Qubits from toroidal compactification

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Motivation: Qubits from homology and cohomology

Main idea: Wrapped membrane configurations defining qubits.
"To wrap or not to wrap that is the qubit" (M. J. Duff).

L. Borsten, D. Dahanayake, M. J. Duff, H. Ebrahim and W. Rubens: "Wrapped Branes as Qubits", Phys. Rev. Lett. **100** 251602 (2008)

We would like to make this idea precise by relating qubits to the **cohomology classes** of the extra dimensions. The talk is based on

P. Lévay: "Qubits from Extra Dimensions", Phys. Rev. **D84** 125020 (2011)
P. Lévay and Sz. Szalay: "STU Attractors from Vanishing Concurrence", Phys. Rev. **D83** , 045005 (2011)

Plan of the talk

1. A qubit from T^2

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2. Three qubits from $T^2 \times T^2 \times T^2$

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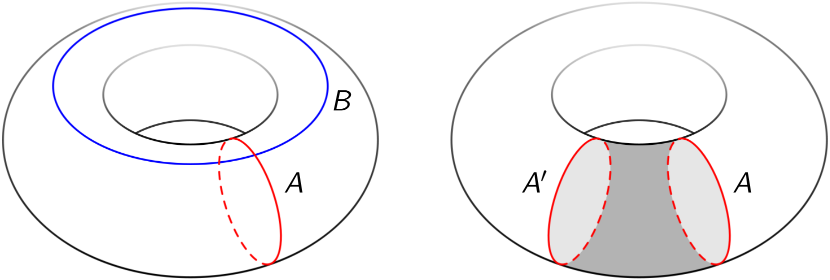
5. Attractors and moduli stabilization from vanishing concurrence
6. Conclusions

Homology base



For a two dimensional cycle S (a closed surface in some space X) one can define a **two-form**,

$$F \equiv \frac{1}{2} F_{ab} dx^a \wedge dx^b$$

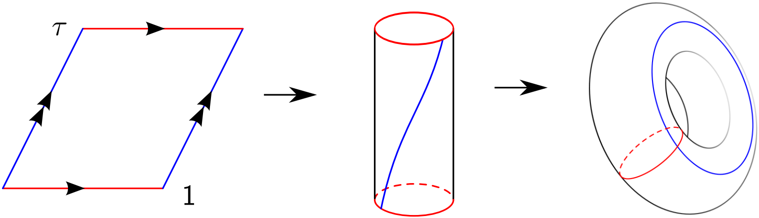
such that the integral $\int_S F$ is a surface integral (flux). If $dF = 0$ (F closed) then the flux is depending merely on the homology class of S.
If $F' - F = dA$ for some $A = A_a dx^a$ then F' and F belong to the same cohomology class of $H^2(X, \mathbb{R})$.

If we have an n dimensional **complex** space X we have complex coordinates dz^a and their conjugates $d\overline{z}^{\overline{a}}$, $a = 1, 2, \dots n$. In this case one can define forms ξ of type (s, t) as

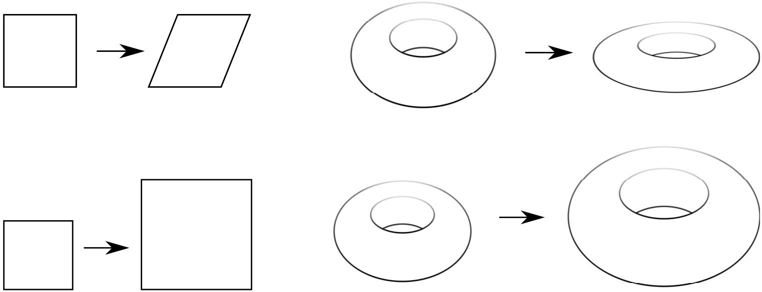
$$\xi = \frac{1}{s!t!}\xi_{a_1 \dots a_s b_1 \dots b_t} dz^{a_1} \wedge \cdots \wedge dz^{a_s} \wedge d\overline{z}^{\overline{b}_1} \wedge \cdots \wedge d\overline{z}^{\overline{b}_t}$$

We can then define cohomology classes $H^{s,t}_{\overline{\partial}}(X, \mathbb{C})$ with elements being **equivalence classes of closed forms** ($\overline{\partial}\xi = 0$) of type (s, t). Two such forms ξ and ξ' are equivalent iff $\xi' = \xi + \overline{\partial}\eta$ with η being a form of type $(s, t - 1)$.

The torus T^2 arising from a lattice of $\mathbb{C}$



Complex structure and Kähler structure deformations



One-qubit systems from deformed tori

Consider T^2 with its deformations labelled by

$$\tau \equiv x - iy \qquad y > 0$$

The complex coordinates on T^2 are $z = u + \tau v$. The space of deformations $\mathcal{M}$ has a Kähler metric

$$ds^2_{\mathcal{M}} = 2G_{\tau\overline{\tau}} d\tau d\overline{\tau} = \frac{dx^2 + dy^2}{2y^2}$$

$$G_{\tau\overline{\tau}} = \partial_\tau \partial_{\overline{\tau}} K = \frac{1}{4y^2} \qquad K = -\log(2y).$$

Define

$$\Omega_0 \equiv dz = du + \tau dv \in H^{1,0}(T^2, \mathbb{C})$$

Then by virtue of

$$\int_{T^2} du \wedge dv = 1$$

$$ie^{-K} = \int_{T^2} \Omega_0 \wedge \overline{\Omega}_0$$

The volume form on T^2 is

$$J = \frac{i}{2y} d\overline{z} \wedge dz = i g_{\overline{z}z} d\overline{z} \wedge dz = du \wedge dv, \qquad e^K = g_{\overline{z}z} = \frac{1}{2y}.$$

Then the Hodge star $*$ is acting as

$$*dz = idz, \qquad *d\overline{z} = -id\overline{z}.$$

In order to reinterpret one-forms on T^2 as **qubits** we use the hermitian inner product

$$\langle \xi | \eta \rangle \equiv \int_{T^2} \xi \wedge *\overline{\eta}, \qquad \xi, \eta \in H^1(T^2, \mathbb{C})$$

Define the one-form Ω as

$$\Omega \equiv e^{K/2} \Omega_0 = \frac{1}{\sqrt{2y}} (du + (x - iy) dv)$$

then the correspondence

$$i\Omega \leftrightarrow |0\rangle, \qquad i\overline{\Omega} \leftrightarrow |1\rangle$$

defines the orthonormal computational base.

Define the flat covariant derivative as

$$D_{\hat{\tau}}\Omega \equiv (\overline{\tau} - \tau)D_\tau\Omega \equiv (\overline{\tau} - \tau)\left(\partial_\tau + \frac{1}{2}\partial_\tau K\right)\Omega = \overline{\Omega}$$

$$D_{\hat{\tau}}\overline{\Omega} \equiv (\overline{\tau} - \tau)\left(\partial_\tau - \frac{1}{2}\partial_\tau K\right)\overline{\Omega} = 0$$

Then

$$D_{\hat{\tau}} \leftrightarrow \sigma_+, \qquad D_{\hat{\overline{\tau}}} \leftrightarrow \sigma_-, \qquad * \leftrightarrow -\sigma_3$$

Hence the covariant derivatives act as **projective bit flip errors**, and the Hodge star as the negative of the **parity check** operator.
Remark: Notice that the symbols $|0\rangle$ and $|1\rangle$ would rather define a **family of basis states** labelled by the complex deformation parameter τ. Hence the notation $|0,\tau\rangle$, $|1,\tau\rangle$ would be more appropriate. Notice also that the operators $D_{\hat{\tau}}$, $D_{\hat{\overline{\tau}}}$, $*$ are acting on the parameters τ on the other hand the ones $\sigma_\pm, \sigma_3$ are rotating the basis states of the Hilbert space. (The Hilbert space structure is associated with the coordinates u and v of T^2 and not with τ of $\mathcal{M}$.)

For a $\Gamma \in H^1(T^2, \mathbb{R})$ let us write

$$\Gamma = p\alpha - q\beta, \qquad \alpha = du, \quad \beta = dv.$$

One can express this in the **Hodge diagonal basis** as

$$\Gamma = -e^{K/2}(p\overline{\tau} + q)i\Omega + e^{K/2}(p\tau + q)i\overline{\Omega}.$$

According to our correspondence between one-forms and qubits we can represent this as a state in the computational base satisfying an extra reality condition

$$|\Gamma\rangle = \Gamma_0|0\rangle + \Gamma_1|1\rangle, \qquad \Gamma_1 = -\overline{\Gamma}_0 = e^{K/2}(p\tau + q). \qquad (1)$$

We note also that after imposing a quantization condition on p and q

$$\Gamma \in H^1(T^2, \mathbb{Z}).$$

The state $|\Gamma\rangle$ is **unnormalized** with norm squared satisfying

$$||\Gamma||^2 = \langle\Gamma|\Gamma\rangle = 2e^K|p\tau + q|^2 = \frac{1}{y}|p\tau + q|^2$$

It is a unitary and a symplectic i.e. $SL(2,\mathbb{R})$ invariant at the same time. The latter means that under the set of combined transformations

$$\tau \mapsto \frac{a\tau + b}{c\tau + d}, \qquad \begin{pmatrix} -p \\ q \end{pmatrix} \mapsto \begin{pmatrix} d & c \\ b & a \end{pmatrix} \begin{pmatrix} -p \\ q \end{pmatrix} \tag{2}$$

$||\Gamma||^2$ remains invariant.

Notice also that in matrix representation the state $|\Gamma\rangle$ can be given the form

$$\begin{pmatrix} \Gamma_0 \\ \Gamma_1 \end{pmatrix} = \frac{1}{\sqrt{2y}} \begin{pmatrix} \overline{\tau} & -1 \\ -\tau & 1 \end{pmatrix} \begin{pmatrix} -p \\ q \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & -1 \\ i & 1 \end{pmatrix} \frac{1}{\sqrt{y}} \begin{pmatrix} y & 0 \\ -x & 1 \end{pmatrix} \begin{pmatrix} -p \\ q \end{pmatrix}$$

With new notational conventions our state can be given the deceptively simple appearance

$$|\Gamma\rangle = \mathcal{S}|\gamma\rangle = US|\gamma\rangle, \qquad |\gamma\rangle = -p|0\rangle + q|1\rangle$$

$T^2 \times T^2 \times T^2$ and three qubits

Coordinates

$$z^a = u^a + \tau^a v^a, \qquad \tau^a = x^a - iy^a \qquad y^a > 0, \qquad a = 1, 2, 3$$

Holomorphic three-form

$$\Omega_0 = dz^1 \wedge dz^2 \wedge dz^3.$$

We have as usual

$$\int_{T^6} \Omega_0 \wedge \overline{\Omega}_0 = i(8y^1y^2y^3) = ie^{-K}$$

$$G_{a\overline{b}} = \partial_a \partial_{\overline{b}} K$$

is a Kähler metric on the manifold $\mathcal{M} \simeq [SL(2,\mathbb{R})/SO(2)]^{\times 3}$ The flat covariant derivatives are

$$D_{\hat{a}}\Omega = (\overline{\tau}^a - \tau^a) D_a \Omega = (\overline{\tau}^a - \tau^a)\left(\partial_a + \frac{1}{2}\partial_a K\right)\Omega$$

$$\Omega = e^{K/2} dz^1 \wedge dz^2 \wedge dz^3, \qquad \overline{\Omega} = e^{K/2} d\overline{z}^1 \wedge d\overline{z}^2 \wedge d\overline{z}^3,$$

$$D_{\hat{1}}\Omega = e^{K/2} d\overline{z}^1 \wedge dz^2 \wedge dz^3, \qquad \overline{D}_{\hat{\overline{1}}}\overline{\Omega} = e^{K/2} dz^1 \wedge d\overline{z}^2 \wedge d\overline{z}^3,$$

$$D_{\hat{2}}\Omega = e^{K/2} dz^1 \wedge d\overline{z}^2 \wedge dz^3, \qquad \overline{D}_{\hat{\overline{2}}}\overline{\Omega} = e^{K/2} d\overline{z}^1 \wedge dz^2 \wedge d\overline{z}^3,$$

$$D_{\hat{3}}\Omega = e^{K/2} dz^1 \wedge dz^2 \wedge d\overline{z}^3, \qquad \overline{D}_{\hat{\overline{3}}}\overline{\Omega} = e^{K/2} d\overline{z}^1 \wedge d\overline{z}^2 \wedge dz^3$$

Now we regard the 8 complex dimensional space $H^3(T^2 \times T^2 \times T^2, \mathbb{C}) \equiv H^{3,0} \oplus H^{2,1} \oplus H^{1,2} \oplus H^{0,3}$ equipped with the Hermitian inner product

$$\langle \varphi | \eta \rangle \equiv \int_{T^6} \varphi \wedge * \overline{\eta}$$

as a Hilbert space isomorphic to $\mathcal{H} \equiv (\mathbb{C}^2)^{\times 3} \simeq \mathbb{C}^8$ of three qubits.

We define the basis states of our computational base to be given by the correspondence

$$-i\Omega \leftrightarrow |000\rangle \quad -iD_{\hat{1}}\Omega \leftrightarrow |001\rangle \quad -iD_{\hat{2}}\Omega \leftrightarrow |010\rangle \quad -iD_{\hat{3}}\Omega \leftrightarrow |100\rangle$$

$$-i\overline{\Omega} \leftrightarrow |111\rangle \quad -i\overline{D}_{\hat{\hat{1}}}\Omega \leftrightarrow |110\rangle \quad -i\overline{D}_{\hat{\hat{2}}}\Omega \leftrightarrow |101\rangle \quad -i\overline{D}_{\hat{\hat{3}}}\Omega \leftrightarrow |011\rangle$$

$$(D_{\hat{1}}, D_{\hat{2}}, D_{\hat{3}}) \leftrightarrow (I \otimes I \otimes \sigma_+, I \otimes \sigma_+ \otimes I, \sigma_+ \otimes I \otimes I)$$

$$(D_{\hat{\hat{1}}}, D_{\hat{\hat{2}}}, D_{\hat{\hat{3}}}) \leftrightarrow (I \otimes I \otimes \sigma_-, I \otimes \sigma_- \otimes I, \sigma_- \otimes I \otimes I)$$

$$* \leftrightarrow -\sigma_3 \otimes \sigma_3 \otimes \sigma_3$$

A wrapping configuration on $T^2 \times T^2 \times T^2$

Now for a real three-form representing a cohomology class we take

$$\Gamma = p^I \alpha_I - q_I \beta^I \in H^3(T^6, \mathbb{Z}),$$

with summation on $I = 0, 1, 2, 3$ and

$$\alpha_0 = du^1 \wedge du^2 \wedge du^3, \qquad \beta^0 = -dv^1 \wedge dv^2 \wedge dv^3$$

$$\alpha_1 = dv^1 \wedge du^2 \wedge du^3, \qquad \beta^1 = du^1 \wedge dv^2 \wedge dv^3$$

with the remaining ones obtained via cyclic permutation. With the choice of orientation

$$\int_{T^6} (du^1 \wedge dv^1) \wedge (du^2 \wedge dv^2) \wedge (du^3 \wedge dv^3) = 1$$

we have

$$\int_{T^6} \alpha_I \wedge \beta^J = \delta_I^J$$

.

A wrapping and moduli dependent three-qubit state

Define

$$Z(\Gamma) \equiv e^{K/2} W(\tau^3, \tau^2, \tau^1)$$

$$W = q_0 + q_1\tau^1 + q_2\tau^2 + q_3\tau^3 + p^1\tau^2\tau^3 + p^2\tau^1\tau^3 + p^3\tau^1\tau^2 - p^0\tau^1\tau^2\tau^3$$

Using the correspondence between three-forms and three-qubit states we can write $\Gamma \leftrightarrow |\Gamma\rangle$ where

$$|\Gamma\rangle = \Gamma_{000}|000\rangle + \Gamma_{001}|001\rangle + \cdots + \Gamma_{110}|110\rangle + \Gamma_{111}|111\rangle,$$

where

$$\Gamma_{111} = -e^{K/2} W(\tau^3, \tau^2, \tau^1) = -\overline{\Gamma}_{000},$$
$$\Gamma_{001} = -e^{K/2} W(\overline{\tau}^3, \overline{\tau}^2, \tau^1) = -\overline{\Gamma}_{110}$$

and the remaining amplitudes are given by cyclic permutation. Let us put the 8 numbers p^I and q_I with $I = 0, 1, 2, 3$ to a $2 \times 2 \times 2$ array γ_{kji} $k, j, i = 0, 1$ as follows

$$\begin{pmatrix} \gamma_{000} & \gamma_{001} & \gamma_{010} & \gamma_{100} \\ \gamma_{111} & \gamma_{110} & \gamma_{101} & \gamma_{011} \end{pmatrix} = \begin{pmatrix} -p^0 & -p^1 & -p^2 & -p^3 \\ -q_0 & q_1 & q_2 & q_3 \end{pmatrix}$$

An alternative form

The three-qubit state $|\Gamma\rangle$ can alternatively be written in the following form

$$|\Gamma\rangle = \mathcal{S}_3 \otimes \mathcal{S}_2 \otimes \mathcal{S}_1 |\gamma\rangle$$

$$|\gamma\rangle = \gamma_{000}|000\rangle + \gamma_{001}|001\rangle + \cdots + \gamma_{110}|110\rangle + \gamma_{111}|111\rangle$$

and the matrix representative of the operator $\mathcal{S}_3 \otimes \mathcal{S}_2 \otimes \mathcal{S}_1$ is

$$\frac{1}{\sqrt{8y^3y^2y^1}} \begin{pmatrix} \overline{\tau}^3 & -1 \\ -\tau^3 & 1 \end{pmatrix} \otimes \begin{pmatrix} \overline{\tau}^2 & -1 \\ -\tau^2 & 1 \end{pmatrix} \otimes \begin{pmatrix} \overline{\tau}^1 & -1 \\ -\tau^1 & 1 \end{pmatrix}$$

One-half the norm $\frac{1}{2}||\Gamma||^2$ is

$$e^K(|W(\tau^3,\tau^3,\tau^1)|^2 + |W(\overline{\tau}^3,\tau^2,\tau^1)|^2 + W(\tau^3,\overline{\tau}^2,\tau^1)|^2 + |W(\tau^3,\tau^2,\overline{\tau}^1)|^2$$

Notice that all amplitudes of $|\Gamma\rangle$ can be obtained from the quantity

$$Z(\Gamma) = e^{K/2} W(z^1, z^2, z^3) = \int_X \Omega \wedge \Gamma = \int_{\mathcal{C}} \Omega$$

where

$$\mathcal{C} = q_I A^I + p^I B_I$$

where A^I, B_I defines a homology basis for $X = T^2 \times T^2 \times T^2$.

How to implement this setting into physics?

Motivation by analogy: From electrodynamics to IIB strings...

$$S_{eldin} = -m\int_{\mathcal{P}} ds + q\int_{\mathcal{P}} A - \frac{1}{4\kappa_0^2}\int F\wedge *F$$

$$S_{\rm 3brane} = -\frac{\sqrt{\pi}}{\kappa_{10}}\int d^4\xi\sqrt{-{\rm Det}\, h_{\alpha\beta}} + \frac{\sqrt{\pi}}{\kappa_{10}}\int C_4 - \frac{1}{8\kappa_{10}^2}\int F_5\wedge *F_5$$

where $\kappa_{10} = 8\pi^{7/2}\alpha'^2$

$$h_{\alpha\beta}(\xi) = G_{\mu\nu}(\xi)\frac{\partial x^\mu}{\partial\xi^\alpha}\frac{\partial x^\nu}{\partial\xi^\beta}, \qquad \alpha,\beta = 0,1,2,3, \qquad \mu,\nu = 0,1,\dots 9$$

$$S_{\rm 3brane} = S_{DBI} + S_{WZ} + S_5$$

If we wish to embed this in IIB we have an extra self-duality constraint.

$$*F_5 = F_5$$

Effective low energy action for IIB SUGRA

$$
\begin{aligned}
S^{IIB}_{\mathrm{bosonic}} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}R \\
&- \frac{1}{4\kappa_{10}^2}\int \frac{1}{\mathrm{Im}\mathcal{T}^2}d\mathcal{T}\wedge * d\overline{\mathcal{T}} - \frac{1}{4\kappa_{10}^2}\int \mathcal{M}_{ij}F_3^i\wedge * F_3^j \\
&- \frac{1}{8\kappa_{10}^2}\int \varepsilon_{ij}C_4\wedge F_3^i\wedge F_3^j - \frac{1}{8\kappa_{10}^2}\int \tilde{F}_5\wedge * \tilde{F}_5
\end{aligned}
$$

$$
\mathcal{T} = C_0 + ie^{-\Phi}, \quad F_3^i = (H_3, F_3)^T, \quad \tilde{F}_5 = F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3
$$

$$
\mathcal{M} = \frac{1}{\mathrm{Im}\mathcal{T}}\begin{pmatrix} |\mathcal{T}|^2 & -\mathrm{Re}\mathcal{T} \\ -\mathrm{Re}\mathcal{T} & 1 \end{pmatrix}.
$$

Here we consider the $C_0 = C_2 = B_2 = \Phi = 0$ case. This means that we should keep the first and the last term from $S^{IIB}_{\mathrm{bosonic}}$ with $\tilde{F}_5 = F_5$.

The total action featuring 3-branes in IIB

$$S^{IIB}_{\mathrm{3branes}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}R - \frac{1}{8\kappa_{10}^2}\int F_5 \wedge *F_5 + S_{DBI} + S_{WZ}$$

$$S_{DBI} = -\frac{\sqrt{\pi}}{\kappa_{10}}\int d^4\xi\sqrt{-\mathrm{Det}\, h_{\alpha\beta}}$$

$$S_{WZ} = \frac{\sqrt{\pi}}{\kappa_{10}}\int C_4, \qquad dC_4 = F_5$$

Note: $\cdots +$ subtleties due to $*F_5 = F_5$!

Compactification from IIB string theory

Take $M_{10} = M_4 \times X$ where $X = T^2 \times T^2 \times T^2$.
We search for **static, spherically symmetric** solutions of this effective theory. A convenient ansatz for the fields featuring this theory is

$$ds^2_{M_4} = g_{\mu\nu} dx^\mu dx^\nu = -e^{2U(r)} dt^2 + e^{-2U(r)} \left(dr^2 + r^2 d\Omega^2 \right)$$

$$ds^2_X = 2 g_{a\overline{b}} dz^a d\overline{z}^{\overline{b}} =: \sum_{a=1}^{3} \frac{1}{y^a(r)} |du^a + \tau^a(r) dv^a|^2$$

$$F_5 = \frac{1}{\sqrt{4\pi}} (\sin\theta d\theta \wedge d\varphi \otimes \Gamma + e^{2U(\varrho)} d\varrho \wedge dt \otimes *_X \Gamma)$$

$$\varrho = \frac{1}{r}, \qquad a, b = 1, 2, 3, \qquad \mu, \nu = 0, 1, 2, 3$$

The meaning of p^I, q_I and $|Z(\Gamma)|$

$$S_{DBI} \geq -\frac{2\sqrt{2\pi}\sqrt{Vol(X)}}{\kappa_{10}} \int |Z(\Gamma)| ds$$

We have equality for **supersymmetric cycles**. This means that the 3-branes have wrapping configurations of minimal volume. In this case in the 4D picture we see a particle with **mass**

$$M_{BPS} = \frac{2\sqrt{2\pi}\sqrt{Vol(X)}}{\kappa_{10}} |Z(\Gamma)| = \frac{1}{\sqrt{G_N}} |Z(\Gamma)|$$

$$S_{DBI} = -M_{BPS} \int ds$$

An analysis of S_{WZ} identifies the wrapping numbers p^I, q_I as quantized magnetic and electric **charges**.

If we take the ansatz discussed above we get the action ($T \equiv \int dt$ is the elapsed time and dot denotes $\frac{d}{d\varrho}$)

$$S_{4D}/T = \frac{1}{2G_N}\int_0^\infty d\varrho \left(\dot{U}^2 + G_{a\overline{b}}\dot{\tau}^a\dot{\overline{\tau}}^{\overline{b}} + G_N e^{2U} V_{BH}\right)$$

with the constraints

$$\dot{U}^2 + G_{a\overline{b}}\dot{\tau}^a\dot{\overline{\tau}}^{\overline{b}} - G_N e^{2U} V_{BH} = 0, \qquad V_{BH} = \frac{1}{2}||\Gamma||^2$$

$$G_{a\overline{b}} = \frac{1}{(y^a)^2}\delta_{a\overline{b}}$$

The action S_{4D} is describing the motion of a particle on $\mathbb{R} \times \mathcal{M}$ subject to the potential $-e^{2U} V_{BH}$. The parameter ϱ plays the role of time. According to the constraint the energy is conserved.

Notice that $V_{BH} = \frac{1}{2}||\Gamma||^2$ i.e. it is just one half the **norm of our three-qubit state** $|\Gamma\rangle$.

The **minima** of $e^{2U} V_{BH}$ are of central importance for finding finite energy solutions.
Since

$$V_{BH} = |Z|^2 + G^{a\overline{b}} D_a Z \overline{D}_{\overline{b}} \overline{Z} = |Z|^2 + 4 G^{a\overline{b}} \partial_a |Z| \overline{\partial}_{\overline{b}} |Z|$$

critical points of $|Z|$ are also **critical points** of V_{BH}. It turns out that such critical points are also **minima**.

Attractors

Up to a surface term S_{4D}/T can alternatively be written as

$$\frac{1}{2G_N}\int_0^\infty d\varrho\left((\dot{U}\pm\sqrt{G_N}e^U|Z|)^2+||\dot{\tau}^a\pm 2\sqrt{G_N}e^U G^{a\overline{b}}\overline{\partial}_{\overline{b}}|Z|||^2\right)$$

Hence the action has a minimum when

$$\dot{U}=-\sqrt{G_N}e^U|Z| \qquad \dot{\tau}^a=-2\sqrt{G_N}e^U G^{a\overline{b}}\overline{\partial}_{\overline{b}}|Z|$$

If we assume $Z_{crit}\equiv\lim_{\varrho\to\infty}Z(\varrho)\neq 0$ then from the first of these we get

$$\lim_{\varrho\to\infty}e^{-U}=\sqrt{G_N}|Z|_c\cdot\varrho$$

Hence the near horizon geometry of the black hole is $AdS_2\times S^2$

$$ds^2_\infty=\left(-\frac{r^2}{G_N|Z|_c^2}dt^2+\frac{G_N|Z|_c^2}{r^2}dr^2\right)+G_N|Z|_c^2\left(d\theta^2+\sin^2\theta d\varphi^2\right)$$

An important reformulation of the first order equations

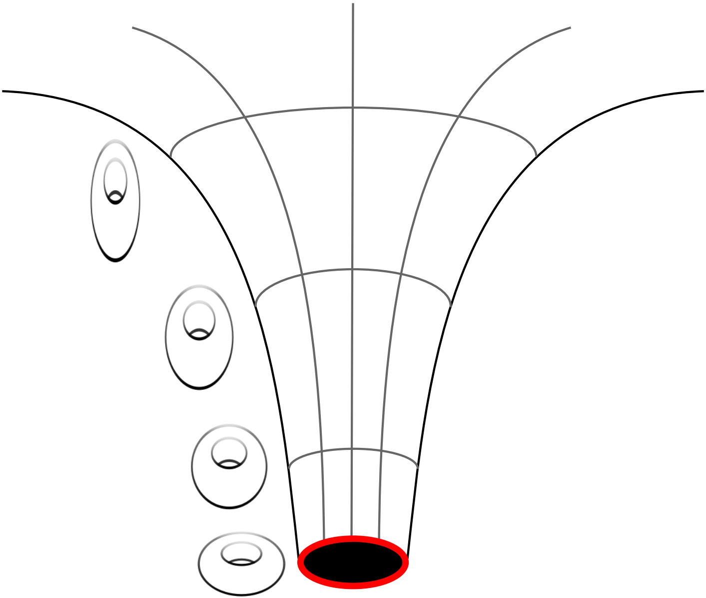
$$\frac{d}{d\varrho}\left[e^{-U}(e^{-i\alpha}|000\rangle - e^{i\alpha}|111\rangle)\right] \simeq -\sqrt{G_N}|\Gamma\rangle$$

$$Z = e^{i\alpha}|Z|, \qquad \varrho = \frac{1}{r}$$

This **first order radial flow** equation clearly shows an interplay between the space-time geometry and the geometry of the extra dimensions. Given the initial conditions at the asymptotically Minkowski region : $\tau^a(0) \equiv \tau^a_0$ and $U(0) = 0$, this dynamical system gives $\tau^a(\infty) \equiv \tau^a_c$ and $U(\infty) \equiv U_c$ at the horizon with geometry $AdS_2 \times S^2$.

Notice that a GHZ-like state is showing up in this equation.

The attractor geometry



The horizon area is

$$A = 4\pi G_N |Z(\infty)|^2$$

hence the thermodynamic Bekenstein-Hawking entropy is

$$S_{BH} = \frac{A}{4G_N} = \pi |Z(\infty)|^2$$

i.e. it is proportional to the **horizon value of the magnitude of the GHZ amplitude of our three-qubit state $|\Gamma\rangle$**

Note:

$$Z(\infty) = Z(\tau^a(\infty, p, q); p, q)$$

Can we express somehow the values of the deformation parameters in terms of the 8 wrapping numbers p, q ? For a statistical interpretation ($S = k \log W$) of black hole entropy this is what we badly need.

An analysis of the first order equations shows that the dynamical system describing the radial flow of the moduli is an **attractor**. This means the independent of the initial conditions at $\varrho = 0$ the moduli flow to a unique value $\tau^a(\infty)$ provided $D(\gamma) < 0$. It can also be shown that the attractor equations giving rise to these **stabilized values for the moduli** are given by the implicit equations between charges and moduli

$$\Gamma_{001} = \Gamma_{010} = \Gamma_{100} = \Gamma_{110} = \Gamma_{101} = \Gamma_{011} = 0.$$

These equations mean that at the horizon a GHZ state of the form

$$\overline{Z}(\infty)|000\rangle - Z(\infty)|111\rangle$$

is formed.

How to find the GHZ amplitudes at the horizon? Here we show two different methods. Here is the first which is 20 years old.

Let us write

$$W(\tau^3, \tau^2, \tau^1) = \gamma_{kji} c^k b^j a^i = \gamma_{kji} \varepsilon^{ii'} \varepsilon^{jj'} \varepsilon^{kk'} c_{k'} b_{j'} a_{i'}$$

$$a_i \leftrightarrow \begin{pmatrix} 1 \\ \tau^1 \end{pmatrix}, \qquad b_j \leftrightarrow \begin{pmatrix} 1 \\ \tau^2 \end{pmatrix} \qquad c_k \leftrightarrow \begin{pmatrix} 1 \\ \tau^3 \end{pmatrix}.$$

Then the BPS attractors are characterized by the equations

$$W(\overline{\tau}^3, \tau^2, \tau^1) = 0, \qquad W(\tau^3, \overline{\tau^2}, \tau^1) = 0, \qquad W(\tau^3, \tau^2, \overline{\tau^1}) = 0$$

and their complex conjugates. In three-qubit notation this corresponds to

$$\Gamma_{001} = \Gamma_{010} = \Gamma_{100} = \Gamma_{110} = \Gamma_{101} = \Gamma_{011} = 0.$$

An alternative form of these conditions is

$$\gamma_{kji} \overline{c}^k b^j a^i = 0, \qquad \gamma_{kji} c^k \overline{b}^j a^i = 0, \qquad \gamma_{kji} c^k b^j \overline{a}^i = 0.$$

Using the fact that γ_{kji} is real these equations taken together with their complex conjugates are equivalent to the vanishing of the 2×2 determinants

$$\mathrm{Det}\left(\gamma_{kji}c^k\right) = 0, \qquad \mathrm{Det}\left(\gamma_{kji}b^j\right) = 0, \qquad \mathrm{Det}\left(\gamma_{kji}a^i\right) = 0$$

provided the imaginary parts of the moduli are non vanishing.
Result: three quadratic equations, keeping only the solutions providing y^1, y^2 and y^3 positive yield the stabilized values for the moduli

$$\tau^a(\infty; p, q) = \frac{(\gamma_0 \cdot \gamma_1)^a + i\sqrt{-D}}{(\gamma_0 \cdot \gamma_0)^a}, \qquad a = 1, 2, 3.$$

$$(\gamma_0 \cdot \gamma_1)^1 \equiv \gamma_{kj0}\varepsilon^{kk'}\varepsilon^{jj'}\gamma_{k'j'1}$$

$$D = (\gamma_0 \cdot \gamma_1)^2 - (\gamma_0 \cdot \gamma_0)(\gamma_1 \cdot \gamma_1)$$

is Cayley's hyperdeterminant. In order to have such solutions $-D$ should be positive and $(\gamma_0 \cdot \gamma_0)$ should be negative.

Black Hole Entropy and the Three-Tangle

Using the stabilized values $\tau^a(\infty)$ in $Z = e^{K/2}W(\tau^3, \tau^2, \tau^1)$ we get

$$|Z|^2 = \sqrt{-D(|\gamma\rangle)} = \sqrt{(\gamma_0 \cdot \gamma_0)(\gamma_1 \cdot \gamma_1) - (\gamma_0 \cdot \gamma_1)^2}$$

Due to the triality symmetry of Cayley's hyperdeterminant products like $(\gamma_0 \cdot \gamma_1)$ can be calculated by using any of the qubits playing a special role. Now the **entropy** is

$$S_{BH} = \pi\sqrt{-D(|\gamma\rangle)}$$

Note that the quantity $\tau_3 = 4|D(|\gamma\rangle)|$ is a genuine entanglement measure of the state $|\gamma\rangle$ in the theory of three-qubit entanglement. For BPS black holes we have $\tau_{123} = -4D$.
The final form of our three-qubit state on the event horizon is

$$|\Gamma\rangle_\infty = (-D)^{1/4}\left(e^{i\alpha}|000\rangle_\infty - e^{-i\alpha}|111\rangle_\infty\right)$$

$$\tan\alpha = \sqrt{-D}\frac{p^0}{2p^1p^2p^3 + p^0(p^0q_0 + p^1q_1 + p^2q_2 + p^3q_3)}$$

The explicit form of D

$$
\begin{aligned}
D &= (p^0q_0+p^1q_1+p^2q_2+p^3q_3)^2 \\
&- 4((p^1q_1)(p^2q_2)+(p^2q_2)(p^3q_3)+(p^3q_3)(p^1q_1)) \\
&+ 4p^0q_1q_2q_3-4q_op^1p^2p^3
\end{aligned}
$$

The second method: attractors from vanishing concurrence

For an *unnormalized* two-qubit density operator ρ regarded as a nonnegative 4×4 Hermitian matrix acting on the composite Hilbert space $\mathcal{H}_{12} = \mathcal{H}_1 \otimes \mathcal{H}_2 = \mathbb{C}^2 \otimes \mathbb{C}^2$ the Wooters concurrence squared is

$$\mathcal{C}_{12}^2 \equiv \tau_{12} = [\max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}]^2$$

Here $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the square-roots of the nonnegative eigenvalues of the matrix

$$\rho\tilde{\rho} \equiv \rho(\varepsilon \otimes \varepsilon)\rho^T(\varepsilon \otimes \varepsilon)$$

For three qubits we have

$$\tau_{ab} = \mathrm{Tr}(\rho_{ab}\tilde{\rho}_{ab}) - \frac{1}{2}\tau_{123}, \qquad a, b \in \{12, 23, 13\}$$

Moduli stabilization from vanishing concurrence

We would like to calculate the Wootters concurrences τ_{ab} for the charge and moduli dependent three-qubit state

$$|\Gamma(r)\rangle = \mathcal{S}_3(r)\otimes\mathcal{S}_2(r)\otimes\mathcal{S}_1(r)|\gamma\rangle$$

$$|\gamma\rangle = \gamma_{000}|000\rangle + \gamma_{001}|001\rangle + \cdots + \gamma_{110}|110\rangle + \gamma_{111}|111\rangle$$

where

$$\mathcal{S}_3\otimes\mathcal{S}_2\otimes\mathcal{S}_1 = \frac{1}{\sqrt{8y^3y^2y^1}}\begin{pmatrix}\overline{\tau}^3 & -1\\ -\tau^3 & 1\end{pmatrix}\otimes\begin{pmatrix}\overline{\tau}^2 & -1\\ -\tau^2 & 1\end{pmatrix}\otimes\begin{pmatrix}\overline{\tau}^1 & -1\\ -\tau^1 & 1\end{pmatrix}$$

$$\begin{pmatrix}\gamma_{000} & \gamma_{001} & \gamma_{010} & \gamma_{100}\\ \gamma_{111} & \gamma_{110} & \gamma_{101} & \gamma_{011}\end{pmatrix} = \begin{pmatrix}-p^0 & -p^1 & -p^2 & -p^3\\ -q_0 & q_1 & q_2 & q_3\end{pmatrix}$$

$$\tau^a(r) = x^a(r) - iy^a(r)$$

The result is

$$\tau_{bc}(|\Gamma(r)\rangle) = \tau_{123}(|\gamma\rangle)[g(\mathcal{M}^a(r), \gamma^a) + 1][g(\mathcal{M}^a(r), \gamma^a) - 1]$$

for a, b, c different where

$$\mathcal{M}^a = \frac{1}{y^a}\begin{pmatrix} 1 & x^a \\ x^a & (x^a)^2 + (y^a)^2 \end{pmatrix}$$

$$\gamma^a = \frac{1}{|\gamma_0 \wedge \gamma_1|}\begin{pmatrix} (\gamma_0 \cdot \gamma_0)_a & (\gamma_0 \cdot \gamma_1)_a \\ (\gamma_0 \cdot \gamma_1)_a & (\gamma_1 \cdot \gamma_1)_a \end{pmatrix}$$

Here

$$|\gamma_0 \wedge \gamma_1| \equiv \sqrt{\gamma_0^2 \gamma_1^2 - (\gamma_0 \cdot \gamma_1)^2}$$

and for any two 2×2 matrices M, N we define

$$g(M, N) \equiv \mathrm{Tr}(M \varepsilon N^T \varepsilon)$$

Moduli stabilization from vanishing concurrence

The space of symmetric real matrices equipped with the symmetric bilinear form $g(\cdot,\cdot)$ is isomorphic to $2 \oplus 1$ dimensional Minkowski spacetime. Indeed one can parametrize such matrices as

$$M = \begin{pmatrix} T - X & Y \\ Y & T + X \end{pmatrix}$$

hence the trace is related to "time" and $g(M, M) = X^2 + Y^2 - T^2$. Since $y^a > 0$ the vectors associated to the matrices γ^a and $\mathcal{M}^a$ are **timelike** and the latter is also **future directed**. It can be easily proved that one has only the following two cases

$$g(\mathcal{M}, \Gamma) < 0, \quad \text{i.e.} \quad Tr(\mathcal{M}) > 0, \quad Tr(\Gamma) > 0$$

or

$$g(\mathcal{M}, \Gamma) > 0, \quad \text{i.e.} \quad Tr(\mathcal{M}) > 0, \quad Tr(\Gamma) < 0$$

Moduli stabilization from vanishing concurrence

The vanishing condition for the concurrence yields the usual set of stabilized values for the moduli

$$\tau^a(r=0;p,q)=\frac{(\gamma_0\cdot\gamma_1)^a\pm i\sqrt{-D}}{(\gamma_0\cdot\gamma_0)^a},\qquad a=1,2,3$$

where the two possible choices for the sign correspond **precisely** to the two possible sets of BPS charge configurations i.e.

$$\gamma_0^2>0,\qquad \gamma_1^2>0,\qquad -D=\gamma_0^2\gamma_1^2-(\gamma_0\cdot\gamma_1)^2>0$$

$$\gamma_0^2<0,\qquad \gamma_1^2<0,\qquad -D=\gamma_0^2\gamma_1^2-(\gamma_0\cdot\gamma_1)^2>0$$

Notice also that due to the CKW inequality

$$S_{BH}=\frac{\pi}{2}\sqrt{\tau_{123}},\qquad \tau_{123}=\tau_{1(23)}=\tau_{2(31)}=\tau_{3(12)}$$

$$\tau_{a(bc)}=2((\mathrm{Tr}\rho_a)^2-\mathrm{Tr}\rho_a^2)$$

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6. There are many more interesting correspondences between entangled systems and the physics of black holes. For a review on this see....

L. Borsten, M. J. Duff and P. Lévay
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