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Hydrodynamics and heavy ions
Unruh effect
Radiation due to one point charge
model
Numerical results
Ellyptic flow due to radiation
Analytical results
Experimental fits
Summary

Elliptic Flow due to Radiation and its Role in Heavy-ion Collissions

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Outline

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   - Hydrodynamics and heavy ions
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- Fermi: thermodynamics for high-energy collisions
- Landau: particle production after expansion
- Several new models since then

Björken model

- fast thermalization
- expansion: 1D, $y = \eta$
- barion number free fluid
- plateau in rapidity spectrum

Landau model

- fast thermalization
- longitudinal + transversal expansion
- bell shapes in rapidity spectrum

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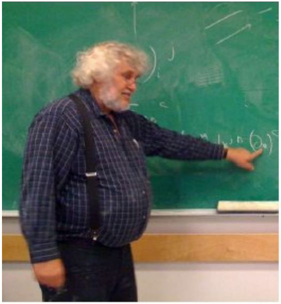
Unruh effect



Unruh effect

A relativistic observer with constant acceleration sees blacbody radiation, non accelerating: monocromatic plane wave. Interpretation: Relativistic Doppler-effect.

- No heath bath
- No Brownian-motion

- Planck spectrum

Unruh temperature

proportional to the accelaration (g) :

$$kT = \frac{\hbar g}{2\pi c}$$

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Radiation from an accelerating point charge

Linear accelaration

An accelarating point charge radiates, it can be interpreted as emission of photons.

T.S Biro Z. Szendi, Zs. Schram (1401.1987 –> EPJ A 2014)

Number of photons: $d^3N = \frac{d^3k}{2k_0(2\pi)^3} \sum |\epsilon \cdot J(k)|^2$

- J(k):source term $J^i(k) = q \int e^{ik\cdot x(\tau)}\, u^i(\tau)\, d\tau,$
- modified source: $\epsilon \cdot J(k) = q \int_{\tau_1}^{\tau_2} e^{ik\cdot x(\tau)} \frac{d}{d\tau} \left(\frac{\epsilon\cdot u}{k\cdot u}\right)\, d\tau$
- $k_i = k_\perp\, (\cosh\eta, \sinh\eta, \cos\psi, \sin\psi)$

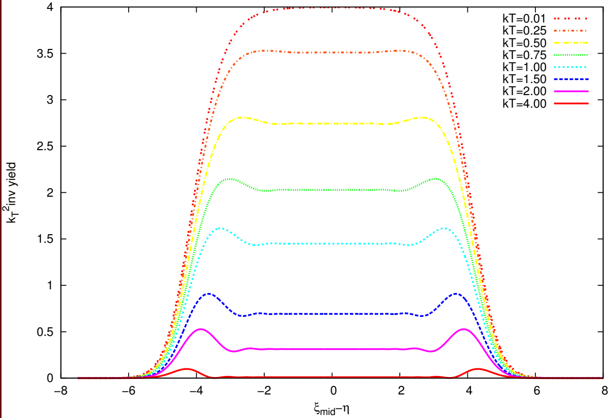
Rapidity distribution

$$k_\perp^2 \frac{dN}{k_\perp dk_\perp d\eta} =$$
$$2\alpha_e \left| \int_{v_1}^{v_2} e^{i\ell k_\perp \gamma v} dv \right|^2$$

Rapidity distr. for large k:

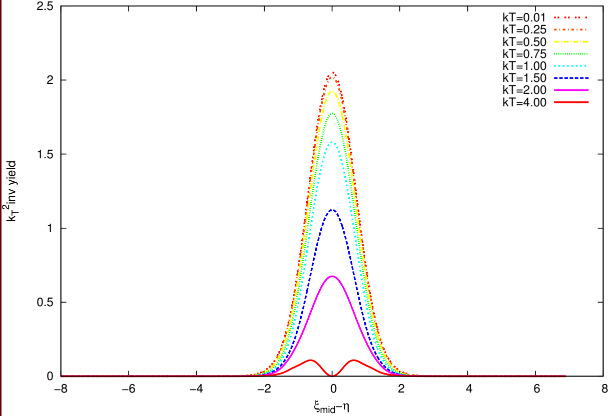
$$\frac{d^2N}{k_\perp dk_\perp d\eta} = N_0\, e^{-\hbar c k_\perp / k_B T}$$
$$T = \frac{\hbar c}{2k_B \ell} = \pi\, T_{Unruh}$$

Semiclassical photon-rapidity spectra I.



T.S Biro Z. Szendi, Zs. Schram (1401.1987 –> EPJ A 2014)

- longer decelaration time
- plateau
- $lk_\perp > 0.5?$

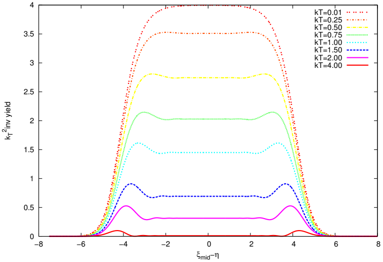


T.S Biro Z. Szendi, Zs. Schram (1401.1987 –> EPJ A 2014)

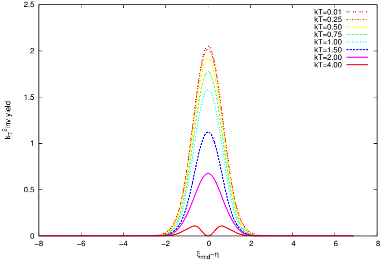
- short deceleration time
- $g = 1$
- bell shape

Bjorken and Landau - a comparison

Bjorken



Landau



- short / long time acceleration
- pleatau vs bell shape
- shift $g\tau = \pi$

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A dipole-like structure

Two point like sources

Two point charges moving in the opposite direction on parallel paths. The yield is:

$$Y \propto \left| A_1 e^{ik_\perp \frac{d}{2}\cos(\alpha-\psi)} + A_2 e^{-ik_\perp \frac{d}{2}\cos(\alpha-\psi)} \right|^2$$

- A_i: amplitudes
- α : detector angle
- ψ : angle of the event
- d : distance

After expending the square:

$$Y \propto |A_1|^2 + |A_2|^2 + A_1 A_2^* \, e^{ik_\perp \frac{d}{2} cos(\alpha-\psi)} + A_1^* A_2 \, e^{-ik_\perp \frac{d}{2} cos(\alpha-\psi)}$$

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Flow coefficients

Flow coefficients

Flow coefficients are the relative amplitudes of $\cos(n\theta)$ terms to the zeroth order term.

Using the Jacobi-Anger formula:

$$e^{ix\cos\Theta} = J_0(x) + 2\sum_{n=1}^{\infty} i^n J_n(x)\cos(n\Theta).$$

v_n

$$v_n = \frac{2R_n\, J_n(k_\perp d)}{|A_1|^2+|A_2|^2+R_0\, J_0(k_\perp d)}$$

- J_n : Bessel functions (first kind)
- $R_n := 2\Re e\left(i^n A_1 A_2^*\right) = 2\,|A_1|\,|A_2|\,\cos\left(\delta + n\frac{\pi}{2}\right)$
- δ : phase factor

Azimuthal anisotrophy for n= 2, v_2

For n = 2 the flow coefficient becomes:

$$v_2 = \frac{-2\,\varepsilon\, J_2(k_\perp d)\, cos(\Delta\varphi)}{1+\varepsilon\, J_0(k_\perp d)\, cos(\Delta\varphi)}$$

with $\varepsilon = \frac{2\,|A_1|\,|A_2|}{|A_1|^2+|A_2|^2} \leq 1$

If the following parametrization is made:

- $A_1 = e^{i\delta_0}$
- γ: ratio of the amplitudes
- $A_2 = e^{i\delta_0+\delta}$
- d : dipole size

The flow coefficient, v_2 becomes:

Final v_2

$$v_2 = \frac{-J_2(k_\perp d)\cos\delta}{\frac{1+\gamma^2}{2\gamma}+J_0(k_\perp d)\cos\delta}$$

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Amplitudes for the dipole

Two sources in the opposite direction:

- velocity from c to 0 - first charge
- second: velocity: from c to above 0 - second charge

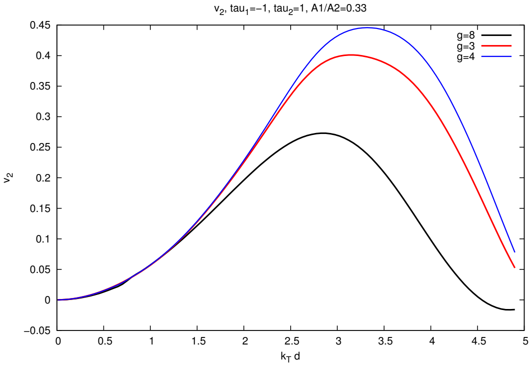
$$A_1 = \int_{-\infty}^{0} \mathrm{d}v \frac{e^{iv\Delta}}{(1+v^2)^{\frac{3}{2}}} = \frac{\Delta}{2}\left(2K_1(\Delta) + i\pi\left(K_1(\Delta) - L_{-1}(\Delta)\right)\right)$$

$$A_2 = \int_{\infty}^{v_2} \mathrm{d}v \frac{e^{iv\Delta}}{(1+v^2)^{\frac{3}{2}}} = -A_1^* + \int_{0}^{v_2} \mathrm{d}v \frac{e^{iv\Delta}}{(1+v^2)^{\frac{3}{2}}},$$

where the parameters are:

- K_n Bessel functions
- $\Delta = \frac{k_\perp}{g}$

Numerical results for v_2



- different accelarating times
- no averaging

Figure : Numerical results for v_2, different lines different accelerations

v_2 analytically

Analytically averaging over the phase factor results in:

Averaged v_2

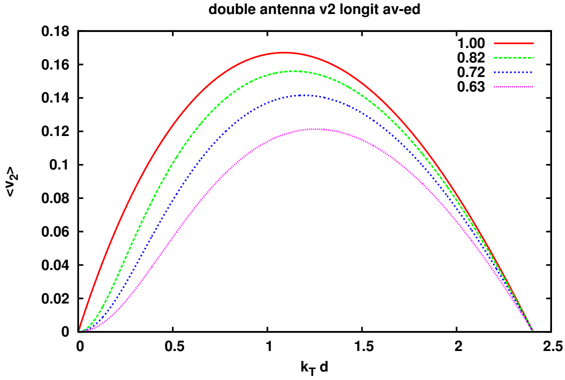
$$\langle v_2 \rangle = \frac{J_2(dk_\perp)}{J_0(dk_\perp)} \left(\frac{1}{\sqrt{1 - \frac{4\gamma^2}{(1+\gamma^2)^2} J_0^2(dk_\perp)}} - 1 \right)$$

- leading term: from dipole
- geometric factor: F

v_2

$$\langle v_2 \rangle_{fit} = F \cdot \langle v_2 \rangle (dk_\perp, \gamma)$$

Analytical results for averaged v_2



- $F = 1$
- Only the dipole term exists
- lines: different aplitude ratios

Figure : Analytical results for averaged v_2

Fits to inclusive photons (ALICE)

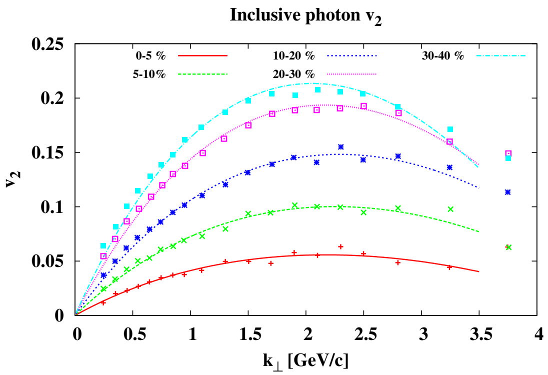


Figure : v_2 for inclusive photons

J. Phys. Conf. Ser. 446, 012028 (2013)

- Pb-Pb collissions
- several centrality classes
- $\sqrt{s_{NN}} = 2.76 TeV$

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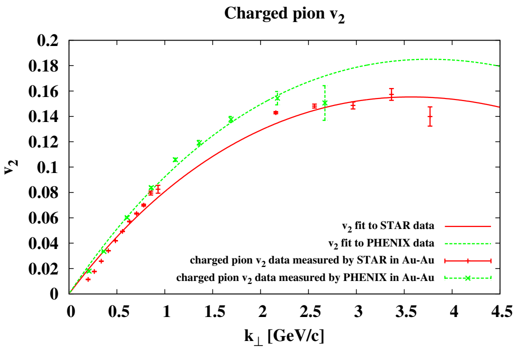
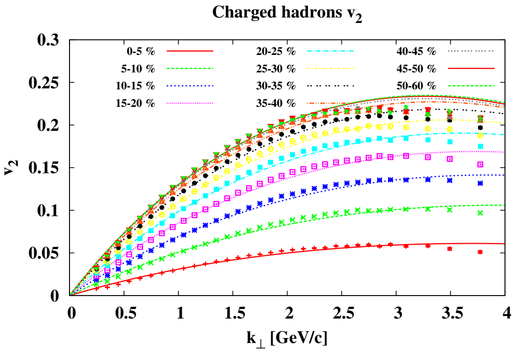


- Au-Au collisions
- $\sqrt{s_{NN}}$ = 62 GeV / 200 GeV
- $\gamma \approx 1$
- $d = 0.06[fm]$

Figure : v_2 of charged pions, data is from STAR (red) and PHENIX (green)

Phys. Rev. C 75, 054906 (2007)

Phys. Rev. Lett. 91, 182301 (2003)



- Au-Au collisions
- $\sqrt{s_{NN}} = 200 GeV$
- $\gamma \approx 1$
- d = 0.07 [fm]

Figure : Fits for hadron data from PHENIX

Phys. Rev. Lett. 105, 062301 (2010), arXiv:1003.5586v2

Summarizing the fit parameters

Summarizing the results from the fits:

- d is stable — d $\approx$ 0.07 / ≈ 0.1 [fm]
- γ is stable — $\gamma \approx$ 1
- fits for different centrality — $0 - 60\%$
- comes close to data

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The model

After Fourier expansion the yield can be written:

$$Y = Y_0 + Y_2\, F\, cos(2\varphi) \qquad \text{with F: } \frac{A^2 - B^2}{A^2 + B^2}$$

How to attach this ellipse to the collisions?

- nuclei disks with radius R
- intersection ellipse
- F = $\frac{G}{2b}$
- b: impact parameter
- subhadronic dipoles

Dipoles can be ordered parallel or perpendicular to the reaction plane, but F remains the same!

Geometric factor II

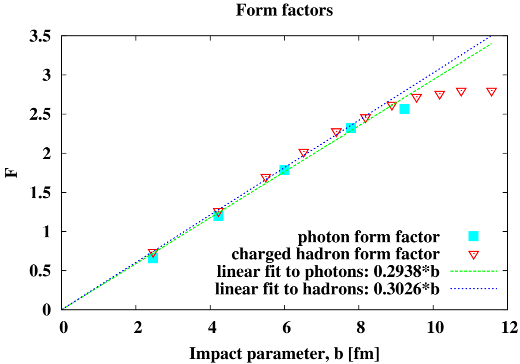

Figure : F(b) rrelation for hadrons and inclusive photons,centrality impact parameter relationship: Phys. Rev. C 88, 044909 (2013

- Linear almost everywhere
- $R \approx 1.67$ [fm]
- collective source?

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One point charge

- no initial fluctuations or Hydro
- charge: linear trajectory
- Landau and Björken like behaviour

Two charges

- Dipole like sturcture
- v_2 calculations
- Fits to experimental data

(Illusory) flow without hydro