

Quark Number Scaling in Fluid Dynamics and Hadronization via Quarkyonic Matter



Hot and Cold Baryonic Matter
HCBM 2010, Budapest

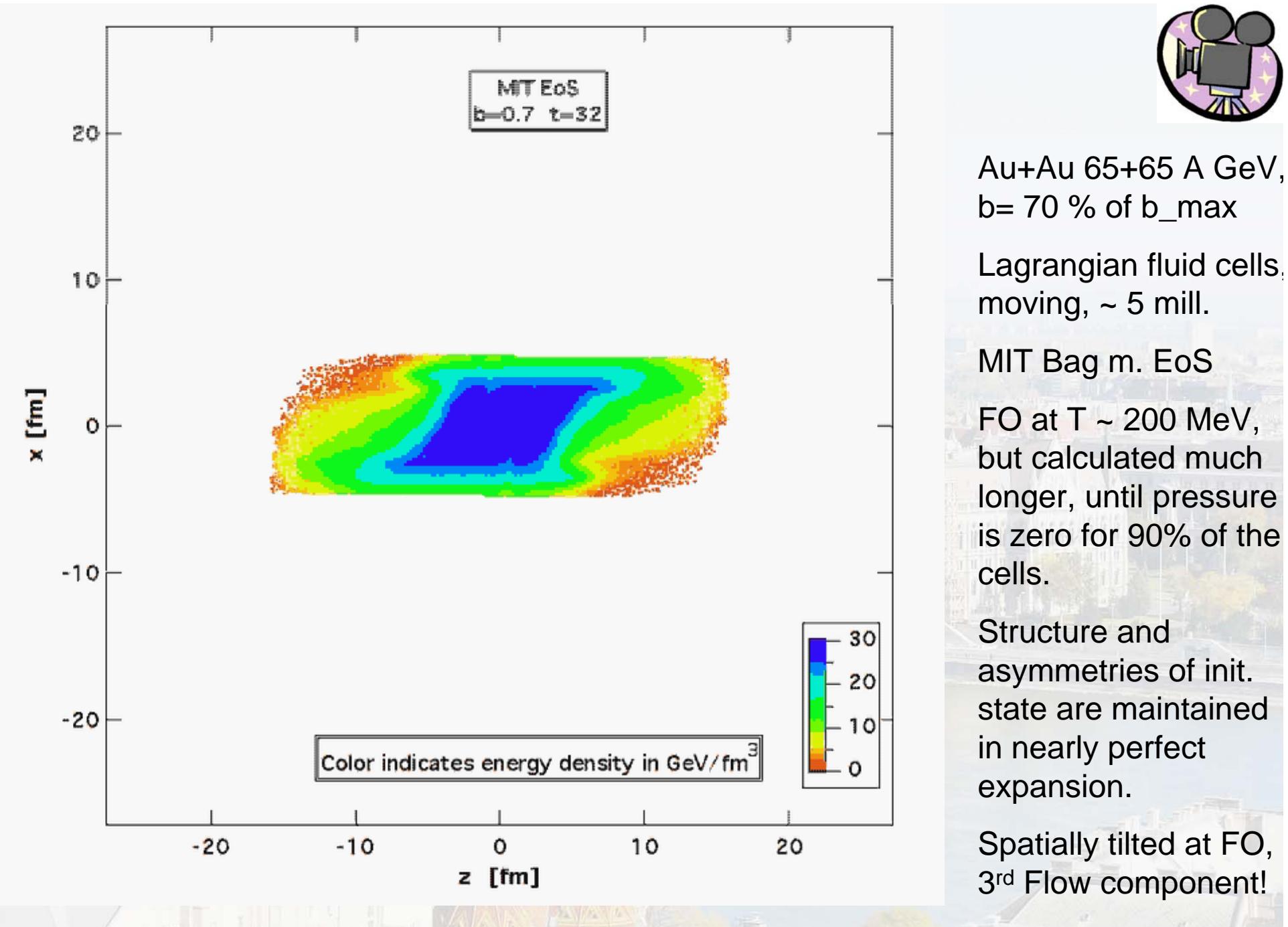
L.P. Csernai
U. Bergen & RMKI Budapest

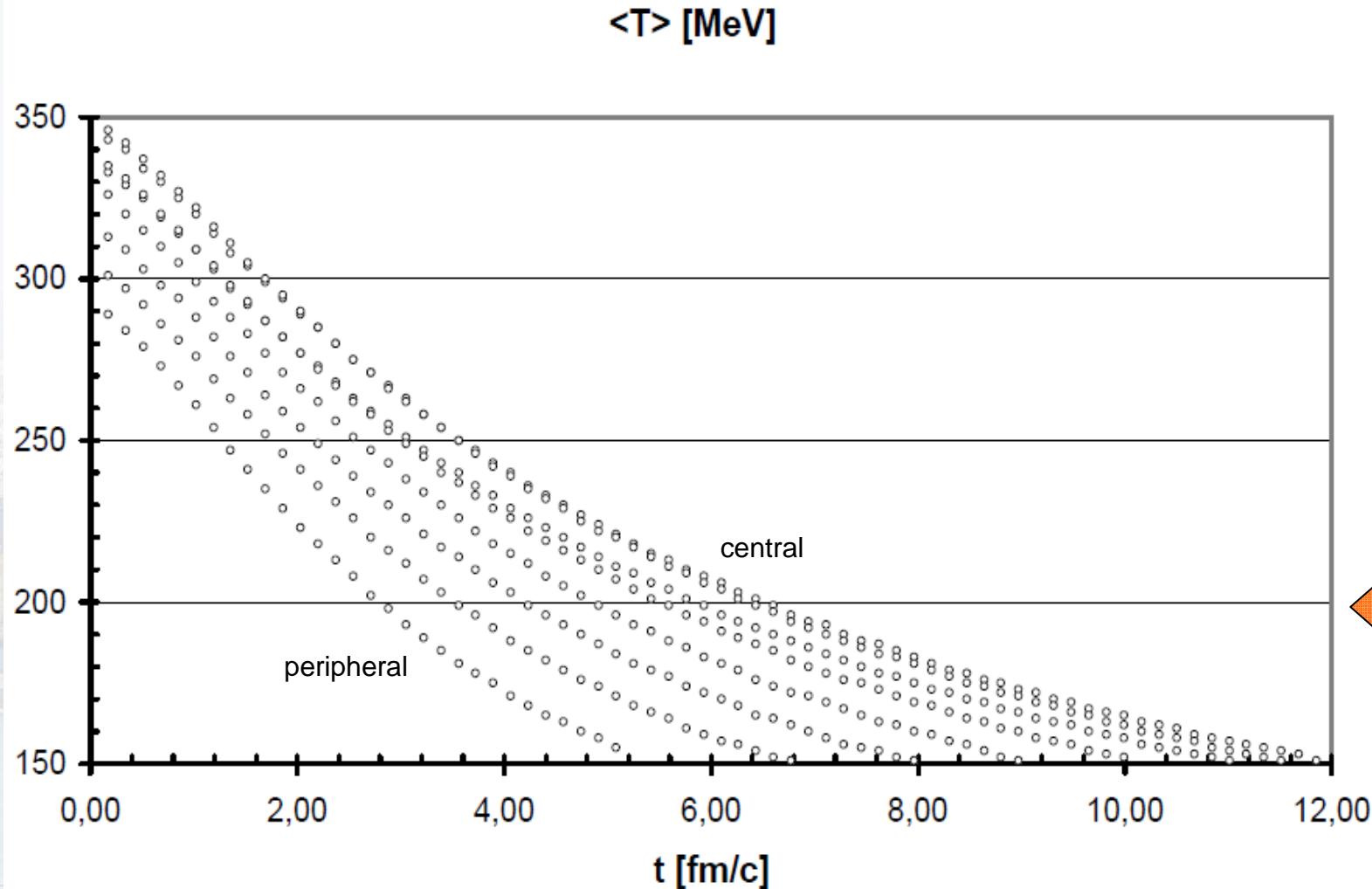
Coauthors

- Sven Zschocke
- Szabolcs Horvat
- Yun Cheng
- Igor Mishustin
- V.K. Magas
- B. Schlei
- D.D. Strottman

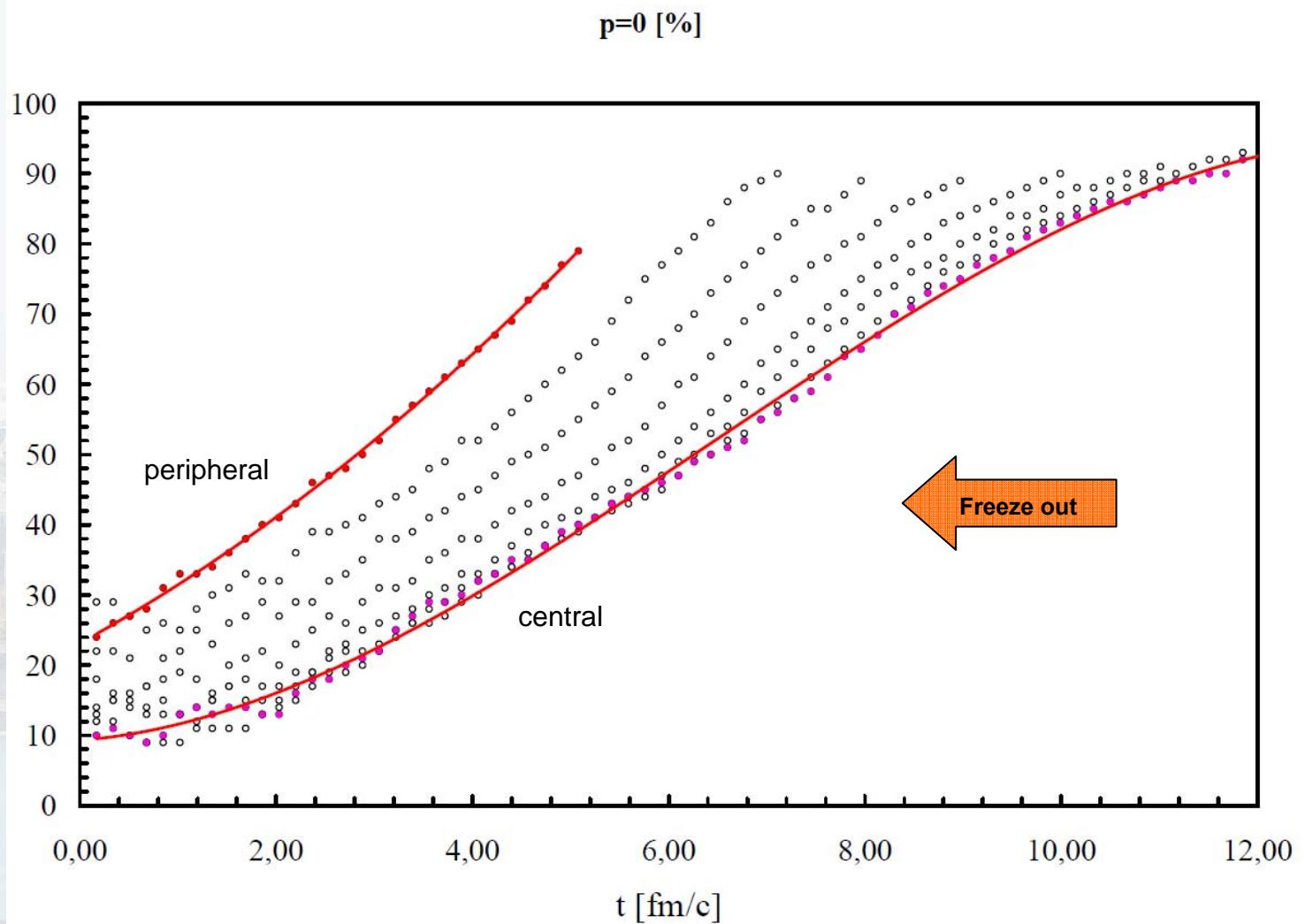
Introduction

- Roy Lacey: NCQ Scaling of flow / v_2
- Tamas Csorgo: Sudden FO and Haronization
- Mei Huang: Quarkyonic matter





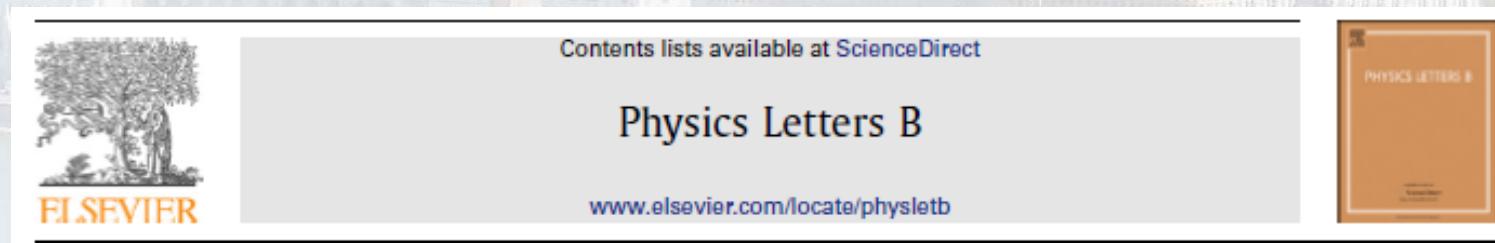
Average temperature versus time in Au+Au collisions at 65+65 AGeV, for impact parameters, $b = 0, 0.1, 0.2, \dots 0.7 b_{\max}$ from the top (0.00) down (0.7).



Percentage of the cells with vanishing pressure ($P=0$) versus time in Au+Au collisions at 65+65 AGeV, for impact parameters, $b = 0, 0.1, 0.2, \dots 0.7 b_{\max}$. The most peripheral collision at the top ($b=0.7$) and the most central one ($b=0.00$) are indicated in red with a trend line.

Extreme states of matter - QGP

- Collective properties – Equation of State (EoS), new phases
- Transport properties – viscosity, dissipation \leftrightarrow EoS
- From collective dynamics in ultra-relativistic collisions,
 v_1 , v_2 , jets, Mach cones



Entropy development in ideal relativistic fluid dynamics with the Bag Model equation of state

Sz. Horvát^{a,b,*}, V.K. Magas^c, D.D. Strottman^{d,e}, L.P. Csernai^{a,d,f}

[Sz. Horvat et al., PLB 692 (2010) 277]

Dissipative expansion in numerical P. hydro

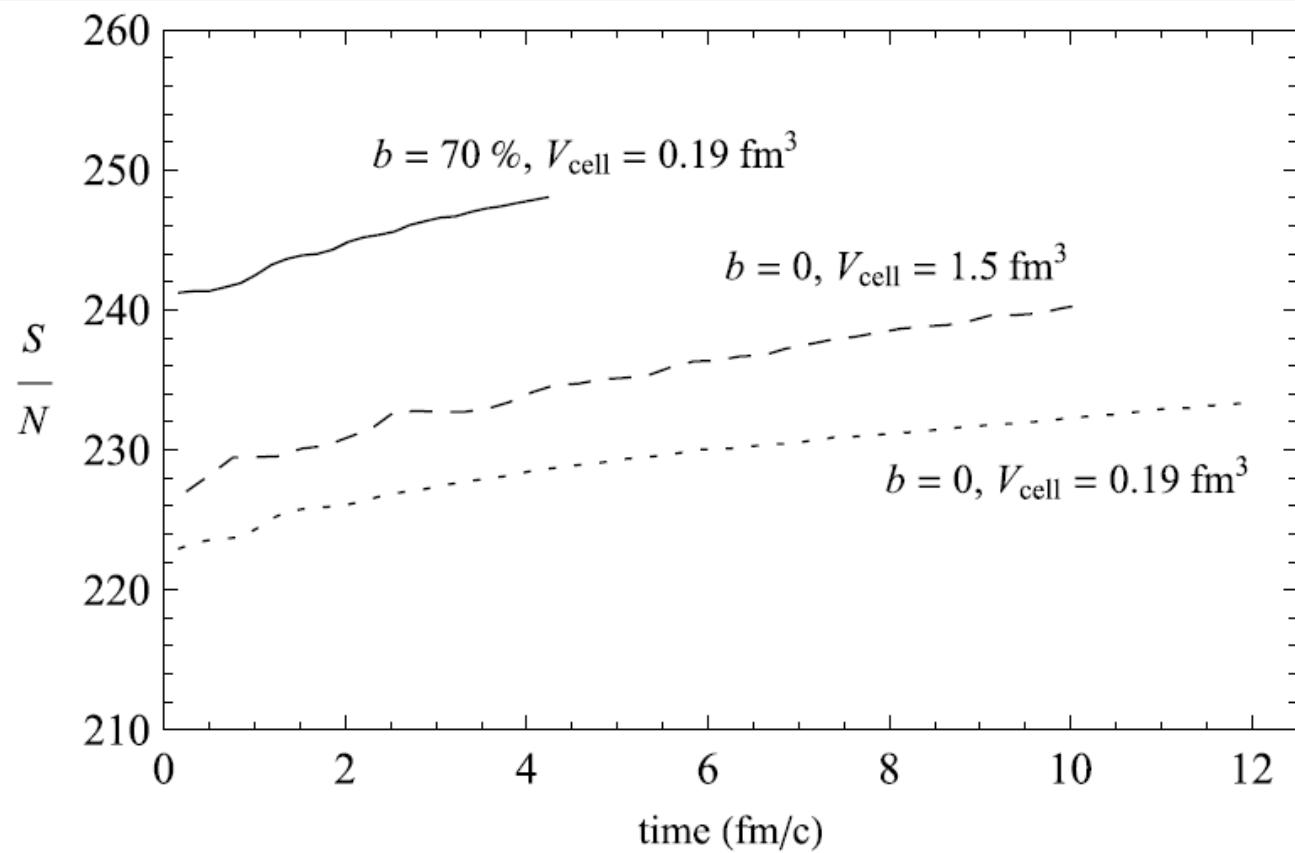


Fig. 1. The mean specific entropy, S/N , is shown for three different FD computations (N is the number of participants). Although the simulations were done for adiabatic expansion of an ideal fluid, the entropy increases due to the numerical viscosity of the method. The difference in initial specific entropy between the two cases describing collisions with impact parameter $b = 0$ is due to coarse graining. V_{cell} denotes the cell size of the computational grid.

[Sz. Horvat et al., PLB 2010]

The energy density of the Bag field

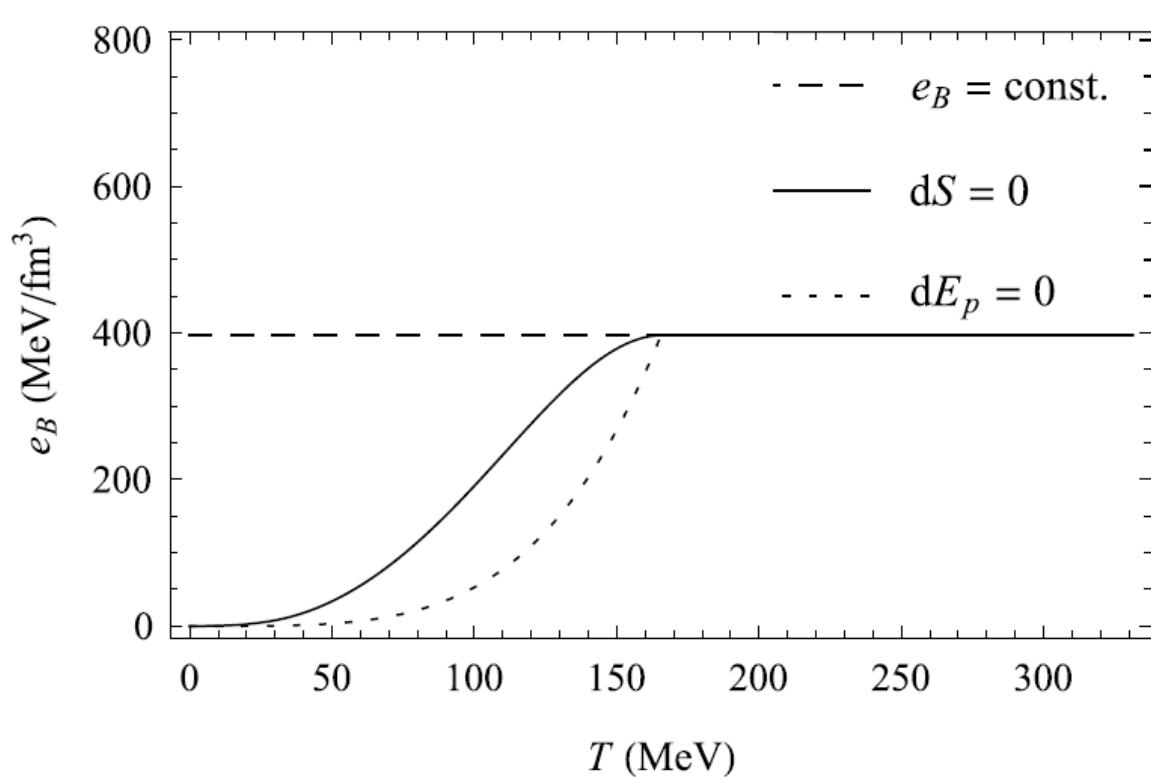
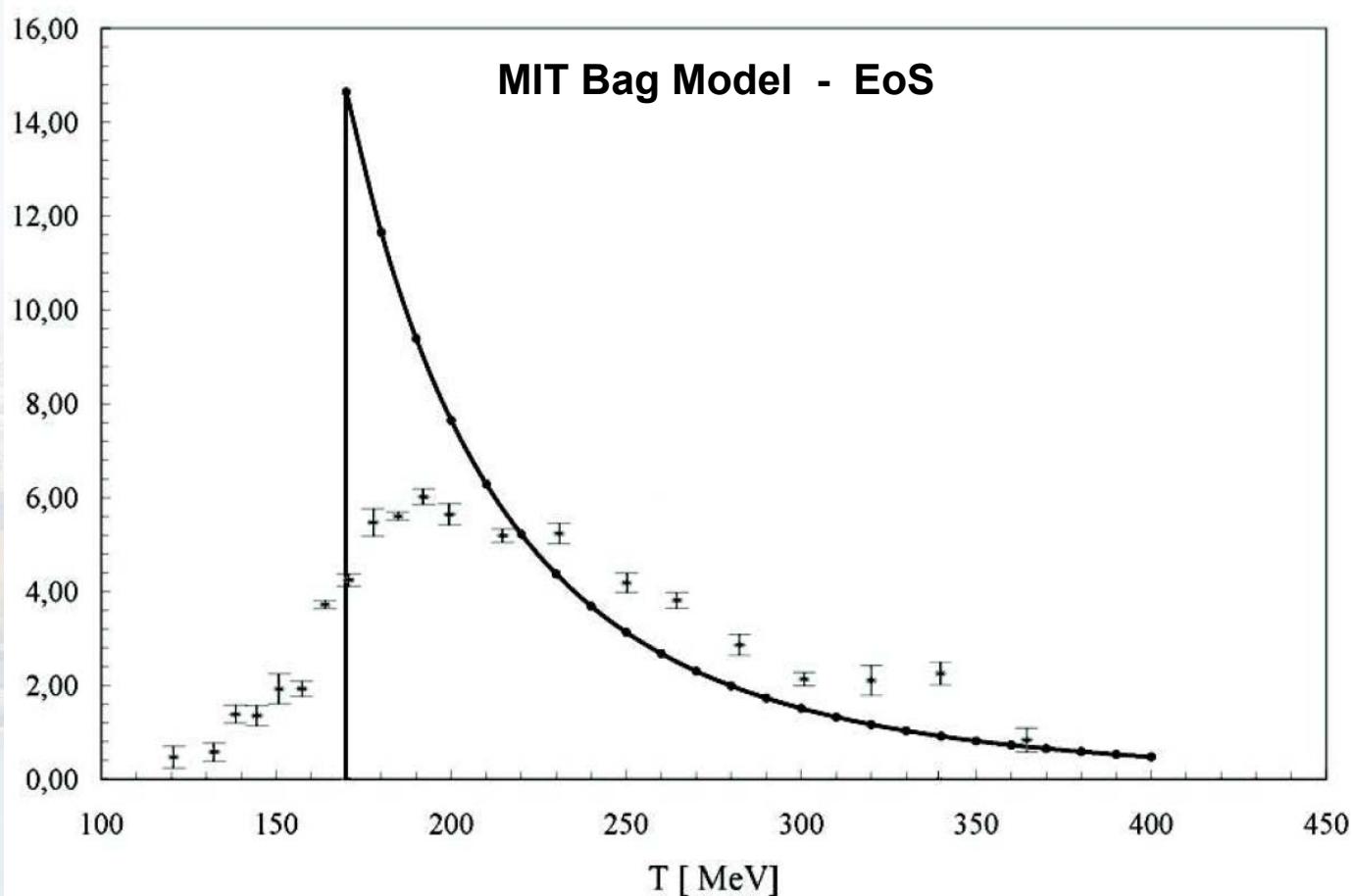


Fig. 2. The energy density of the Bag field, e_B , as a function of the temperature of the partonic gas, T , for adiabatic expansion (full line, $dS = 0$), constant energy of the Bag field (dashed line), and without change of the energy of the parton gas (dotted line). Case $dE_p = 0$ assumes that the total energy of the ideal gas component remains constant, thus the total energy of the Bag field remains constant as well during the final expansion of the system, while its energy density decreases. Realistically, the Bag field must disappear in the transition, so the real trajectory must lay below the dotted line. This disappearance of the Bag field may start before the total pressure drops to zero.

[Sz. Horvat et al., PLB 2010]

Interaction Measure



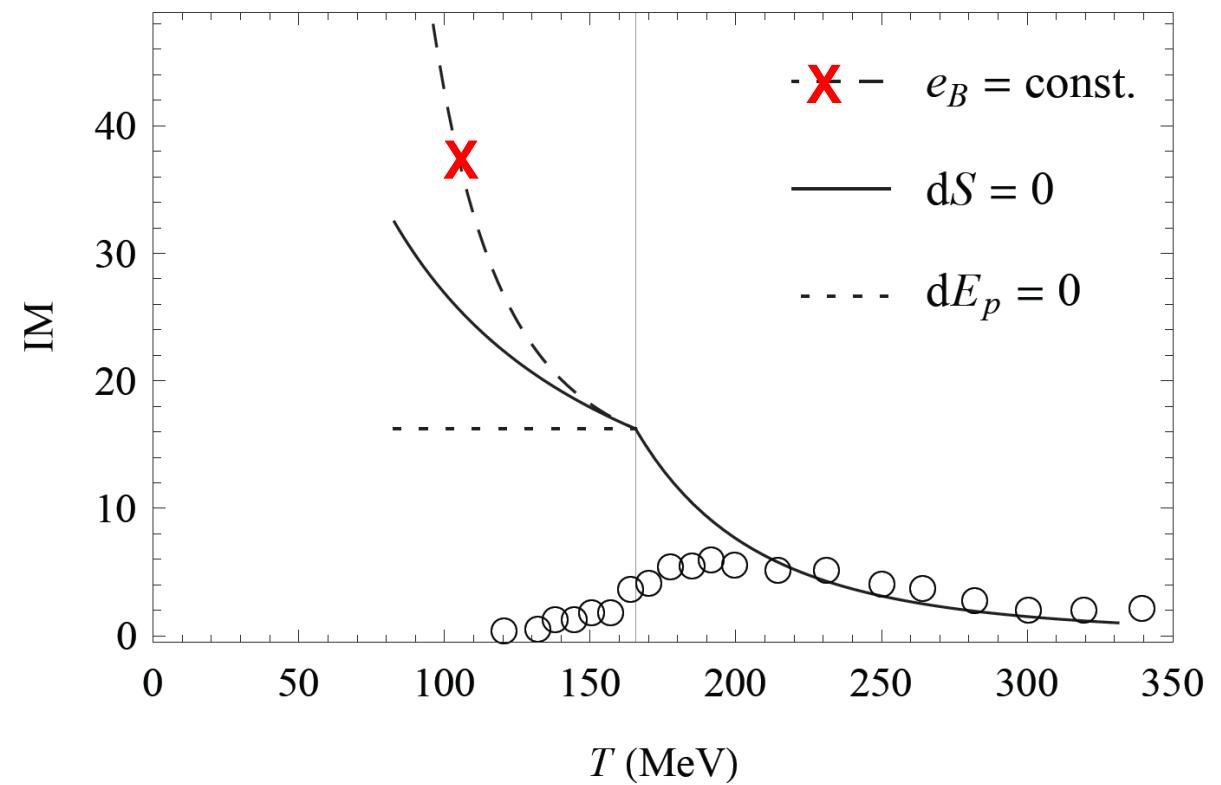
Clusterization in QGP due to dynamical stretching of the plasma
[Mishustin, CPOD 2007]

Dynamical viscous pressure ~ bulk stress → $p < 0$ → cavitation ~ bubble / droplet formation
[Rajogopal, Tripuraneni 2009]

Interaction measure, $(e-3p)/T^4$, from the MIT Bag model and from Lattice QCD [MILC]. The bag model is acceptable above $T=200\text{MeV}$. The bag model behaviour around T_c with a fix B leads to negative pressure.

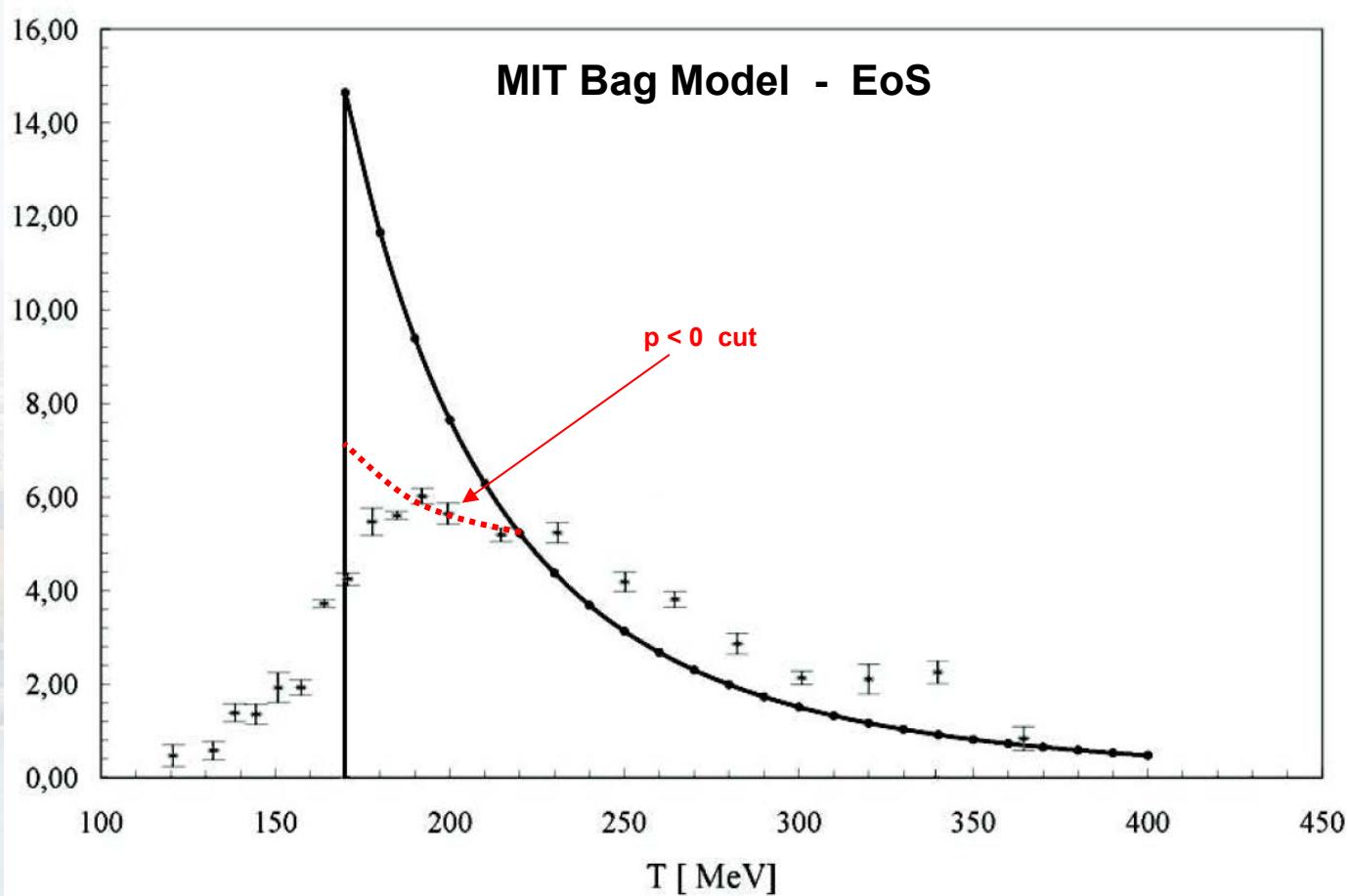
EoS – Surface of an expanding system

[Sz. Horvat et al., PLB 2010]



IM from the MIT Bag model and lattice QCD calculation (circles) [MILC 2005]. There is relatively good agreement above a temperature of 200 MeV. At $T=165$ MeV the pressure drops to zero. The Bag energy density must decrease, the change of T and s in adiabatic (full) and dissipative (dotted) expansion are shown.
→ Final stage EoS depends on hadronization mechanism !

Interaction Measure



Clusterization in QGP due to dynamical stretching of the plasma
[Mishustin, CPOD 2007]

Dynamical viscous pressure ~ bulk stress → $p < 0$ → cavitation ~ bubble / droplet formation
[Rajogopal, Tripuraneni 2009]

Interaction measure, $(e-3p)/T^4$, from the MIT Bag model and from Lattice QCD [MILC]. The bag model is acceptable above $T=200$ MeV. The bag model behavior around T_c with a fix B leads to negative pressure.

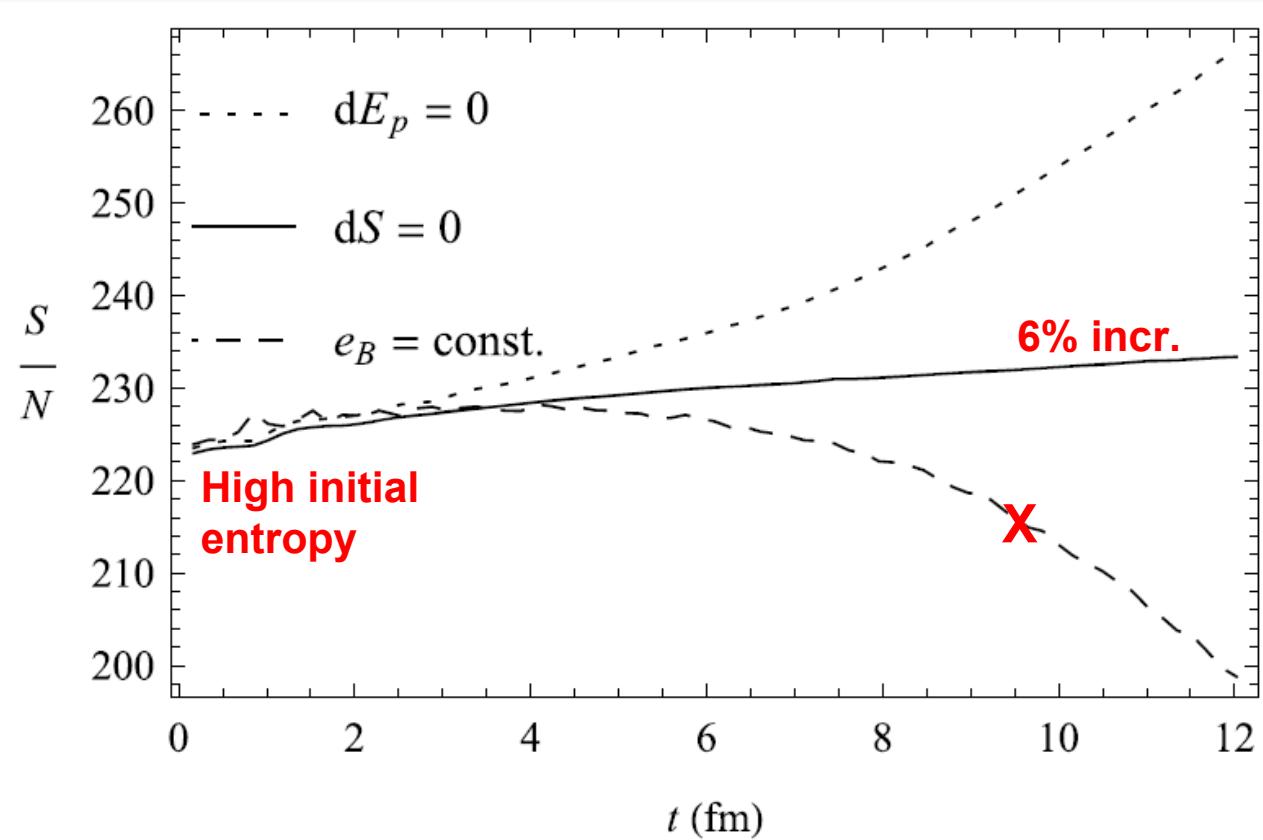


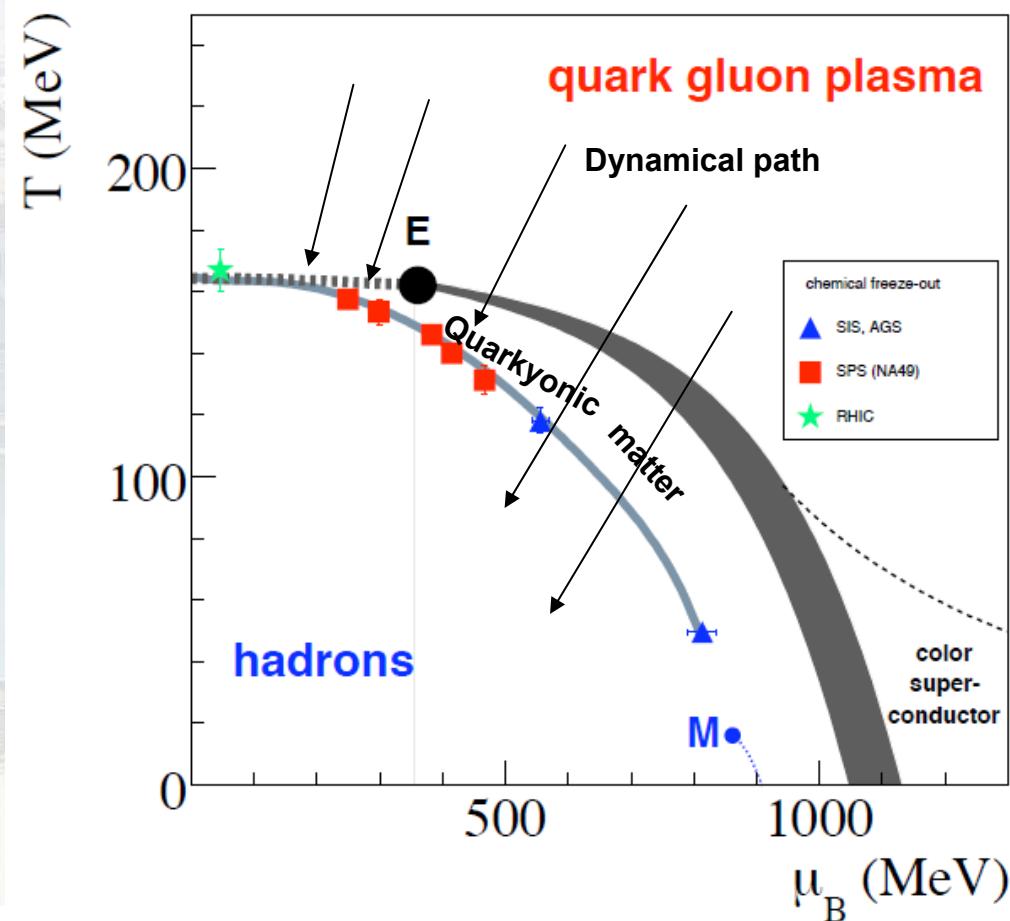
Fig. 4. Results for an Au + Au collision at 65 + 65 A GeV energy at impact parameter $b = 0$, from a CFD calculation with the Particle in Cell (PiC) method with cell size $dx = dy = dz = 0.575$ fm. The mean specific entropy of the Au + Au system, S/N , as a function of time in the numerical fluid dynamics simulation of a heavy ion collision. Solid line: adiabatic expansion of the ideal gas component, dashed line: $e_B = B = \text{const}$, dotted line: $E_p = \text{const}$. The slight entropy increase in the “adiabatic” case is due to numerical viscosity.

[Sz. Horvat et al., PLB 2010]

Fluid Dynamics

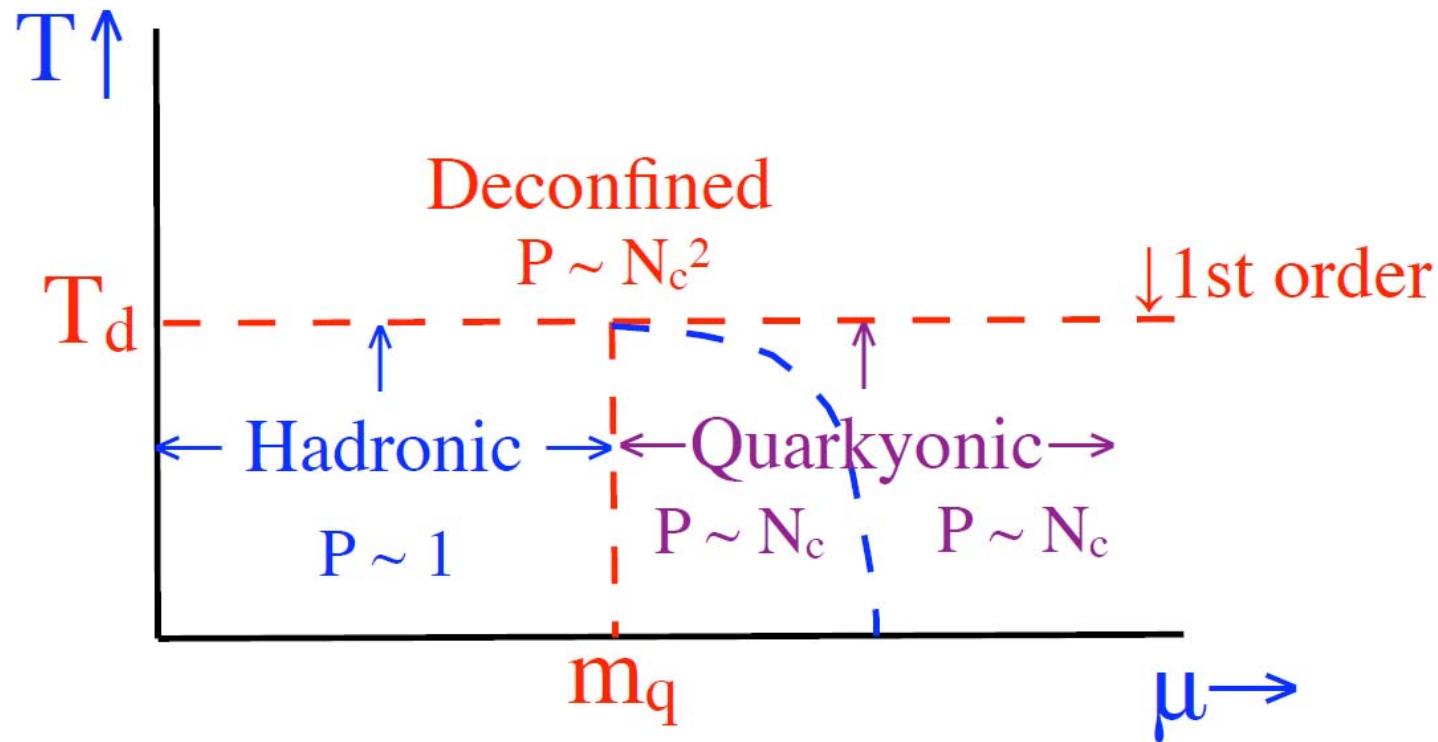
↔

Equation of State & Transport Properties



Quarkyonic Matter [McLerran, Pisarski]

Quarks exist, gainng mass, gluons are absorbed



[Mei Huang,
HCBM 2010]

Elliptic flow/ Sources of v2

- 1) Anisotropic flow from initial state eccentricity (finite $b \rightarrow$ spatial v anticorrelation)
- 2) Initial state surface layer [RC Hwa, CB Yang]
- 3) Viscous damping of the flow
- 4) EoS of the matter
- 5) Recombination from local anisotropic $f(x_p)$ and the collision integral [D Molnar, CM Ko et al.,]
- 6) FO asymmetry of final state influences v_2
- (!) MD models may include 1, (2), (3), 5, (6)
- → Description of NCQ scaling is a complex issue !!!



ELSEVIER

22 MARCH 2001

PHYSICS LETTERS B

Physics Letters B 503 (2001) 58–64

www.elsevier.nl/locate/npe

Radial and elliptic flow at RHIC: further predictions

P. Huovinen^a, P.F. Kolb^{b,c}, U. Heinz^b, P.V. Ruuskanen^d, S.A. Voloshin^e



(!) FO T = Const.

P. Huovinen et al. / Physics

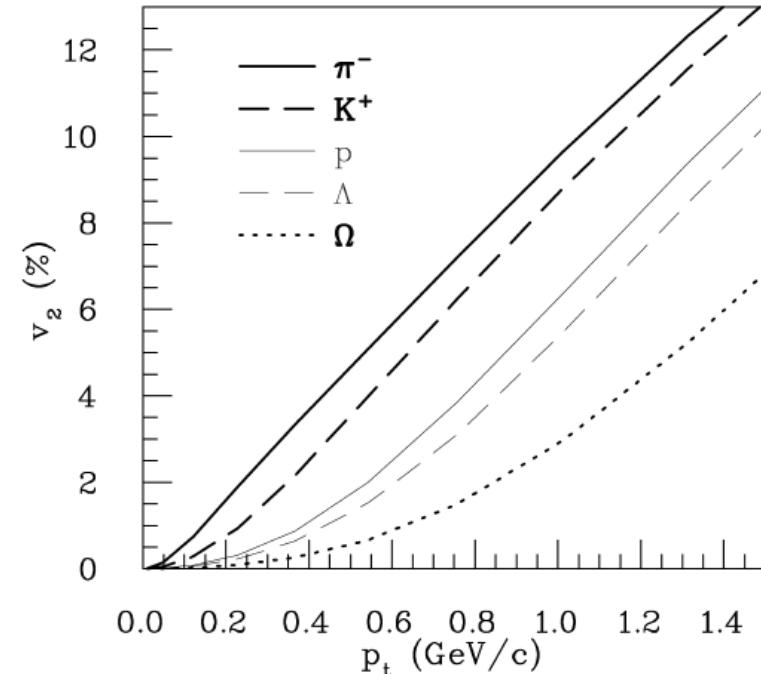


Fig. 3. p_t -differential elliptic flow at midrapidity for various hadrons from minimum bias Au+Au collisions at $\sqrt{s} = 130 A$ GeV for EOS Q(120).

Linear pt dependence of flow (?)

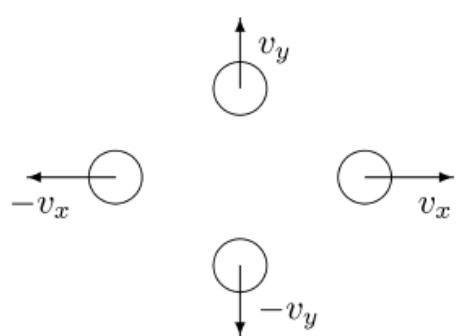


Fig. 6. Simple source of four fireballs.

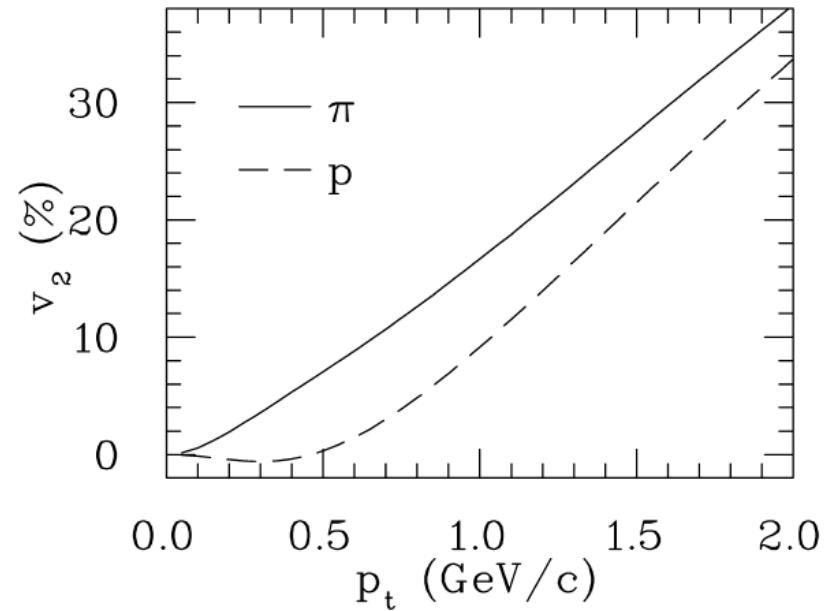


Fig. 7. Transverse momentum dependence of elliptic flow for midrapidity pions and protons from the schematic source in Fig. 6, for $T = 140$ MeV, $v_x = 0.6$, and $v_y = 0.5$.

$$v_2(y, p_t) = \frac{I_2(\gamma_x v_x p_t / T) - e^{\frac{E}{T}(\gamma_x - \gamma_y)} I_2(\gamma_y v_y p_t / T)}{I_0(\gamma_x v_x p_t / T) + e^{\frac{E}{T}(\gamma_x - \gamma_y)} I_0(\gamma_y v_y p_t / T)}.$$

(!) FO T = Const.

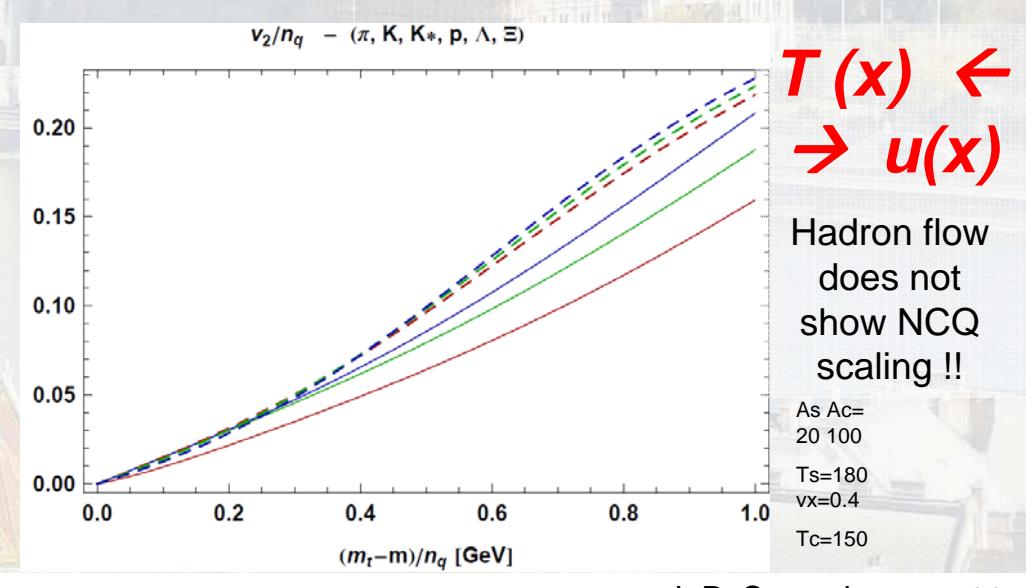
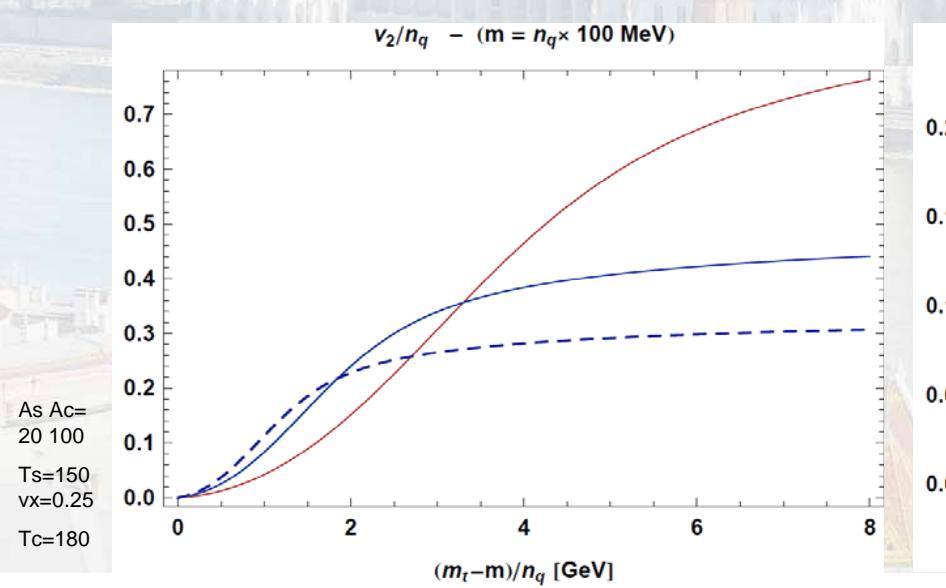
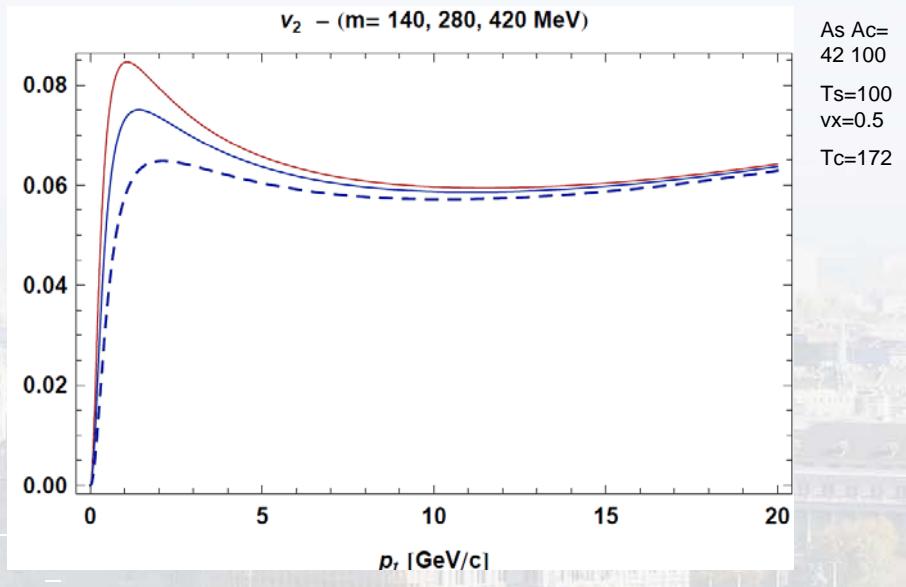
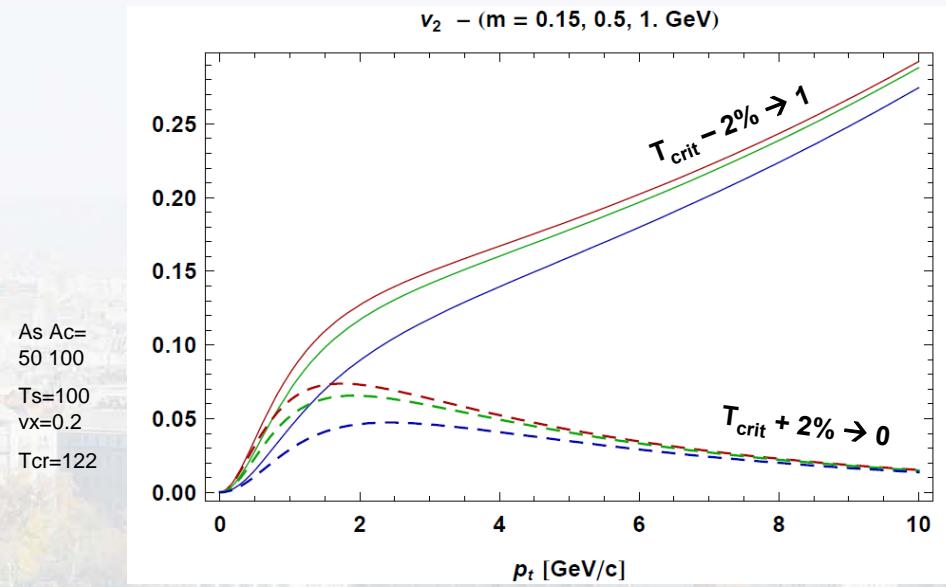
V2 from few source models [Huovinen et al. 2001] → $v_2(p_t)$ rises linearly at high p_t (Bjorken Model)

NCQ - Importance of Initial State

Take 3 sources only:



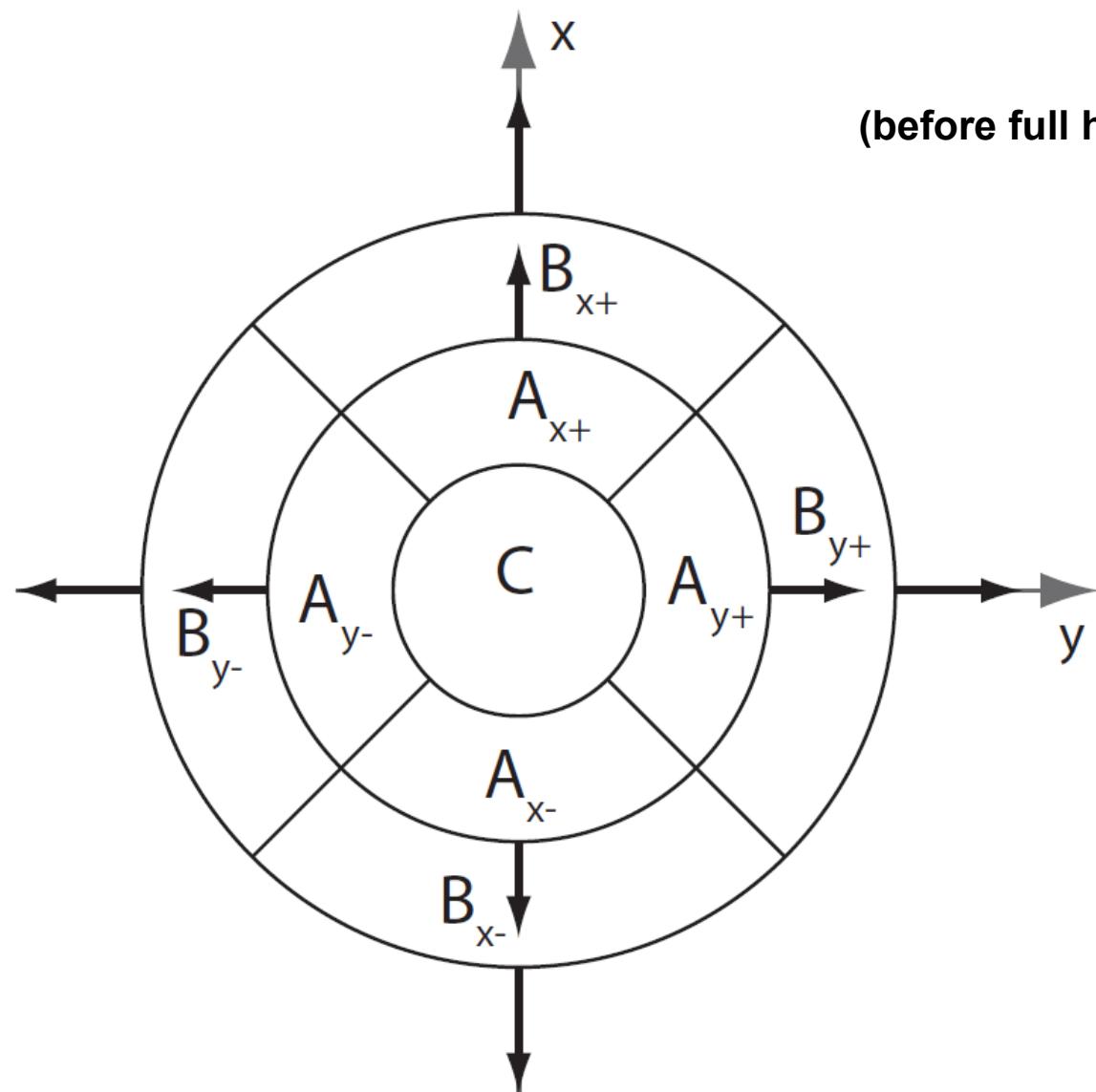
FO [w/Mishustin]



Note that:

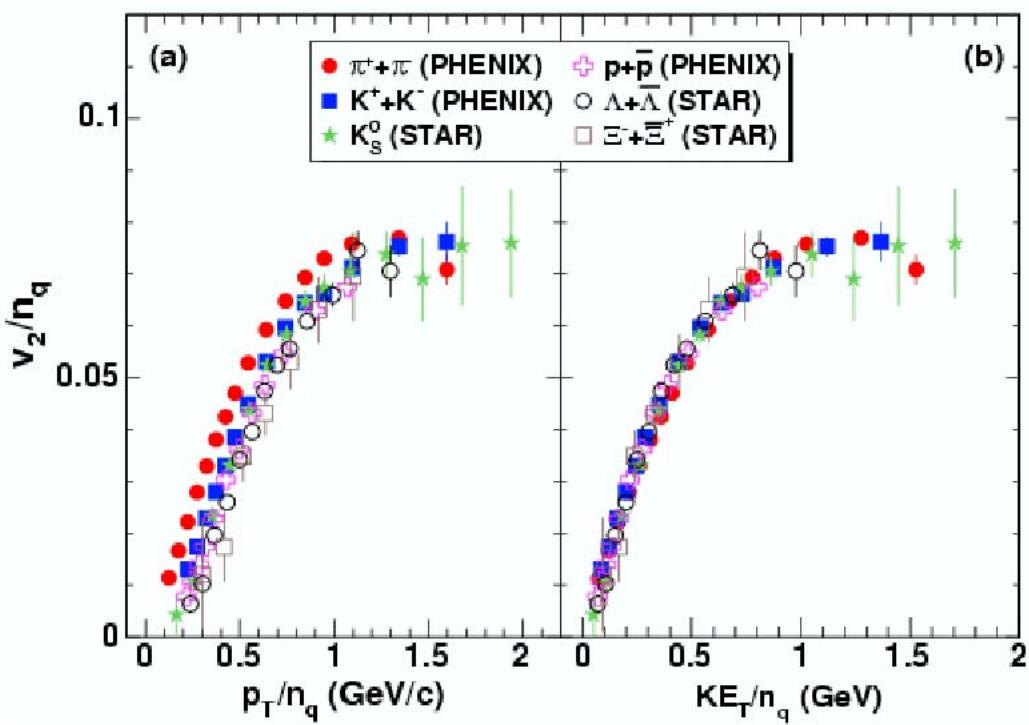
- Thermal equilibrium among different mass particles does not lead to NCQ scaling.
- Sources of different T do not lead to linearly increasing $V_2(p_t)$ spectra.

Next step of few source models



CNQ scaling

Constituent quark number scaling of v_2 (KE_T)



Collective flow of hadrons can be described in terms of constituent quarks.

Observed n_q – scaling →

Flow develops in quark phase, there is no further flow development after hadronization

R. A. Lacey (2006), nucl-ex/0608046.

Hadronization via recombination

Momentum distribution of mesons in simple recombination model:

$$\frac{d^3 N}{dp^3} \propto \int \prod_{i=1}^2 d^3 x_i d^3 p_i f_q(x_1, p_1) f_q(x_2, p_2) W_M(p, p_1, p_2, x_1, x_2)$$

Local $f_q(p_\mu u^\mu)$ is centered at the local u , & meson Wigner function:

$$W_M(p, p_1, p_2, x_1, x_2) = \Phi_M(x_1 - x_2, p_1 - p_2) \delta(p_T - p_{T1} - p_{T2})$$

momentum conservation

comoving quark and antiquark:

$$\Phi_M \propto \delta^3(x_1 - x_2) \delta^3(p_1 - p_2)$$

for the momentum distribution of mesons we get:

$$\frac{d^3 N_M}{p_T d p_T dy d\phi} \propto \int d^3 x f_q(x, p_t/2)^2$$

flow moments:

for baryons, $2 \rightarrow 3$

$$v_n(p_T) = \frac{\int dy d\phi \cos n\phi \frac{d^3 N}{p_T d p_T dy d\phi}}{\int dy d\phi \frac{d^3 N}{p_T d p_T dy d\phi}}$$

[MolnarD-NPA774(06)257]

→ Elliptic flow of mesons:

$$v_{2,M}(p_T) = \frac{2v_{2,q}(p_T/2)}{1+2v_{2,q}^2(p_T/2)} \quad \frac{v_{2,M}(p_T)}{2} = v_{2,q}(p_T/2)$$

For baryons:

$$v_{2,B}(p_T) = \frac{3v_{2,q}(p_T/3)+3v_{2,q}^3(p_T/3)}{1+6v_{2,q}^2(p_T/3)} \quad \frac{v_{2,B}(p_T)}{3} = v_{2,q}(p_T/3)$$

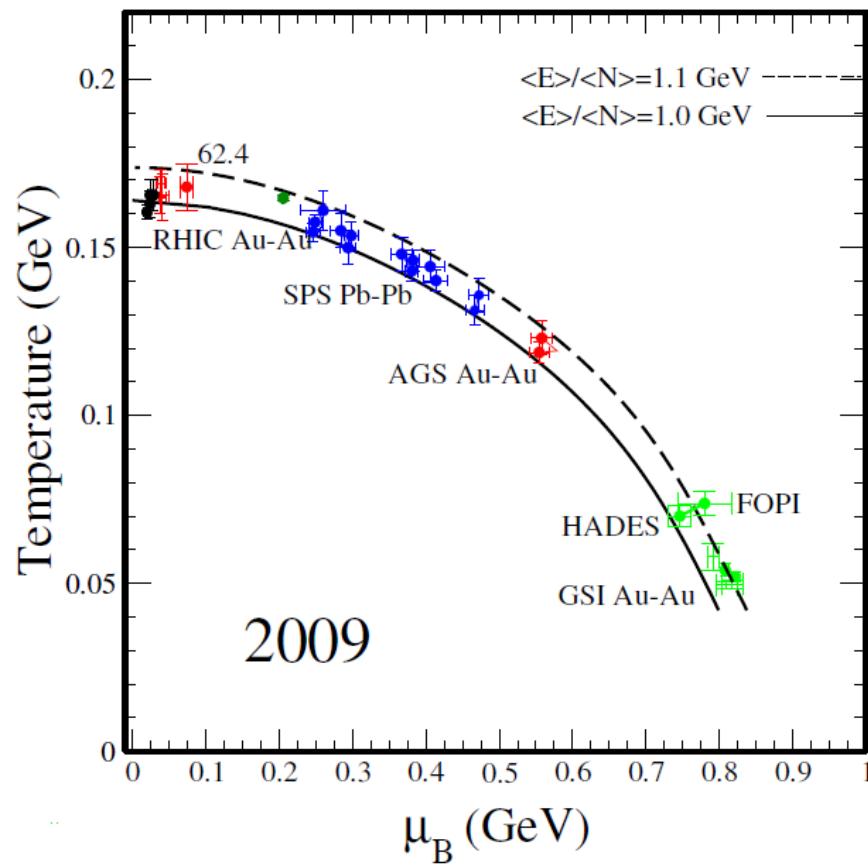
Scaling Variables of Flow:

1st step: Flow asymmetry: $V_2/n_q \rightarrow V_2$ scales with n_q i.e., flow develops in QGP phase, following the common flow velocity, u , of all q-s and g-s. Mass here does not show up (or nearly the same mass for all constituent quarks).

Then flow asymmetry does not change any more.

In a medium p_T is not necessarily conserved, $K E_T = m_T - m$ might be conserved → scaling in the variable $K E_T$ [J. Jia & C. Zhang, 2007]

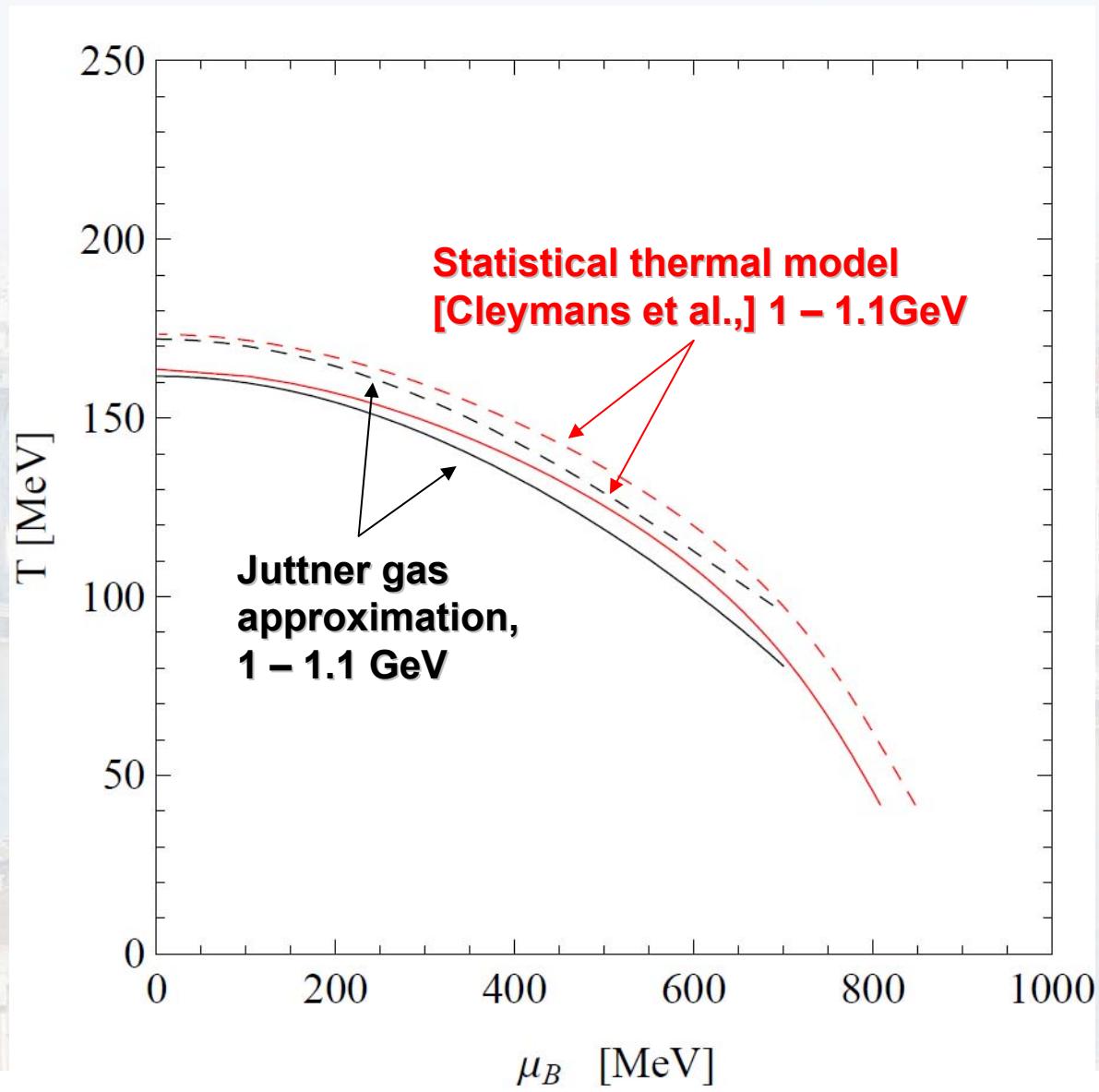
Observed Hadron FO



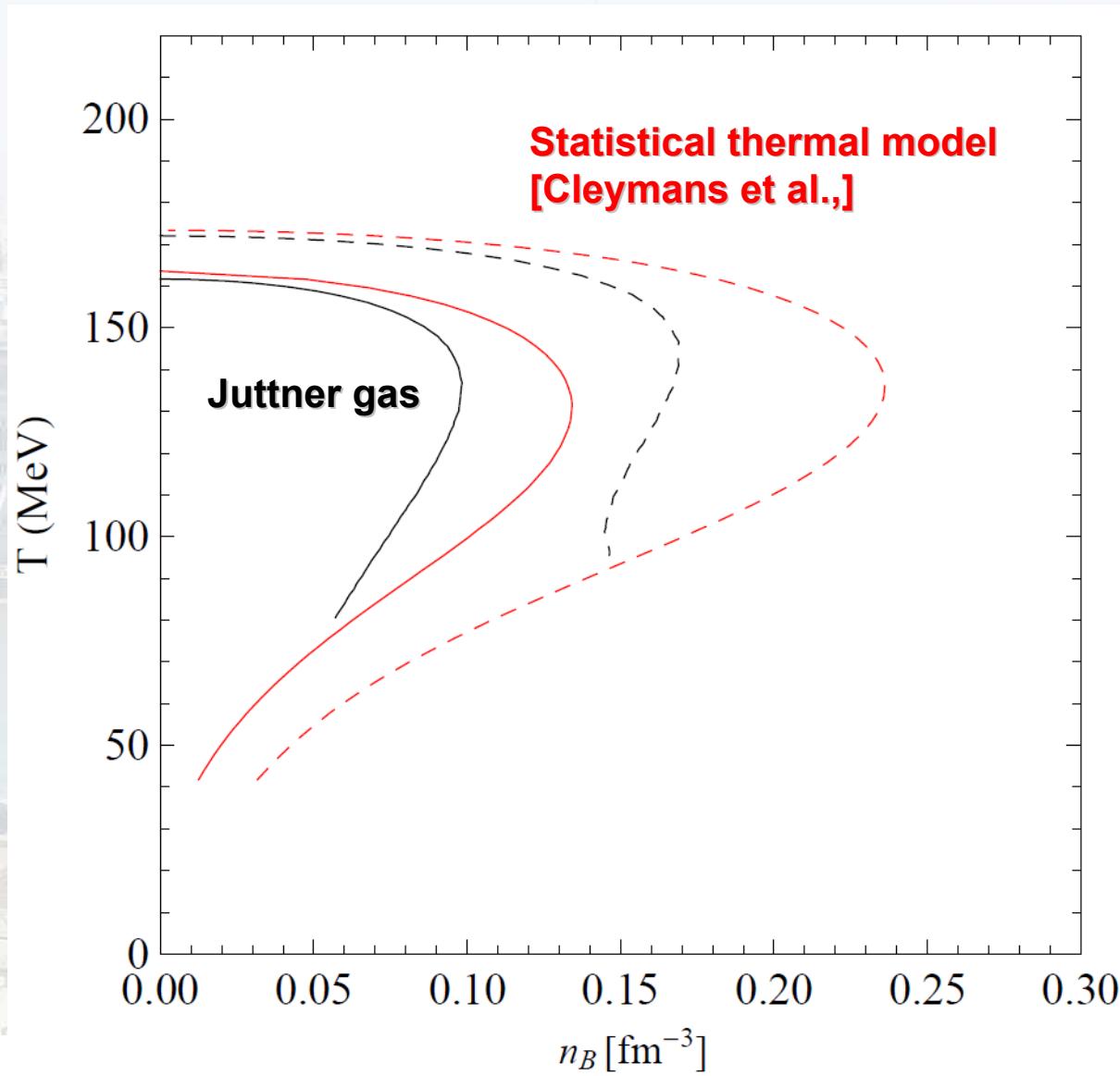
[Cleymans et al., PRL
81 (1998), PRC59
(1999), PRC73 (2006)]

Fig. 1. Results for the chemical freeze-out temperature and baryon chemical potential. Curves obtained for constant values of $E/N = 1.0$ (full line) and 1.1 GeV (dashed line) are also shown [12].

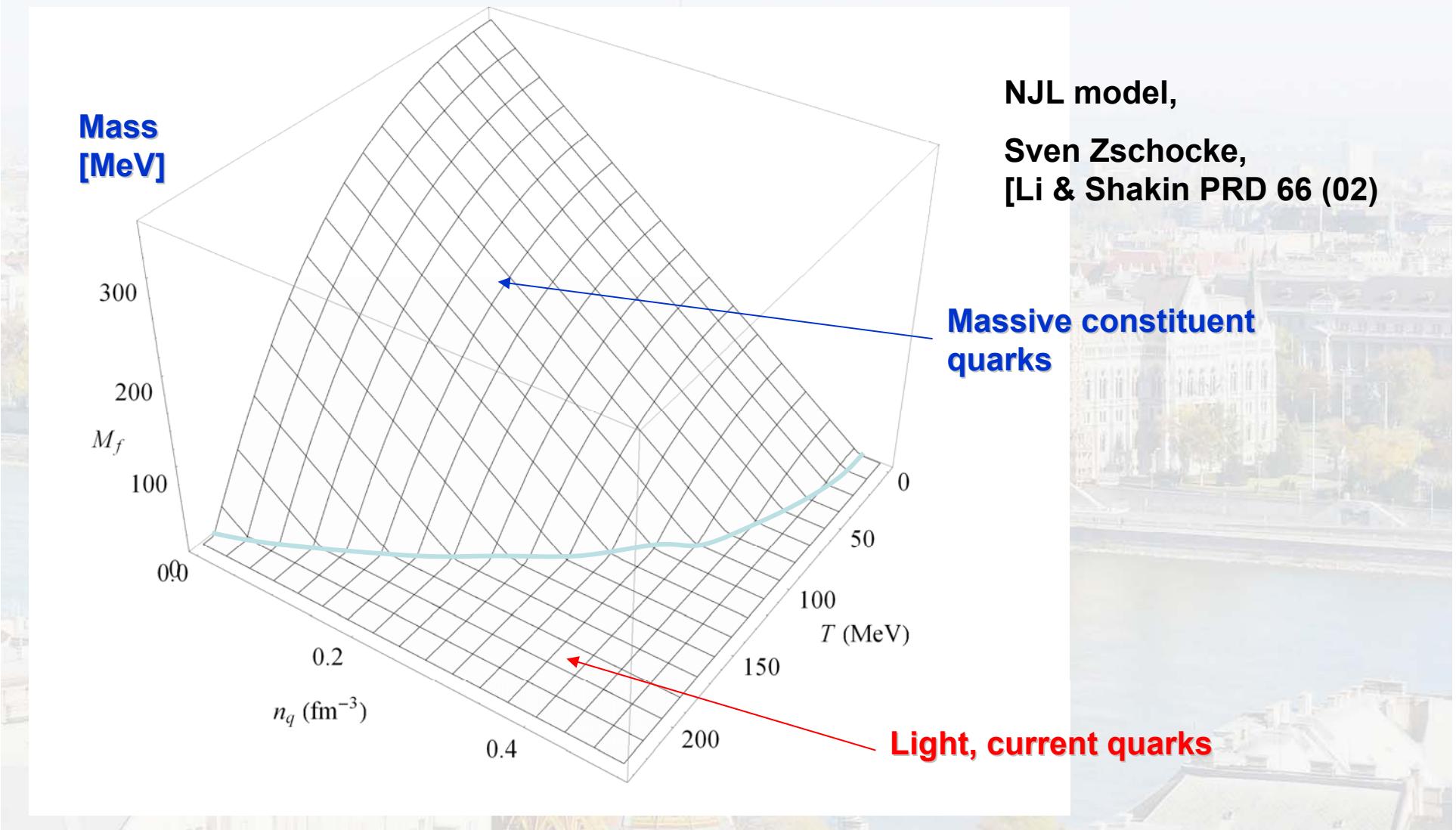
Observed Hadron FO



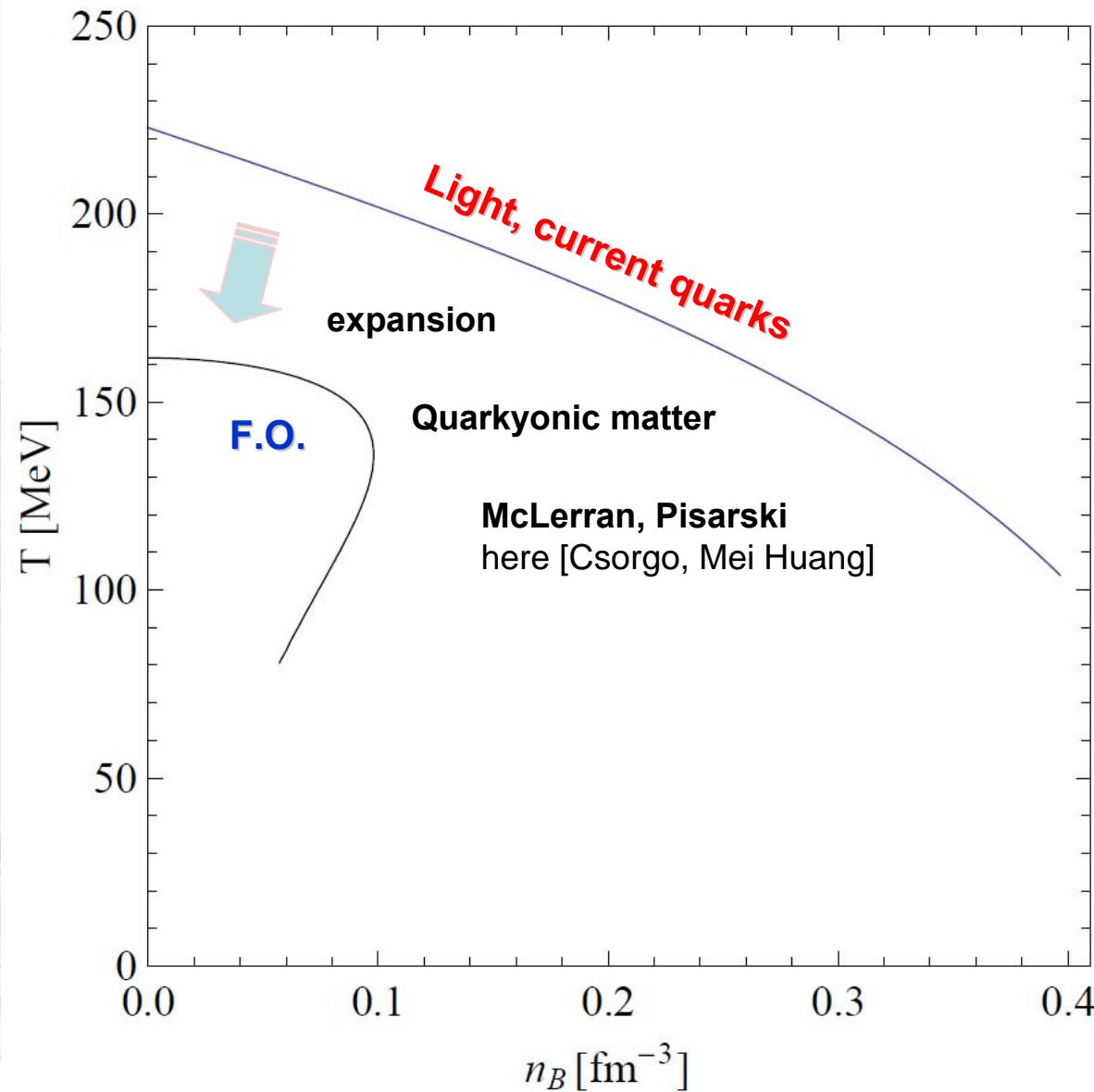
FO points on the T, n_B plane



Mass change of constituent quarks



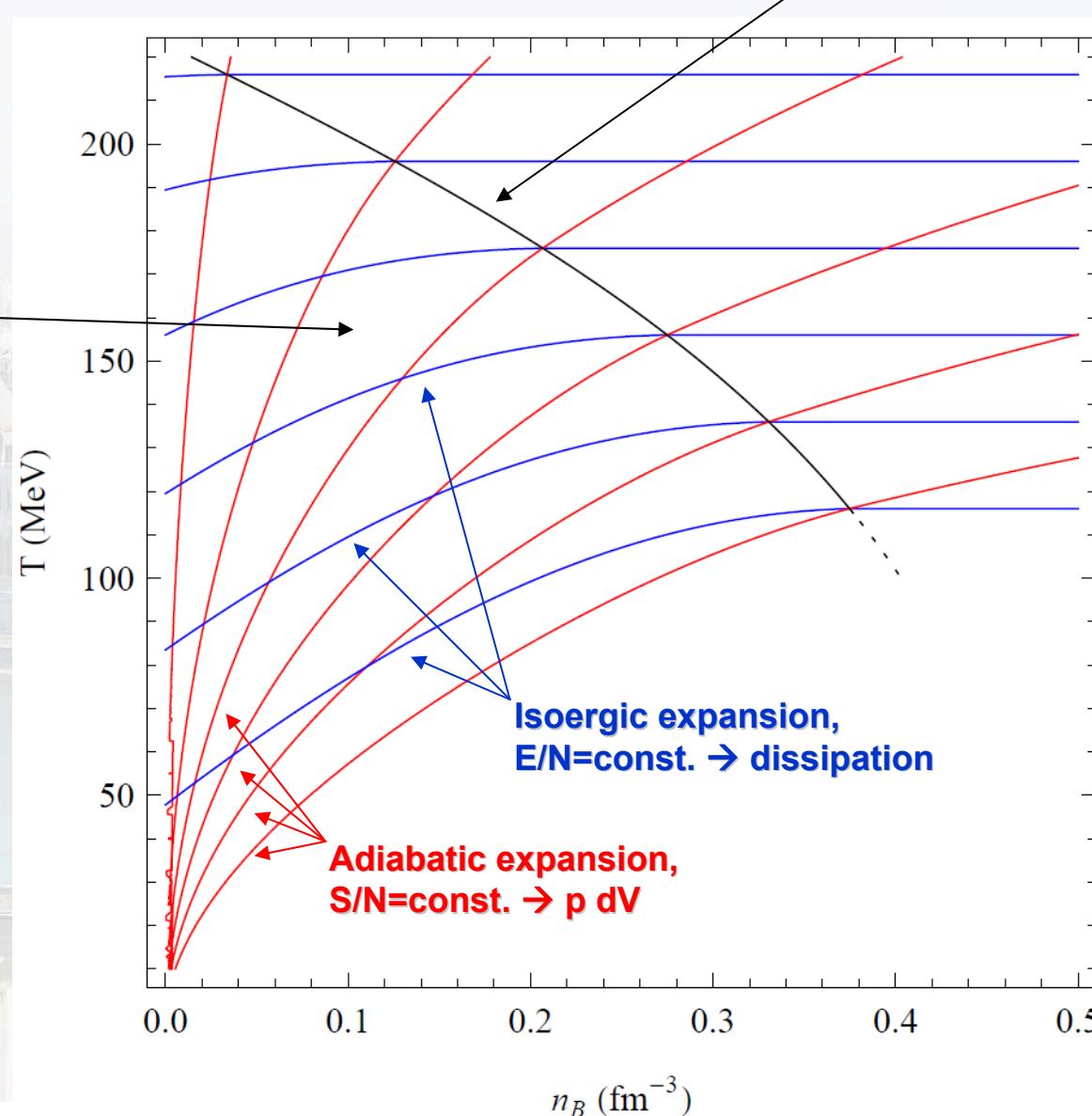
Expansion and mass gain



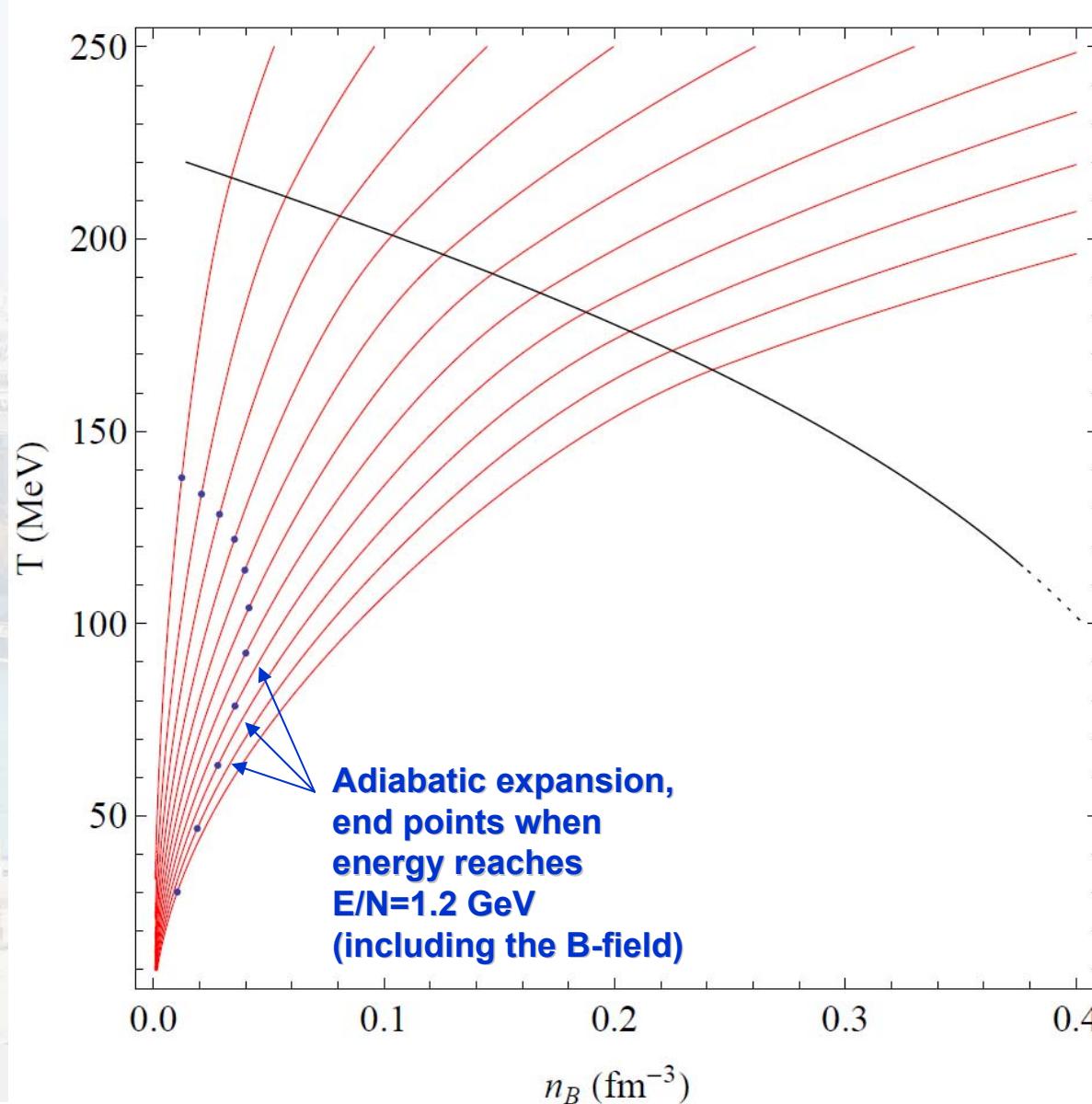
Expansion mechanisms

Light current quarks,
 nq freezes out

Constituent
quarks, mass
increases



End point of adiabatic expansion



Endpoints are still above the FO energy of $E_H/N_H \sim 1 \text{ GeV}$.

Viscous dissipation & rapid recombination to mesons and baryons, with using part of the latent heat,

can increase the final T to the observed FO temperatures.

Status of progress

- After chemical FO, we have break chiral symmetry and quark masses change
- Recombination to Hadrons (B & M) happens when Thermal eq. also breaks. →
- Temperatures of M & B will be different
- We have to adjust dissipation rates and recombination rates & then
- Check if the resulting distribution shows NCQ scaling

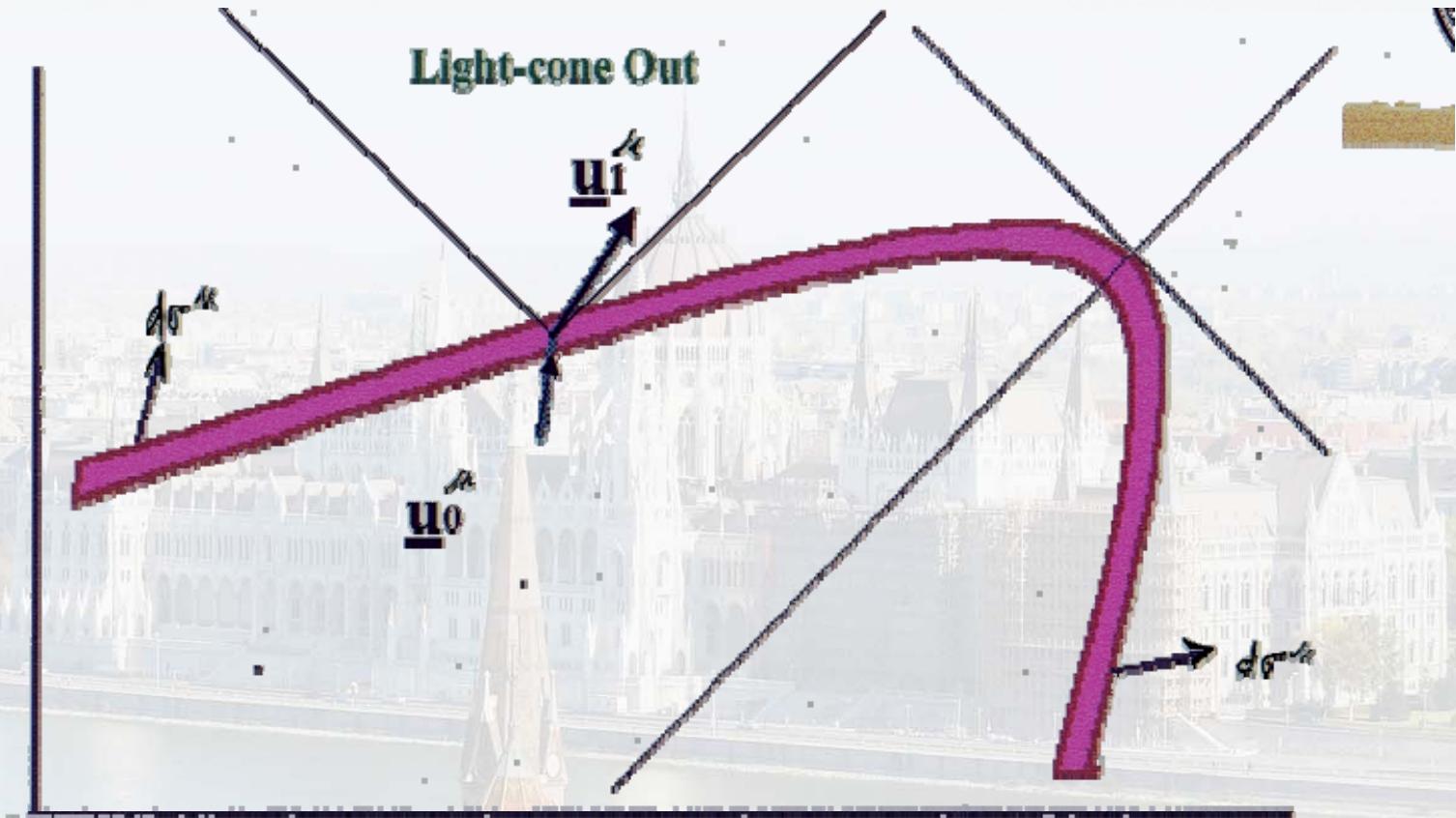
Freeze Out

Rapid and simultaneous FO and “hadronization”

- **Improved Cooper-Frye FO:**
- - Conservation Laws: $[T^{\mu\nu}\Lambda_\nu] = 0, [N^\nu\Lambda_\nu] = 0$ [L.P. Csernai,
Sov. JETP, 65 (1987) 216.]
- - Post FO distribution: $\Theta(p^\nu\Lambda_\nu) f(p) > 0$ [Cancelling Juttner or
Cut Juttner distributions.]
- **Hadronization ~ CQ-s**
- - Pre FO: Current q and \bar{q} , QGP
- - Post FO: Constituent q and \bar{q}
- - N_q and $N_{\bar{q}}$ are conserved in FO!!!
- **Choice of F.O. hyper-surface / layer**

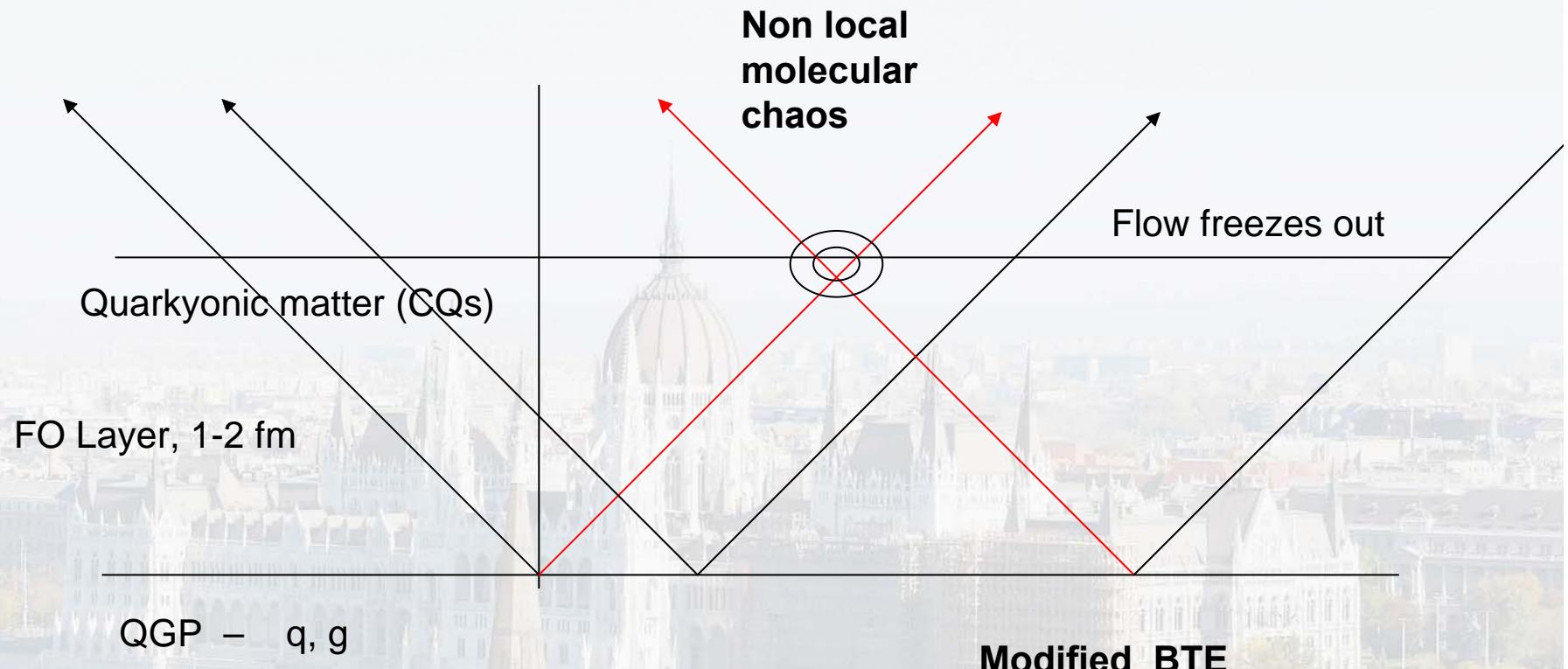
M3

Conservation Laws across hypersurface



- Energy-, Momentum-, Baryon- current
- [A. Taub, 1948, L.P. Csernai, 1987]

M3



In the FO layer the main free path increases, local molecular chaos assumption does not hold, (large effective viscosity)

Current quarks are gaining mass, while gluons are absorbed, forming constituent quarks (CQs) with mass, m_o . Final flow develops with joint flow velocity, u , for all CQs.

These then recombine to hadrons, in this process E_T is conserved but, p_t and u change depending on what hadrons are formed.

Matching stages of heavy-ion collision models

Yun Cheng,^{1,2,3,*} L. P. Csernai,^{1,2,4} V. K. Magas,⁵ B. R. Schlei,⁶ and D. Strottman^{2,7}

$$\begin{aligned} [N^\mu d\sigma_\mu] &= 0; \\ [T^{\mu\nu} d\sigma_\mu] &= 0; \\ [S^\mu d\sigma_\mu] &\geq 0, \end{aligned}$$

Taub adiabat [6,7],

$$j^2 = [P](d\sigma^\mu d\sigma_\mu)/[X], \quad [P] = [(e + P)X]/(X_1 + X_0).$$

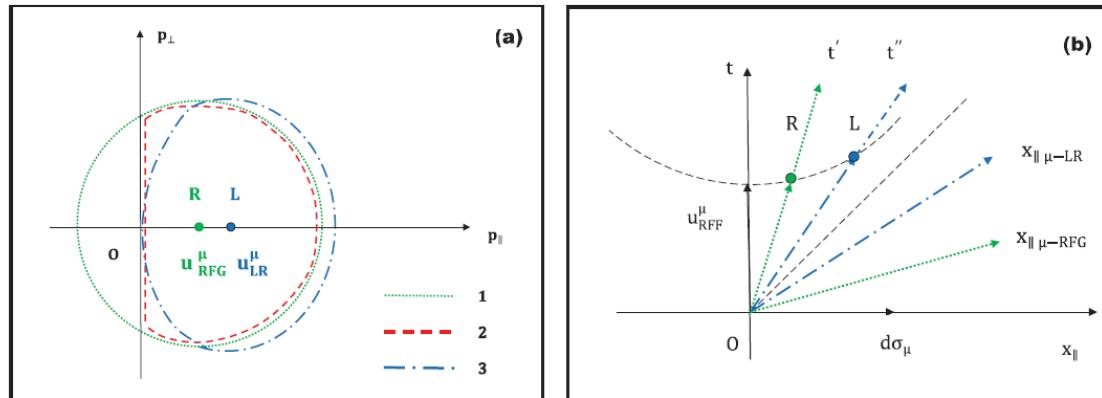
$$\underline{A_0^\mu A_{0\mu}} = (e - P)\underline{A_0^\mu d\sigma_\mu} + e P \underline{(d\sigma^\mu d\sigma_\mu)}, \quad (18)$$

which can be solved straightforwardly if the EoS, $P = P(n, e)$,

Spec. case: with an EoS of $P = e/3$, Eq. (18) leads to a quadratic equation

$$\underline{d\hat{\sigma}^\mu d\hat{\sigma}_\mu e^2} + 2\underline{a^\mu d\hat{\sigma}_\mu e} - 3\underline{a^\mu a_\mu} = 0,$$

where $a^\mu \equiv A_0^\mu/D$ is the energy momentum transfer four



FAIR

SUMMARY

- **Initial state** is decisive and can be tested by v1 & v2
 - v2 dominates in more peripheral collisions
- **Viscosity** is important both in hydro and in the initial dynamics
 - Numerical viscosity should be taken in correction
- **F.O.** : entropy condition → space like FO is weak at RHIC / LHC &
 - important at FAIR
 - → bulk viscosity limits space like F.O. \gg FAIR
- CNQ scaling indicates QGP, simplifies F.O. description to Const. Quarks.
This requires, however, Modified BTE description
- F.O. leads to acceleration ! (simplified approach eliminates this)



The END

