Critical Opalescence: A Smoking Gun Signature for a Critical Point

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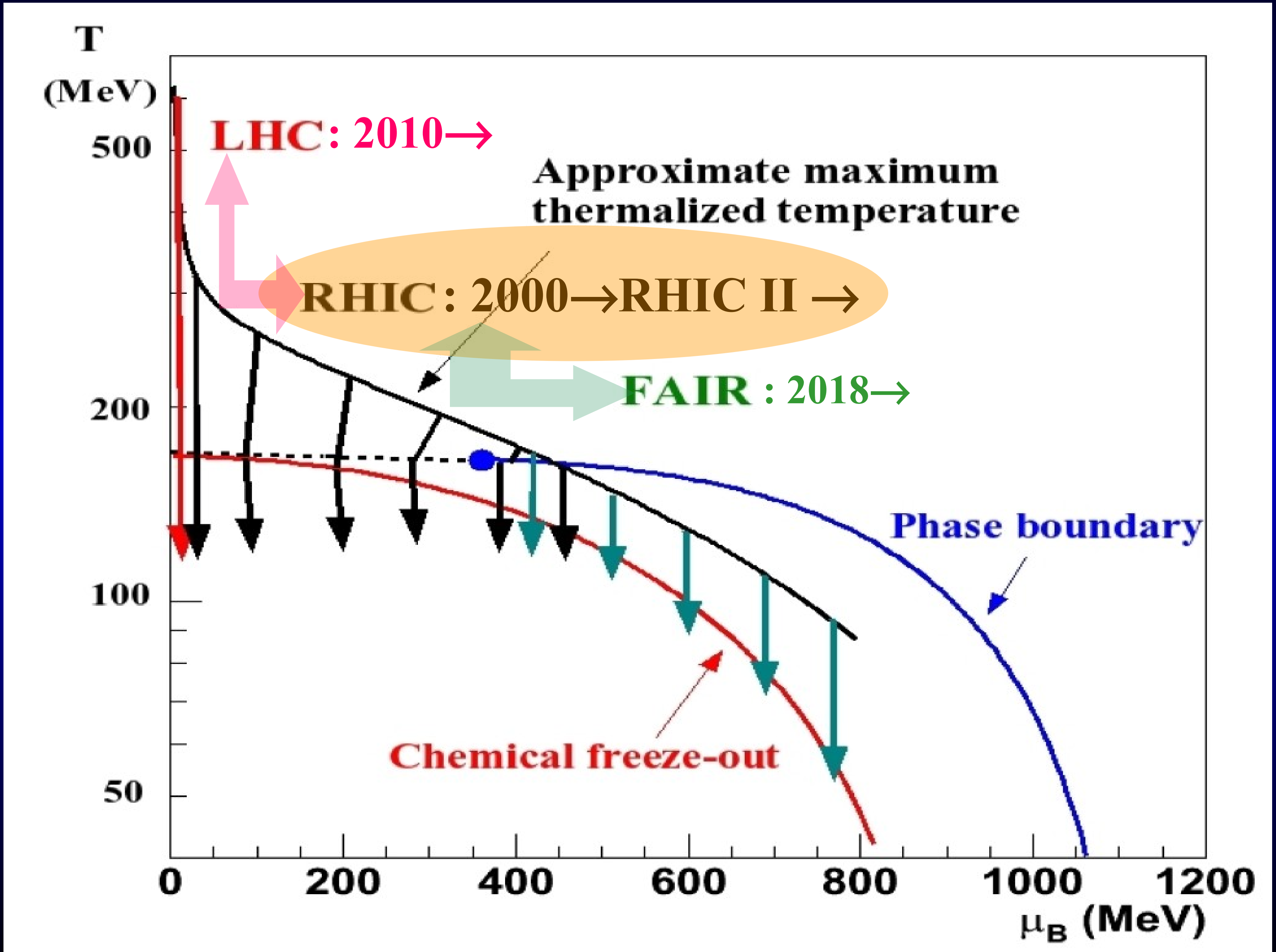
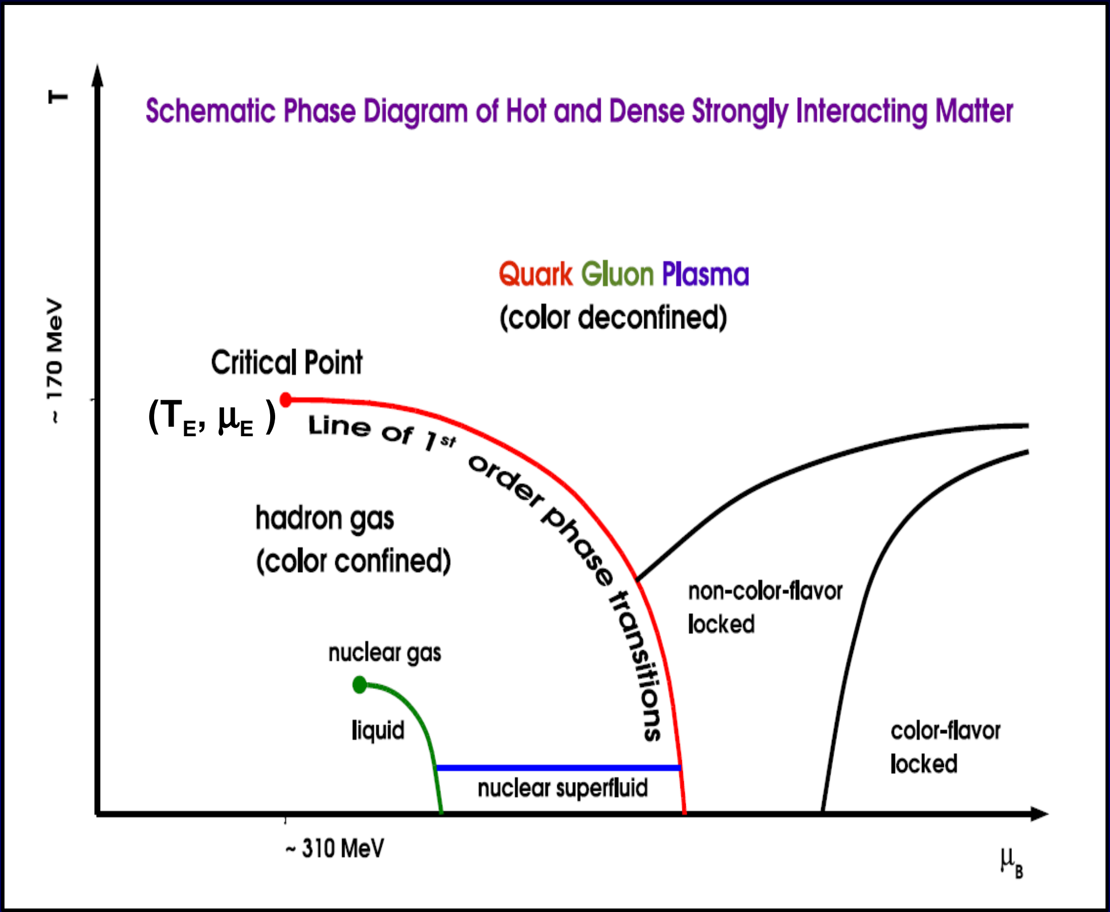
based on
arXiv:0903.0669 [nucl-th]
arXiv:0911.5015 [nucl-th]

World Context: Search for the CEP



4 steps 4 definitive CEP measurements



What about
- random fields?
- experimentally measurable order pars ?
- 1st order PT: speed of sound, latent heat?

<u>1. Identify:</u>
What type of transition
chiral ?
deconfinement ?
quarkionic ? liquid-gas?

<u>2. Locate:</u>
Where is (T_E, μ_E)?
At what centrality, $\sqrt{s_{NN}}$?
critical opalescence
onset of 1^{st} order PT

<u>3. Characterize:</u>
measure
order parameters,
critical exponents,
universality classes.

<u>4. Controll:</u>
Cross-checks for
consistency,
significance,
quality.

Correlations for VARIOUS Quark Matters

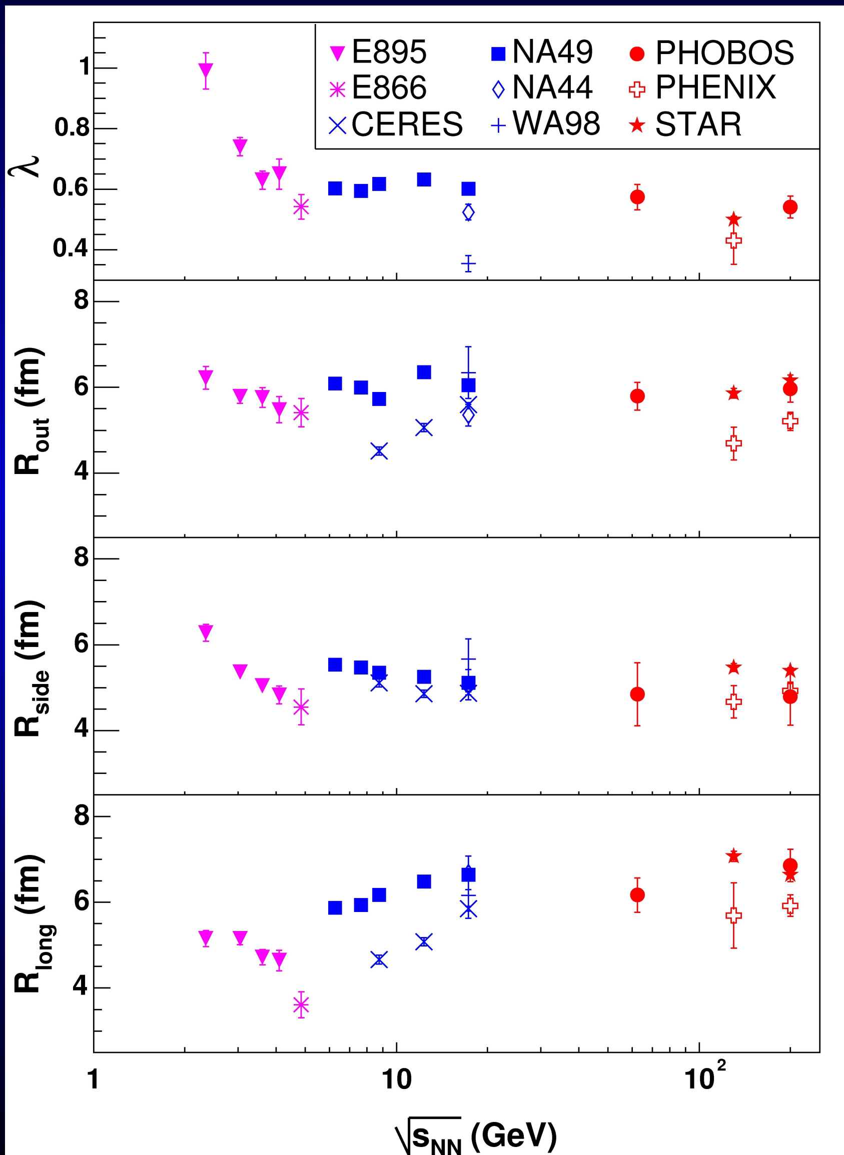
Transition to hadron gas may be:

1st order (strong)
2nd order (Critical Point, CP)
Cross-over
Non-equilibrium, e.g. from a supercooled state (scQGP)

Type of phase transition:	its correlation signature:
Strong 1st order QCD phase transition: (Pratt, Bertsch, Rischke, Gyulassy)	$R_{out} >> R_{side}$
2nd order QCD phase transition: (T. Cs, S. Hegyi, T. Novák, W.A. Zajc)	non-Gaussian shape α(Lévy) decreases to 0.5
Cross-over quark matter-hadron gas: (lattice QCD, Buda-Lund hydro fits)	hadrons appear from a region with $T > T_c$
Supercooled QGP (scQGP) -> hadrons: (T. Cs, L.P. Csernai)	pion flash ($R_{out} \sim R_{side}$) same freeze-out for all particles strangeness enhancement no mass-shift of ϕ

Excitation of 3d Gaussian fit parameters

STAR, Phys.Rev.C71:044906,2005



These data indicate

$R_{out} \sim R_{side}$

hence exclude:

Strong, equilibrium
1st order phase transit.
> 50 hydro models

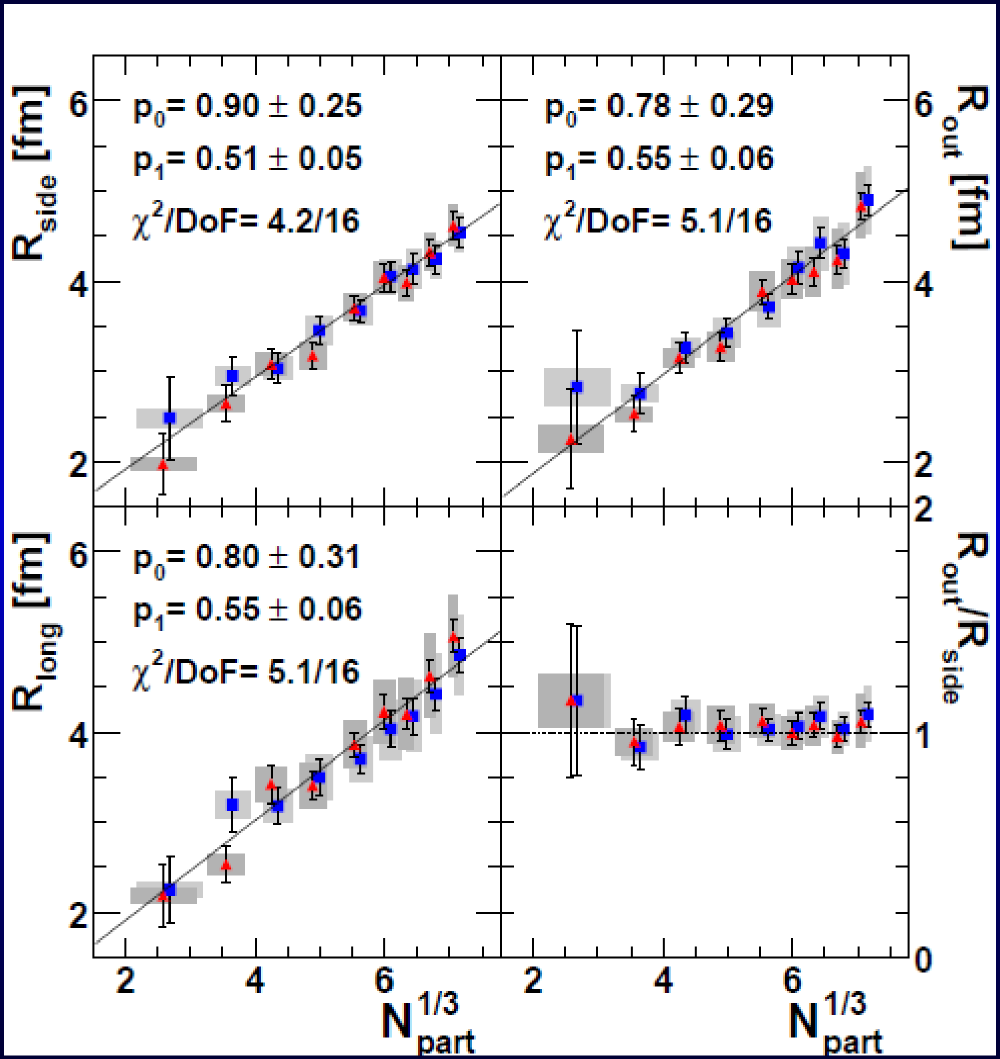
For a second order PT:

check excitation function of
non-Gaussian parameter α
(RHIC, FAIR, also @LHC!)

New analysis /
new data are needed

HBT Radii
independent of energy
perhaps initial volume ?
subtle m_t dependencies?

Excitation of 3d Gaussian fit parameters



PHENIX, Phys. Rev. Lett. 93, 152302 (2004)

Gaussian HBT Radii
scale as initial volume,
with $N^{1/3}_{part}$

These data extend $R_{out} \sim R_{side}$
to broad centrality, m_t range
hence exclude:

Strong, equilibrium
1st order phase transitions
and > 50 hydro models

For a second order PT:

check excitation function of
non-Gaussian parameter α

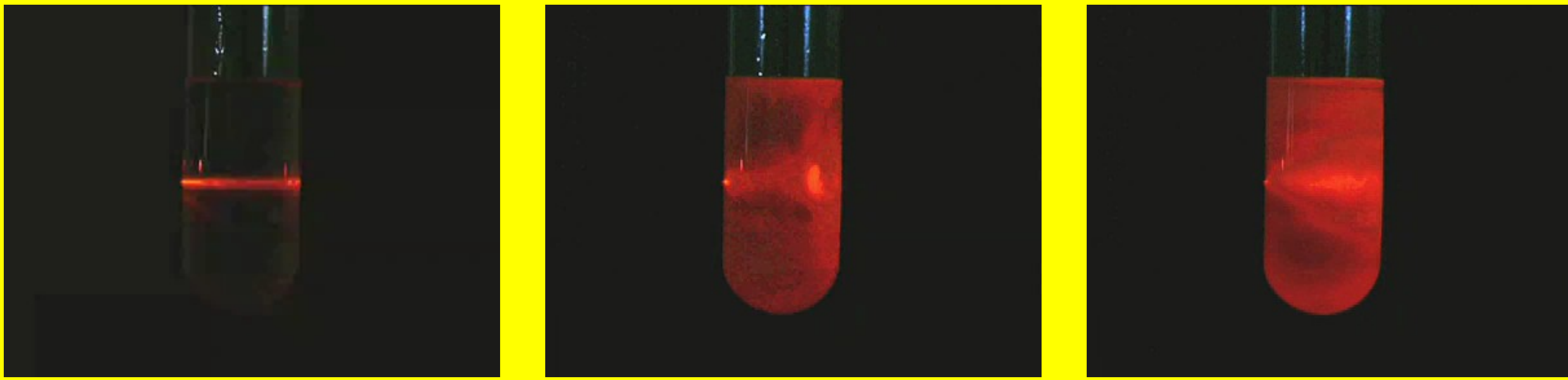
New analysis and/or
new data are needed
RHIC beam energy scan
also at FAIR, LHC

Critical Opalescence

Critical Opalescence: a laboratory method to observe a 2nd order PT

correlation length diverges, clusters on all scales appear incl. the wavelength of the penetrating (laser) probe

side view: http://www.msm.cam.ac.uk/doitpoms/tlplib/solidsolutions/videos/laser1.mov



front view: **matter becomes opaque at the critical point (CP)**

$T \gg T_c$ **$T \approx T_c$** **$T = T_c$**

Opical opacity κ and attenuation length λ

$$\frac{\partial I}{\partial x} = -\kappa I$$

$$I = I_0 \exp(-\kappa x) = I_0 \exp(-x/\lambda)$$

$$\kappa = \frac{I(\text{generated}) - I(\text{transmitted})}{I(\text{generated})\Delta x}$$

$$R_{AA} = \frac{I(\text{transmitted})}{I(\text{generated})} = \frac{I(\text{measured})}{I(\text{expected})}$$

$$I(\text{measured}) = \frac{1}{N_{event}^{AA}} \frac{d^2 N_{AA}}{dy\, dp_t}$$

$$I(\text{expected}) = \frac{\langle N_{coll} \rangle}{\sigma_{inel}^{NN}} \frac{d^2 \sigma_{NN}}{dy\, dp_t}$$

$$\boxed{\kappa = -\frac{\ln(R_{AA})}{R_{HBT}}}$$

Opical opacity κ and attenuation length λ

$$\kappa = -\frac{\ln(R_{AA})}{R_{HBT}}$$

$$\lambda = \frac{1}{\kappa} = -\frac{R_{HBT}}{\ln(R_{AA})}$$

Centrality	0-5 %	20-30 %	30-40 %	40-50 %	50-60 %
Opacity κ (fm^{-1})	0.35 ± 0.04	0.27 ± 0.03	0.26 ± 0.04	0.12 ± 0.02	0.15 ± 0.05
Attenuation λ (fm)	2.9 ± 0.3	3.7 ± 0.4	3.8 ± 0.6	8.1 ± 1.5	6.5 ± 2.0

Table 1: Examples of opacities κ and attenuation lengths λ in $\sqrt{s_{NN}} = 200$ GeV Au+Au reactions, evaluated from nuclear modification factors measured at $p_t = 4.75$ GeV/c in ref. [36] and using HBT radii of ref. [37], averaged over both directions and charge combinations, at the same centrality class as R_{AA}.

No max in opacity or min in attenuation length is seen wrt centrality

Contrast: for a 5 GeV γ on lead, λ = 6.2 mm

RHIC perfect fluid is more opaque (to jets),

than lead (to γ) - by 12 orders of magnitude

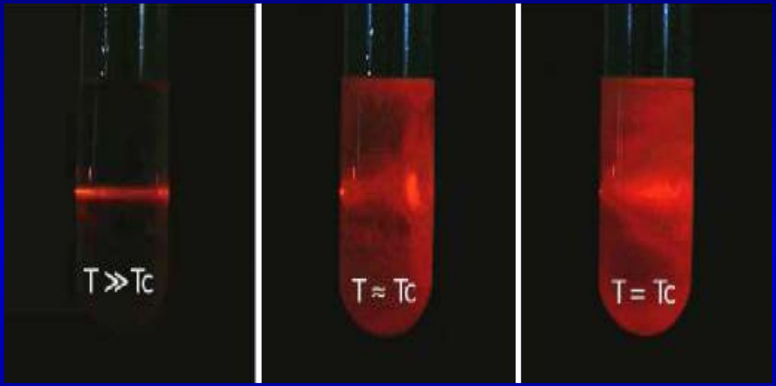
for details, see: arXiv:0903.0669v3, arXiv:0911.5015 [nucl-th]

2nd order phase transitions by critical opacescence with R_{AA} and R_{HBT}

Critical Opalescence: a smoking gun signature of a 2[nd] order PT

New experimental definition of opacity / attenuation length:

A combination of attenuation (R_{AA}) and lengthscale (e.g. R_{HBT}) is needed



$$I = I_0 \exp(-\kappa x) = I_0 \exp(-x/\lambda)$$

$$\kappa = -\frac{\ln(R_{AA})}{R_{HBT}}$$

Use optical opacity and look for maximal opalescence!

Alternative lengthscale measurement: $R(HBT) = p_0 + p_1 N_{part}^{1/3}$

Estimate Npart and take p_0 and p_1 from HBT measurements

Possible: azimuthally sensitive R_{AA} and R_{HBT} : azimuthally sensitive opacity

Further refinements: beyond Gaussian approximation, e.g. use R(Lévy)

Characterizing critical phenomena

Theoretical order parameter of QCD - quark condensate:

Experimental order parameter is needed:

- for chiral dynamics, signal of in-medium mass-shift
- for deconfinement, signal of quark degrees of freedom

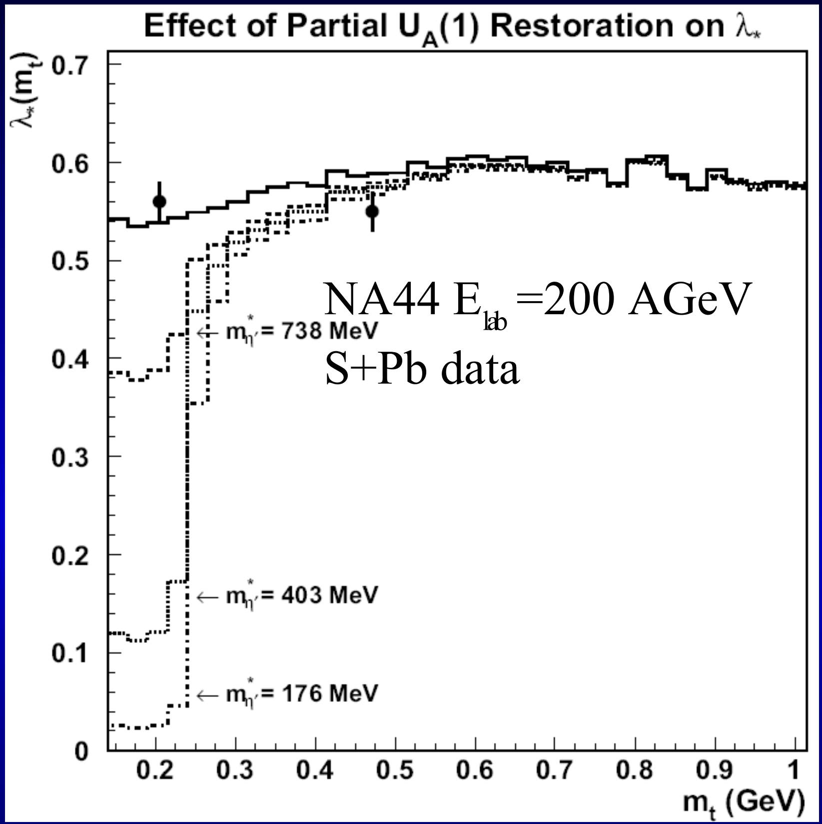
Understandable in laymen's terms: quark scaling of particle ratios

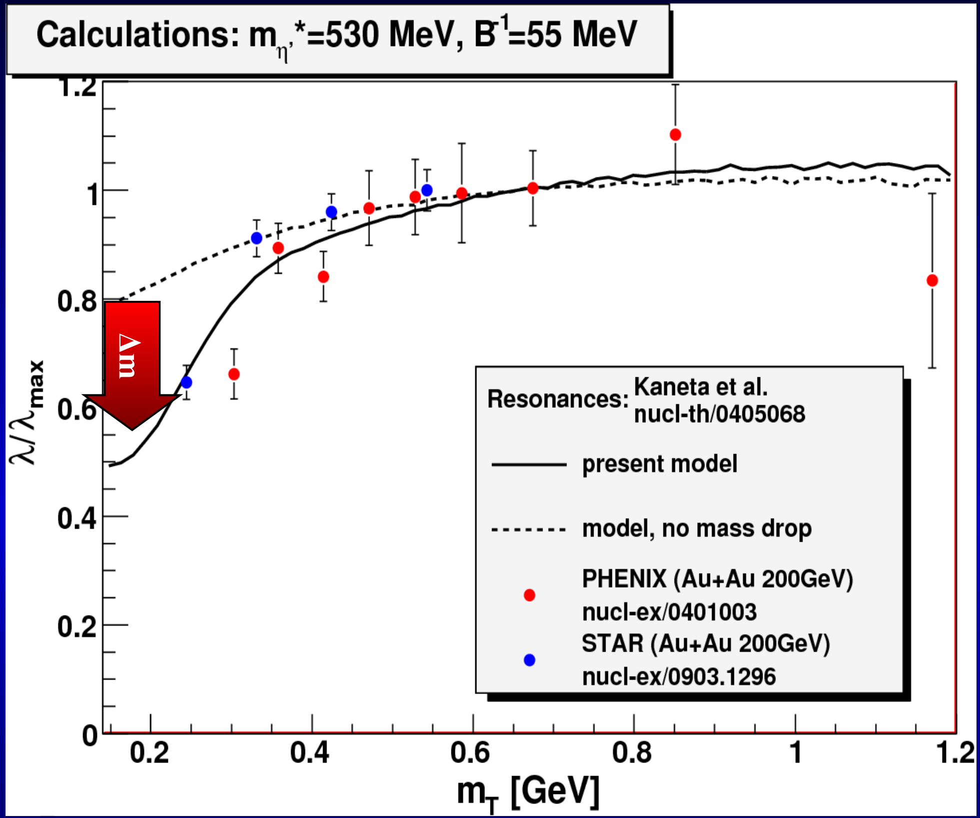
$$
\begin{aligned}
\frac{\overline{\Lambda}|\overline{\Sigma}}{\Lambda|\Sigma} &= \frac{\overline{p}}{p}\left[\frac{K}{\overline{K}}\right], \\
\frac{\overline{\Xi}}{\Xi} &= \frac{\overline{p}}{p}\left[\frac{K}{\overline{K}}\right]^2, \\
\frac{\overline{\Omega}}{\Omega} &= \frac{\overline{p}}{p}\left[\frac{K}{\overline{K}}\right]^3.
\end{aligned}
\quad \Longleftrightarrow \quad
\begin{aligned}
\frac{\overline{p}}{p} &= \left[\frac{\overline{Q}}{Q}\right]^3, \\
\frac{\overline{\Lambda}|\overline{\Sigma}}{\Lambda|\Sigma} &= \left[\frac{\overline{Q}}{Q}\right]^2\left[\frac{\overline{S}}{S}\right], \\
\frac{\overline{\Xi}}{\Xi} &= \left[\frac{\overline{Q}}{Q}\right]\left[\frac{\overline{S}}{S}\right]^2, \\
\frac{\overline{\Omega}}{\Omega} &= \left[\frac{\overline{S}}{S}\right]^3, \\
\frac{\overline{K}}{K} &= \frac{\overline{Q}\,S}{Q\,\overline{S}}.
\end{aligned}
$$

J. Zimányi, T. Biró, T.Cs, P. Lévai, Phys.Lett.B472:243-246

A. Bialas, Phys.Lett.B442:449-452,1998

Order parameter for chiral symmetry from HBT, using $\lambda(m_t, \sqrt{s_{NN}})$





Idea: Signals of mass decrease of η' at low p_t

No signal @SPS in S+Pb NA44 data

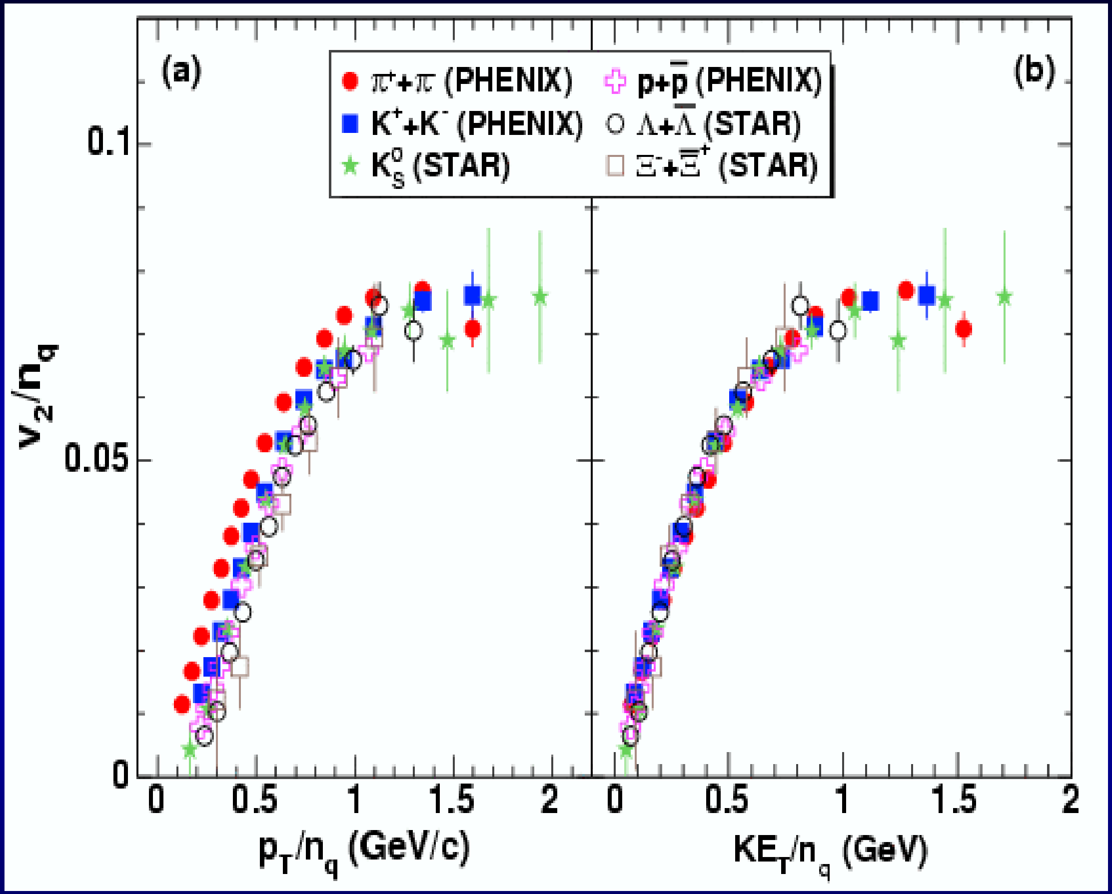
Signal @ RHIC: in 200 GeV STAR and PHENIX Au+Au data

$\Delta m(\eta') > 200$ MeV @ 99.9 CL

R. Vértesi, T.Cs, J. Sziklai, arXiv:0905.2803

Suggests: chiral symmetry restoration may start above top SPS energy

Order parameter for deconfinement from identified particle v_2



Idea: look for the break down of the quark number scaling
If scaling: quark degrees of freedom are active (exp. view)

Measure: v_2/n_q as a function of KE_T/n_q of identified particles
needs high statistics PID measurement at low $\sqrt{s_{NN}}$

Critical Exponents at 2nd order PT

Relevant and important quantities:
critical exponents, universality classes

Reduced temperature: $t = (T - T_c)/T_c$

Exponent of specific heat: $C(T) \sim |t|^{-\alpha} + \text{less singular.}$

Exponent of order parameter: $\langle|\phi|\rangle \sim |t|^{\beta} \quad \text{for } t < 0$

Exponent of correlation length : $\xi \sim |t|^{-\nu}$

Exponent of susceptibility: $\int d^3x \; G_{\alpha\beta}(x) \sim t^{-\gamma}$

Critical Exponents (2)

Exponent of the Fourier-transformed correlation function:

$$G_{\alpha\beta}(k \to 0) \sim k^{-2+\eta}$$

Exponent of order parameter in external field:

$$\langle|\phi|\rangle(t=0, H \to 0) \sim H^{1/\delta}$$

There are thus 6 critical exponents, α, β, γ, δ, ν, η

but only 2 are independent:

$$\begin{aligned} \alpha &= 2 - d\nu \\ \beta &= \frac{\nu}{2}(d-2+\eta) \\ \gamma &= (2-\eta)\nu \\ \delta &= \frac{d+2-\eta}{d-2+\eta} \, . \end{aligned}$$

Exponents <-> universality class!

What is the universality cl of QCD CEP?

Rajagopal, Wilczek (1992):
3d Ising model

$$\eta = 0.03 \pm 0.01, \quad \nu = 0.73 \pm 0.02,$$

$$\alpha = -0.19 \pm 0.06, \quad \beta = 0.38 \pm 0.01, \quad \gamma = 1.44 \pm 0.04, \quad \delta = 4.82 \pm 0.05.$$

Csörgő, Hegyi, Zajc (2004):
random field 3d Ising model

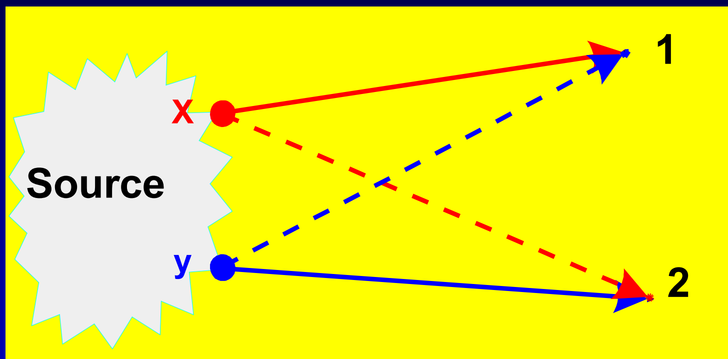
$$\eta = 0.50 \pm 0.05, \quad \nu = 1.1 \pm 0.2.$$

$$\alpha = -1.3 \pm 0.6, \quad \beta = 0.6 \pm 0.1, \quad \gamma = 2.2 \pm 0.4, \quad \delta = 4.7 \pm 0.3.$$

Random fields change the universality class
but only data can decide which one is realized

Two particle Interferometry

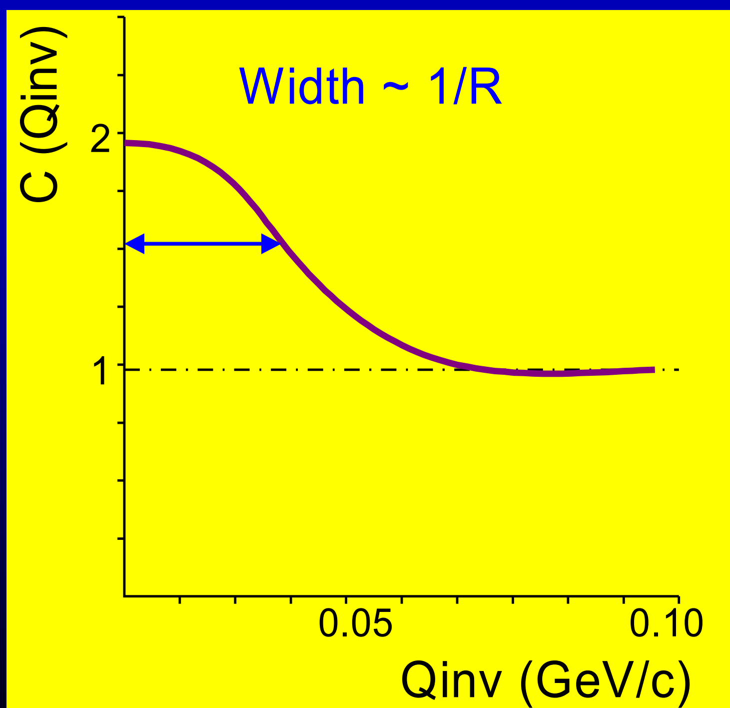
for non-interacting identical bosons



$$A_{12} = \frac{1}{\sqrt{2}}\left[e^{ip_1\cdot(r_1 - x)}e^{ip_2\cdot(r_2 - y)} + e^{ip_1\cdot(r_1 - y)}e^{ip_2\cdot(r_2 - x)}\right]$$

so that

$$\mathcal{P}_{12} = \int d^4x\, d^4y\, |A_{12}|^2 \rho(x)\rho(y) = 1 + |\tilde{\rho}(q)|^2 \equiv C_2(q)$$



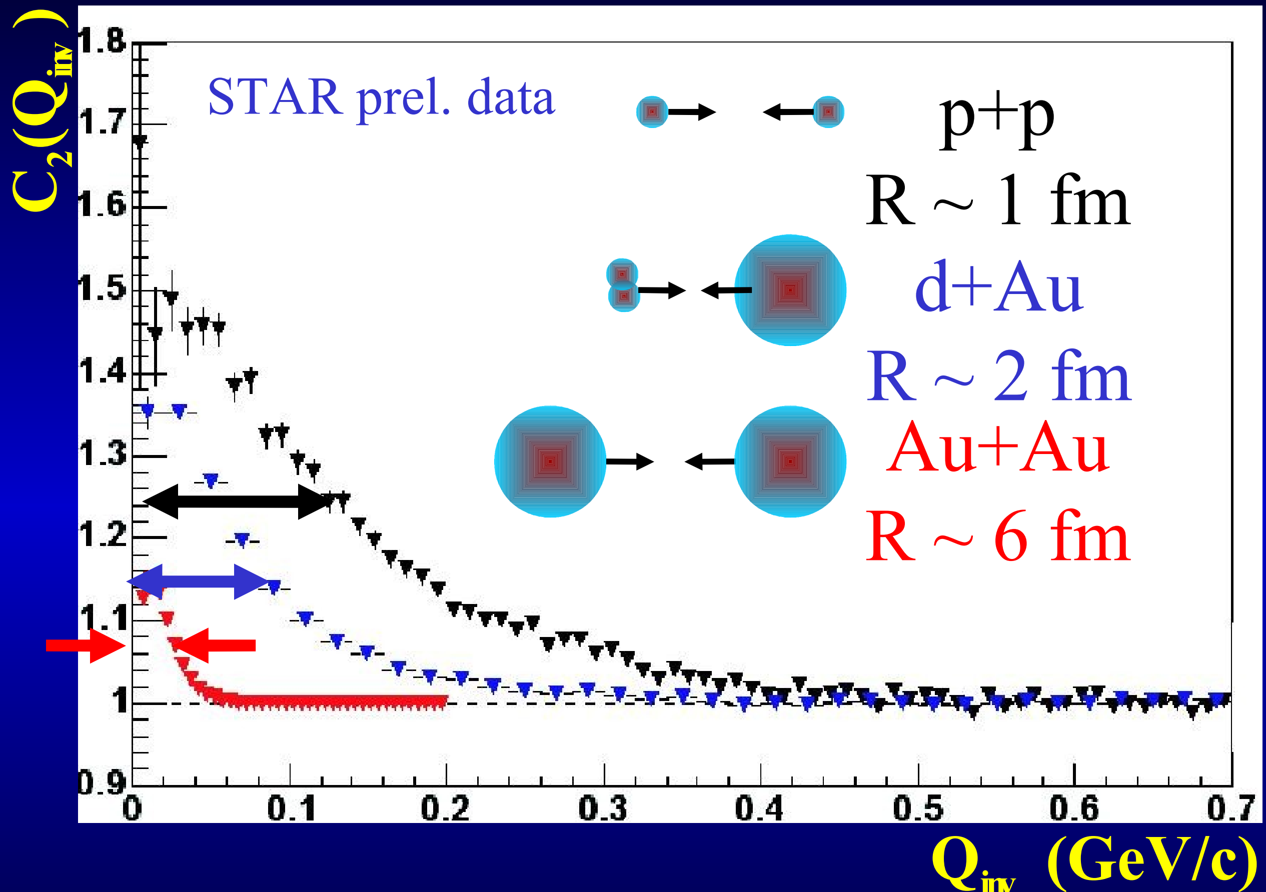
emission function

$$C(p_1, p_2) = 1 + \frac{\left|\int d^4x \cdot S(x,K) \cdot e^{iq\cdot x}\right|^2}{\left|\int d^4x \cdot S(x,K)\right|^2}$$

$q = p_1 - p_2$

$K = \frac{1}{2}(p_1 + p_2)$

Correlations for various collisions



Correlations have more information (3d shape analysis)
Use advanced techniques & extract it (~ medical imaging)

CEP: Scale invariant (Lévy) sources

Fluctuations appear on many scales,
final position is a sum of many random shifts:

$$x = \sum_{i=1}^{n} x_i, \qquad f(x) = \int \Pi_{i=1}^{n} \mathrm{d}x_i \, \Pi_{j=1}^{n} f_j(x_j) \, \delta(x - \sum_{k=1}^{n} x_k).$$

correlation function measures a Fourier-transform,
that of an n-fold convolution:

$$\tilde{f}(q) = \int \mathrm{d}x \exp(iqx) f(x), \qquad \tilde{f}(q) = \prod_{i=1}^{n} \tilde{f}_i(q),$$

Lévy: generalized central limit theorems
adding one more step in the convolution does not change the shape

$$\begin{aligned} \tilde{f}_i(q) &= \exp\left(iq\delta_i - |\gamma_i q|^{\alpha}\right), & \prod_{i=1}^{n} \tilde{f}_i(q) &= \exp\left(iq\delta - |\gamma q|^{\alpha}\right) \\ \gamma^{\alpha} &= \sum_{i=1}^{n} \gamma_i^{\alpha}, & \delta &= \sum_{i=1}^{n} \delta_i. \end{aligned}$$

Correlation functions for Lévy sources

Correlation funct of stable sources:

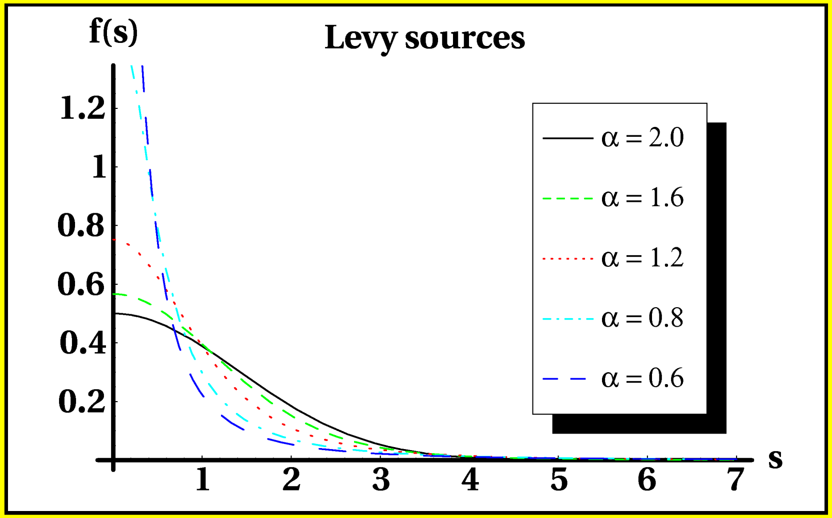
$$C(q;\alpha) = 1 + \lambda \exp\left(-|qR|^{\alpha}\right)$$

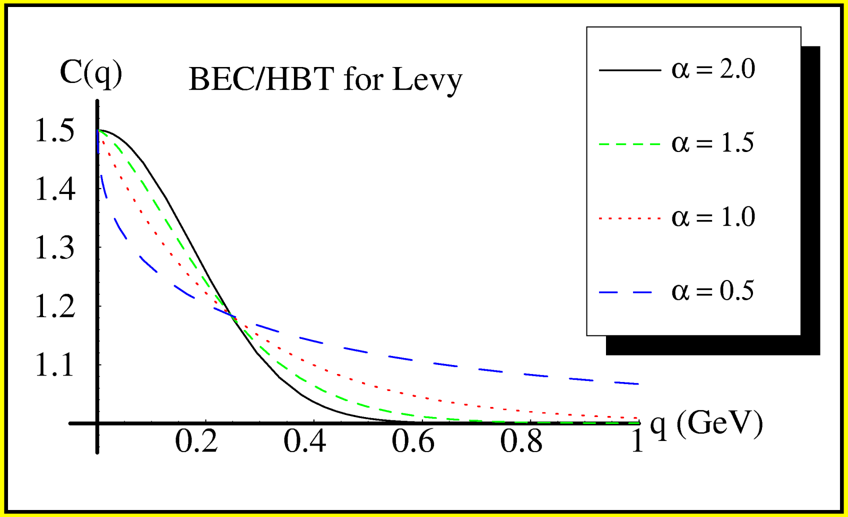
R: scale parameter

α: shape parameter or Lévy index of stability

$\alpha = 2$ Gaussian, $\alpha = 1$ Lorentzian sources

Further details: T. Cs, S. Hegyi and W. A. Zajc, EPJ C36 (2004) 67





Correlation signal of the CEP

If the source distribution
at CEP is a Lévy, it decays as:

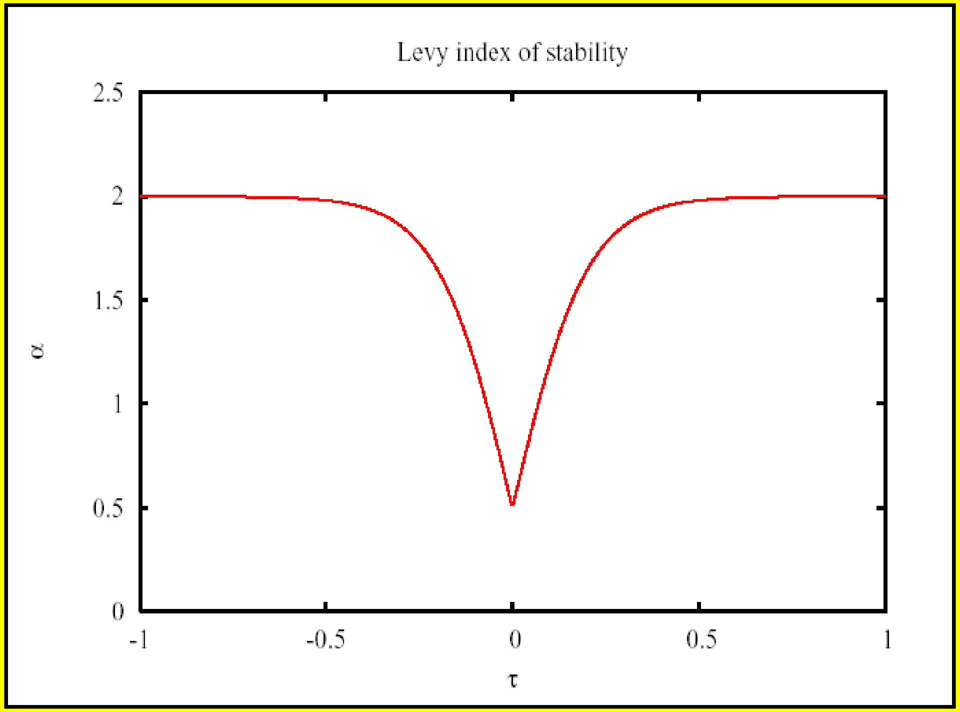
$$\rho(R) \propto R^{-(1+\alpha)}$$

at CEP, the tail decreases as:

$$\rho(R) \propto R^{-(d-2+\eta)}$$

hence:

$$\alpha(\text{Lévy}) = \eta(\text{3d Ising}) = 0.50 \pm 0.05$$



α as a function of $\tau = |T - T_c| / T_c$

T. Cs, S. Hegyi, T. Novák, W.A.Zajc,
Acta Phys. Pol. B36 (2005) 329-337

Critical exponents, universality class

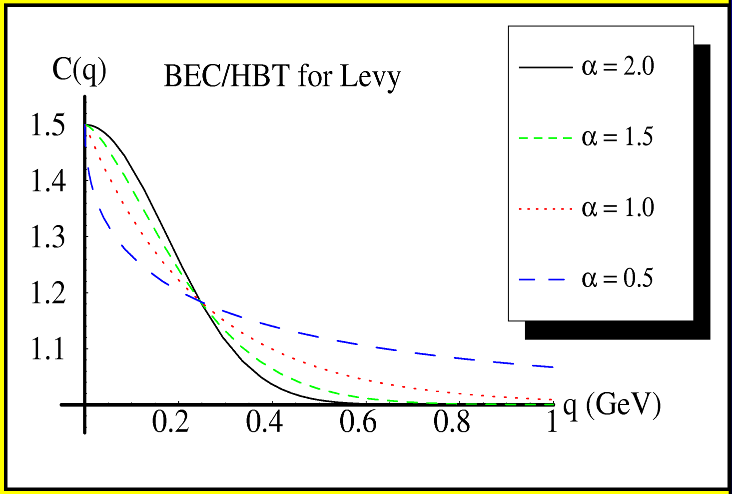
Soft Bose-Einstein /HBT correlations:
measure Lévy index of stability, α

$$C(q;\alpha) = 1 + \lambda \exp\left(-|qR|^{\alpha}\right)$$

$\alpha = \eta$: critical exponent of the correlation function
Note: pt scale below T_c ~ 170 MeV is essential !!

Universality class argument:
α decreases from 2 (or 1.4) to 0.5 (random field 3d Ising)
or even smaller value (0.03 in 3d Ising) at Critical Point

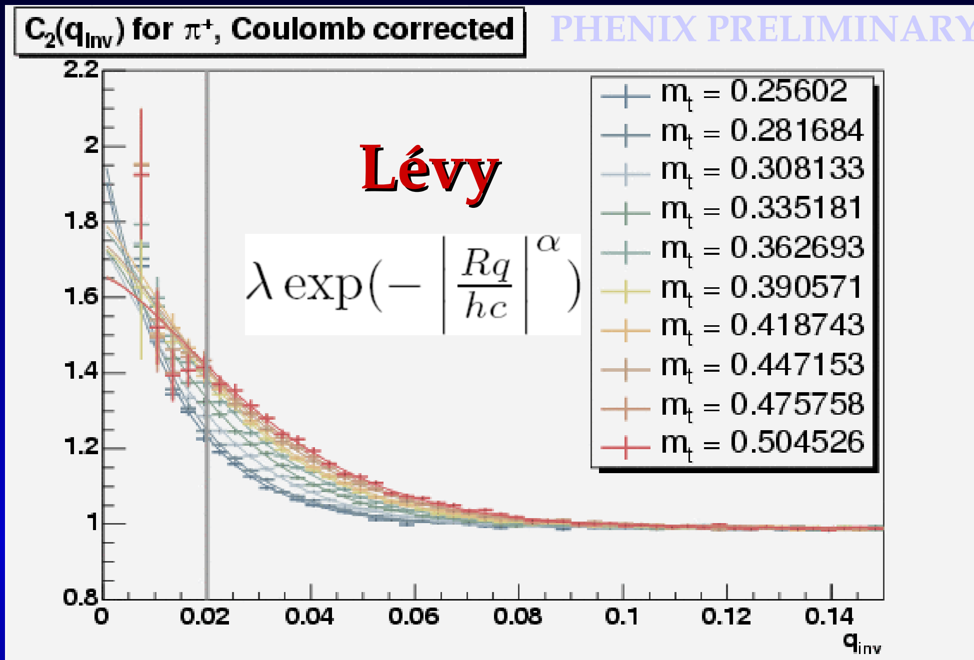
T. Cs, S. Hegyi and W. A. Zajc, EPJ C36 (2004) 67
T. Cs, S. Hegyi, T. Novák, W.A.Zajc,
Acta Phys. Pol. B36 (2005) 329-337

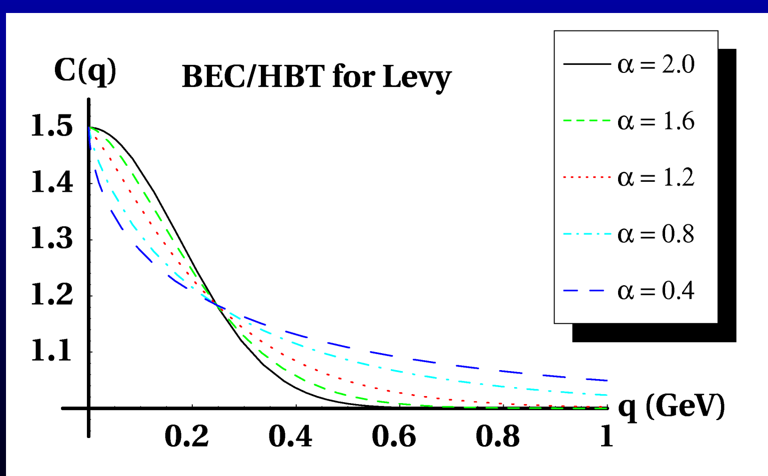


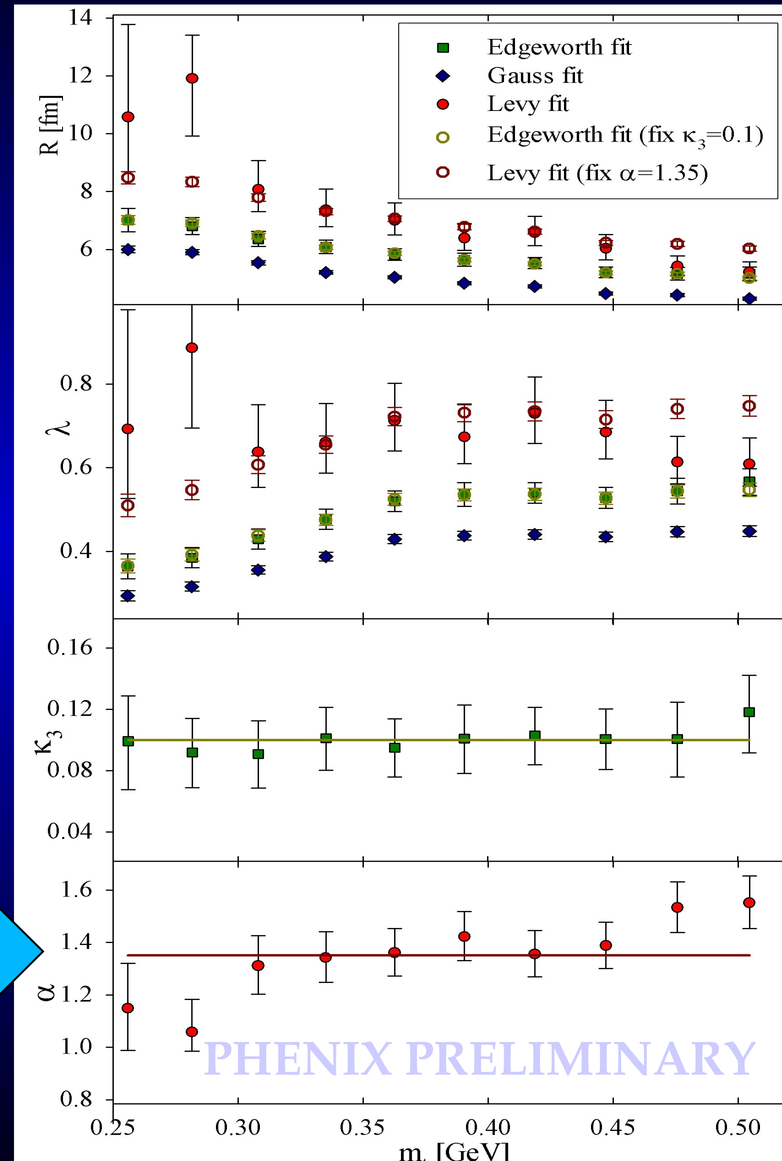
yielding one of the 6 exponents that CHARACTERIZE a 2[rd] order PT

see J. Mitchell's CPOD'09 talk and nucl-ex/0509042

Lévy fits to PHENIX prel. Au+Au @ 200 GeV







Summary

4 steps for a definitive result on CEP:
- **identify type of phase transition**
- **locate**
- **characterize**
- **cross-check**

Concept of optical opacity:
- **both attenuation measure, R_{AA}**
- **and lengthscale measure R_{HBT} needed**

perfect fluid at RHIC:
12 orders of magn more opaque than Pb for γ

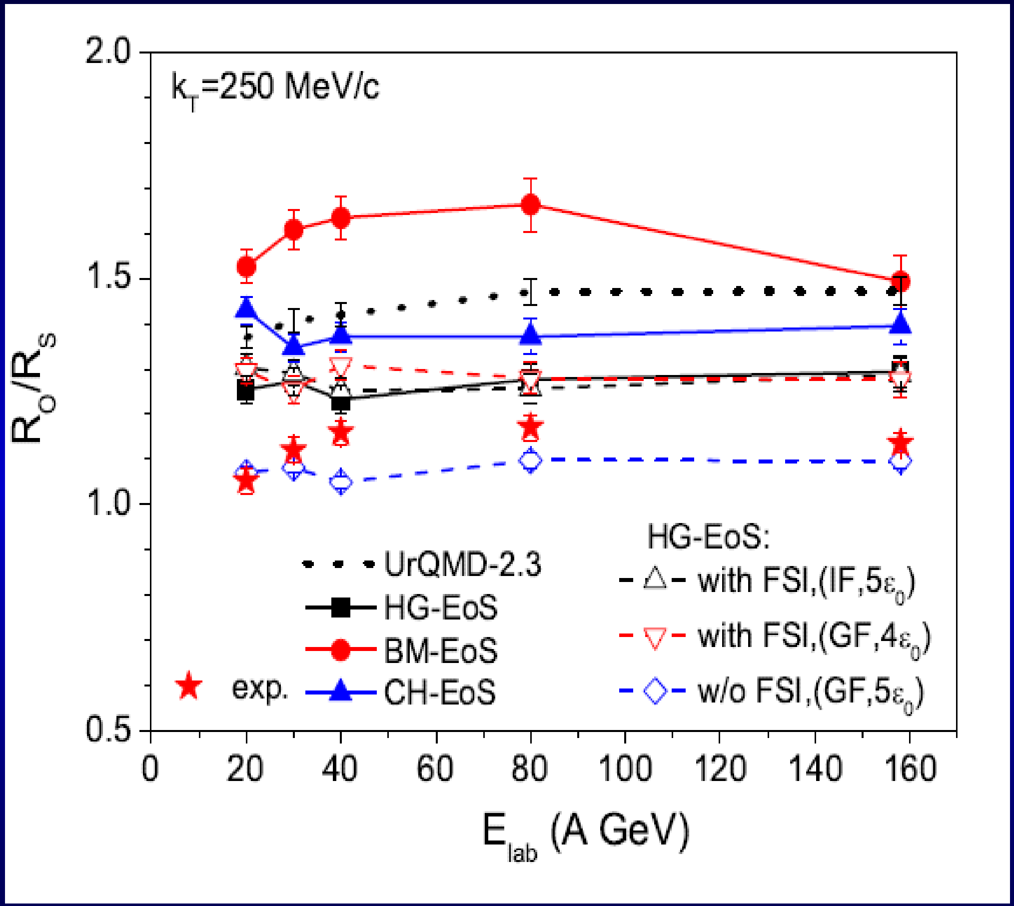
Critical Opalescence:
Smoking gun signature to locate CEP

Levy stable Bose-Einstein/HBT correlations

→ **critical exponent η**

Backup slides

Signal of first order phase transitions from HBT R_{out} / R_{side} $(m_t, \sqrt{s_{NN}})$



H. Petersen, QM 2009 talk:
HG= hadron gas EoS +hydro
BM= bag model EoS + hydro
CH= chiral EoS with CP+hydro
arXiv: 0812.0375

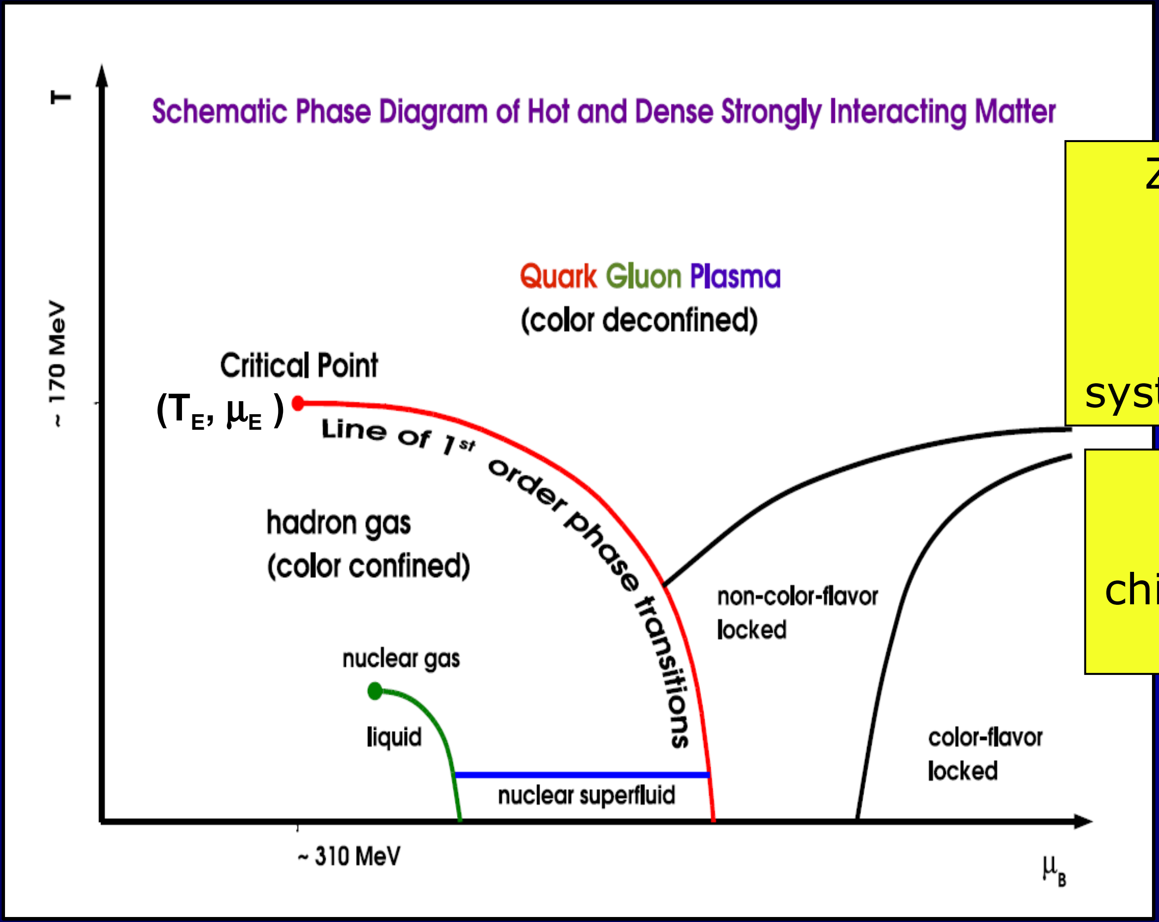
Comment:
Rischke's hydro ⟺ NA49 data

HBT puzzle is not RHIC specific

R_{out} / R_{side} sensitive to the EoS

Idea: look first order phase transition where $R_{out} / R_{side} >> 1$
Measure: Gaussian HBT radii for pions (if possible kaons too)

Lattice QCD: EoS of QCD Matter



Z. Fodor, S.D. Katz:
T_E=162±2 MeV,
μ_E = 360±40 MeV
(hep-lat/0402006
systematics: this meeting

deconfinement
and
chiral phase transitions
in general differ

At the Critical End Point, the chiral phase transition is of 2nd order.
Stepanov, Rajagopal, Shuryak:
(Static) universality class of QCD = 3d Ising model
PRL 81 (1998) 4816

Measure by two-particle correlations

Single particle spectrum:
averages over space-time information

$$E\frac{dN}{d\mathbf{p}} = \int dx^4 S(x,\mathbf{p})$$

Correlations:
sensitivity to space-time information

$$C_2(\mathbf{q}) = \frac{dN_2 / d\mathbf{p}_1 d\mathbf{p}_2}{(dN_1 / d\mathbf{p}_1)(dN_1 / d\mathbf{p}_2)} \approx \int d\mathbf{r} \,|\, \Phi\, (\mathbf{r},\mathbf{q})\,|^2\; S(\mathbf{r},\mathbf{q})$$

FSI **Source function**

Intensity interferometry, HBT technique, femtoscopy ….

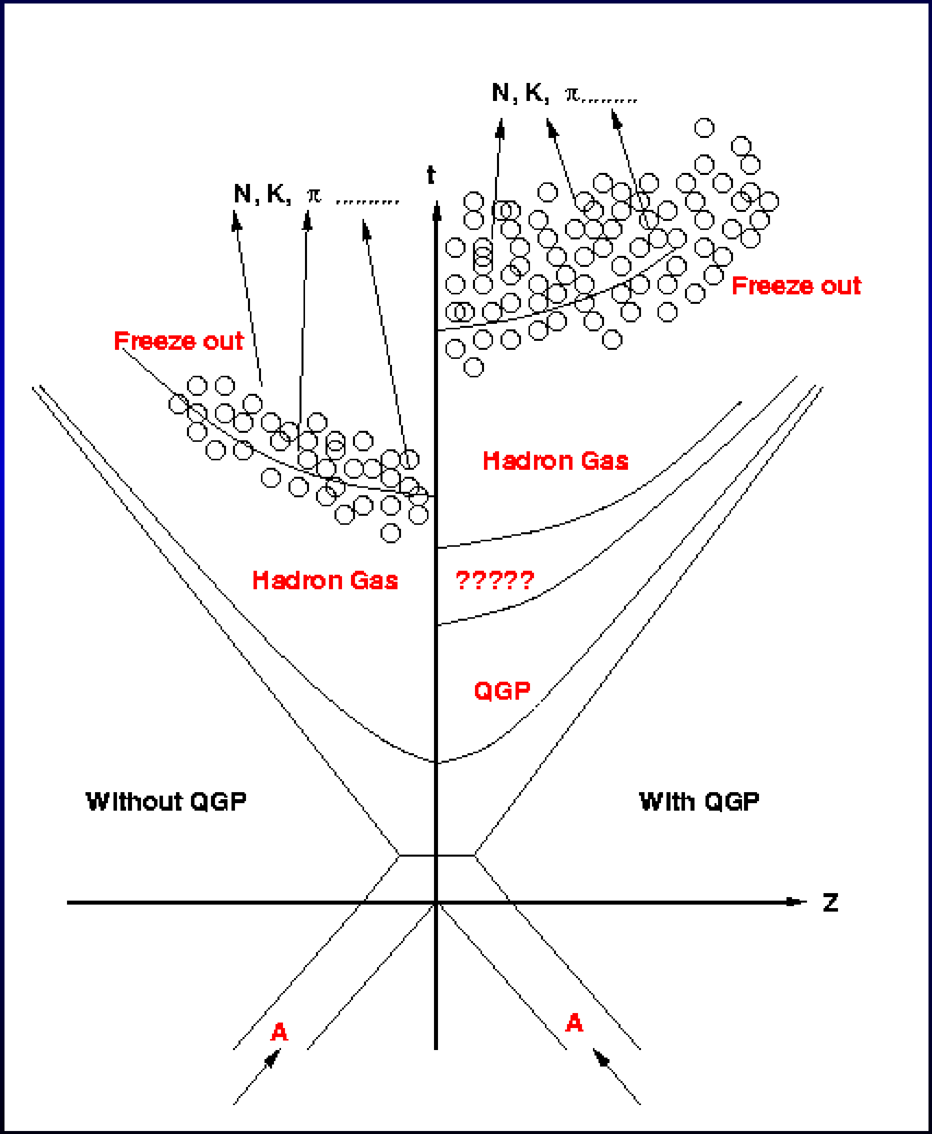
Search for a 1st order QCD PT

QGP has more degrees of freedom than pion gas

Entropy should be conserved during fireball evolution

Hence:
Look in *hadronic* phase for signs of:
- **Large size,**
- **Large lifetime,**
- **Softest point of EOS**

possible signal of a 1st order phase trans.



But are the correlation data Gaussian?

1 dimensional correlations:

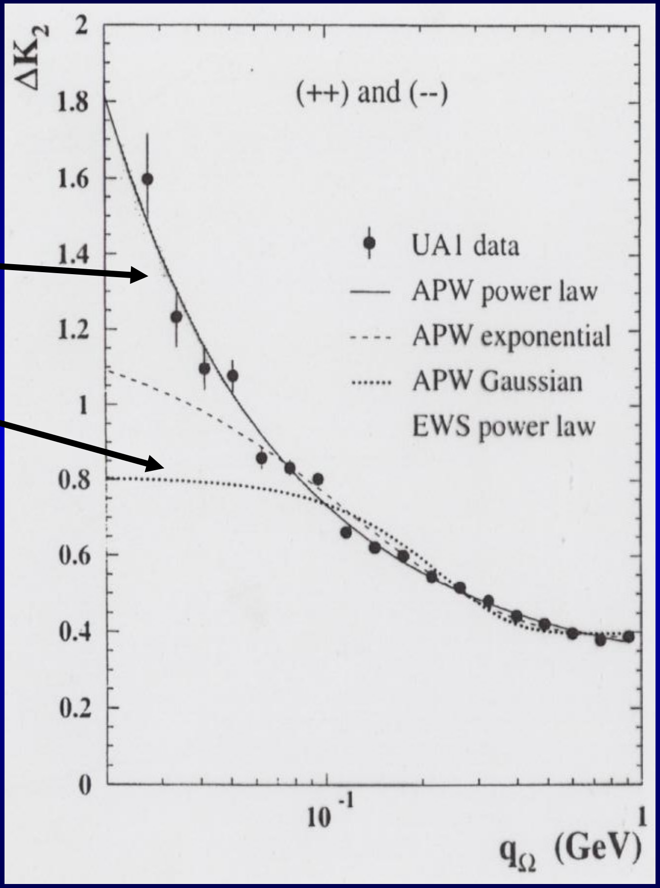
typically more peaked than a Gaussian

if a Gaussian fit does not describe the data

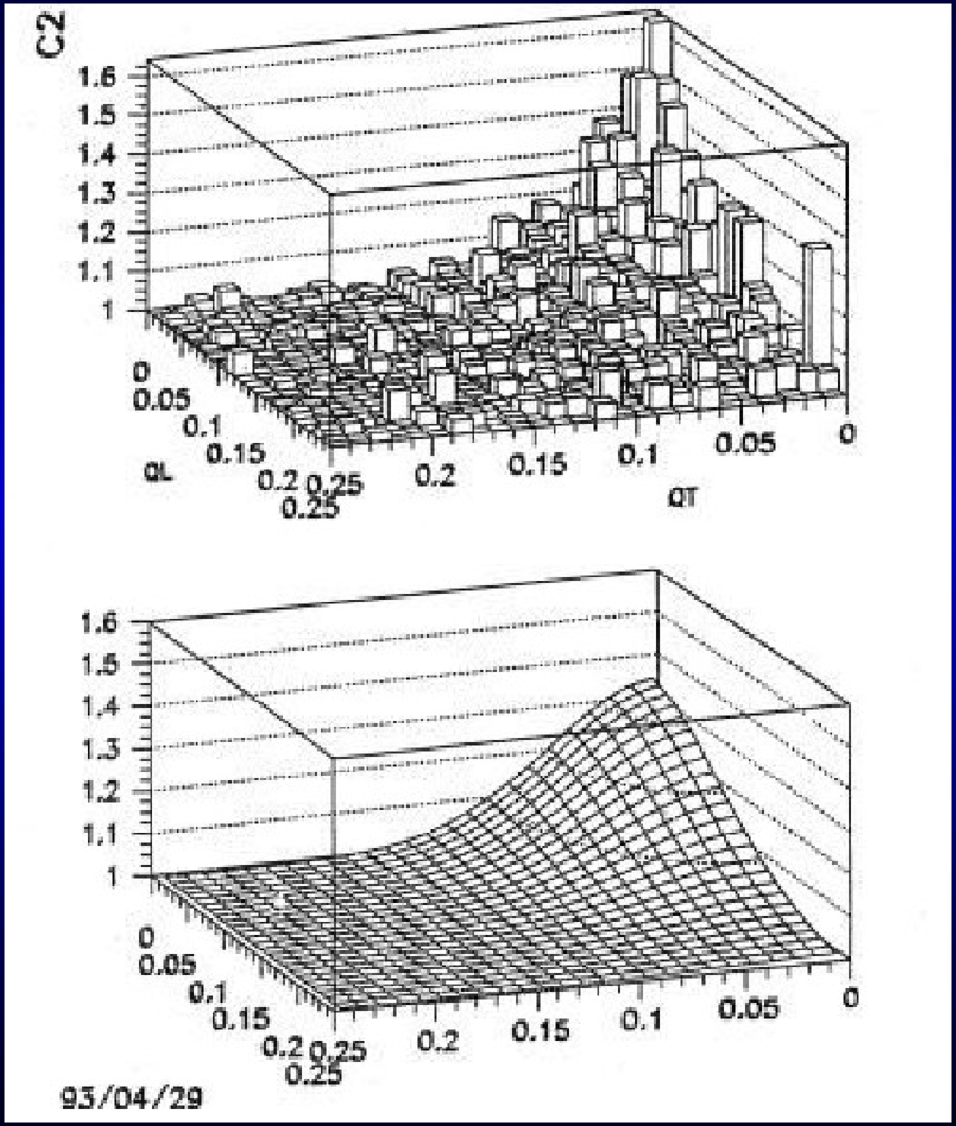
then have the parameters any meaning?

Example: like sign correlation data of the UA1 collaboration

p + pbar @ E_{cms} = 630 GeV



Non-Gaussians, 2d E802 Si+Au data

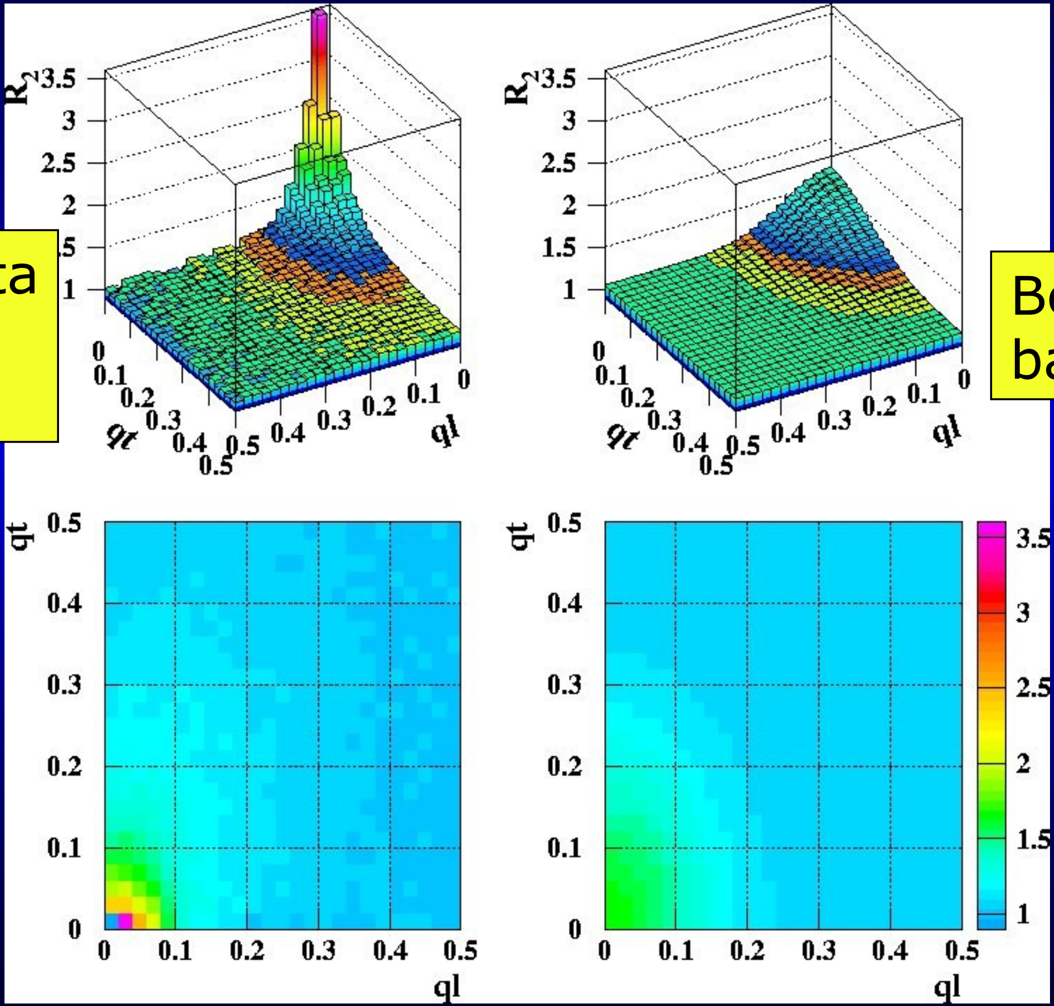


E802 Si+Au data,
sqrt(s_{NN}) = 5.4 GeV
Z. Fodor, S.D. Katz:
T_E=162±2 MeV,
μ_E = 360±40 MeV
(hep-lat/0402006
systematics: this meeting

Best Gaussian:
bad shape

Non-Gaussian structures, 2d, UA1 data

UA1 ($p+\bar{p}$) data
B. Buschbeck
PLB (2006)



Best Gaussian
bad shape

Femptoscopy signal of sudden hadronization

Buda-Lund hydro:
RHIC data
follow the
predicted
(1994-96)
scaling of HBT radii

T. Cs, L.P. Csernai
hep-ph/9406365
T. Cs, B. Lörstad
hep-ph/9509213

Hadrons with $T>T_c$:
1st order PT excluded
hint of a cross-over
M. Csanád, T. Cs, B. Lörstad and A. Ster,
nucl-th/0403074