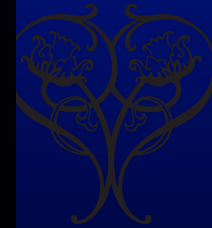
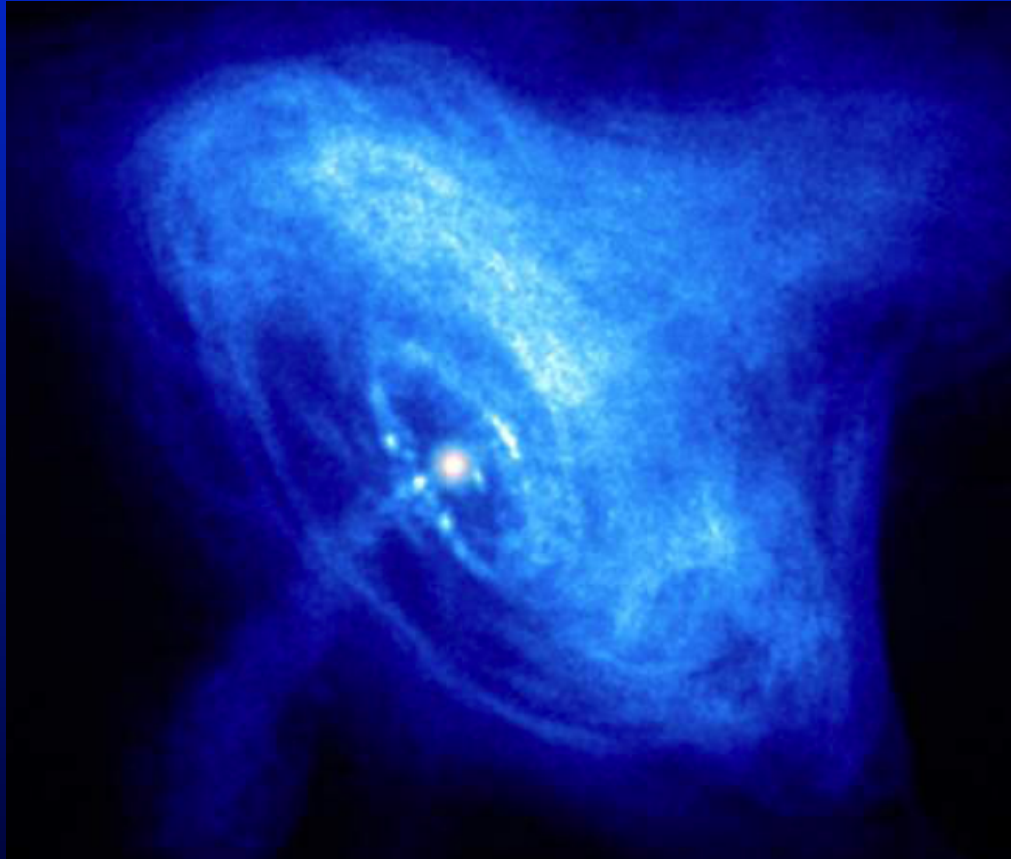


Neutron star interiors

Gordon Baym
University of Illinois



Wigner Colloquium
Wigner Research Centre for Physics
24 May 2016



Wigner students, Princeton 1930's



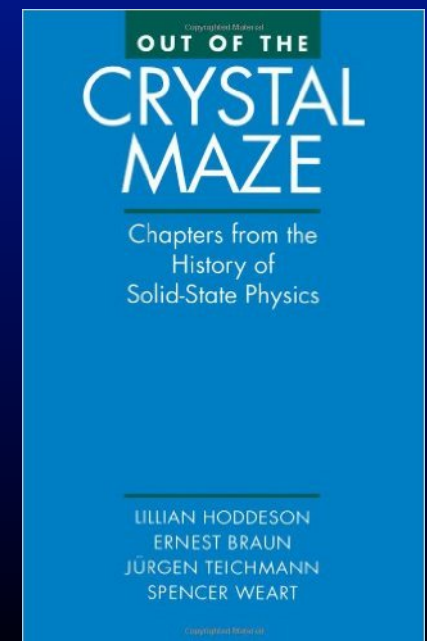
Fred Seitz, 1911-2008
Wigner-Seitz method for metals



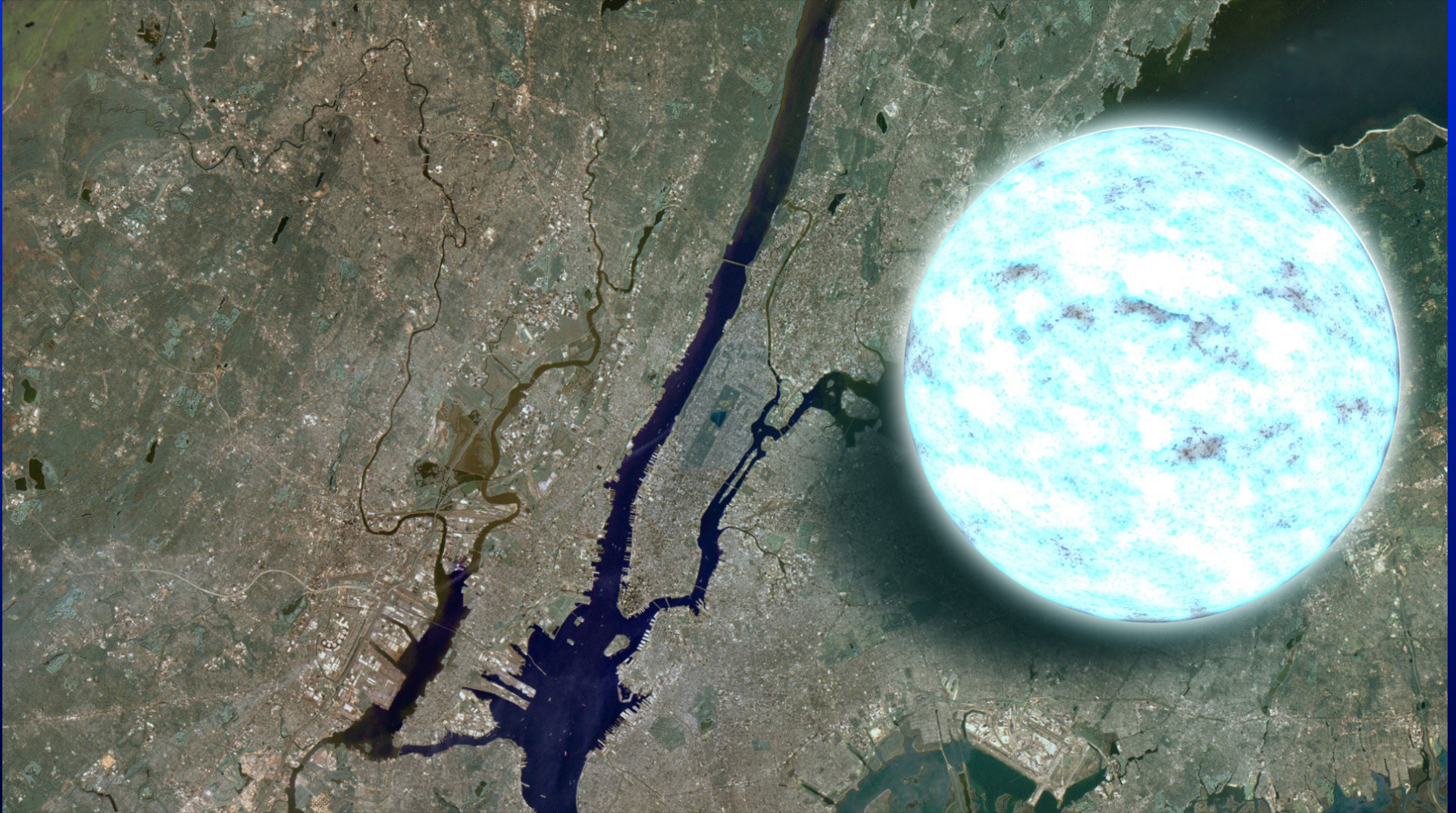
John Bardeen, 1908-1991
Transistor
BCS theory of superconductivity



Conyers Herring, 1914-2009
Band theory of solids,
“Handrails”



Compress the sun (radius 700,000 km)
down to a radius of 10-12 km



Neutron star over New York City

Neutron stars

Mass $\sim 1.3\text{-}2 M_{\text{sun}}$

Baryon no. $\sim 10^{57}$

$$\sim (Gm_p^2/\hbar c)^{-3/2}$$

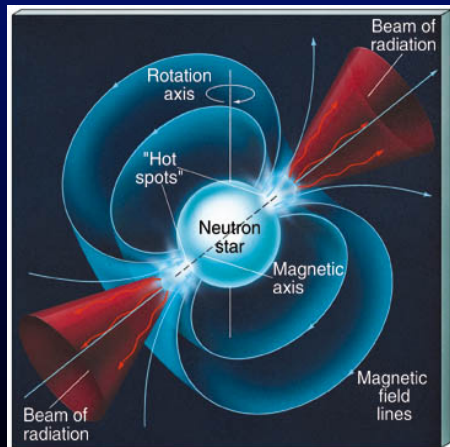
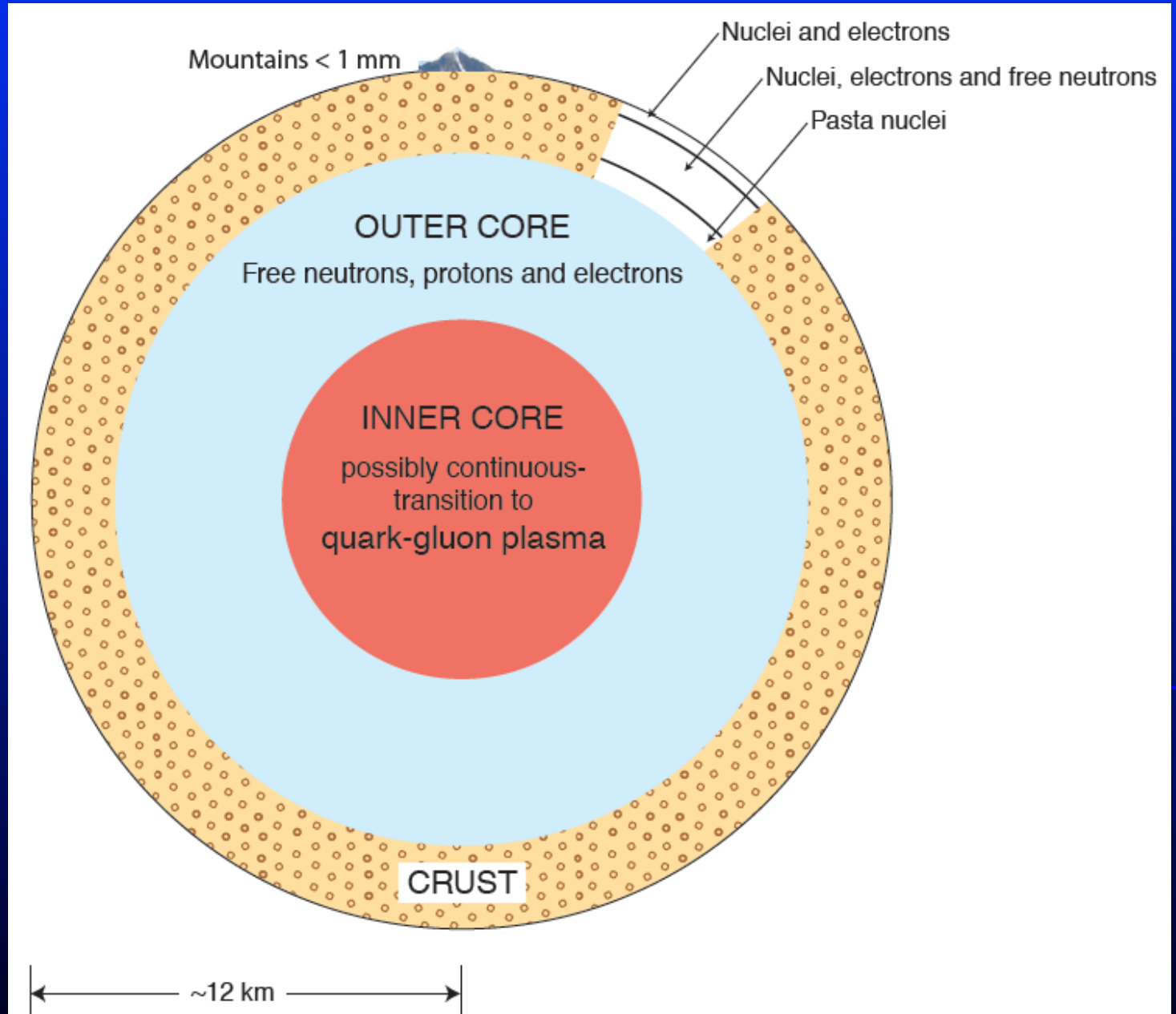
Radius $\sim 10\text{-}12 \text{ km}$

Temperature
 $\sim 10^6\text{-}10^9 \text{ K}$

Surface gravity
 $\sim 10^{14}$ that of Earth

Surface binding
 $\sim 1/10 mc^2$

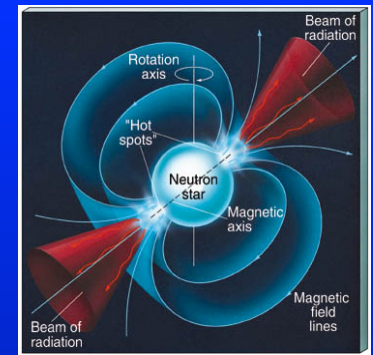
Magnetic fields
 $\sim 10^6 - 10^{15} \text{ G}$



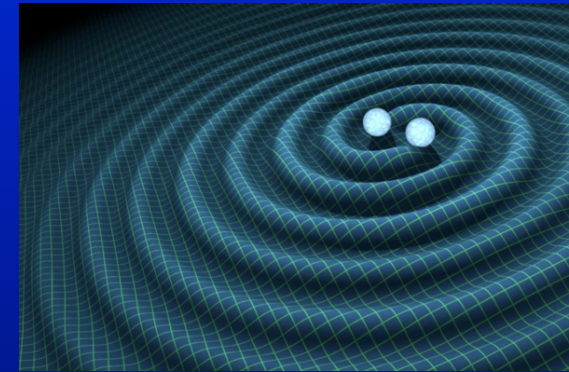
m³

Made in gravitational collapse of massive stars
(supernovae)

Central element in variety of compact energetic systems:
pulsars, binary x-ray sources, soft gamma repeaters



Merging neutron star-neutron star and neutron star
-black hole sources of gamma ray bursts, and
gravitational radiation



Matter in neutron stars is densest in universe:

ρ up to $\sim 5-10 \rho_0$ ($\rho_0 = 3 \times 10^{14} \text{g/cm}^3 =$ density of matter in atomic nuclei)
[cf. white dwarfs: $\rho \sim 10^5-10^9 \text{g/cm}^3$]

Supported against gravitational collapse by nucleon/quark degeneracy
pressure

Astrophysical laboratory for study of cold high density matter
complementary to accelerator experiments (RHIC, LHC)

What are states in interior? Onset of quark degrees of freedom!

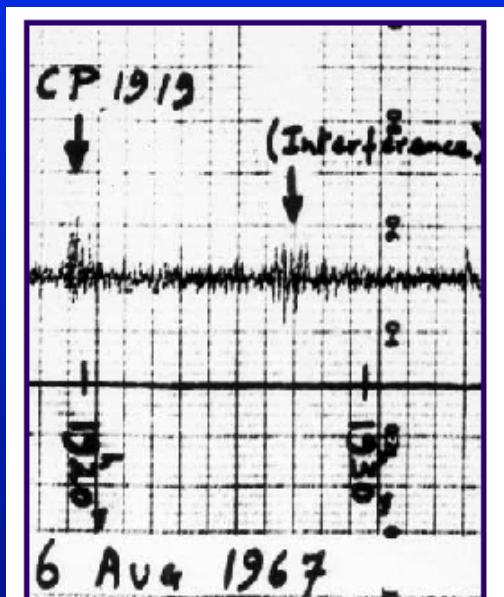
First proposal of neutron stars

Neutron discovered
by Chadwick in early
1932

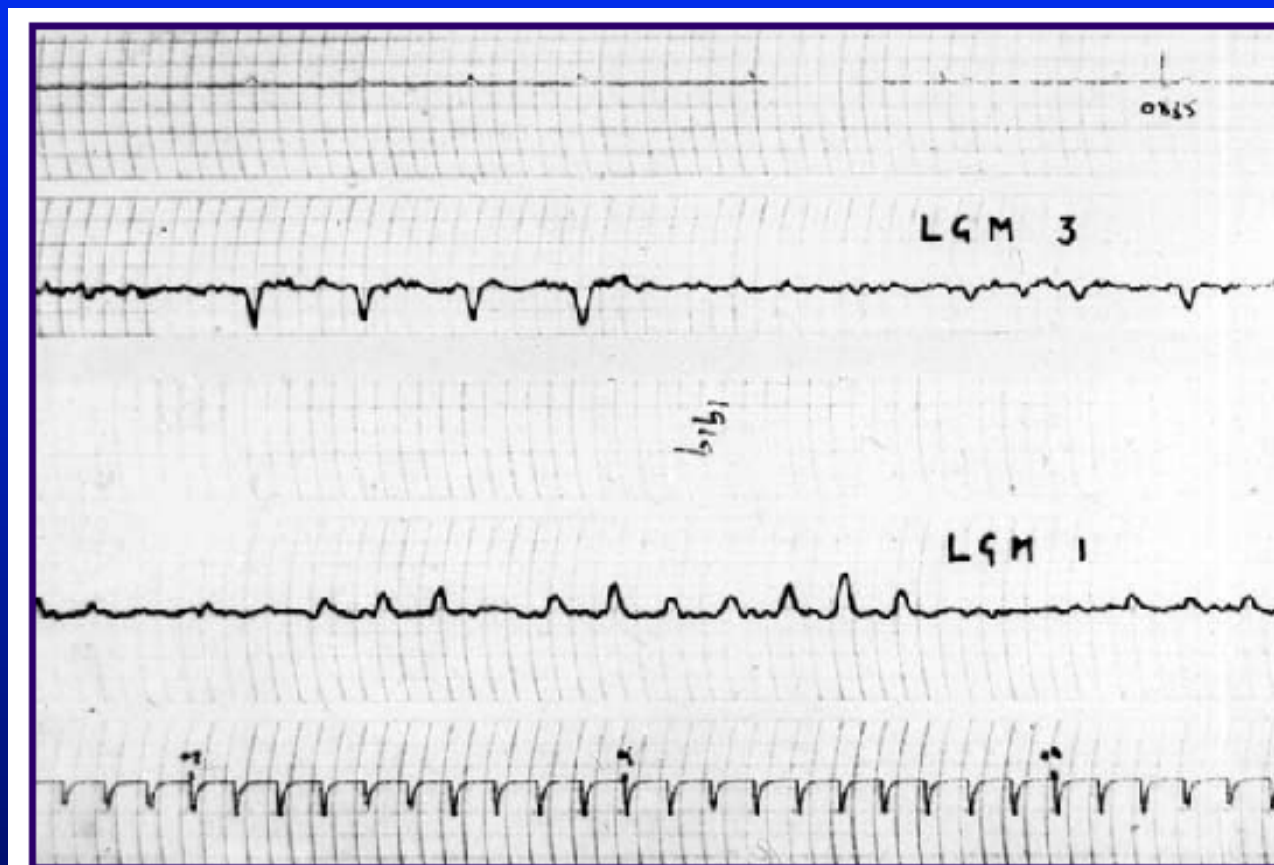
Baade and Zwicky at
Stanford APS Meeting,
15-16 Dec. 1933
Phys. Rev. 45, 138
(1934)

38. Supernovae and Cosmic Rays. W. BAADE, *Mt. Wilson Observatory*, AND F. ZWICKY, *Californic Institute of Technology*.--Supernovae flare up in every stellar system (nebula) once in several centuries. The lifetime of a supernova is about twenty days and its absolute brightness at maximum may be as high as $M_{vis} = -14^M$. The visible radiation L_v of a supernova is about 10^3 times the radiation of our sun, that is, $L_v = 3.78 \times 10^{41}$ ergs/sec. Calculations indicate that the total radiation, visible and invisible, is of the order $L_r = 10^7 L_v = 3.78 \times 10^{48}$ ergs/sec. The supernova therefore emits during its life a total energy $E_r \cong 10^6 L_r = 3.78 \times 10^{53}$ ergs. If supernovae initially are quite ordinary stars of mass $M < 10^{34}$ g, E_r/c^2 is of the same order as M itself. In the *supernova process mass in bulk is annihilated*. In addition the hypothesis suggests itself that *cosmic rays are produced by supernovae*. Assuming that in every nebula one supernova occurs every thousand years, the intensity of the cosmic rays to be observed on the earth should be of the order $\sigma = 2 \times 10^{-3}$ erg/cm² sec. The observational values are about $\sigma = 3 \times 10^{-3}$ erg/cm² sec. (Millikan, Regener). With all reserve we advance the view that supernovae represent the transitions from ordinary stars into *neutron stars*, which in their final stages consist of extremely closely packed neutrons.

1967 First pulsar detection: 1919+21 Bell & Hewish



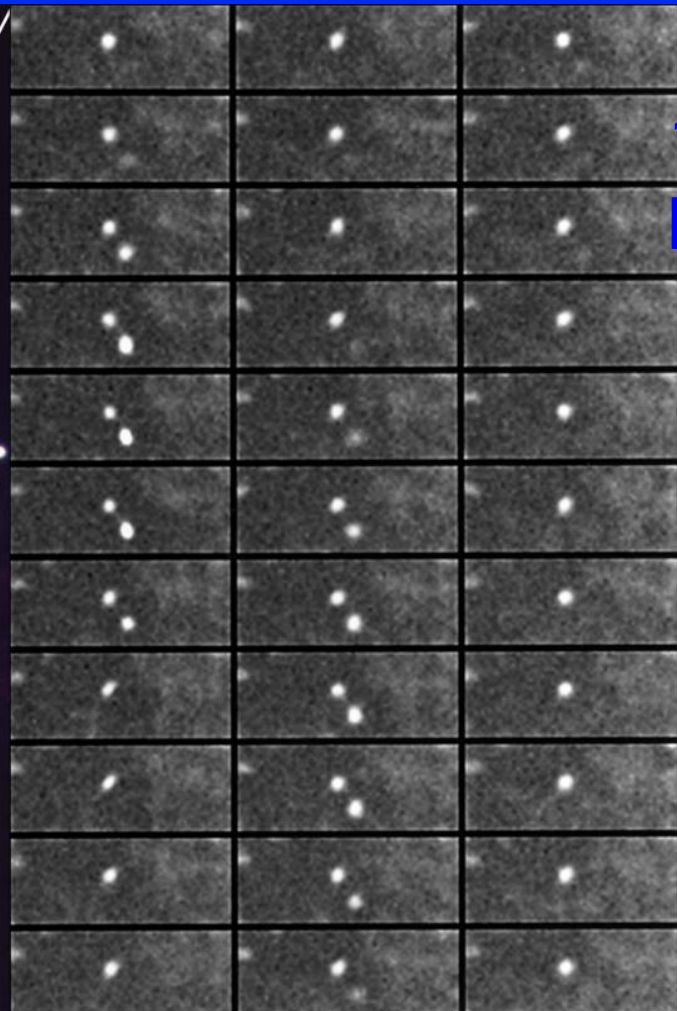
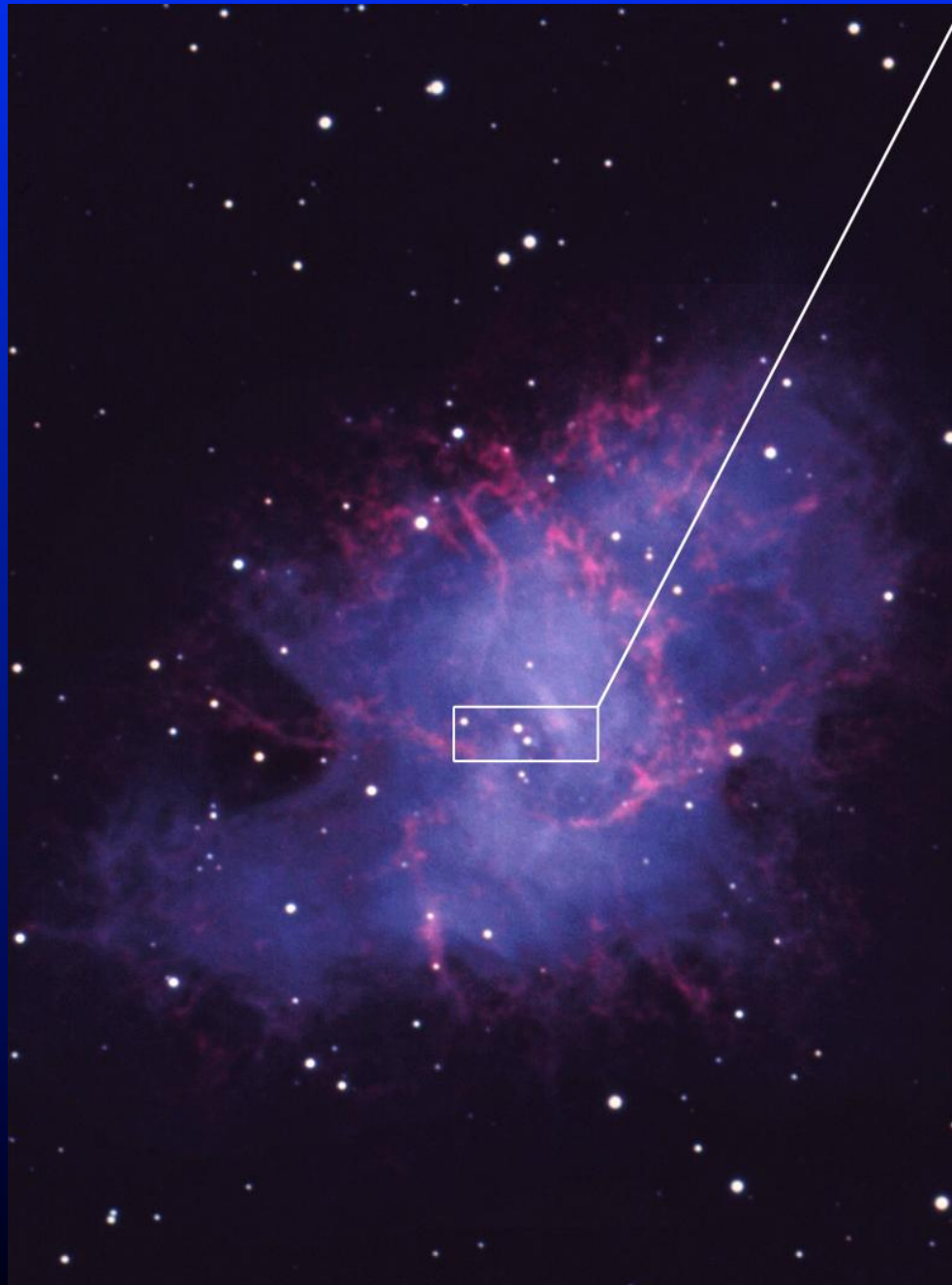
2: The first detection of the first pulsar, occupying about one-quarter inch of chart paper. About five minutes later is a short burst of low level interference. This signal has been high-pass filtered, to remove the telescope's interference pattern. (Mullard Radio Astronomy Observatory.)



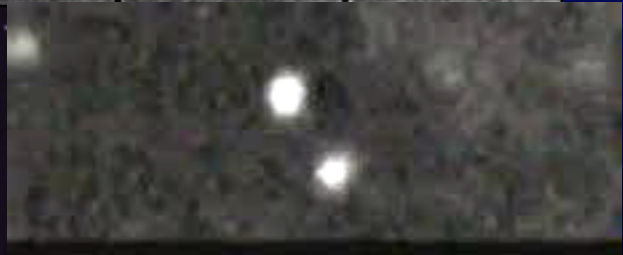
3: The bottom trace is of broadcast one-second time pips. The middle trace shows the first recording to reveal the pulsed nature of the pulsar PSR1919. The top trace is of the third pulsar discovered, PSR0834.

“It was highly unlikely that there would be two lots of little green men on opposite sides of the universe both deciding to signal at the same time to a rather inconspicuous star on a rather curious frequency.” Jocelyn Bell

Crab Pulsar (period = 33 msec) Supernova July 4, 1054



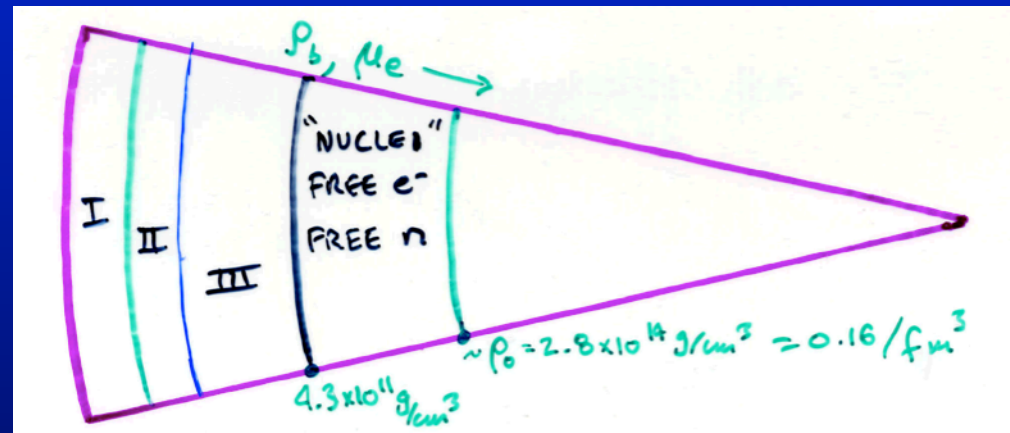
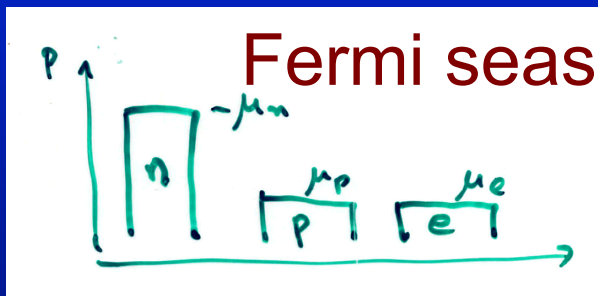
1 msec
per frame



Nuclei before neutron drip

$e^- + p \rightarrow n + \nu$: makes nuclei neutron rich
 as electron Fermi energy increases with depth
 $n \rightarrow p + e^- + \bar{\nu}$: not allowed if e^- state already occupied

Beta equilibrium: $\mu_n = \mu_p + \mu_e$

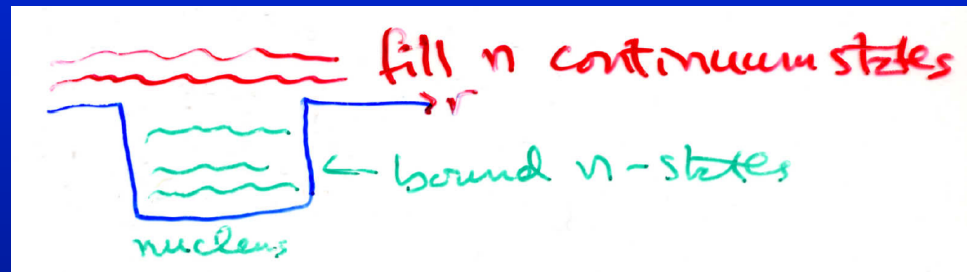


- I: ^{56}Fe ^{62}Ni ^{64}Ni
- II: ^{84}Se ^{82}Ge ^{80}Zn ^{78}Ni ^{76}Fe } N=50
- III: ^{124}Mo ^{122}Zr ^{120}Sr ^{118}Kr } N=82

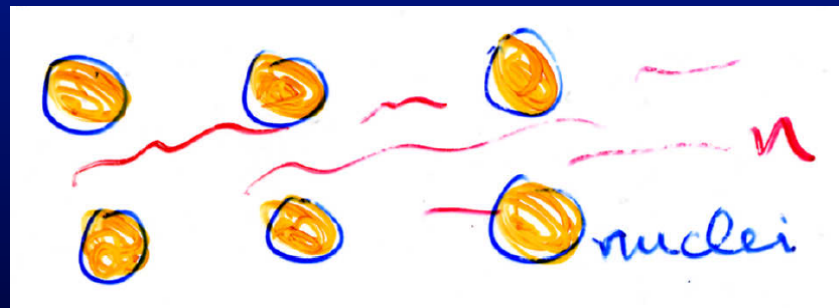
Shell structure (spin-orbit forces) for very neutron rich nuclei?
 Do N=50, 82 remain neutron magic numbers? Proton shell structure?
 Being explored at rare isotope accelerators: **RIKEN Rare Ion Beam Facility**, and later GSI (**MINOS**), FRIB, RAON (KoRIA)

Neutron drip

Beyond density $\rho_{\text{drip}} \sim 4.3 \times 10^{11} \text{ g/cm}^3$ neutron bound states in nuclei become filled. Further neutrons must go into continuum states. Form degenerate neutron Fermi sea.



Neutrons in neutron sea are in equilibrium with those inside nucleus (common μ_n)



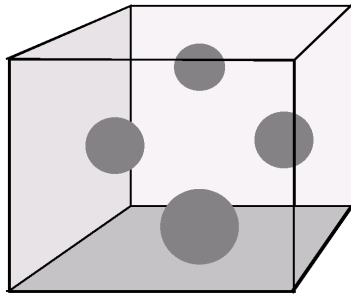
Protons appear not drip, but remain in bound states until nuclei merge in interior liquid.

Pasta Nuclei in inner crust

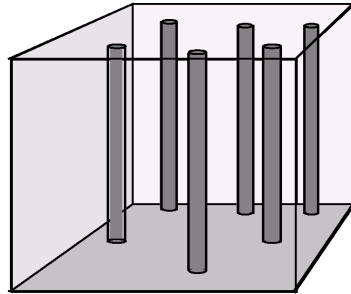
Lorentz, Pethick, and Ravenhall PRL (1993)

When Coulomb wins over surface energies

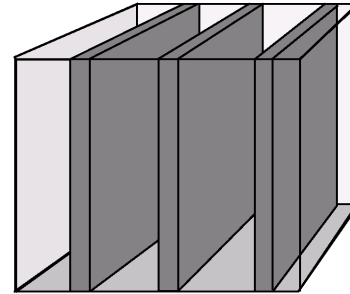
"Pasta"



(a) Meatballs



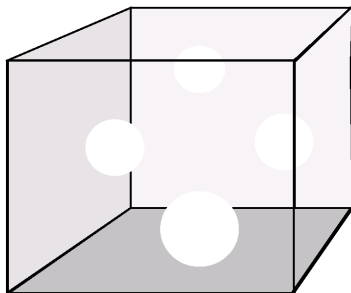
(b) Spaghetti



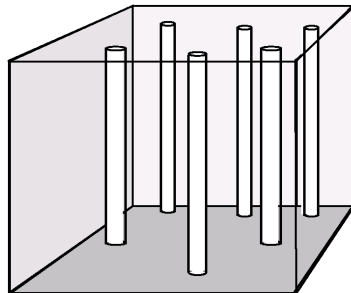
(c) Lasagna

F K Lamb

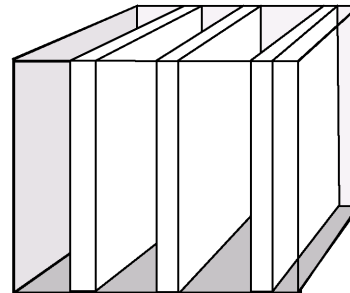
"Antipasta"



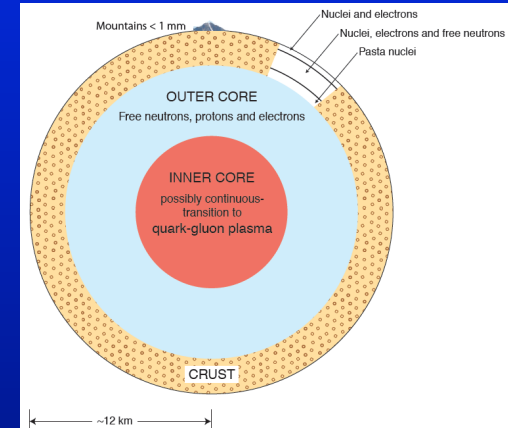
(f) Cheese



(e) Anti-spaghetti



(d) Anti-lasagna



Involves over half the mass of the crust !!
Effects crust bremsstrahlung of neutrinos, pinning of n vortices, ...

Properties of liquid interior near nuclear matter density

Determine N-N potentials from

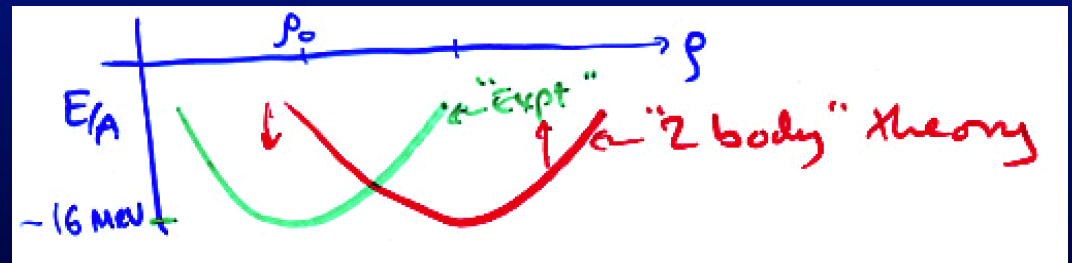
- scattering experiments $E < 300$ MeV
- deuteron, 3 body nuclei (${}^3\text{He}$, ${}^3\text{H}$)

ex., Paris, Argonne, Urbana 2 body potentials

Solve Schrödinger equation by variational techniques

Large theoretical extrapolation from low energy laboratory nuclear physics at near nuclear matter density

Two body potential alone:

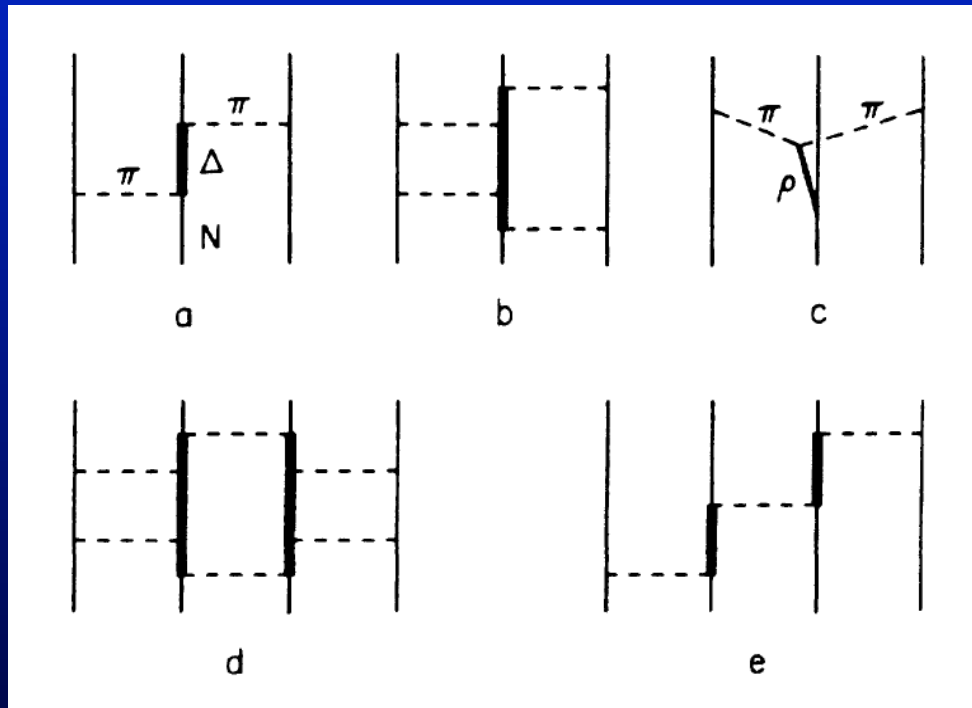
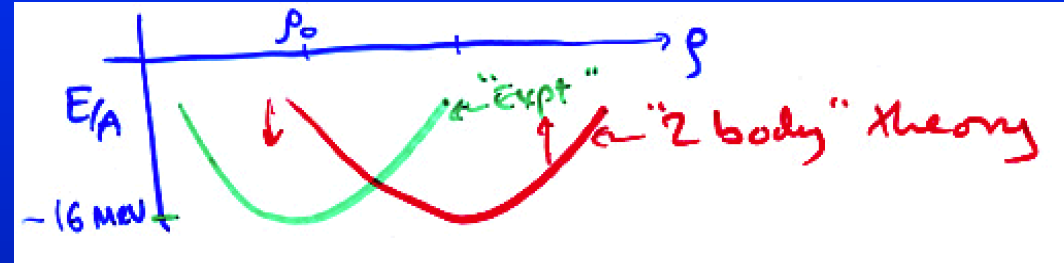


Underbind ${}^3\text{H}$: Exp = -8.48 MeV, Theory = -7.5 MeV

${}^4\text{He}$: Exp = -28.3 MeV, Theory = -24.5 MeV

Importance of 3 body interactions

Attractive at low density
Repulsive at high density

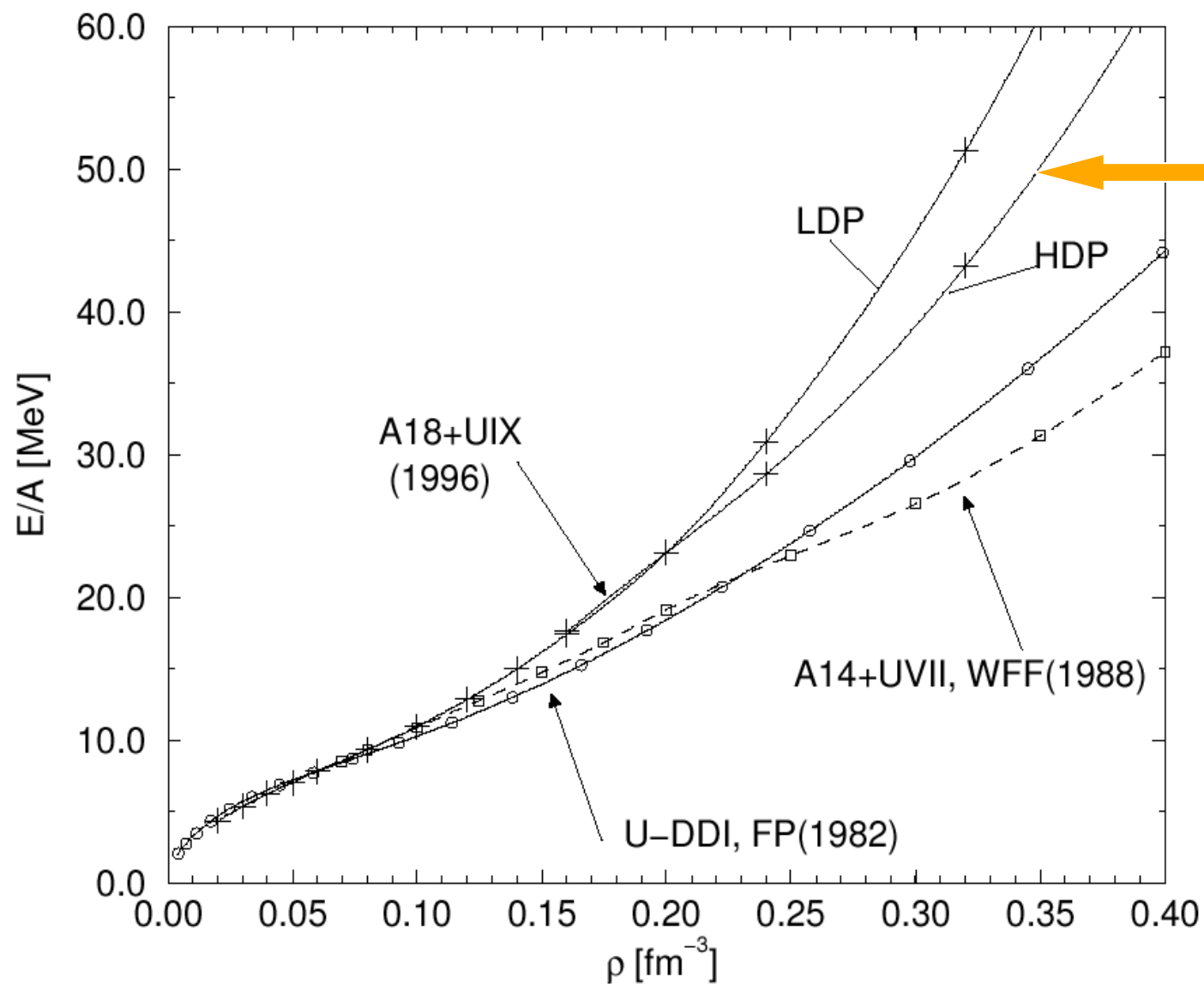


Various processes that lead to three and higher body intrinsic interactions (not described by iterated nucleon-nucleon interactions).

Stiffens equation of state at high density
Large uncertainties!

Energy per nucleon in pure neutron matter

Akmal, Pandharipande and Ravenhall, *Phys. Rev. C*58 (1998) 1804



π^0
condensate

Neutron star models

Equation of state: $E = \text{energy density} = \rho c^2$
 $n_b = \text{baryon density}$
 $P(\rho) = \text{pressure} = n_b^2 \partial(E/n_b)/\partial n_b$

Tolman-Oppenheimer-Volkoff equation of hydrostatic balance:

$$\frac{\partial P(r)}{\partial r} = - \frac{G (\rho(r) + P(r)/c^2)}{r^2 (1 - 2m(r)G/rc^2)} (m(r) + 4\pi P(r)r^3/c^2)$$

$$m(r) = \int_0^r \rho(r') 4\pi r'^2 dr'$$

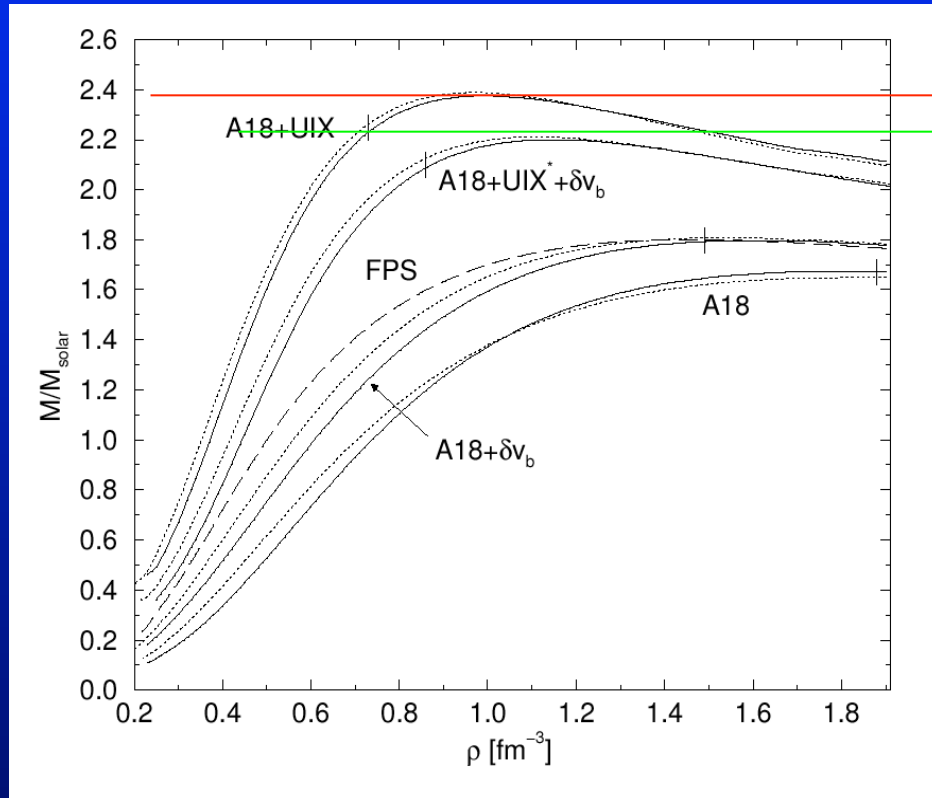
general relativistic corrections

= mass within radius r

- 1) Choose central density: $\rho(r=0) = \rho_c$
- 2) Integrate outwards until $P=0$ (at radius R)
- 3) Mass of star

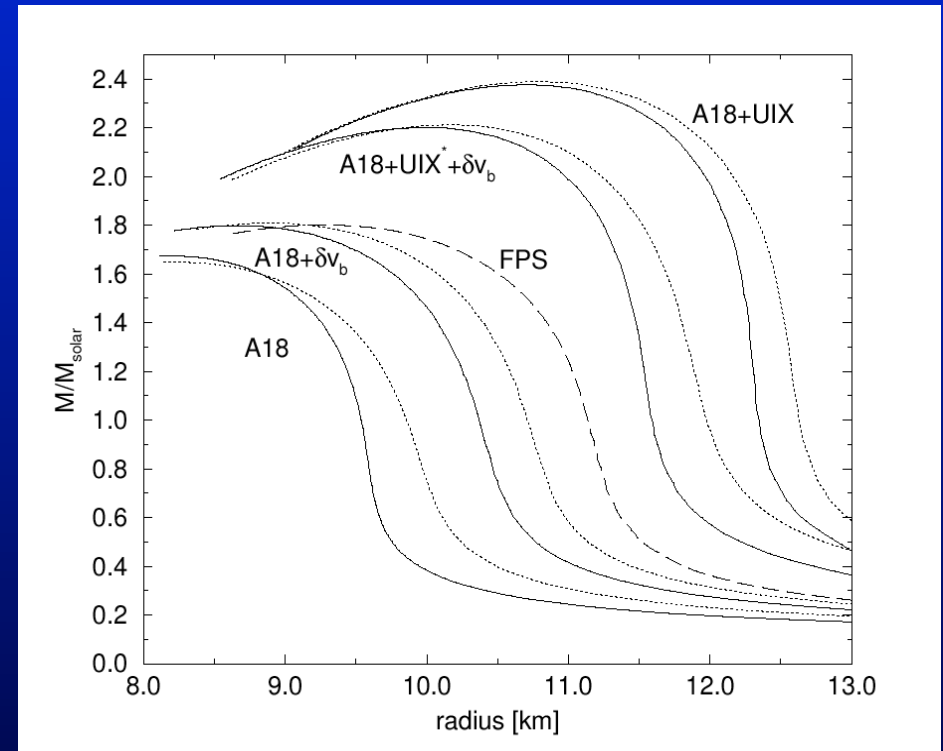
$$M = \int_0^R \rho(r) 4\pi r^2 dr$$

Neutron star models using *static interactions between nucleons*



Mass vs. central density

Maximum neutron star mass



Mass vs. radius

Well beyond nuclear matter density



"I name this place Terra Incognita."

Well beyond nuclear matter density

Onset of new degrees of freedom: mesonic, Δ 's, quarks and gluons, ...

Properties of matter in this extreme regime determine maximum neutron star mass.

Large uncertainties!

Hyperons: Σ , Λ , ...

Meson condensates: π^- , π^0 , K^-

Quark matter

in bulk

in droplets

Color superconductivity

Strange quark matter

absolute ground state of matter??

strange quark stars?


$$\langle u, d \rangle$$


$$\langle u, d \rangle = \langle d, s \rangle = \langle s, u \rangle$$

Fundamental limitations of equation of state based on nucleon-nucleon interactions alone:

Accurate for $n \sim n_0 = \text{nuclear saturation density} = 0.16/\text{fm}^3$

$n \gg n_0$:

- can forces be described with static few-body potentials?
- Force range $\sim 1/2m_\pi \Rightarrow$ relative importance of 3 (and higher) body forces $\sim n/(2m_\pi)^3 \sim 0.3 n/n_0$. Estimate from chiral effective field theory possibly lower.
- No well defined expansion in terms of 2,3,4,...body forces.
- Can one even describe system in terms of well-defined "asymptotic" laboratory particles? Early percolation of nucleonic volumes!

Must take quarks degrees of freedom seriously at densities $n \gg n_0$

Quark degrees of freedom

Quarks = fractionally charged spin-1/2 fermions,
baryon no. = 1/3, with internal SU(3) **color** degree of freedom.

Flavor	Charge/e	Mass(MeV)
u	2/3	(2.1-3.5) 5
d	-1/3	(2.1-3.5) 10
s	-1/3	(54-92) 150
c	2/3	1300
b	-1/3	4200
t	2/3	175000



Hadrons are composed of quarks:

$$\text{proton} = u + u + d$$

$$\text{neutron} = u + d + d$$

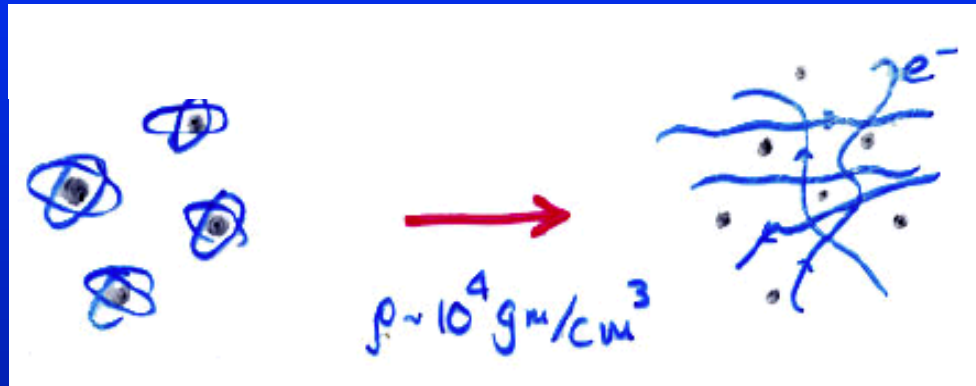
$$\pi^+ = u + \bar{d}, \text{ etc.}$$

Form of baryons in the early universe at $t < 1\mu \text{ sec}$ ($T > 100 \text{ MeV}$).

Basic degrees of freedom in deep interiors of neutron stars.

Compress matter to form new states

Atoms



Plasma

Nuclei

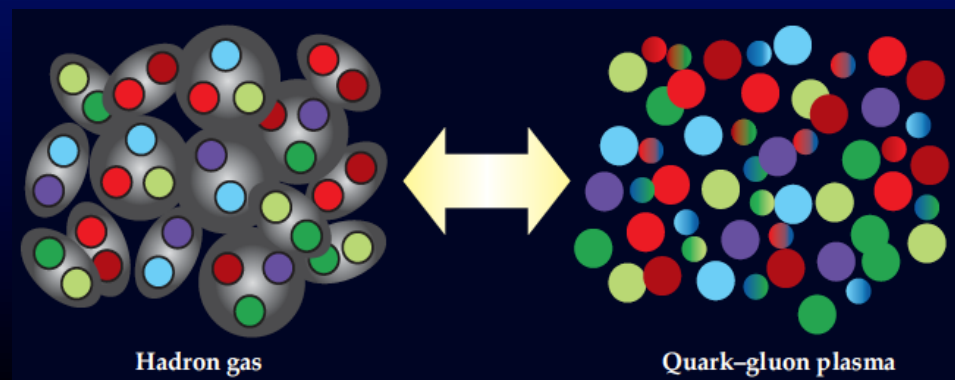


Nuclear matter

$$\rho \sim 2.5 \times 10^{14} \text{ gm/cm}^3 = \rho_{\text{nm}} = 0.17 \text{ baryons/fm}^3$$

(1 fm = 10^{-13} cm)

Hadrons
(n, p, ...)

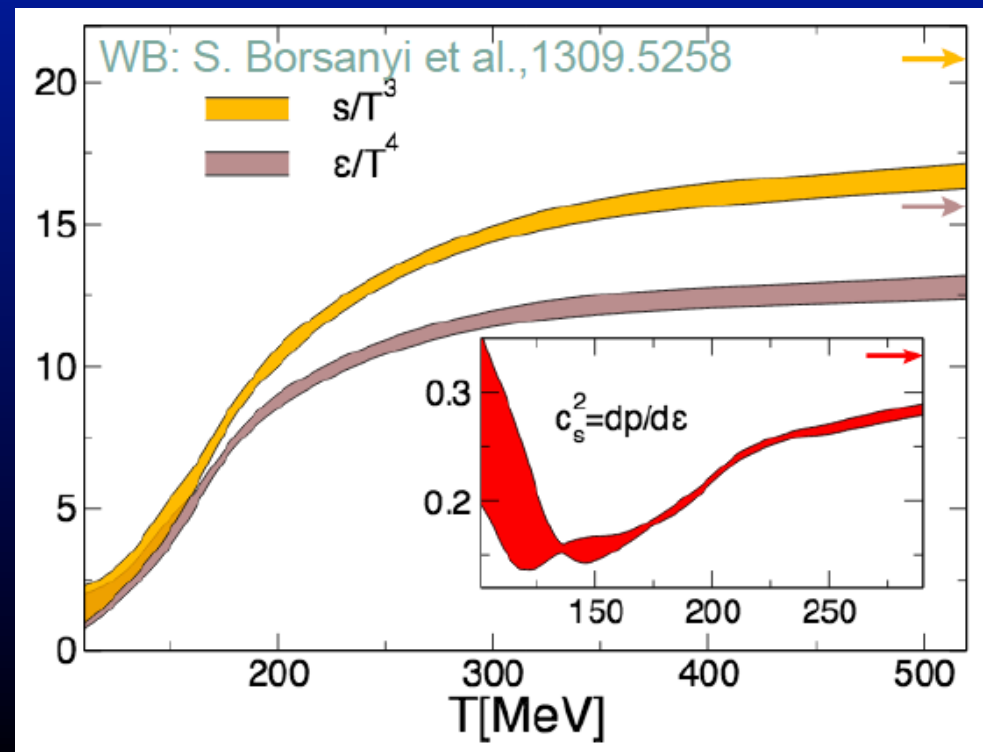
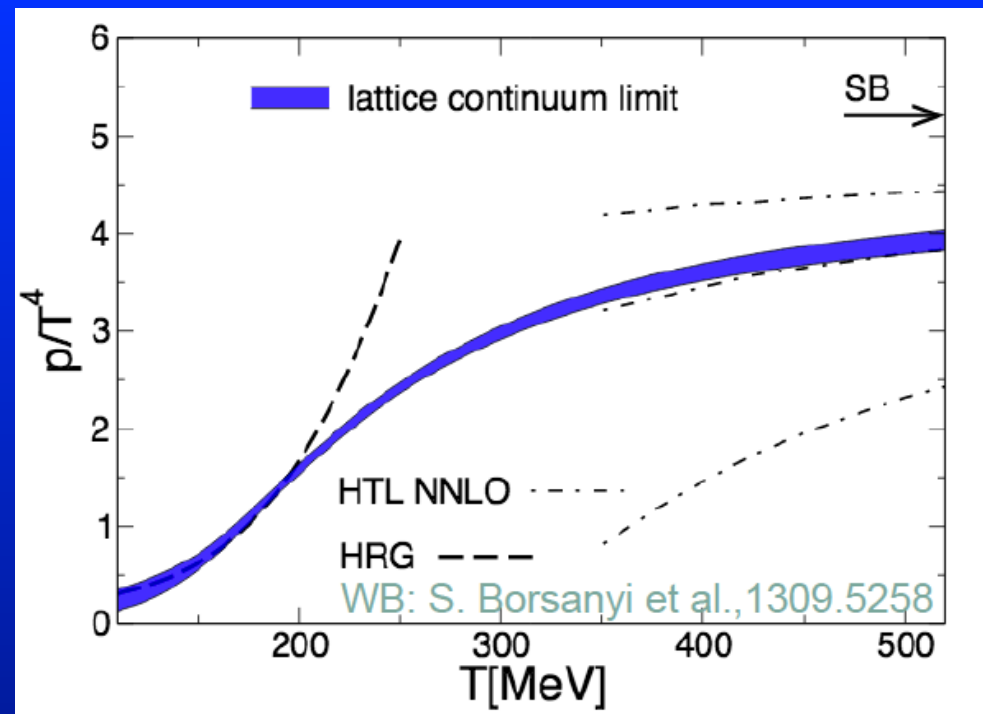


Quark matter

Lattice gauge theory calculations of equation of state of QGP

Limited, because of “fermion sign problem” to zero baryon density and nearby.

Can't systematically calculate yet for realistic chemical potentials



Learning about dense matter from neutron star observations

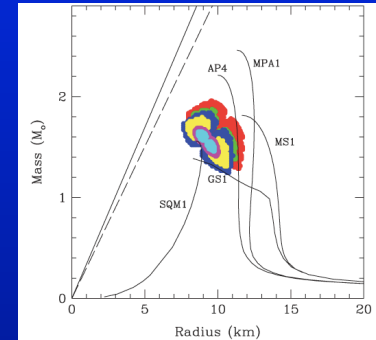


Challenges to nuclear theory!!

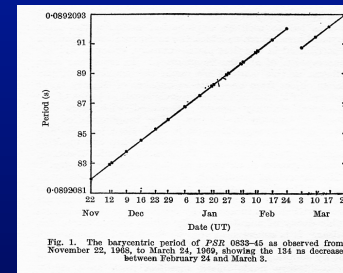
Learning about dense matter from neutron star observations

Masses of neutron stars

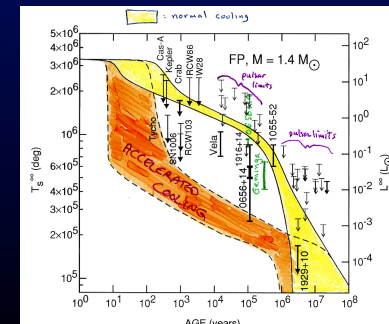
Binary systems: stiff e.o.s
 Thermonuclear bursts in X-ray binaries => Mass vs. Radius, strongly constrains eq.of state



Glitches: probe n,p superfluidity and crust

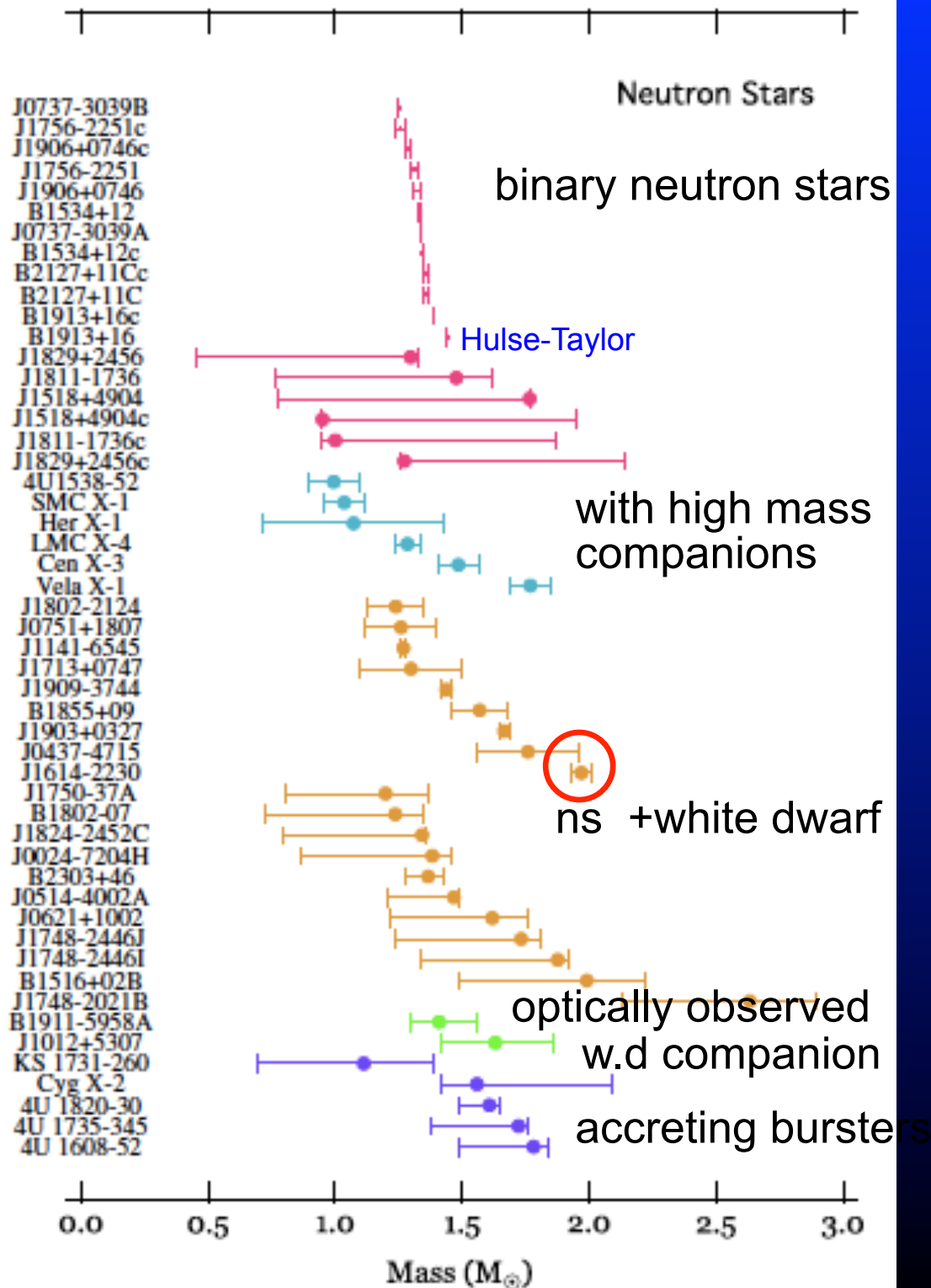


Cooling of n-stars: search for exotica

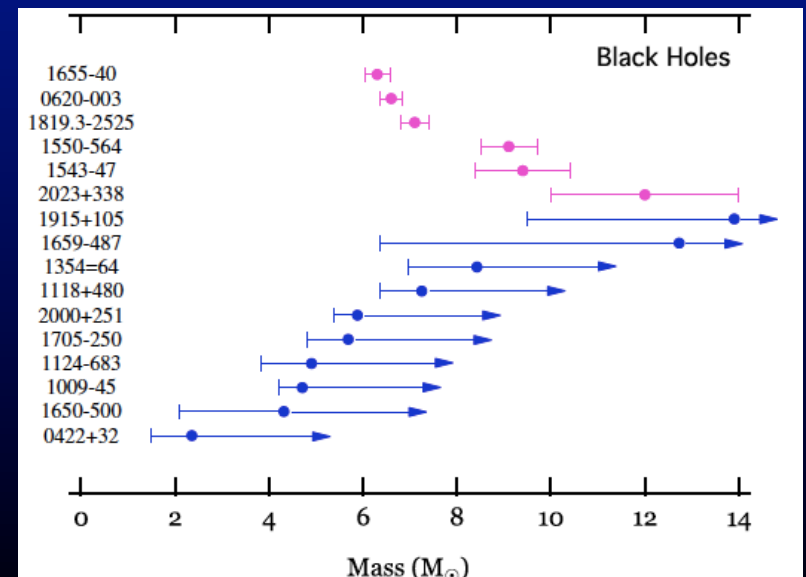


Neutron star masses

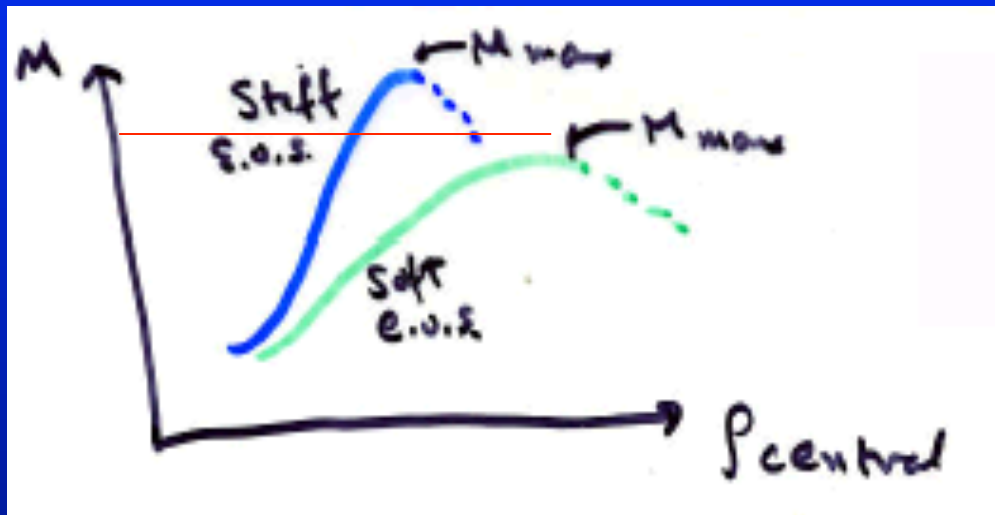
F. Özel et al, Ap.J.757, 1 (2012)



Galactic black hole masses



The equation of state is very stiff



Softer equation of state =>
lower maximum mass and
higher central density

Binary neutron stars $\sim 1.4 M_{\odot}$: consistent with soft eq. of state

PSR J1614-2230 : $M_{\text{neutron star}} = 1.97 \pm 0.04 M_{\odot}$

PSR J0348+0432: $M_{\text{neutron star}} = 2.01 \pm 0.04 M_{\odot}$

require very stiff equation of state! How possible?

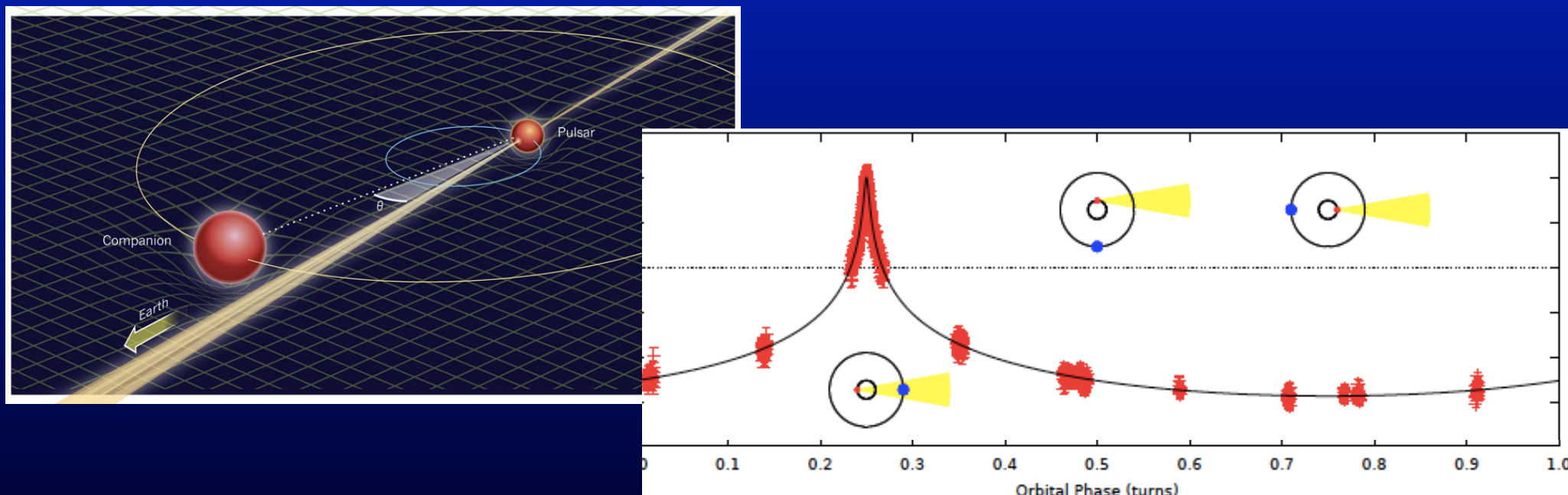
High mass neutron star, PSR J1614-2230 -- in neutron star-white dwarf binary

Demorest et al., Nature 467, 1081 (2010); Ozel et al., ApJ 724, L199 (2010).

Spin period = 3.15 ms; orbital period = 8.7 day

Inclination = $89:17^\circ \pm 0:02^\circ$: **edge on**

$M_{\text{neutron star}} = 1.97 \pm 0.04 M_\odot$; $M_{\text{white dwarf}} = 0.500 \pm 0.006 M_\odot$



(Gravitational) Shapiro delay of light from pulsar
when passing the companion white dwarf

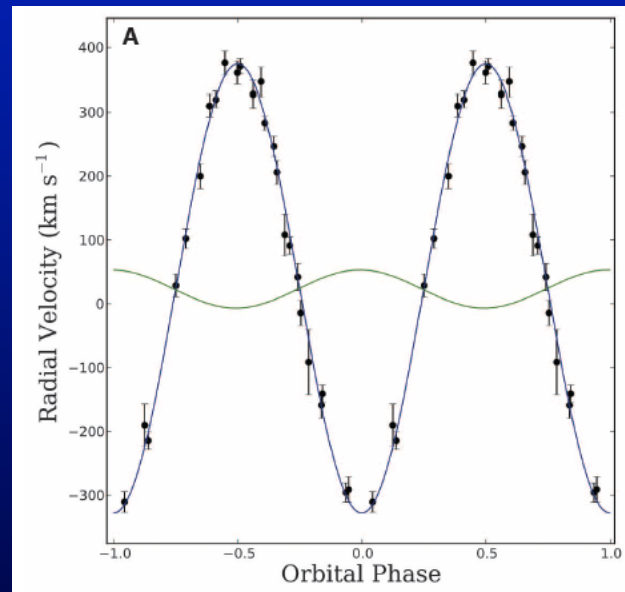
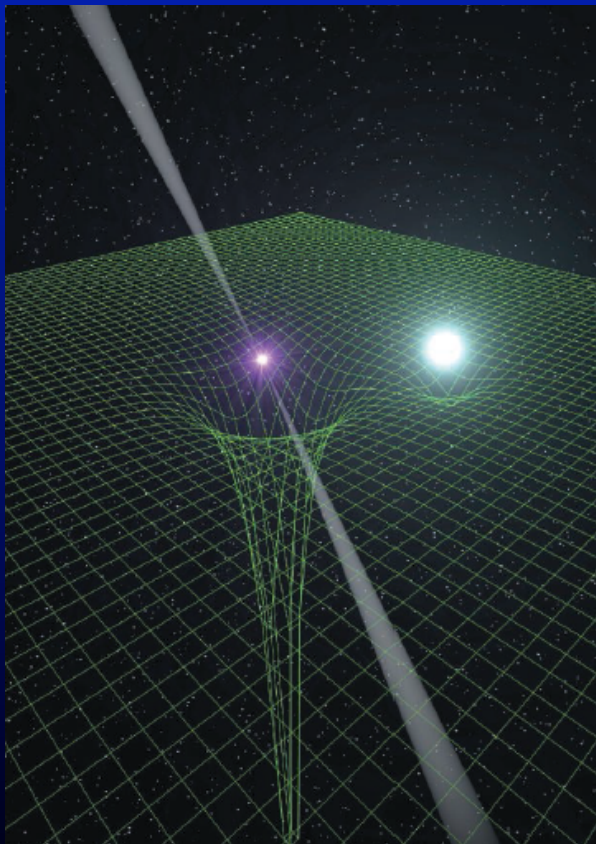
Second high mass neutron star, PSRJ0348+0432 -- in neutron star-white dwarf binary

Antonidas et al., Science 340 1233232 (2013)

Spin period = 39 ms; orbital period = 2.46 hours

Inclination = 40.2°

$M_{\text{neutron star}} = 2.01 \pm 0.04 M_\odot$; $M_{\text{white dwarf}} = 0.172 \pm 0.003 M_\odot$



Significant gravitational radiation

$$\dot{P}/\dot{P}_{\text{GR}} = 1.05 \pm 0.18$$

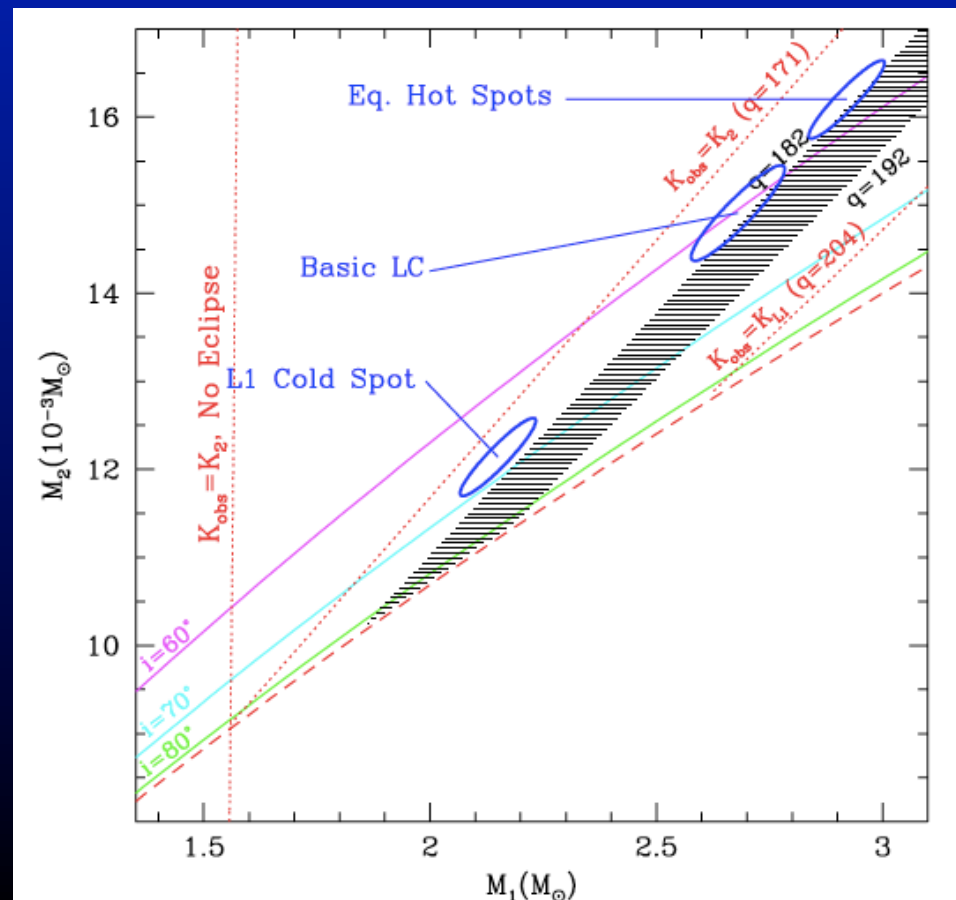
400 Myr to coalescence!

Possible third high mass neutron star in “black widow pulsar” PSR J1311-3430 – neutron star - He star binary

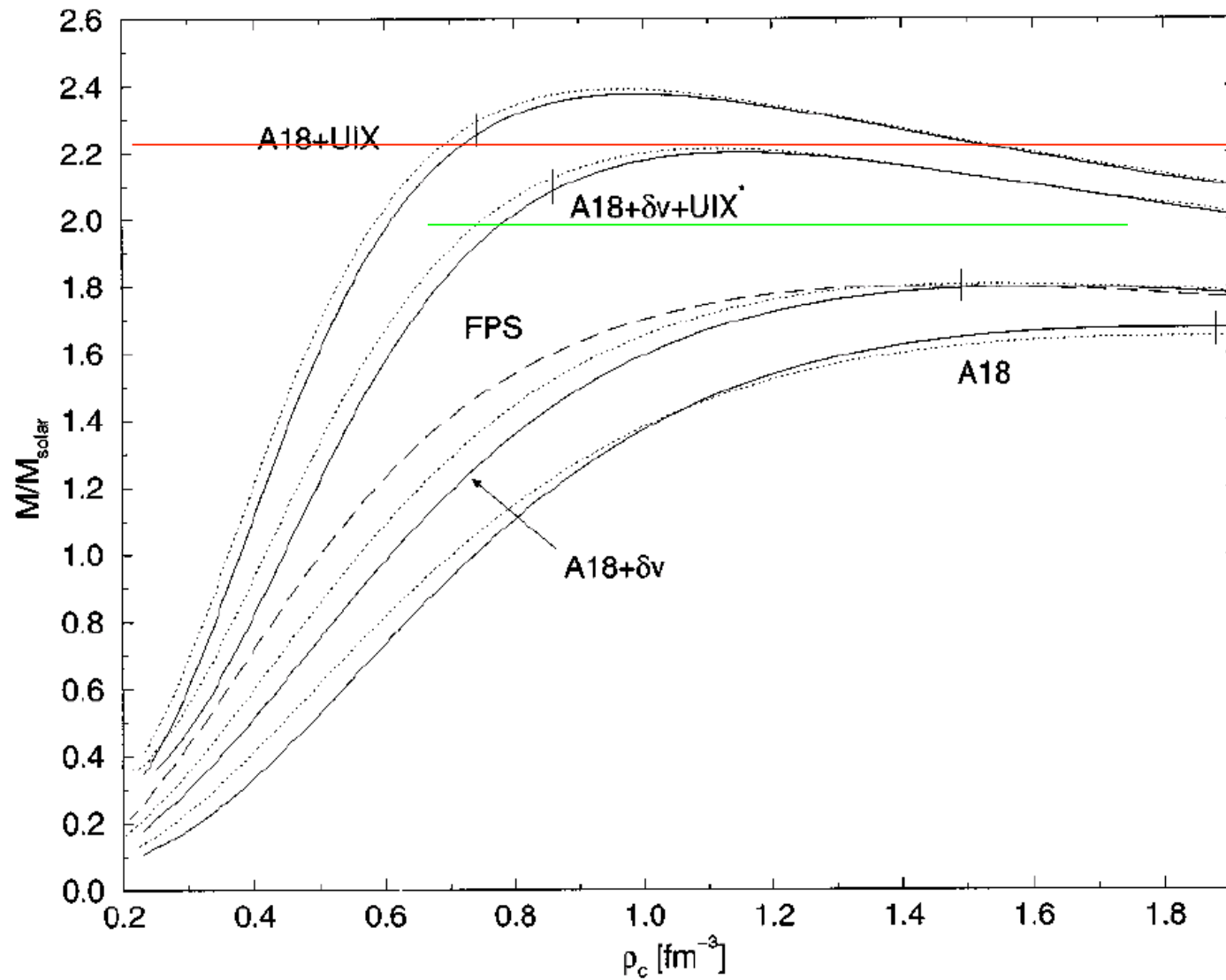
Romani et al., *Ap. J. Lett.*, 760:L36 (2012), *Ap. J.* 804:115R (2015)

$$M_{\text{neutron star}} \sim 1.8 - 2.7 M_{\odot}; \quad M_{\text{companion}} \sim 0.01 M_{\odot}$$

Uncertainties arising from internal dynamics of companion



Neutron star masses vs. central density

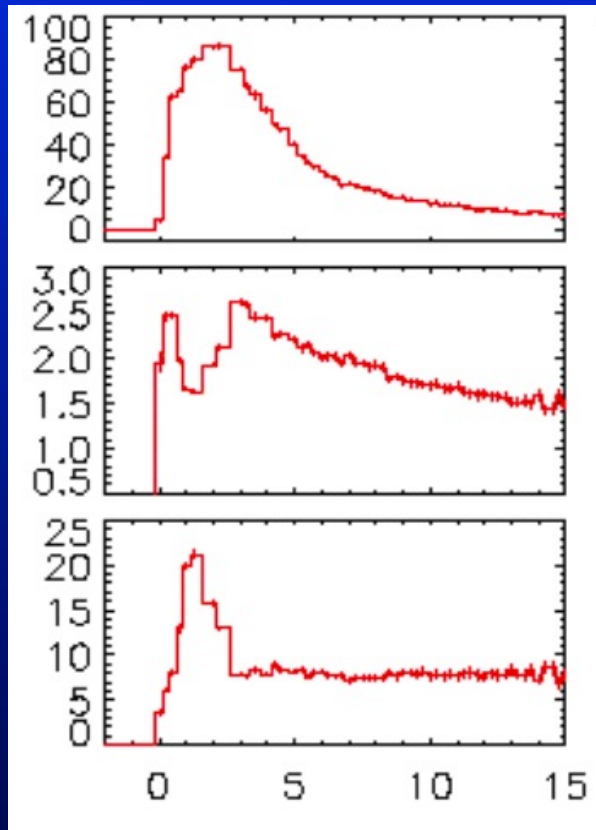


Akmal, Pandharipande and Ravenhall, 1998

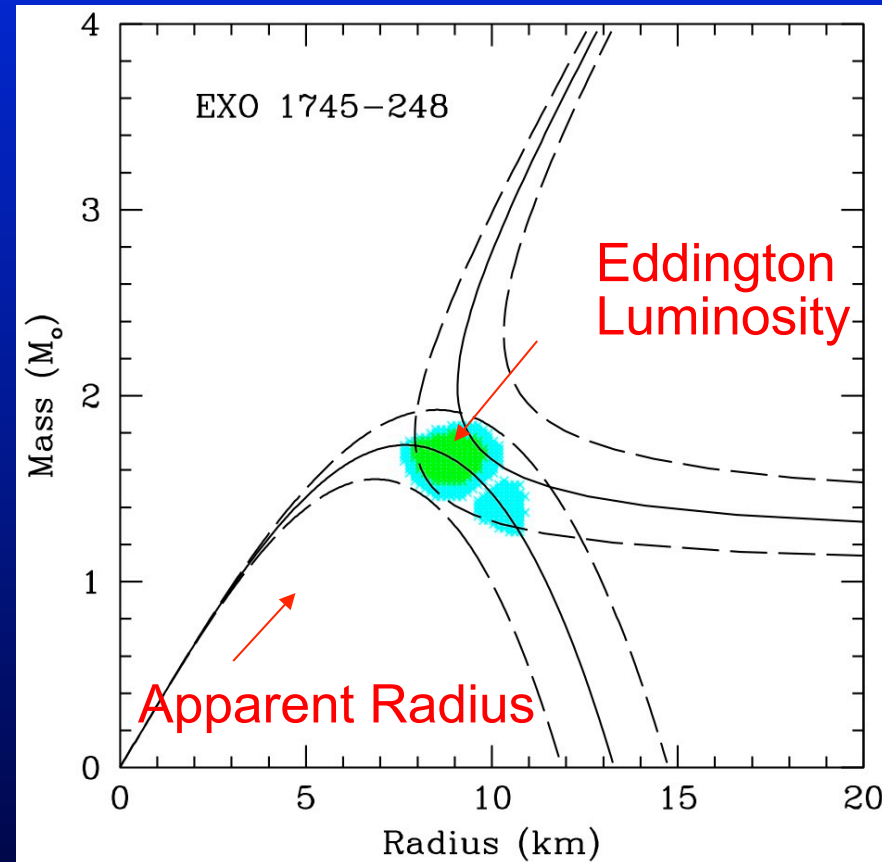
Measuring masses and radii of neutron stars in thermonuclear bursts in X-ray binaries

Özel et al., 2006-2016

Steiner et al. 2010-2013



Time (s)



Measurements of *apparent* surface area, flux at Eddington limit (radiation pressure = gravity), combined with distance to star, constrains M and R .

M vs R from bursts (Özel et al., Steiner et al.)

EXO 1745-248

$\alpha = 0.14 \pm 0.01$

$R_{ph} = R$

4U 1820-30

$\alpha = 0.18 \pm 0.02$

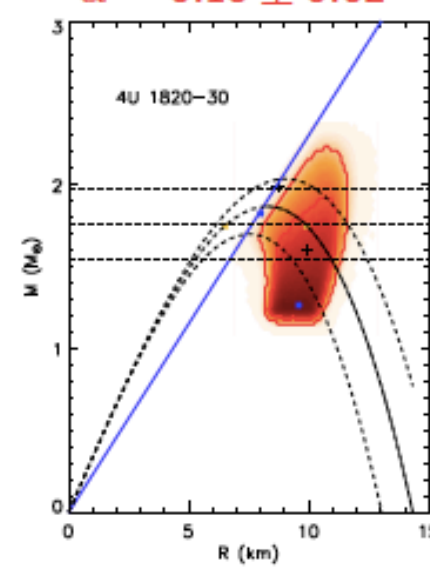
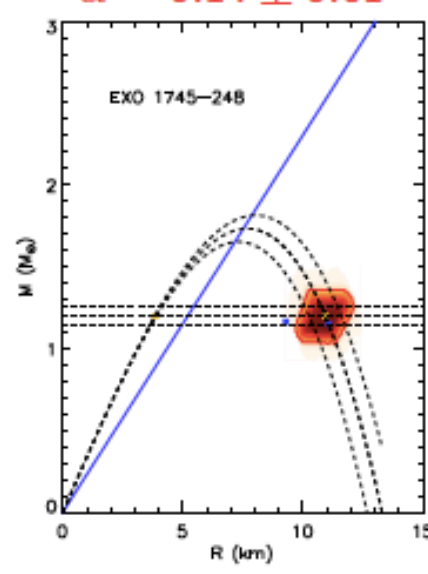
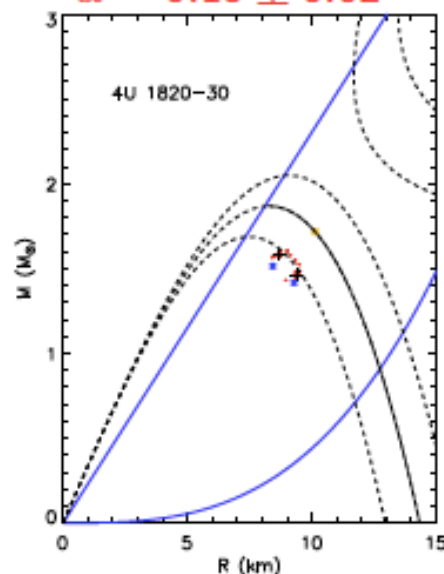
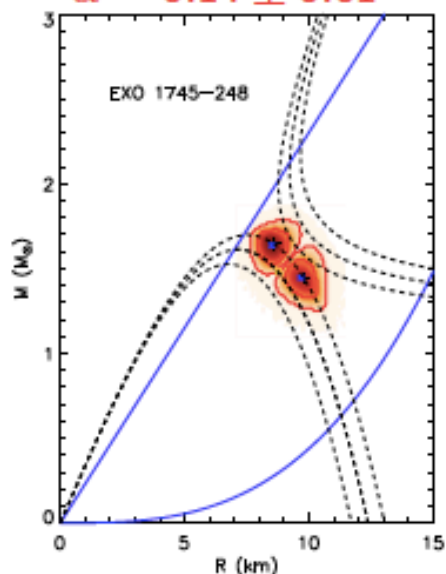
EXO 1745-248

$\alpha = 0.14 \pm 0.01$

$R_{ph} > R$

4U 1820-30

$\alpha = 0.18 \pm 0.02$



4U 1608-52

$\alpha = 0.26 \pm 0.10$

Özel et al. 2009, 2010, 2011

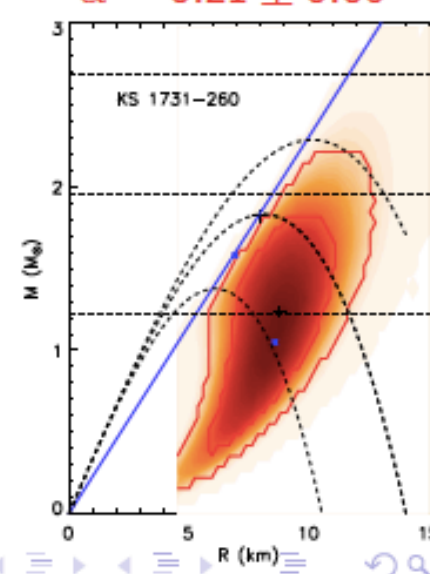
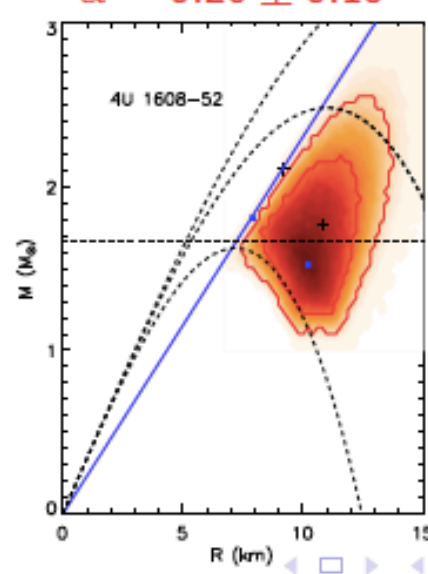
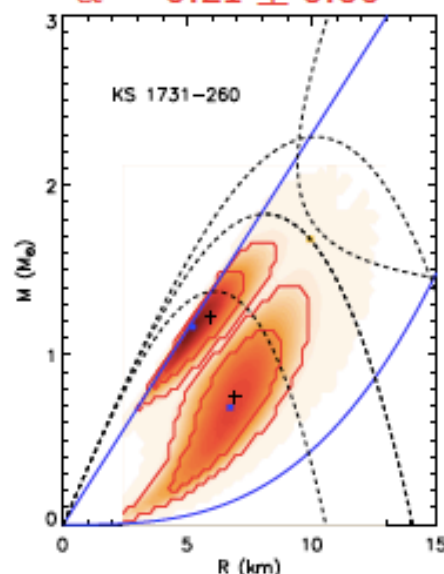
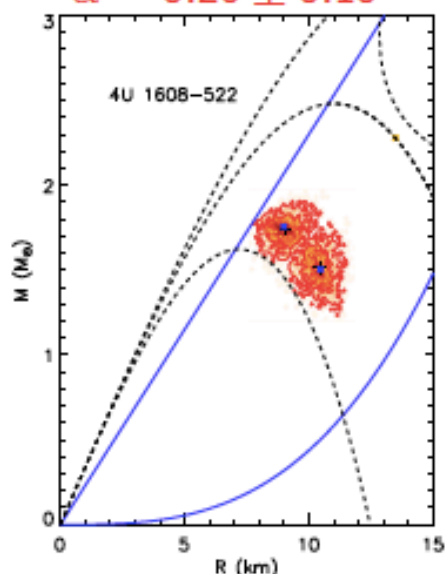
$\alpha = 0.21 \pm 0.06$

4U 1608-52

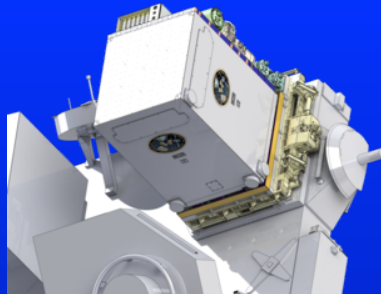
$\alpha = 0.26 \pm 0.10$

Steiner, Lattimer & Brown 2010, 2011

$\alpha = 0.21 \pm 0.06$



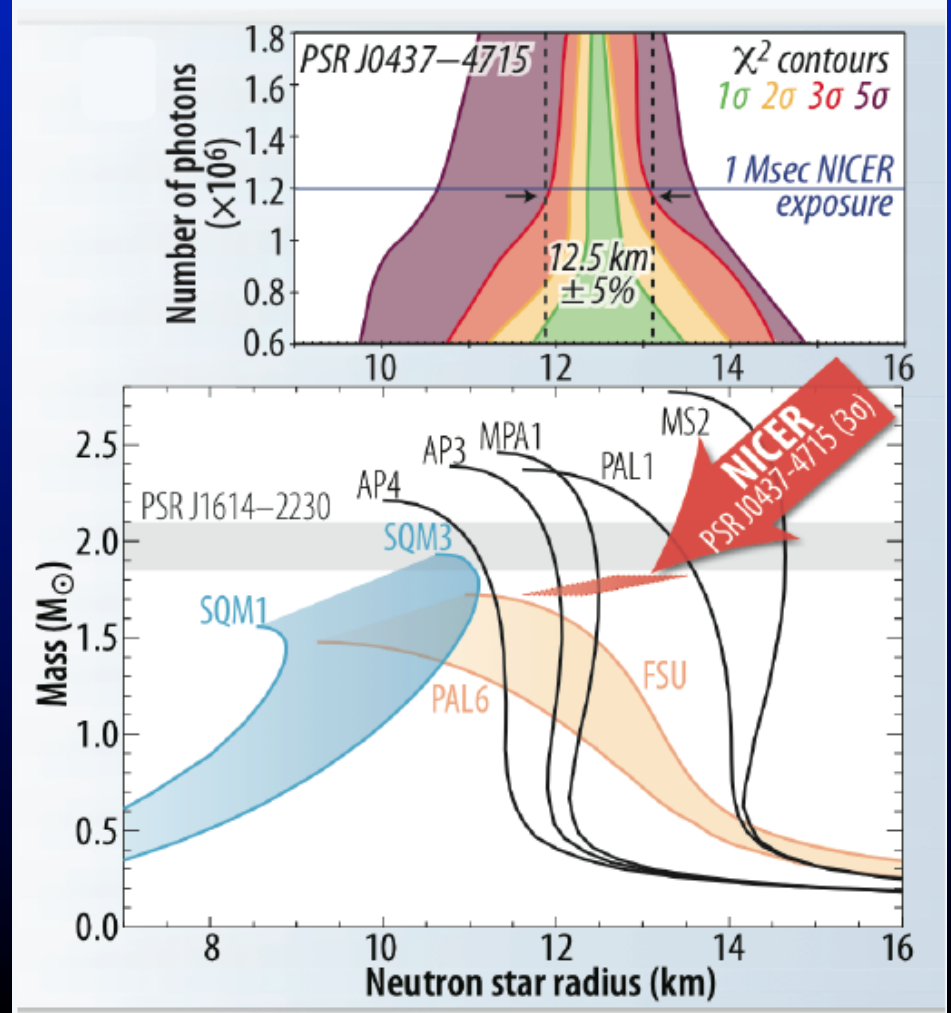
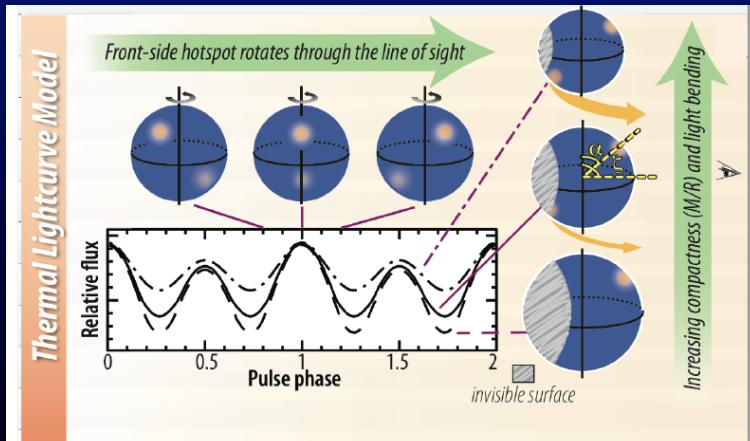
NICER = neutron star interior composition explorer



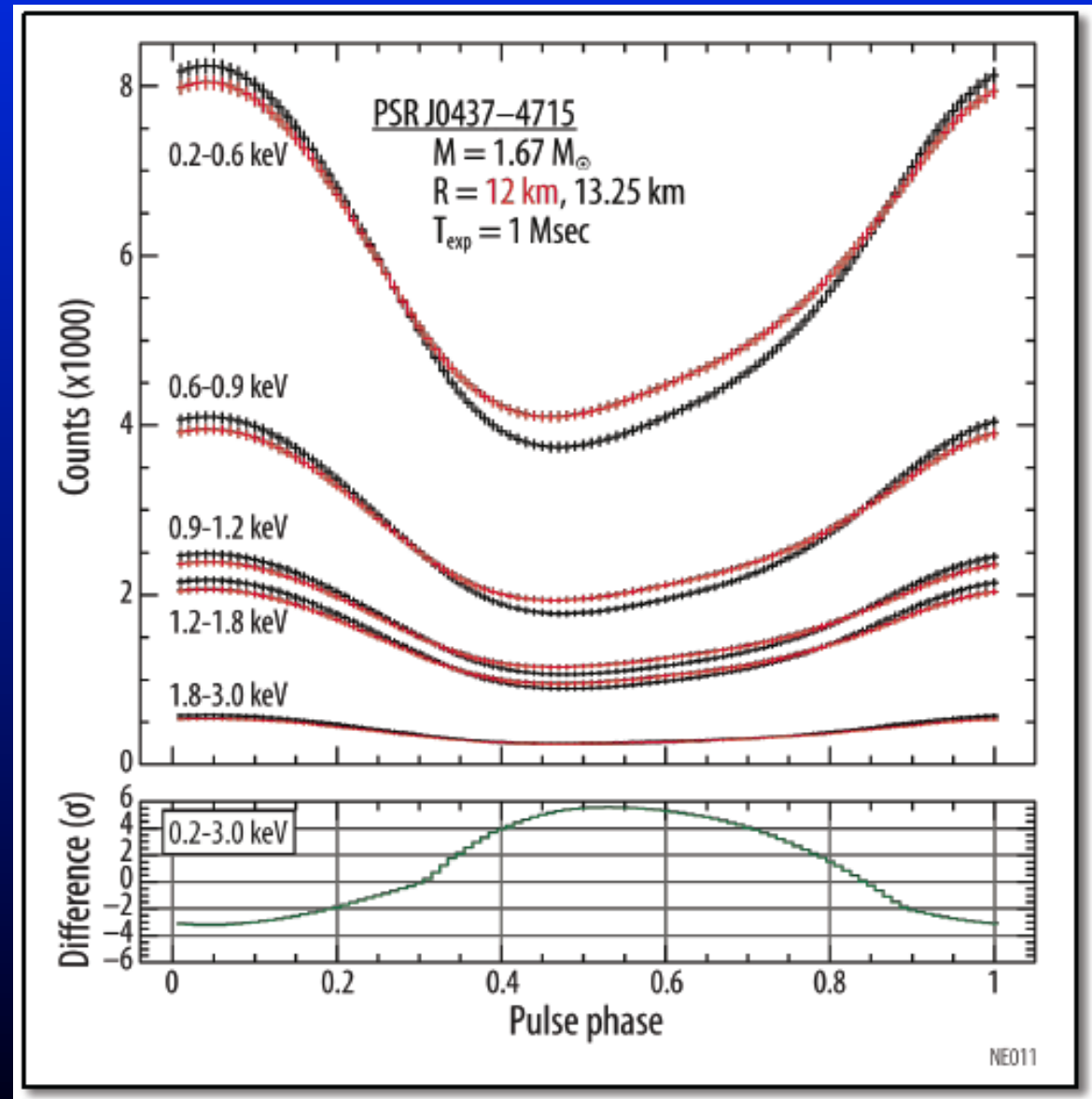
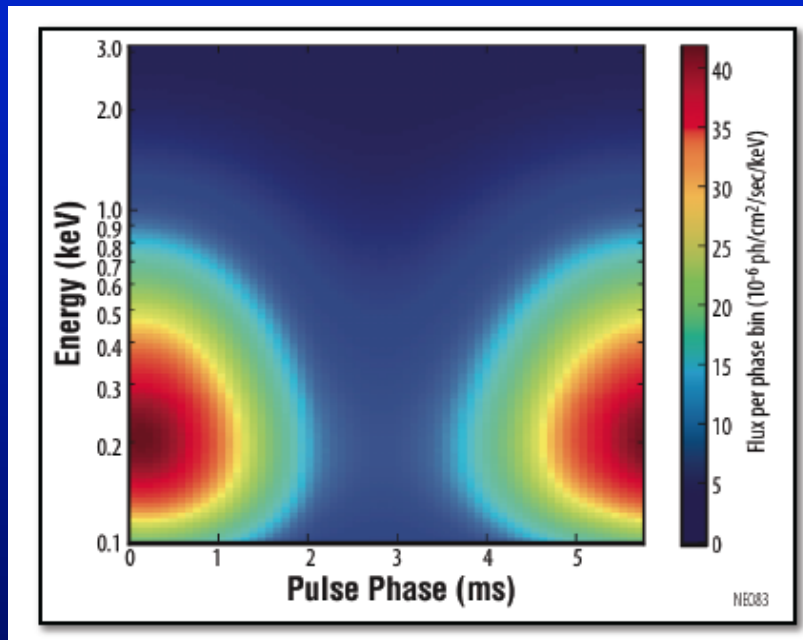
To be flown to Int. Space Station (Space-X) fall 2016

X-ray timing (GPS to 300nsec) and spectroscopy (0.12-12 KeV)

- Measure radii and masses
- Pulsar timing stability
- Radiation spectra and luminosities



Measure radii by pulsar phase-resolved spectroscopy

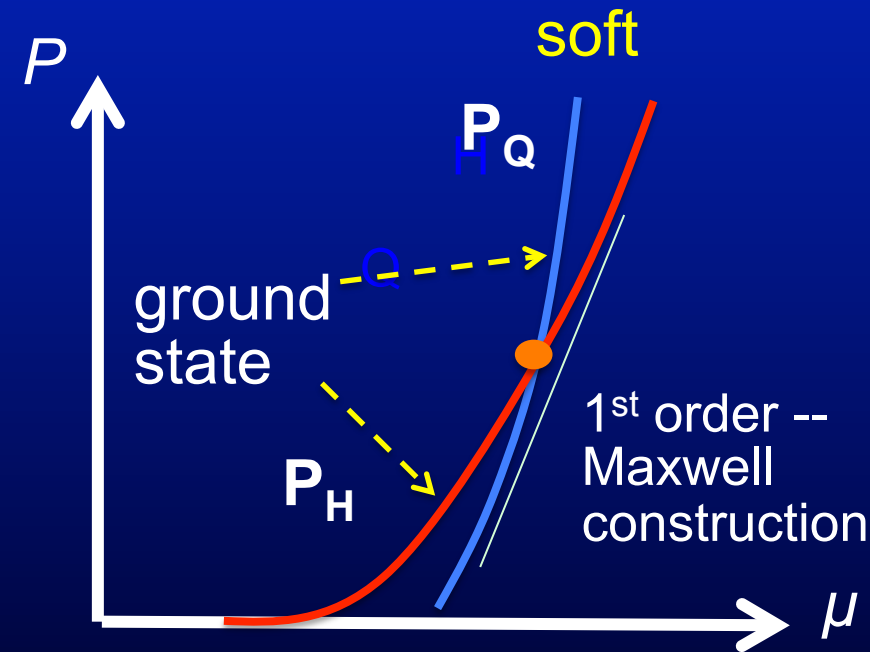
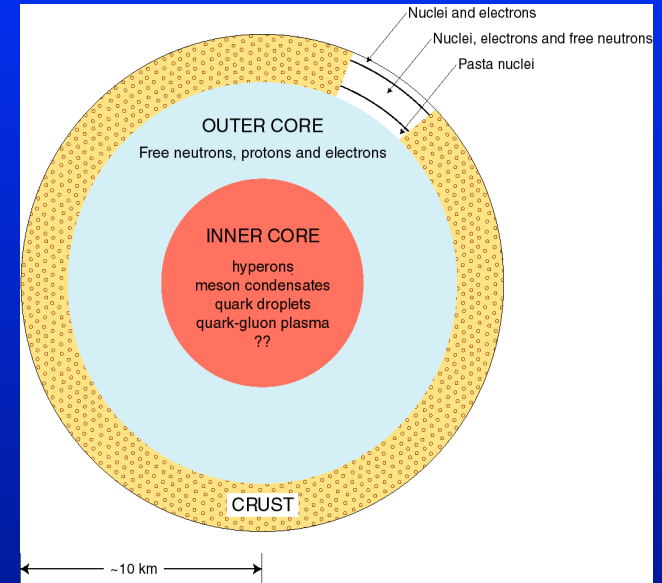


Quark matter cores in neutron stars

Canonical picture: compare calculations of eqs. of state of hadronic matter and quark matter.

GB & S.A. Chin (1976)

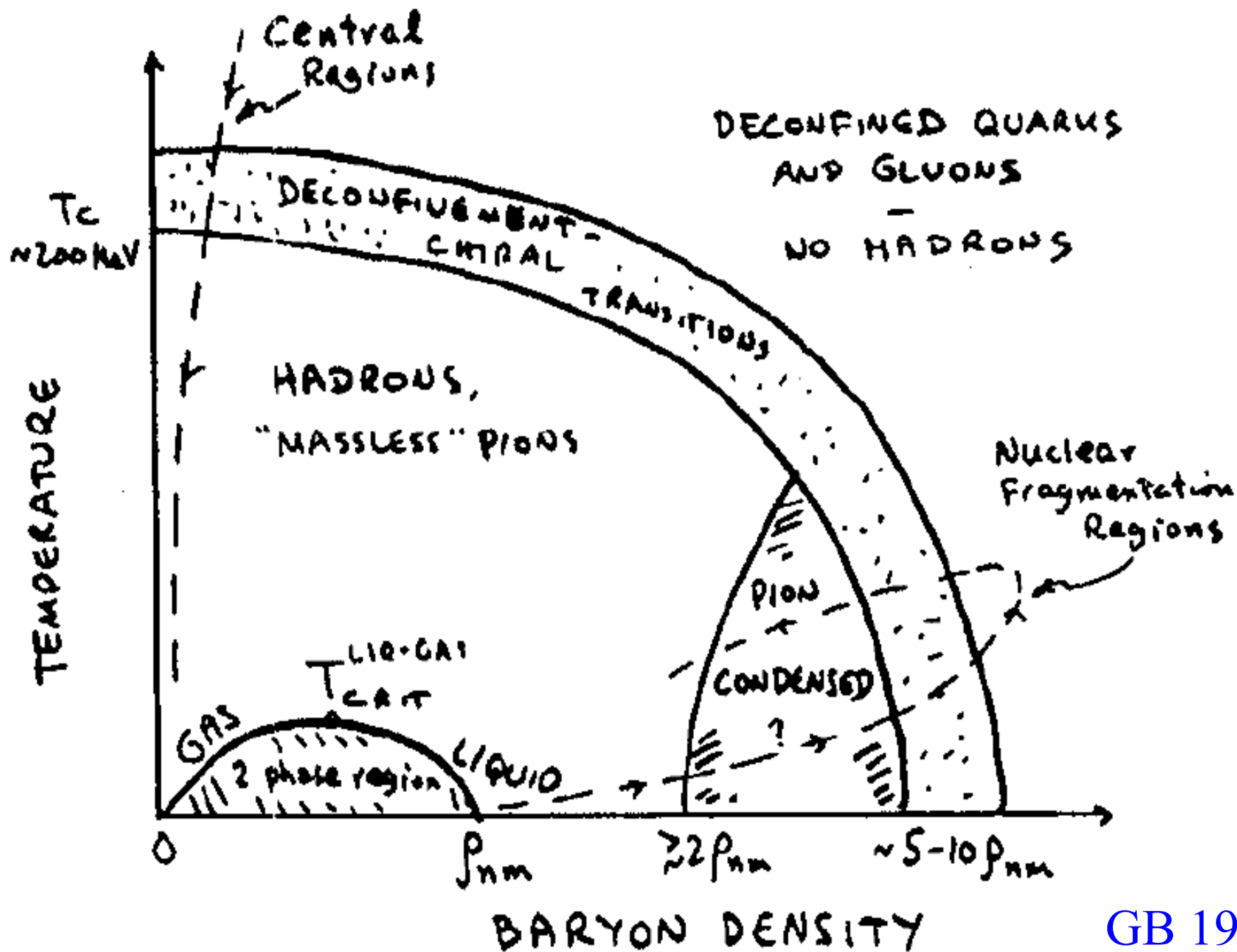
Crossing of thermodynamic potentials
 \Rightarrow first order phase transition.



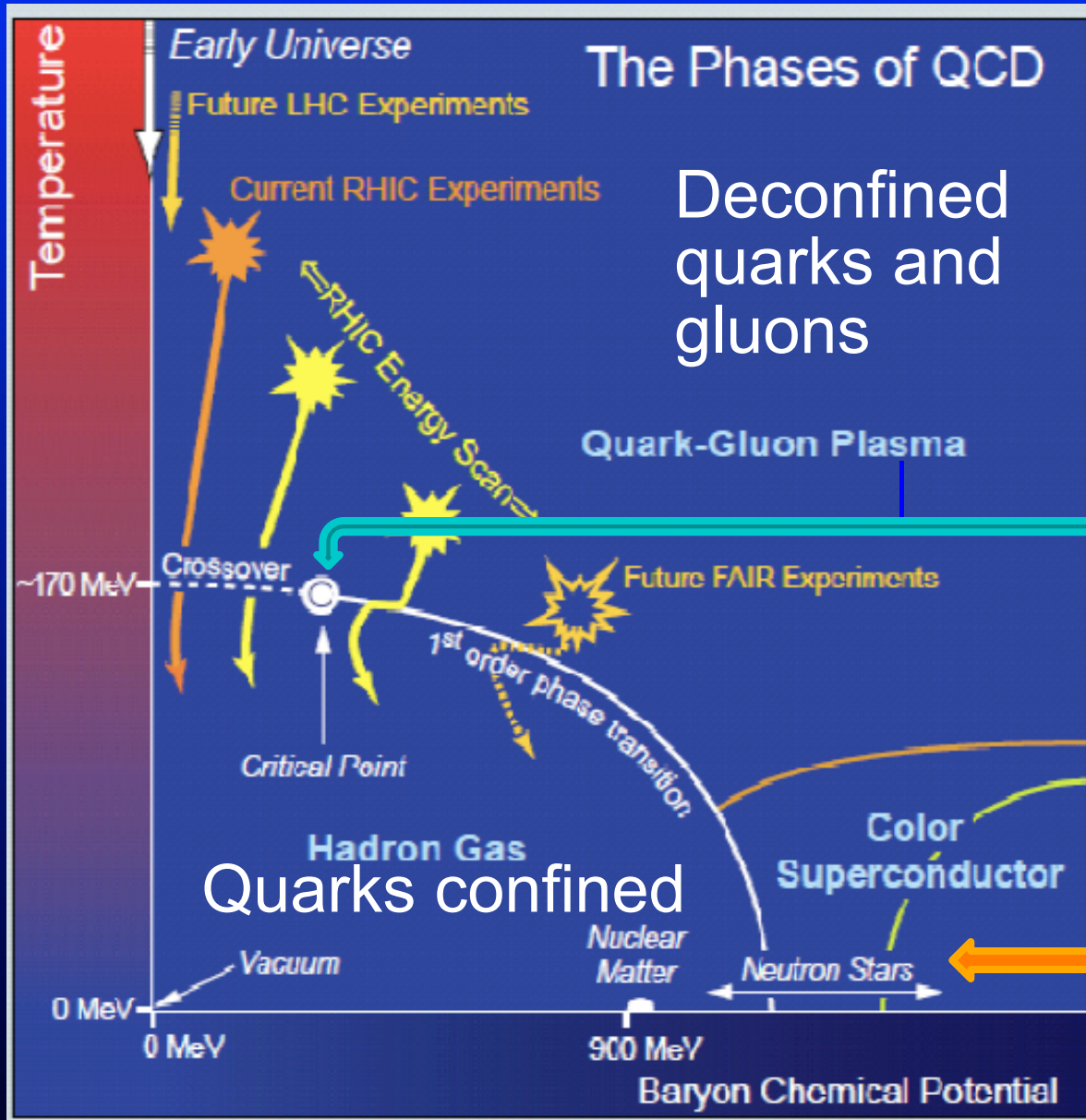
ex. nuclear matter using 2 & 3 body interactions, vs. pert. expansion or bag models.

Typically conclude transition at $n \sim 10n_{nm}$ -- would not be reached even in high mass neutron stars \Rightarrow **no quark matter cores**

PHASE DIAGRAM OF NUCLEAR MATTER



More modern phase diagram



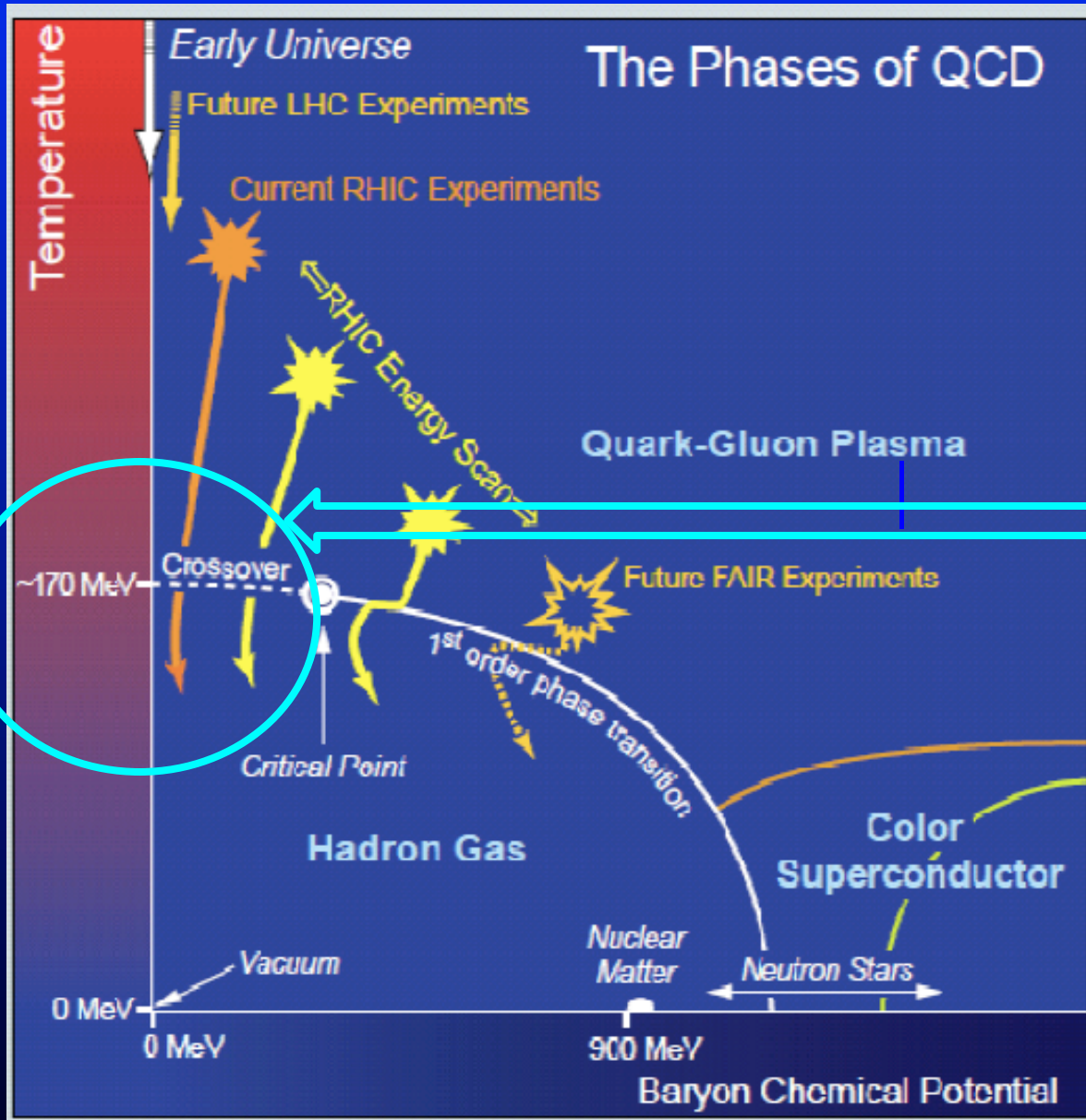
Asakawa-Yazaki critical point (1989)

Search in RHIC & SPS energy scans.

States of color superconductivity – diquark BCS pairing

2SC / Color flavor locked (Alford, Rajagopal, Wilczek, ...)

Crossover at zero net baryon density

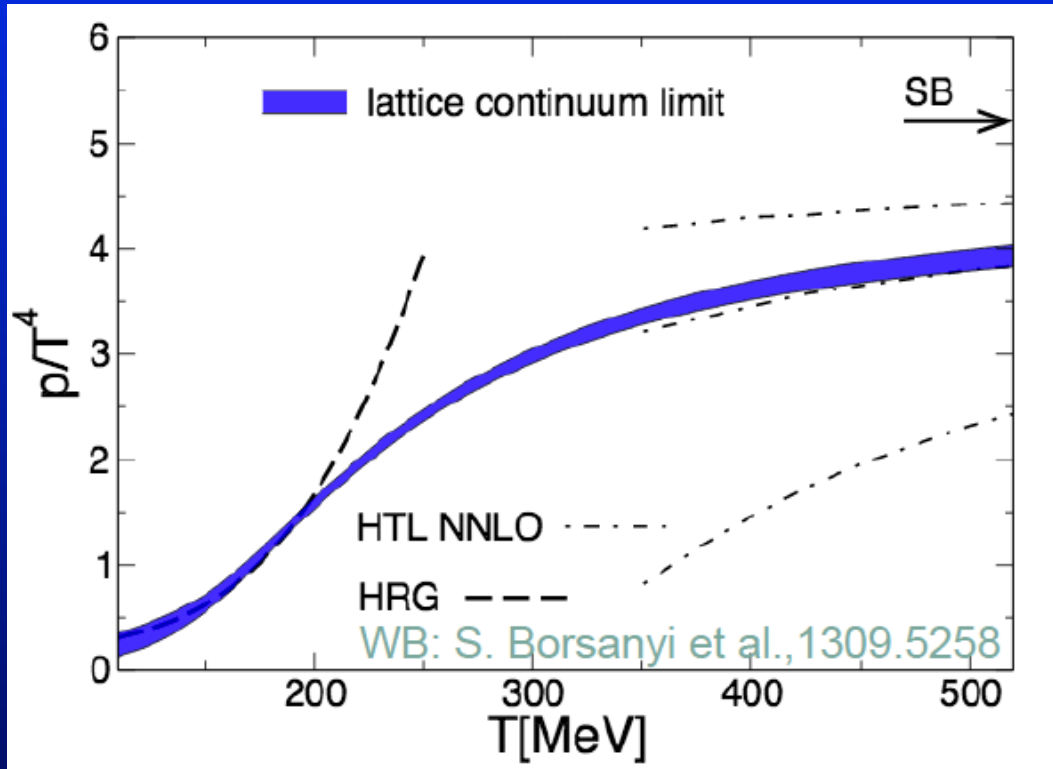


QCD lattice gauge theory -- for finite light quark masses -- predicts crossover from confined phase at lower T to deconfined phase at higher T .

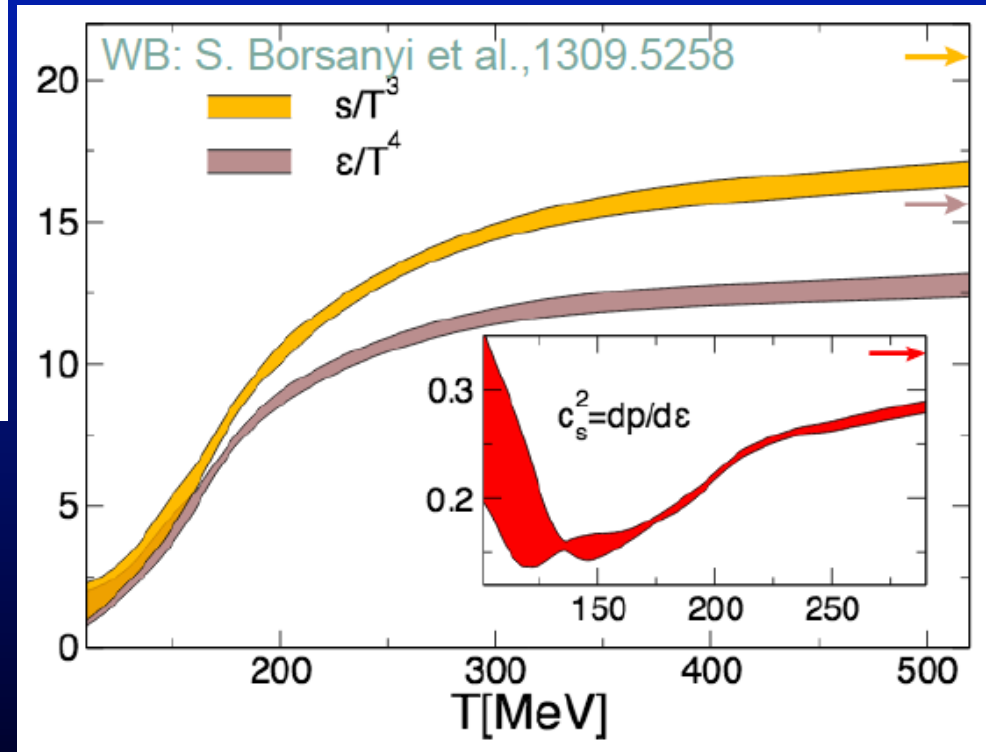
Do quarks roam freely in the deconfined phase? If so, they must also roam freely at lower T .

Are there really quarks running about freely in this room?

Crossover at zero net density: see no evidence of phase transition in pressure, entropy, or energy density.



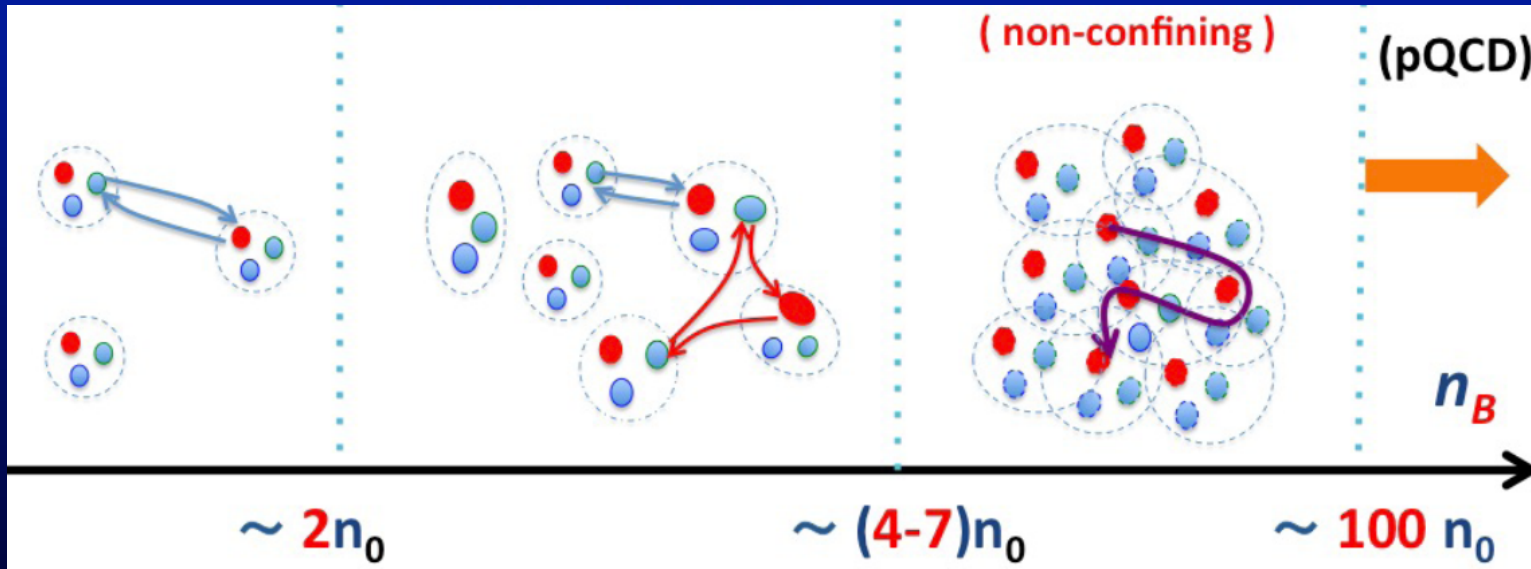
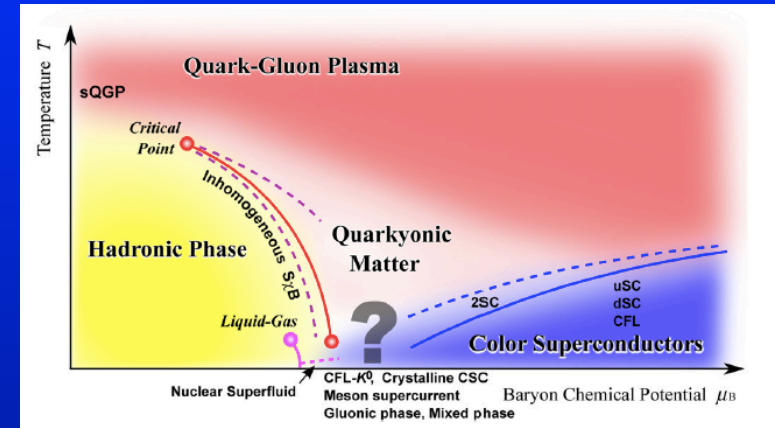
Wuppertal-Budapest
lattice collaboration



WB: S. Borsanyi et al., PLB (2014)
HotQCD: A. Bazavov et al., PRD (2014)

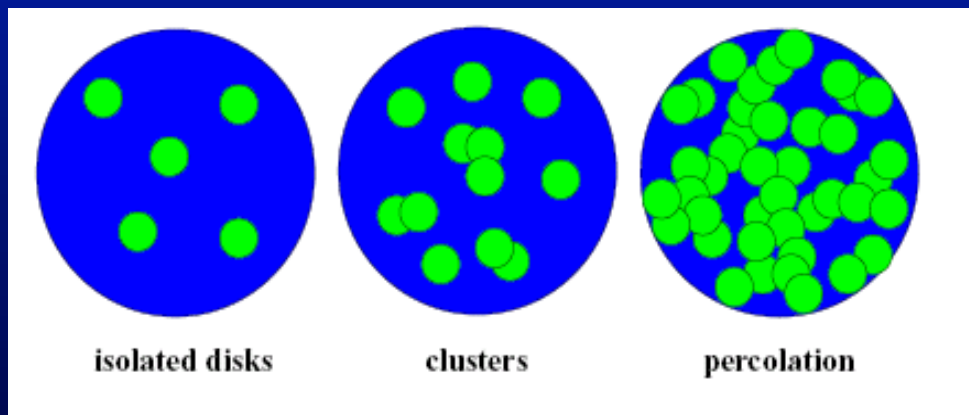
No free quarks even above the crossover!

In confined region quarks are inside hadrons. Also have quarks and antiquarks in the QCD forces between hadrons. With higher density or temperature, form larger clusters, which percolate at the crossover. In deconfined regime clusters extending across all of space.



Percolation of clusters along the density axis, at zero temperature. n_0 is the density of matter inside a large nucleus.

Percolation

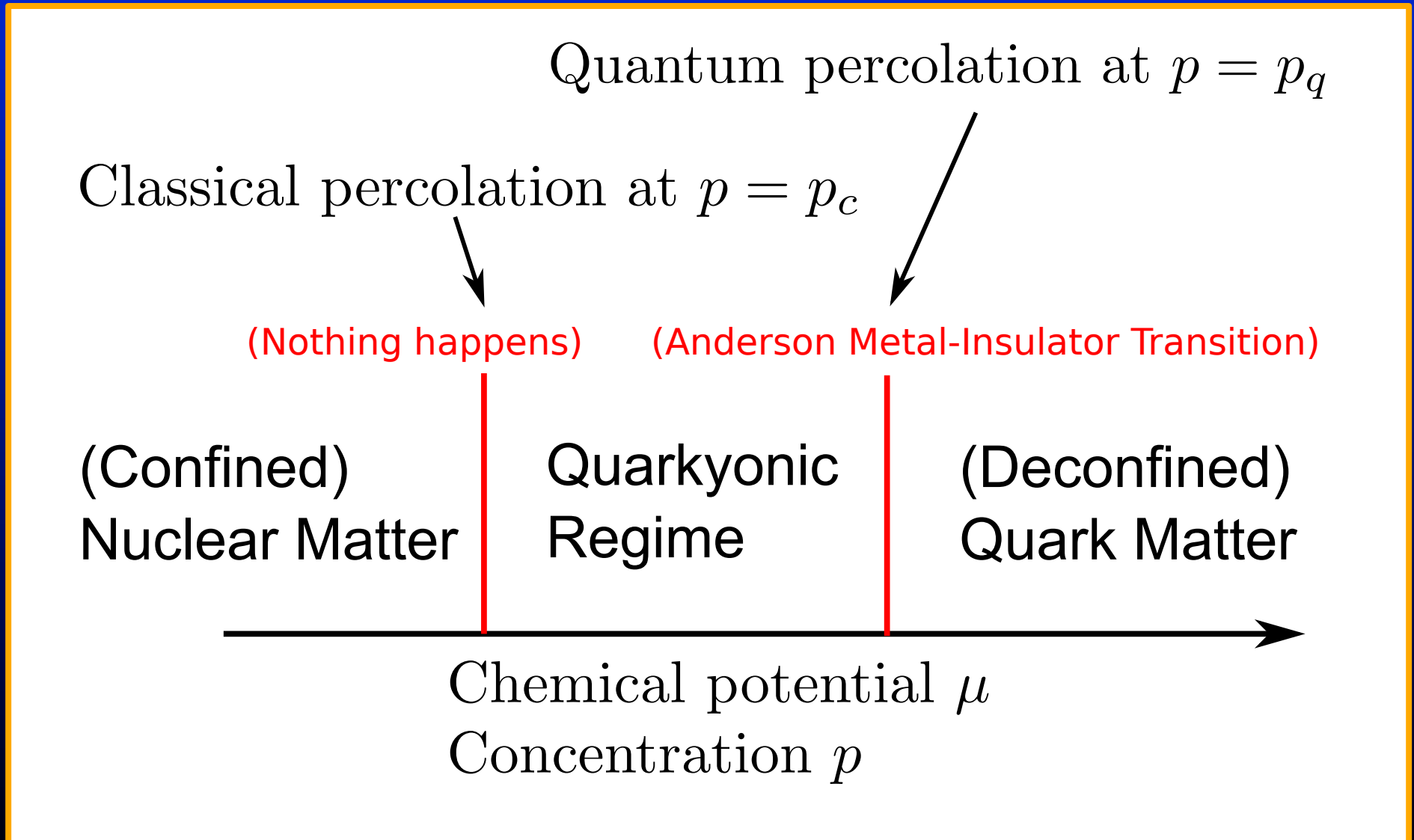


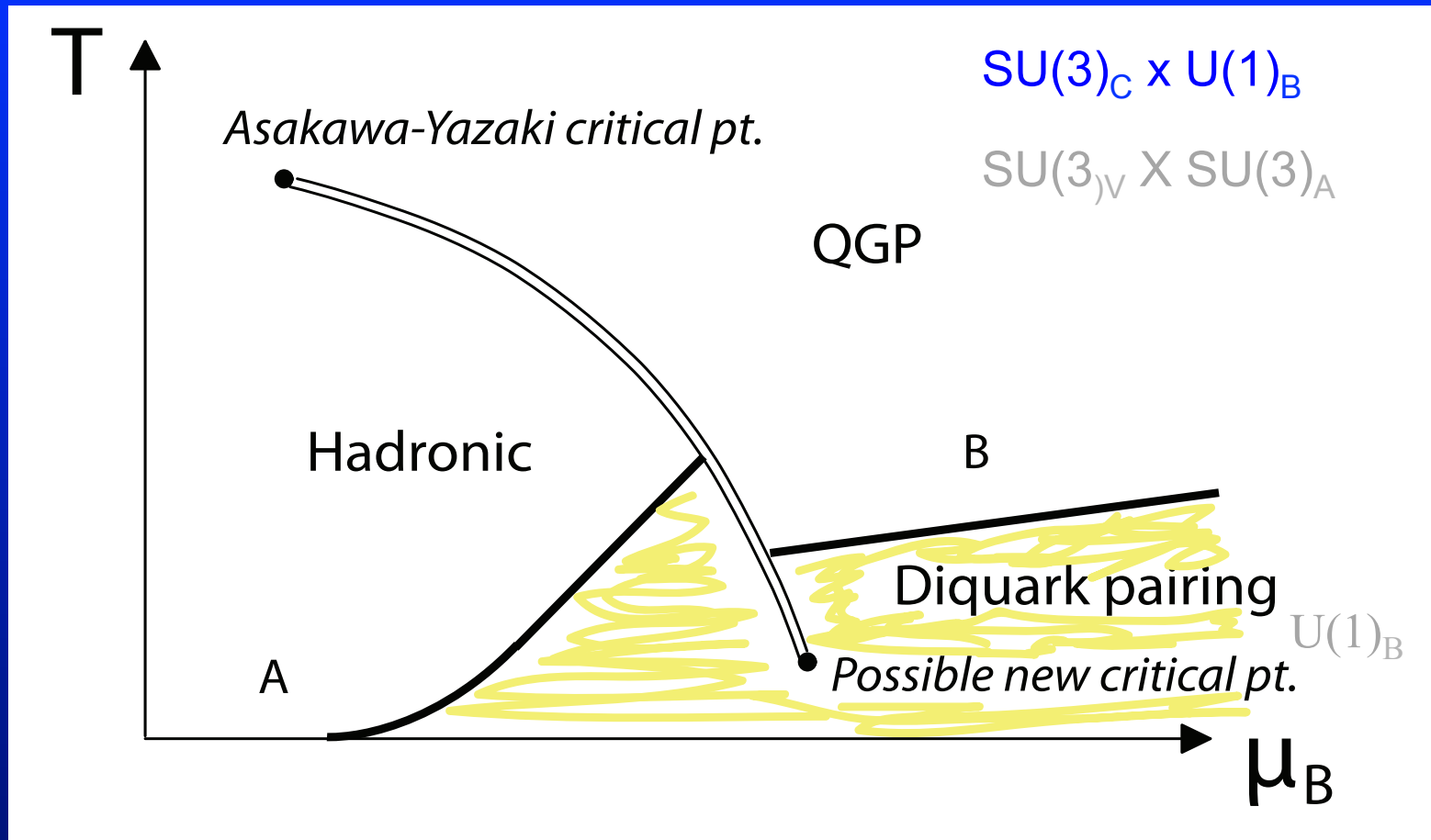
$$n_{\text{perc}} \sim 0.34 \left(\frac{3}{4}\pi r_n^3 \right) \text{ fm}^{-3}$$

Quarks can still be bound even if deconfined.

Distinguish classical (geometric) percolation from quantum percolation

Deconfinement as (inverse) Anderson localization
(Kenji Fukushima):

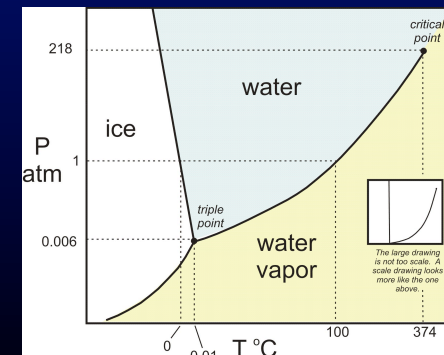




Critical points similar to those in liquid-gas phase diagram (H_2O)

Can go continuously from A to B around the upper (A-Y) critical point.

In lower shaded region have BCS pairing of quarks, and possibly other states (meson condensates).



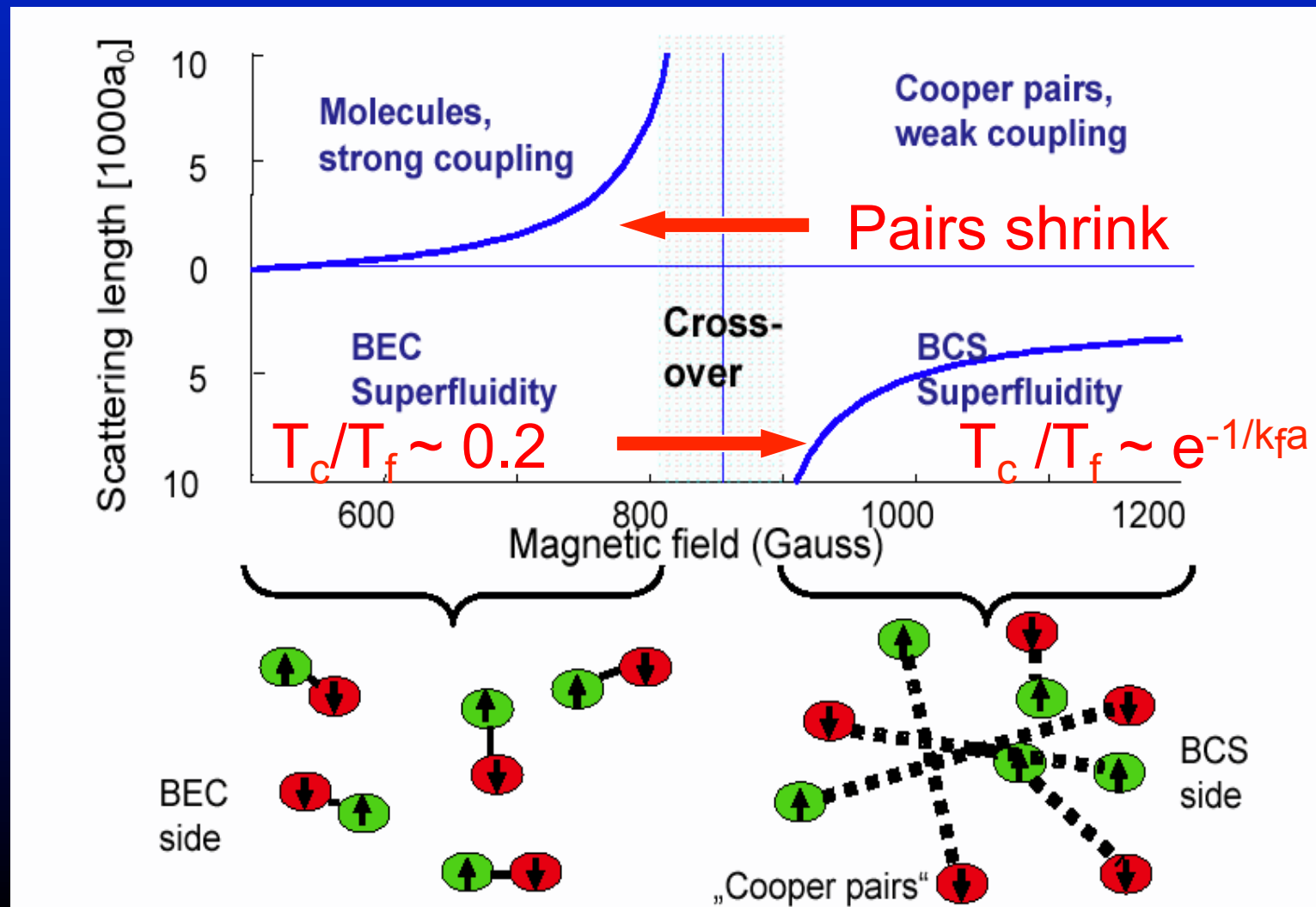
BEC-BCS crossover in Fermi systems

Continuously transform from molecules to Cooper pairs:

D.M. Eagles (1969)

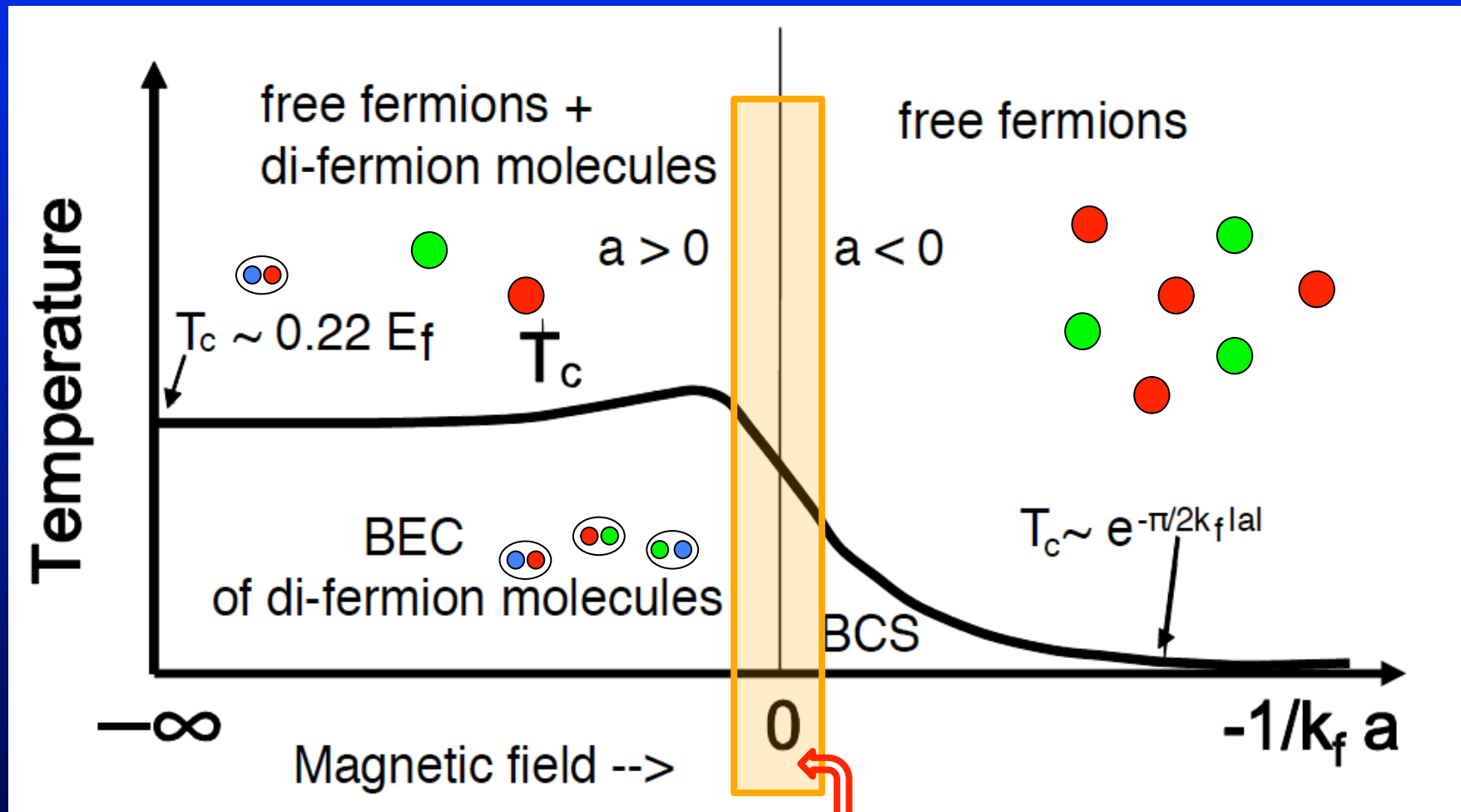
A.J. Leggett, J. Phys. (Paris) C7, 19 (1980)

P. Nozières and S. Schmitt-Rink, J. Low Temp Phys. 59, 195 (1985)



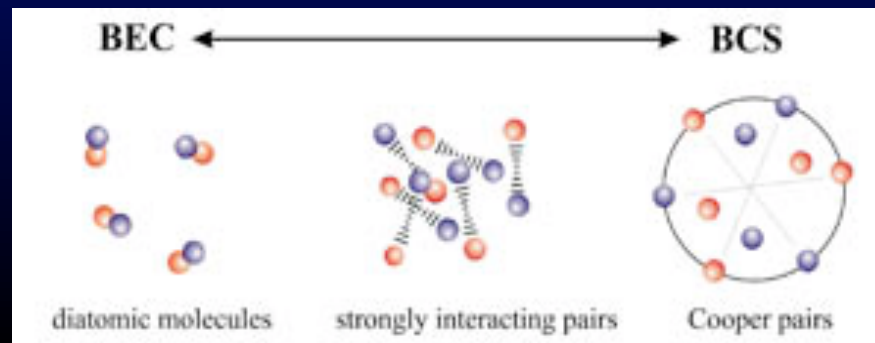
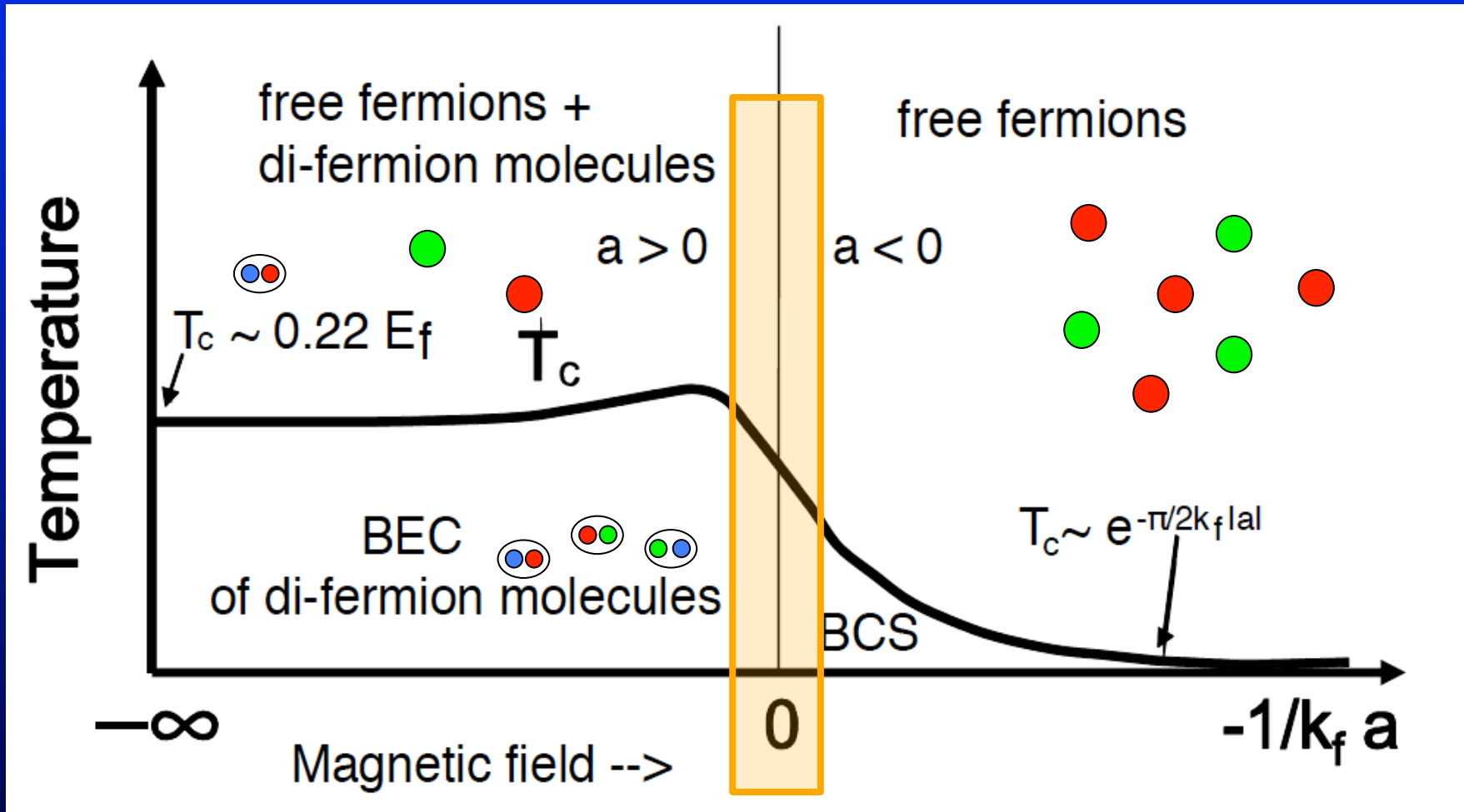
⁶Li

Phase diagram of ultracold atomic fermion gases: in temperature and strength of the particle interactions



Unitary regime (**Feshbach resonance**) – BEC-BCS crossover. No phase transition through crossover

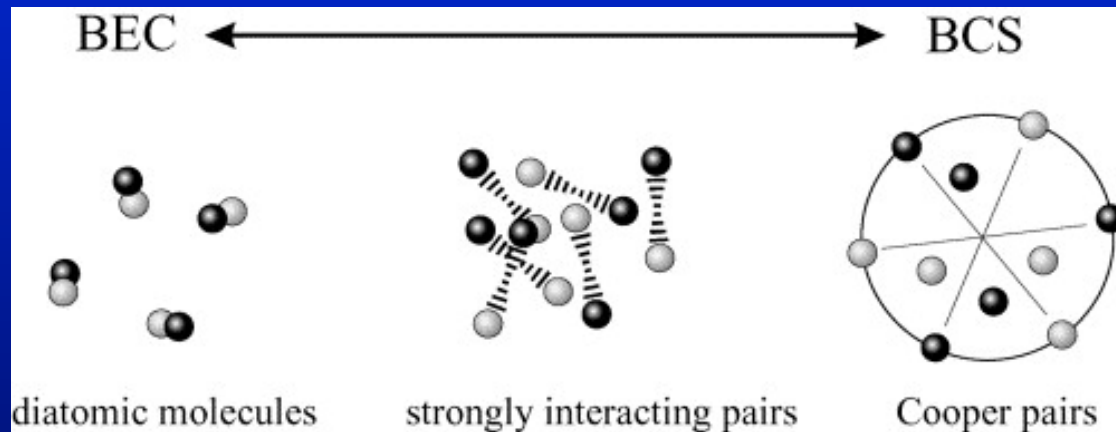
Phase diagram of ultracold atomic fermion gases: in temperature and strength of the particle interactions



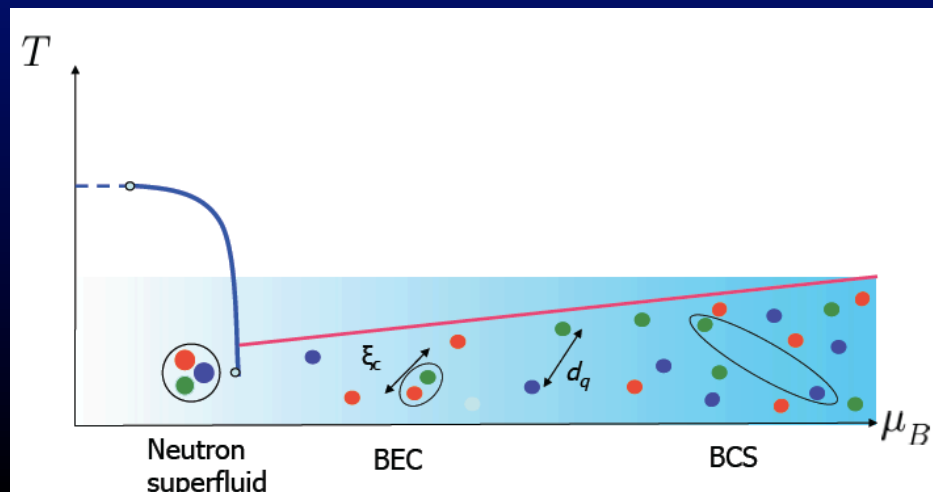
Smooth evolution of states in atomic clouds and nuclear matter

GB, T.Hatsuda, M.Tachibana, & Yamamoto. *J. Phys. G: Nucl. Part. 35*, 10402 (2008)
 H. Abuki, GB, T. Hatsuda, & N. Yamamoto, *Phys. Rev. D* 81, 125010 (2010)

Evolution of Fermi atoms with weakening attraction between atoms:



Similarly, as nuclear matter becomes denser have “continuous” evolution from hadrons (nucleons) to quark pairs (diquarks) to quark matter:

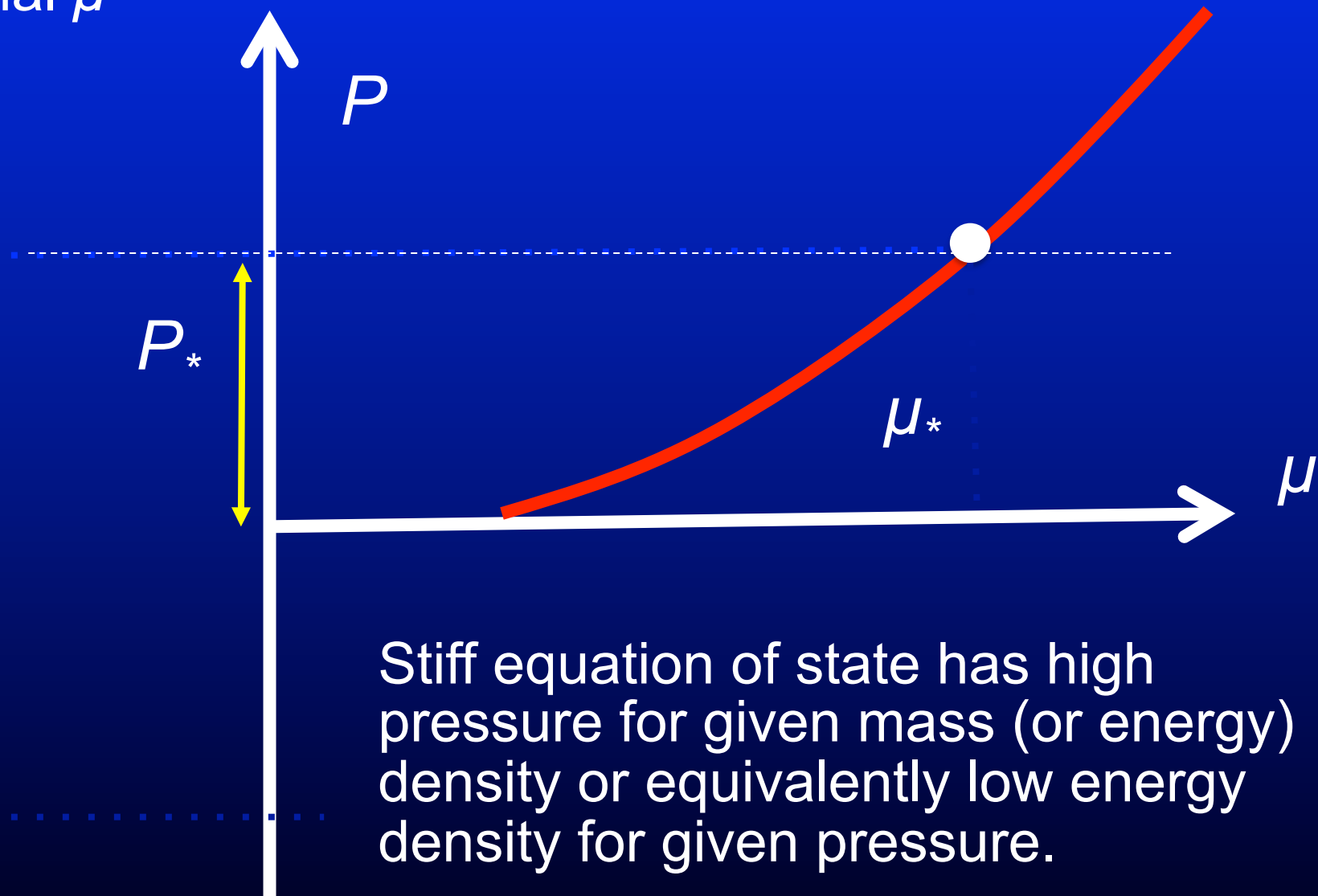


denser →

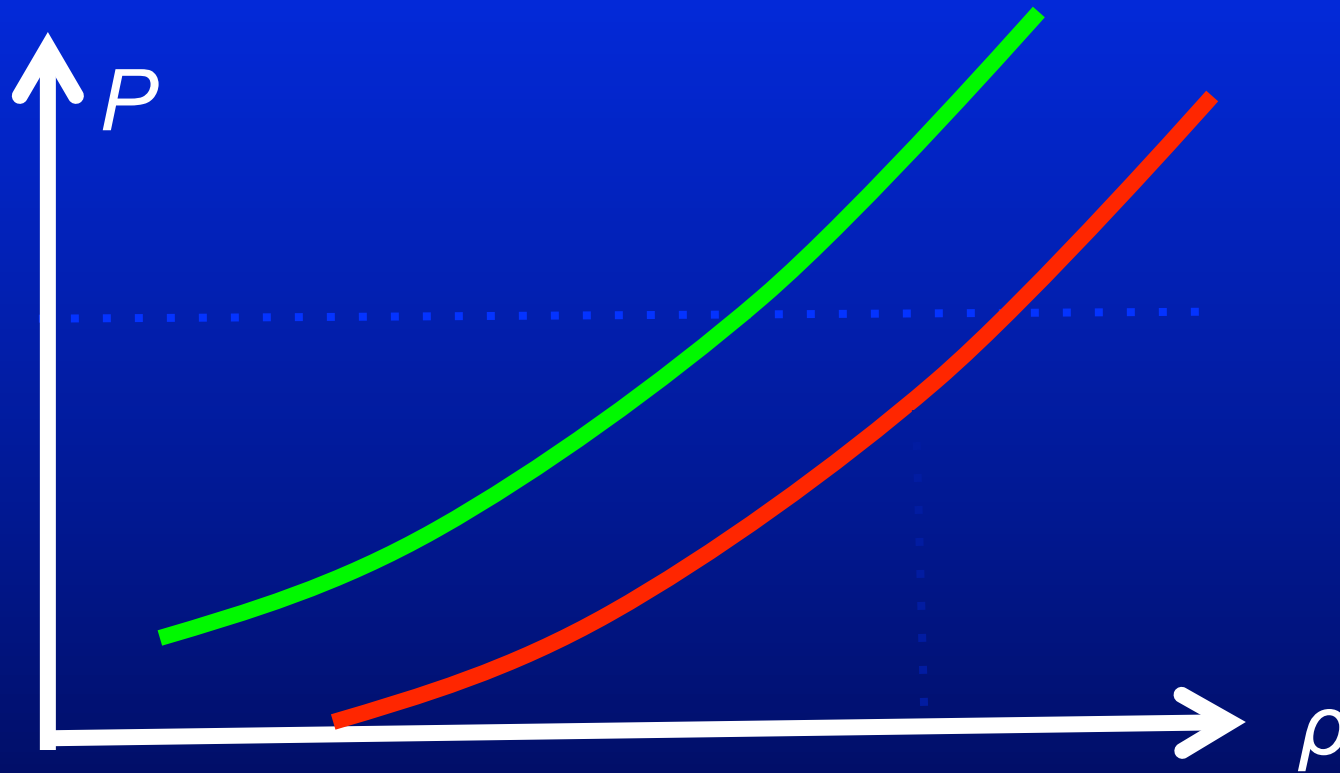
K. Masuda, T. Hatsuda, & T. Takatsuka, *Ap. J.* 764, 12 (2013)

How can QCD give large mass neutron stars?

Pressure P is a continuous function of baryon chemical potential μ



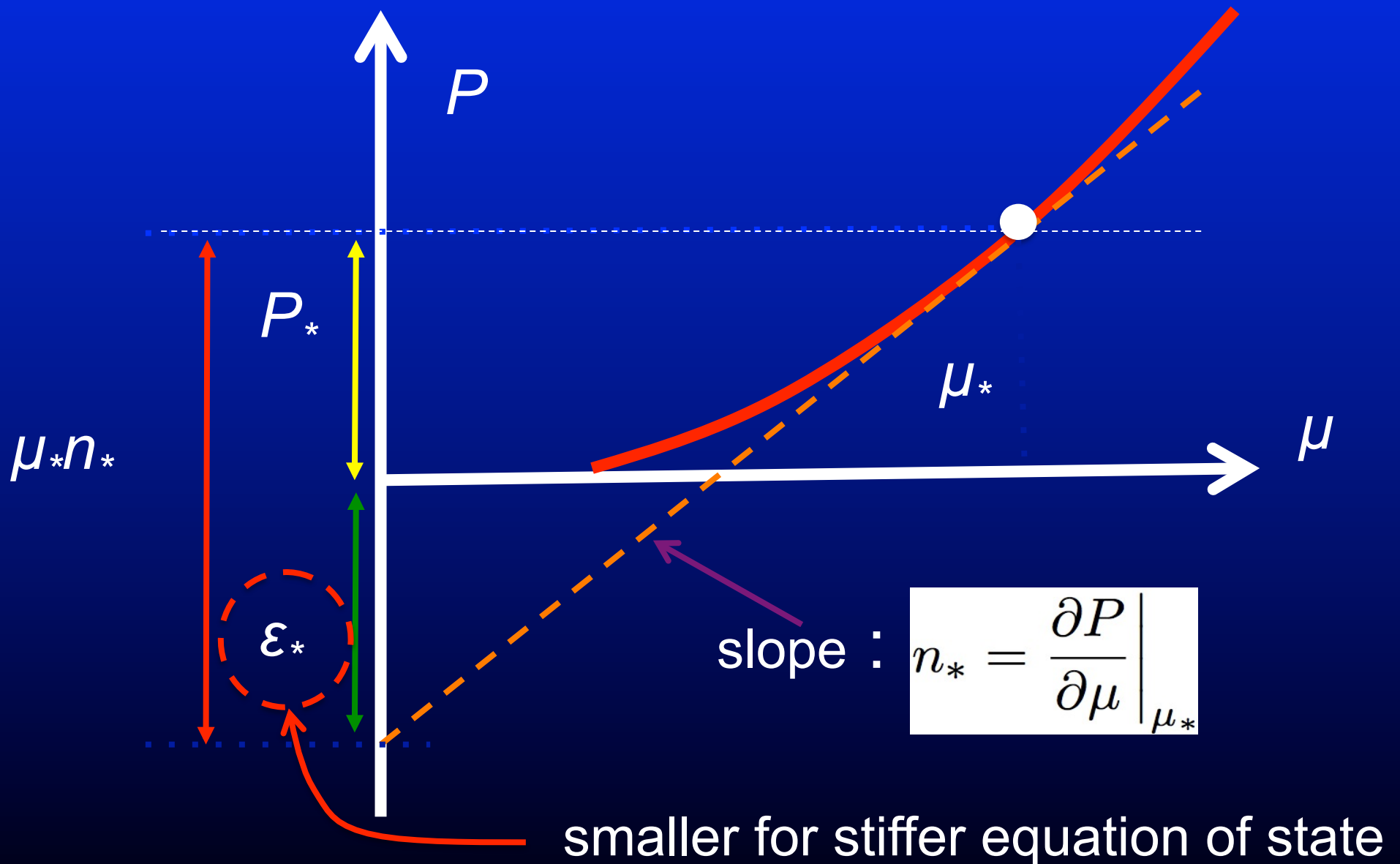
Stiffer equations of state given more massive neutron stars,
with lower central densities



Green equation of state is stiffer than red.
Has larger pressure for given mass density ρ ,
and has **smaller ρ for given pressure P**

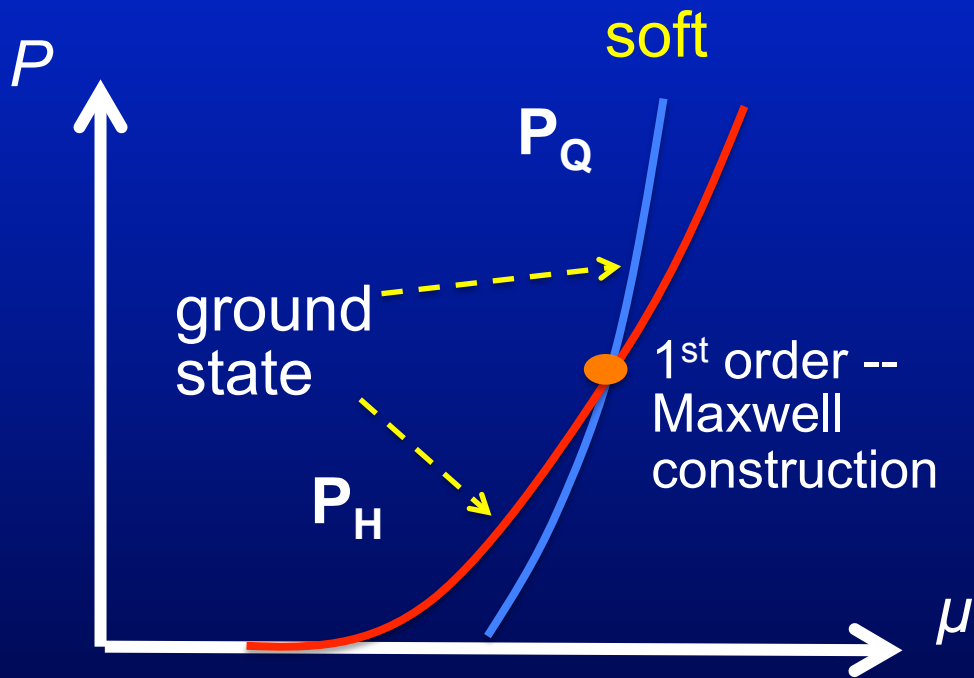
How can QCD give large mass neutron stars?

$$\text{Energy or mass density } \varepsilon = \rho c^2 = \mu n - P$$



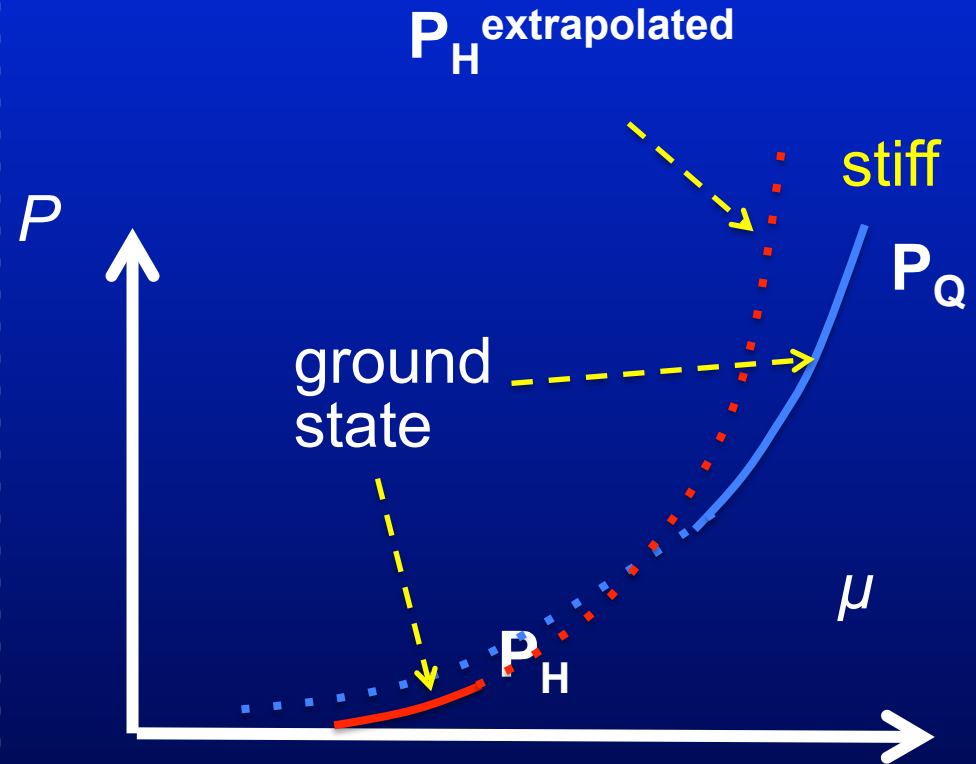
Hybrid eqs. of state
are intrinsically softer

Phase with larger P at given μ
thermodynamically preferred



Assumes hadronic state at high densities – not possible when hadrons substantially overlap

Continuous eqs. of state can
be much stiffer



Hadrons only at low density
and quark matter at high density.
In between???

Model calculations of neutron star matter within NJL model

NJL Lagrangian

$$\mathcal{L} = \bar{q}(i\gamma_\mu \partial^\mu - m_q + \mu\gamma_0)q + \mathcal{L}^{(4)} + \mathcal{L}^{(6)}$$

$$\mathcal{L}_X^{(4)} = G \sum_{a=0}^8 [(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma_5\tau_a q)^2]$$

chiral interactions

$$\mathcal{L}_d^{(4)} = H \sum_{A,A'=2,5,7} [(\bar{q}i\gamma_5\tau_A\lambda_{A'}C\bar{q}^T)(q^T Ci\gamma_5\tau_A\lambda_{A'}q)]$$

BCS pairing interactions

$\mathcal{L}^{(6)}$ = Kobayashi-Maskawa-'t Hooft six quark axial anomaly

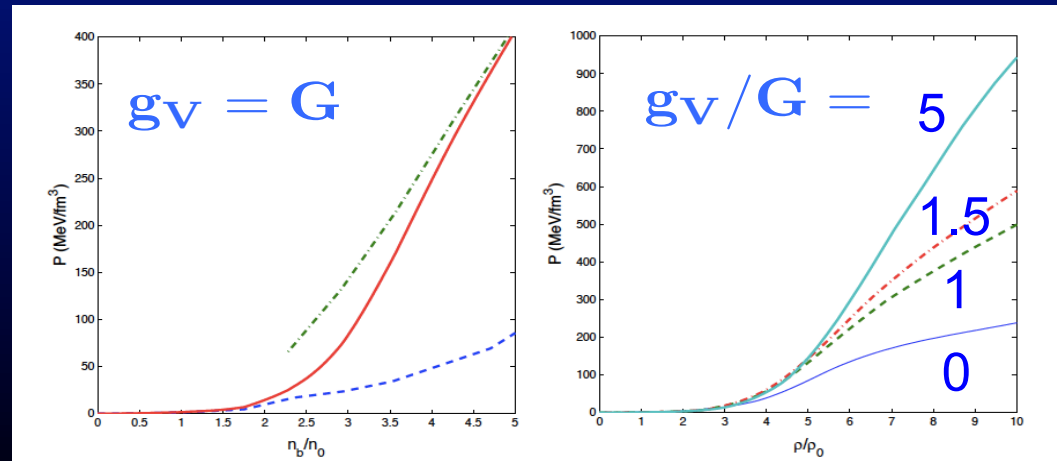
plus universal repulsive quark-quark vector coupling

$$\mathcal{L}_V^{(4)} = -g_V (\bar{q}\gamma^\mu q)^2 \quad T. Kunihiro$$

*K. Masuda, T. Hatsuda,
& T. Takatsuka, Ap. J.764,
12 (2013)*

*GB, T. Kojo, T. Hatsuda,
C.J. Pethick, T. Takatsuka,
Y. Song (to be published)*

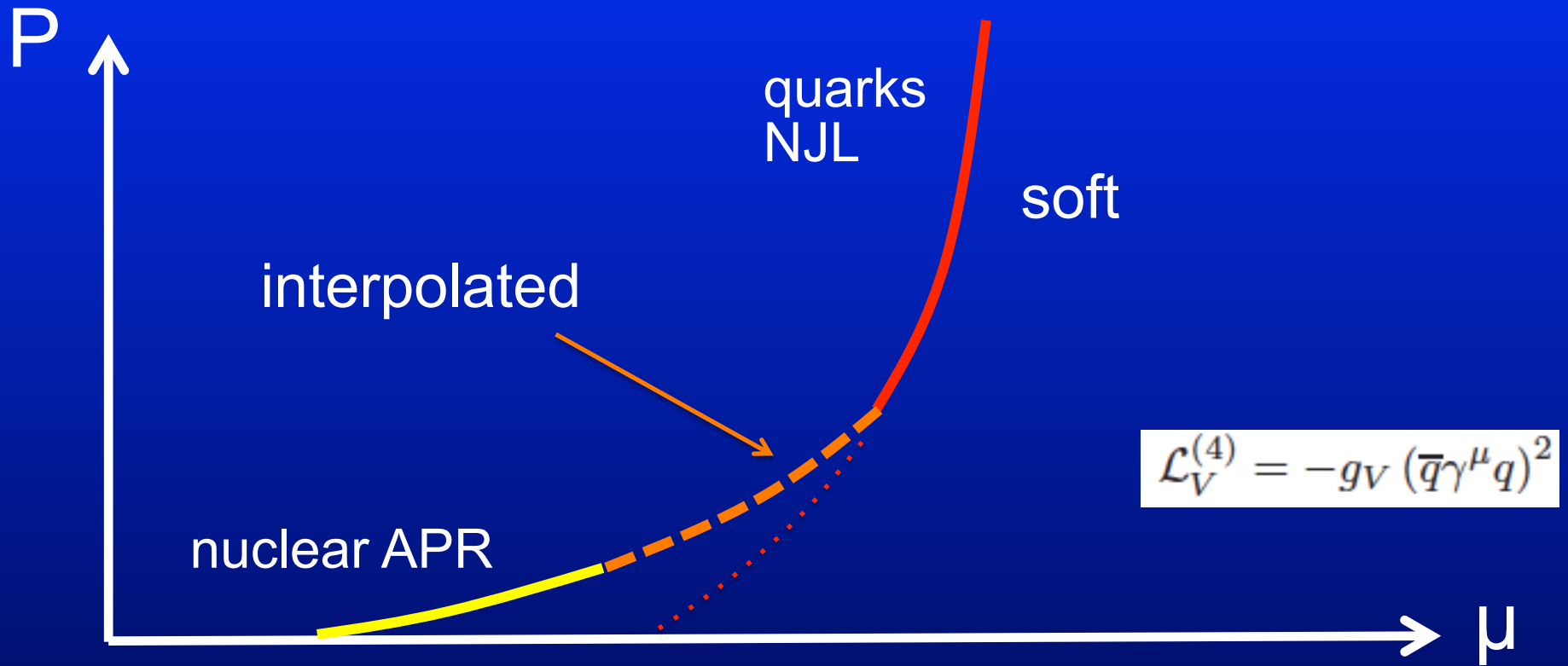
pressure



baryon density

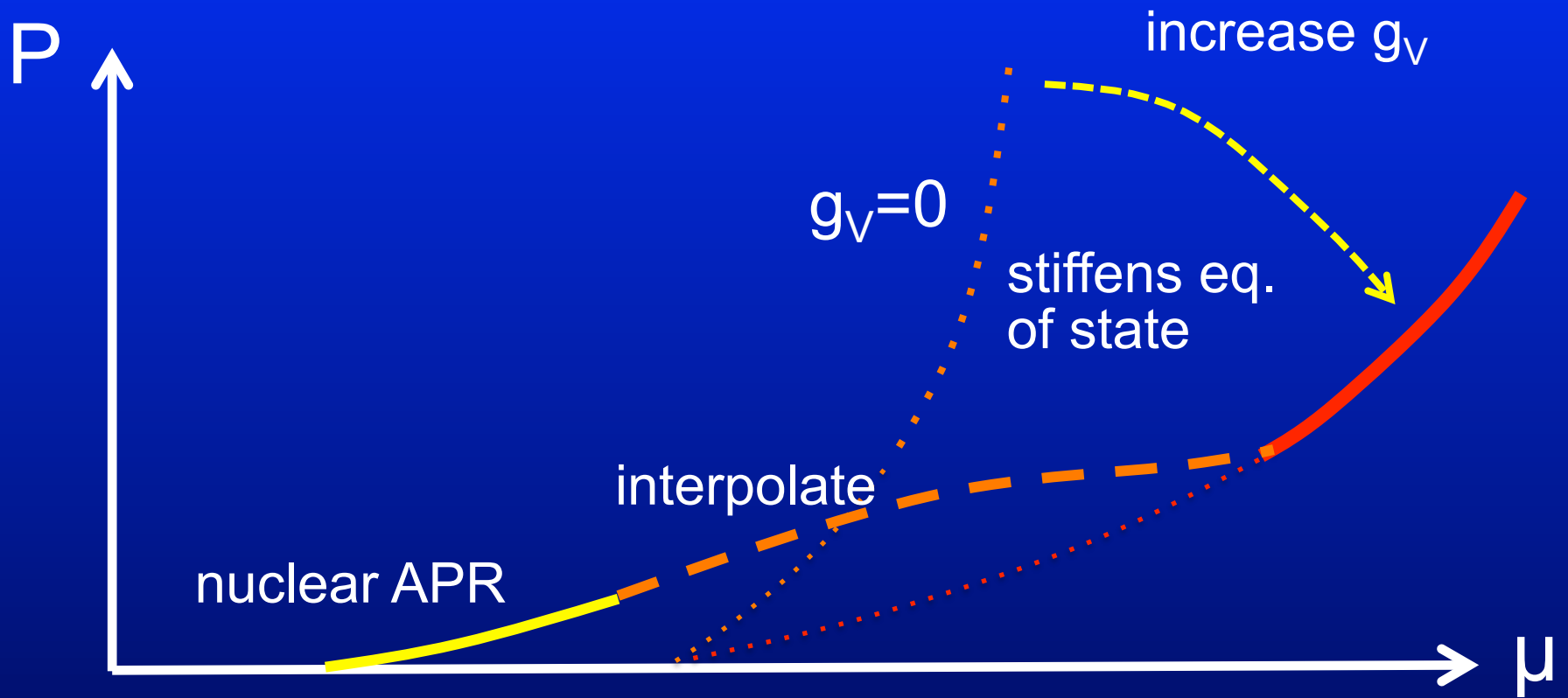
mass density

Minimal model: $g_V = 0$



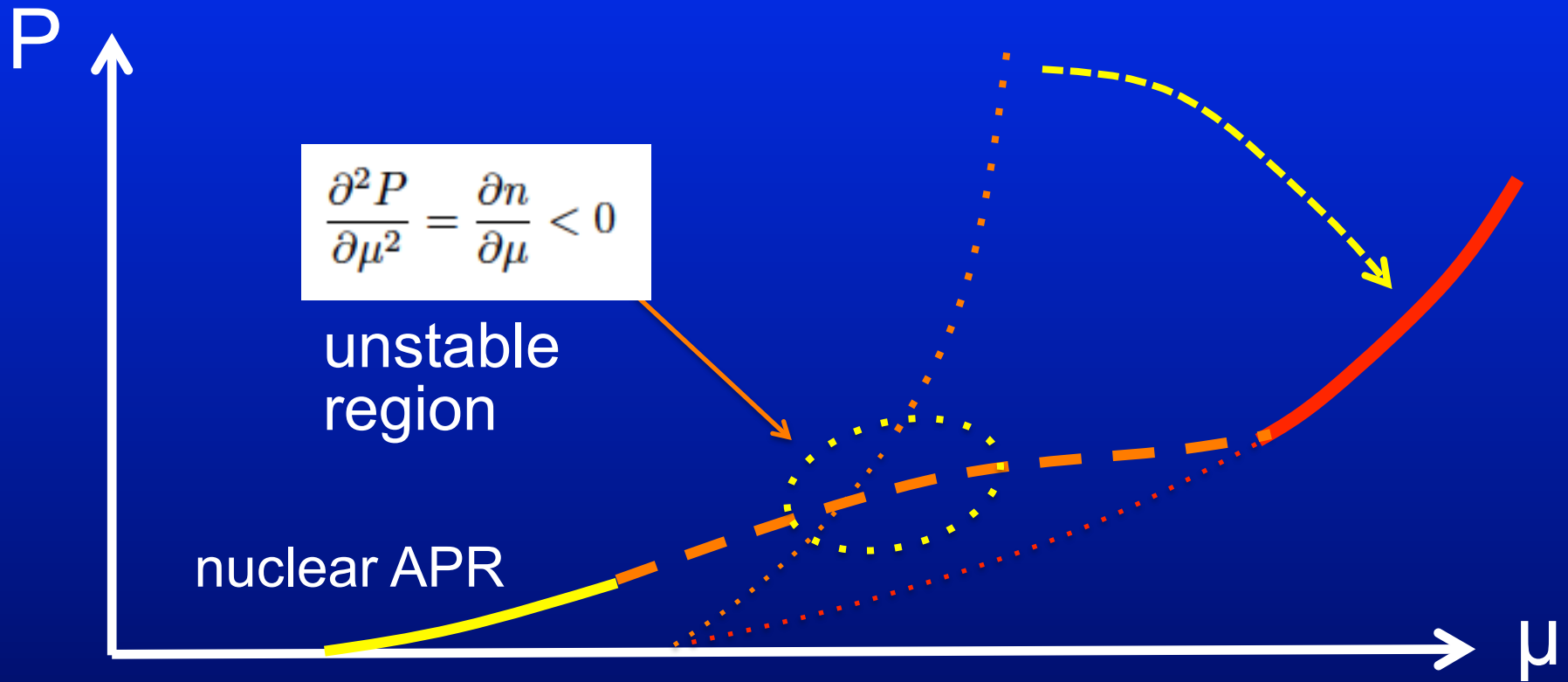
Soft quark equation of state does not allow high mass neutron stars

Vector interaction stiffens eq. of state



Shift of pressure in quark phase towards higher μ

Vector interaction stiffens eq. of state



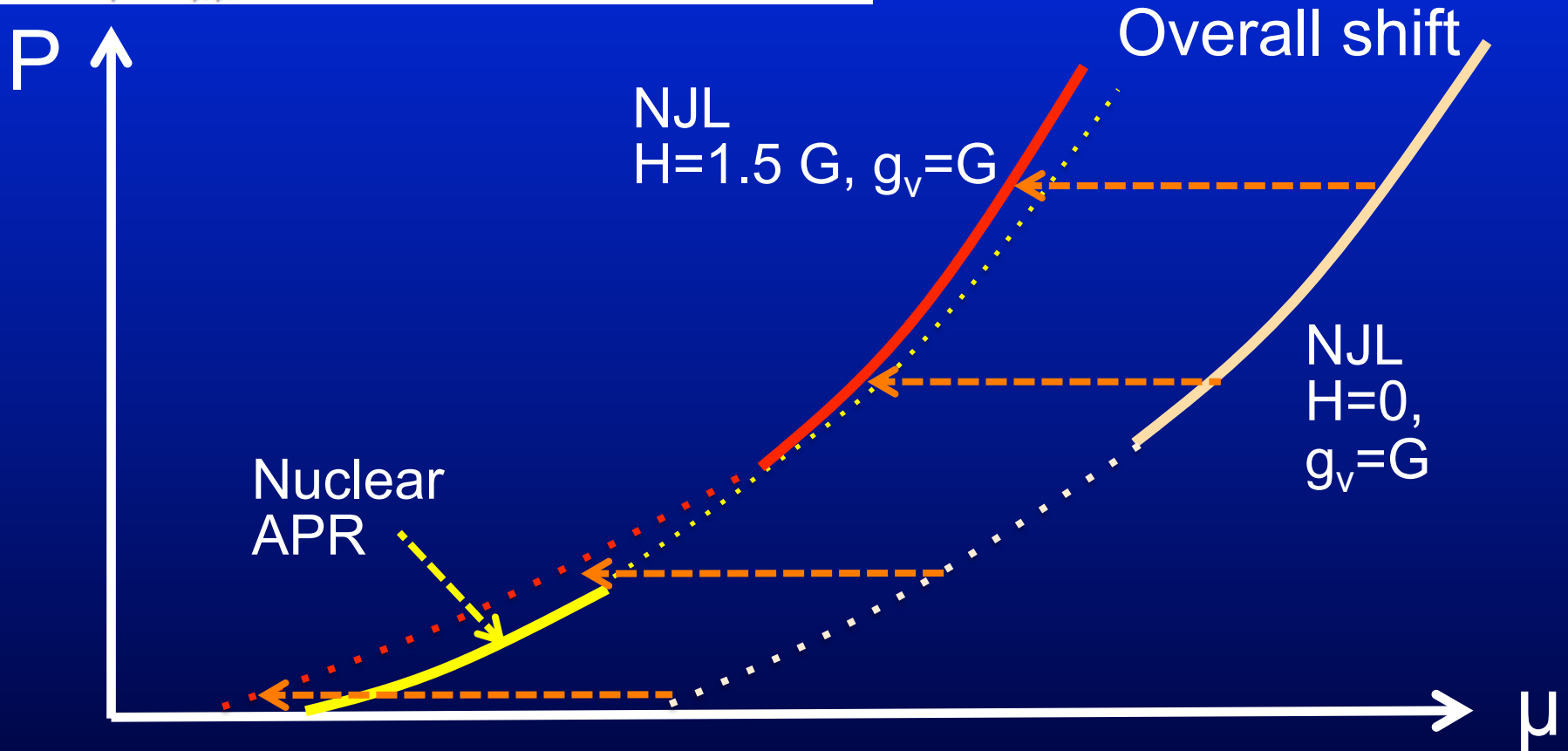
Larger g_v leads to unphysical thermodynamic instability



**In this house, we obey
the laws of thermodynamics!**

Restore stability with increased BCS (diquark) pairing interaction, H

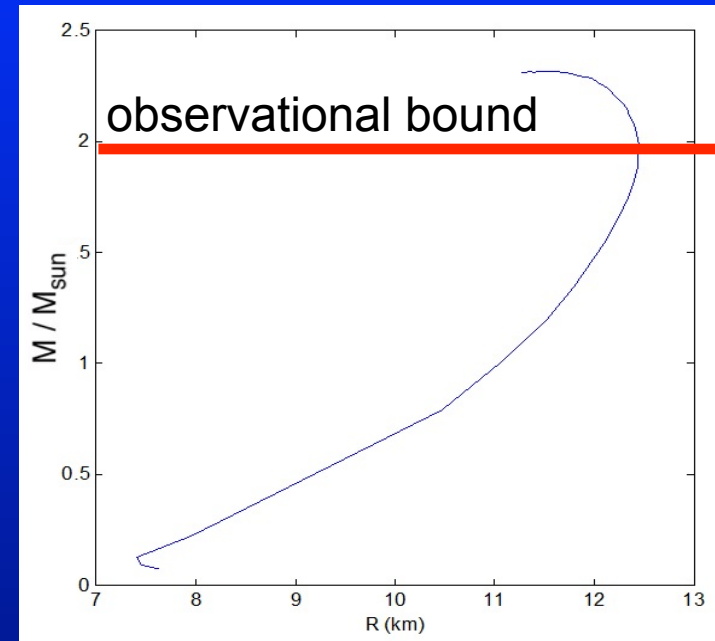
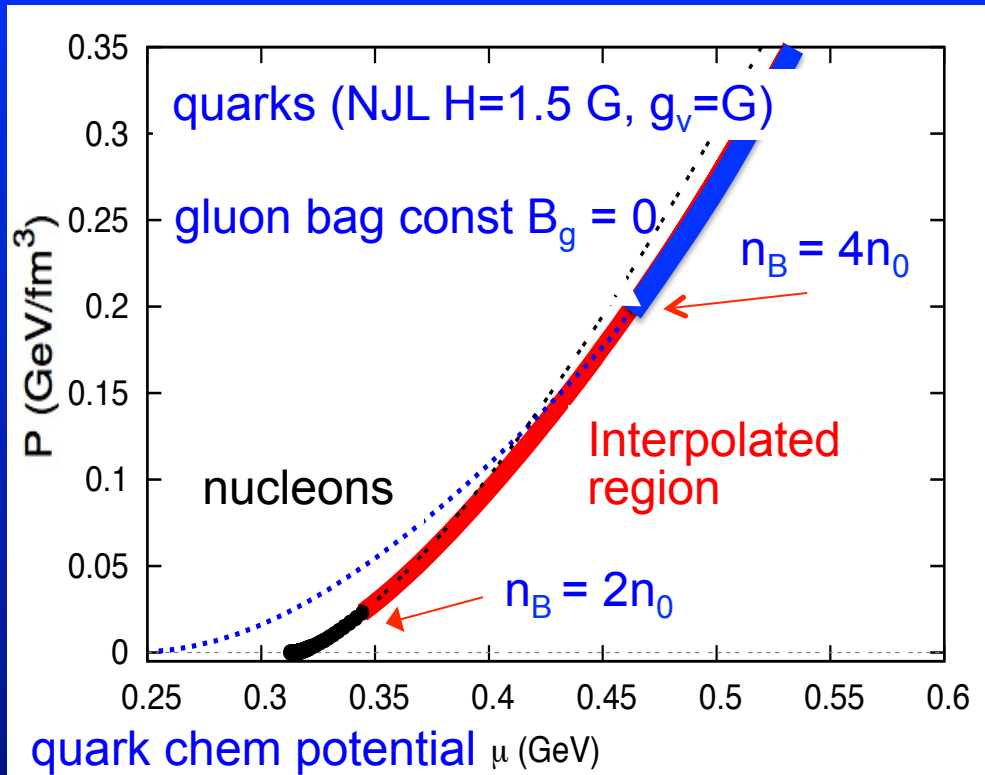
$$\mathcal{L}_d^{(4)} = H \sum_{A,A'=2,5,7} [(\bar{q}i\gamma_5\tau_A\lambda_{A'}C\bar{q}^T)(q^TCi\gamma_5\tau_A\lambda_{A'}q)]$$



Increased BCS pairing (onset of stronger 2-body correlations) as quark matter comes nearer to becoming confined

Sample “unified” equation of state

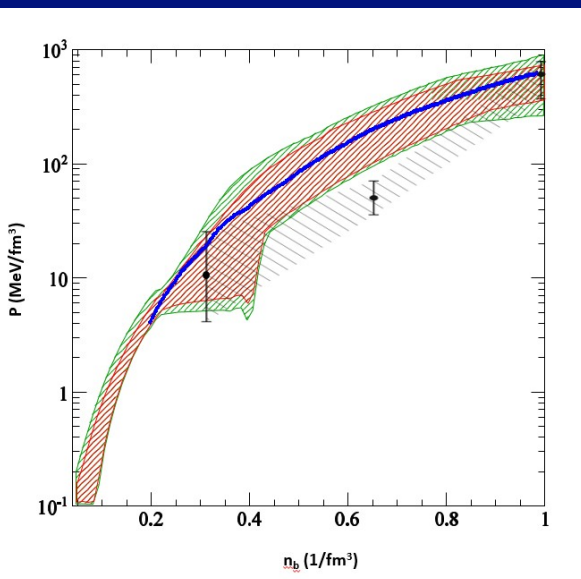
T. Kojo et al.



M-R relation similar to results of nucleonic (APR) equation of state but with correct high density degrees of freedom

≤ Reasonable agreement with eq. of state inferred from M vs. R measurements.

Quark eqs. of state can be stiffer than previously thought: allow for n.s. masses $> 2 M_{\odot}$, and with substantial quark cores in neutron stars!!!



Summary

For $2 n_0 < n_B < 7-8 n_0$ matter is intermediate between purely hadronic and purely quark: “quarkyonic”

Quark model eqs. of state can be stiffer than previously thought, allowing for neutron star masses $> 2 M_\odot$

Interaction parameters of order vacuum values $H \sim g_V \sim G_S^{vac}$

Gluonic bag constant is small; gluons remain non-perturbative at densities in neutron stars. Else significant softening on equation of state. Vacuum gluon condensate persists.

But we are not quite there yet:

Uncertainties in nuclear matter equation of state (APR, etc.)

Uncertainties in interpolating from nuclear matter to quark matter lead to errors in maximum neutron star masses and radii

Uncertainties in the vector coupling g_v and pairing forces

Going beyond the NJL model

Need to produce finite temperature equation of state (≤ 50 MeV) for modelling neutron star -- neutron star (or black hole) mergers as sources of gravitational radiation.

K. Masuda, T. Hatsuda, and T. Takatsuka, Prog. Theor. Exp. Phys. 2016, 021D01