Observables from a rotating hydrodynamical model

Márton NAGY[1], Viktor Könye[1], Tamás Csörgő[2]

[1]Eötvös University, Dept. of Atomic Physics
[2]MTA Wigner RCP, Budapest, Hungary

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- Rotation in high-energy heavy-ion collision
- Buda-Lund model
- A rotating exact solution of fireball hydrodynamics
- Observables: exact results & generalization of the BL model

TÁMOP 4.2.4.A

Introduction

Motivation:

In non-central collisions:
initial angular momentum
leads to final state tilt of fireball

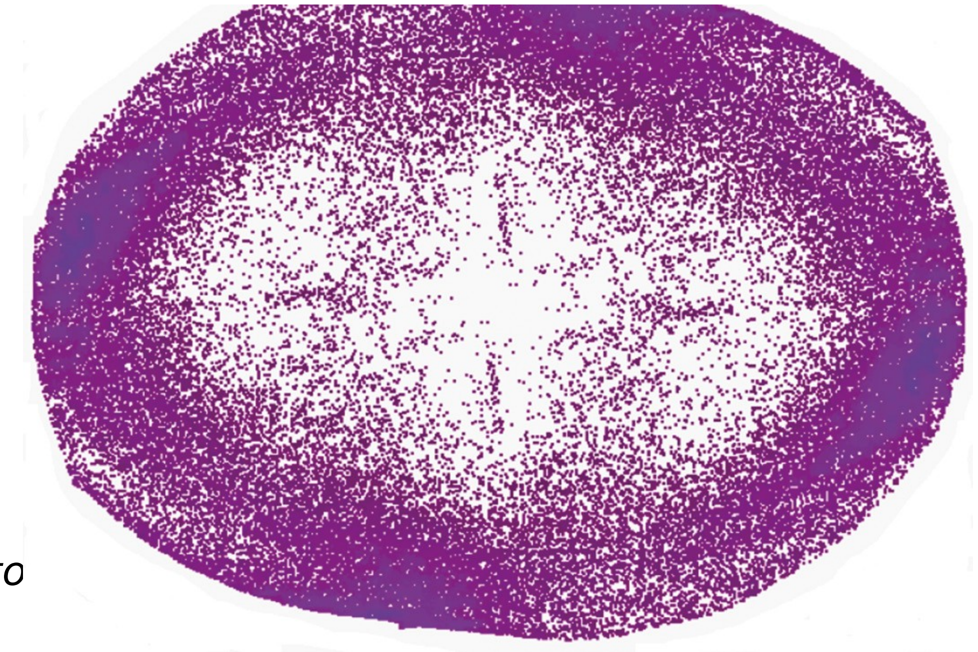
Exact hydrodynamical solutions:

insight into the dynamics
using simple analytic formulas

Determination of rotation:

important question,
theoretically, experimentally.

Looking for rotating hydro solutions to follow time evolution, calculate observables, tell if there is rotation



Example from EPN 43/22 (2012) 91 (L. Cifarelli, L.P. Csernai, H. Stöcker)

This work follows the footsteps of:

Csizmadia, Csörgő, Lukács, PLB443 (1998) 21
Csörgő, Acta Phys. Polon. B37 (2006) 483 & references therein
Csörgő, Akkelin, Hama, Lukács, Sinyukov, PRC67 (2003) 034094
Csörgő, Nagy, PRC89 (2014) 044901

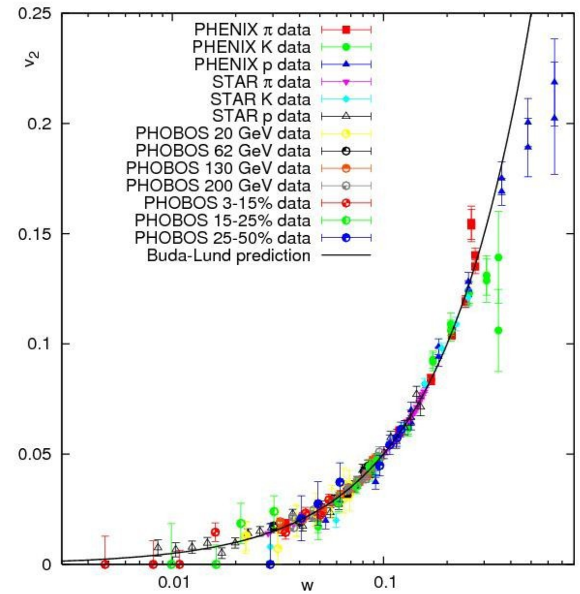
Buda-Lund hydrodynamics

Buda-Lund model:

Csörgő, Lörstad, PRC54 (1996) 1390 (cylindrical)

Csanád, Csörgő, Lörstad, Nucl.Phys.A742 (2004) 80

...

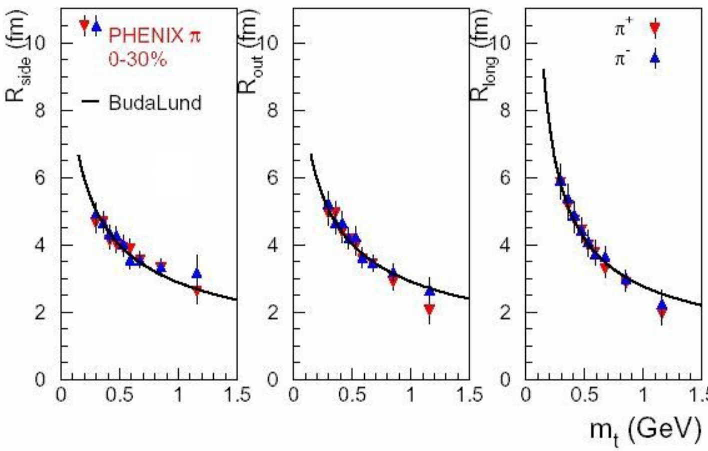


Buda-Lund model:

- core & halo picture for Bose-Einstein correlations
- simple formulas! (with saddle-point integration)
- parametrization that gives back certain exact solutions of hydrodynamics (in certain limiting cases), both NR and R

Successful in description of soft observables & scalings!

- Scaling in HBT radii and azimuthal anisotropies suggest Hubble-like flows



Basic equations (NR)

Basic equations (non-relativistic, perfect fluid)

$$\partial_t n + \mathbf{v}\nabla n = -n\nabla\mathbf{v}$$

mass conservation

$$\partial_t \varepsilon + \mathbf{v}\nabla\varepsilon = -(\varepsilon + p)\nabla\mathbf{v}$$

energy conservation

$$\partial_t \mathbf{v} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{nm_0}\nabla p$$

Euler equation

Follow only from local thermal equilibrium & local conservation laws.

Equation of State (EoS):

$$p = nT \qquad \varepsilon = \kappa(T)p$$

Frames: inertial K, co-rotating: K'

tilt angle (x-z plane): ϑ

angular velocity: $\omega = \begin{pmatrix} 0 \\ \dot{\vartheta} \\ 0 \end{pmatrix}$

Equations in rotating frame:

$$\left(\frac{\kappa}{T} + \frac{d\kappa}{dT}\right)\left(\partial_t + \mathbf{v}'\nabla'\right)T + \nabla'\mathbf{v}' = 0$$

$$\partial_t n + \mathbf{v}'\nabla' n = -n\nabla'\mathbf{v}'$$

$$\partial_t \mathbf{v}' + (\mathbf{v}'\nabla')\mathbf{v}' = -\frac{1}{nm_0}\nabla' p + f'$$

$$f' = 2\mathbf{v}'\times\omega + \mathbf{r}'\times\dot{\omega} + \omega\times(\mathbf{r}'\times\omega)$$

A Hubble-like rotating solution

Hubble-type velocity:

$$\mathbf{v}' = \begin{pmatrix} \frac{\dot{X}}{X} r_x' + g \frac{Z}{X} r_z' \\ \frac{\dot{Y}}{Y} r_y' \\ \frac{\dot{Z}}{Z} r_z' - g \frac{X}{Z} r_x' \end{pmatrix}$$

scaling variable:
(rotating ellipsoids)

$$s = \frac{r_x'^2}{X^2} + \frac{r_y'^2}{Y^2} + \frac{r_z'^2}{Z^2}$$

„volume”:

$$V = XYZ$$

coordinate transformation:

$$r_x' = r_x \cos\vartheta - r_z \sin\vartheta$$

$$r_z' = r_x \sin\vartheta + r_z \cos\vartheta$$

$$r_y' = r_y$$

Solution of the continuity equations:

$$n(\mathbf{r},t) = n_0 \frac{V_0}{V} e^{-s/2}$$

for constant κ :

$$T(\mathbf{r},t) = T(t) = T_0 \left(\frac{V_0}{V(t)} \right)^{\frac{1}{\kappa}}$$

for non-constant κ :

$$T(\mathbf{r},t) = T(t)$$

$$\frac{V_0}{V} = \exp\left(\int_{T_0}^{T} \mathrm{d}\beta \left(\frac{d\kappa}{d\beta} + \frac{\kappa}{\beta} \right) \right)$$

Similar to & generalization of:

Csörgő, Acta Phys. Polon. B37 (2006) 483 & references therein
Csörgő, Akkelin, Hama, Lukács, Sinyukov, PRC67 (2003) 034094
Csörgő, Nagy, PRC89 (2014) 044901

Arriving at a parametric solution

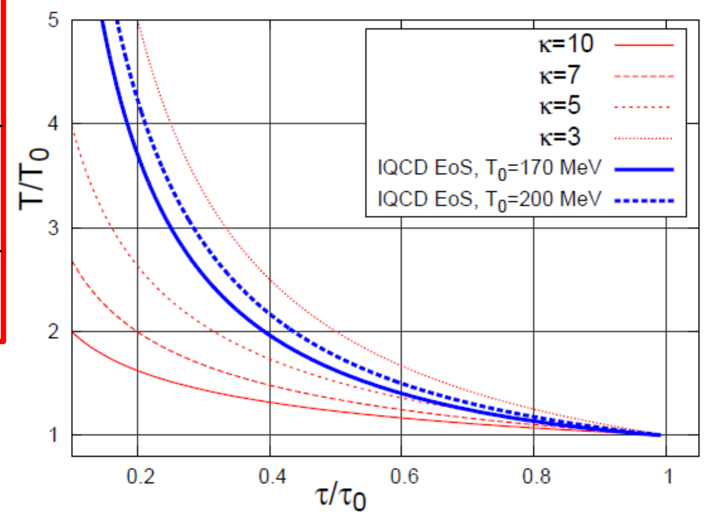
Remaining functions:

Axes: $X(t)$, $Y(t)$, $Z(t)$
Auxiliary function: $g(t)$
Angular velocity: $\dot{\vartheta}(t)$

Solution if

$$g(t) = \frac{\chi_0}{(X+Z)^2}$$

$$\dot{\vartheta}(t) = \frac{\chi_0}{(X+Z)^2}$$



„Motion" of axes:
Governed by a Lagrangian!

In the constant κ case:

$$L = \frac{m}{2}\left(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2\right) - \frac{1}{\kappa}\left(\frac{V_0}{V}\right)^{1/\kappa} - \frac{m\chi_0^2}{(X+Z)^2}$$

Equations of motion:
Ordinary p.d.e.-s
Solution is easy numerically

Conserved quantities: total particle number: $N_0 = n_0 V_0 (2\pi)^{3/2}$

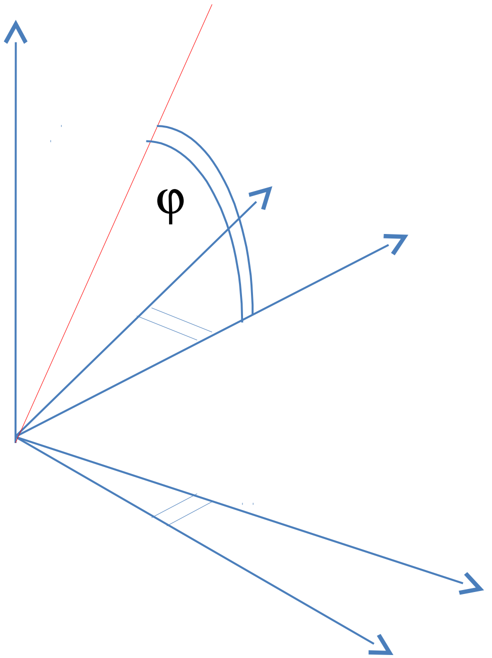
total angular momentum (y component): $M_y = mN_0\chi_0$

total energy: $E = \frac{m}{2}\left(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2\right) + \frac{1}{\kappa}\left(\frac{V_0}{V}\right)^{1/\kappa} + \frac{m\chi_0^2}{(X+Z)^2}$

Observables

Hydro evolution until freeze-out:
Dependent on EoS

Freeze-out criterion:
Same freeze-out temperature
Same time everywhere
Analytic results!



Source function:

$$S(\mathbf{r},\mathbf{p}) = \frac{n(t_f,\mathbf{r})}{(2\pi m T_f)^{3/2}} \exp\left\{-\frac{(\mathbf{p}-m\mathbf{v}(t_f,\mathbf{r}))^2}{2mT_f}\right\}$$

Definitions & recipes:

Single-particle spectrum:

$$\frac{\mathrm{d}n}{\mathrm{d}^3\mathbf{p}} = \int \mathrm{d}^3\mathbf{r}\, S(\mathbf{r},\mathbf{p})$$

Bose-Einstein correlation:

$$C(\mathbf{K},\mathbf{q}) = 1 + \lambda \frac{\left|\tilde{S}(\mathbf{K},\mathbf{q})\right|^2}{\left|\tilde{S}(\mathbf{K},0)\right|^2}$$

Single-particle spectrum

Definition:

$$\frac{\mathrm{d}n}{\mathrm{d}^3\mathbf{p}} = \int \mathrm{d}^3\mathbf{r}\, S(\mathbf{r},\mathbf{p})$$

Result:

$$\frac{\mathrm{d}n}{\mathrm{d}^3\mathbf{p}} \propto \exp\left(-\frac{p_x^2}{2mT_x} - \frac{p_y^2}{2mT_y} - \frac{p_z^2}{2mT_z} - \frac{\beta_{xz}}{m} p_x p_z \right)$$

Coefficients:

$$T'_{0x} = T_f + m\left(\dot{X}^2 + \Phi^2\right)$$

$$T'_{0x} = T_f + m\left(\dot{Z}^2 + \Phi^2\right)$$

$$\beta'_{xz} = \frac{m\Phi(\dot{X}-\dot{Z})}{T'_{0x}T'_{0z} - m^2\Phi^2(\dot{X}-\dot{Z})^2}$$

$$T'_x = T'_{0x} + \frac{m^2\Phi^2}{T'_{0z}}(\dot{X}-\dot{Z})^2$$

$$T'_z = T'_{0z} + \frac{m^2\Phi^2}{T'_{0x}}(\dot{X}-\dot{Z})^2$$

$$T_y = T_f + m\dot{Y}^2 \qquad \Phi = \frac{\chi_0}{X_f + Z_f}$$

$$\frac{1}{T_x} = \frac{\cos^2\vartheta_f}{T'_x} + \frac{\sin^2\vartheta_f}{T'_z} + \beta'_{xz}\sin(2\vartheta_f)$$

$$\frac{1}{T_z} = \frac{\sin^2\vartheta_f}{T'_x} + \frac{\cos^2\vartheta_f}{T'_z} - \beta'_{xz}\sin(2\vartheta_f)$$

$$\beta_{xz} = \frac{\sin(2\vartheta_f)}{2}\left(\frac{1}{T'_z} - \frac{1}{T'_x}\right) + \beta'_{xz}\cos(2\vartheta_f)$$

Avg. spectrum & flow coefficients

Results: (same as in *Csörgő et al., PRC67 (2003) 034094*):

up to 2nd order in :

$$\frac{1}{2\pi p_T}\frac{\mathrm{d}n}{\mathrm{d}p_T\mathrm{d}y} \propto \exp\left(-\frac{p_z^2}{2mT_z}-\frac{p_T^2}{2mT_{\mathrm{eff}}}\right)\times$$

$$\times\left[I_0(w)+\frac{\nu^2}{4}\left(I_0(w)+I_1(w)\right)\right]$$

$$v_1 = \frac{\nu}{2}\left[1+\frac{I_1(w)}{I_0(w)}\right]$$

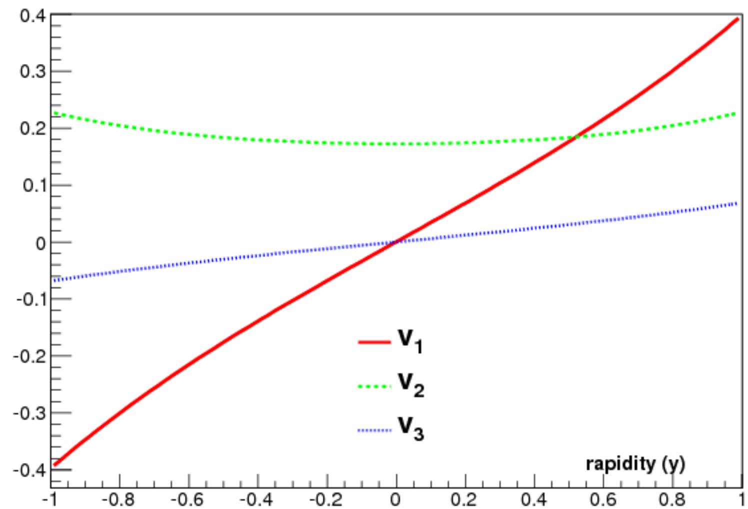
$$v_2 = \frac{I_1(w)}{I_0(w)}+\frac{\nu^2}{8}\left[1+\frac{I_2(w)}{I_0(w)}-2\frac{I_1^2(w)}{I_0^2(w)}\right]$$

$$v_3 = \frac{\nu}{2}\left[\frac{I_1(w)}{I_0(w)}+\frac{I_2(w)}{I_0(w)}\right]$$

$$w = \frac{p_T^2}{4m}\left(\frac{1}{T_y}-\frac{1}{T_x}\right)$$

$$T_{\mathrm{eff}} = \frac{1}{2}\left(\frac{1}{T_y}+\frac{1}{T_x}\right)$$

$$\nu = -\frac{\beta_{xz}}{m}p_z p_T$$



Universal scaling of v_2 is preserved

(*Csanád et al., Nucl.Phys. A742 (2004) 80*)

Two-particle correlations

Bose-Einstein correlations (raw): *Coulomb correction treatment important!*

Definition:

$$C(\mathbf{K},\mathbf{q}) = 1 + \lambda \frac{\left|\tilde{S}(\mathbf{K},\mathbf{q})\right|^2}{\left|\tilde{S}(\mathbf{K},0)\right|^2}$$

$$\tilde{S}(\mathbf{K},\mathbf{q}) = \int \mathrm{d}^3\mathbf{r}\, e^{iq\mathbf{r}} S(\mathbf{r},\mathbf{K})$$

Result: Gaussian

$$C(\mathbf{K},\mathbf{q}) = 1 + \lambda \exp\left(-\sum_{ij} q_i q_j R_{ij}^2\right)$$

$$R'^2_x = \frac{X^2 T_f T'_{0z}}{T'_{0x} T'_{0z} - m^2\Phi^2(\dot{X}-\dot{Z})^2}$$

$$R'^2_y = \frac{Y^2 T_f}{T_y}$$

$$R'^2_z = \frac{Z^2 T_f T'_{0x}}{T'_{0x} T'_{0z} - m^2\Phi^2(\dot{X}-\dot{Z})^2}$$

$$R'^2_{xz} = \frac{m T_f XZ\Phi(\dot{Z}-\dot{X})}{T'_{0x} T'_{0z} - m^2\Phi^2(\dot{X}-\dot{Z})^2}$$

$$R_x^2 = R'^2_x \cos^2\vartheta_f + R'^2_z \sin^2\vartheta_f + R'^2_{xz}\sin(2\vartheta_f)$$

$$R_y^2 = R'^2_y$$

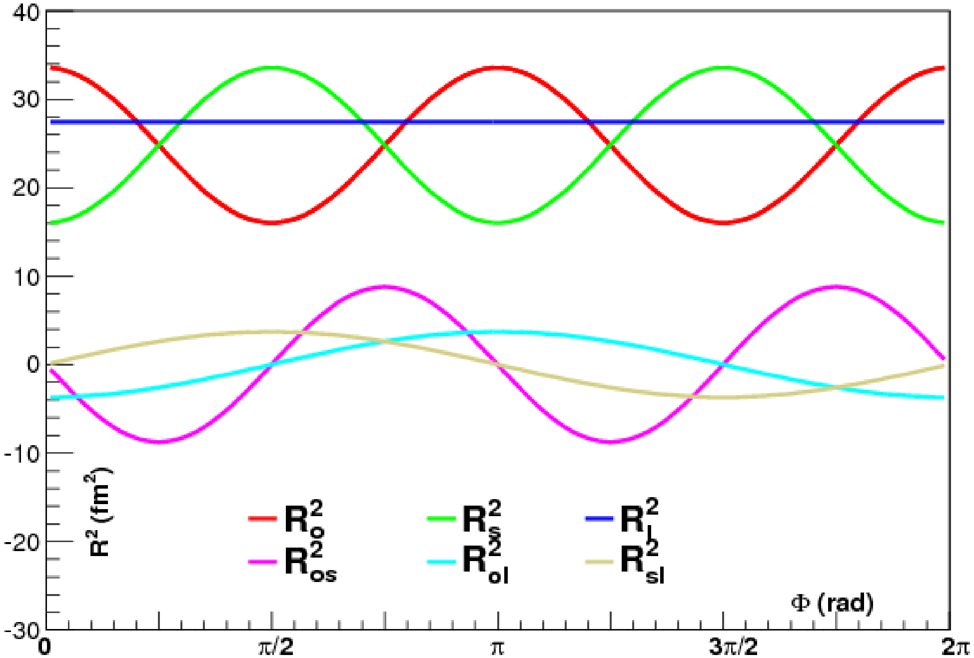
$$R_x^2 = R'^2_x \sin^2\vartheta_f + R'^2_z \cos^2\vartheta_f - R'^2_{xz}\sin(2\vartheta_f)$$

$$R_{xz}^2 = \frac{\sin(2\vartheta_f)}{2}\left(R'^2_x - R'^2_z\right) + \cos(2\vartheta_f) R'^2_{xz}$$

Two-particle correlations

Bertsch-Pratt parametrization:

$$C(\mathbf{K},\mathbf{q}) = 1 + \lambda \exp\left(- \sum_{i,j=o,s,l} q_i q_j R_{ij}^2 \right)$$



HBT Radii:

$$R_o^2 = R_x^2 \cos^2\varphi + R_y^2 \sin^2\varphi$$

$$R_s^2 = R_x^2 \sin^2\varphi + R_y^2 \cos^2\varphi$$

$$R_l^2 = R_z^2$$

$$R_{os}^2 = \left(R_y^2 - R_x^2\right)\sin\varphi\cos\varphi$$

$$R_{ol}^2 = R_{xz}^2 \cos\varphi$$

$$R_{sl}^2 = -R_{xz}^2 \sin\varphi$$

Oscillations: s, o, os, ol, sl !

ol, sl values vanish for no tilt:

No 2nd order oscillations of ol, sl!

Rotating Buda-Lund model

Unrealistic features (because of NR approximation)

No transverse mass dependence of HBT radii!

Not realistiv transverse mass dependence of flow parameters!

A relativistic generalization: BL model for rotating flows

$$S(x,p)\mathrm{d}^4x := \frac{g_s}{(2\pi)^3}\frac{p^\mu \mathrm{d}^4\Sigma_\mu(x)}{B(x,p)+s_q} \qquad B(x,p)=\exp\left[\frac{p^\nu u_\nu(x)}{T(x)}-\frac{\mu(x)}{T(x)}\right]$$

$$u^\mu=\gamma\left(1,\frac{\mathbf{r}}{t}\right) \qquad p^\mu \mathrm{d}^4\Sigma_\mu(x) := p^\mu u_\mu H(\tau)\mathrm{d}^4x \qquad s=\frac{r_x^2}{2X^2}+\frac{r_y^2}{2Y^2}+\frac{r_z^2}{2Z^2}$$

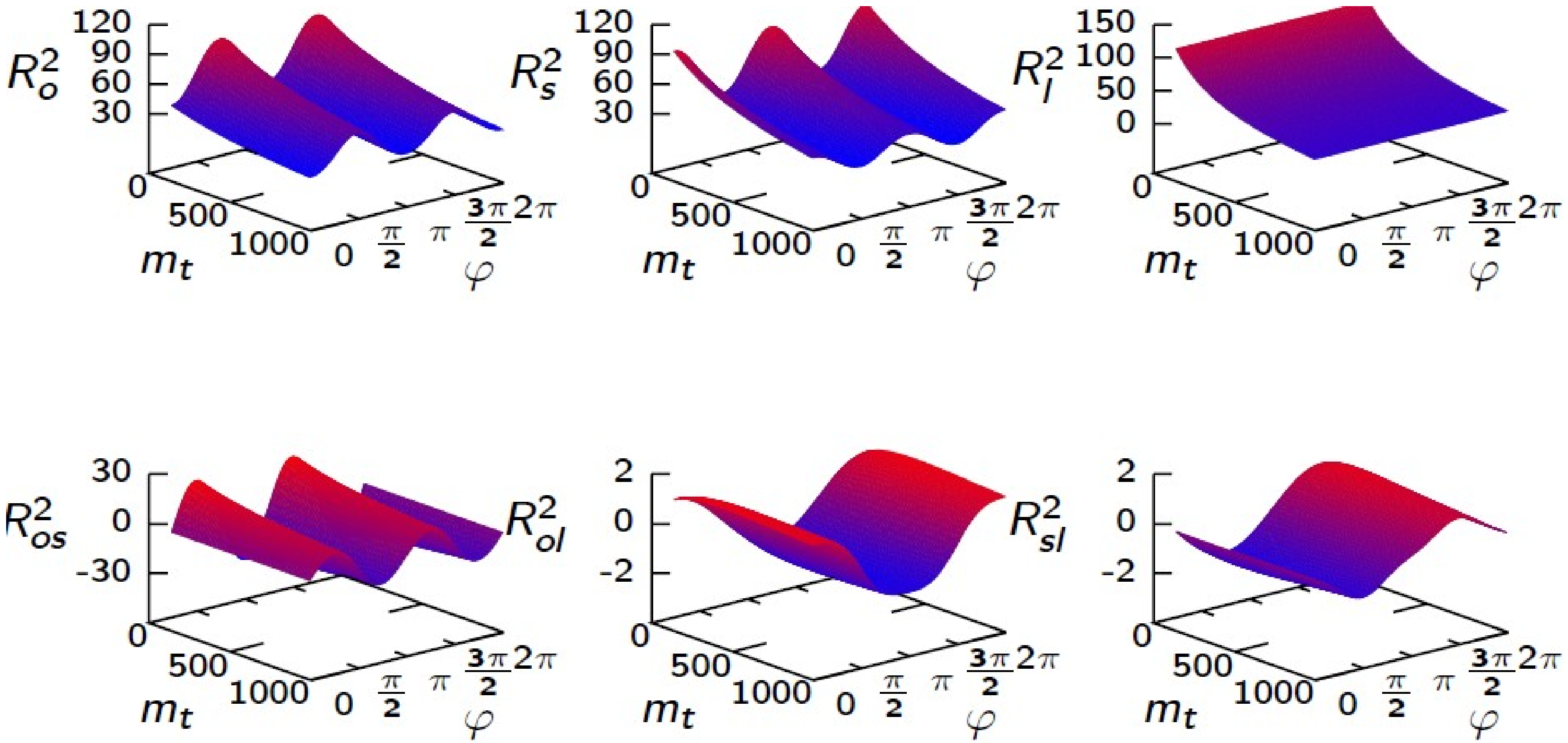
$$H(\tau) := \frac{1}{\sqrt{2\pi\Delta\tau^2}}\exp\left[-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right]$$

So far, traditional BL. New feature: rotation! Velocity:

$$u^\mu=\begin{pmatrix} G \\ r_x\frac{\dot{X}}{X}+r_z\frac{g_*}{Z} \\ r_y\frac{\dot{Y}}{Y} \\ r_z\frac{\dot{Z}}{Z}-r_x\frac{g_*}{X}\end{pmatrix}$$

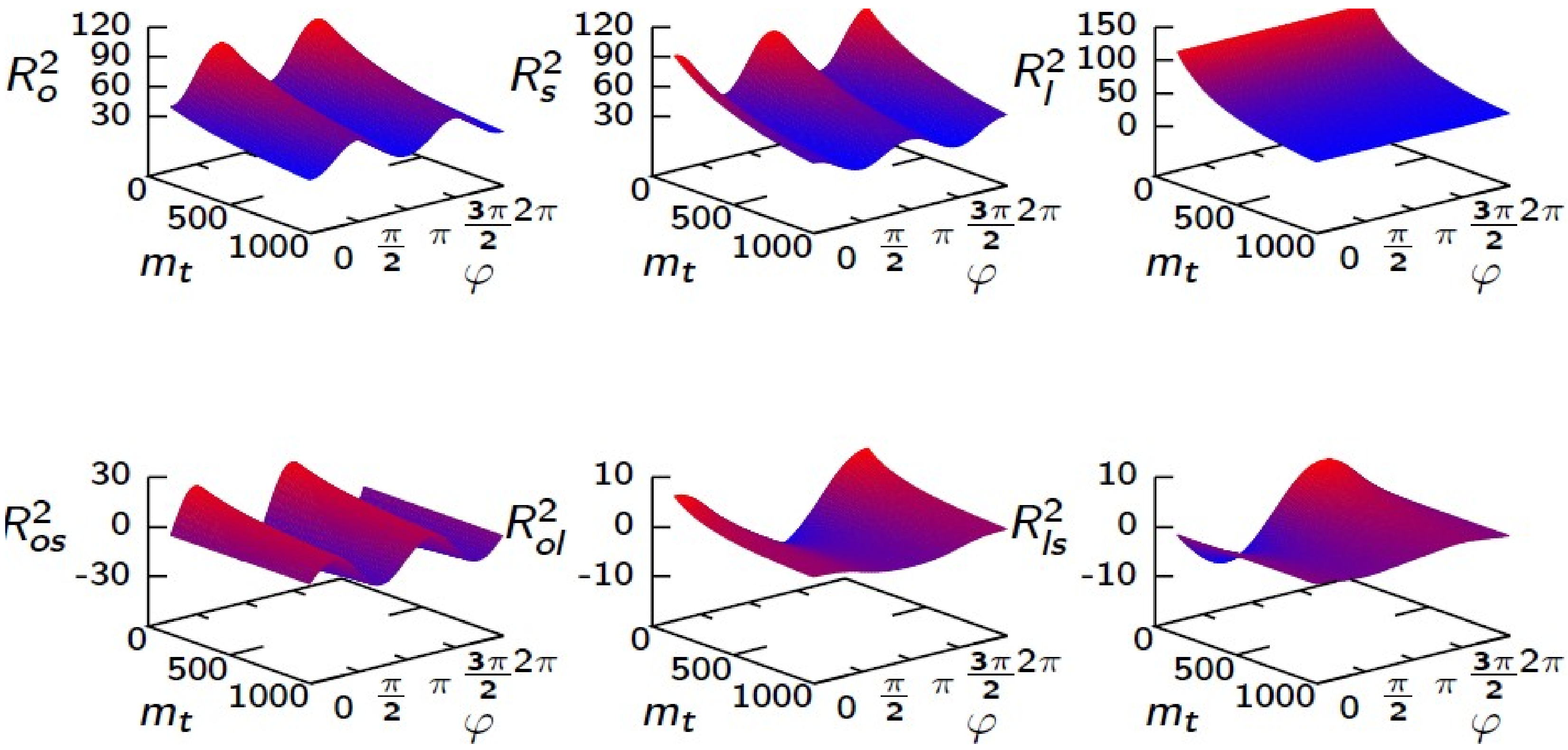
Rotating Buda-Lund model

HBT radii vs transverse mass: (non-rotating but tilted case):



Rotating Buda-Lund model

HBT radii vs transverse mass: (rotating AND tilted case):



Summary and outlook

Non-relativistic hydrodamical equations:

Extended the class of known parametric solutions

Found rotating, 3-axis ellipsoidal solutions, for arbitrary EoS

Calculated observables

Simplified treatment

Needs at least some relativistic generalization (eg. HBT radii now do not depend on transverse momentum) Relativistic generalization of solutions: hard task…

Relativistic generalization of Buda-Lund model:

Incorporated into successful model a new class of velocity fields suggested by NR solution

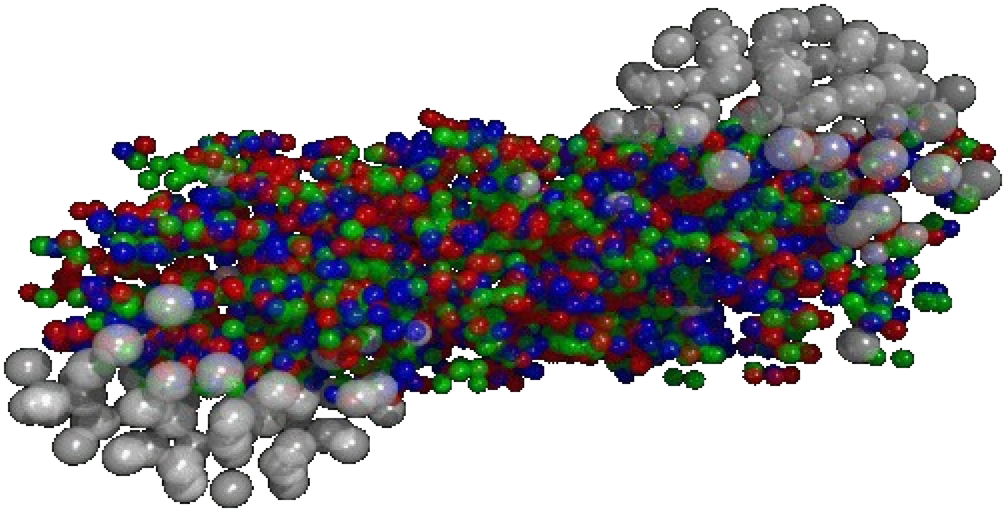
Observables calculated; approximations needed?

Measurement of tilt angle:

A final state variable; initial angle known to be zero?

Possible to investigate EoS!

Thank you for your attention!



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