Facets of the QCD Phase Diagram

- The “Perfect Fluid”
  (with J. Liao)

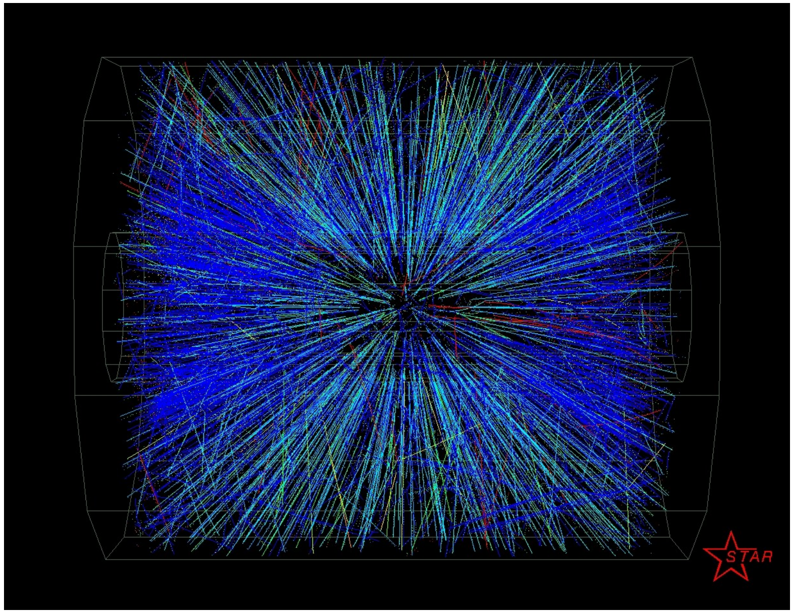
- Have we seen local parity violation?
  (with A.Bzdak and J. Liao)

Part 1, The “Perfect” Fluid

Based on:
J. Liao and V.K, arXiv:0909.3105, Phys.Rev.C80:034904,2009.

The Perfect Liquid?

VS



J. Liao and V.K, arXiv:0909.3105, Phys.Rev.C80:034904,2009.

“Minimum” viscosity

AdS/CFT correspondence:

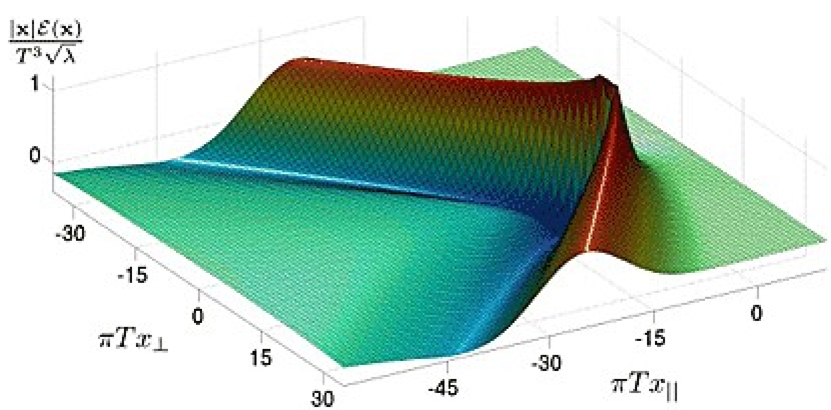
Maldacena et al, hep-th/9905111v3
Kovtun, Son, hep-th/0405231v2

$$\frac{\eta}{s} \geqslant \frac{1}{4\pi}$$

$\eta = \textit{shear viscosity}$

$s = \textit{entropy density}$

Holds for a large class of strongly coupled gauge theories



Kovtun, arXiv:0706.0368

Kinetic theory + waving hands:

$$\eta \sim n \bar{v} m \lambda$$

$$\bar{v} m = p\,, \quad p\lambda > \hbar\,, \quad n \sim s$$

$$\frac{\eta}{n} \sim \frac{\eta}{s} \geqslant 1$$

“Quantum” bound

More detailed derivation: Danielewicz and Gyulassy (85)

Viscosity

Navier Stokes Equation: $$\rho\left[\frac{\partial \vec{v}}{\partial t}+(\vec{v}\,\vec{\nabla})\,\vec{v}\right]= -\vec{\nabla}\,p \;+\; \eta\nabla^{2}\vec{v}+\ldots$$

"Inertia" "Force" "Friction"

	Viscosity [kg/m s]	Kinematic Viscosity $\nu=\frac{\eta}{\rho}$ [m^2 / s]
Water	0.001	0.10
Air	0.000018	1.5

The kinematic viscosity (friction/inertia) controls how good a fluid is

The perfect fluid?

- Is there a quantum bound on eta/s?
- Does the “quantum bound” on eta/s provide a limit on fluidity?
- Has RHIC produced such a system? I assume so...
- How about other substances
  - Water, Palinka, liquid Helium, cold quantum gases???
- How does one define fluidity?
- How do I compare systems on the atomic/molecular scale with those at quark/gluon scale?

Defining Fluidity

Hydrodynamics works for a big variety of systems:

- Liquids (Water)
- Gases (Air, sound)
- Interstellar Dust (Star formation)
- QGP ?

Problem: How to compare substances at vastly different length scales?

- Interstellar Dust: $n^{-1/3} \sim 10^{-4}$ m
- Water: $n^{-1/3} \sim 3\ 10^{-10}$ m
- Air : $n^{-1/3} \sim 3\ 10^{-9}$ m
- QGP : $n^{-1/3} < 10^{-15}$ m

Typical criterion for applicability of fluid dynamics:

Knudsen Number : $\dfrac{\textit{Mean Free Path}}{\textit{typical lengthscale of variation}}$

Kinetic theory??? (→ Mean Free Path)

Different length-scales??? (→ typical lengthscale of variation)

Obviously not what we need

Defining Fluidity

1) Extract "effective mean free path" solely from fluid-dynamics
2) Calibrate with "inter-particle" distance

Effective mean free path:

Analyze sound modes and determine minimum wavelength

$$\omega = c_s k + \frac{i}{2} k^2 \frac{\frac{4}{3}\eta}{w/c^2}$$

Damping ~ k^2:
Hydro always works
in long wavelength limit

Enthalpy density:

$$w = \epsilon + p = Ts + \mu n \approx Ts + mn$$

$w \rightarrow mn$ Non-relativistic limit: **mass** density controls inertia

$w \rightarrow Ts$ Relativistic limit: entropy density controls inertia

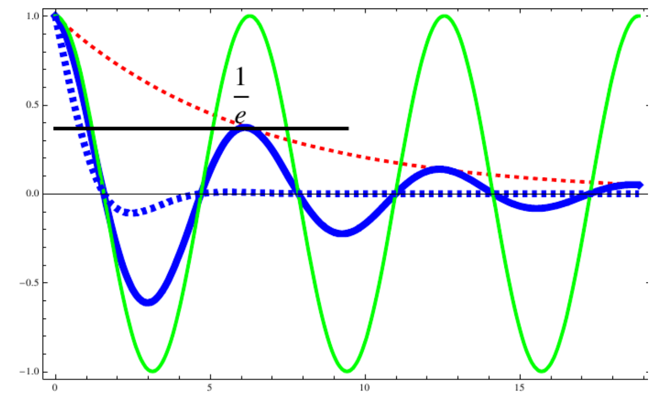
$\frac{\eta}{s}$ cannot be a universal quality measure

Fluidity measure

Effective mean free path: Analyze sound modes

$$\omega = c_s k + \frac{i}{2} k^2 \frac{\frac{4}{3}\eta}{w/c^2}$$

Require: $$\frac{|\Im(\omega)|}{|\Re(\omega)|} \equiv \frac{L_\eta}{\Lambda} \ll 1$$



Provides a minimal wavelength $\Lambda = L_\eta$

Dilute (kinetic limit): $L_\eta \to \lambda_{mfp}$

$$L_\eta = \frac{\eta}{w\, c_s}$$

Enthalpy density

$$w = \epsilon + p = Ts + \mu n \approx Ts + m n$$

Fluidity measure

$$L_\eta = \frac{\eta}{w\, c_s}$$

Calibrate with "inter-particle distance" d:

$$d \Leftrightarrow \langle \epsilon(x)\epsilon(0) \rangle$$

Non-relativistc systems $d = n^{(-1/3)}$

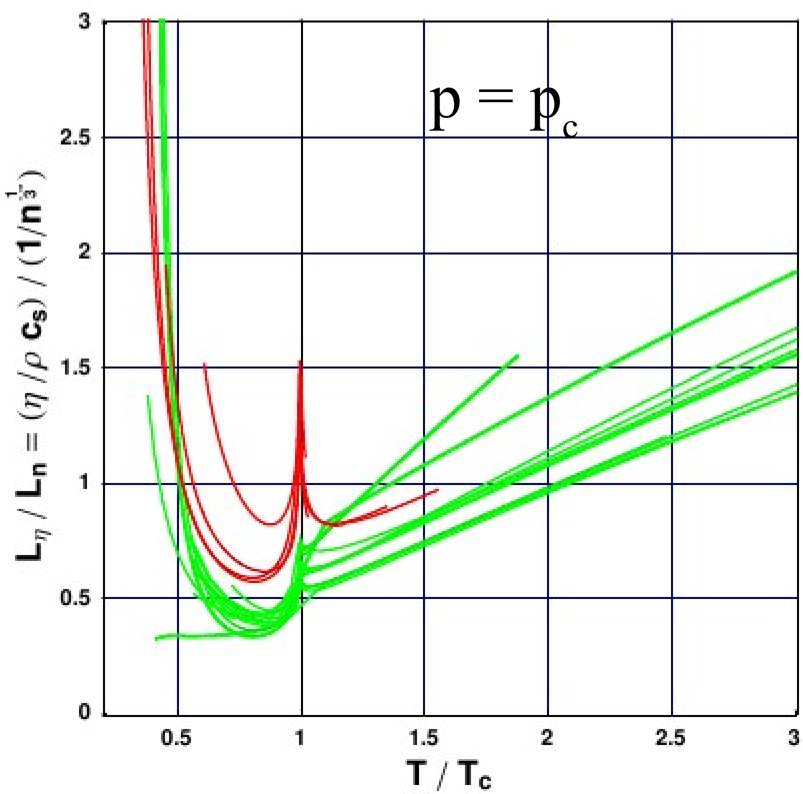
Fluidity measure:

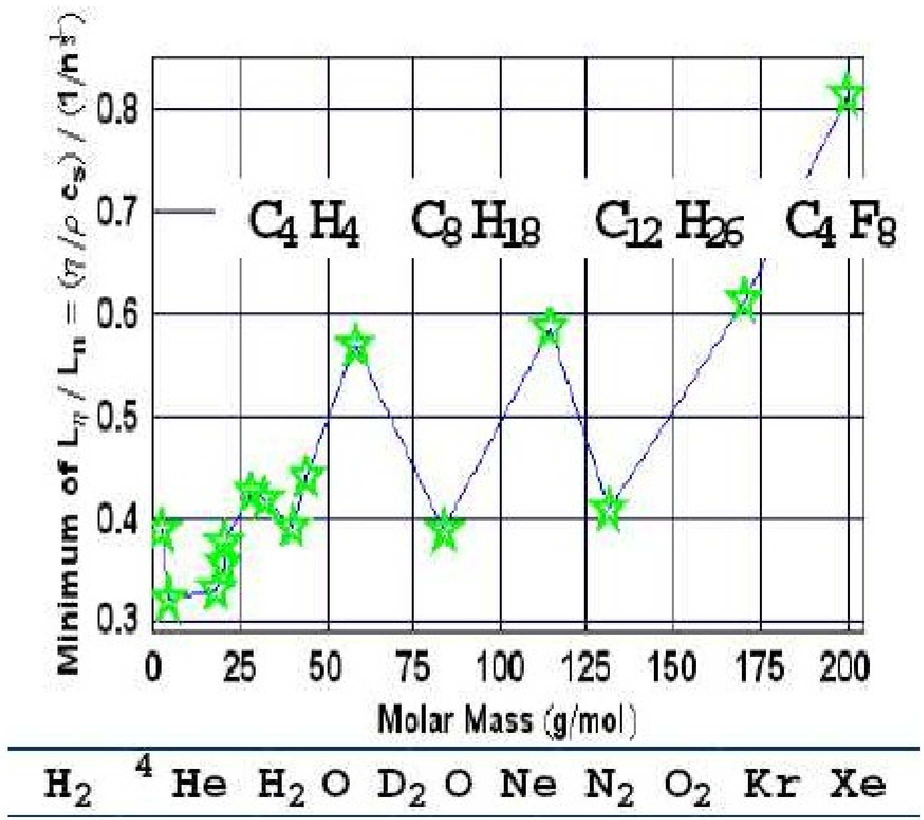
$$F = \frac{L_\eta}{d} = \frac{\eta}{w\, c_s}\frac{1}{d} = \frac{\eta}{w\, c_s} n^{1/3}$$

Depends only on *intrinsic* properties of substance
Well defined: NO kinetic theory needed!

Fluidity measure

16 substances with M_{mol}, T_c, p_c spanning 2 Orders of Magnitude

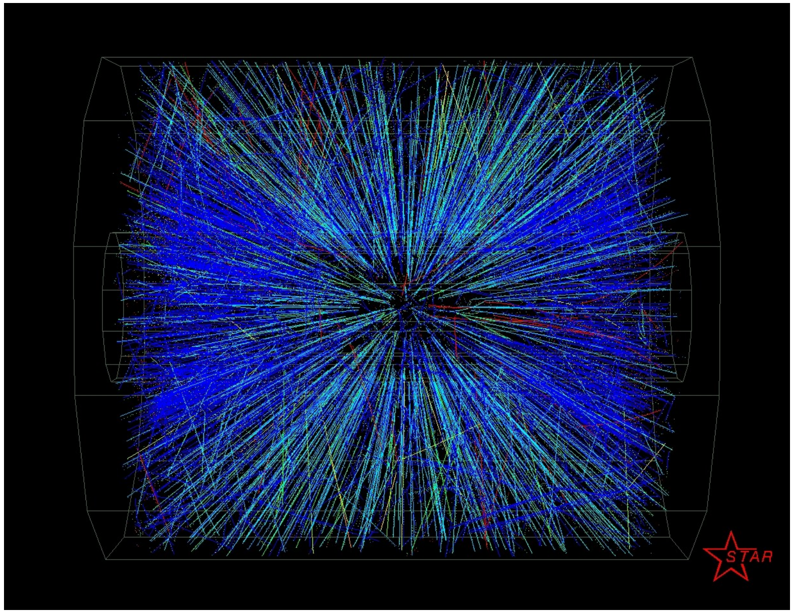




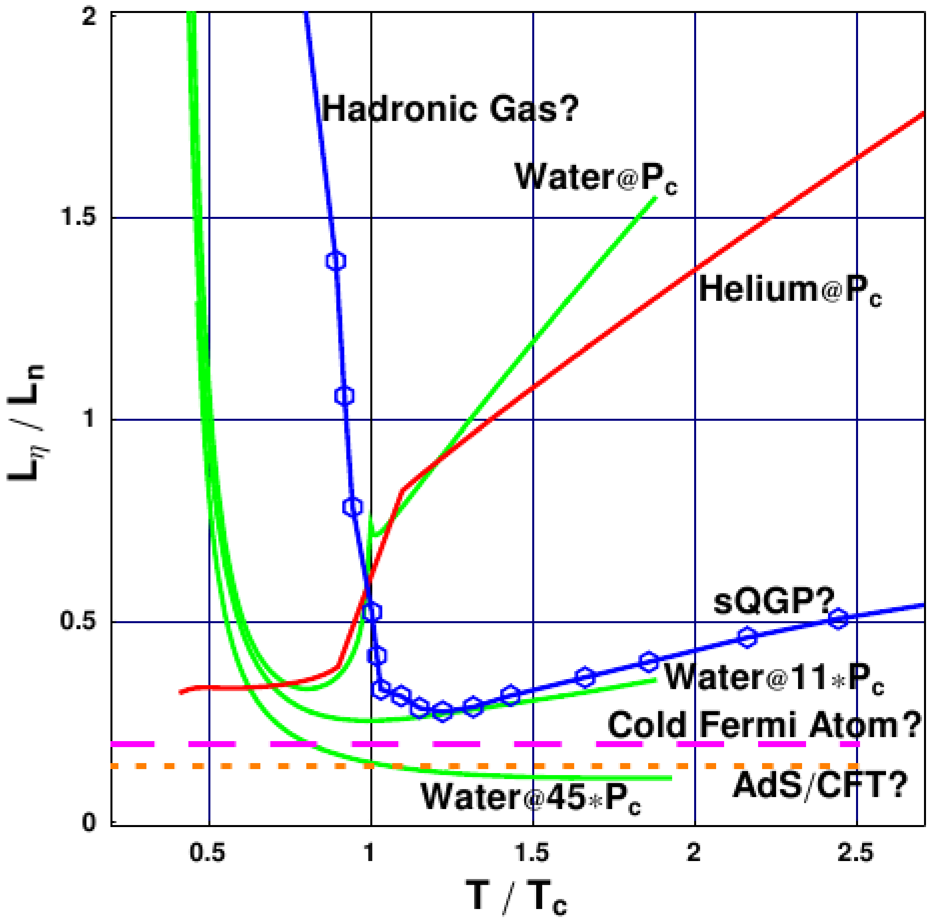
A good fluid is a good fluid!!!!!

So who is the winner?

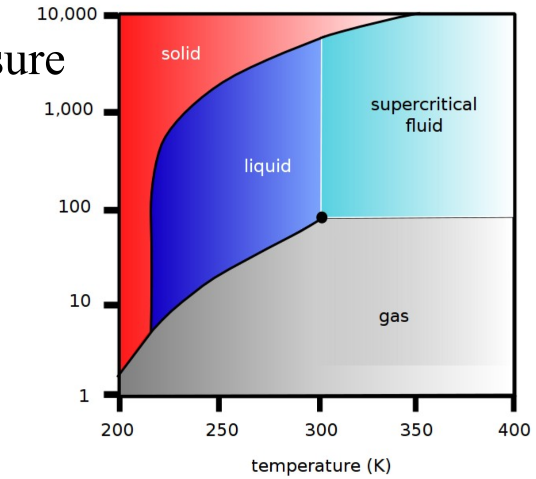
VS



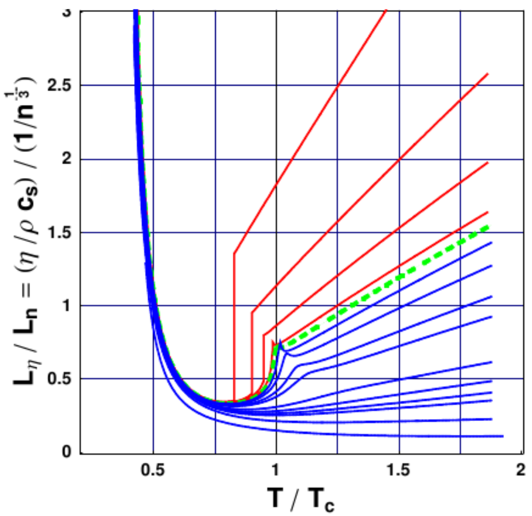
None of the above
Super-critical fluids!!!



pressure



CO_2



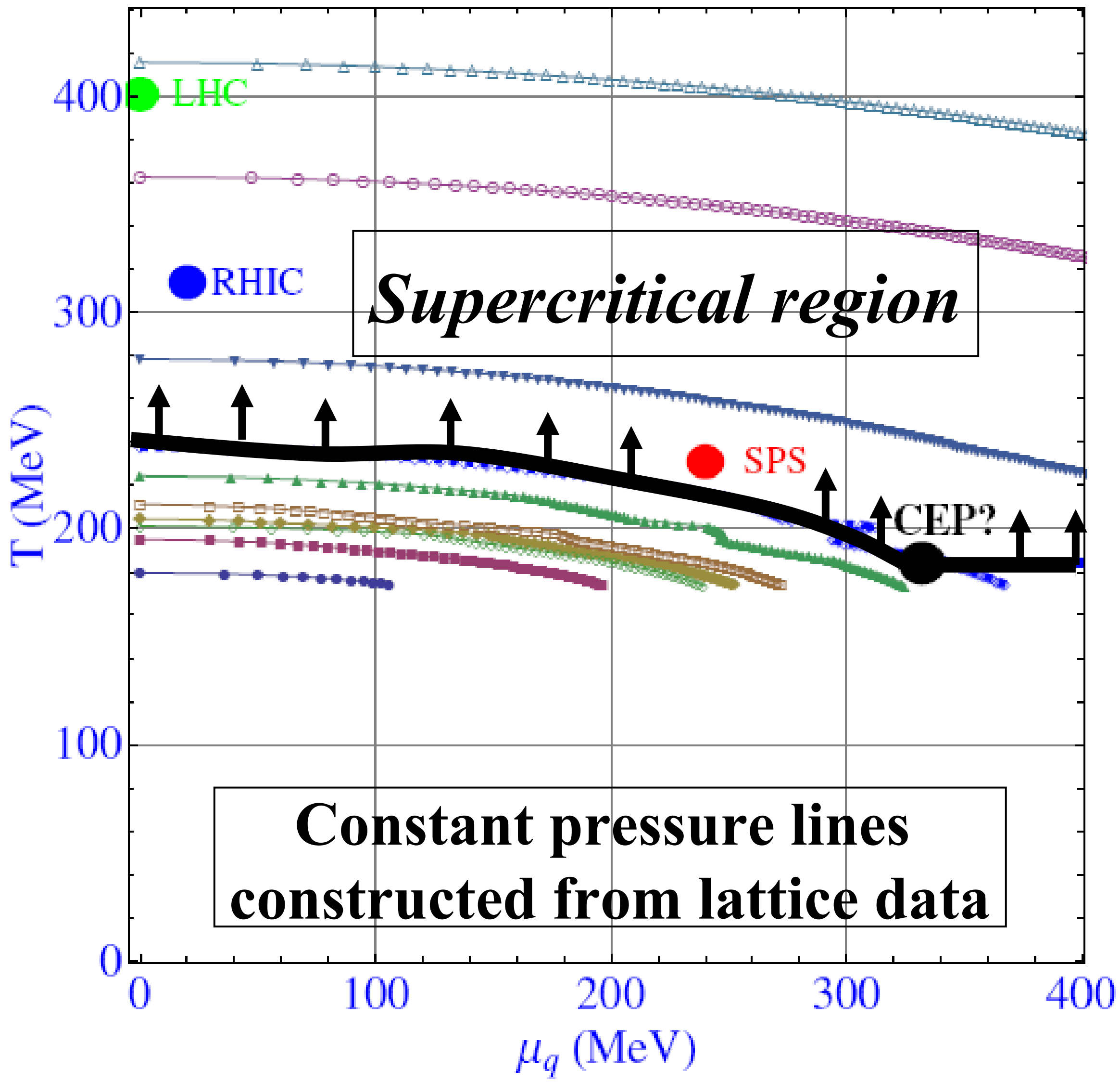
Water

Used in dry cleaning, decaffeinating coffee, ...

An the winner is...

Consequences for the QGP????

RHIC and the Dry-Cleaner

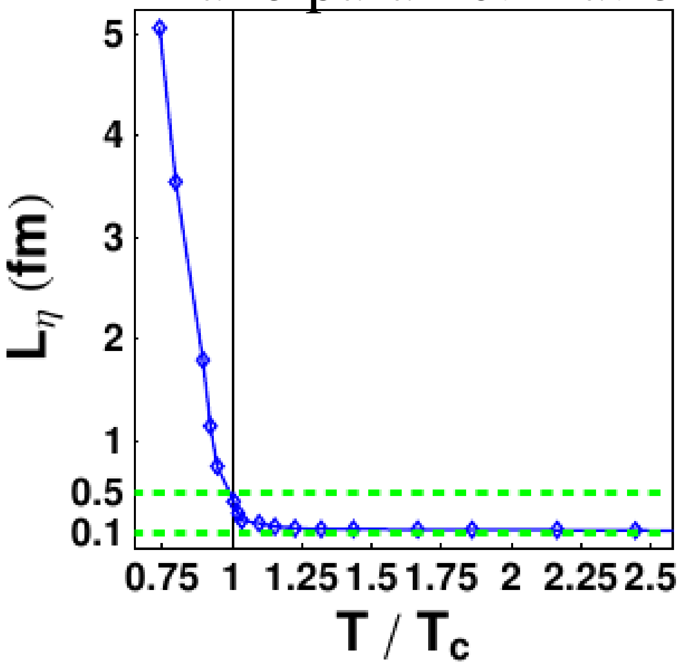


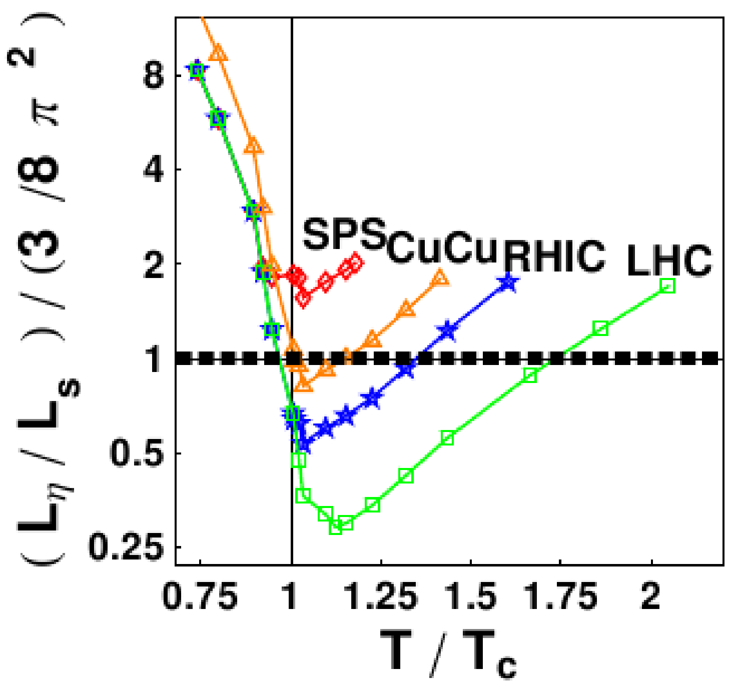
If there is a QCD critical point RHIC-QGP would be in Super critical region

Predict: even better hydro behavior at LHC

Hydro Performance

Based on Gyulassy
Hirano parametrization





Simple exansion:

$$L_s = 2\tau \text{ for } \tau < R$$
$$L_s = 2R \text{ for } \tau > R$$

Summary Part 1

- A good fluid is a good fluid
- QGP nothing special
- eta/s only meaningful for relativistic fluids without phase transition
- Supercritical fluids win the race
- QGP may be a supercritical fluid
    - Predict better hydro description at LHC

Part 2:
Have we seen Local Parity Violation at RHIC?

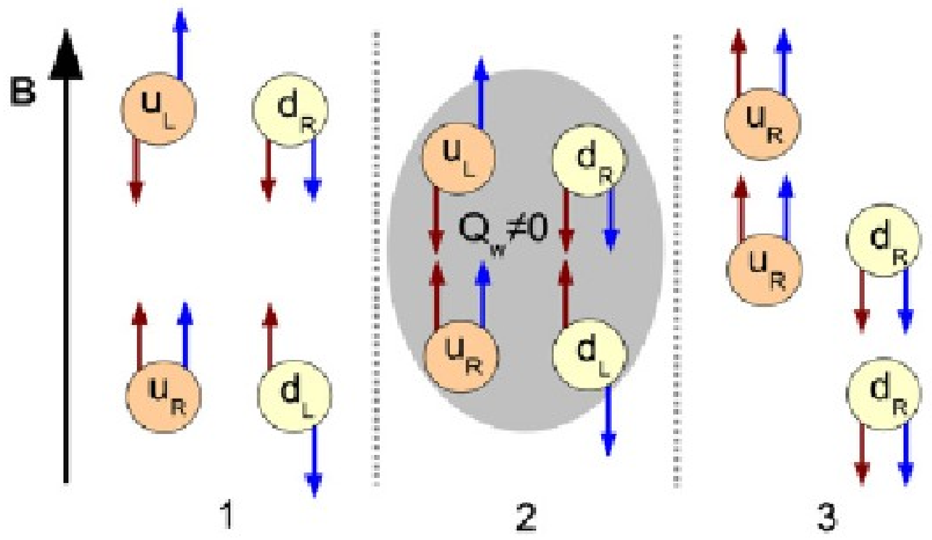
Topology vs. Trigonometry

Based on: Adam Bzdak, VK and Jinfeng Liao, PRC81 031901(R) (2010), [arXiv:0912.5050]

J. Liao, VK, and A. Bzdak, arXiv:1005.5380 (alternative observable)
A. Bzdak,VK, and J. Liao, in preparation (transverse momentum conservation)

The CME @ RHIC

Kharzeev, McLerran, Warringa (08)



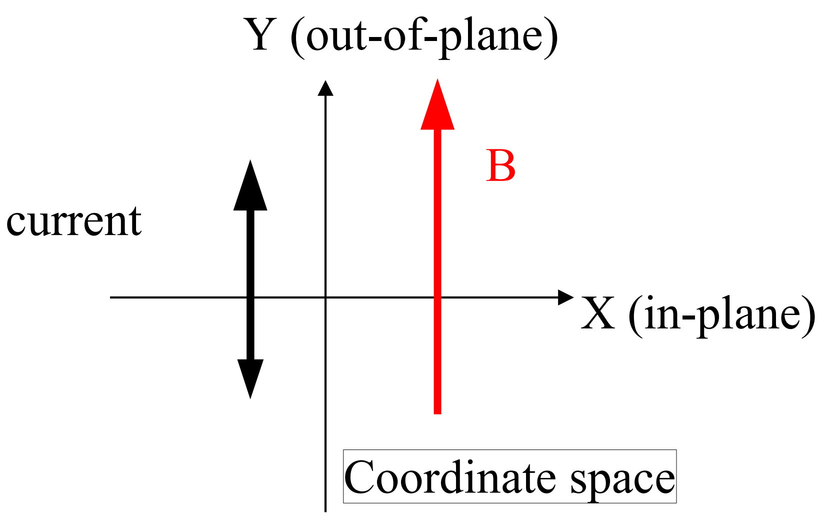
$$\vec{j} \propto (N_L - N_R)\,\vec{B} \propto Q\,\vec{B}$$

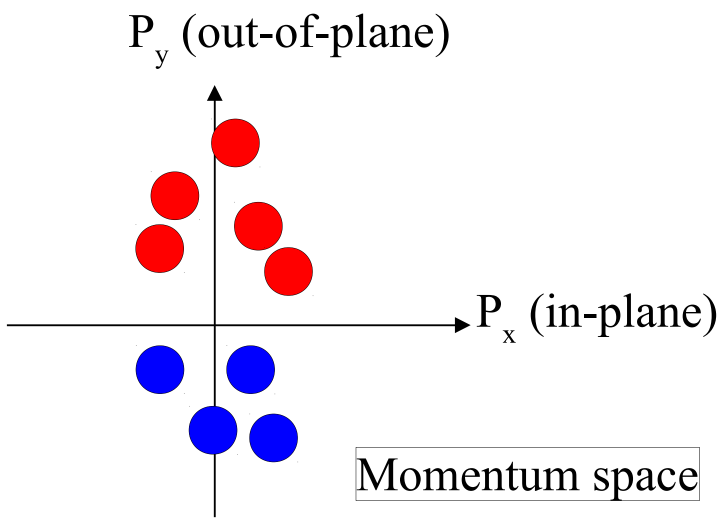
Q:
Theta vacua; sphaleron transition;
Flipping → ***exp. difficulty***

B:
Non-central; strong at very early time;
Out-of-plane → ***exp. difficulty***

J. Liao, BNL workshop, April 2010

The basic observable





<u>Charge Separation</u> *or*
<u>Electric Dipole in Pt Space</u>
(along out-of-plane direction)

Complications:

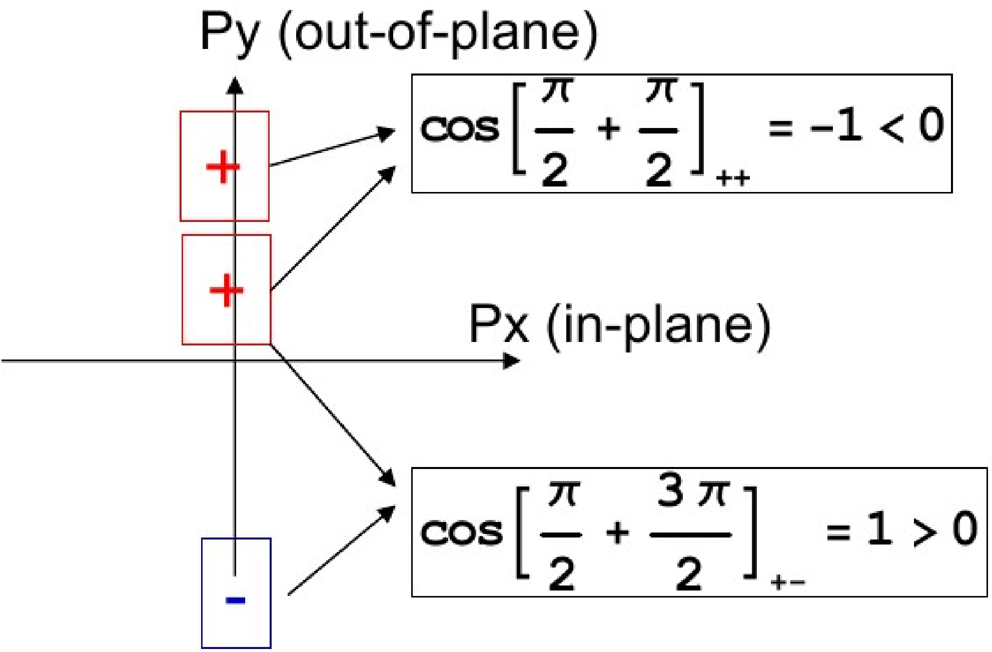
- hard to identify direction of magnetic field (reaction plane)P. E-by-E
- Direction of dipole either parallel **OR** anti-parallel to magnetic field

→ **only *<u>variance</u>* of parity-odd operator can be observed**

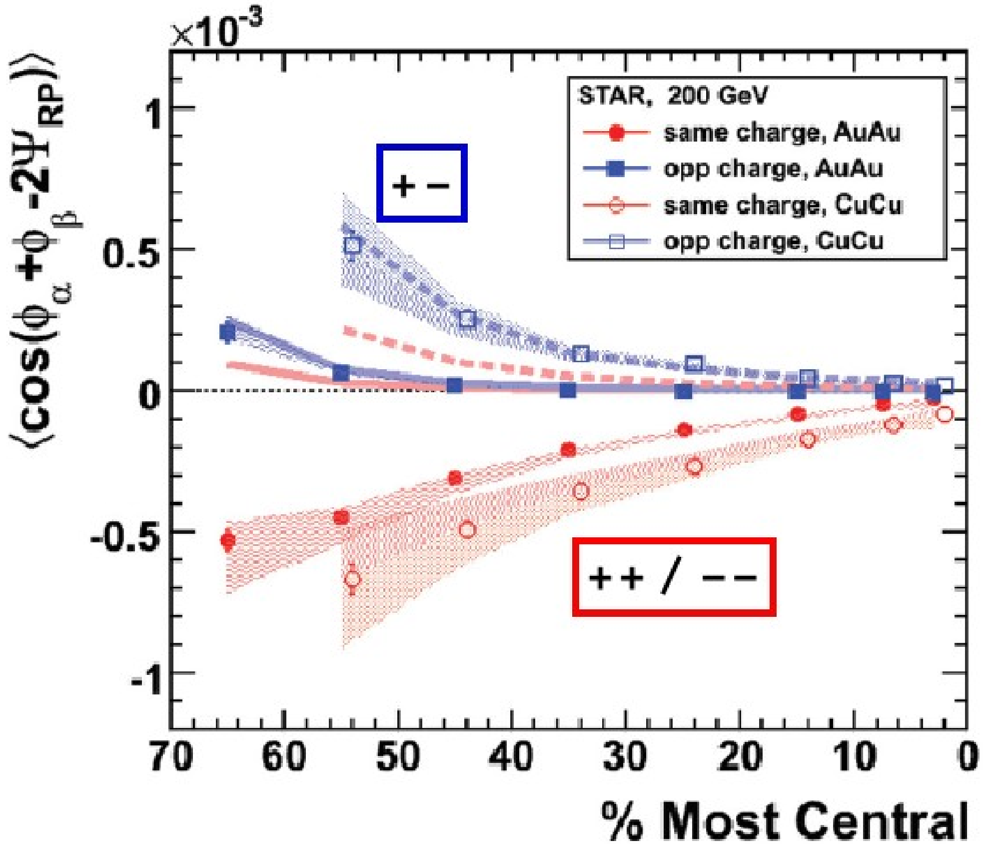
The STAR measurement
(which everybody discusses)

3P correlations: *Voloshin (04)*

$$\langle\cos(\phi_a + \phi_\beta - 2\Psi_{RP})\rangle = \langle\cos(\phi_a + \phi_\beta - 2\phi_c)\rangle / v_{2,c}.$$



CME expectation

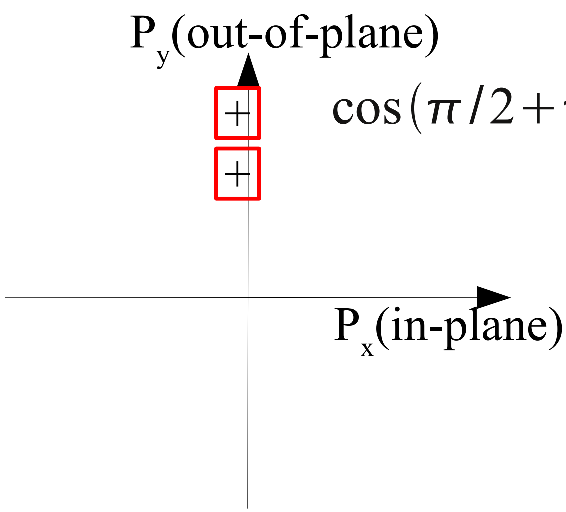


STAR (09)

The STAR measurement

(a closer look)

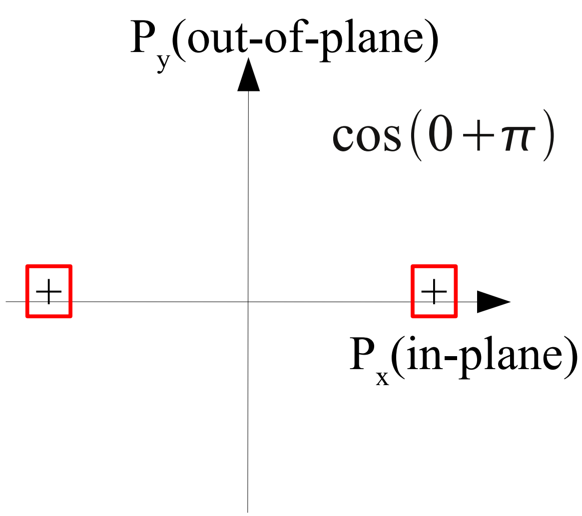
Concentrate on same sign pairs for the moment



$\cos(\pi/2+\pi/2)$

Set $\Psi_{R.P.}=0$

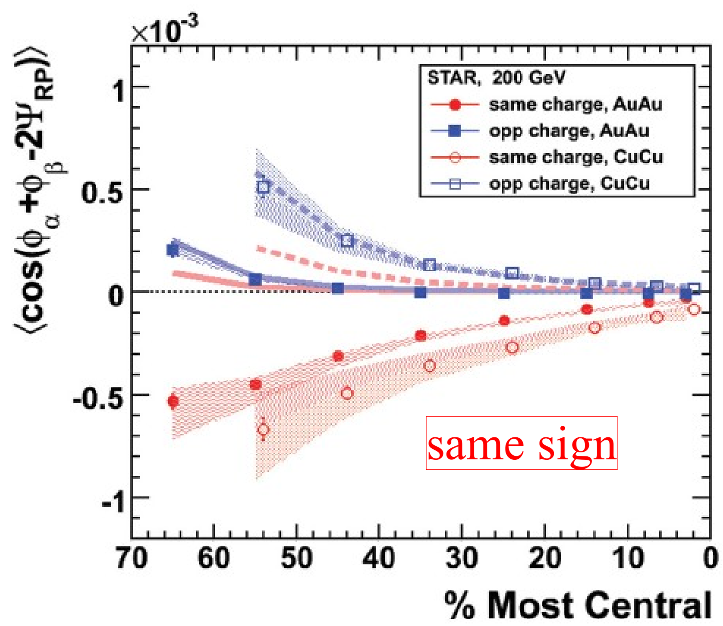
$$\langle\cos(\phi_1+\phi_2-\Psi_{R.P})\rangle_{++}=\langle\cos(\phi_1+\phi_2)\rangle_{++}$$



$\cos(0+\pi)$

$$\langle\cos(\phi_1+\phi_2-2\Psi_{R.P.})\rangle_{++}<0$$

for **both** configurations

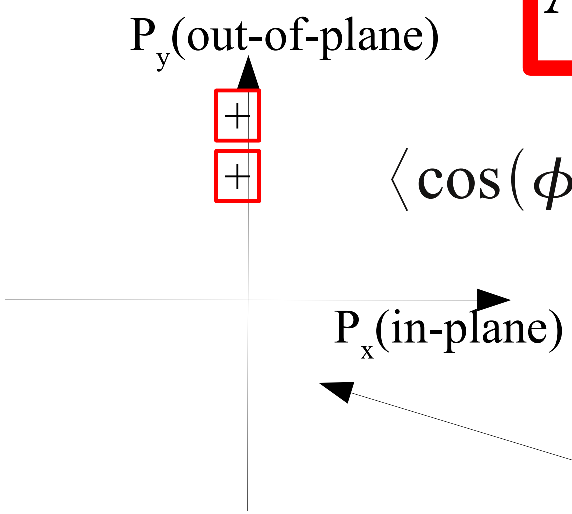


How to distinguish?

The STAR measurement

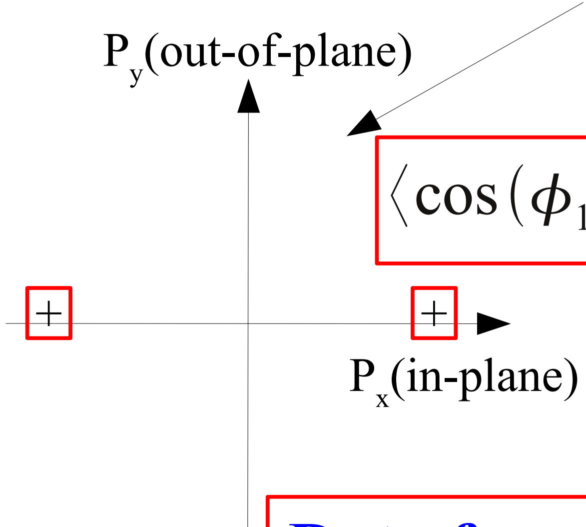
(which not so many discuss)

Add $\langle \cos(\phi_1 - \phi_2) \rangle$ to the mix

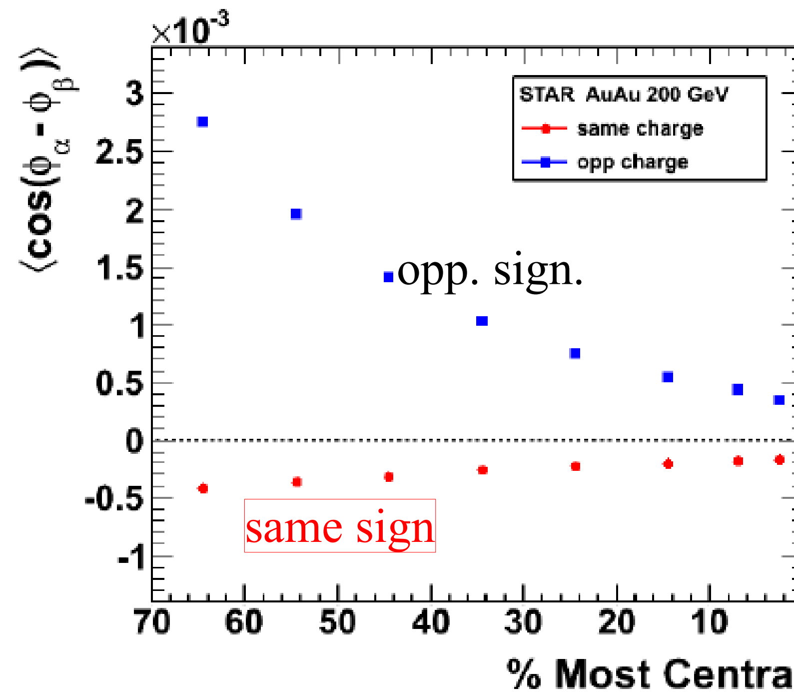


$$\langle \cos(\phi_1 - \phi_2) \rangle_{++} > 0$$

$$\langle \cos(\phi_1 + \phi_2 - 2\Psi_{R.P.}) \rangle_{++} < 0$$



$$\langle \cos(\phi_1 - \phi_2) \rangle_{++} < 0$$

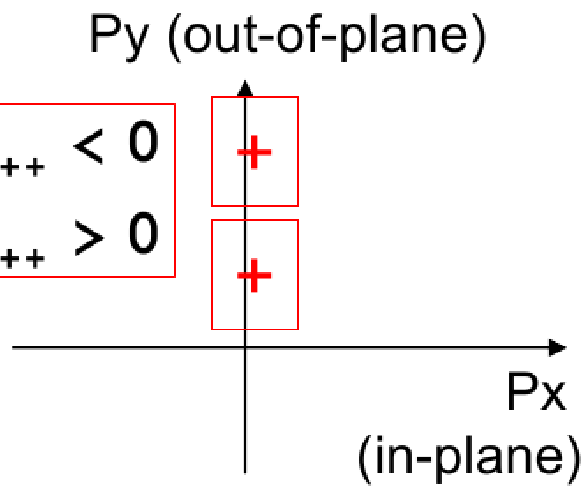


Data favor in-plane back-to-back correlation

In-plane v.s. Out-of-plane

$\cos\left[\phi_\alpha + \phi_\beta\right]_{++} < 0$

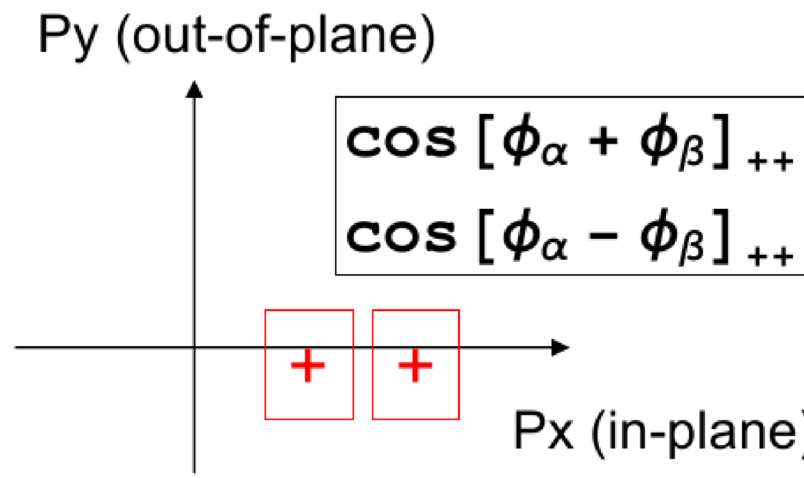
$\cos\left[\phi_\alpha - \phi_\beta\right]_{++} > 0$



CME

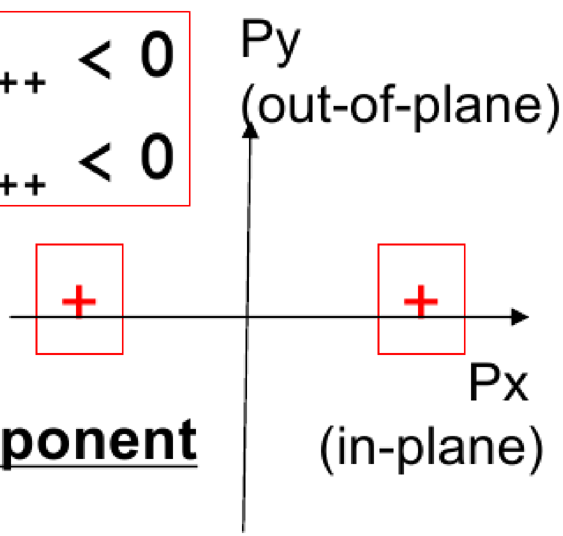
$\cos\left[\phi_\alpha + \phi_\beta\right]_{++} > 0$

$\cos\left[\phi_\alpha - \phi_\beta\right]_{++} > 0$



$\cos\left[\phi_\alpha + \phi_\beta\right]_{++} < 0$

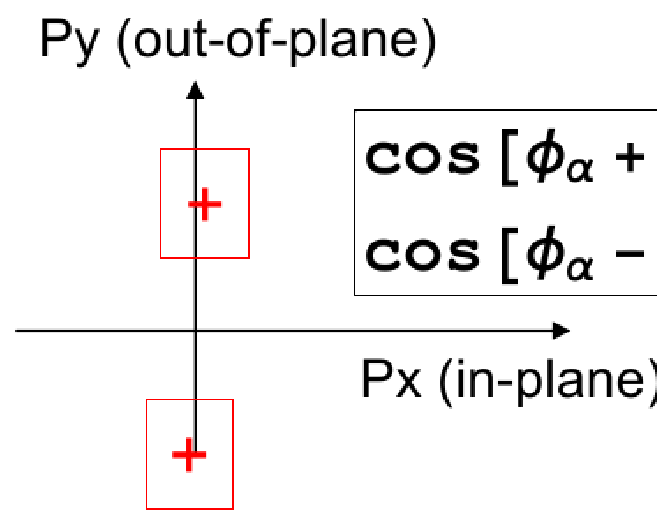
$\cos\left[\phi_\alpha - \phi_\beta\right]_{++} < 0$



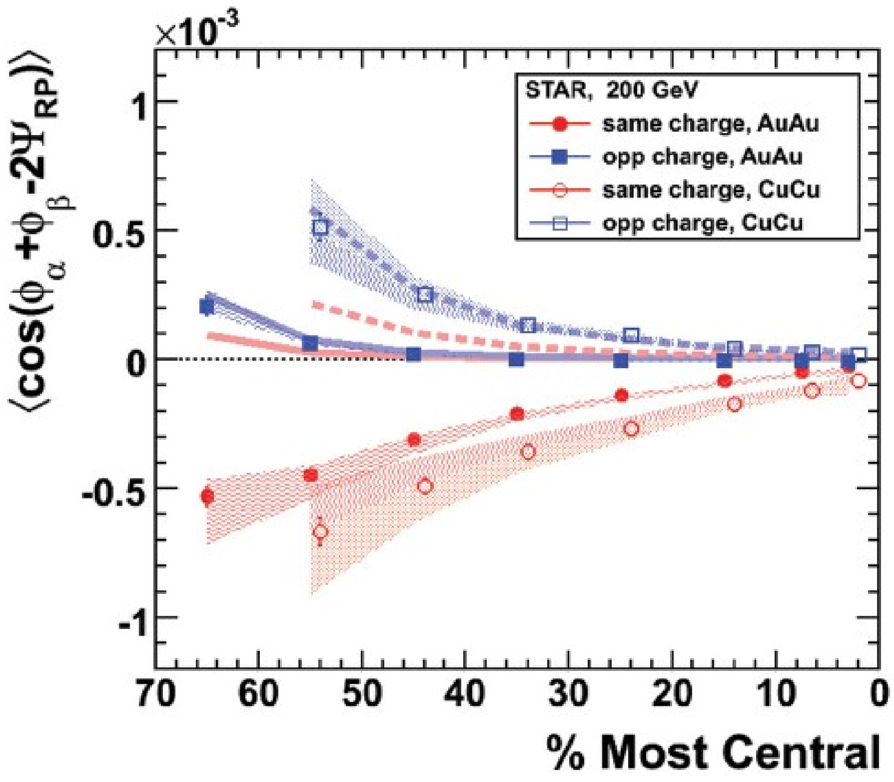
Dominant component seen in DATA

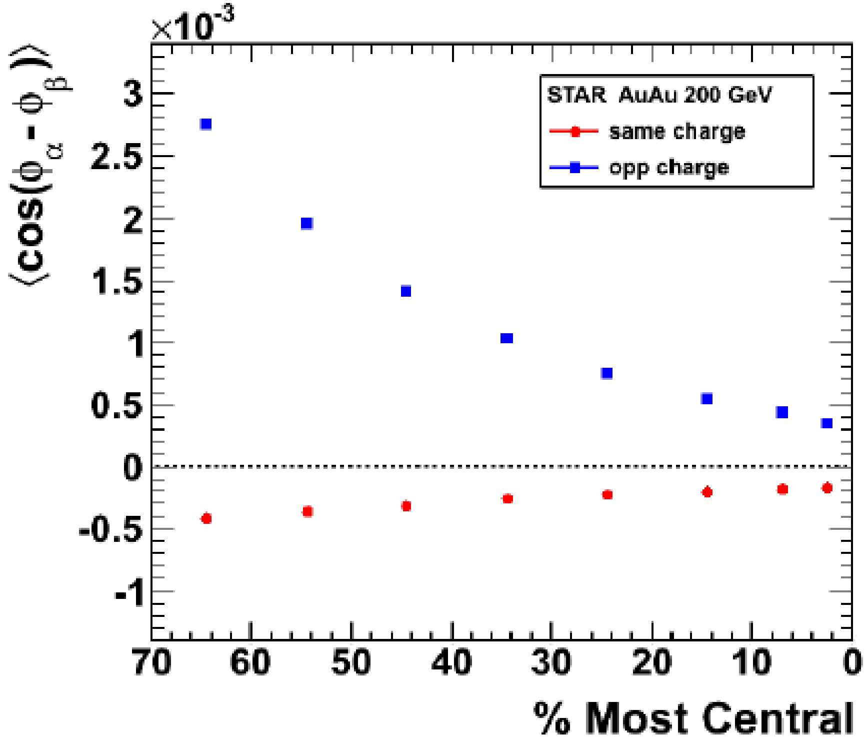
$\cos\left[\phi_\alpha + \phi_\beta\right]_{++} > 0$

$\cos\left[\phi_\alpha - \phi_\beta\right]_{++} < 0$



The bottom line





$$\langle\cos(\phi_1-\phi_2)\rangle_{++}\approx\langle\cos(\phi_1+\phi_2)\rangle_{++}$$

Using simple math

$$\cos(\phi_1+\phi_2)=\cos(\phi_1)\cos(\phi_2)-\sin(\phi_1)\sin(\phi_2)$$
$$\cos(\phi_1-\phi_2)=\cos(\phi_1)\cos(\phi_2)+\sin(\phi_1)\sin(\phi_2)$$

STAR measures for same sign pairs in Au+Au:

$$\langle\cos(\phi_1+\phi_2)\rangle_{++}\approx\langle\cos(\phi_1-\phi_2)\rangle_{++}$$

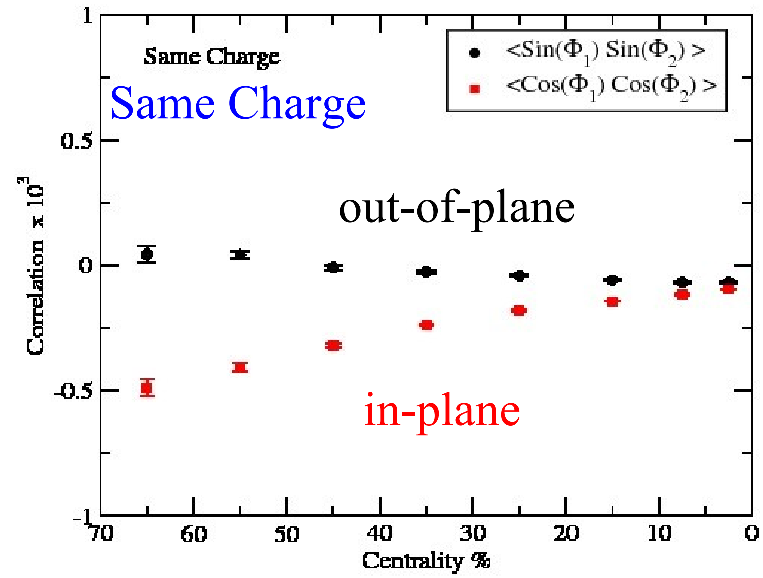
Therefore:

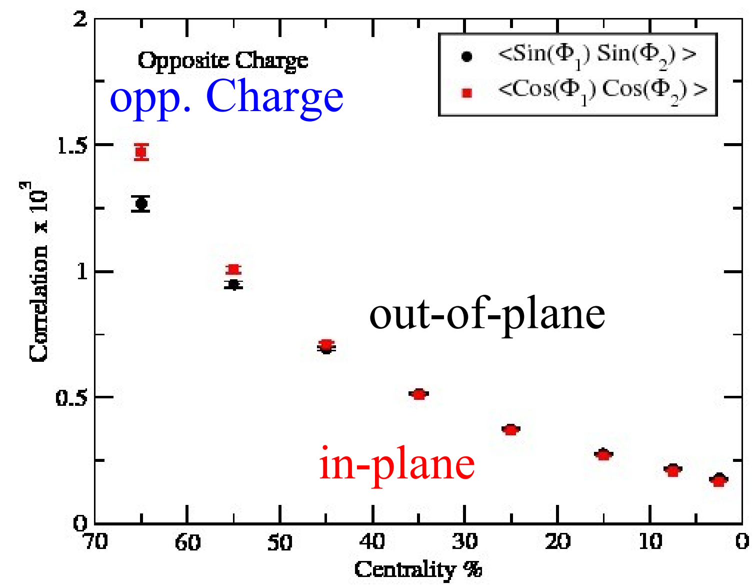
$$\langle\sin(\phi_1)\sin(\phi_2)\rangle_{++}\approx 0$$

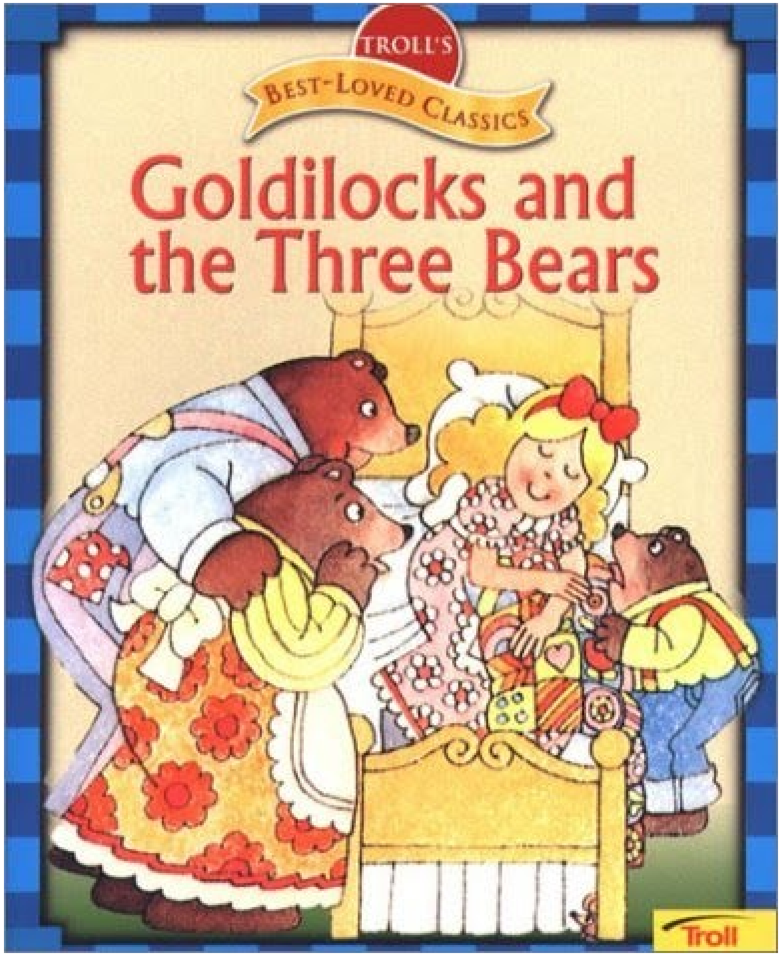
No out-of-plane correlation for *same* charge pairs

Opposite charge:

$$\langle\cos(\phi_1)\cos(\phi_2)\rangle_{+-}\approx\langle\sin(\phi_1)\sin(\phi_2)\rangle_{+-}>0$$







Saving the chiral magnetic effect...

The argument in the STAR papers:

B_{in}, B_{out} = Background (in and out of plane)
P = Parity violating signal

$$\langle \sin(\phi_1)\sin(\phi_2)\rangle_{++} = P + B_{out}$$

$$\langle \cos(\phi_1)\cos(\phi_2)\rangle_{++} = B_{in}$$

$$B_{out} \approx B_{in}$$

Thus:

$$\langle \cos(\phi_1+\phi_2)\rangle_{++} = \langle \cos(\phi_1)\cos(\phi_2) - \sin(\phi_1)\sin(\phi_2)\rangle_{++} \approx P$$

The data show

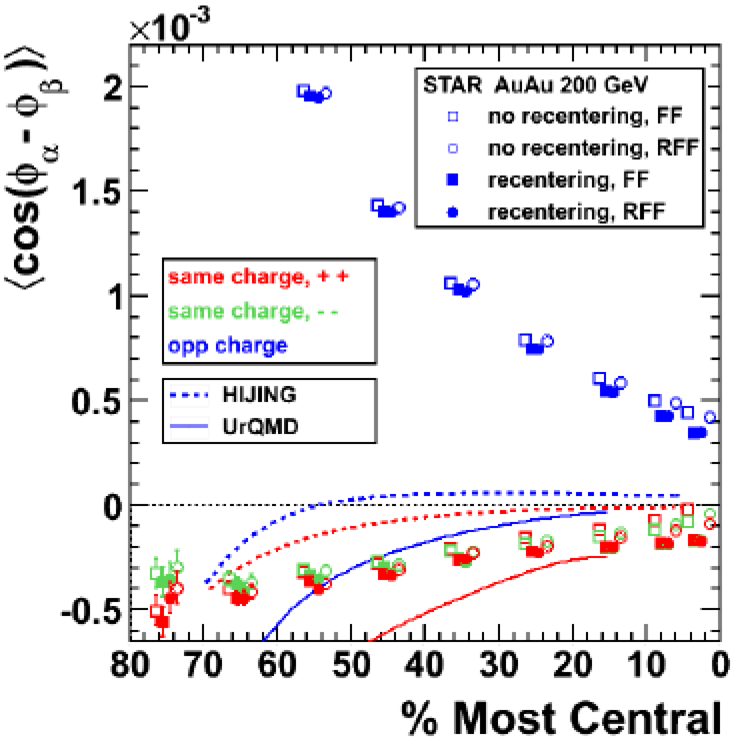
$$\langle \sin(\phi_1)\sin(\phi_2)\rangle_{++} \approx 0$$

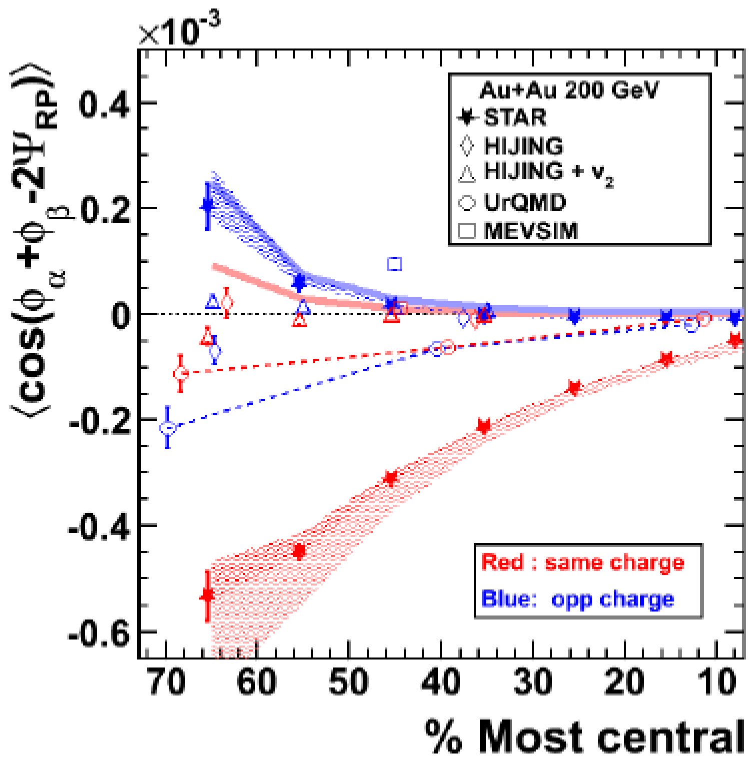
Thus existence of CME would require:

$$B_{out} \approx -P$$

- "Juuuuust right scenario" a.k.a fine tuning !!!
- We need to understand the background
- We need *differential* information on $\langle \cos(\phi_1-\phi_2)\rangle_{++}$

Do we understand the background





NO!

How realistic is the assumption $B_{\text{out}} \approx B_{\text{in}}$?

Implies: "background" correlations are *independent* of reaction plane

Two particle density:

$$\rho_2(\phi_1, \phi_2) = \rho_1(\phi_1)\rho_1(\phi_2)\left[1 + C_2(\phi_1, \phi_2)\right]$$

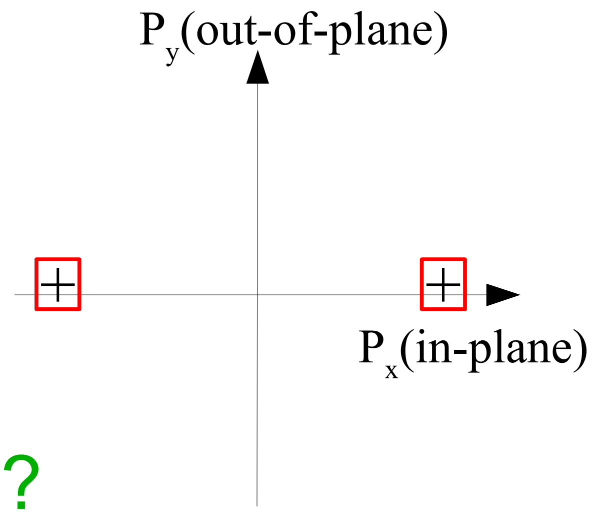
$$\rho_1(\phi) \propto 1 + 2 v_2 \cos(2\phi)$$

Reaction plane dependence **always** enters via v_2!
Almost ANY two particle correlation function C_2 contributes to $\langle \cos(\phi_1 + \phi_2) \rangle$

Sources discussed in context of CME:
Clusters (STAR, F. Wang), Resonances (STAR), .Anomaly (Asakawa et al)...

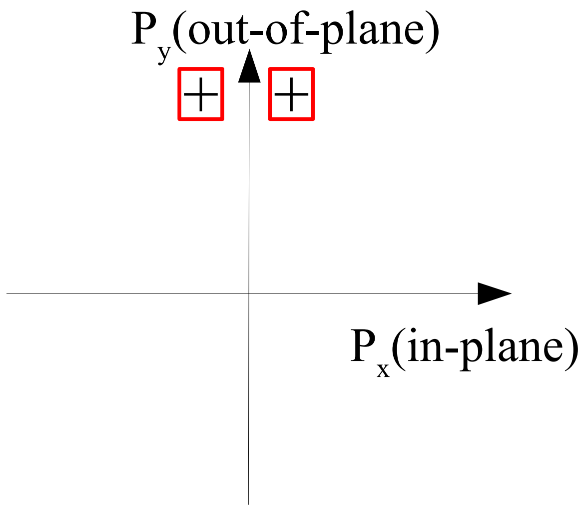
Summary, Part 2

- Data favor in-plane back-to-back correlation for same charge
  - CME predicts out of plane

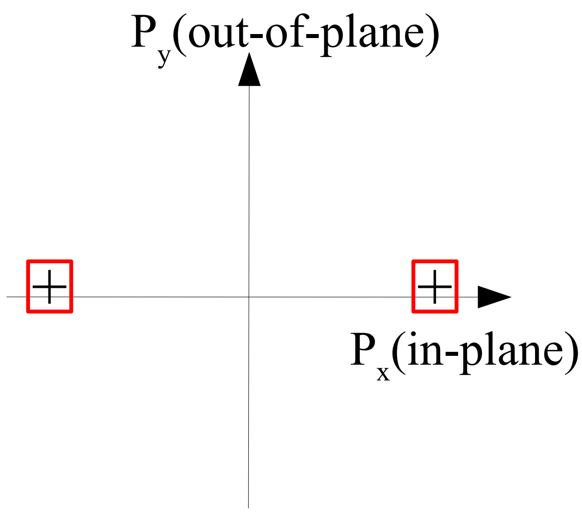


- Presence of CME requires fine-tunining: Background = - CME ?
- Need differential information on $\langle\cos(\phi_1-\phi_2)\rangle$
  - Be aware that p_t- and eta dependence may be different
- How about proton-proton????

Summary, Part 2



Local Parity violations
Predicts THIS



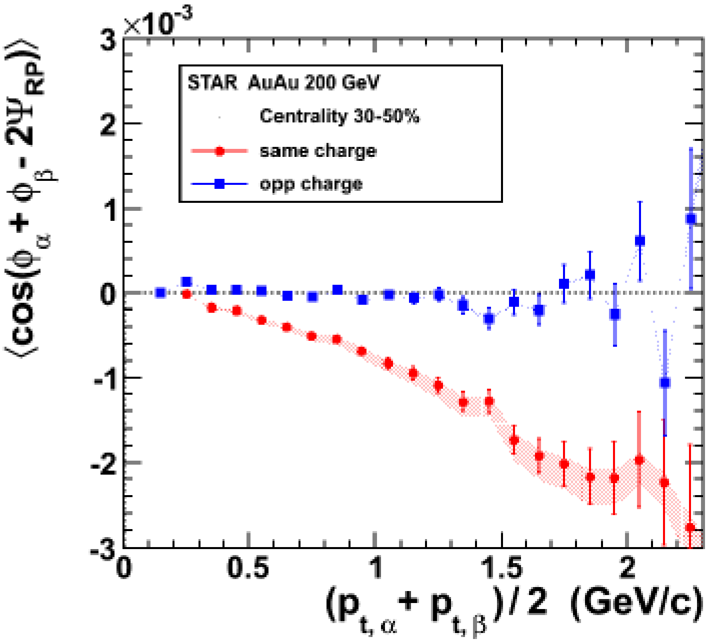
STAR measures THAT

Conclusion, Part 2

NO definitive evidence for local parity violation!

Yet !?

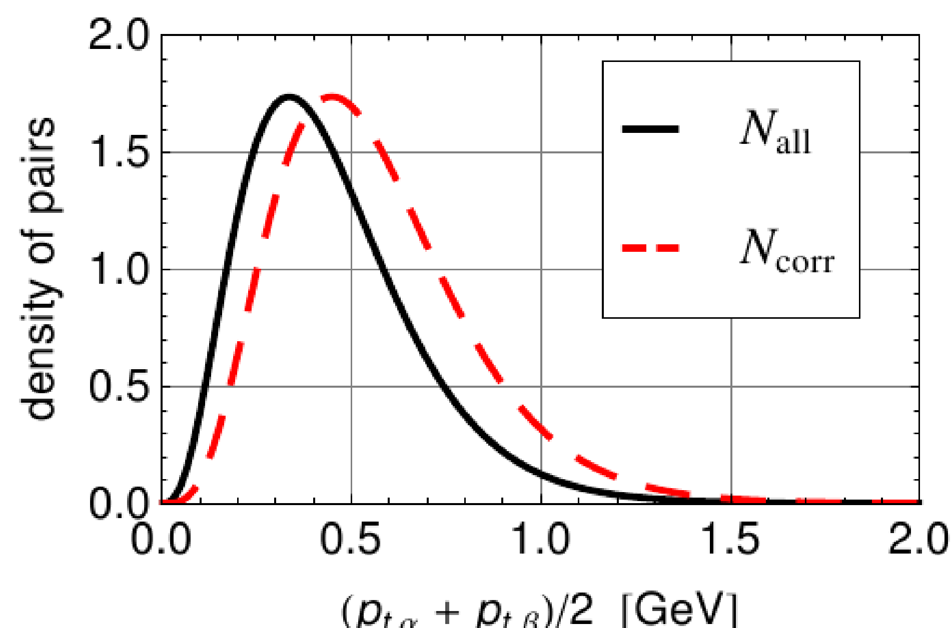
P_t-dependence



$$\langle \cos(\phi_1 + \phi_2 - \Psi_{R.P}) \rangle = \frac{N_{\text{correlated}}}{N_{\text{all}}}$$

$$N_{all}(p_+) \propto \int d^2p_{t,\alpha} d^2p_{t,\beta} \exp\left(-\frac{p_{t,\alpha}}{T}\right) \exp\left(-\frac{p_{t,\beta}}{T}\right) \delta\left(2p_+ - [p_{t,\alpha} + p_{t,\beta}]\right)$$
$$\propto p_+^3 e^{-2p_+/T}$$

Correlated pairs are
only moderately harder than
thermal pairs



Alternative observable

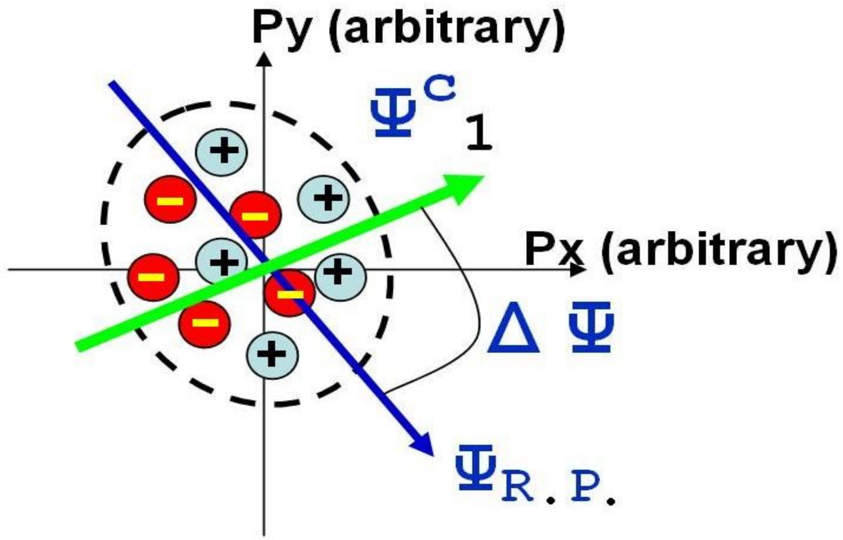
$$f(\phi, q)_{\chi} \propto 1 + 2 v_2 \cos(2\phi - \Psi_{R.P.}) + 2\chi q d_1 \cos(\phi - \Psi_C)$$

Quadrupole moment

Reaction plane angle

Charged dipole moment

Charged dipole angle



Extract dipole moment and angle

In each event : N particles, $(\eta_i, p_{t\,i}, \phi_i, q_i)$

Recall Q2 analysis for elliptic flow:
looking for the maximal quadruple

$$Q_2 \cos 2\Psi_2 = \sum_i \cos 2\phi_i$$

$$Q_2 \sin 2\Psi_2 = \sum_i \sin 2\phi_i$$

In analogy, Q^c_1 analysis:
looking for the maximal dipole

$$Q_1^c \cos \Psi_1^c = \sum_i q_i \cos \phi_i$$

$$Q_1^c \sin \Psi_1^c = \sum_i q_i \sin \phi_i$$

In each event ⇒ (Q_2, Ψ_2) & (Q^c_1, Ψ^c_1)

Relative orientation !

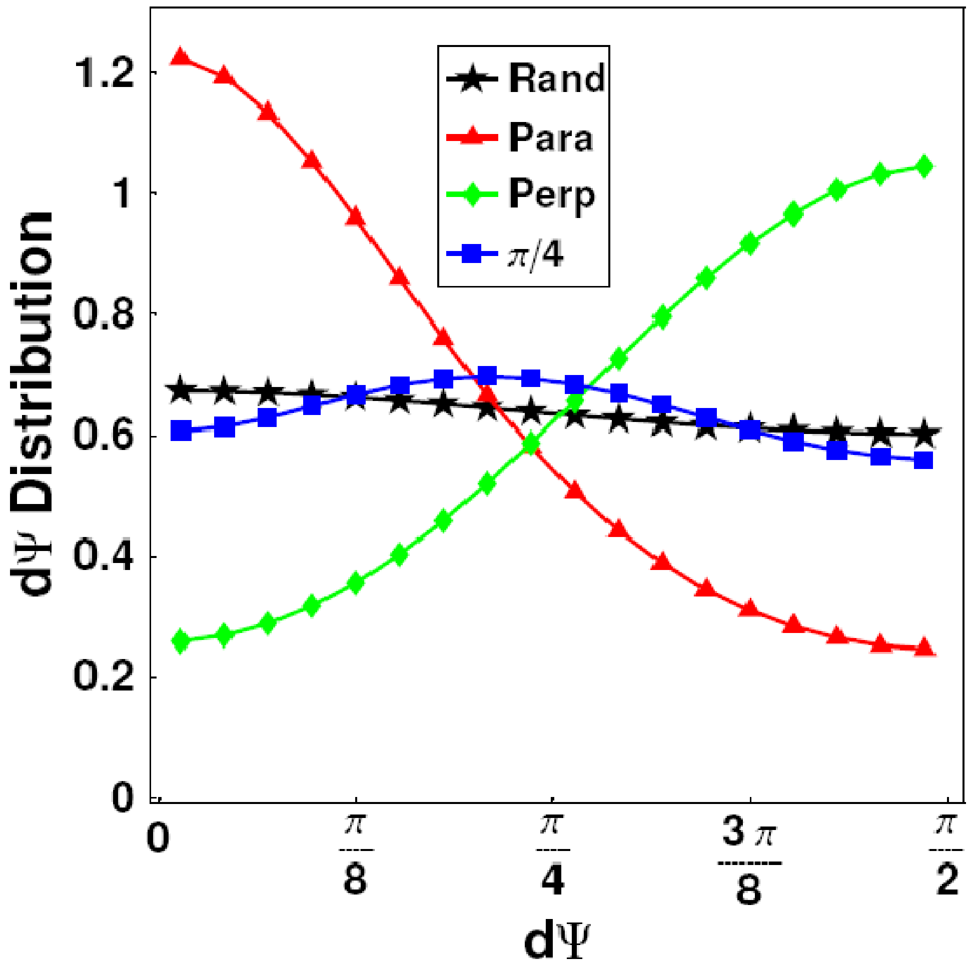
Discriminating the Angular Pattern

Azimuthal distribution: *JL, Koch, Bzdak (to appear)*

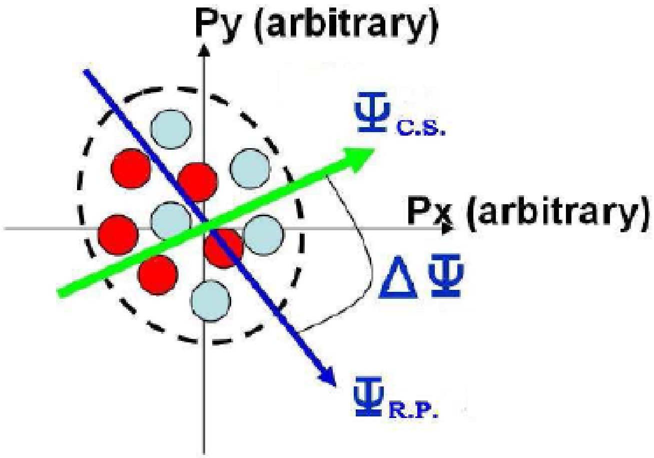
$$f_\chi(\phi, q) \propto 1 + 2\,v_2 \cos(2\phi - 2\Psi_{R.P.}) + 2\,q\,\chi\,d_1 \cos(\phi - \Psi_{C.S.})$$

Monte Carlo sampling of events according the above with different relative orientation:
(a) Random (b) parallel
(c) perpendicular (d) 45deg.
Then applying the
joint Q2 & Q^c_1 analysis !
[details: 200 +, 200 -, v2 =0.1 , d1=0.05]

* Great discriminating power, may clarify the situation;
* Robust: surviving tests with built-in cluster type correlations.



Measure the Relative Orientation



$$Q_1^c \cos\Psi_1^c \equiv \sum_i q_i \cos\phi_i$$
$$Q_1^c \sin\Psi_1^c \equiv \sum_i q_i \sin\phi_i$$

$$Q_2 \cos 2\Psi_2 \equiv \sum_i \cos 2\phi_i$$
$$Q_2 \sin 2\Psi_2 \equiv \sum_i \sin 2\phi_i$$

in a given event

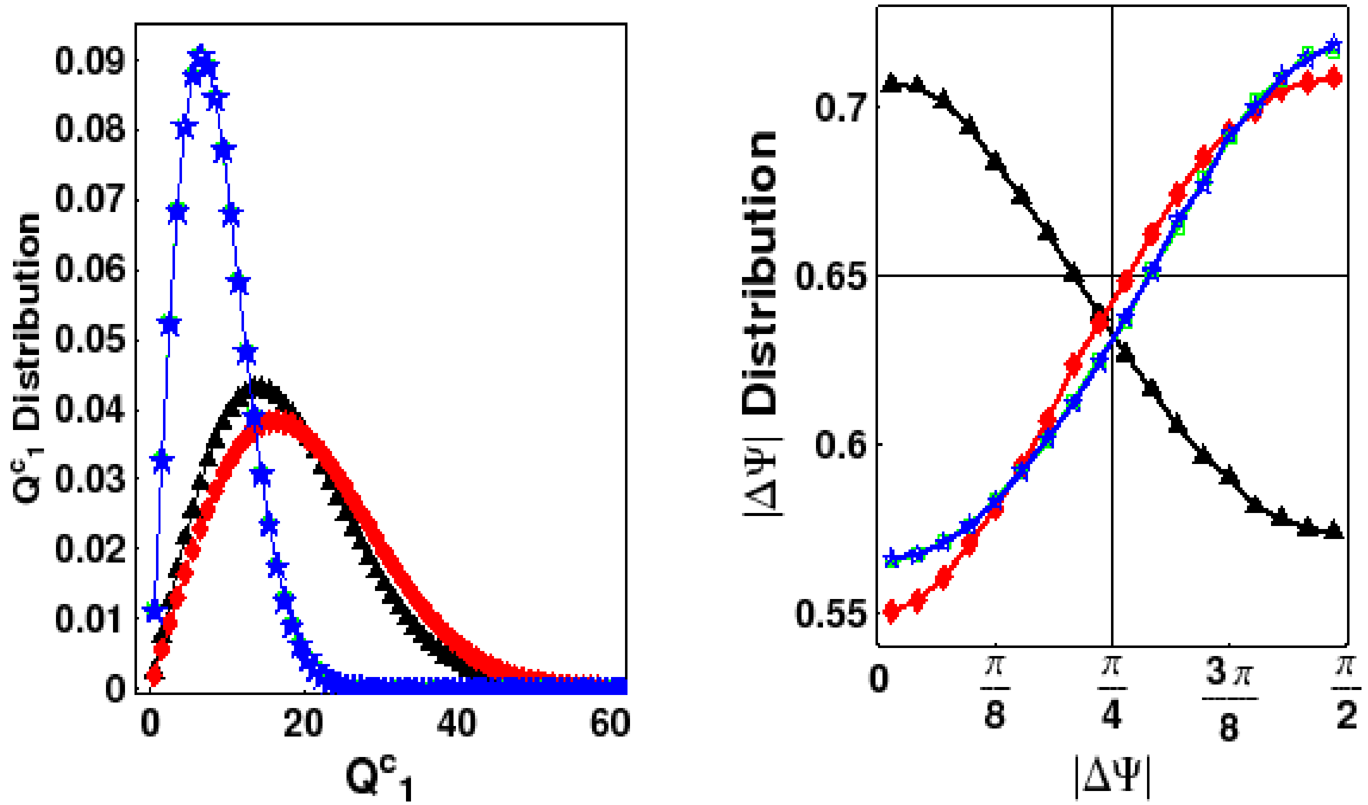
$$\cos[2*(\Psi^c_1 - \Psi_2)] = \frac{N_{ch} + 2\sum q_i q_j \cos(\phi_i - \phi_j) + \sum \cos(2\phi_i - 2\phi_j) + \sum q_i q_j \cos(\phi_i + \phi_j - 2\phi_k)}{[N_{ch} + \sum \cos(2\phi_i - 2\phi_j)]^{1/2} * [N_{ch} + \sum q_i q_j \cos(\phi_i - \phi_j)]}$$

$$< \cos[2*(\Psi^c_1 - \Psi_2)] >$$

-1 — +1

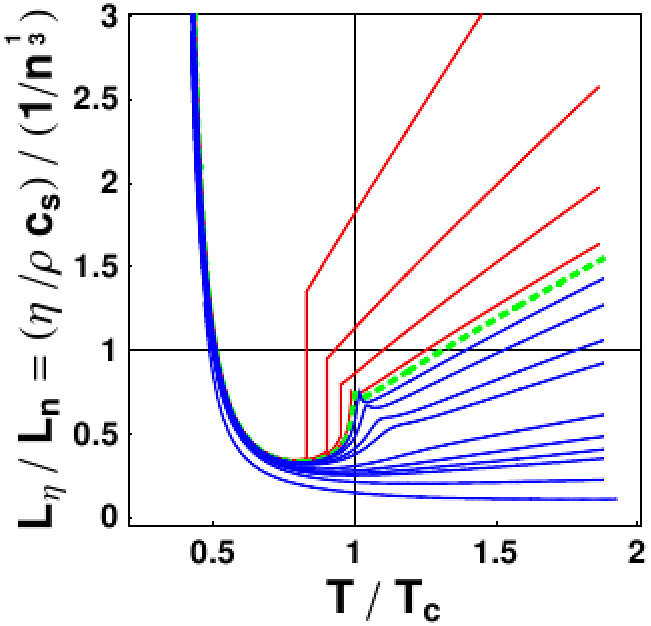
Out-of-plane — In-plane

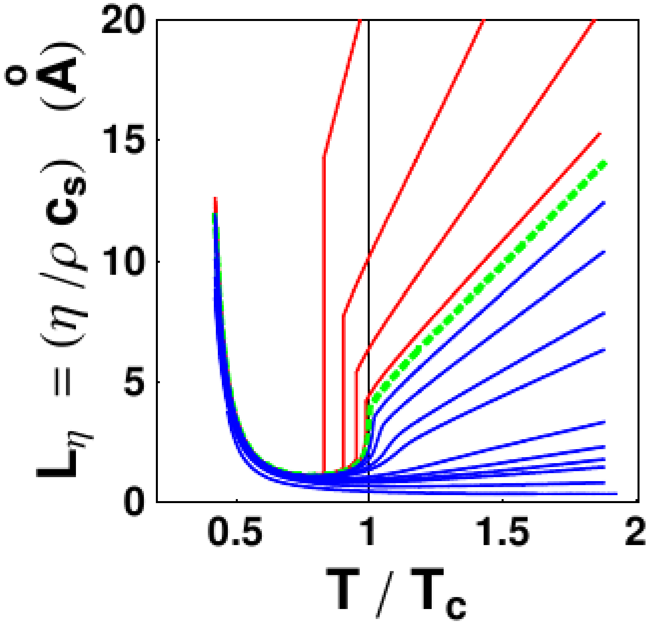
- involves all 2P,3P correlations in a complicated way
- current data can NOT tell about it: please measure !



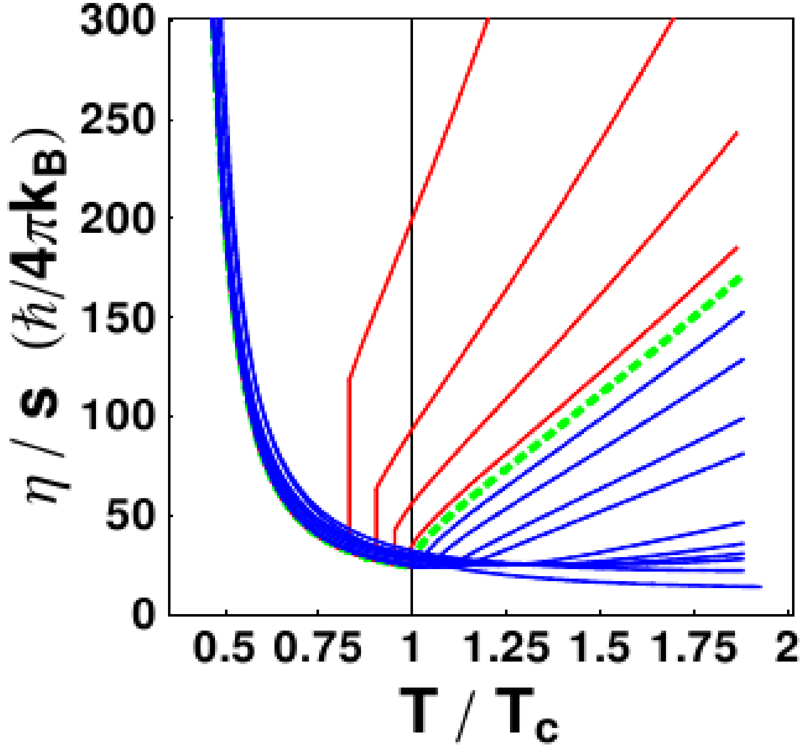
Correlations can lead to similar dipole angel; but reduce magnitude

Supercritical Water





Supercritical Water



My usual question

Why do we think that at large T the system should become weakly coupled???

coupling: $g \propto \frac{1}{\log(T/\lambda)}$

"density" $\rho \propto T^3$

Mean free Path: $l_{\text{mfp}} = \frac{1}{\sigma\rho} \propto \frac{1}{g^2\rho} \propto \frac{\log(T/\lambda)}{T^3} \rightarrow 0$

$$\frac{l_{\text{mfp}}}{d} \propto l_{\text{mfp}} T \propto \frac{\log(T/\lambda)}{T^2} \rightarrow 0$$

Virial expansion leads to similar conclusions.....

Minimum viscosity

AdS/CFT correspondence:

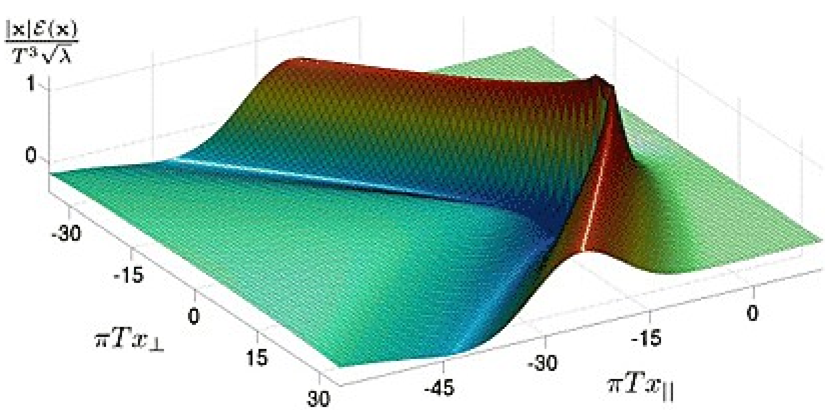
Maldacena et al, hep-th/9905111v3
Kovtun, Son, hep-th/0405231v2

$$\frac{\eta}{s} \geqslant \frac{1}{4\pi}$$

$\eta = shear\ viscosity$

$s = entropy\ density$

Holds for a large class of strongly coupled gauge theories



Kovtun, arXiv:0706.0368

Kinetic theory + waving hands:

$$\eta \sim n \bar{v} m \lambda$$

$$\bar{v} m = p \,, \quad p \lambda > \hbar \,, \quad n \sim s$$

$$\frac{\eta}{n} \sim \frac{\eta}{s} \geqslant 1$$

Quantum bound

More detailed derivation: Danielewcz and Gyulassy (85)