = \frac{p^0}{E}, \qquad P(n) = \binom{n+r-1}{r-1} \tilde{p}^n (1-\tilde{p})^r$$

Averaging over *n* fluctuations

$$p^0 \frac{d\sigma}{d^3p} = \sum_n P(n) n \left\{ p^0 \frac{d\sigma}{d^3p} \right\}^{n=fix} \sim \left(1 + \frac{\tilde{p}}{1-\tilde{p}} x \right)^{-r-3}$$

Motivationtion for such N distribution

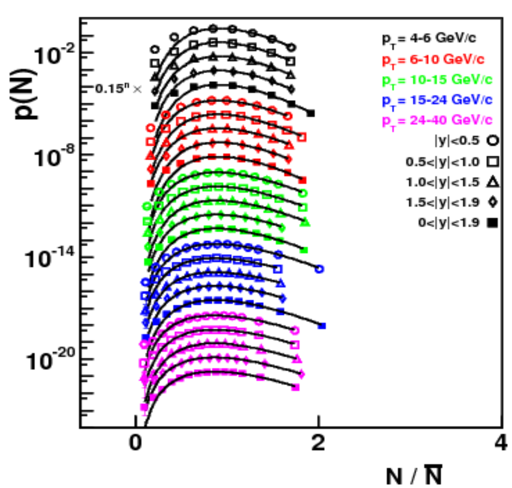
- **Small angle coherent radiation of n gauge bosons causes**



- scale evolution,
- large log-s and
- Poissonian n distribution

$$\propto \frac{\left(\alpha_s/2\pi\right)^n}{n!}\log^n\left(\frac{Q^2}{Q_0^2}\right) \propto \frac{\lambda^n}{n!}$$

- ***However, in jets, we see NBD / Γ(n) type n fluctuations:***



pp → jets @ 7 TeV

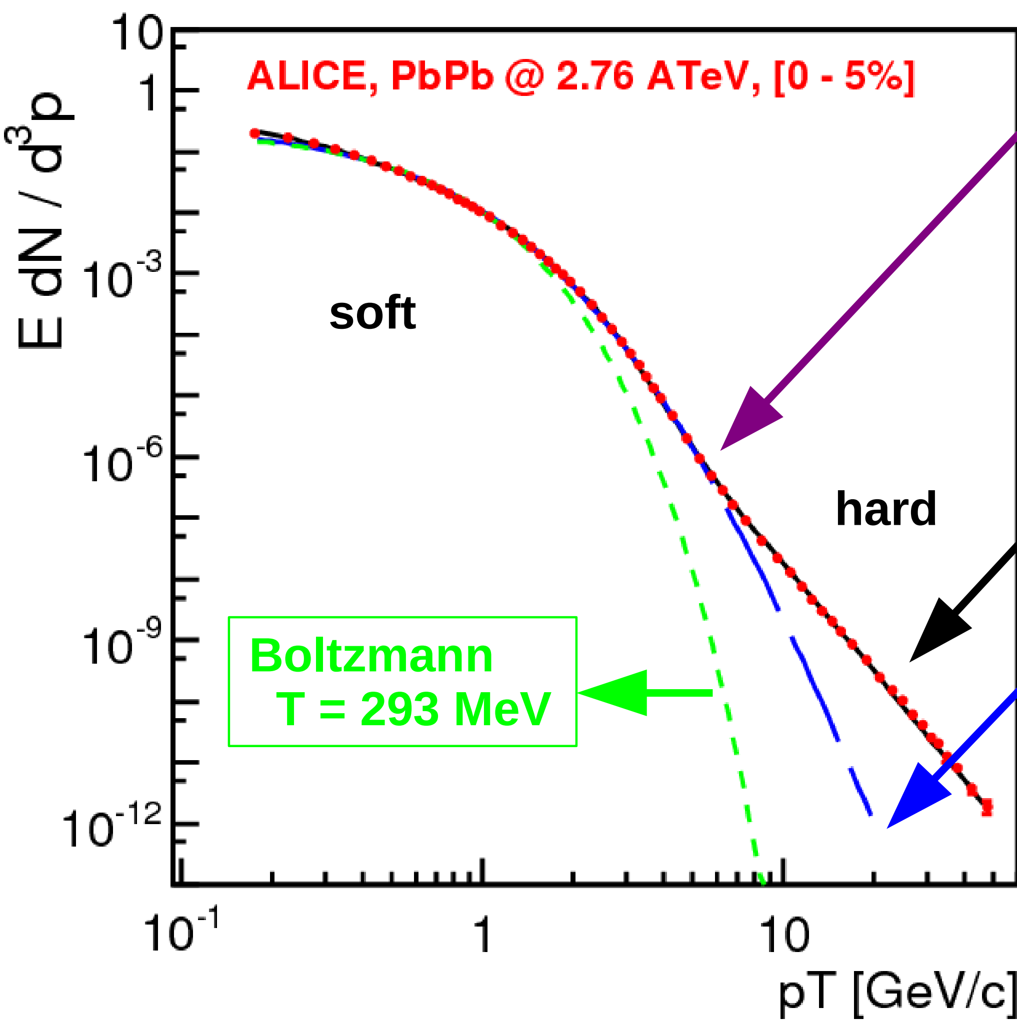
Urmossy et. al., *PLB*, **718**, 125-129, (2012)

- ***This may be resolved by:***

$$f(\lambda) \propto \left(\frac{\lambda}{\lambda_0}\right)^{b-1} e^{-b\lambda/\lambda_0}$$

$$\rightarrow \left\langle \frac{\lambda^n}{n!} e^{-\lambda} \right\rangle_\lambda \propto \left(\frac{n}{n_0}\right)^{a-1} e^{-an/n_0}$$

The power of the spectrum changes drastically at pT ~ 6 GeV/c.



$\sim \; p_T^{-6.08}$

Tsallis $\sim \; p_T^{-13.7}$

A ***hard + soft*** model:

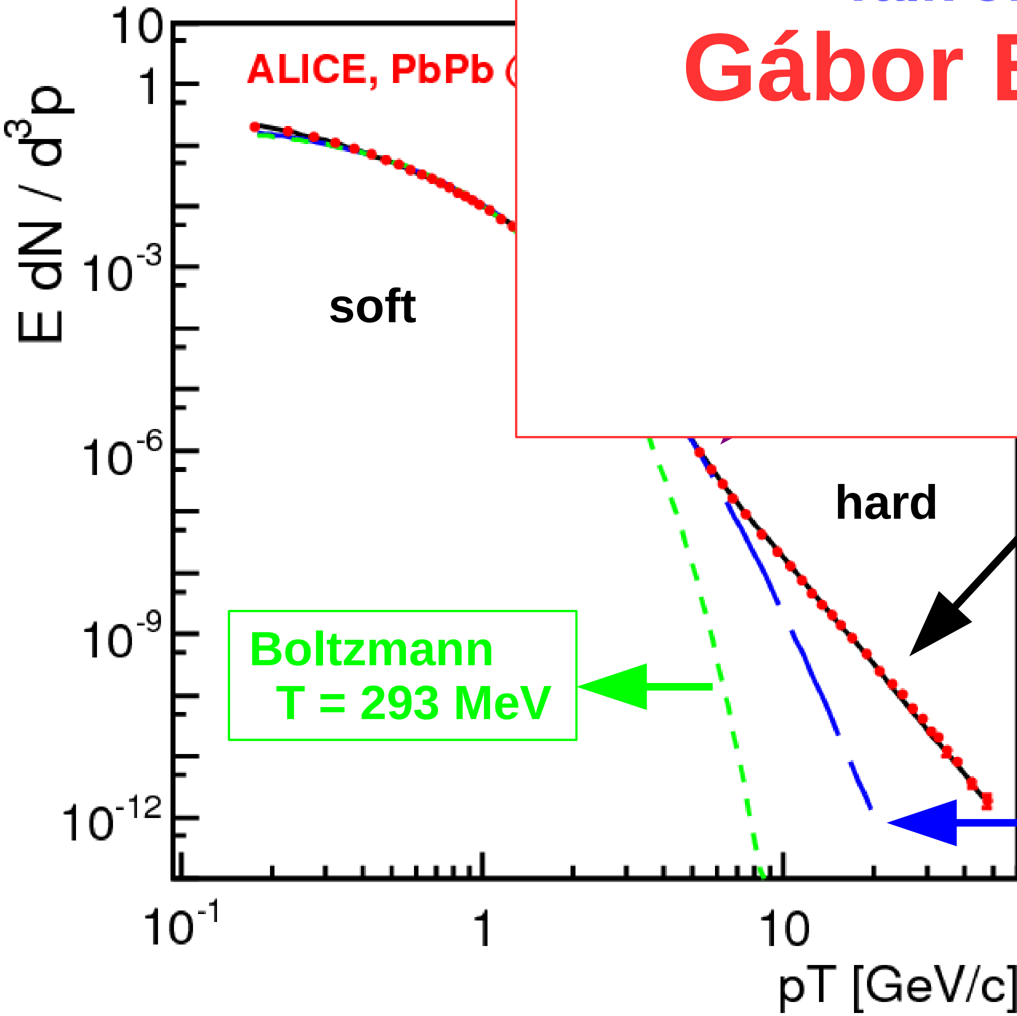
$$E\frac{dN}{d^3\boldsymbol{p}} = E\frac{dN}{d^3\boldsymbol{p}}^{hard} + E\frac{dN}{d^3\boldsymbol{p}}^{soft}$$

1.

The power o
lly at pT ~ 6 GeV/c.

Talk of

Gábor Bíró



$\sim \quad p_T^{-6.08}$

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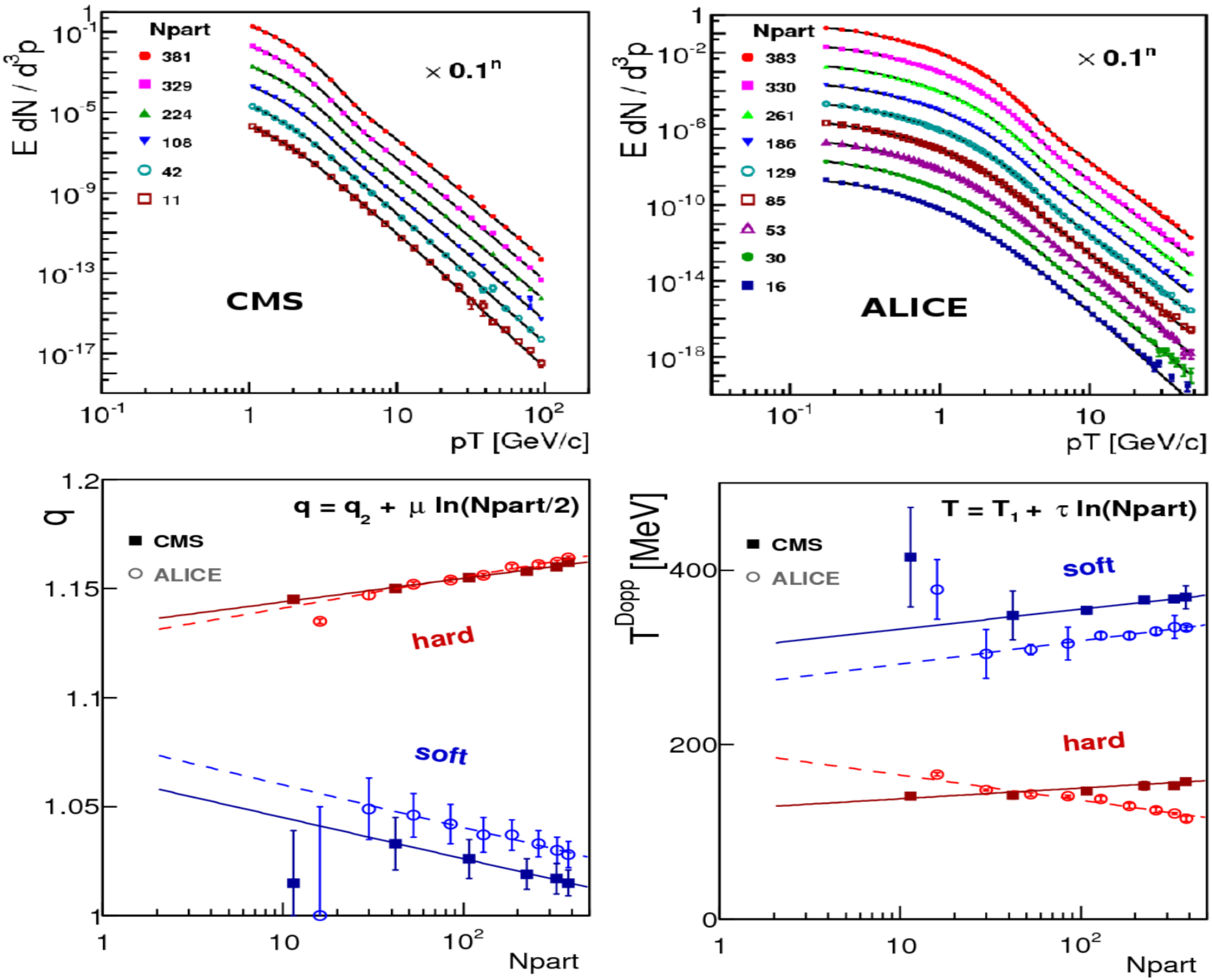
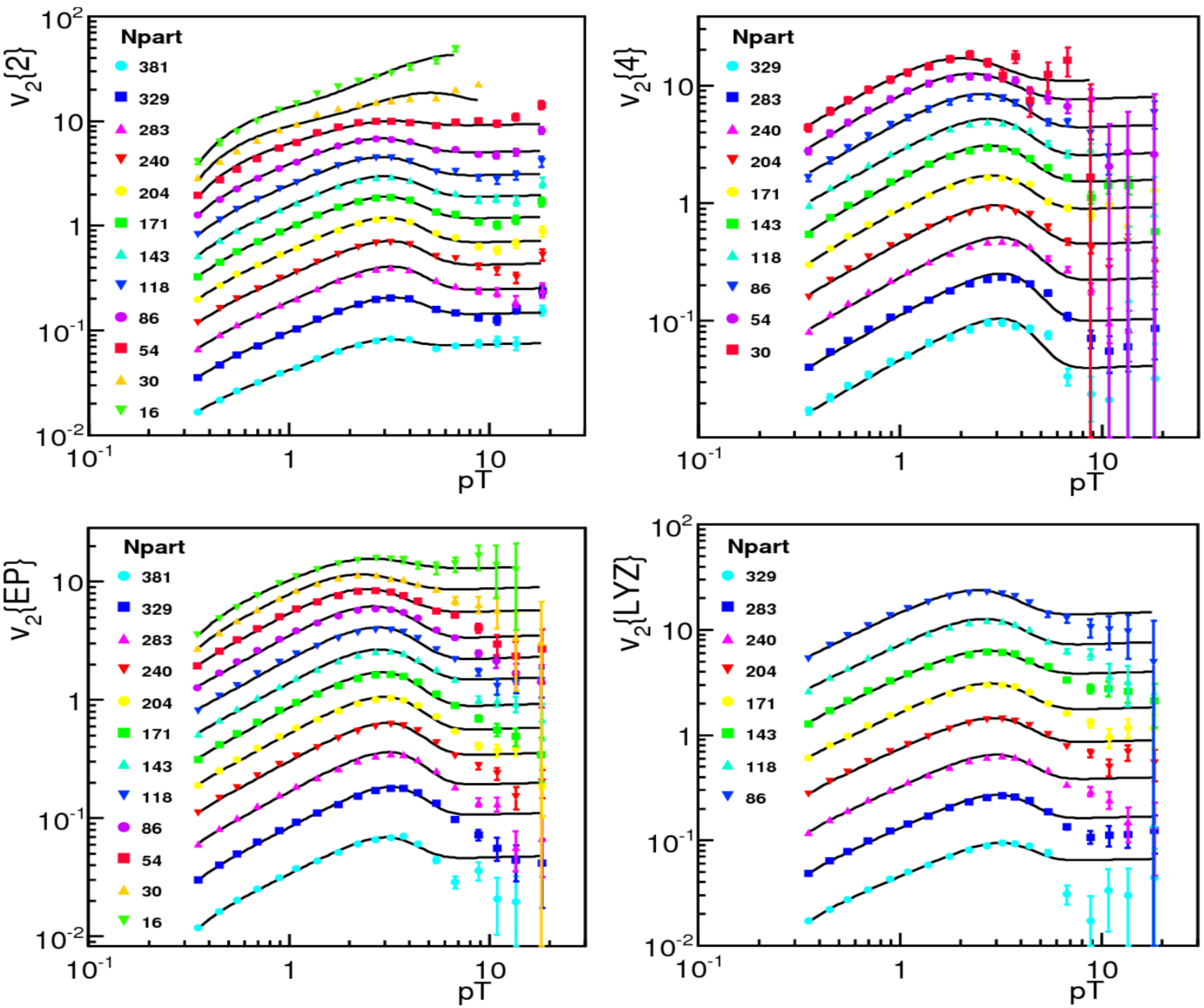


PbPb → $h^{\pm}$

CMS

arXiv:1405.3963



v2 of $h^{\pm}$

arXiv:1405.3963

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Jet-fragmentation in e^+e^- & pp collisions

The cross-section of the creation of hadrons $h_1, \dots, h_N$ in a jet of N hadrons

$$d\sigma^{h_1,\dots,h_N} = |M|^2 \delta^{(4)}\left(\sum_i p^{\mu}_{h_i} - P^{\mu}_{tot}\right) d\Omega_{h_1,\dots,h_N}$$

- **If the jet is very narrow, hadrons fly *nearly co-linearly* (*quasi 1 Dim*).**
- ***If* we neglect hadron masses ($m_i = 0$), the conservation of P_μ reduces to the conservation of E**
- **If $|M| \approx$ *constans*, *we arrive at a 1D microcanonical ensemble*:**

$$d\sigma^{h_1,\dots,h_N} \propto \delta\left(\sum_i \epsilon_{h_i} - E_{tot}\right) d\Omega_{h_1,\dots,h_N} \propto E_{tot}^{N-1}$$

- U.K. et.al., *Phys. Lett. B*, **718**, 125-129, (2012)
- U.K. et.al., *Phys.Lett.B*, **701,** 111-116 (2011)
- U.K. et.al., *Acta Physica Polonica B*, **5**:(2), pp. 363-368, (2012)
- T.S. Biró et.al., *Acta Physica Polonica B*, **43**:(4), pp. 811-820, (2012)

Jet-fragmentation in e^+e^- & *pp* collisions

Thus, the *energy distribution* of hadrons in a jet of *N* (massless) hadrons is:

$$f_N(z) = A_N(1-z)^{N-2}, \quad z = \epsilon_h / E_{jet}$$

- **The *hadron multiplicity* in a jet *fluctuates* as *(exp. observation):***

$$p(N) \propto (N-N_0)^{\alpha-1} e^{-\beta(N-N_0)}$$ ***or NBD***

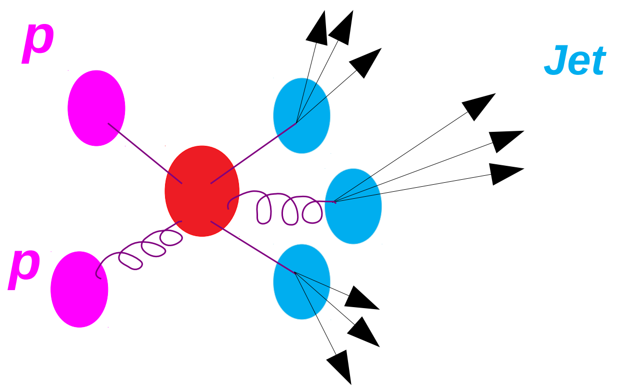
- **Thisway, the *multiplicy averaged hadron distribution* (*fragmentation function*):**

$$\frac{d\sigma}{dz} = \sum_{N=N_0}^{\infty} f_N(z) N\, p(N) \sim \frac{(1-z)^{\nu(N_0)}}{\left(1-\frac{(q-1)}{T/E_{jet}}\ln(1-z)\right)^{1/(q-1)}}$$

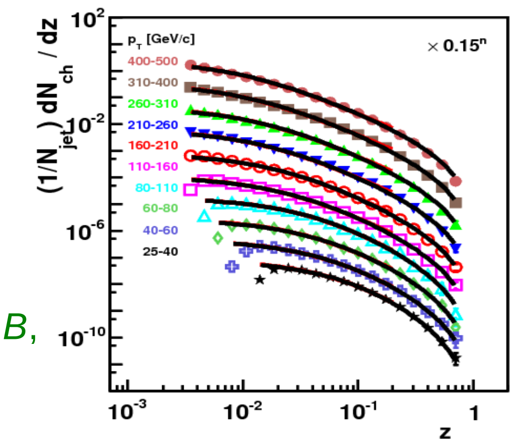
$$q=1+1/(\alpha+2), \quad T=E_{jet}\beta/(\alpha+2), \quad \nu=N_0-2$$

Confrontation with measurements

pp → jets @LHC (pT = 25–500 GeV/c)

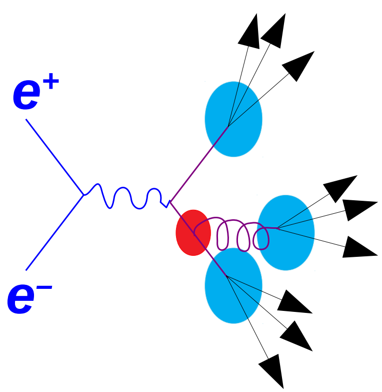


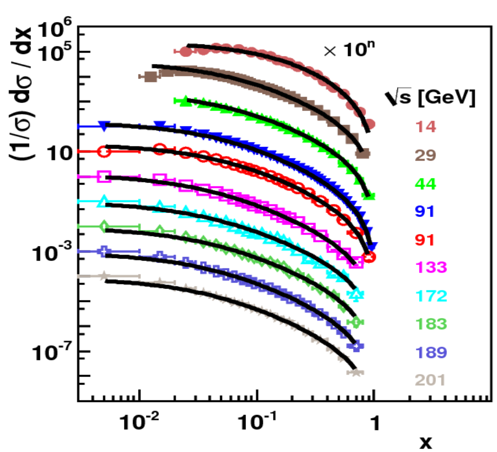
$$\frac{dN}{dz} \propto \left[1 - a\ln\left(1-z\right)\right]^{-b}$$



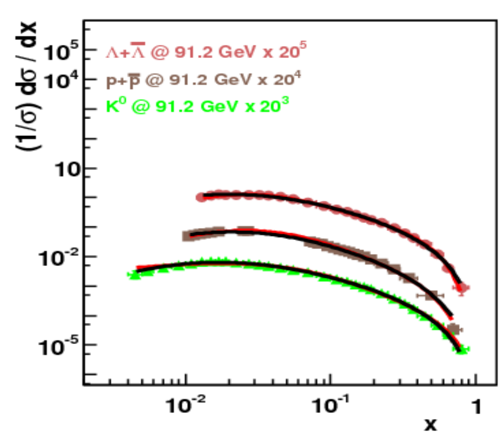
Urmossy et.al. *Phys. Lett. B*, **718**, 125-129, (2012)

e$^+$e$^-$ annihilation @LEP (√s = 14–200 GeV)

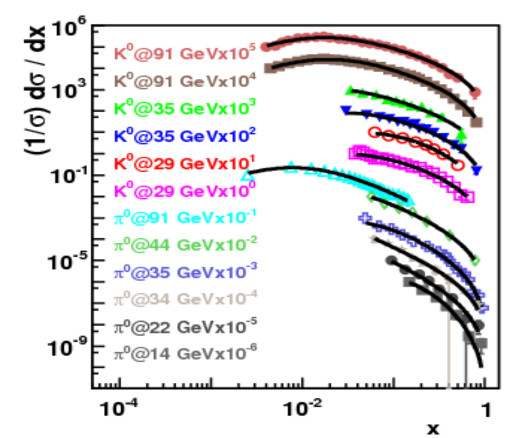




Urmossy et. al., *Phys. Lett. B*, **701**, 111-116 (2011)



Urmossy et.al., *Acta Phys. Polon. Supp.* **5** (2012) 363-368



T. S. Biró et.al., *Acta Phys. Polon. B*, **43** (2012) 811-820

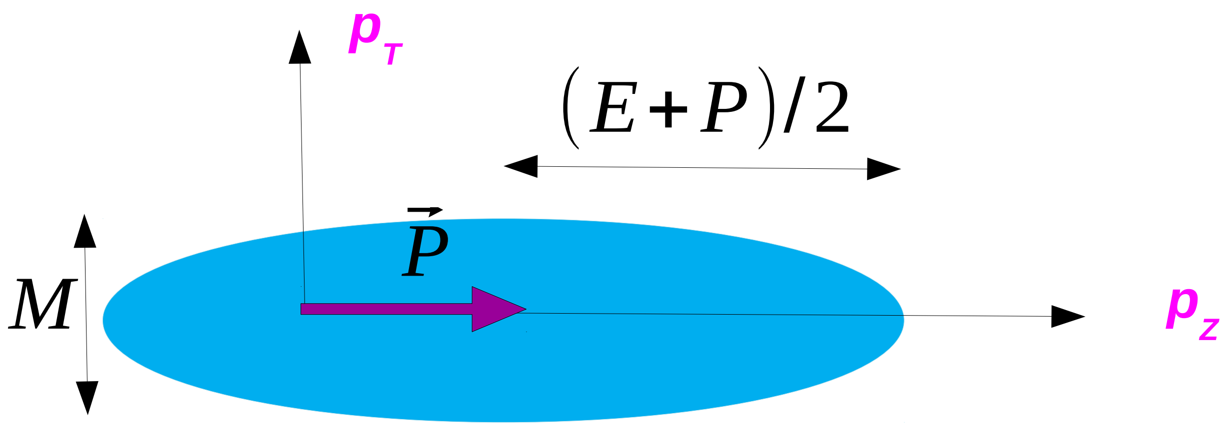
Extension to 3D

To obtain the transverse structure of jets of momentum *P*

$$d\sigma^{h_1,\dots,h_n} \sim \delta\left(\sum_i p^\mu_{h_i} - P^\mu_{tot}\right) d\Omega_{h_1,\dots,h_n} \propto \left(P_\mu P^\mu\right)^{n-2} = M^{2n-4}$$

Thus, the haron distribution in a jet of *n* hadron is

$$p^0 \frac{d\sigma}{d^3 p}^{n=fix} \propto \left(1-x\right)^{n-3}, \qquad x = \frac{P_\mu p^\mu}{M^2/2}$$



Averaging over *n* fluctuations

$$p^0 \frac{d\sigma}{d^3p} = A\left\{\left(1+\frac{\tilde{p}}{1-\tilde{p}}x\right)^{-r-3} - \sum_{3}^{n_0-1} P(n)\, n\, f_n(x)\right\}, \qquad P(n) = \binom{n+r-1}{r-1}\tilde{p}^n(1-\tilde{p})^r$$

pp → jets @LHC (pT = 400–500 GeV/c)

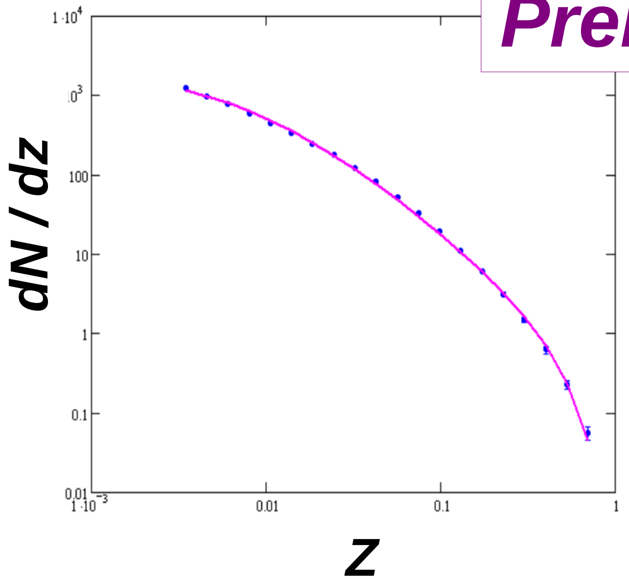
Fitted parameters

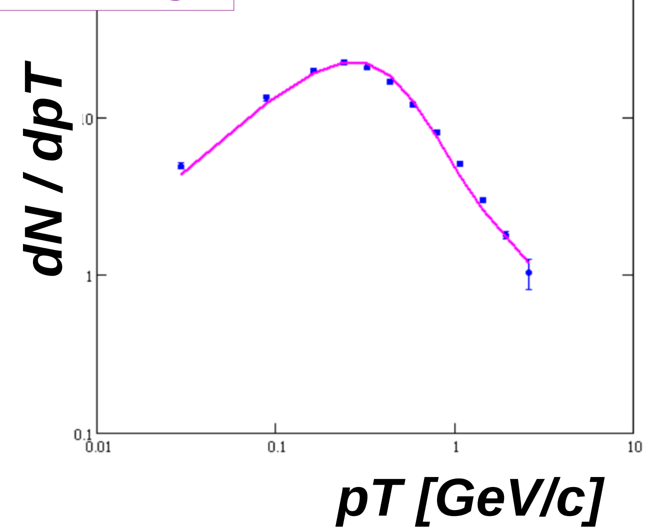
q = 1.536

T = 4.543*10^{-3}

M = 112.349

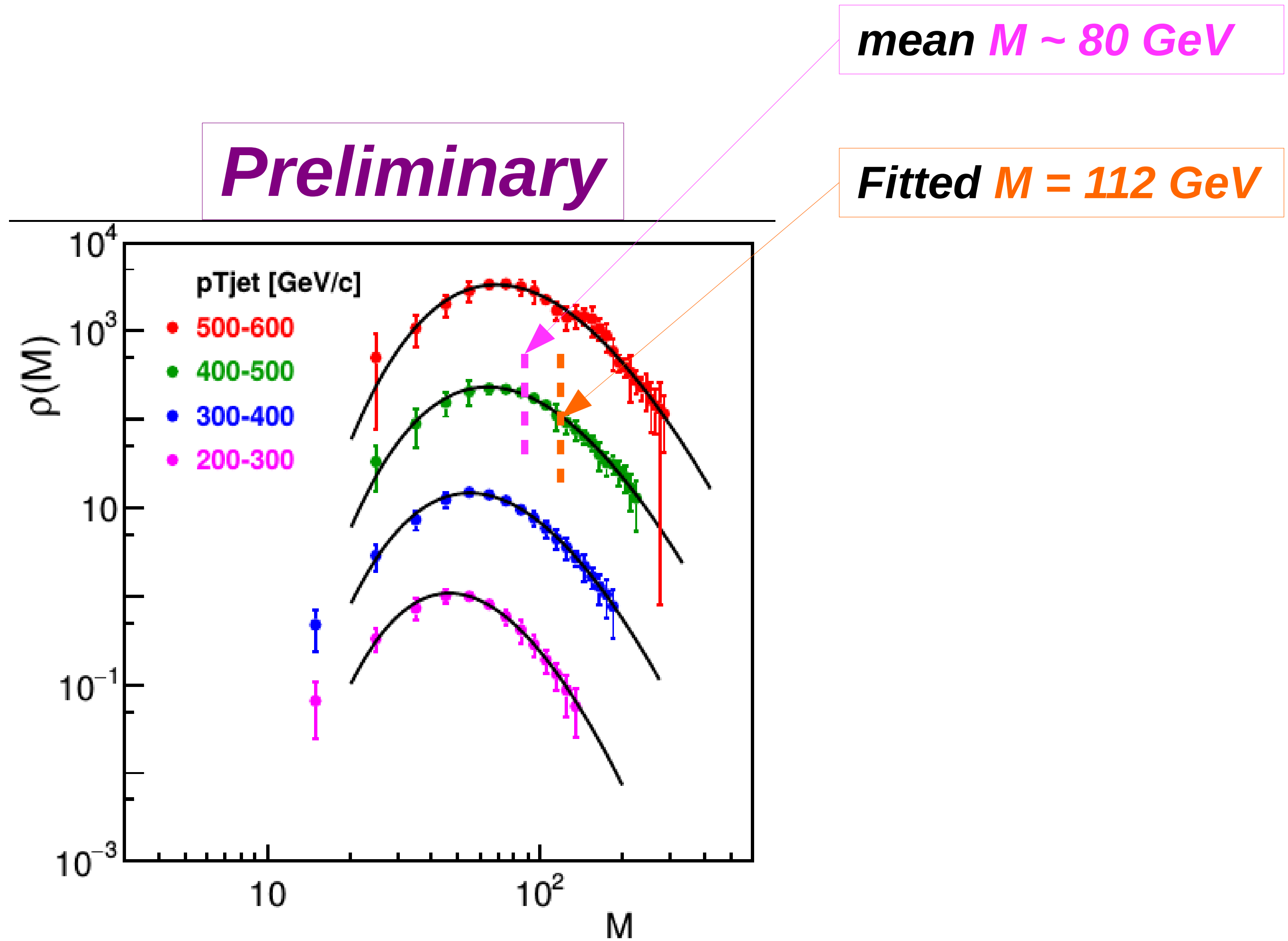
Preliminary





The jet mass distribution in pp @ LHC

$$\rho(M) \propto \frac{\ln^b(M)}{M^c}$$



Data: ATLAS, *JHEP* 1205 (2012) 128

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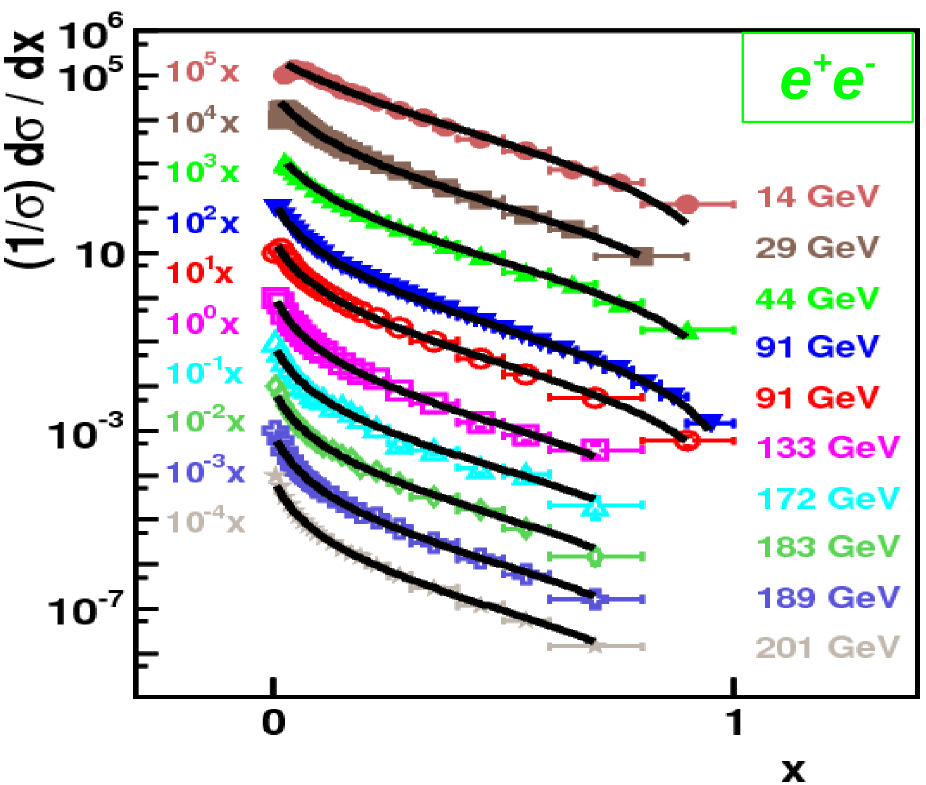
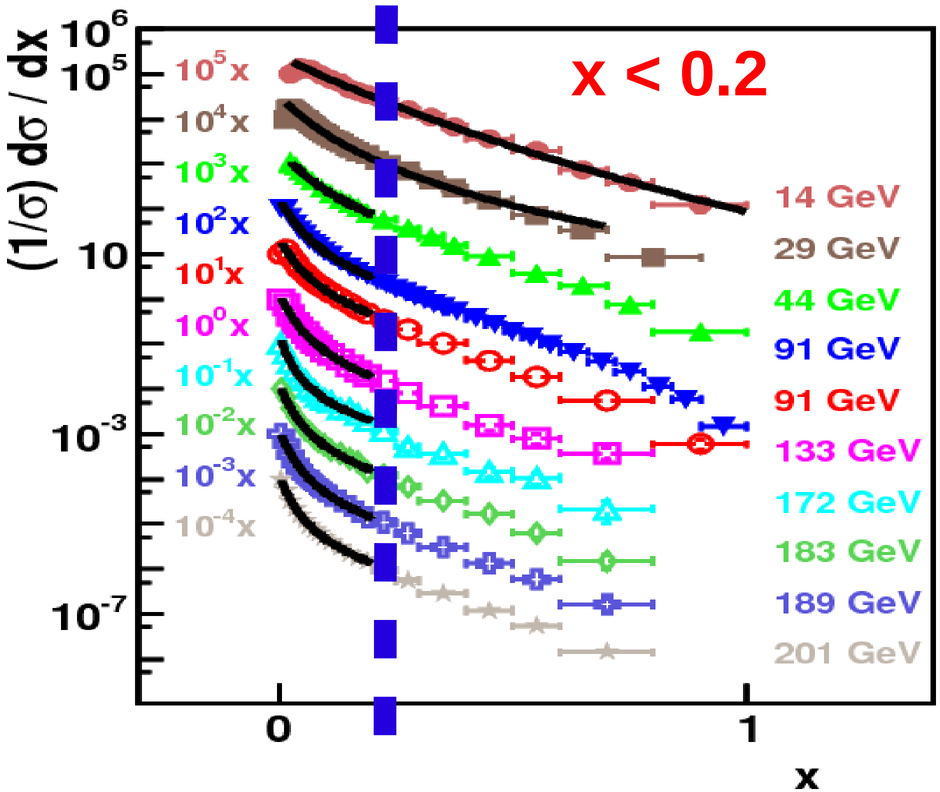
$x \ll 1$ limit: microcanonical → canonical Tsallis distribution



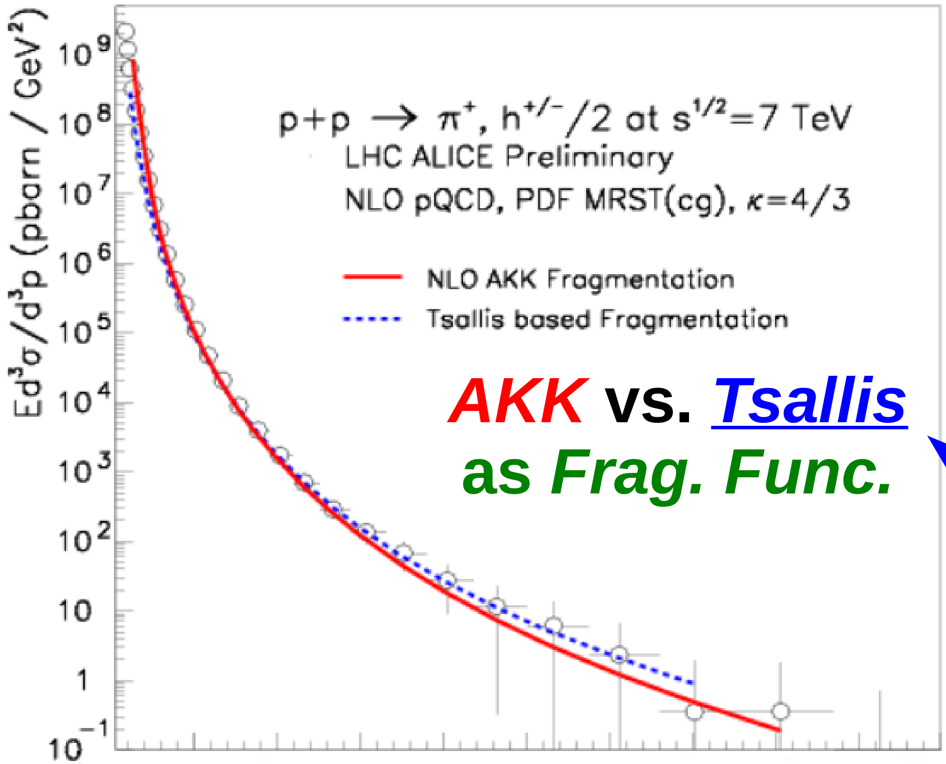


$$\frac{d\sigma}{dx} \propto \left(1-\frac{q-1}{T/(\sqrt{s}/2)}\ln(1-x)\right)^{-1/(q-1)} \quad \Rightarrow \quad \left(1+\frac{q-1}{T/(\sqrt{s}/2)}x\right)^{-1/(q-1)}$$

Urmossy et.al., *Phys.Lett.B*, **701,** 111-116 (2011), arXiv:1101.3023

Application in a pQCD calculation

π^+ spectrum in pp --> $\pi^\pm$X @ √s=7 TeV (NLO pQCD)



AKK vs. Tsallis
as Frag. Func.

$$D_{p_i}^{\pi^+}(z) \sim \left(1+(q_i-1)\,z/T_i\right)^{-1/(q_i-1)}$$

Barnaföldi et. al., *Proceedings of the Workshop Gribov '80* (2010)

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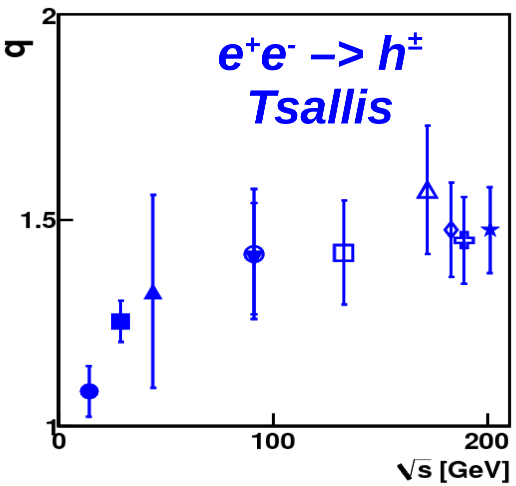
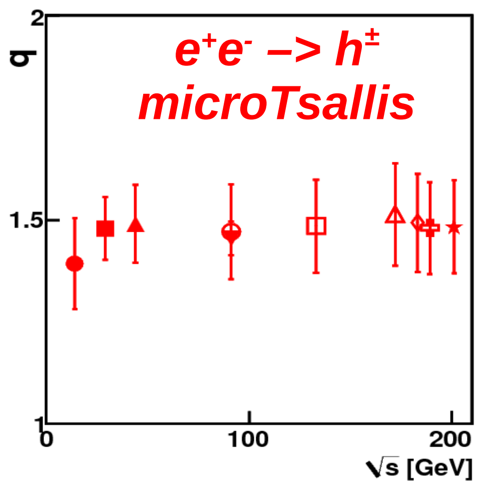
Scale Evolution

Fitts:

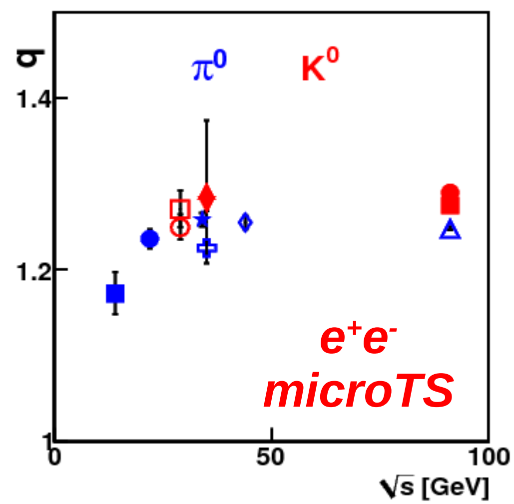
1)



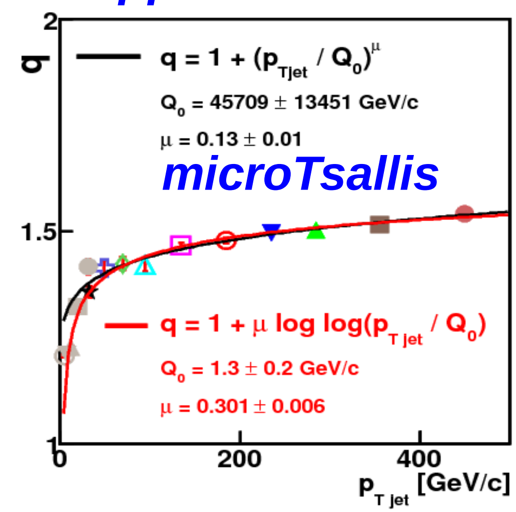
2)



3)

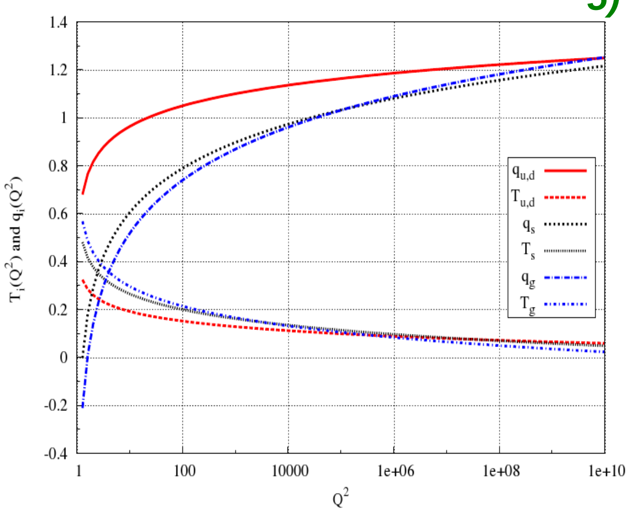


pp –> Jet –> h± 4)



Theory: Scale evolution of *q*, *T* from fits to AKK Frag. Funcs:

5)



$$D^{\pi^+}_{p_i}(z) \;\sim\; \left(1+(q_i-1)\,z/T_i\right)^{-1/(q_i-1)}$$

$$q_i \;=\; q_{0i}+q_{1i}\ln\left(\ln\left(Q^2\right)\right)$$

1-2) U.K. etal., *Phys.Lett. B*, **701** (2011) 111-116

3) T. S. Biró etal., *Acta Phys.Polon. B*, **43** (2012) 811-820

4) U.K. etal., *Phys.Lett. B*, **718** (2012) 125-129

5) Barnaföldi etal., *Gribov-80 Conf*: C10-05-26.1, p.357-363

Scale Evolution

Fitts:

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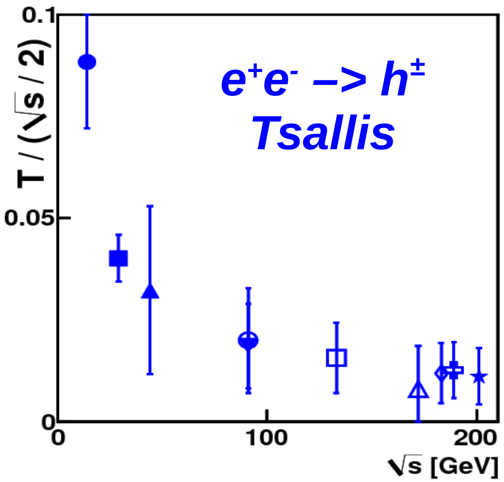

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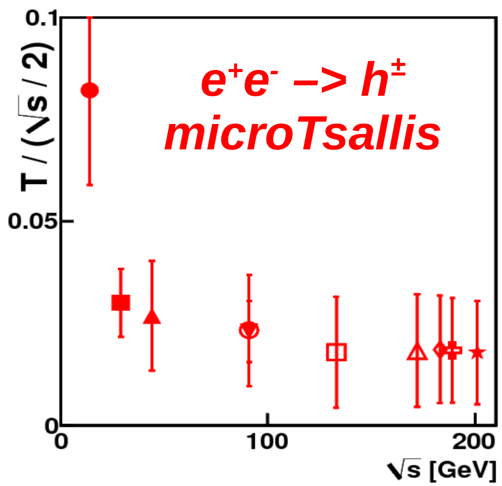

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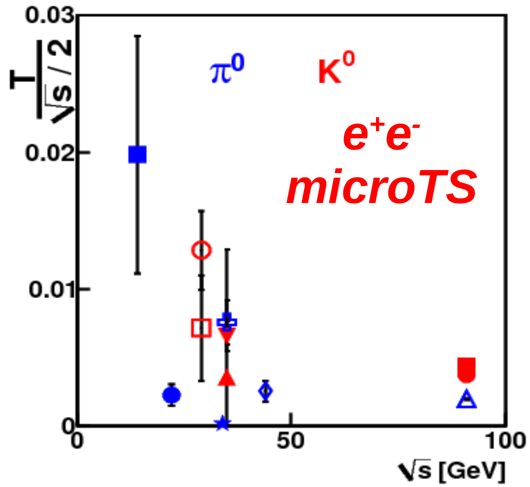

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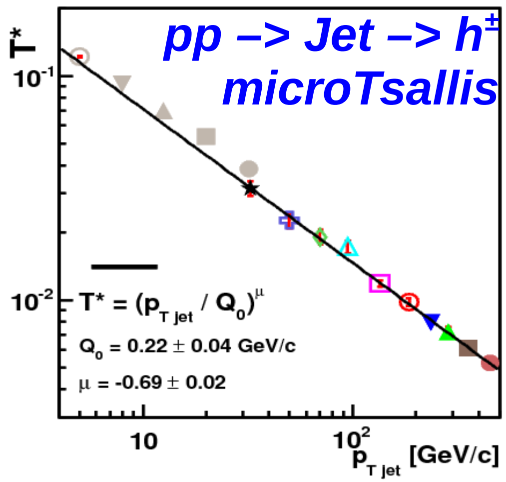

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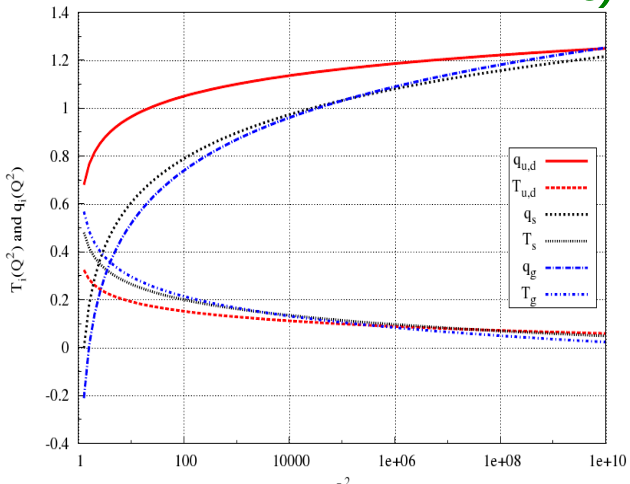


$$D_{p_i}^{\pi^+}(z) \sim \left(1+(q_i-1)\,z/T_i\right)^{-1/(q_i-1)}$$

$$T_i = T_{0i} + T_{1i}\ln\left(\ln(Q^2)\right)$$

1-2) U.K. etal., *Phys.Lett. B*, **701** (2011) 111-116

3) T. S. Biró etal., *Acta Phys.Polon. B*, **43** (2012) 811-820

4) U.K. etal., *Phys.Lett. B*, **718** (2012) 125-129

5) Barnaföldi etal., *Gribov-80 Conf*: C10-05-26.1, p.357-363

Branching in the Φ^3 Theory

Resummation of branchings with DGLAP

$$\frac{d}{dt} f(x,t) = g^2(t) \int_x^1 \frac{dz}{z} P(z) f(x/z,t), \qquad t = \ln(Q^2/Q_0^2)$$

$$P(z) = z(1-z) - \frac{1}{12}\delta(1-z)$$

The branching *starts* with: $f(x) = \delta(1-x)$

Thus, the *distribution of partons* in a jet, *prior to hadronisation* is

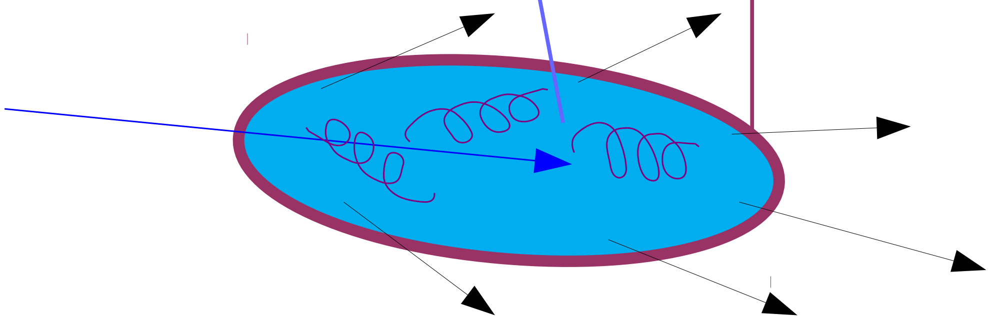
$$f(x) \sim \delta(1-x) + \sum_{k=1}^{\infty} \frac{b^k}{k!(k-1)!} \sum_{j=0}^{k-1} \frac{(k-1+j)!}{j!(k-1-j)!} x \ln^{k-1-j}\left[\frac{1}{x}\right]\left[(-1)^j + (-1)^k x\right]$$

$$b = \beta_0^{-1} \ln\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right)$$

Hadronisation of the Jet

The *hadron distribution*:

$$\frac{dN}{dx} = \int_x^1 \frac{dz}{z} f(z,t)\, d(x/z,t')$$



Used *hadronisation functions*:

$$d(x) = n(n-1)(1-x)^{n-2}$$

$$d(x) = \left(1+\frac{q-1}{T}x\right)^{-1/(q-1)}$$

Conclusion

Jet-fragmentation might be statistical?

It could be checked by measuring the dN/dz fragmentation function in jets of fix multipliciy.

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It could be checked by measuring the dN/dz fragmentation function in jets of fix multiplicity.

What causes the transition of Poissonian multiplicity distributions at the parton level in jets to NBD/Gamma-type multiplicity distributions observed at the hadronic level?

Back-up Slides........................

Transverse spectra in pp

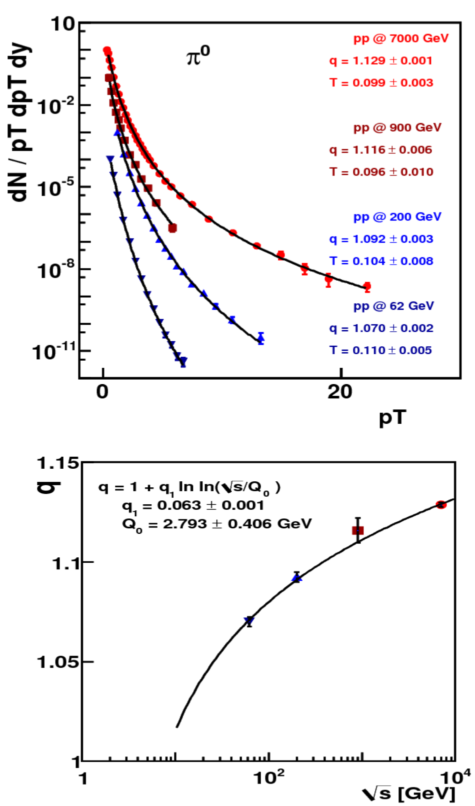
Hadron spectra in pp collisions can be described by the Tsallis distribution:

$$\frac{dN}{d^3\boldsymbol{p}} \propto \left[1 + \frac{q-1}{T}(m_T - m)\right]^{-1/(q-1)} .$$

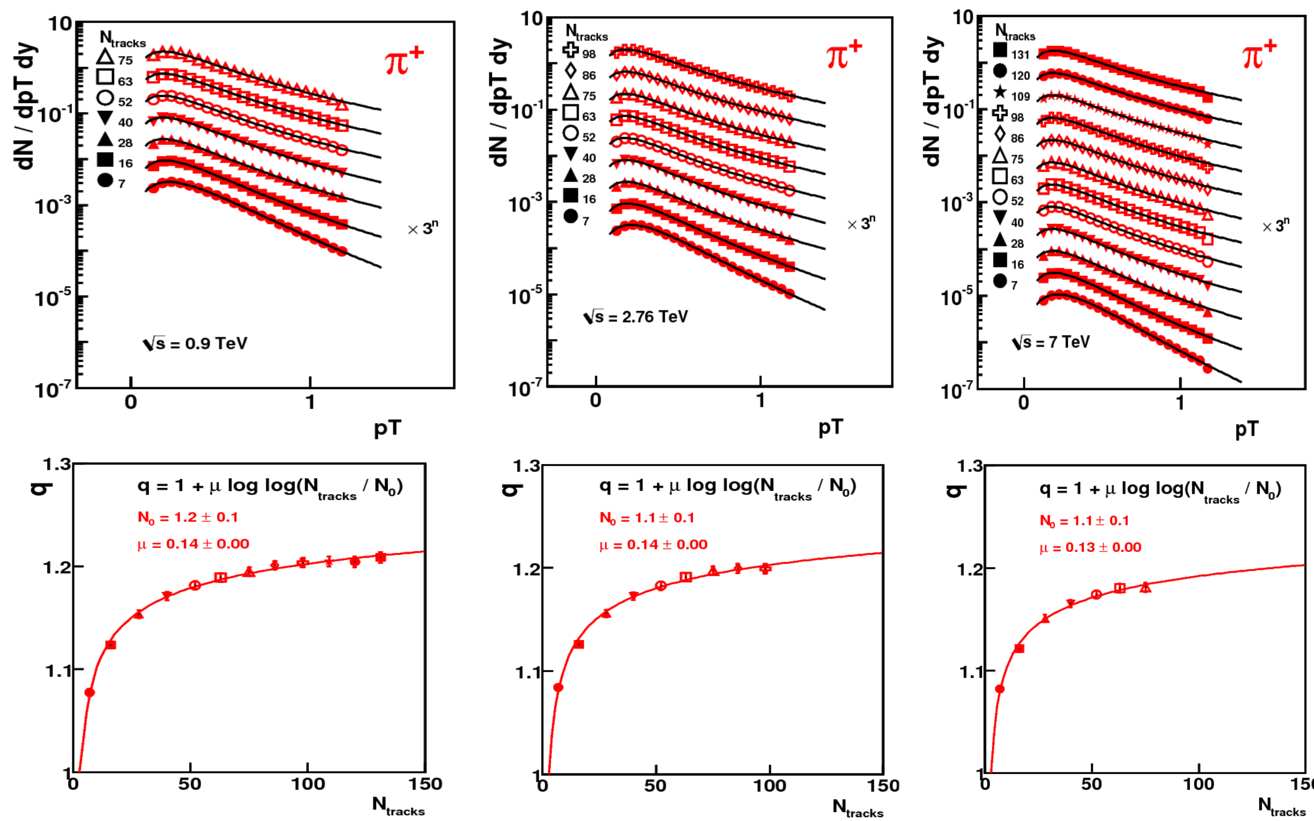
π spectra in pp collisions depends similarly on √s and on the multiplicity N

$$q(s) = 1 + q_1 \ln\ln(\sqrt{s}/Q_0) ,$$
$$q(N) = 1 + \mu \ln\ln(N/N_0) .$$

√s = fix



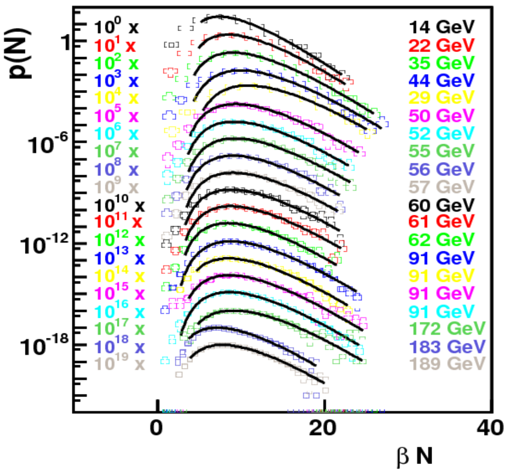
N = fix



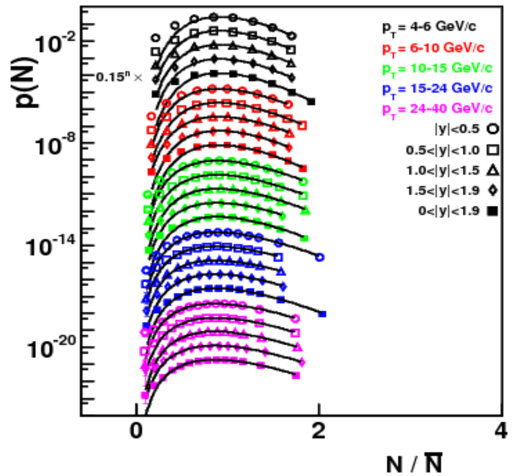
Motivation

Particle Multiplicity fluctuates according to the Gamma-distribution

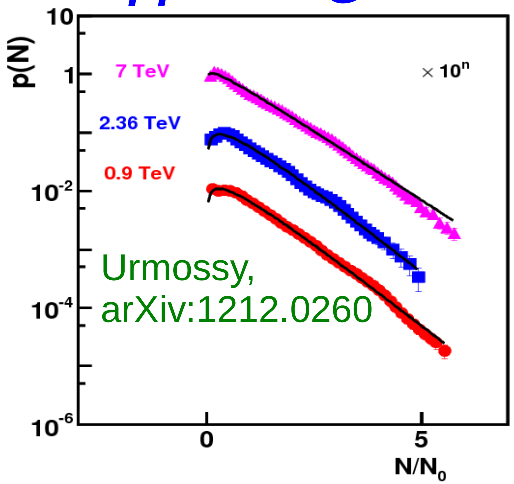
$e^-e^+ \to h^\pm$



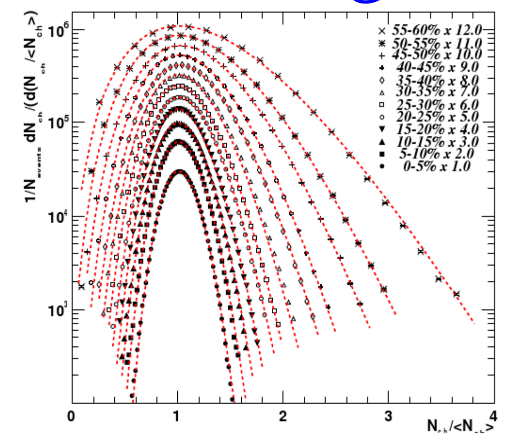
pp → jets @ 7 TeV



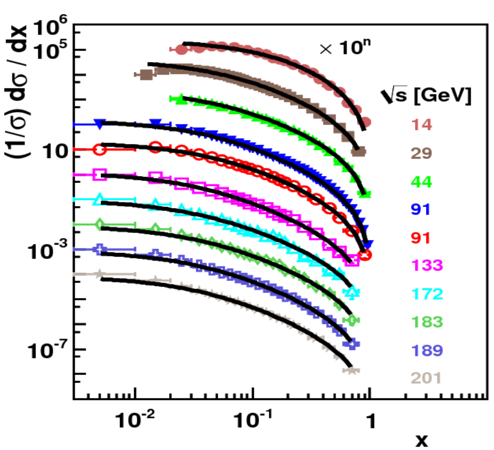
pp → h± @ LHC



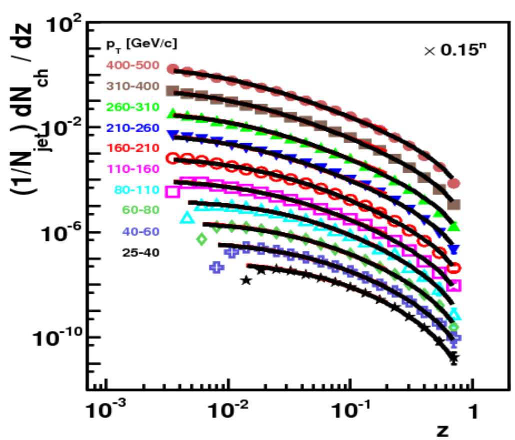
AuAu → h± @RHIC



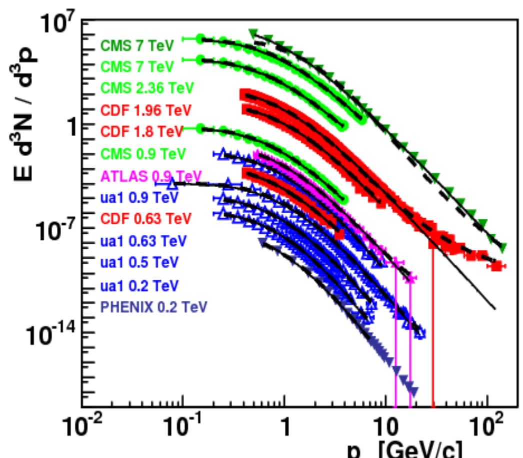
Power-law hadron spectra



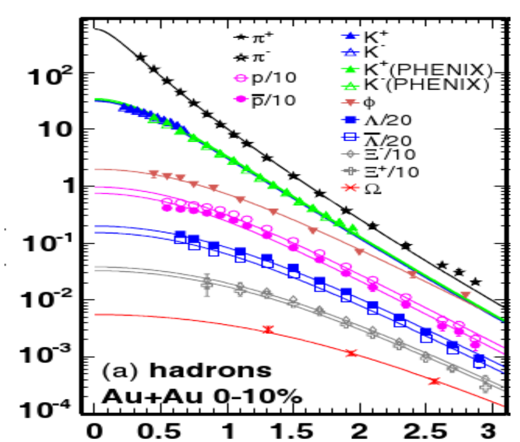
Urmossy et.al.,*PLB*, **701**: 111-116 (2011)



Urmossy et. al., *PLB*, **718**, 125-129, (2012)



Barnaföldi etal, *J. Phys.: Conf. Ser.*, **270,** 012008 (2011)



J. Phys. G: Nucl. Part. Phys. **37** 085104 (2010),

***KNO – scaling* (Koba-Nielsen-Olesen) of Multiplicity distributions :**

$$p(N) = \frac{1}{\langle N(s)\rangle}\Psi\left(\frac{N-N_0}{\langle N(s)\rangle}\right)$$

- A. Rényi, Foundations of Probability, Holden-Day (1970).
- A. M. Polyakov, Zh. Eksp. Teor. Fiz. 59, 542 (1970).
- Z. Koba, H. B. Nielsen, P. Olesen, Nucl. Phys. B 40, 317 (1972).
- S. Hegyi, Phys. Lett. B: 467, 126-131, 1999.
- S. Hegyi, Proc. ISMD 2000, Tihany, Lake Balaton, Hungary, 2000
- Yu.L. Dokshitzer, Phys. Lett. B, 305, 295 (1993); LU-TP/93-3 (1993).

A functional form that is consistent with experiments:

$$p(N) \propto (N-N_0)^{\alpha-1} e^{-\beta(N-N_0)}$$

The average hadron distributions are

$$\frac{d\sigma}{d^D x} = \sum f_N(x) N\, p(N)$$

(1) Statistical description of hadron spectra:

$$E\frac{dN}{d^3\mathbf{p}} = \sum_{sources} f\left[u_\mu p^\mu\right]$$

(2) Space-time dependence only through $u_\mu(x)$ Bjorken + Blast Wave

$$u_\mu = \left(\gamma\cosh\zeta, \gamma\sinh\zeta, \gamma v\cos\alpha, \gamma v\sin\alpha\right), \qquad \zeta = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$$

$$v(\alpha) = v_0 + \sum_1^N \delta v_m \cos(m\alpha)$$

Then, the spectrum and the v2 are

$$\left.\frac{dN}{p_T dp_T dy}\right|_{y=0} \propto f\left[E(v_0)\right] + O\left(\delta v^2\right)$$

$$v_2 \propto \delta v_2 \left(v_0 m_T - p_T\right)\frac{f'\left[E(v_0)\right]}{f\left[E(v_0)\right]} + O\left(\delta v^2\right) \qquad E(v_0) = \gamma_0\left(m_T - v_0 p_T\right)$$

Then, the spectrum and the v2 are

$$\frac{dN}{p_T\,dp_T\,dy}_{y=0} \propto f\left[E(v_0)\right] + O(\delta v^2) \qquad E(v_0) = \gamma_0 (m_T - v_0 p_T)$$

$$v_2 \propto \delta v_2 (v_0 m_T - p_T) \frac{f'\left[E(v_0)\right]}{f\left[E(v_0)\right]} + O(\delta v^2)$$

Boltzmann-distribution:

$$f\left[E(v_0)\right] \propto e^{-E(v_0)/T}$$

$$v_2 \propto p_T - v_0 m_T$$

Tsallis-distribution:

$$f\left[E(v_0)\right] \propto \left[1 + (q-1)\frac{E(v_0) - m}{T}\right]^{-1/(q-1)}$$

$$v_2 \propto \frac{p_T - v_0 m_T}{1 + \frac{q-1}{T}\left[\gamma_0 (m_T - v_0 p_T) - m\right]}$$

[1] Barnaföldi etal, (Hot Quarks 2014) J. Phys. Conf. Ser. 612 (2015) 1, 012048

[2]Urmossy etal, (WPCF 2014) arXiv:1501.05959, Conference: C14-08-25.8

[3] Urmossy etal, (High-pT 2014), arXiv:1501.02352

Urmossy etal, arXiv:1405.3963

The *hard + soft* model:

$$E\frac{dN}{d^3\mathbf{p}} = E\frac{dN}{d^3\mathbf{p}}^{hard} + E\frac{dN}{d^3\mathbf{p}}^{soft}$$

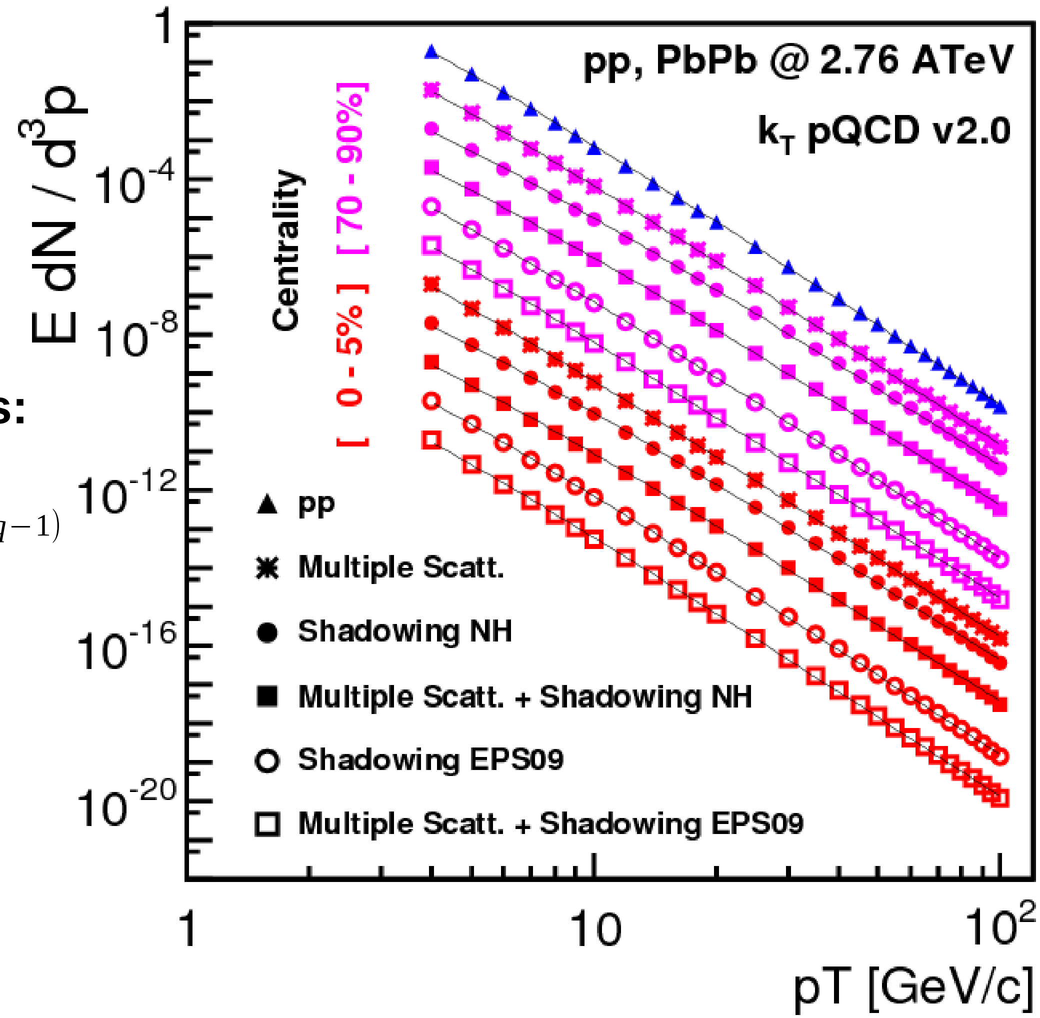
Both types of yields are Tsallis distributions:

$$E\frac{dN}{d^3\mathbf{p}}^{soft/hard} = A\left[1 + \frac{q-1}{T}\left[E^{co} - m\right]\right]^{-1/(q-1)}$$

$$E^{co} = \gamma\left(m_T - v\,p_T\right)$$

The hard yield is Tsallis, because it fits pQCD spectra in PbPb @ 7 TeV.

The remaining soft yield can be described by another Tsallis too.



arXiv:1405.3963

Tsallis from N fluctuations

If in a collision, the hadron distribution is *Boltzmann-Gibbs*,

$$f(\epsilon) = A\exp(-\beta\epsilon), \qquad E/n = DT$$

But *multiplicity fluctuates from collision to collision, while E = constant*

$$p(n) \propto n^{\alpha-1}\exp(-\alpha n/\langle n\rangle)$$

the *average spectrum is the Tsallis* distributions:

$$\frac{dN}{d^3p} = \int dn\, p(n) f_n(\epsilon) \propto \left(1+\frac{D\langle n\rangle}{\alpha E}\epsilon\right)^{-(\alpha+D+1)}$$

$$\propto \left(1+\frac{q-1}{T}\epsilon\right)^{-1/(q-1)}$$

Tsallis Distribution from Fluctuations

Moreover,

If the hadron distribution is *Boltzmann-Gibbs*,

$$f(\epsilon) = A \exp(-\beta \epsilon), \qquad E/n = DT$$

but the *total transverse energy fluctuates event-by-event while n = fix*

$$p(E) \propto E^{-\alpha-2} \exp(-\alpha \langle E \rangle / E)$$

the *average distribution* becomes the *Tsallis* distribution:

$$\frac{dN}{d^3 p} = \int dE \, p(E) f_E(\epsilon) \propto \left(1 + \frac{D n}{\alpha \langle E \rangle} \epsilon\right)^{-(\alpha + D + 1)}$$

Tsallis Distribution from Fluctuations

SuperStatistics (**C. Beck, G. Wilk,** *Eur. Phys. J. A*, **40**, 267 and 299-312, (2009)):

If the hadron distribution is *Boltzmann-Gibbs*,

$$f(\epsilon) = A \exp(-\beta \epsilon)$$

but the *temperature fluctuates* *event-by-event* or *position-to-position* as

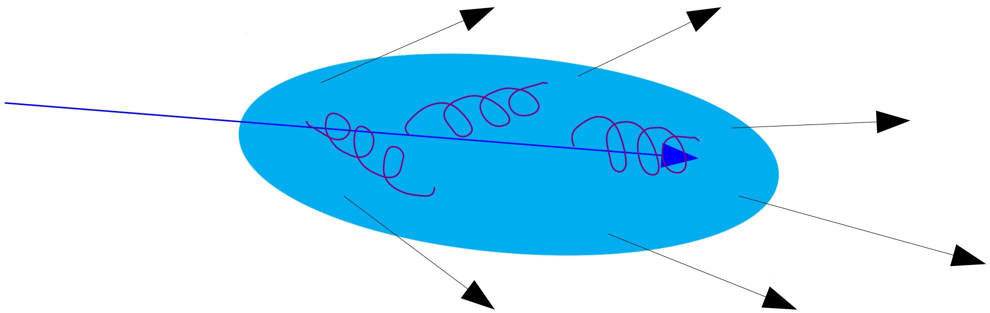
$$p(\beta) \propto \beta^{\alpha-1} \exp(-\alpha \beta / \langle \beta \rangle)$$

the *average distribution* becomes the *Tsallis* distribution:

$$\frac{dN}{d^3 p} = \int d\beta\, p(\beta) f_{\beta}(\epsilon) \propto \left(1 + \frac{\langle \beta \rangle \epsilon}{\alpha}\right)^{-(\alpha + D + 1)}$$

Motivation

- ***If the hadronisation of a single jet was statistical (microcanonical),***

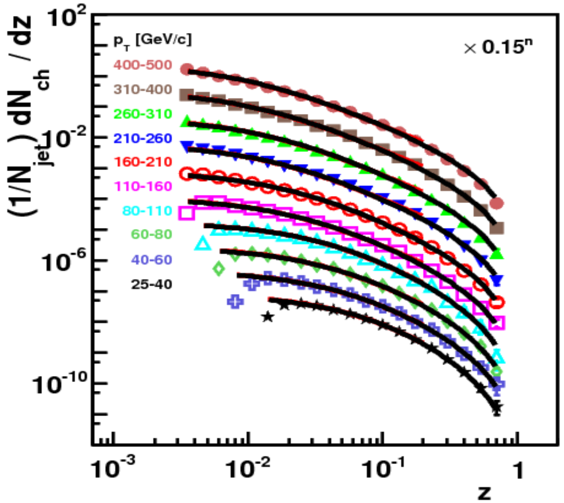


- ***N fluctuations of the form of***

$$p(n) \propto \left(\frac{n}{n_0}\right)^{a-1} e^{-a n/n_0}$$

turn statistical distributions into cut-power laws:

$$\frac{d\sigma}{dz} \propto \left(1 - \frac{q-1}{T}\ln(1-z)\right)^{-1/(q-1)}$$



pp → jets @ 7 TeV

Urmossy et. al., *PLB*, **718**, 125-129, (2012)

What is T?

If in a single event / jet, we have equipartition:

$$1\,event: \quad \frac{E_{event}}{N_{event}} = D\,T_{event}$$

On the average, we have:

$$\frac{E}{N} = \frac{\int \epsilon f_{TS}(\epsilon)}{\int f_{TS}(\epsilon)} = \frac{D\,T}{1-(q-1)(D+1)}$$

(m ≈ 0 particles)