Chiral phase boundary and thermodynamical quantities from an improved vector meson extended Polyakov quark meson model

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Overview

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   - Vector meson extended PQM model – Polyakov loop
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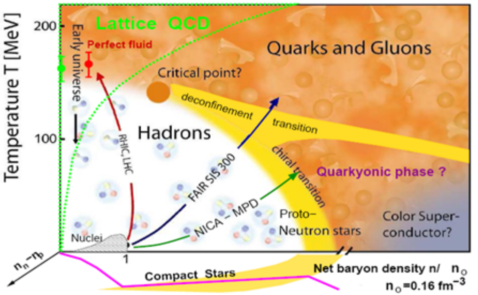
QCD phase diagram

Phase diagram in the $T - \mu_\mathrm{B} - \mu_\mathrm{I}$ space



- At $\mu_\mathrm{B} = 0$
  $T_c = 151(3)$ MeV
  Y. Aoki, *et al.*, PLB **643**, 46 (2006)
- Is there a CEP?
- At $T = 0$ in μ_B where is the phase boundary?
- Behavior as a function of $\mu_\mathrm{I}/\mu_\mathrm{S}$?

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)

Chiral symmetry, chiral models

If the quark masses are zero (chiral limit) $\Longrightarrow$ QCD invariant under the following global transformation (chiral symmetry):

$$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A = SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

$U(1)_V$ term $\longrightarrow$ baryon number conservation

$U(1)_A$ term $\longrightarrow$ broken through axial anomaly

$SU(3)_A$ term $\longrightarrow$ broken down by any quark mass

$SU(3)_V$ term $\longrightarrow$ broken down to $SU(2)_V$ if $m_u = m_d \neq m_s$
$\longrightarrow$ totally broken if $m_u \neq m_d \neq m_s$ (realized in nature)

Since QCD is very hard to solve $\longrightarrow$ low energy effective models $\longrightarrow$ reflecting the global symmetries of QCD $\longrightarrow$ degrees of freedom: observable particles instead of quarks and gluons

Linear realization of the symmetry $\longrightarrow$ linear sigma model

Lagrangian (2/1)

$$\mathcal{L} = \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2$$
$$+ c_1(\det\Phi + \det\Phi^\dagger) + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2)$$
$$+ \mathrm{Tr}\left[\left(\frac{m_1^2}{2}\mathbb{1} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\})$$
$$+ \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger)$$
$$+ \bar{\Psi} i\gamma_\mu D^\mu\Psi - g_F\bar{\Psi}(\Phi_S + i\gamma_5\Phi_{PS})\Psi,$$

$$\begin{aligned}
D^\mu\Phi &= \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3,\Phi],\\
L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\},\\
R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\},\\
D^\mu\Psi &= \partial^\mu\Psi - iG^\mu\Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a.
\end{aligned}$$

+ Polyakov loop potential

Lagrangian (2/2)

the matter and external fields are

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8}(\rho_i^\mu - b_i^\mu)T_i, \quad L^\mu = \sum_{i=0}^{8}(\rho_i^\mu + b_i^\mu)T_i, \quad \Delta = \sum_{i=0}^{8}\delta_i T_i$$

$$\Psi = (u, d, s)^{\mathrm{T}}$$

non strange – strange base:

$$\xi_N = \sqrt{2/3}\xi_0 + \sqrt{1/3}\xi_8,$$
$$\xi_S = \sqrt{1/3}\xi_0 - \sqrt{2/3}\xi_8, \qquad \xi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle\sigma_{N/S}\rangle \equiv \bar{\sigma}_{N/S}$

Included fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \quad (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_S = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} \quad (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars: $a_0(980 \text{ or } 1450), K_S(800 \text{ or } 1430)$,

2 of $f_0(500, 980, 1370, 1500, 1710)$

Structure of scalar mesons

	Mass (MeV)	width (MeV)	decays
$a_0(980)$	980 ± 20	50 – 100	$\pi\pi$ dominant
$a_0(1450)$	1474 ± 19	265 ± 13	$\pi\eta$, $\pi\eta'$, $K\bar{K}$
$K_s(800) = \kappa$	682 ± 29	547 ± 24	$K\pi$
$K_s(1430)$	1425 ± 50	270 ± 80	$K\pi$ dominant
$f_0(500) = \sigma$	400–550	400 – 700	$\pi\pi$ dominant
$f_0(980)$	980 ± 20	40 – 100	$\pi\pi$ dominant
$f_0(1370)$	1200–1500	200 – 500	$\pi\pi \approx 250$, $K\bar{K} \approx 150$
$f_0(1500)$	1505 ± 6	109 ± 7	$\pi\pi \approx 38$, $K\bar{K} \approx 9.4$
$f_0(1710)$	1722 ± 6	135 ± 7	$\pi\pi \approx 30$, $K\bar{K} \approx 71$

Possible scalar states: $\bar{q}q$, tetraquarks, glueballs

scalar $\bar{q}q$ nonet content: 1 a_0, 1 K_s, and 2 f_0: $a_0^{\bar{q}q} \to a_0(1450)$, $K_s^{\bar{q}q} \to K_s(1430)$, $f_0^{L,\bar{q}q} \to f_0(1370)$, $f_0^{L,\bar{q}q} \to f_0(1710)$

Parganlija et al., PRD87, 014011

tetraquarks: $f_0(500)$, $f_0(980)$, $a_0(980)$, $K_s(800)$?

glueballs: $f_0(1500)$?

Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star-} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra is not $\longrightarrow$ SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \bar{\sigma}_{N/S} \qquad \bar{\sigma}_{N/S} \equiv <\sigma_{N/S}>$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)]$:

$$\begin{aligned}
\eta_N - f_{1N}^\mu &: \quad -g_1\bar{\sigma}_N f_{1N}^\mu \partial_\mu \eta_N, \\
\pi - a_1^\mu &: \quad -g_1\bar{\sigma}_N(a_1^{\mu+}\partial_\mu\pi^- + a_1^{\mu 0}\partial_\mu\pi^0) + \mathrm{h.c.}, \\
\eta_S - f_{1S}^\mu &: \quad -\sqrt{2}g_1\bar{\sigma}_S f_{1S}^\mu \partial_\mu \eta_S, \\
K_S - K_\mu^\star &: \quad \frac{ig_1}{2}(\sqrt{2}\bar{\sigma}_S - \bar{\sigma}_N)(\bar{K}_\mu^{\star 0}\partial^\mu K_S^0 + K_\mu^{\star -}\partial^\mu K_S^+) + \mathrm{h.c.}, \\
K - K_1^\mu &: \quad -\frac{g_1}{2}(\bar{\sigma}_N + \sqrt{2}\bar{\sigma}_S)(K_1^{\mu 0}\partial_\mu \bar{K}^0 + K_1^{\mu +}\partial_\mu K^-) + \mathrm{h.c.}.
\end{aligned} \tag{1}$$

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp\left[i \int_0^\beta d\tau\, G_4(\vec{x}, \tau) \right]$$

$\hookrightarrow$ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement transition

low T: confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$
high T: deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

- Polyakov gauge: $G_4(\vec{x}, \tau) = G_4(\vec{x})$, plus gauge rotation to diagonal form in color space
- further simplification: $\vec{x}$-independence

$$\hookrightarrow \quad L = e^{i\beta G_4} = \mathrm{diag}(a, b, c) \left(\overset{!}{\in} SU(N_c) \right); \quad a, b, c \in \mathbb{Z}$$

$\hookrightarrow$ use this to calculate partition function of free quarks on constant gluon background

Effects of Polyakov loops on FD statistics

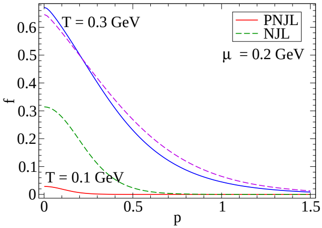
Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p-\mu_q) \longrightarrow f_\Phi^+(E_p) = \frac{\left(\bar\Phi + 2\Phi e^{-\beta\left(E_p-\mu_q\right)}\right) e^{-\beta\left(E_p-\mu_q\right)} + e^{-3\beta\left(E_p-\mu_q\right)}}{1+3\left(\bar\Phi + \Phi e^{-\beta\left(E_p-\mu_q\right)}\right) e^{-\beta\left(E_p-\mu_q\right)} + e^{-3\beta\left(E_p-\mu_q\right)}}$$

$$f(E_p+\mu_q) \longrightarrow f_\Phi^-(E_p) = \frac{\left(\Phi + 2\bar\Phi e^{-\beta\left(E_p+\mu_q\right)}\right) e^{-\beta\left(E_p+\mu_q\right)} + e^{-3\beta\left(E_p+\mu_q\right)}}{1+3\left(\Phi + \bar\Phi e^{-\beta\left(E_p+\mu_q\right)}\right) e^{-\beta\left(E_p+\mu_q\right)} + e^{-3\beta\left(E_p+\mu_q\right)}}$$

$\Phi, \bar\Phi \to 0 \Longrightarrow f_\Phi^\pm(E_p) \to f(3(E_p \pm \mu_q))$ $\quad \Phi, \bar\Phi \to 1 \Longrightarrow f_\Phi^\pm(E_p) \to f(E_p \pm \mu_q)$

three-particle state appears: mimics confinement of quarks within baryons



the effect of the Polyakov loop is more relevant for $T < T_c$

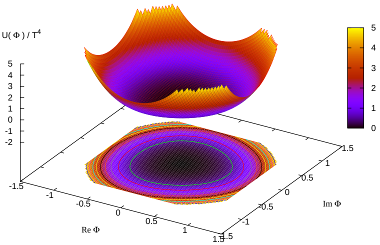
at $T = 0$ there is no difference between models with and without Polyakov loop: $\Theta(3(\mu_q - E_p)) \equiv \Theta((\mu_q - E_p))$

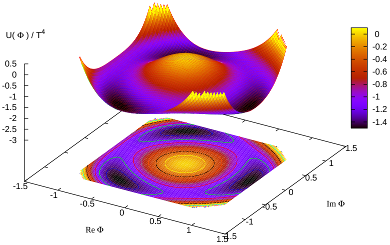
H. Hansen et al., PRD75, 065004

Polyakov loop potential

"Color confinement"
$\langle\Phi\rangle = 0 \longrightarrow$ no breaking of $\mathbb{Z}_3$
one minimum

"Color deconfinement"
$\langle\Phi\rangle \neq 0 \longrightarrow$ spontaneous breaking of $\mathbb{Z}_3$
minima at $0, 2\pi/3, -2\pi/3$
one of them spontaneously selected





from H. Hansen et al., PRD75, 065004 (2007)

Form of the potential

I.) Simple polynomial potential invariant under $\mathbb{Z}_3$ and charge conjugation: R.D.Pisarski, PRD 62, 111501

$$\frac{\mathcal{U}^{\mathrm{YM}}_{\mathrm{poly}}(\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2$$

with $b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2} + a_3\frac{T_0^3}{T^3}$

II.) Logarithmic potential coming from the $SU(3)$ Haar measure of group integration K. Fukushima, Phys. Lett. **B591**, 277 (2004)

$$\frac{\mathcal{U}^{\mathrm{YM}}_{\mathrm{log}}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi\bar{\Phi} + b(T)\ln\left[1-6\Phi\bar{\Phi}+4\left(\Phi^3+\bar{\Phi}^3\right)-3\left(\Phi\bar{\Phi}\right)^2\right]$$

with $a(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2}$, $\quad b(T) = b_3\frac{T_0^3}{T^3}$

$\mathcal{U}^{\mathrm{YM}}\left(\Phi,\bar{\Phi}\right)$ models the free energy of a pure gauge theory
$\longrightarrow$ the parameters are fitted to the pure gauge lattice data

Improved Polyakov loop potential

Previous potentials describe successfully the first order phase transition of the pure $SU(3)$ Yang–Mills

$\hookrightarrow$ taking into account the gluon dynamics (quark polarization of gluon propagator) $\longrightarrow$ QCD glue potential

$\hookrightarrow$ can be implemented by changing the reduced temperature

$$t_{\text{glue}} \equiv \frac{T - T_c^{\text{glue}}}{T_c^{\text{glue}}}, \quad t_{\text{YM}} \equiv \frac{T^{\text{YM}} - T_c^{\text{YM}}}{T_c^{\text{YM}}}$$

$$t_{\text{YM}}(t_{\text{glue}}) \approx 0.57 t_{\text{glue}}$$

$$\frac{\mathcal{U}^{\text{glue}}}{T^4}(\Phi, \bar{\Phi}, t_{\text{glue}}) = \frac{\mathcal{U}^{\text{YM}}}{(T^{\text{YM}})^4}(\Phi, \bar{\Phi}, t_{\text{YM}}(t_{\text{glue}}))$$

L. M. Haas *et al.*, PRD 87, 076004 (2013)

Field equations for the condensates ($\phi_{N/S}$)

Hybrid approach: fermions at one-loop, mesons at tree-level
$\longrightarrow$ calculate Ω the grand canonical potential

$$\Omega(T,\mu_q) = U^{\text{tree}}_{\text{meson}}(\langle\phi\rangle) + \Omega^{vac}_{\bar{q}q} + \Omega^{T}_{\bar{q}q}(T,\mu_q) + \mathcal{U}^{\text{glue}}(\Phi,\bar{\Phi},t_{\text{glue}}(T))$$

$$\imath.) \qquad \frac{\partial\Omega}{\partial\bar{\sigma}_N} = \frac{\partial\Omega}{\partial\bar{\sigma}_S}\bigg|_{\bar{\sigma}_N=\phi_N,\bar{\sigma}_S=\phi_S} = 0$$

$$m_0^2\phi_N + \left(\lambda_1 + \frac{1}{2}\lambda_2\right)\phi_N^3 + \lambda_1\phi_N\phi_S^2 - h_N + \frac{g_F}{2}N_c\left(\langle u\bar{u}\rangle_T + \langle d\bar{d}\rangle_T\right) = 0$$

$$m_0^2\phi_S + (\lambda_1+\lambda_2)\,\phi_S^3 + \lambda_1\phi_N^2\phi_S - h_S + \frac{g_F}{\sqrt{2}}N_c\langle s\bar{s}\rangle_T = 0$$

$$\langle q\bar{q}\rangle_T = -4m_q\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_q(p)}\left(1 - f^-_\Phi(E_q(p)) - f^+_\Phi(E_q(p))\right)$$

Field equations for the Polyakov-loop variables

$$ii.) \qquad \frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \bar{\Phi}}\Big|_{\bar{\sigma}_N=\phi_N, \bar{\sigma}_S=\phi_S} = 0,$$

$$- \frac{d}{d\Phi}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right) + \frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^-(p)}}{g_q^-(p)} + \frac{e^{-2\beta E_q^+(p)}}{g_q^+(p)}\right) = 0$$

$$- \frac{d}{d\bar{\Phi}}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right) + \frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^+(p)}}{g_q^+(p)} + \frac{e^{-2\beta E_q^-(p)}}{g_q^-(p)}\right) = 0$$

$$g_q^+(p) = 1 + 3\left(\bar{\Phi} + \Phi e^{-\beta E_q^+(p)}\right) e^{-\beta E_q^+(p)} + e^{-3\beta E_q^+(p)}$$

$$g_q^-(p) = 1 + 3\left(\Phi + \bar{\Phi} e^{-\beta E_q^-(p)}\right) e^{-\beta E_q^-(p)} + e^{-3\beta E_q^-(p)}$$

$$E_q^\pm(p) = E_q(p) \mp \mu_B/3, \ \ E_{u/d}(p) = \sqrt{p^2 + m_{u/d}^2}, E_s(p) = \sqrt{p^2 + m_s^2}$$

Curvature masses

$$\mathcal{M}^2_{i,ab} = \frac{\partial^2\Omega(T,\mu_f)}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\bigg|_{\min} = m^2_{i,ab} + \Delta_0 m^2_{i,ab} + \Delta_T m^2_{i,ab},$$

$m^2_{i,ab} \longrightarrow$ tree-level mass matrix,
$\Delta_{0/T} m^2_{i,ab} \longrightarrow$ fermion vacuum/thermal fluctuation,

$$\Delta_0 m^2_{i,ab} = \frac{\partial^2\Omega^{\text{vac}}_{q\bar{q}}}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\bigg|_{\min} = -\frac{3}{8\pi^2}\sum_{f=u,d,s}\left[\left(\frac{3}{2}+\log\frac{m_f^2}{M^2}\right)m^{2(i)}_{f,a}m^{2(i)}_{f,b} + m_f^2\left(\frac{1}{2}+\log\frac{m_f^2}{M^2}\right)m^{2(i)}_{f,ab}\right],$$

$$\Delta_T m^2_{i,ab} = \frac{\partial^2\Omega^{\text{th}}_{q\bar{q}}}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\bigg|_{\min} = 6\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_f(p)}\bigg[\left(f_f^+(p)+f_f^-(p)\right)\left(m^{2(i)}_{f,ab} - \frac{m^{2(i)}_{f,a}m^{2(i)}_{f,b}}{2E_f^2(p)}\right) + \left(B_f^+(p)+B_f^-(p)\right)\frac{m^{2(i)}_{f,a}m^{2(i)}_{f,b}}{2TE_f(p)}\bigg],$$

where $m^{2(i)}_{f,a} \equiv \partial m_f^2/\partial\varphi_{i,a}$, $m^{2(i)}_{f,ab} \equiv \partial^2 m_f^2/\partial\varphi_{i,a}\partial\varphi_{i,b}$

Curvature masses

Table : The first and second derivatives of the squared quark mass with respect to the scalar (S) and pseudoscalar (P) meson fields evaluated at the minimum in the $N-S$ basis. In the first two columns a summation over $\ell \in \{u, d\}$ is understood.

i	ab	$m^{2\,(i)}_{\ell,a} m^{2\,(i)}_{\ell,b}/g^4$	$m^{2\,(i)}_{\ell,ab}/g^2$	$m^{2\,(i)}_{s,a} m^{2\,(i)}_{s,b}/g^4$	$m^{2\,(i)}_{s,ab}/g^2$
S	11	$\frac{1}{2}\phi_N^2$	1	0	0
S	44	0	$\frac{Z^2_{K^\star}\phi_N}{\phi_N-\sqrt{2}\phi_S}$	0	$\frac{-\sqrt{2}Z^2_{K^\star}\phi_S}{\phi_N-\sqrt{2}\phi_S}$
S	NN	$\frac{1}{2}\phi_N^2$	1	0	0
S	SS	0	0	ϕ_S^2	1
P	11	0	Z_π^2	0	0
P	44	0	$\frac{Z^2_K\phi_N}{\phi_N+\sqrt{2}\phi_S}$	0	$\frac{\sqrt{2}Z^2_K\phi_S}{\phi_N+\sqrt{2}\phi_S}$
P	NN	0	$Z^2_{\eta_N}$	0	0
P	SS	0	0	0	$Z^2_{\eta_S}$

Determination of the parameters

14 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F) \longrightarrow$ determined by the min. of χ^2:

$$\chi^2(x_1,\ldots,x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1,\ldots,x_N) - Q_i^{\rm exp}}{\delta Q_i}\right]^2 ,$$

$(x_1,\ldots,x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1,\ldots,x_N) \longrightarrow$ from the model, $Q_i^{\rm exp} \longrightarrow$ PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Curvature masses $\rightarrow$ 16 physical quantities: $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities: $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$

Features of our approach

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables with $\mathcal{U}^{\mathrm{YM}}$ or $\mathcal{U}^{\mathrm{glue}}$
- constituent quarks
- Four order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}) \longrightarrow$ four coupled T/μ_B-dependent equations
- Fermion vacuum fluctuations
- Fermion thermal fluctuations
- Fermion contributions to the tree-level meson masses $\longrightarrow$ curvature masses
- $+$ Thermal pion fluctuations for the pressure and other thermodynamical quantities

Consequence of scalar mesons sector

	Mass (MeV)	width (MeV)	decays
$a_0(980)$	980 ± 20	$50 - 100$	$\pi\pi$ dominant
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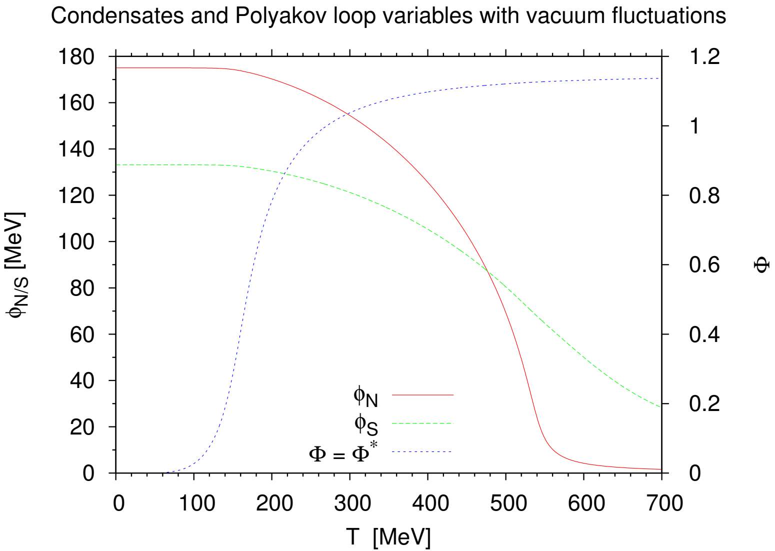
$f_0(500) = \sigma$	400–550	$400 - 700$	$\pi\pi$ dominant
$f_0(980)$	980 ± 20	$40 - 100$	$\pi\pi$ dominant
$f_0(1370)$	1200–1500	$200 - 500$	$\pi\pi \approx 250$, $K\bar{K} \approx 150$
$f_0(1500)$	1505 ± 6	109 ± 7	$\pi\pi \approx 38$, $K\bar{K} \approx 9.4$
$f_0(1710)$	1722 ± 6	135 ± 7	$\pi\pi \approx 30$, $K\bar{K} \approx 71$

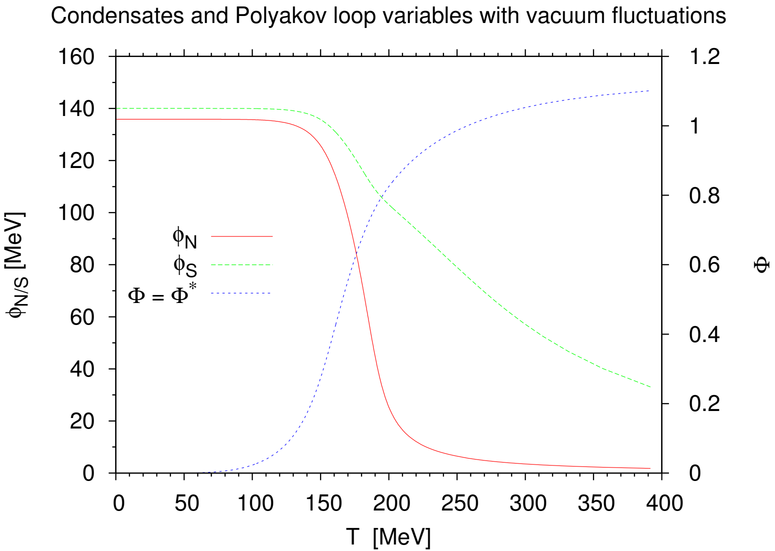
$\hookrightarrow$ We have 40 assginement possibilities!

Different parameterizations can give different thermodynamical behavior

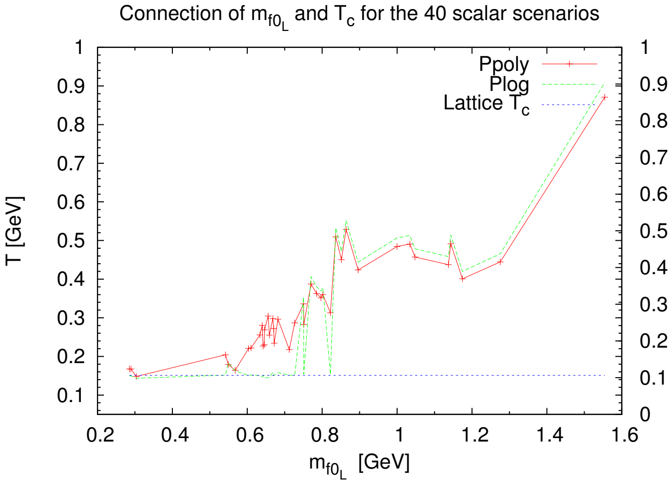
With high mass scalars, $m_{f_0^L} = 1326$ MeV



With low mass scalars, $m_{f_0^L} = 402$ MeV

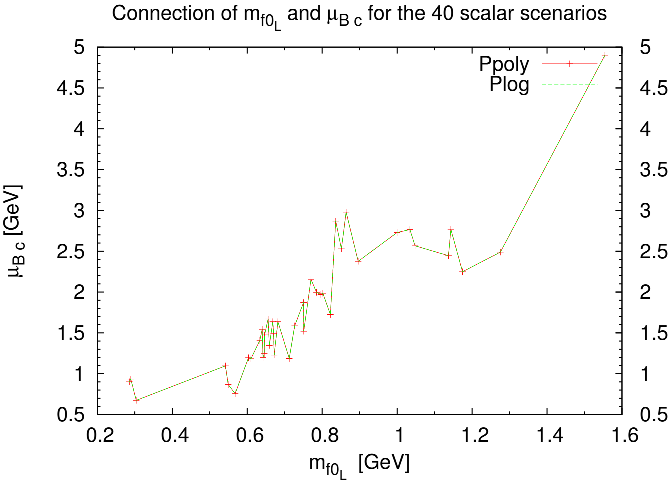


T_c at $\mu_B = 0$ for various parameterizations



40 parameterizations $\longrightarrow$ all the possible combinations for the masses of the scalar sector

μ_{B_c} at $T = 0$ for various parameterizations

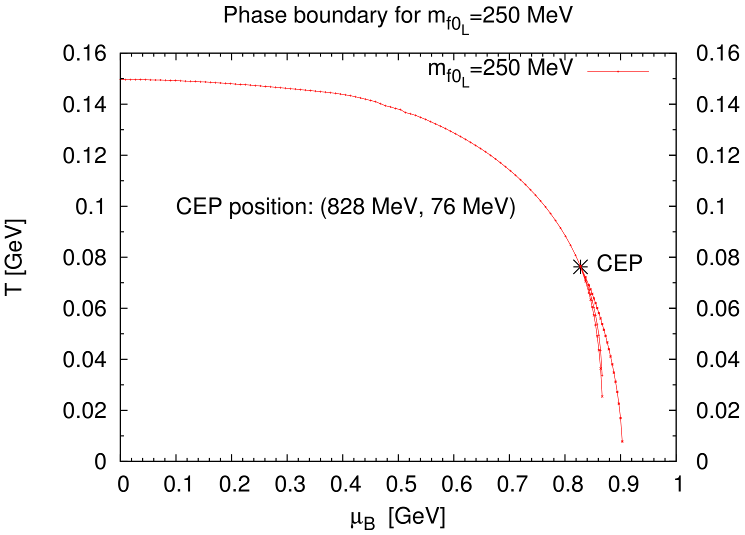


40 parameterizations $\longrightarrow$ all the possible combinations for the masses of the scalar sector

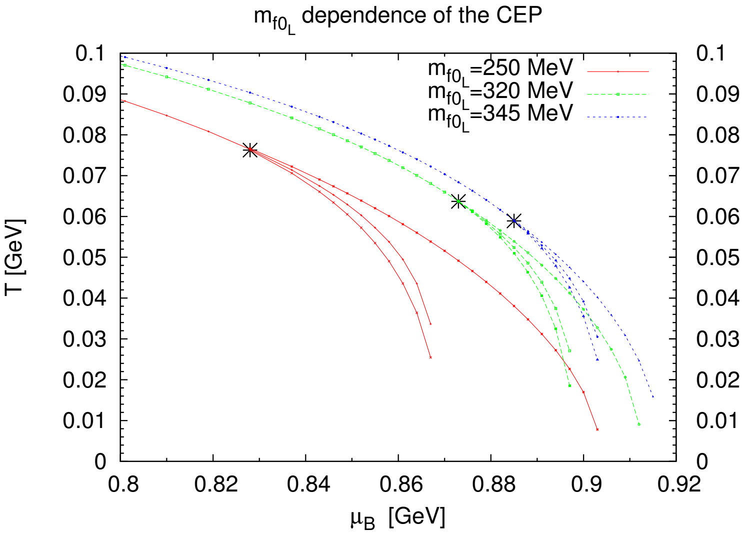
Remarks

- In all 40 cases the best χ^2 solution was chosen
- Only parameterizations, which produced $m_{f_0^L} \lesssim 800$ MeV can have $T_c \approx 151$ MeV (lattice data)
- Only parameterizations, which produced $m_{f_0^L} \lesssim 400$ MeV can have 1[st] order transition in $\mu_B \implies$ there is CEP
- If $T_c \approx 150$ MeV and the CEP exists $\implies m_{a_0}$ and m_{K_S} are also below 1 GeV
- Important note: scale dependence $\longrightarrow$ can change the numbers

The phase boundary



CEP for different f_0^L masses



Calculation of thermodynamical quantities

pressure: $p = \frac{\partial(T \ln Z)}{\partial V} = -\Omega$

entropy density: $s = \frac{1}{V}\frac{\partial(T \ln Z)}{\partial T} = \frac{\partial p}{\partial T}$

energy density $(\mu_B = 0)$: $\epsilon = -p + Ts$

mesonic thermal 1-loop contribution to the pressure:

$$p_{\text{meson}} = -\Omega^{\text{1-loop},T}_{meson} = -NT \int \frac{d^3p}{(2\pi)^3} \ln\left(1 - e^{-\beta\omega(p)}\right)$$

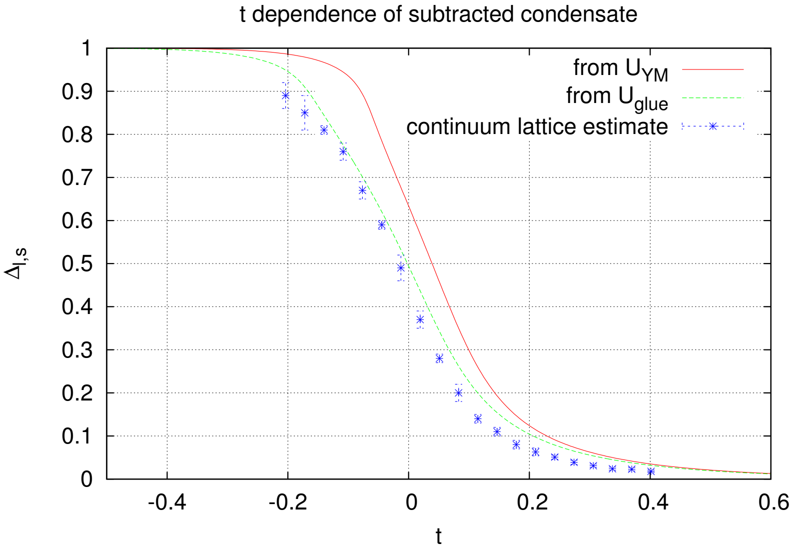
where, $\omega(p) = \sqrt{p^2 + m^2}$

to compare with the lattice $\longrightarrow$

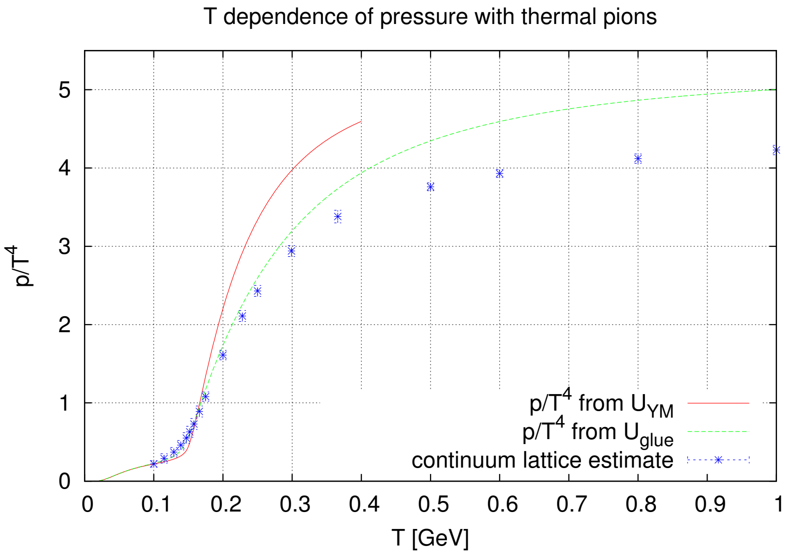
subtracted condensate: $\Delta_{l,s} = \frac{\Phi_N - \frac{h_N}{h_S}\cdot\Phi_S|_T}{\Phi_N - \frac{h_N}{h_S}\cdot\Phi_S|_{T=0}}$

interaction measure: $I/T^4 = (\epsilon - 3p)/T^4$

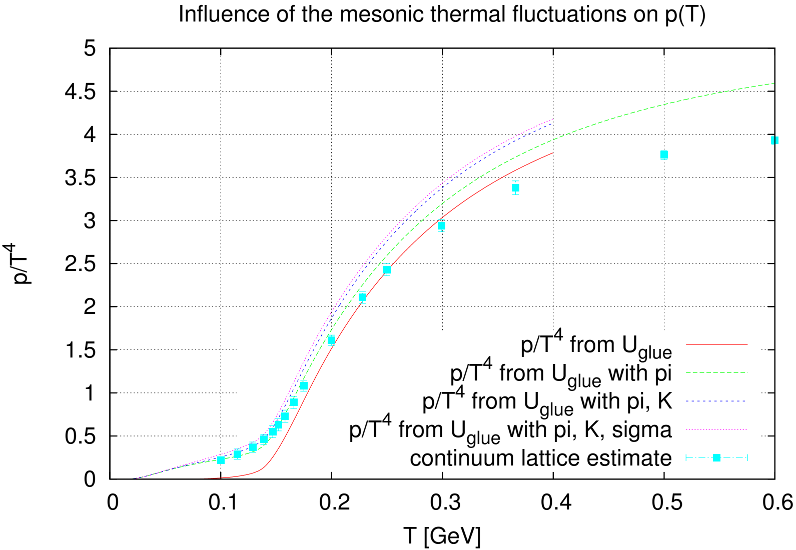
The subtracted condensate



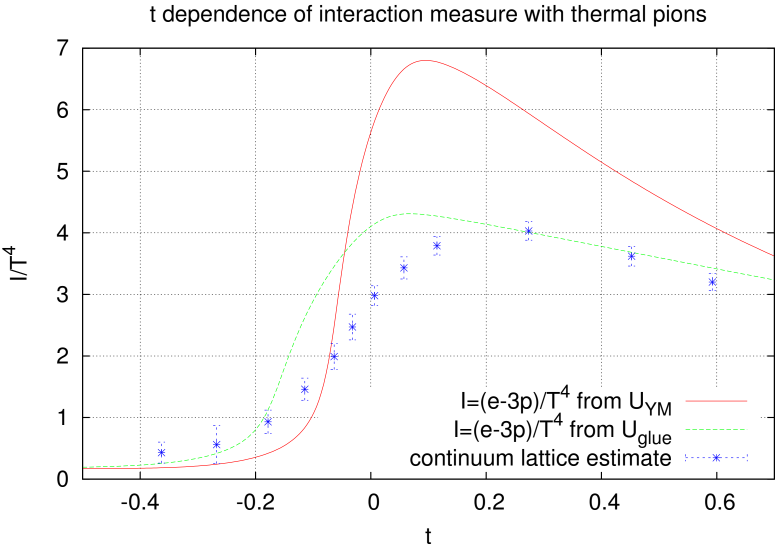
The normalized pressure



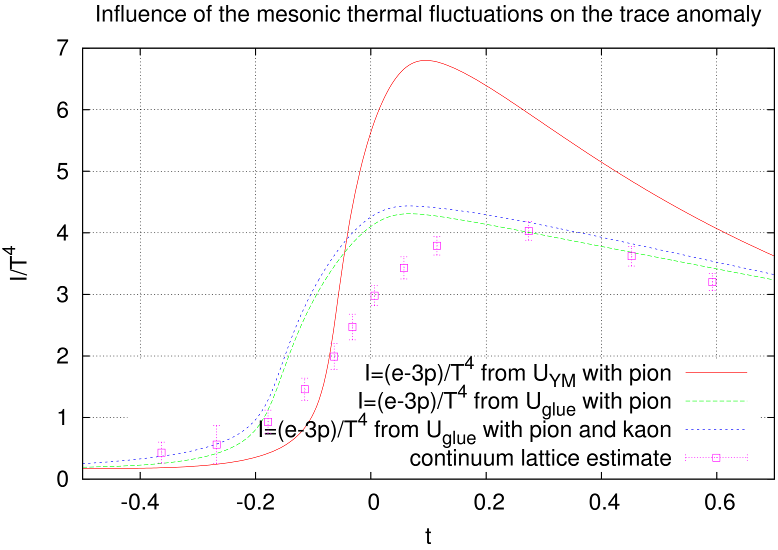
Different mesonic thermal fluctuations to the pressure



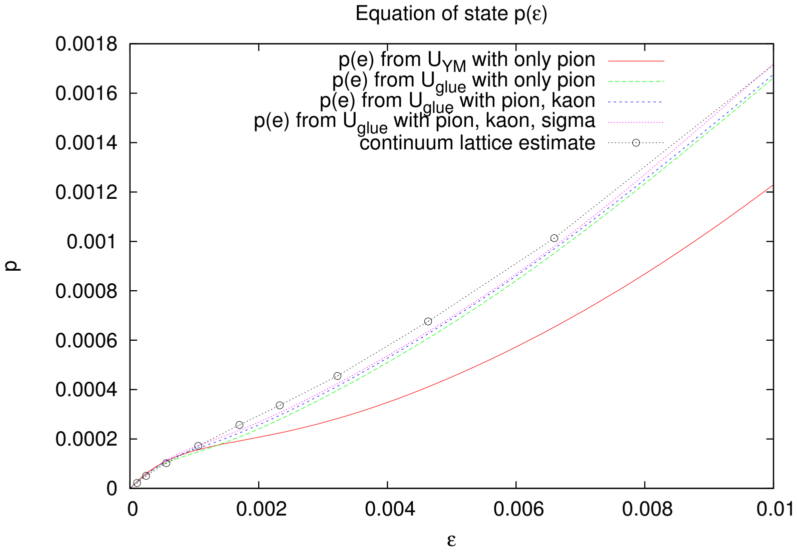
Interaction measure or trace anomaly



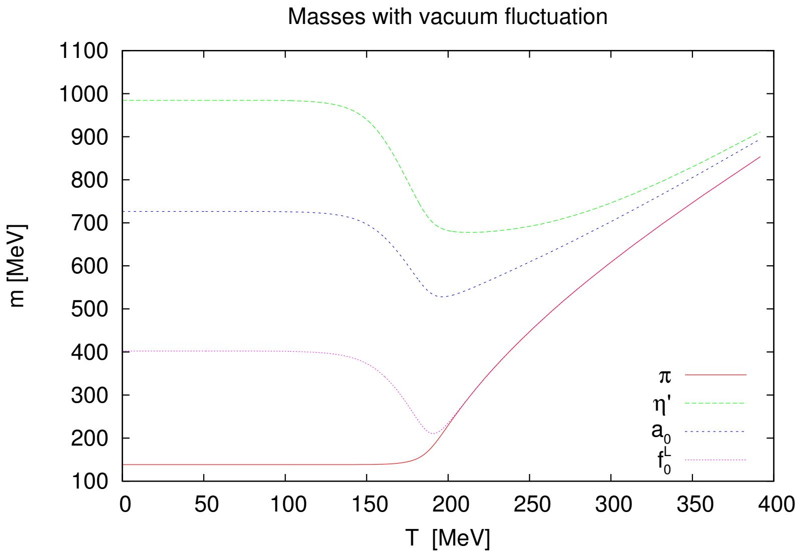
Influence of thermal mesons on the interaction measure



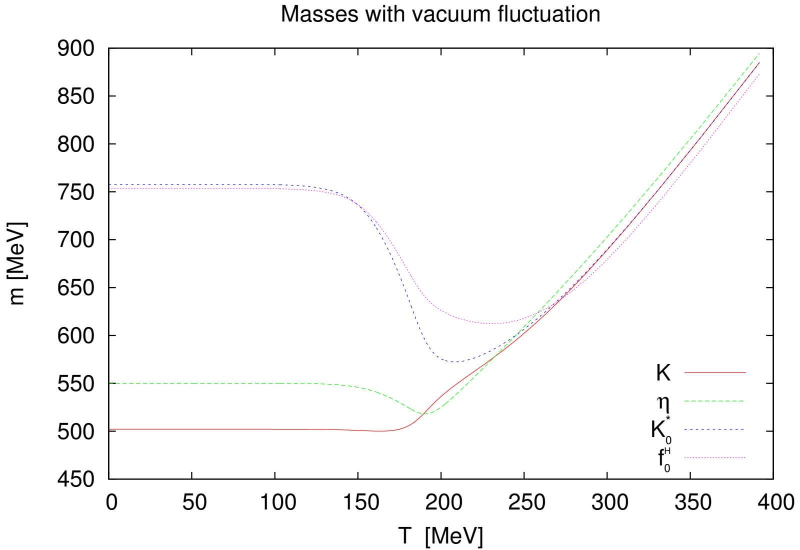
Equation of state at $\mu_B = 0$



π, η', a_0, f_0^L masses



K, η, $K^\star$, f_0^H masses



Summary

- Thermodynamics of a vector meson extended Polyakov quark meson was investigated
- We used a hybrid approach: fermion vacuum/thermal fluctuations (has the largest contribution), bosons at tree–level (except in pressure and such)
- We investigated the 40 possible scalar parameterization scenarios
- At finite T/μ_B there was 4 coupled equations for the 4 order parameters
- Various thermodynamical quantities were calculated, which show quiet good agreement with lattice results, if we use the improved Polyakov potential

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- At finite T/μ_B there was 4 coupled equations for the 4 order parameters
- Various thermodynamical quantities were calculated, which show quiet good agreement with lattice results, if we use the improved Polyakov potential
- Model can be used to give predictions at finite densities (masses, decay widths, thermodynamical quantities)

Thank you for your attention!

μ_B dependence of the π, η, η', K masses