Investigation of shear stress and shear flow within a partonic transport model

Felix Reining, Zhe Xu and Carsten Greiner

Institut für Theoretische Physik, Goethe–Universität Frankfurt, Max–von–Laue–Straße 1, 60438 Frankfurt am Main, Germany

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

Abstract

Experimental data of ultrarelativistic heavy-ion-collisions at RHIC indicate that the Quark-Gluon-Plasma behaves like an ideal fluid above and close to T_C, and thus can be described by hydrodynamic models. Fundamental for this ansatz is the smallness of the viscosity coefficients. Using the partonic cascade BAMPS (Boltzmann Approach of Multi–Parton Scatterings) [1, 2] we investigate flow-gradients and shear viscosity in a seady fluid. Especially we calculate the viscosity-to-entropy-ratio of a gluonic medium with pQCD-crosssections.

Shear stress and shear flow

We consider a particle system embeded between two plates, as seen in Fig. 1. The two plates move with a velocity v_{wall} in opposite directions. The shearstress T^{xz} is proportional to the gardient of the flow velocity.

$$T^{xz} = -\eta \frac{\partial v_z}{\partial x}, \qquad (1)$$

where η is the shear viscosity. A gradient of the flow velocity $v_z(x)$ is established due to the interactions among particles. A naive solution of $v_z(x)$ is a linear function, which is shown in Fig. 2. However this is an approximate solution, which is only valid for nonrelativistic fluids. For a relativistic fluid we find that

$$v_z(x) = \tanh(cx) \qquad (2)$$

with a constant c.

$$y_z(x) = cx \qquad (3)$$

is defined as rapidity, which is only shifted by a constant under a Lorentz transformation. This indicates that the gradient of $v_z(x)$ looks the same in the comoving frame of each layerbeing parallel to the plates. $v_z(x)$ is shown in Fig. 2 for am example with $v_{wall} = 0.964$.

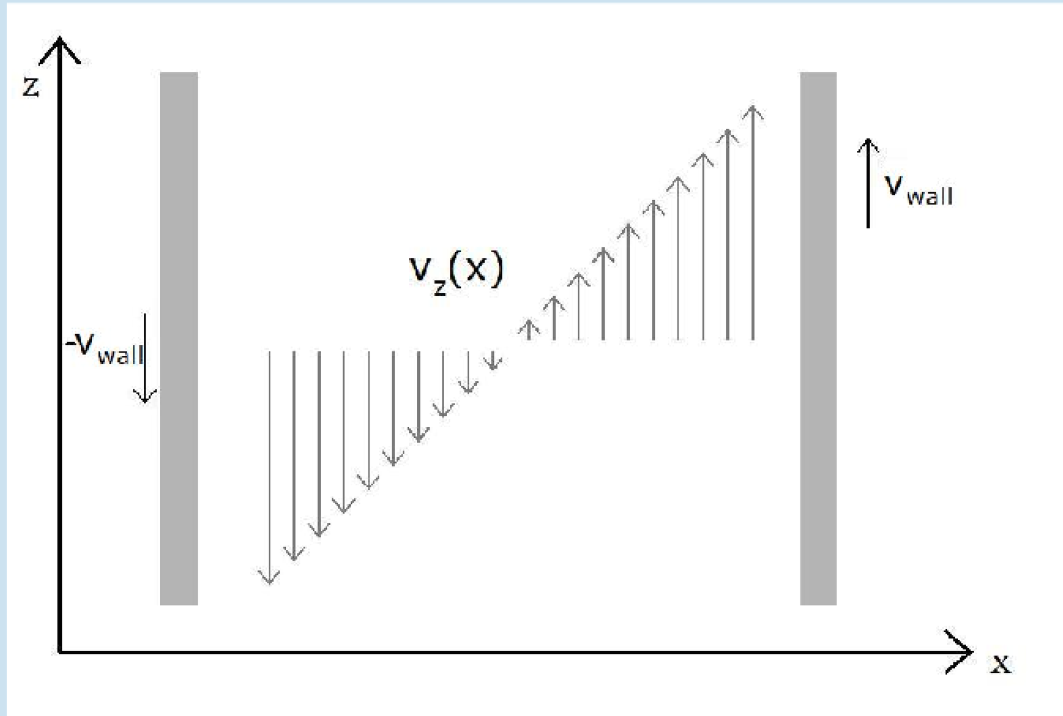

Figure 1: The classical definition of viscosity. Two plates moving in opposite directions with velocity $\pm v_{wall}$. A flowgradient is established between the plates. The viscosity is proportional to the Force.

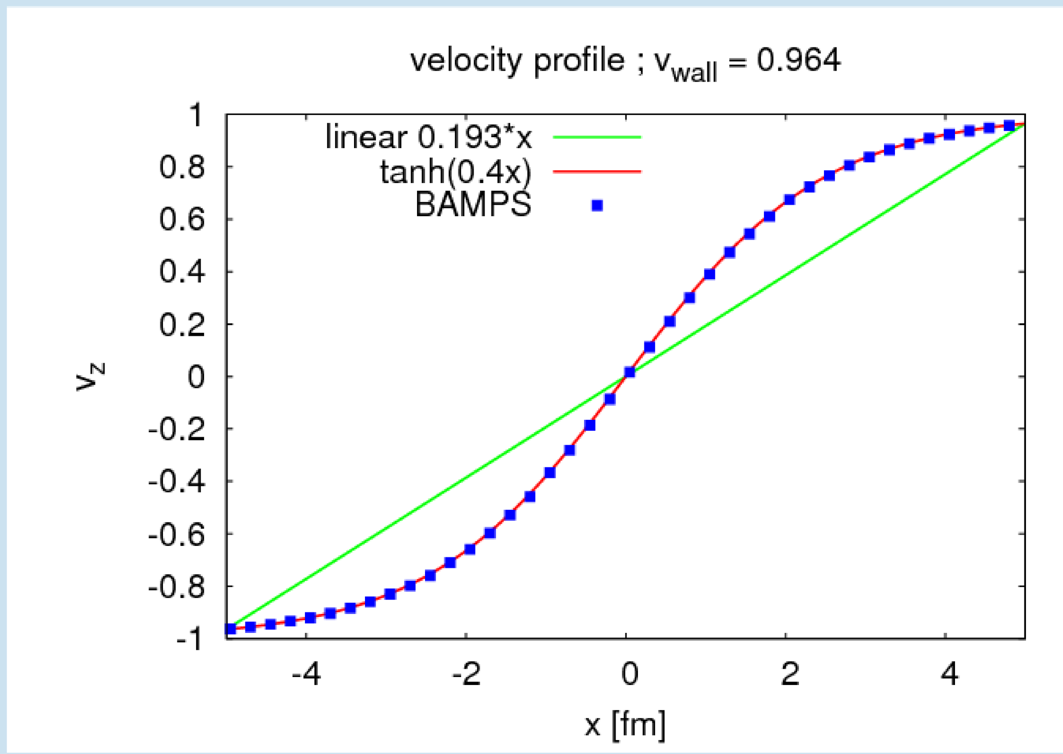

Figure 2: Velocityprofile in z-direction. BAMPS results for $v_{wall} = \pm 0.964c$ and mean free path $\lambda_{mfp} = 0.01fm$.

Finite-size-effects

We assume that the static gradient is self influencing. This means, that the value at one point depends on all other points in space. As all interaction in our model happens through particle exchange one can assume, that the influence decreases exponentially with distance as long als the interaction-probabilities at each place, which is basicly the crosssection, stays constant. The influence of $y_z(x\prime)$ on $y_z(x)$ is

$$\propto y_z(x\prime) \times exp^{-\frac{|x-x\prime|}{\lambda_{mfp}}}. \qquad (4)$$

Taking all places into accout, is basically an integration over Eq. 4

$$y_z(x) = \frac{1}{2\lambda_{mfp}} \int_{-\infty}^{\infty} dx\prime y_z(x\prime) \times exp^{-\frac{|x-x\prime|}{\lambda_{mfp}}}. \qquad (5)$$

Fixing the value outside an intervall $[-L/2, L/2]$ to $\pm y_{wall}$, leads to the solution for this differental equation:

$$y_z(x) = \frac{2y_{wall}}{L + 2\lambda_{mfp}} x = cx \qquad (6)$$

Results from BAMPS show a very good agreement in Fig.3.

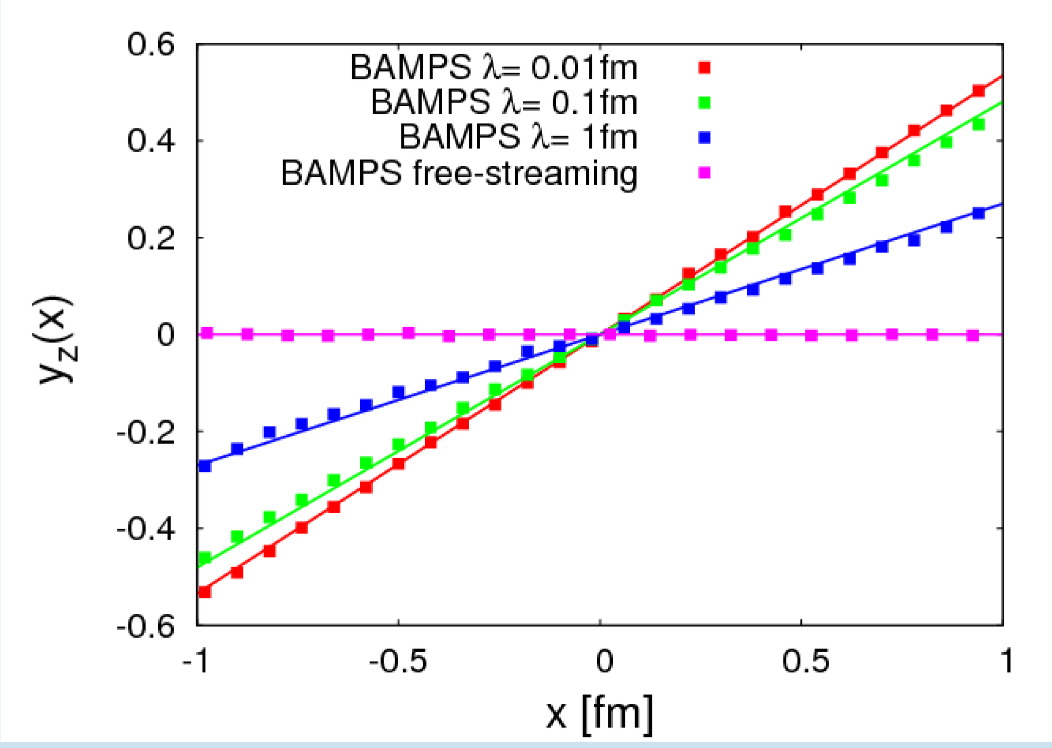

Figure 3: Changing of the rapidity gradient. BAMPS results for for different λ_{mfp} in agreement with Eq. 6

The cascade

We use the Cascade BAMPS as described in [1, 2] with elastic isotropic crossections σ_{22} .The mean-free-path λ_{mfp} is kept constant using

$$\lambda_{mfp} = \frac{1}{n\sigma_{22}} \qquad (7)$$

where n is the local particle density.

When a particle hits one ofthe plates, it will be reflected into the box, but with a new momentum distributed equally to a thermalized gas with a certain velocity v_{wall} in z-direction. Flowvelocity and rapidity are calculated within BAMPS and shown in Fig. 2 and 3

Shear viscosity

To calculate the shear viscosity η we use the Navier-Stokes-Approximation

$$\pi^{\mu\nu} = 2\eta \nabla^{<\mu} u^{\nu>} \qquad (8)$$

where u^μ is the fourvelocity. With

$$u^\mu = (\gamma, 0, 0, \gamma v) = (\cosh(c_{grad}x), 0, 0, \sinh(c_{grad}x)) \qquad (9)$$

Eq. 8 goes to

$$\pi^{\mu\nu} = -\eta\gamma c_{grad} \qquad (10)$$

In comparision to our calculations we use the relation from deGroot [5]

$$\eta^{NS} \approx 0.8436 \frac{T}{\sigma_{tr}} \qquad (11)$$

for isotropic 2 ⇔ 2 crossections where σ_{tr} is given [4]

$$\sigma_{tr} = \frac{2}{3}\sigma_{22} = \frac{2}{3}\frac{1}{n\lambda} \qquad (12)$$

Finally we have

$$\eta^{NS} \approx 1.2654 T n \lambda \qquad (13)$$

For given Temperature T, particle density n and mean-free-path λ_{mfp} one can calculate η^{NS} according to Eq. 13. In Fig. 4 we see a perfect agreement.

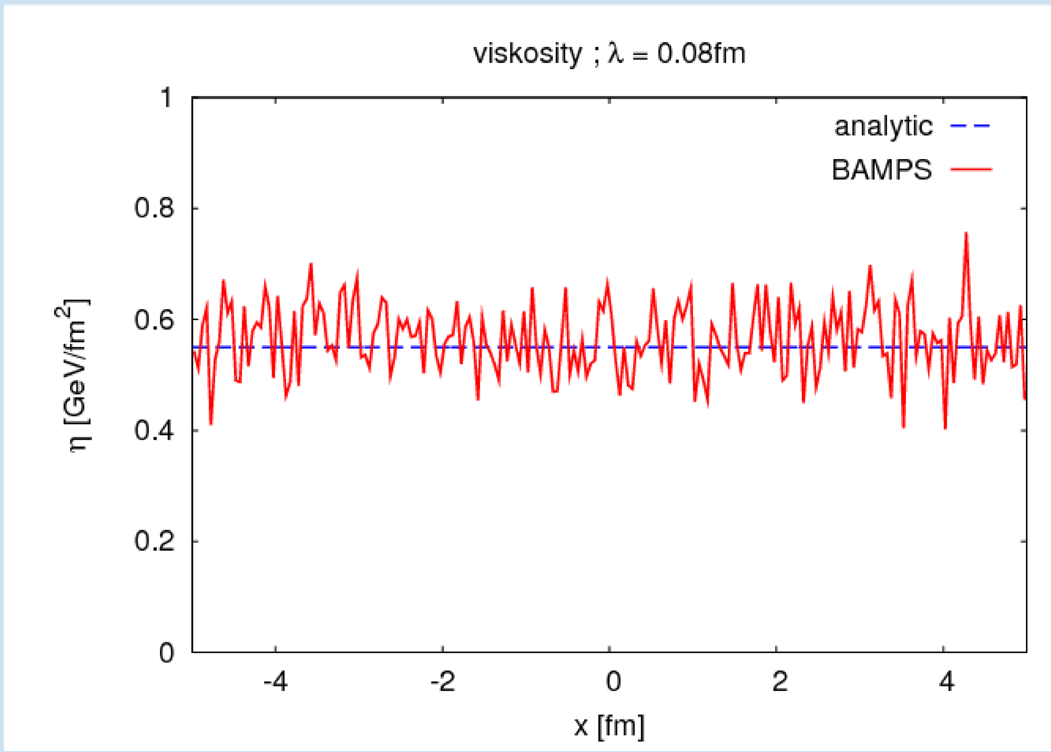

Figure 4: Viscosity over boxposition. BAMPS results fit the analytic value very good

Implementing elastic and inelastic pQCD-crosssections, we calculate the viscosity of a gluonic medium and the entropy density

$$s = 4n$$

We obtain the shear viscosity to entropy ratio for $\alpha_s = 0.3$:

$$\frac{\eta}{s} = 0.118 \qquad (14)$$

which is in very good agreement with Xu and Greiner [4], Muronga and El [6] and Wesp [7]

Summary

The gradient of a shearr flow is analytically derived and cofirmed numerically within BAMPS . The influence of finite-size effects to the flowvelocity is analysed. Using this Navier-Stokes-Approximation we calculated viscosity η and the viscosity to entropy ratio $\frac{\eta}{s}$ for both isotropic elastic scattering and inelastic scattering with pQCD-crosssections.

References

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reining@th.physik.uni-frankfurt.de