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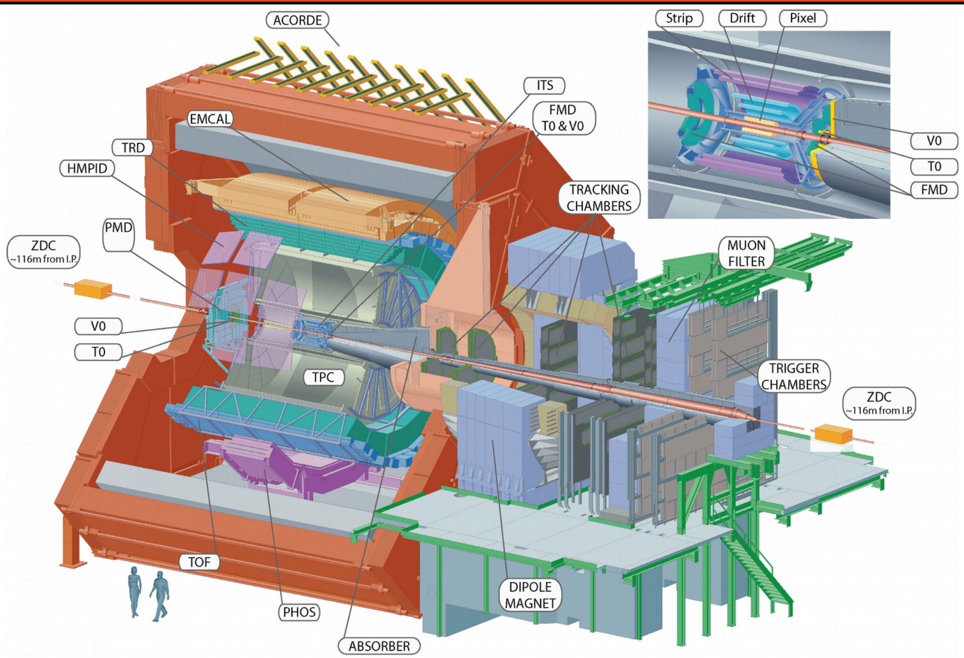
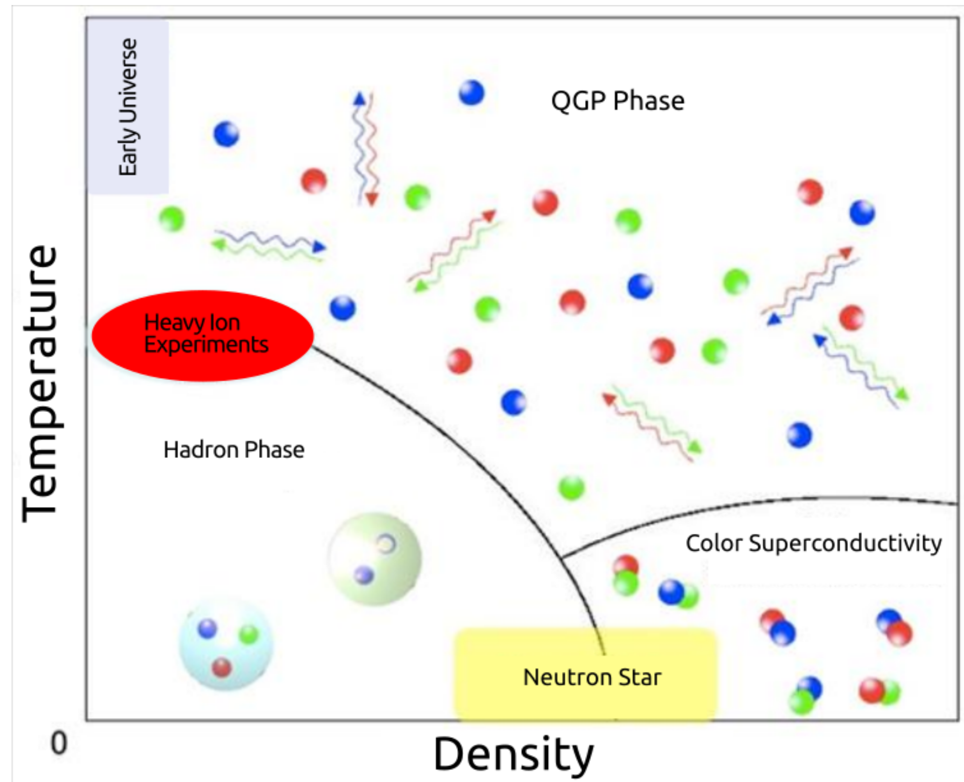
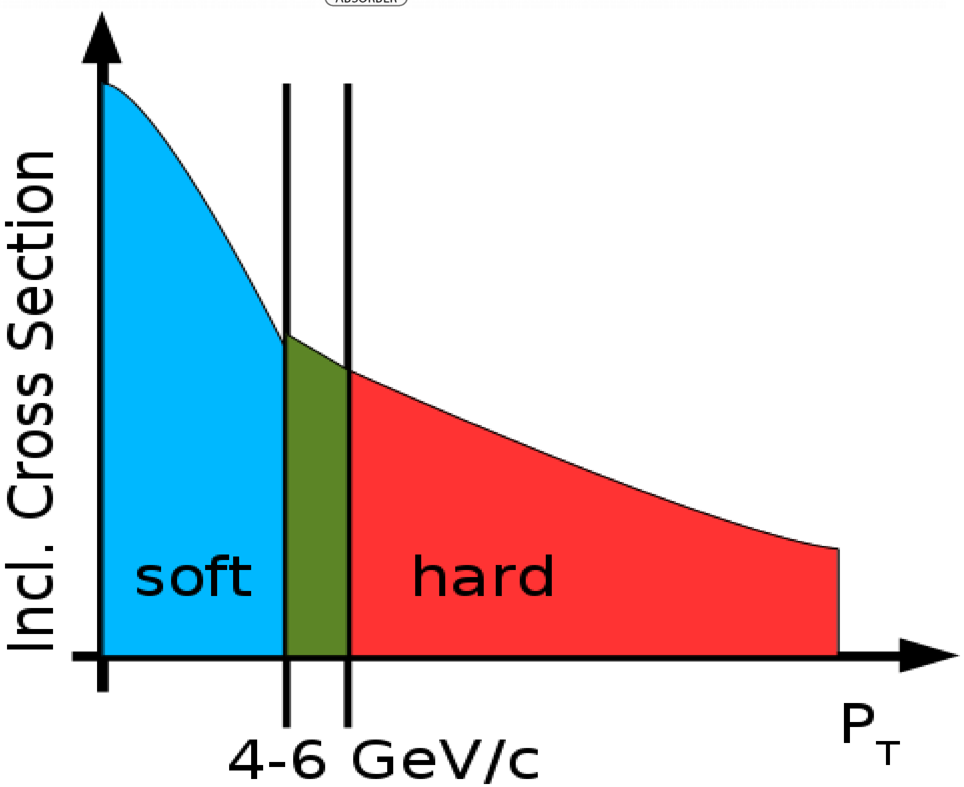
Analysing Identified Hadron Spectra in the Soft+Hard Approach

- **Motivation**
  - Why to use Tsallis – Pareto-like distributions?
- **Non-extensive statistical approach**
  - Fits of experimental spectra from e^+e^-, pp
  - Non-extensive statistical approach
- **Can Tsallis – Pareto fit spectra of HIC?**
  - The soft+hard model and its applications
  - Spectra fit and extraction of q and T
  - Asimuthal anisotropy from the model







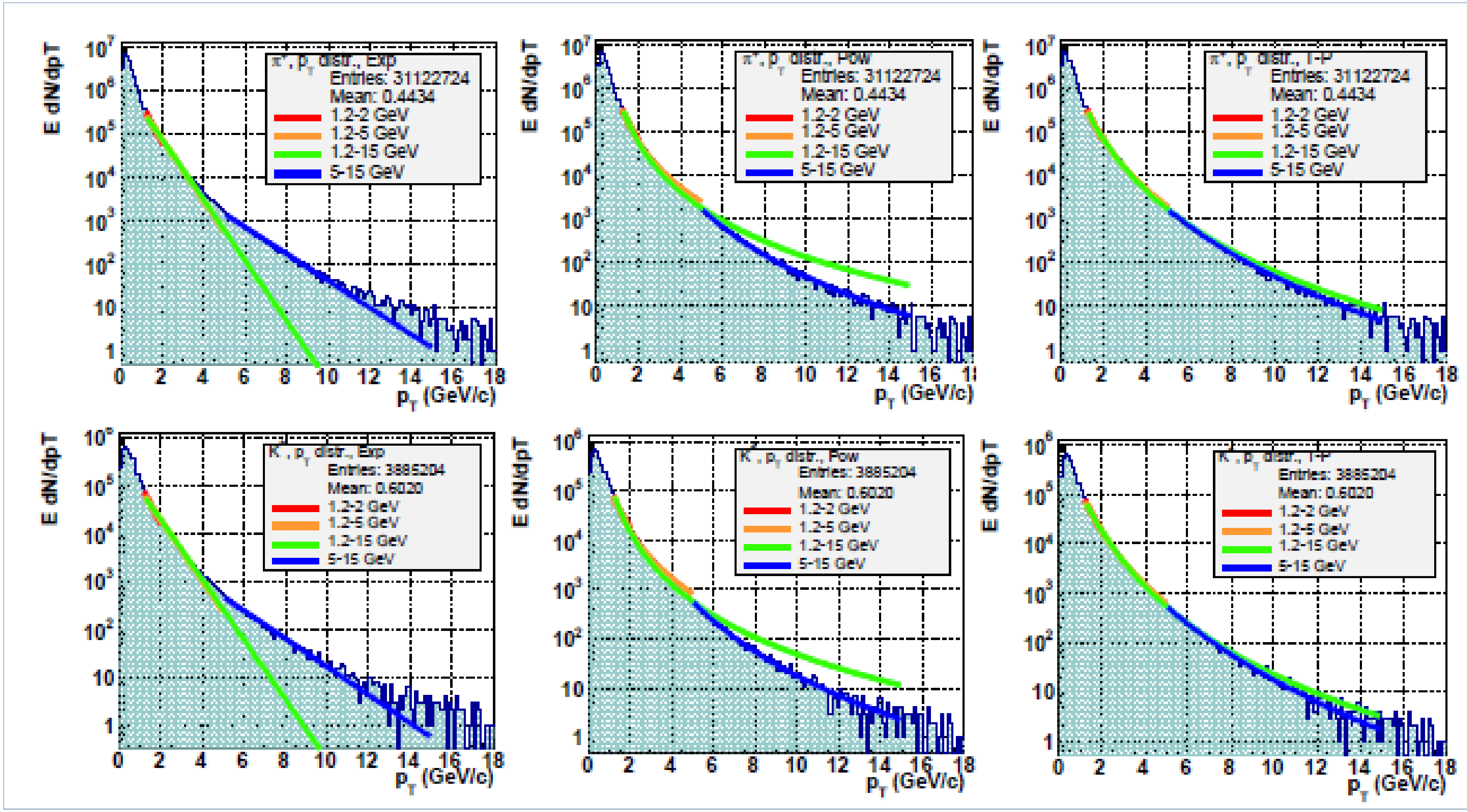
- ALICE: A Large Ion Colliding Experiment
- Analysing QGP: only through observable quantities
- Hadron spectra: a break around 4-6 GeV/c

- **Why use Tsallis–Pareto distribution?**
  - Is it true Boltzmann-Gibbs fits better at low momenta?
  - Is it true Power-law distribution is better at high momenta?
  - Is it true Tsallis – Pareto fits the whole mumentum range?
  - Can we apply this for any system: ee, pp, pA, AA?
- **A 'known' case:**
  - PYTHIA6.4: π, K and p production in proton-proton @ 14 TeV
  - Fits of Boltzmann-Gibbs, Power law, and Tsallis–Pareto distributions
  - Low momenta: [1.2 GeV/c : 2.0 GeV/c] or [1.2GeV/c : 5.0 GeV/c]
  - High momenta: [5.0 GeV/c : 15.0 GeV/c]
  - Full range: [1.2 GeV/c : 15.0 GeV/c]

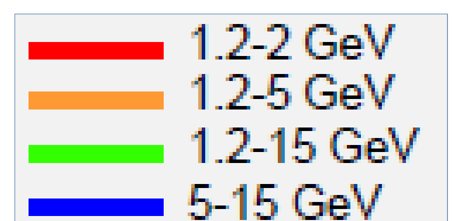
Boltzmann–Gibbs | Power Law | Tsallis–Pareto

Pions

Kaons



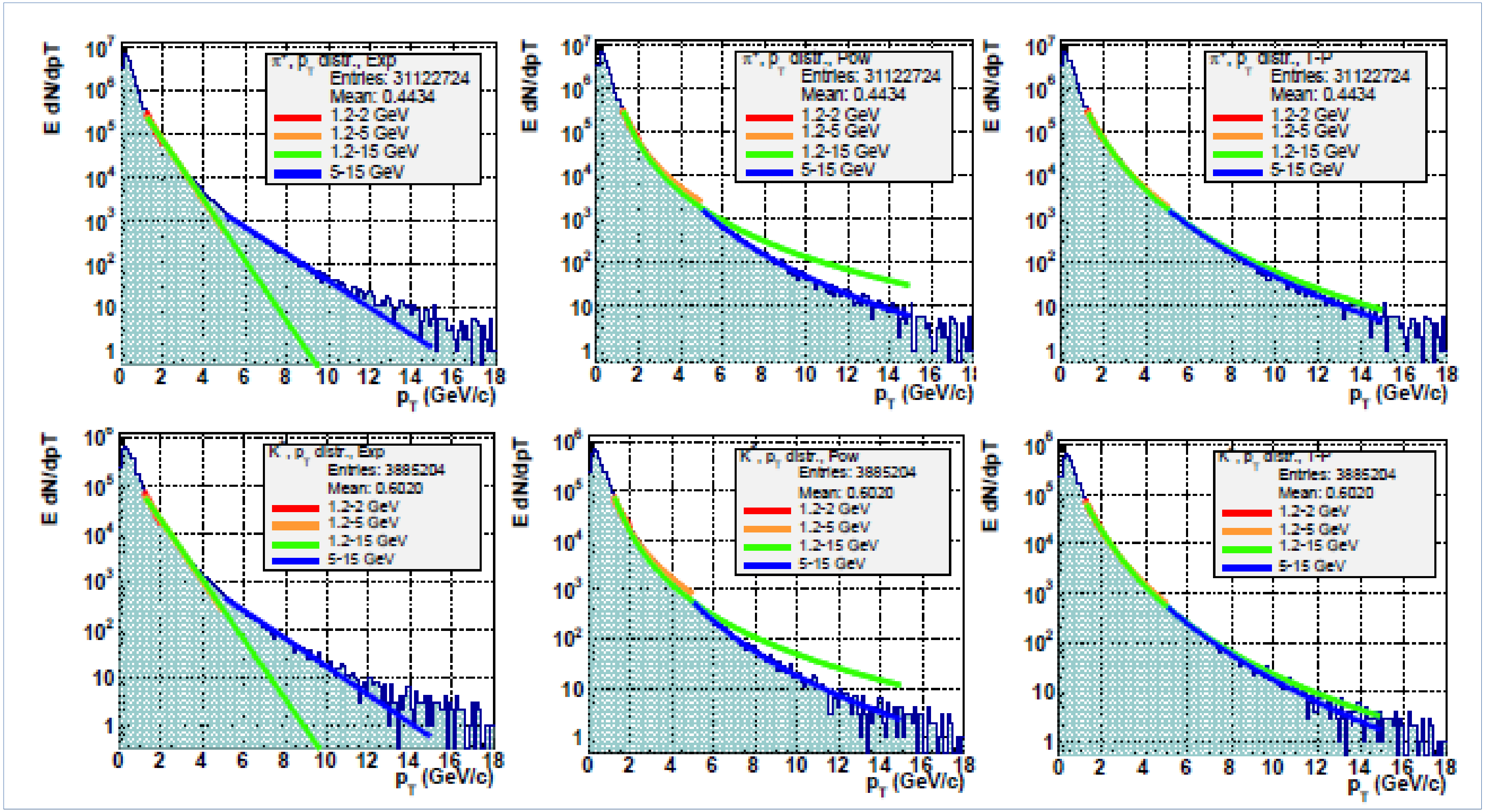
The fitted momentum regions:



Boltzmann–Gibbs | Power Law | Tsallis–Pareto

Pions

Kaons



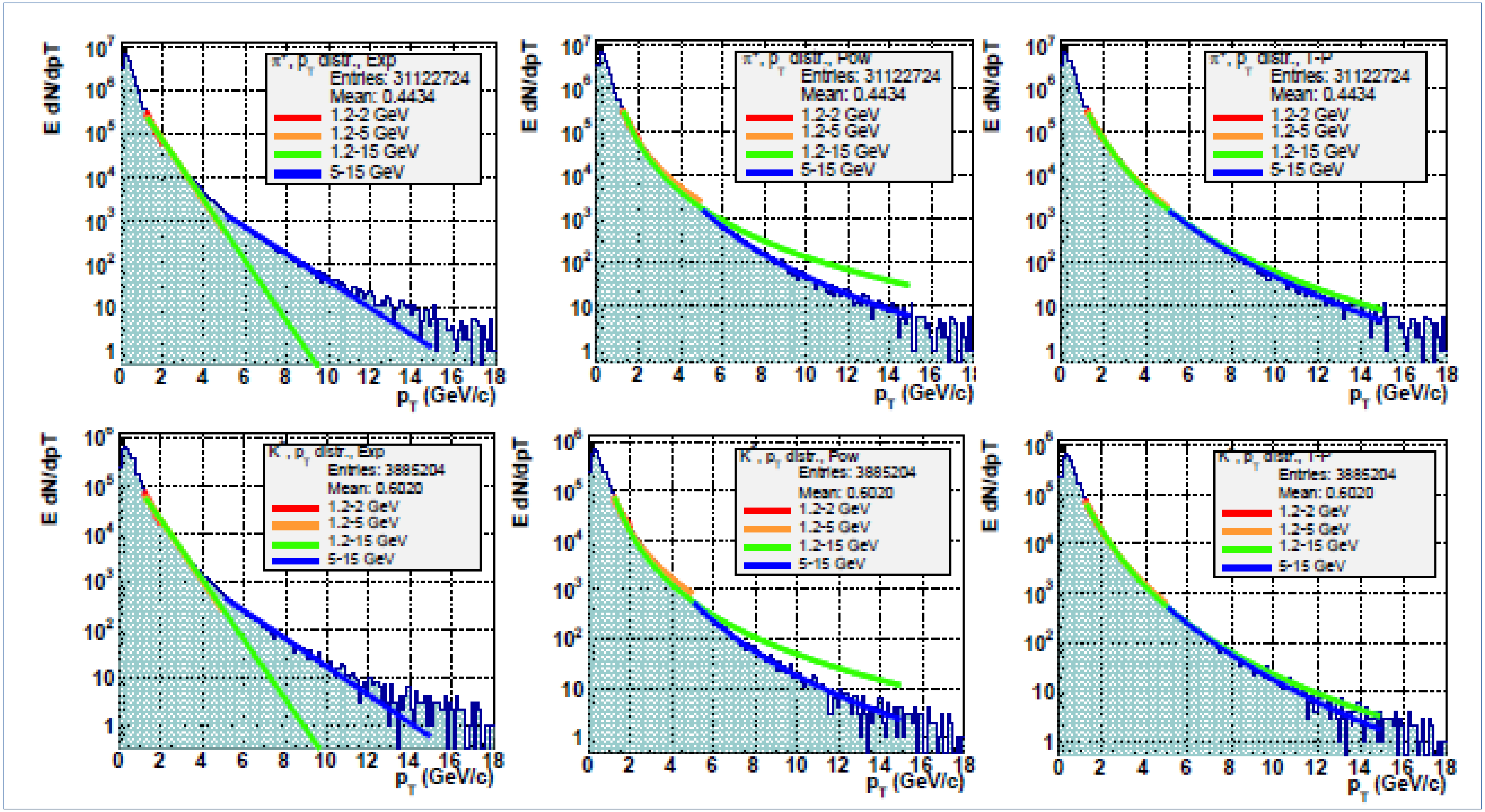
χ^2 **values:**

	[1,2:2] GeV/c	[1,2:5] GeV/c	[1,2:15] GeV/c	[5:15] GeV/c
Exp	112,37/29,81/27,34	623,89/130,48/109,26	254,12/61,71/48,13	3,01/1,44/1,45
Pow	1,71/0,98/0,47	161,27/55,68/56,08	214,12/76,92/77,26	1,37/1,144/0,91
TP	0,45/1,19/0,56	12,21/5,55/11,06	10,39/4,37/7,77	1,14/0,97/0,91

Boltzmann–Gibbs | Power Law | Tsallis–Pareto

Pions

Kaons



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TP	0,45/1,19/0,56	12,21/5,55/11,06	10,39/4,37/7,77	1,14/0,97/0,91

- Why fit Tsallis–Pareto distribution?
  - Yes, it is true Boltzmann-Gibbs fits better at low momenta.
  - Yes, it is true Power-law distribution is better at high momenta.
  - Yes, it is true Tsallis – Pareto fits the whole mumentum range.
  - Can we apply this for any system: ee, pp, pA, AA?

- Need for
  - High-p_T PID hadron data
  - High statistic data
  - Spectra in several multiplicity bins

- Extensive Boltzmann – Gibbs statistics

$$S_{12} = S_1 + S_2$$
$$E_{12} = E_1 + E_2$$
$$\Rightarrow \quad S_B = -\sum_i p_i \ln p_i$$

- Non-extensivity → generalized entropy

$$\hat{L}_{12} = \hat{L}_1(S_1) + \hat{L}_2(S_2)$$
$$L_{12} = L_1(E_1) + L_2(E_2)$$
$$\Rightarrow \quad S_T = \frac{1}{1-q} \sum_i (p_i^q - p_i)$$

- Tsallis entropy

$$S_{12} = S_1 + S_2 + (q-1) S_1 S_2 \quad \Rightarrow \quad \hat{L}(S) = \frac{1}{q-1} \ln\left(1 + (q-1)S\right)$$

from here: Tsallis – Pareto distribution

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

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- Tsallis – Pareto distribution

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

$$q = \frac{\langle S'(E)^2 + S''(E) \rangle}{\langle S'(E) \rangle^2}$$

$$\frac{1}{T} = \langle S'(E) \rangle$$

$$q = \frac{\langle n(n-1) \rangle}{\langle n \rangle^2}$$

$$q = 1 + \frac{\Delta T^2}{T^2} - \frac{1}{C}$$

$$\frac{E}{\langle n \rangle} = DT_{BG}$$

$$\frac{E}{\langle n \rangle} = \frac{\int \varepsilon f_{TS}(\varepsilon)}{\int f_{TS}(\varepsilon)} = \frac{DT}{1-(q-1)(D+1)}$$

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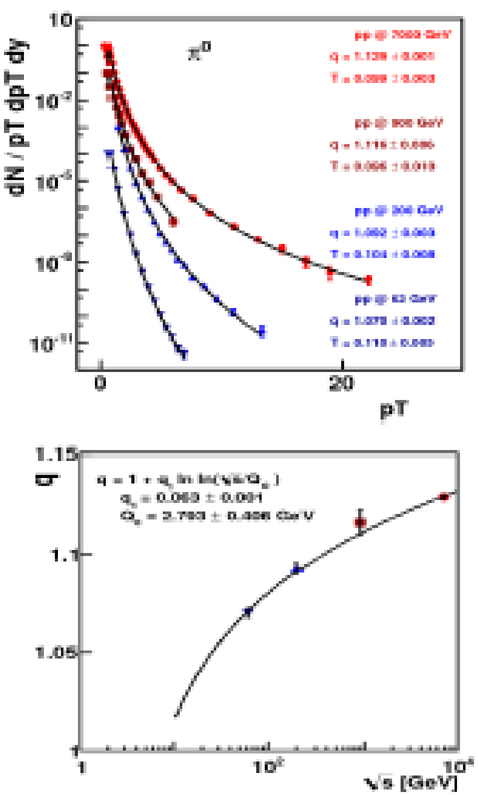
Hadron spectra in *pp* collisions can be described by the Tsallis distribution

$$\frac{dN}{d^3\mathbf{p}} \propto \left[1 + \frac{q-1}{T}(m_T - m)\right]^{-1/(q-1)} .$$

π spectra in *pp* collisions depends similarly on $\sqrt{s}$ and on the multiplicity N

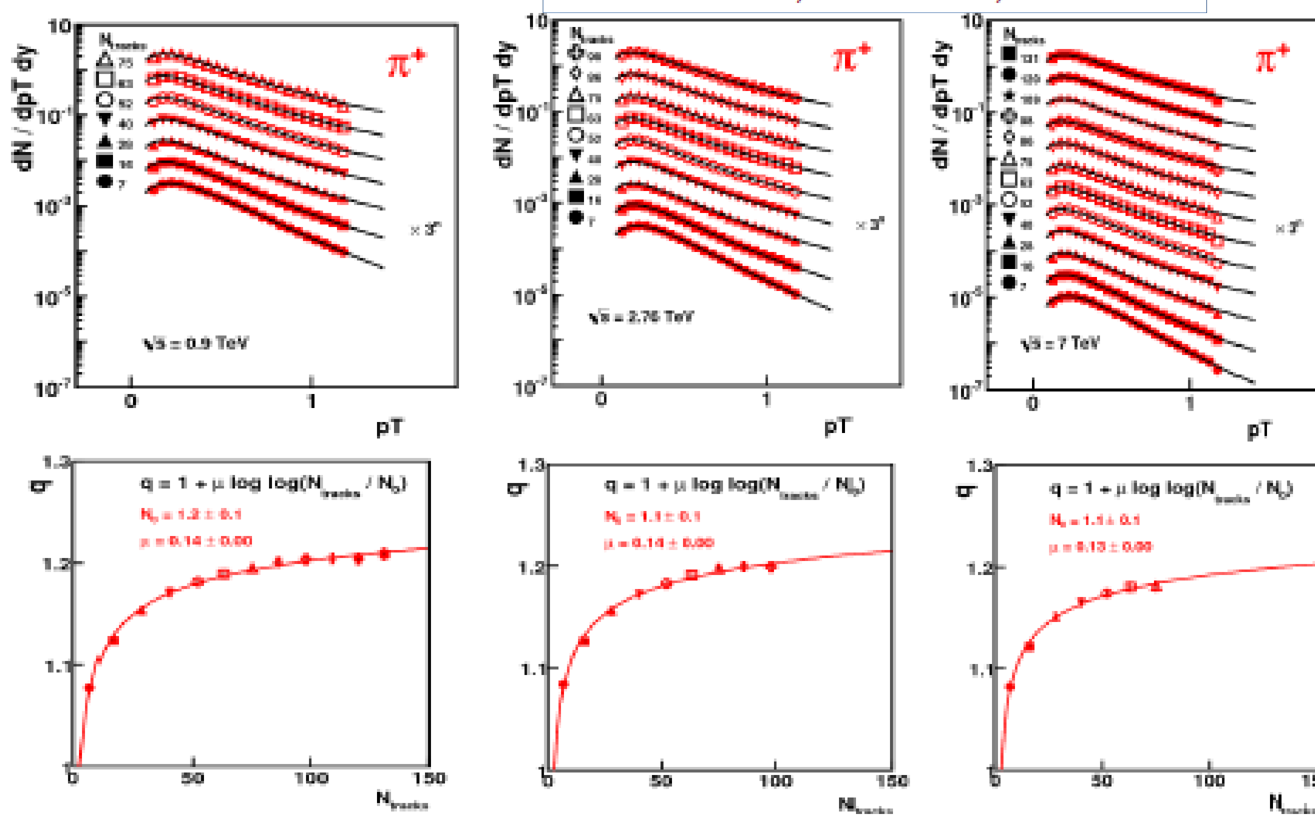
$$q(s) = 1 + q_1 \ln\ln(\sqrt{s}/Q_0) ,$$
$$q(N) = 1 + \mu \ln\ln(N/N_0) .$$

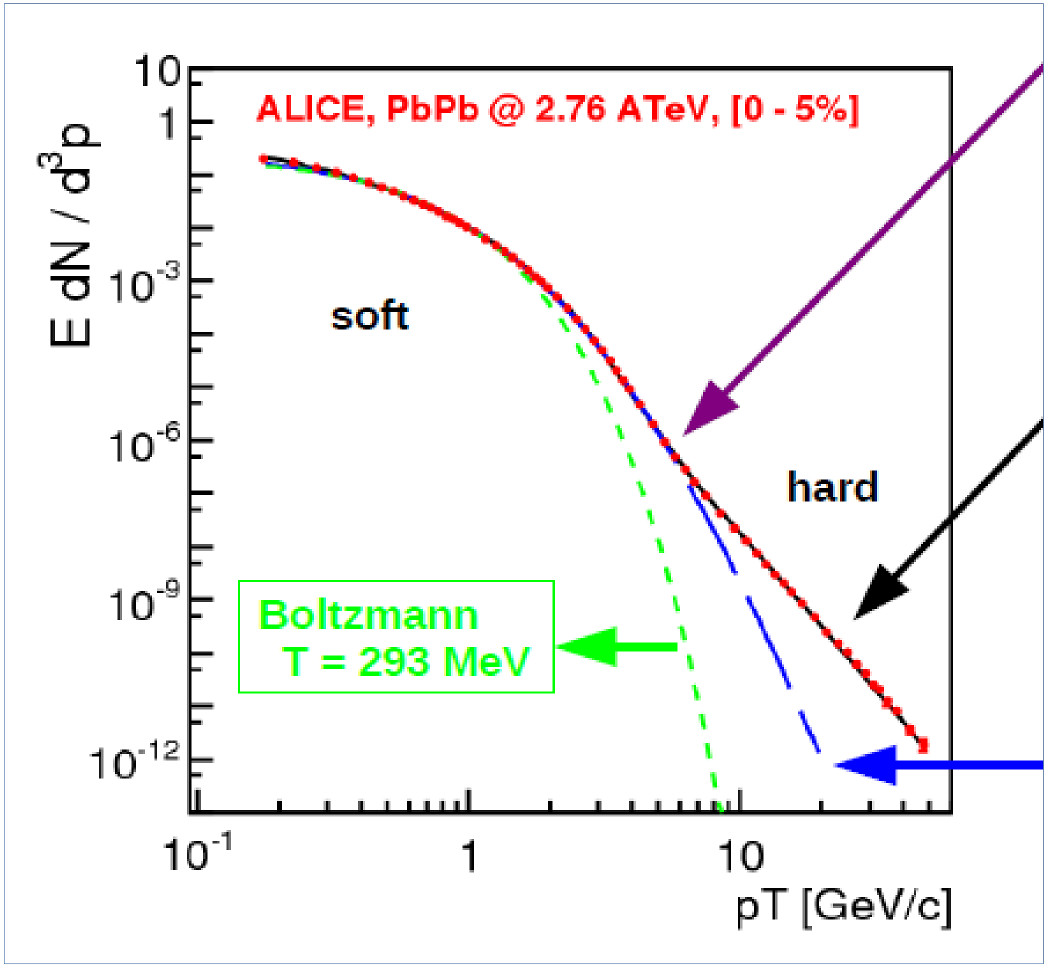
√s = fix



N = fix

arXiv:1405.3963, 1501.02352, 1501.05959





The power of the spectra changes at around 6 GeV/c

Power-law with $p^{-6.08}$ dependence

Tsallis spectra with $p^{-13.7}$

- Simplest approximation: soft ('bulk') + hard ('jet') contribution

$$p^0 \frac{dN}{d^3p} = p^0 \frac{dN}{d^3p}^{hard} + p^0 \frac{dN}{d^3p}^{soft}$$

- Identified hadron spectra is given by double Tsallis–Pareto:

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = f_{hard} + f_{soft} \qquad f_i = A_i \left[1 + \frac{(q_i - 1)}{T_i}\left[\gamma_i(m_T - v_i p_T) - m\right]\right]^{-1/(q_i-1)}$$

in where parameters are given by

  - Lorentz factor $\gamma_i = 1/\sqrt{1 - v_i^2}$
  - Transverse mass $m_T = \sqrt{p_T^2 + m^2}$
  - Doppler temperature $T_i^{Dopp} = T_i \sqrt{\frac{1 + v_i}{1 - v_i}}$

- Finally we assume N_{part} scaling for the parameters

$$q_i = q_{2,i} + \mu_i \ln\left(N_{part}/2\right)$$

$$T_i^{Dopp} = T_{1,i} + \tau_i \ln\left(N_{part}\right)$$

ArXiv: 1405.3963, 1501.02352, 1501.05959

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1. **Single Tsallis** fit over the whole p_T range, the fitted parameters are: $\boldsymbol{q}$. $\boldsymbol{T}$. $\boldsymbol{A}$. $\boldsymbol{v}$

2. **Single Tsallis** fit over the whole p_T range, the fitted parameters are: $\boldsymbol{T}$. $\boldsymbol{A}$. $\boldsymbol{v}$ ($\boldsymbol{q}$ fixed)

3. **Two single Tsallis** fit:

   1) **First Tsallis**: the range is $[p_0 - \varepsilon; p_{max}]$, the fitted parameters are:

      $\boldsymbol{q}_{hard}$, $\boldsymbol{A}_{hard}$ ($\boldsymbol{v}_{hard}$ and $\boldsymbol{T}_{hard}$ fixed)

   2) **Second Tsallis**: the range is $[p_{min}; p_0 + \varepsilon]$, the fitted parameters are:

      $\boldsymbol{T}_{soft}$, $\boldsymbol{A}_{soft}$, $\boldsymbol{v}_{soft}$ ($\boldsymbol{q}_{soft}$ fixed)

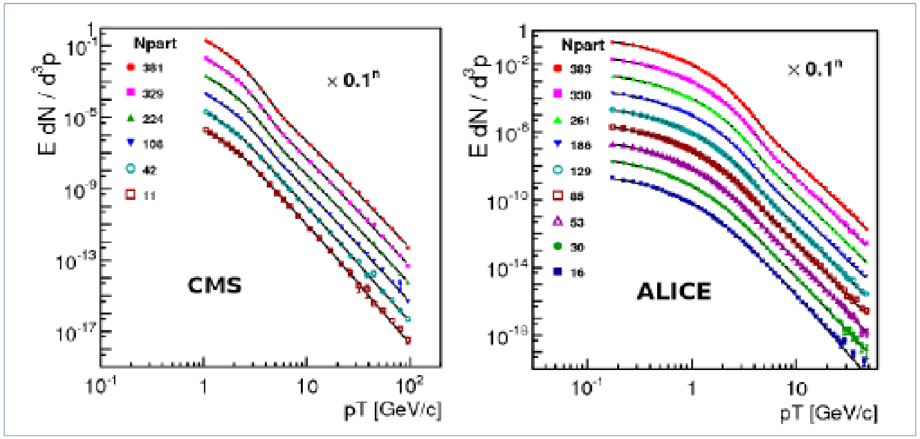
4. **Double Tsallis** fit, the range is $[p_0 - \varepsilon; p_{max}]$, the fitted parameters are the **hard** parameters

5. **Double Tsallis** fit, the range is $[p_{min}; p_0 + \varepsilon]$, the fitted parameters are the **soft** parameters

6. **Double Tsallis** fit, the range is $[p_{min}; p_{max}]$, the fixed parameters are $\boldsymbol{q}_{soft}$, $\boldsymbol{T}_{soft}$ and $\boldsymbol{v}_{soft}$

7. **Double Tsallis** fit, the range is $[p_{min}; p_{max}]$, **all** of the parameters are fitted

Fit of pp and central/peripheral PbPb data



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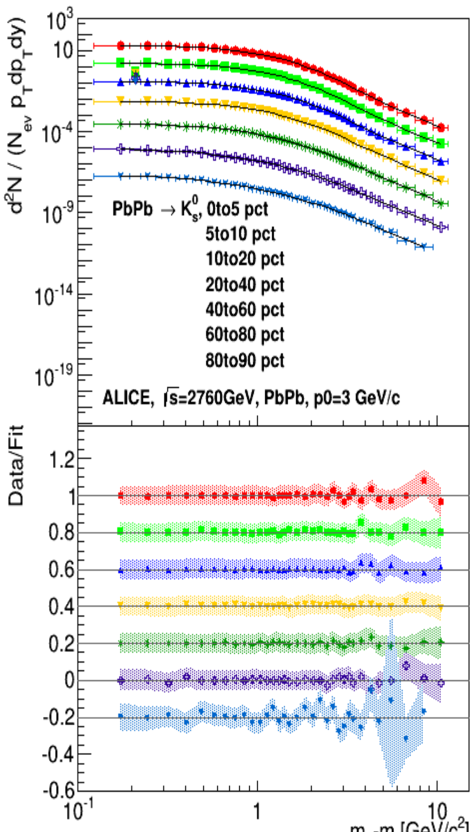
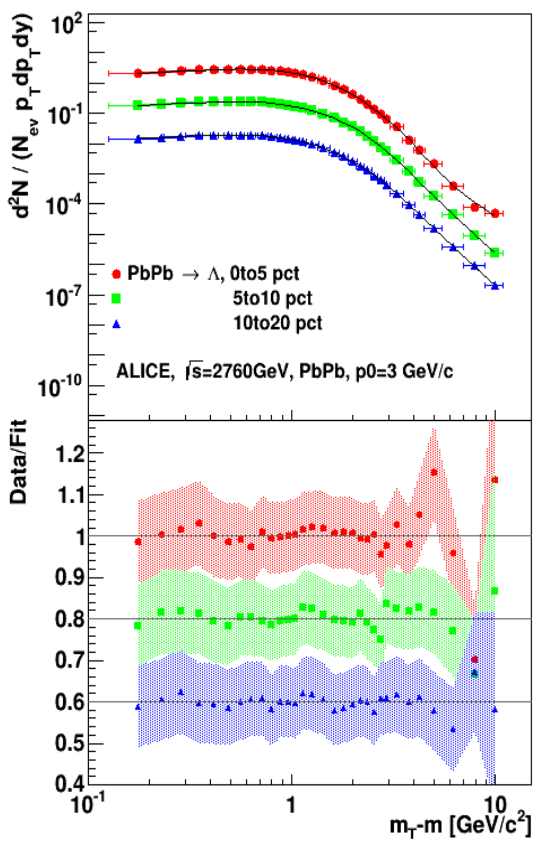
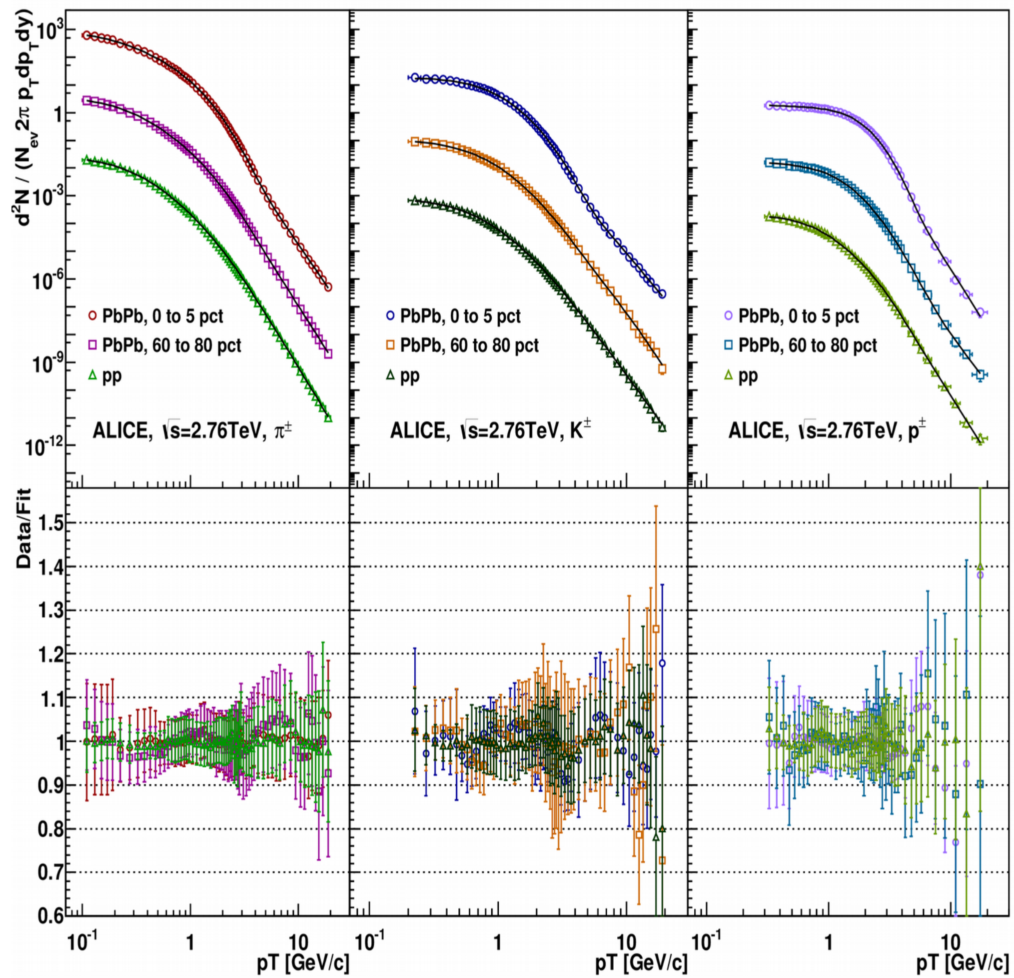






$$q_i = q_{2,i} + \mu_i \ln\left(N_{part}/2\right)$$

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = f_{hard} + f_{soft}$$

$$f_i = A_i \left[1 + \frac{(q_i - 1)}{T_i}\left[\gamma_i(m_T - v_i p_T) - m\right]\right]^{-1/(q_i-1)}$$

$$T_i^{Dopp} = T_{1,i} + \tau_i \ln\left(N_{part}\right)$$

$$T_i^{Dopp} = T_i\sqrt{\frac{1+v_i}{1-v_i}}$$

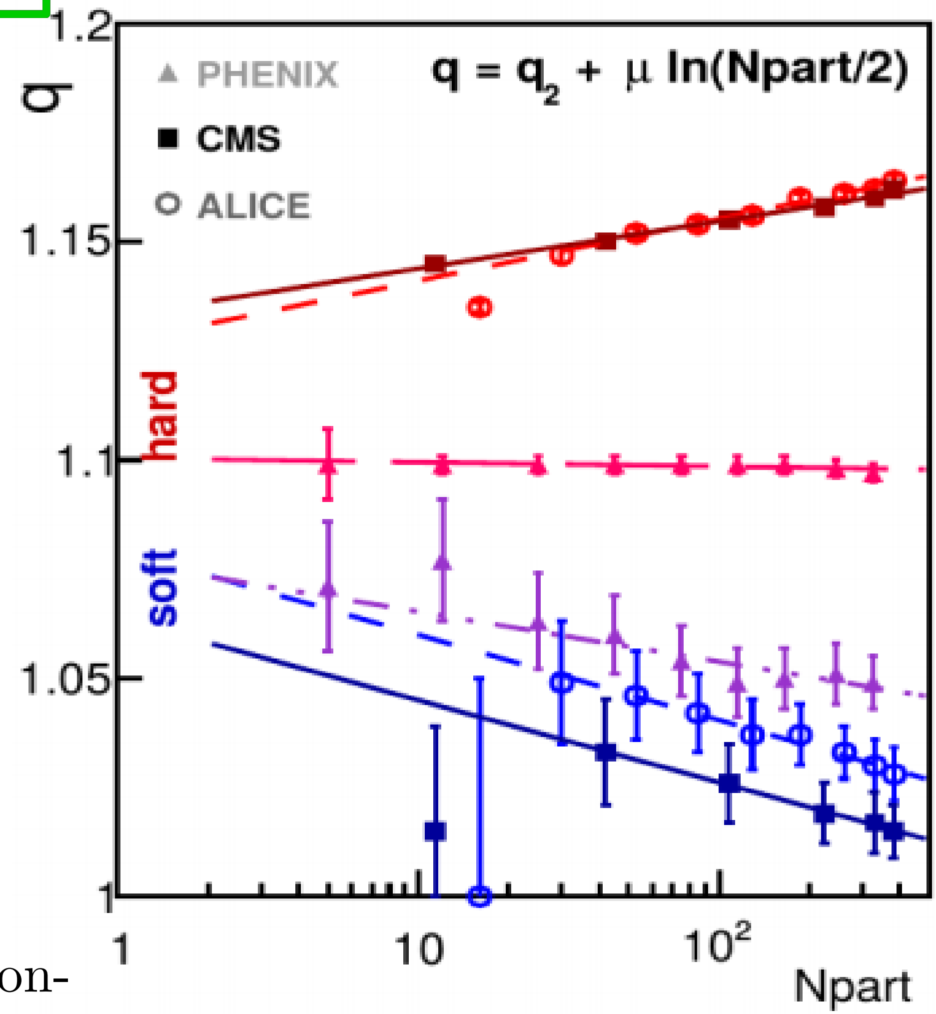
	$q_{2,soft}$	$q_{2,hard}$	μ_{soft}	μ_{hard}
CMS	1.058 ± 0.025	1.136 ± 0.001	-0.008 ± 0.005	0.005 ± 0.0003
ALICE	1.074 ± 0.018	1.131 ± 0.002	-0.009 ± 0.004	0.006 ± 0.0006
PHENIX	1.073 ± 0.016	1.100 ± 0.002	-0.005 ± 0.004	0.000 ± 0.0006

	T_1^{soft} [MeV]	T_1^{hard} [MeV]	τ_{soft} [MeV]	τ_{hard} [MeV]
CMS	310 ± 20	126 ± 5	9.9 ± 3.7	5.3 ± 0.8
ALICE	266 ± 16	194 ± 2	11.5 ± 2.9	-12.5 ± 0.5
PHENIX	165 ± 26	192 ± 20	9.3 ± 5.5	18.7 ± 4.6

- Scaling of the $q_i = q_{2,i} + \mu_i \ln(N_{part}/2)$
  - Soft component, $q \to 1$
    - LHC: decreasing
    - RHIC: decreasing

    Higher N_{part} result BG statistics
  - Hard component, $q > 1.1$
    - LHC: slight increasing
    - RHIC: constant

    Without the soft part result clearer non-extensive behaviour, like e^+e^-



ArXiv: 1405.3963, 1501.02352, 1501.05959

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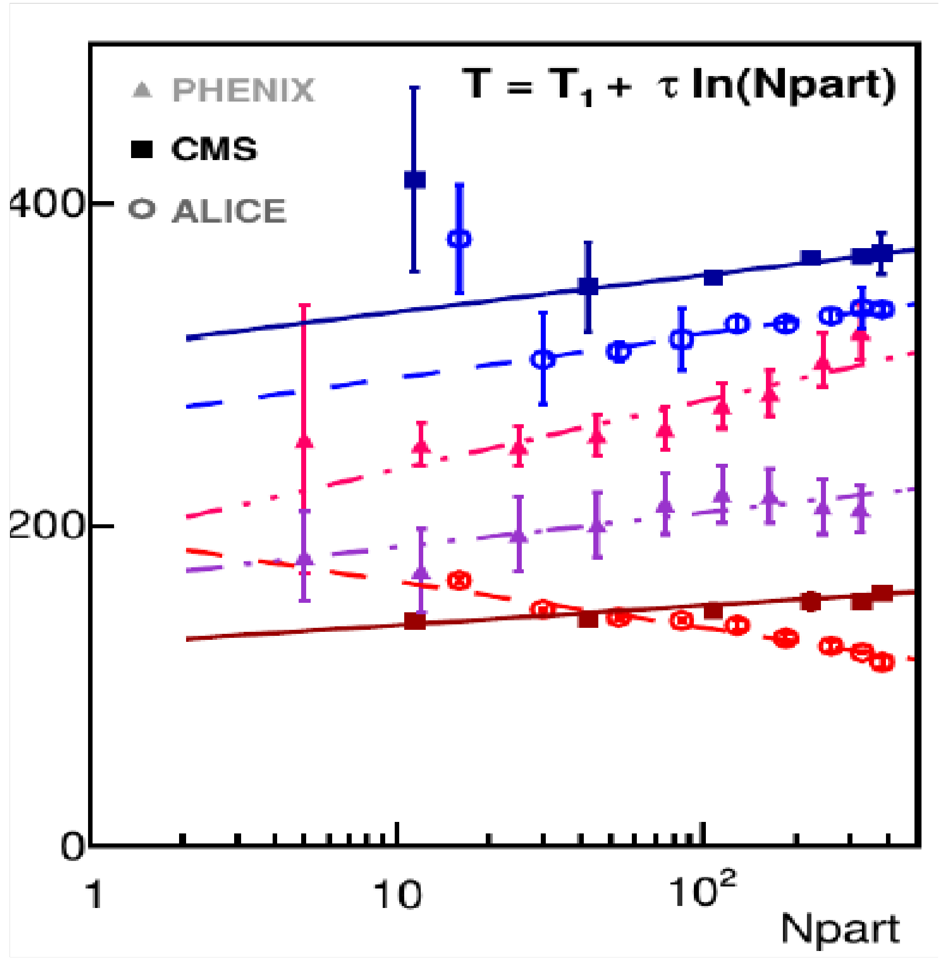
- Scaling of the $T_i^{Dopp} = T_{1,i} + \tau_i \ln(N_{part})$
  - Soft component, *T~200-400 MeV*
    - LHC: constant/increasing
    - RHIC: slightly increasing

    Higher N_{part} results bit higher T
  - Hard component, *T ~100-300 MeV*
    - LHC: decreasing
    - RHIC: increasing

    N_{part} scaling seems sensitive...



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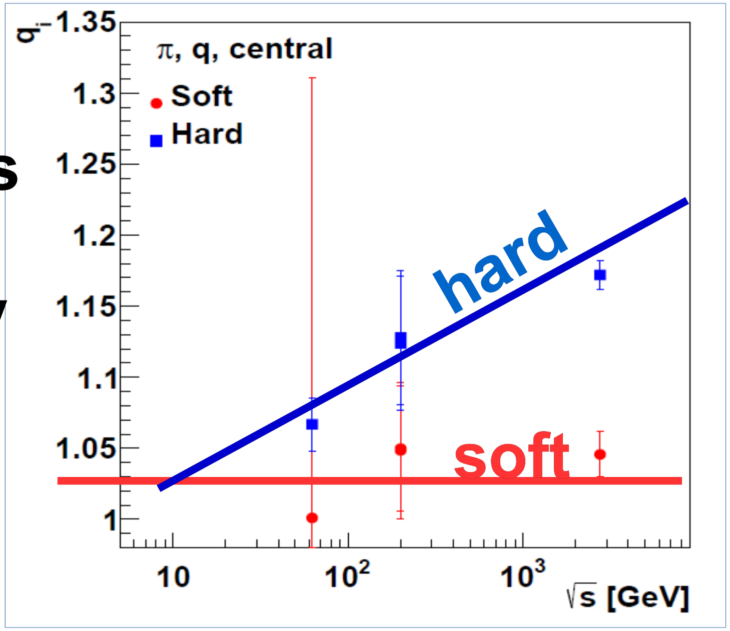
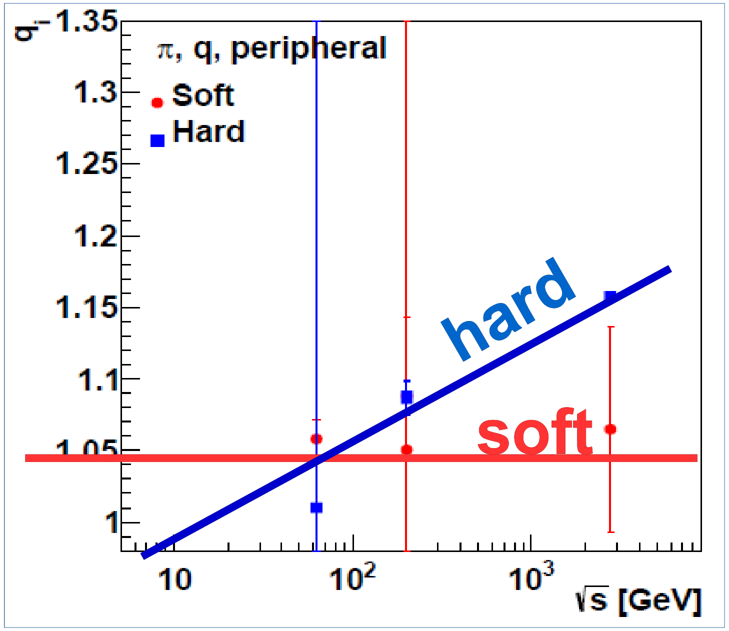
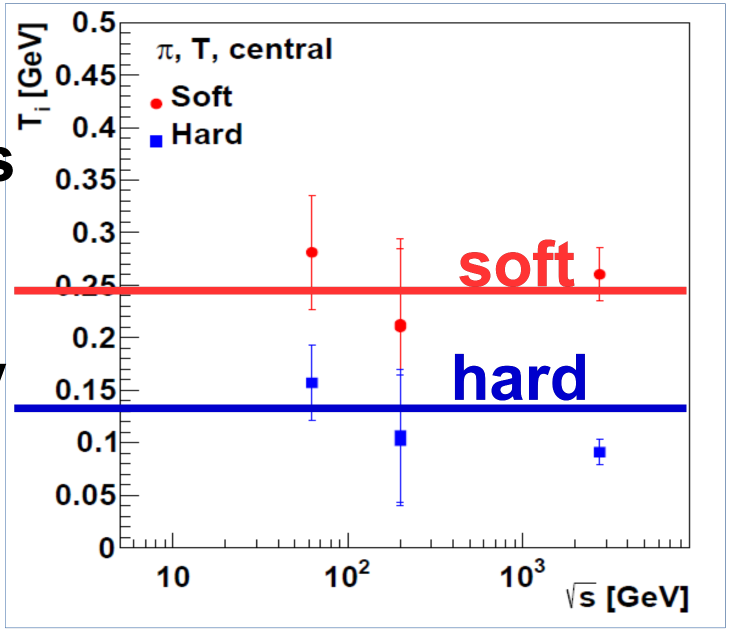
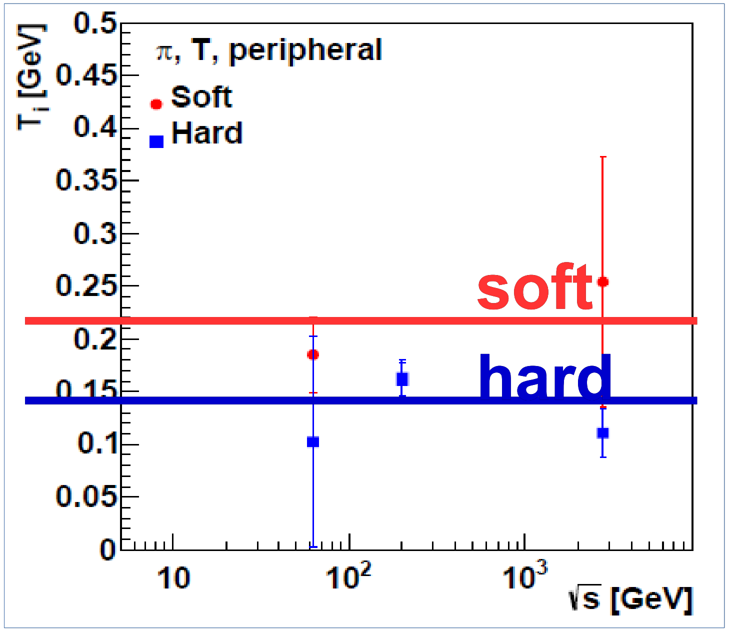
***q* measures non-extensivity**





***T* measures average *E* per multiplicity**



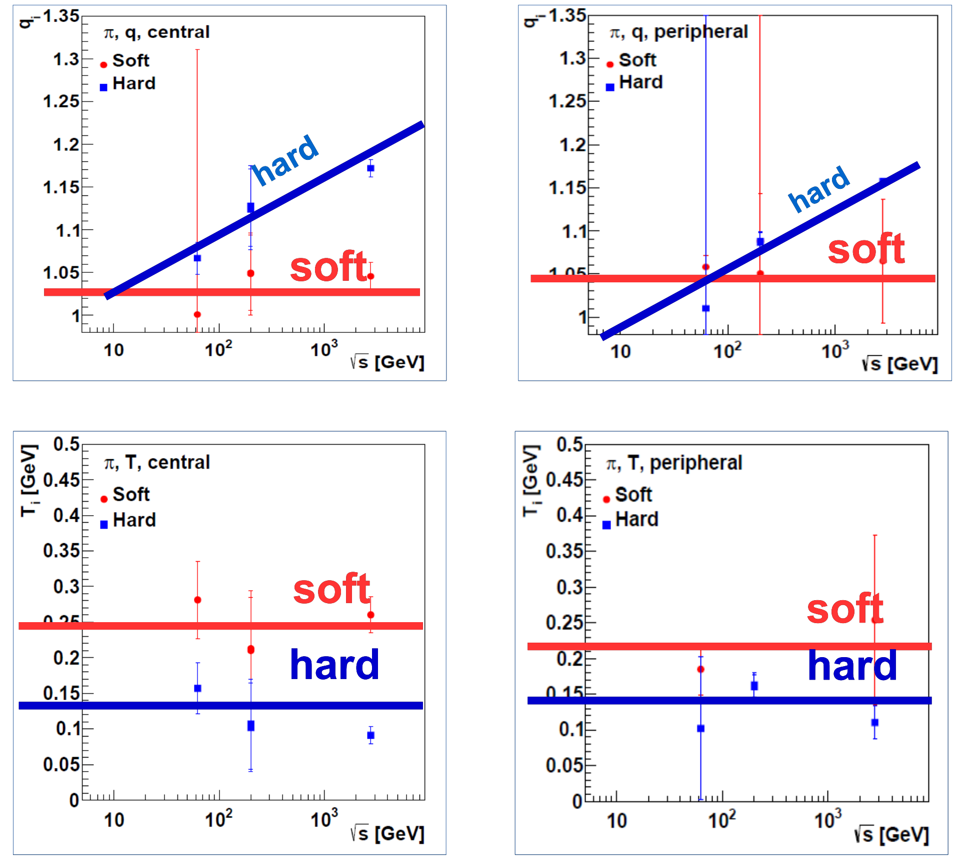


- Energy dependence
  - Parameter q
    - HARD: clearly increasing
    - SOFT: no relevant change
  - Parameter T
    - HARD: central decreasing

      peripheral const?

      $T_{centr} = T_{periph}$
    - SOFT: similar trend

      T_{centr} ~100 MeV higher



- Parameters q & T present different values for centr./periph.
- Above RHIC soft is BG-like and hard is more TP-like.

- Spectra originating from hadronic sources

$$p^0 \frac{dN}{d^3p}\bigg|_{y=0} = \int_{-\infty}^{+\infty} d\zeta \int_0^{2\pi} d\alpha f[u_\mu p^\mu] \quad \Rightarrow \quad \frac{dN}{2\pi p_T dp_T dy}\bigg|_{y=0} = \int_0^{2\pi} \frac{d\varphi}{2\pi} p^0 \frac{dN}{d^3p}\bigg|_{y=0}$$

where we used parameters and assumptions:

- Hadron momenum: $p^\mu = (m_T \cosh y, m_T \sinh y, p_T \cos\varphi, p_T \sin\varphi)$
- Cylindric symmetry: $u^\mu = (\gamma \cosh\zeta, \gamma \sinh\zeta, \gamma v \cos\alpha, \gamma v \sin\alpha)$
  - where $\zeta = \frac{1}{2} \ln[(t+z)/(t-z)]$ and $\gamma = 1/\sqrt{1-v^2}$
  - Co-moving energy: $u_\mu p^\mu|_{y=0} = \gamma\left[m_T \cosh\zeta - v p_T \cos(\varphi - \alpha)\right]$
  - Transverse flow: $v(\alpha) = v_0 + \sum_{m=1}^{\infty} \delta v_m \cos(m\alpha) \equiv v_0 + \delta v(\alpha)$
  - Taylor expansion: $f[u_\mu p^\mu]|_{y=0} = \sum_{m=0}^{\infty} \frac{[\delta v(\alpha)]^m}{m!} \frac{\partial^m}{\partial v_0^m} f[u_\mu p^\mu]|_{y=0}^{v(\alpha)=v_0}$

- Spectra originating from hadronic sources

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = \int_0^{2\pi} \frac{d\varphi}{2\pi} p^0 \left.\frac{dN}{d^3p}\right|_{y=0} = \sum_{m=0}^{\infty} \frac{a_m}{m!} \frac{\partial^m}{\partial v_0^m} f[E(v_0)] \approx f[E(v_0)] + O(\delta v^2)$$

where $E(v_0) = \gamma_0(m_T - v_0 p_T)$ and $a_m = \int_0^{2\pi} d\alpha [\delta v(\alpha)]^m$

- Azimuthal anisotropy:

$$v_n = \frac{\int_0^{2\pi} d\varphi \cos(n\varphi) p^0 \left.\frac{dN}{d^3p}\right|_{y=0}}{\int_0^{2\pi} d\varphi p^0 \left.\frac{dN}{d^3p}\right|_{y=0}} \approx \frac{\delta v_n \gamma_0^3}{2} \frac{(v_0 m_T - p_T) f'[E(v_0)]}{f[E(v_0)]} + O(\delta v^2)$$

  - Boltzmann–Gibbs: $f \sim \exp[-\beta E(v_0)]$ → $v_n^{BG} \approx \frac{\delta v_n \beta \gamma_0^3}{2}(p_T - v_0 m_T) + O(\delta v^2)$
  - Tsallis–Pareto: $f \sim [1 + (q-1)\beta E(v_0)]^{-1/(q-1)}$ → $v_n^{TS} \approx \frac{\delta v_n \beta \gamma_0^3}{2} \frac{p_T - v_0 m_T}{1 + (q-1)\beta\gamma_0(m_T - v_0 p_T)} + O(\delta v^2)$

- Spectra originating from hadronic sources

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = \int_0^{2\pi} \frac{d\varphi}{2\pi} p^0 \left.\frac{dN}{d^3p}\right|_{y=0} = \sum_{m=0}^{\infty} \frac{a_m}{m!} \frac{\partial^m}{\partial v_0^m} f[E(v_0)] \approx f[E(v_0)] + O(\delta v^2)$$

where $E(v_0) = \gamma_0(m_T - v_0 p_T)$ and $a_m = \int_0^{2\pi} d\alpha [\delta v(\alpha)]^m$

- Azimuthal anisotropy:

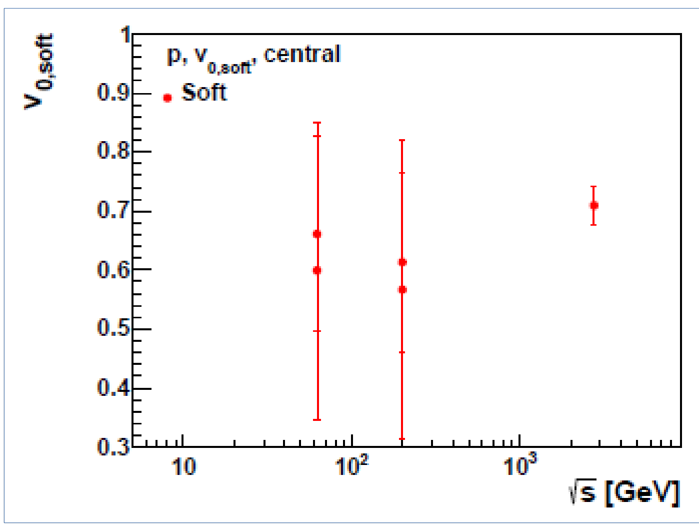
$$v_n = \frac{\int_0^{2\pi} d\varphi \cos(n\varphi) p^0 \left.\frac{dN}{d^3p}\right|_{y=0}}{\int_0^{2\pi} d\varphi p^0 \left.\frac{dN}{d^3p}\right|_{y=0}} \approx \frac{\delta v_n \gamma_0^3}{2} \frac{(v_0 m_T - p_T) f'[E(v_0)]}{f[E(v_0)]} + O(\delta v^2)$$

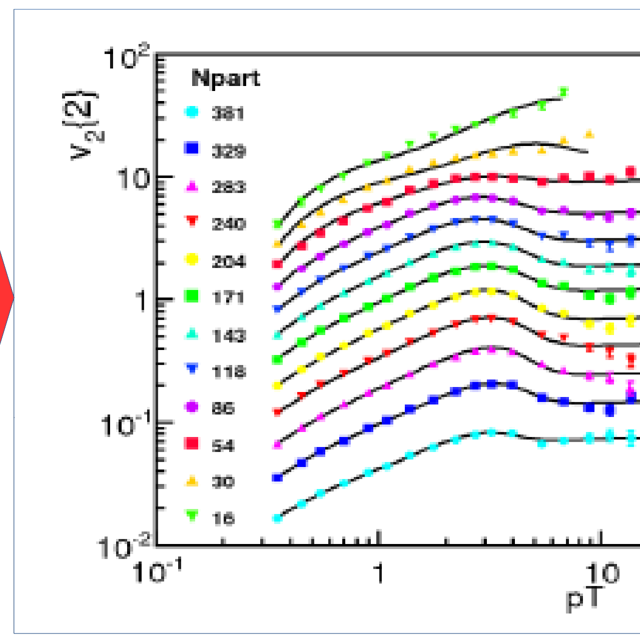
- Using the soft+hard model:

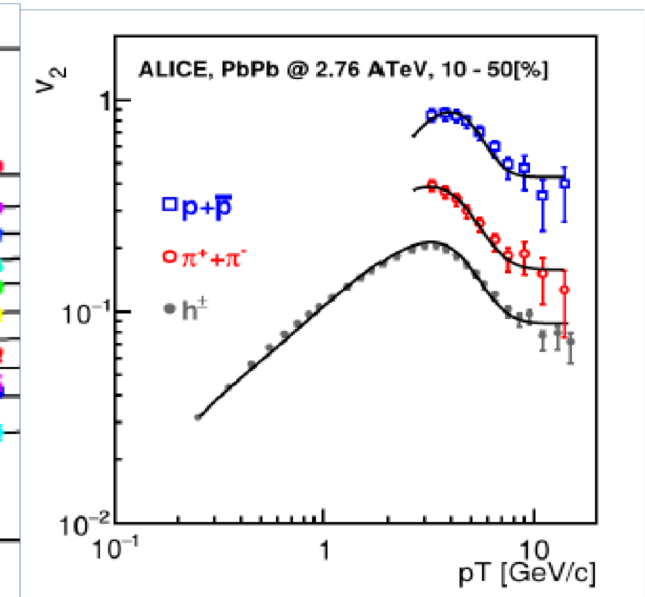
$$v_2 = \frac{w_{hard} f_{hard} + w_{soft} f_{soft}}{f_{hard} + f_{soft}}$$ with the coefficient $$w_i = \frac{\delta v_i \gamma_i^3}{2T_i} \frac{p_T - v_i m_T}{1 + \frac{q_i - 1}{T_i}[\gamma_i(m_T - v_i p_T) - m]}$$

- Using the soft+hard model:

$$v_2 = \frac{w_{hard} f_{hard} + w_{soft} f_{soft}}{f_{hard} + f_{soft}}$$ with the coefficient $$w_i = \frac{\delta v_i \gamma_i^3}{2T_i} \frac{p_T - v_i m_T}{1 + \frac{q_i - 1}{T_i}[\gamma_i(m_T - v_i p_T) - m]}$$







ArXiv: 1405.3963, 1501.02352, 1501.05959

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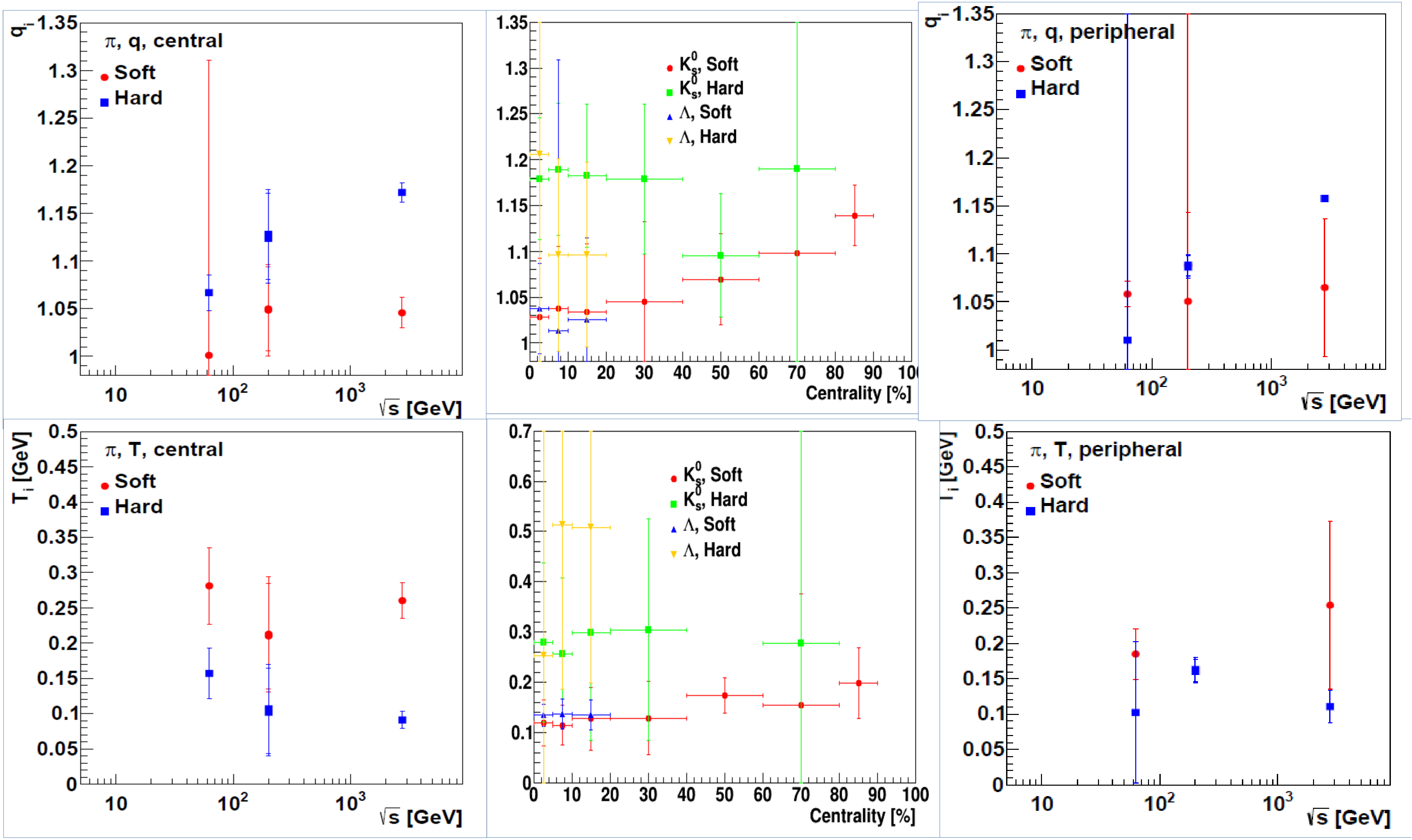
- **Non-extensive statistical approach in e^+e^- & pp**
  - Obtained Tsallis/Rényi entropies from the first principles.
  - Providing phyiscal meaning of $q=1-1/C + \Delta T^2/T^2$
  - Boltzmann Gibbs limit $C \to \infty$ & $\Delta T^2/T^2 \to 0$ $(q \to 1)$,
  - Tsallis – Pareto fits on spectra in e^+e^-, pp
  - Not working for larger system, like pA, AA and no flow.
- **Application of 'soft+hard' model in AA**
  - Tsallis – Pareto + Exp does not working.
  - Double Tsallis – Pareto measures non-extensitivity
  - SOFT: $q \to 1$, suggest Boltzmann Gibbs (QGP)
  - HARD: $q > 1.1$, Tsallis – Pareto like
  - Asimuthal anisotropy can be obtained too.

Thank you for your attention!


π, q, central
Soft
Hard
q_i
√s [GeV]
K_s^0, Soft
K_s^0, Hard
Λ, Soft
Λ, Hard
Centrality [%]
π, q, peripheral
π, T, central
T_i [GeV]
π, T, peripheral

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