Hadronisation in a Parton Cascade Model with Non-additive Energy Composition

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with

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Hot and Cold Baryonic Matter

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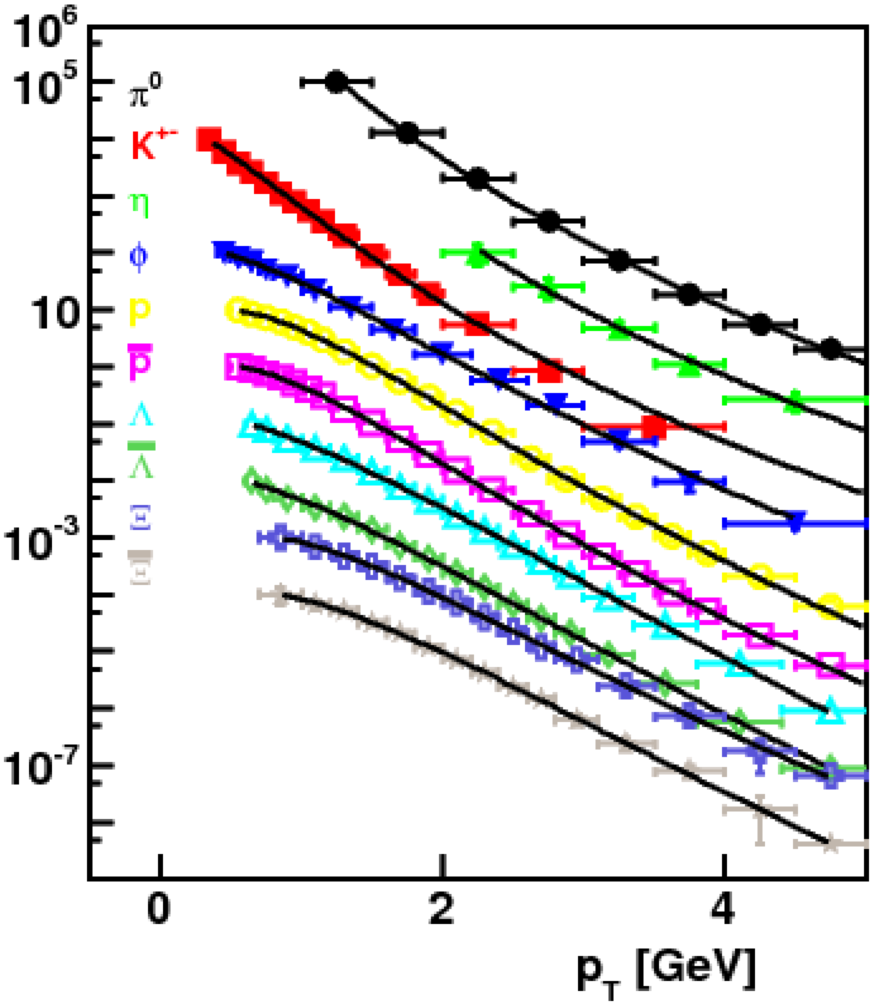
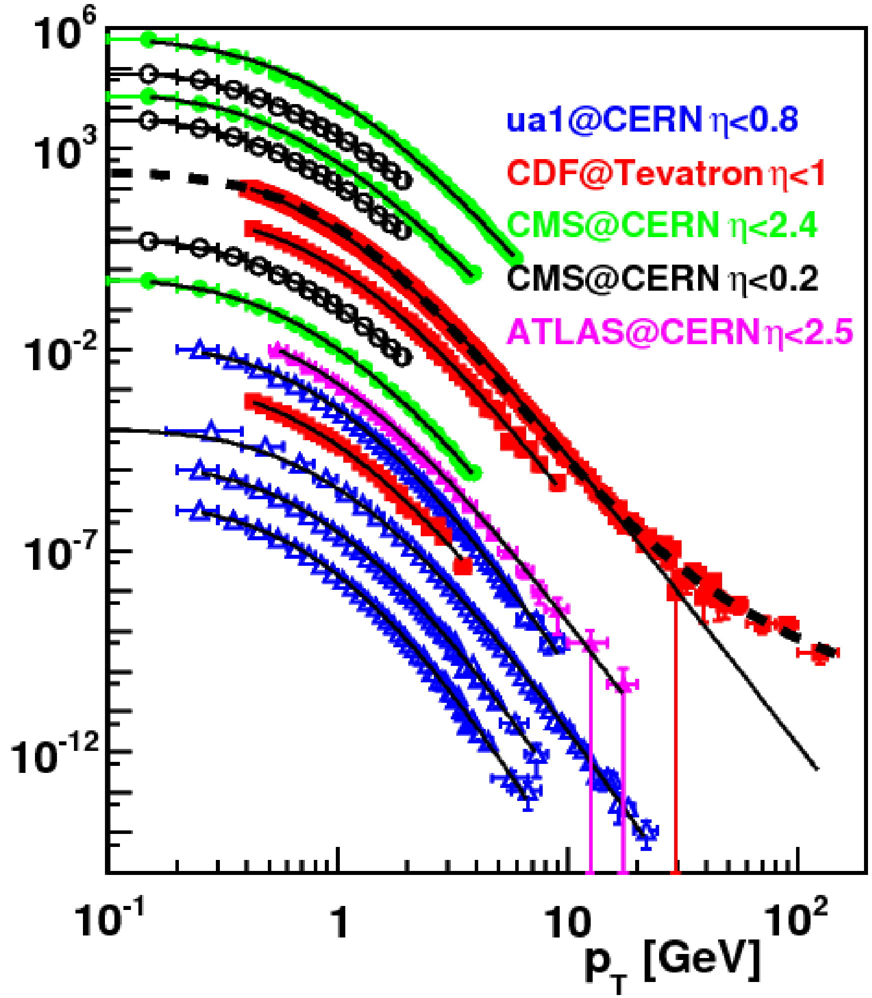
Appetiser

Particle spectra take *Cut power-law* shape:

AuAu --> h+X
at *√s = 200 GeV*



pp --> h$^\pm$+X
at *√s = 0.2–2.36 TeV*



Menu:

- ***Thermal quark recombination with Tsallis distribution in AuAu collisions***
- ***Generalised statistics (based on the Theorem of Large Deviations) resulting in Tsallis distribution***
- ***π spectrum from a non-extensive 'quasi-quark cascade model' simulation***

Thermal Quark Recombination with Tsallis Distribution in AuAu Collisions

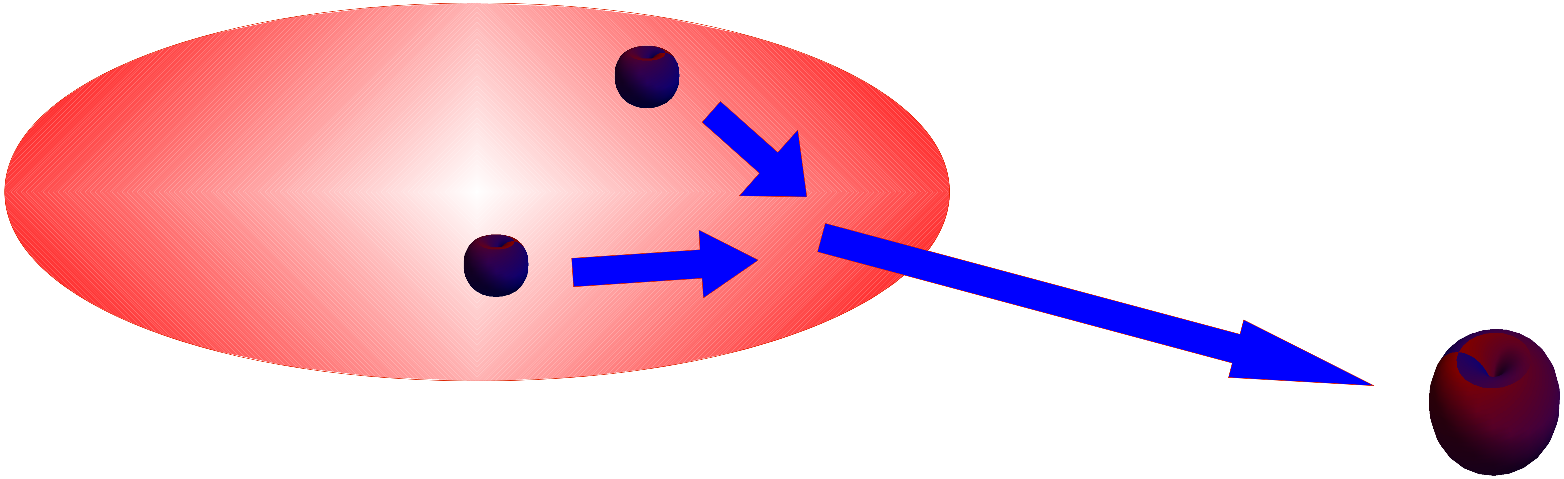
Thermal Re-hadronisation By Quark Coalescence

n **on-shell quarks produce a hadron very rapidly:**

$$F_h(P_h) = \int \prod d^3 p_i \;\; f_q(E_1) \dots f_q(E_n) C(p_i, P_h, M_h)$$

The hadron formation requires ***energy-momentum conservation*** **and that the incoming quarks have** ***small relative momenta***

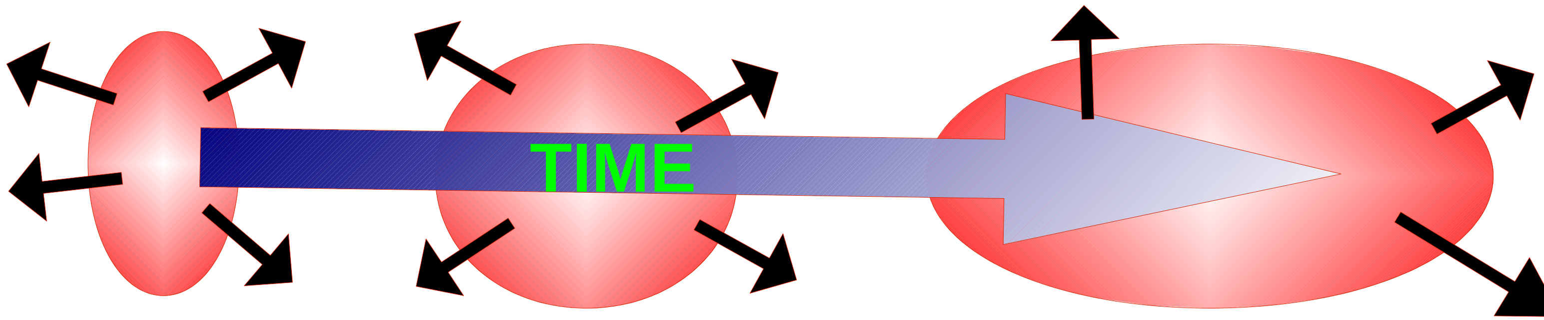
$$C(p_i, P_h, M_h) = \delta^4\left(\sum p_i^\mu - P_h^\mu\right) \prod \delta^3\left(\vec{p}_i - \vec{P}_h / n\right)$$



Hadron Evaporation from the Expanding Plasma

The 3-volume of the QGP evolves in time and spits out hadrons:

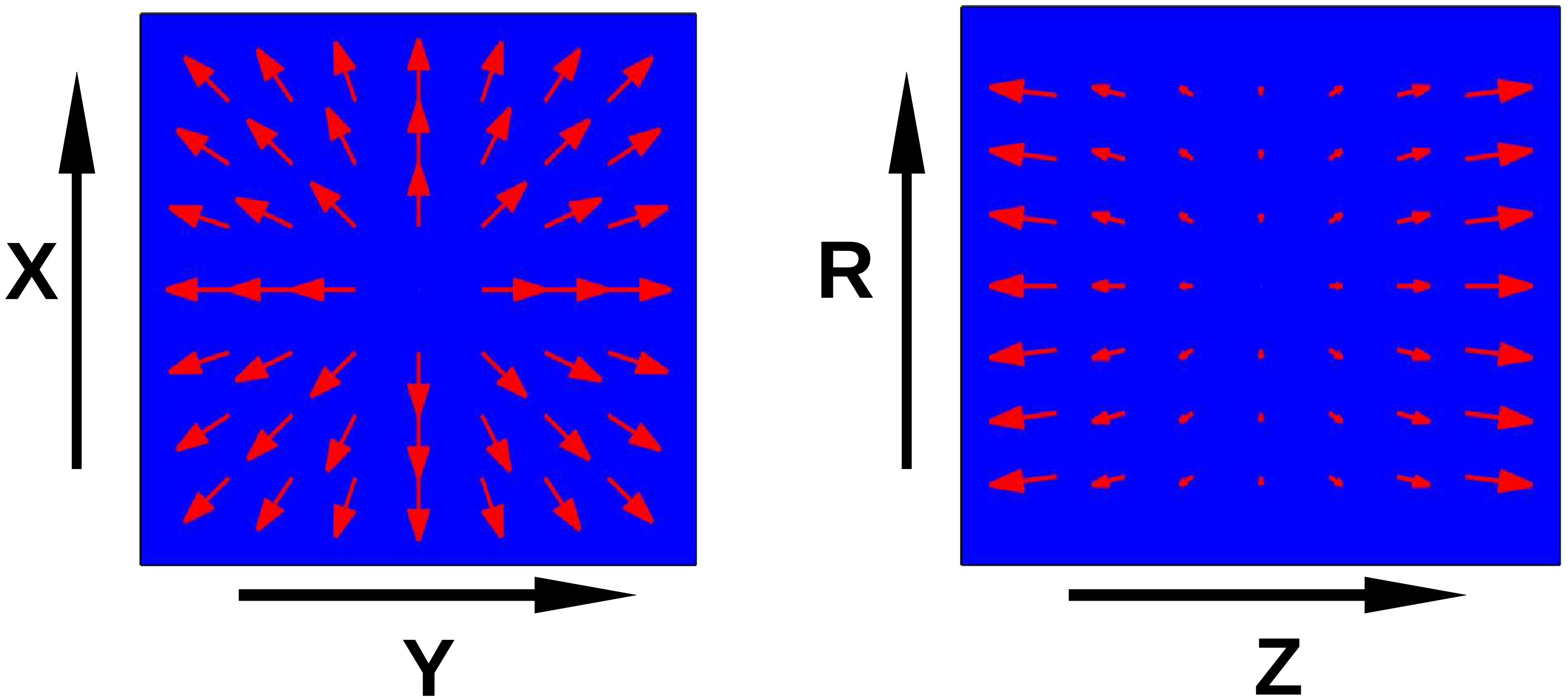
$$V_3^{QGP}(t) \rightarrow \Sigma^{\mu}$$



The number of hadrons leaving the plasma with momentum, p^{μ} is the flux of the hadron current through the hadronisation hyper-surface.

$$E\frac{dN}{d^3p} = \int d\Sigma^{\mu} J_{\mu}$$ **with hadron current** $$J^{\mu} = \frac{p^{\mu}}{p^0} F_h(E_h)$$

Flow Profile of the Expanding Plasma



Spacetime coordinates: $r\,,\ \ \alpha\,,\ \ \zeta = \frac{1}{2}\ln\left(\frac{t-z}{t+z}\right),\ \ \tau = \sqrt{t^2 - z^2}$

Flow profile:

$$u^{\mu}(\zeta,\alpha) = (\gamma\cosh\zeta\,,\gamma\sinh\zeta\,,\gamma\,v\cos\alpha\,,\gamma\,v\sin\alpha),\ \ \gamma = \frac{1}{\sqrt{1-v^2}}$$

The reasons for this type of parametrisation, and the choice of such a flow profile are

- **Cylindrical symmetry $\rightarrow$ r, α**

- **In non-viscous hydro temperature is a function of proper time T(s)**

- **The expansion is longitudinally dominated, so**

$$s=\sqrt{t^2-r^2-z^2} \;\rightarrow\; \tau=\sqrt{t^2-z^2}$$

  is a good approximation.

- **It is reasonable to say, the plasma evaporates hadrons from its surface, until its whole volume hadronises abruptly at a certain longitudinal propertime, when its temperature**

$$T(\tau_0)=T_c$$

Parametrisation of the spacetime, flow and hadron momentum

$$x^\mu = (\tau \cosh\zeta, \tau \sinh\zeta, r\cos\alpha, r\sin\alpha)$$

$$u^\mu = (\gamma\cosh\eta, \gamma\sinh\eta, \gamma v\cos\phi, \gamma v\sin\phi)$$

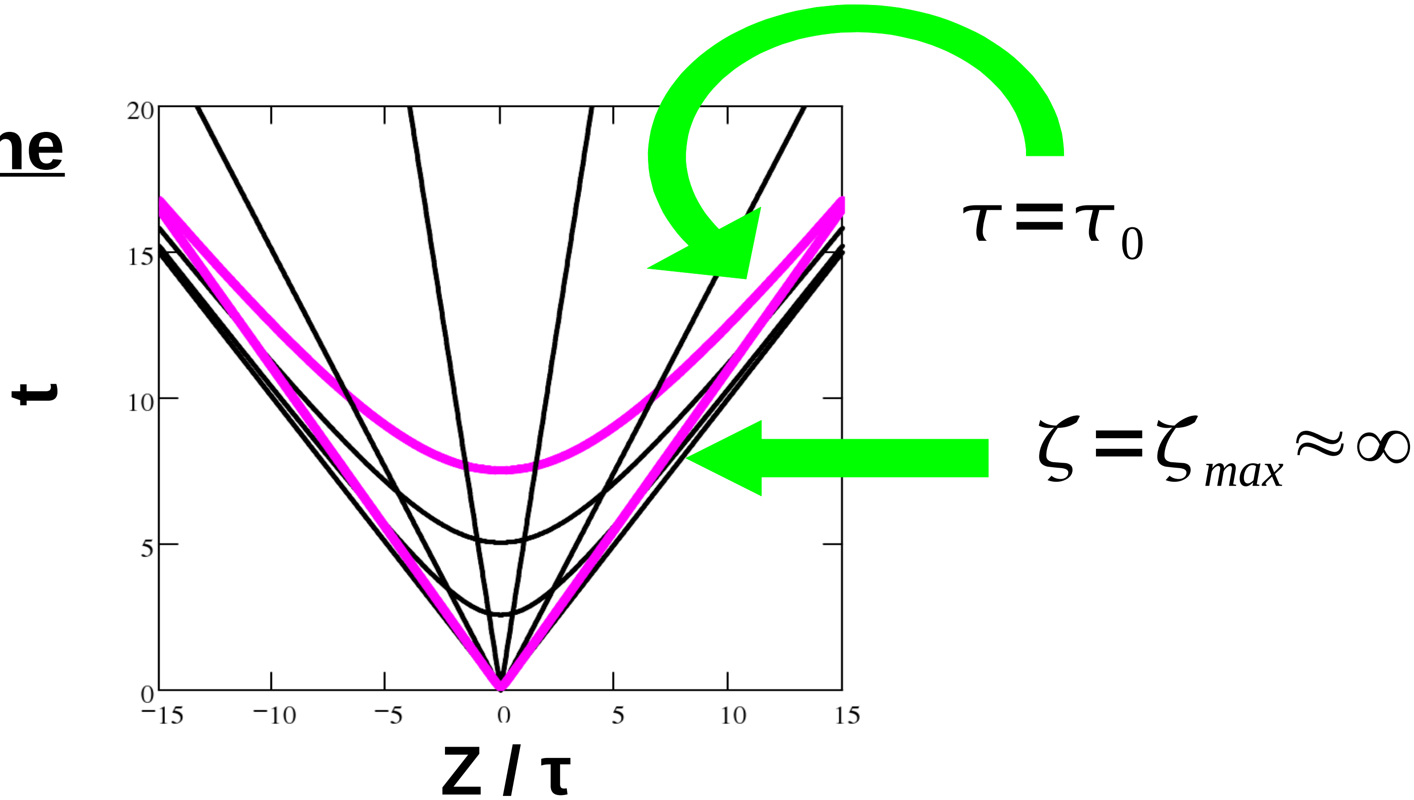
$$p^\mu = (m_T \cosh y, m_T \sinh y, p_T \cos\varphi, p_T \sin\varphi)$$

The integration measure in the hadron yield calculation:

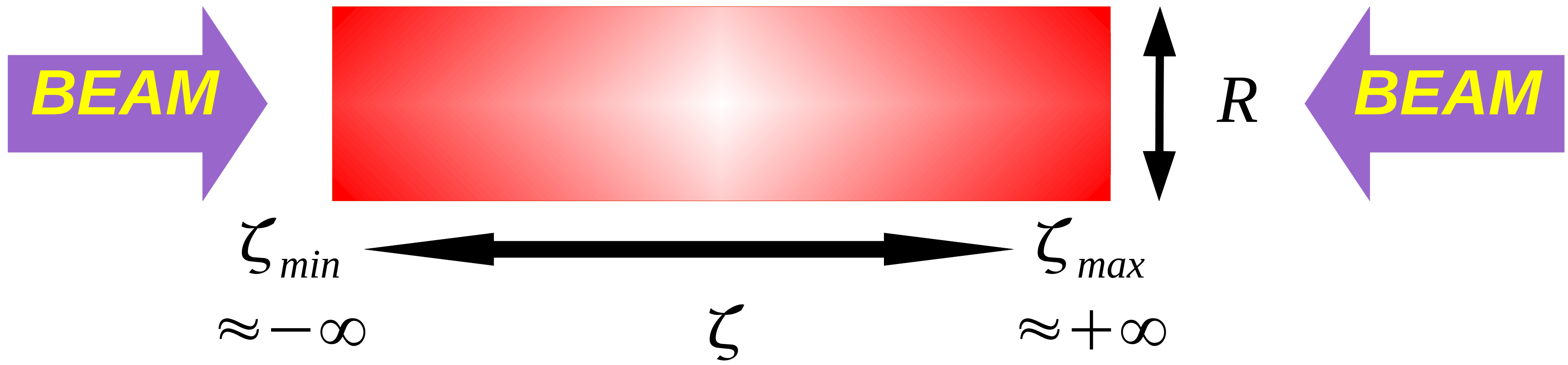
$$\begin{aligned} p_\mu d\Sigma^\mu = {} & m_T \cosh(y-\zeta)\tau r\, d\zeta\, dr\, d\alpha \\ & + m_T \sinh(y-\zeta) r\, d\tau\, dr\, d\alpha \\ & + p_T \cos(\varphi-\alpha)\tau r\, d\tau\, d\zeta\, d\alpha \\ & + p_T \sin(\varphi-\alpha)\tau\, d\tau\, d\zeta\, dr \end{aligned}$$

The Hadronisation hyper-surface

In the t-z plane



In the r-ζ plane



Used thermal hadron distributions:

1. $F_h(E) = A\, e^{-\beta E}$ ***Boltzmann-Gibbs***
2. $F_h(E) = A\left(1+(q-1)E/T\right)^{-1/(q-1)}$ ***Cut power-law***

with *E*, the hadron energy in the frame co-moving with the plasma flow,

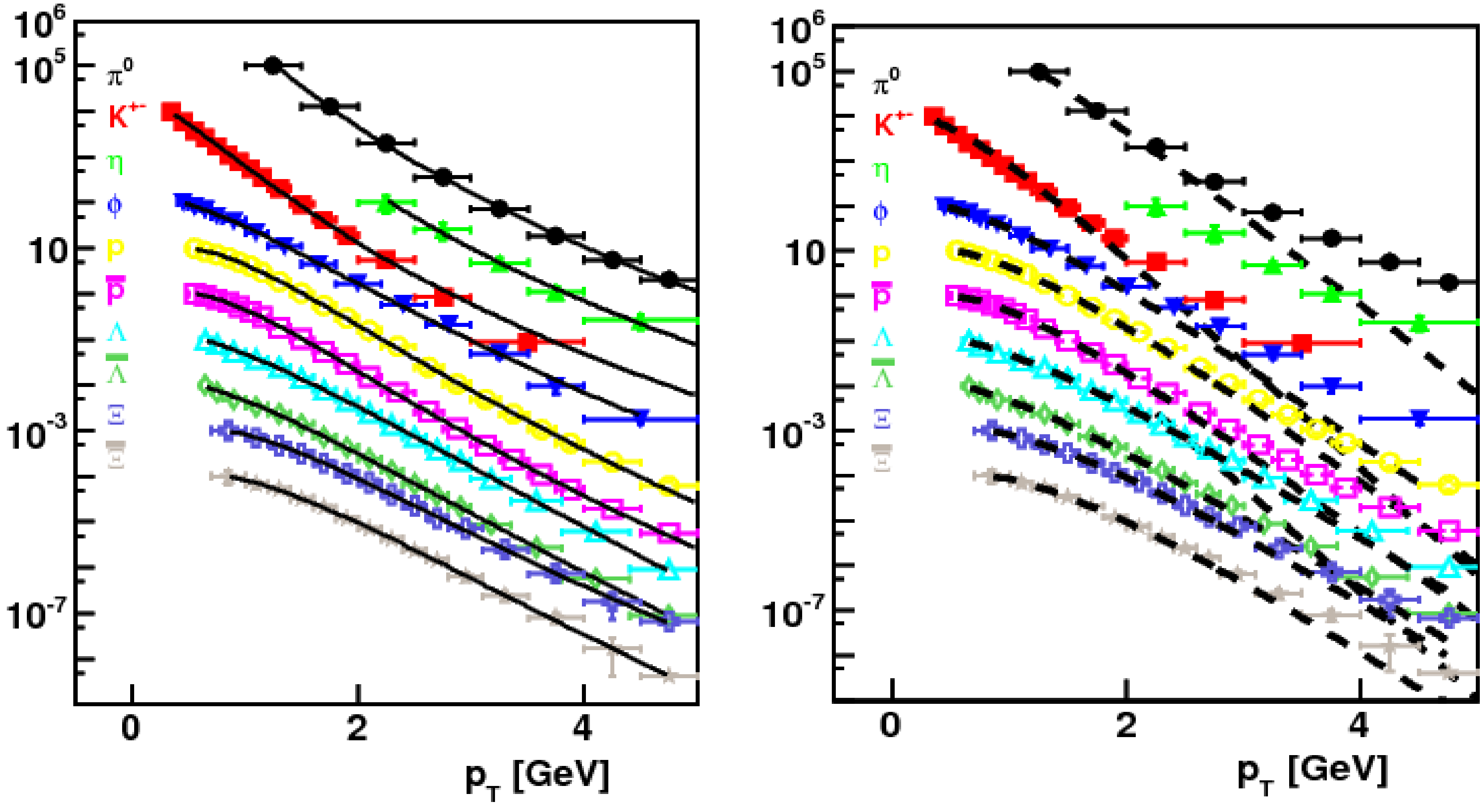
$$E = p_\mu u^\mu$$

The resulting transverse hadron yielsd are:

1. $$p^0 \frac{dN}{d^3p} \sim \xi\, m_T K_1(\beta\gamma m_T) I_0(\beta\gamma v p_T) + (1-\xi)\, p_T K_0(\beta\gamma m_T) I_1(\beta\gamma v p_T) \sim e^{-\beta\gamma(m_T - v p_T)}$$

2. $$p^0 \frac{dN}{d^3p} \sim \frac{\xi\, m_T G_0(p_T) + (1-\xi)\, p_T G_2(p_T)}{\left(1+(q-1)\beta\gamma(m_T - v p_T)\right)^{1/(q-1)}} \sim p_T^{-q/(q-1)}$$

Cut power-law **and** ***exponential fits*** **to identified particle spectra, produced in** ***AuAu --> h+X*** **at** ***√s = 200 GeV*** **.**



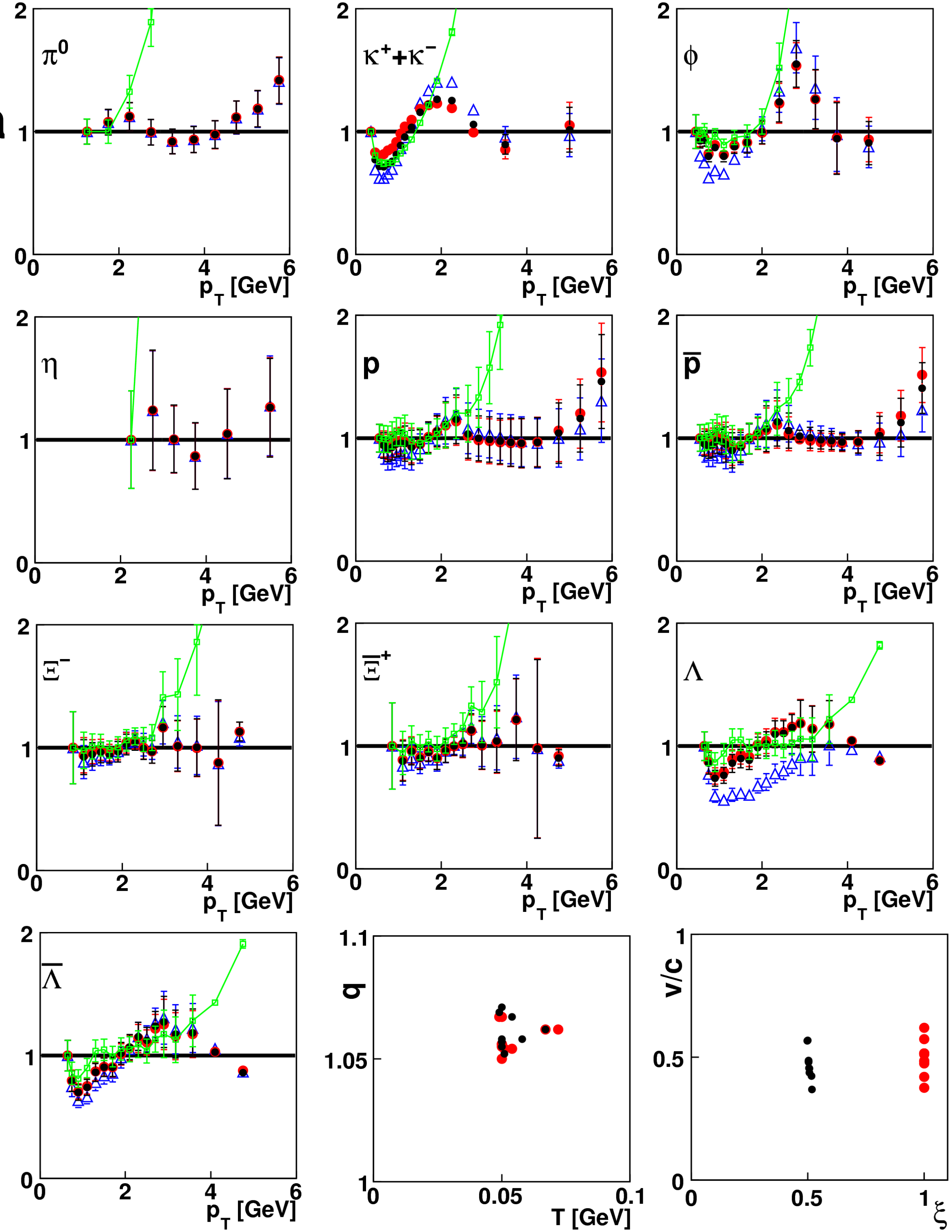
Data / Theory plots for Identified particle spectra In *Au + Au - > h + X* Reactions at
$s = (200\ \text{GeV})^2$
Collision energy

■ **Boltzmann–Gibbs**

▲ **power-law, surface terms only, $\xi = 0$.**

● **power-law, volume terms only, $\xi = 1$.**

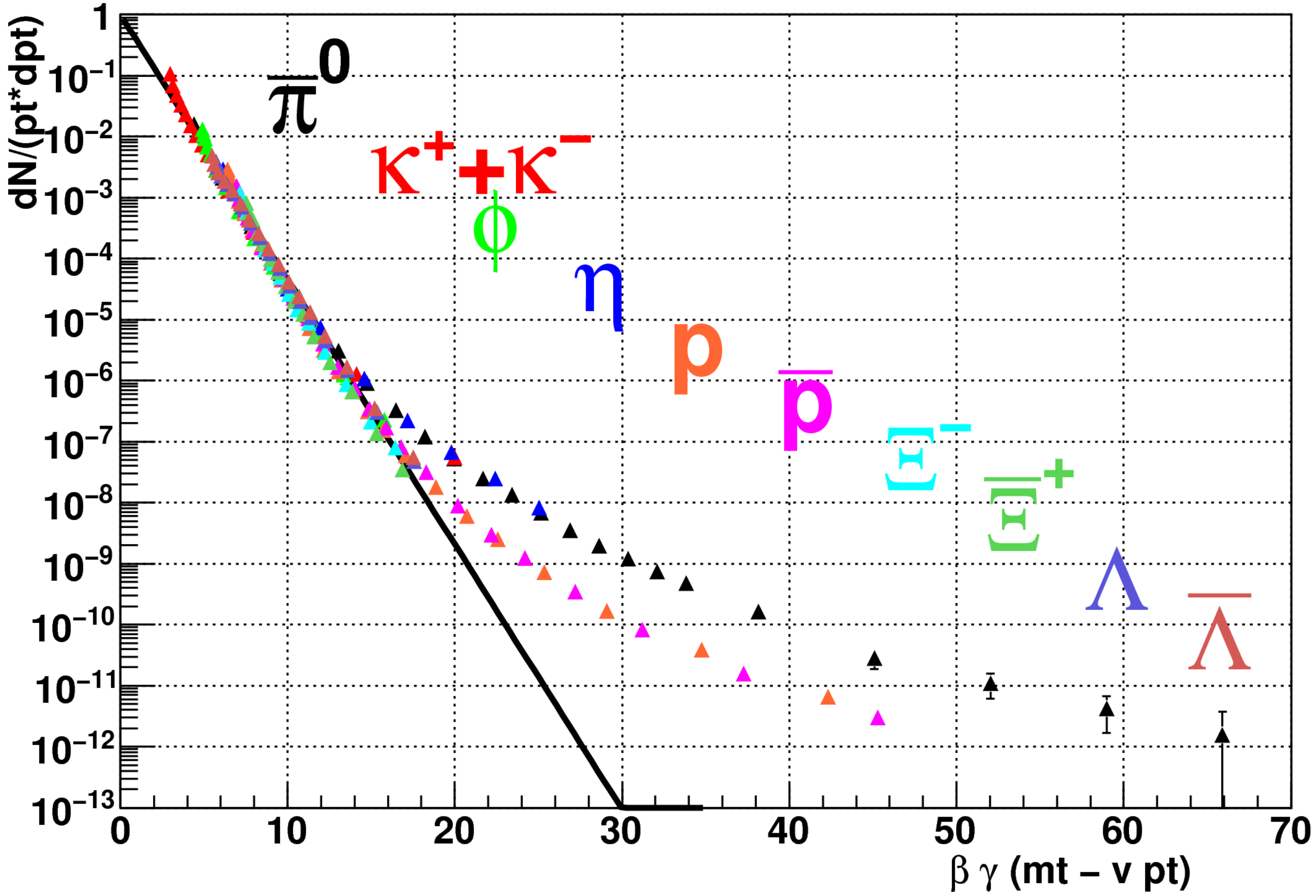
● **power-law, ξ = surface/volume ratio is fitted. ($\xi \approx 0.5$)**



K. Urmossy, T. S. Biro, *Phys. Lett. B*, **689**: 14-17, 2010.

Boltzmann-Gibbs Fits to Au Au - > h X at s = (200 GeV)^2

T = 51 ± 10 MeV, q = 1.062 ± 7.65 × 10−3, v = 0.5 ± 0.1



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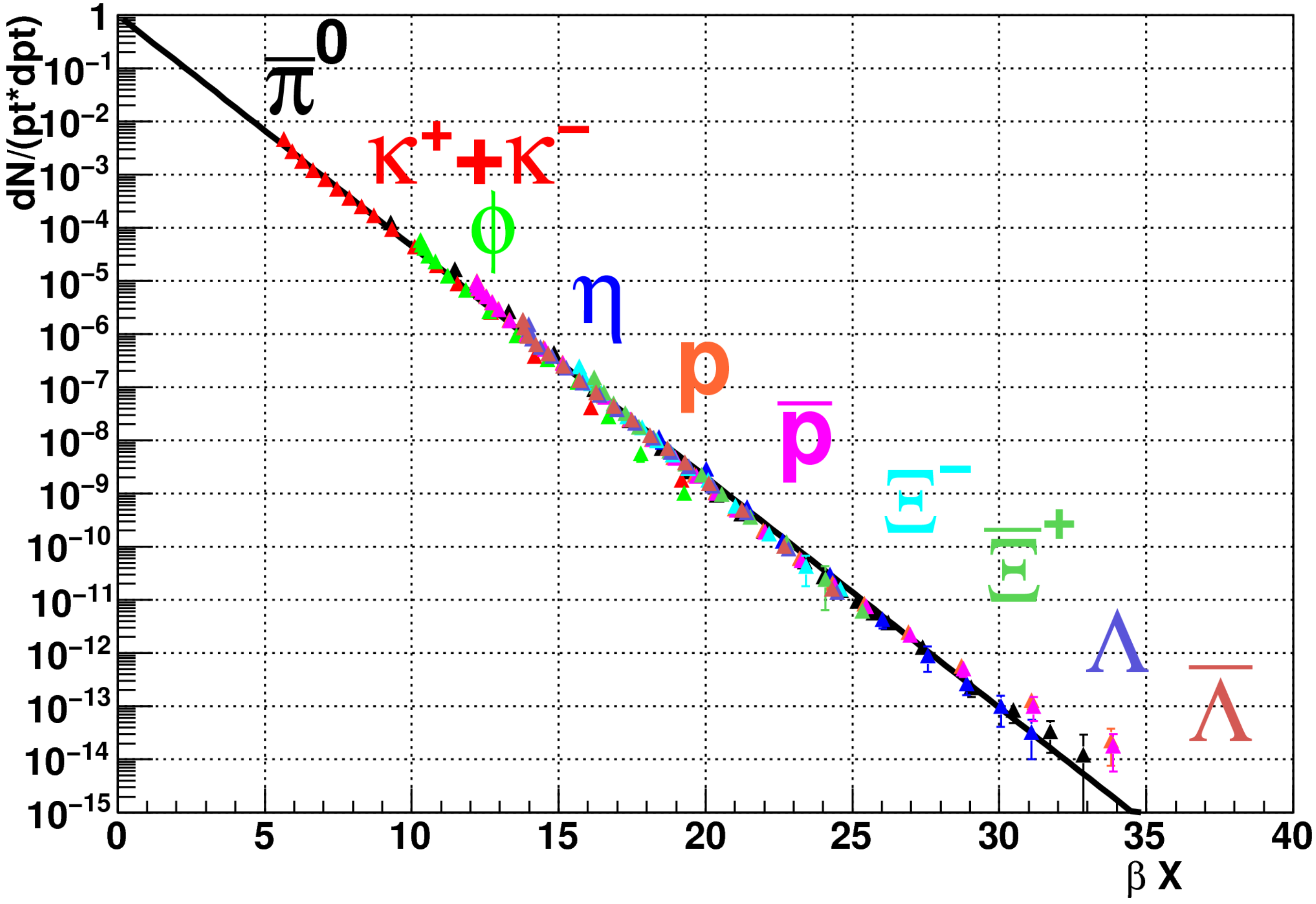


$$X = \frac{1}{q-1} \log\left[1 + (q-1)\,\gamma\,(m_T - v\,p_T)/T\right]$$

Generalised statistics (based on the Theorem of Large Deviations) resulting in Tsallis distribution

The 1-particle distribution of quarks in the QGP

1, Choose ensamble!

The QGP produced in a heavy-ion collision is NOT in equilibrium with its surroundings or with any heat bath. So if it is ergodic and is in equilibrium, it is *micro-canonical* with density operator $\hat{\rho} = \delta\left(\hat{H} - E_0\right)$

***2,* So the 1-particle distribution, we look for is**

$$f\left(\epsilon_k\right) = \langle a_k^\dagger a_k \rangle = \int D\Psi \, a_k^\dagger a_k \int_{-\infty}^{+\infty} ds \, e^{is\left\{H[\Psi] - E_0\right\}} = \ldots\ldots$$

***3,* Instead, we try to model the system with *statistically independent quasiparticles*, and call for the help of probability theory.**

Theorem of Large Deviations

Let ξ_i be *independent, identically distributed* random variables, representing the single particle energies

$$f(\epsilon) = p\left(\xi_1 = \epsilon \mid \sum_1^N \xi_i = E\right) = e^{-\beta\epsilon}/Z, \qquad \beta \leftarrow \frac{E}{N} = \int \epsilon f(\epsilon)$$

It is a pure mathmatical statment. It does not involve the introduction of any sort of entropy formula.

No need for equilibrium ;-)

Max. Entropy principle gives the same 1-PD.

$$-\int f \ln f - \beta\left(\int \epsilon f(\epsilon) - E/N\right) = min \;\rightarrow\; f(\epsilon) \sim e^{-\beta\epsilon}$$

The use of TLD for interacting particles

If we can express interactions with an associative rule among single particle energies

$$E_{12}=h(E_1,E_2) \quad \rightarrow \quad E_N=h\circ h\circ\ldots\circ h(E_1,E_2,\ldots,E_N)$$

Such a rules can be turned into a additive one

$$L(E_N)=L(E_1)+\ldots+L(E_N)$$

by a monotonic function ('Formal Logarithm'), *L(E), and the TLD holds for the new variable* $\eta=L(\xi)$

$$p\left(\xi_1=\epsilon \mid h\circ\ldots\circ h(\xi_1,\ldots,\xi_N)=E\right)$$
$$= \; p\left(\eta_1=L(\epsilon) \mid \sum\eta_i=L(E)\right)$$
$$= \; e^{-\beta\eta}/Z=e^{-\beta L(\epsilon)}/Z, \qquad \beta\leftarrow\frac{L(E)}{N}=\int L(\epsilon)f(\epsilon)$$

The Max Entropy principle gives us the same result

$$-\int f \ln f - \beta\left(\int L(E) f(E) - L(E_0)/N\right) = min \rightarrow f(E) = \frac{e^{-\beta L(E)}}{Z(\beta)}$$

Examples for energy addition rules (*aE_1E_2: interaction*)

1) $E_1 \oplus E_2 = E_1 + E_2 \;\rightarrow\; L(\epsilon) = \epsilon \;\rightarrow\; f(\epsilon) \sim e^{-\beta\epsilon}$ **Boltzmann**

2) $E_1 \oplus E_2 = E_1 + E_2 + aE_1E_2 \;\rightarrow\; L(\epsilon) = \frac{1}{a}\log(1+a\epsilon)$

$\rightarrow\; f(\epsilon) \sim (1+a\epsilon)^{-1/\beta a}$ **Tsallis when**

$a = (q-1)/T$

Temperature and mean 1-particle energy

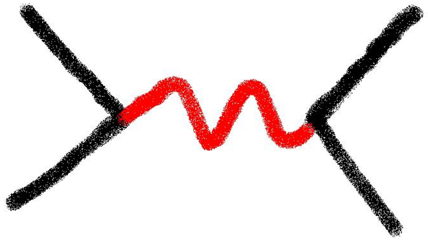
$$3T = \frac{E/N}{1+a\; E/N}$$

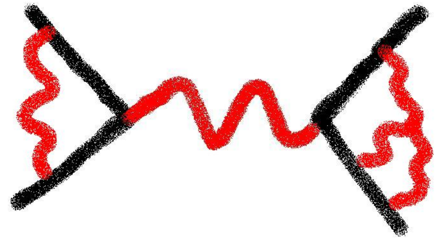
Non-extensivity

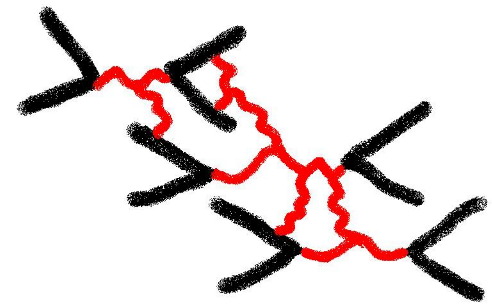
$$E \sim L^{-1}(N L(\bar{\epsilon})) \rightarrow \frac{(a\bar{\epsilon})^N}{a}$$

π Spectrum From a Non-extensive 'Quasi-Quark Cascade Model' Simulation

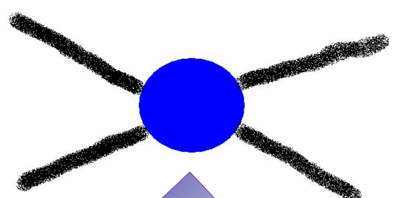
Instead of struggling with perturbation







We try out a simple rule

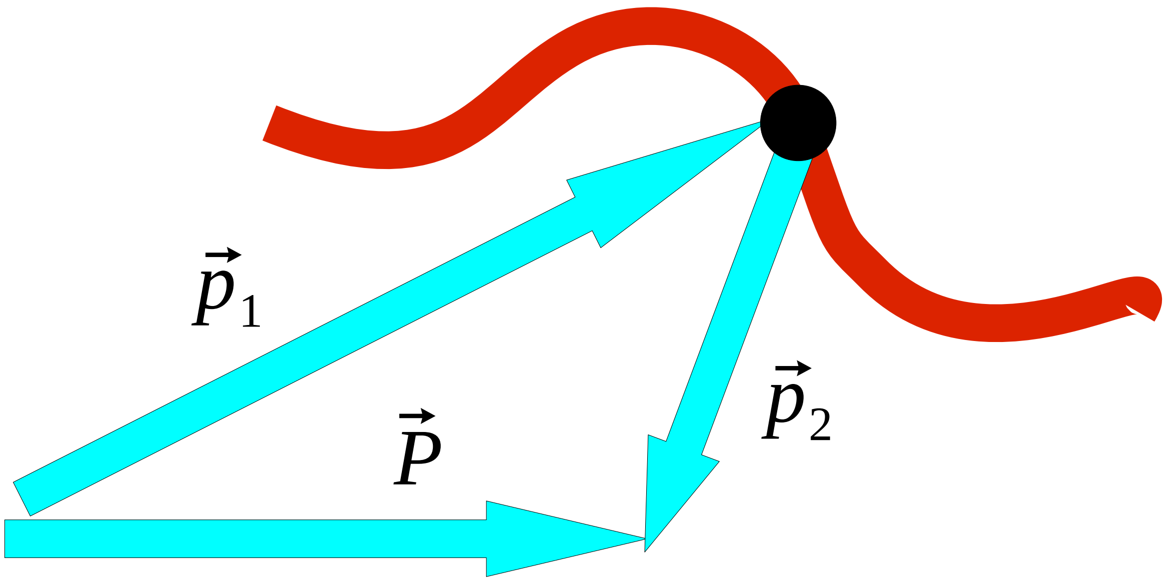


$$\delta\left(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4\right)$$
$$\times\delta\left(E_1\oplus E_2-E_3\oplus E_4\right)$$

With

$$E_1\oplus E_2=$$
$$=E_1+E_2+aE_1E_2$$

For massless out going particles this means



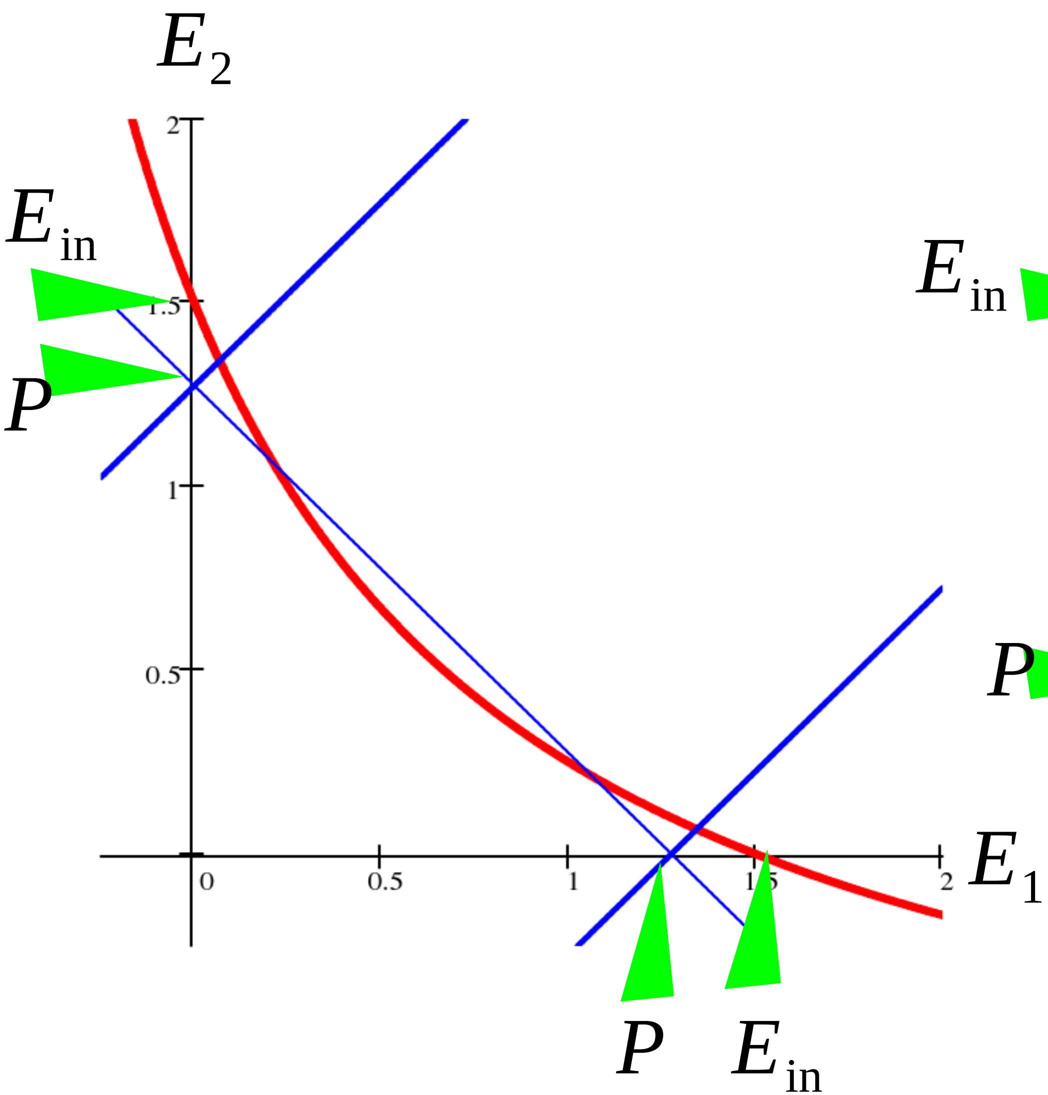
Along this line

$$E_{out} = p_1 + p_2 + a\, p_1\, p_2$$
$$= E_{\mathrm{in}} = const$$

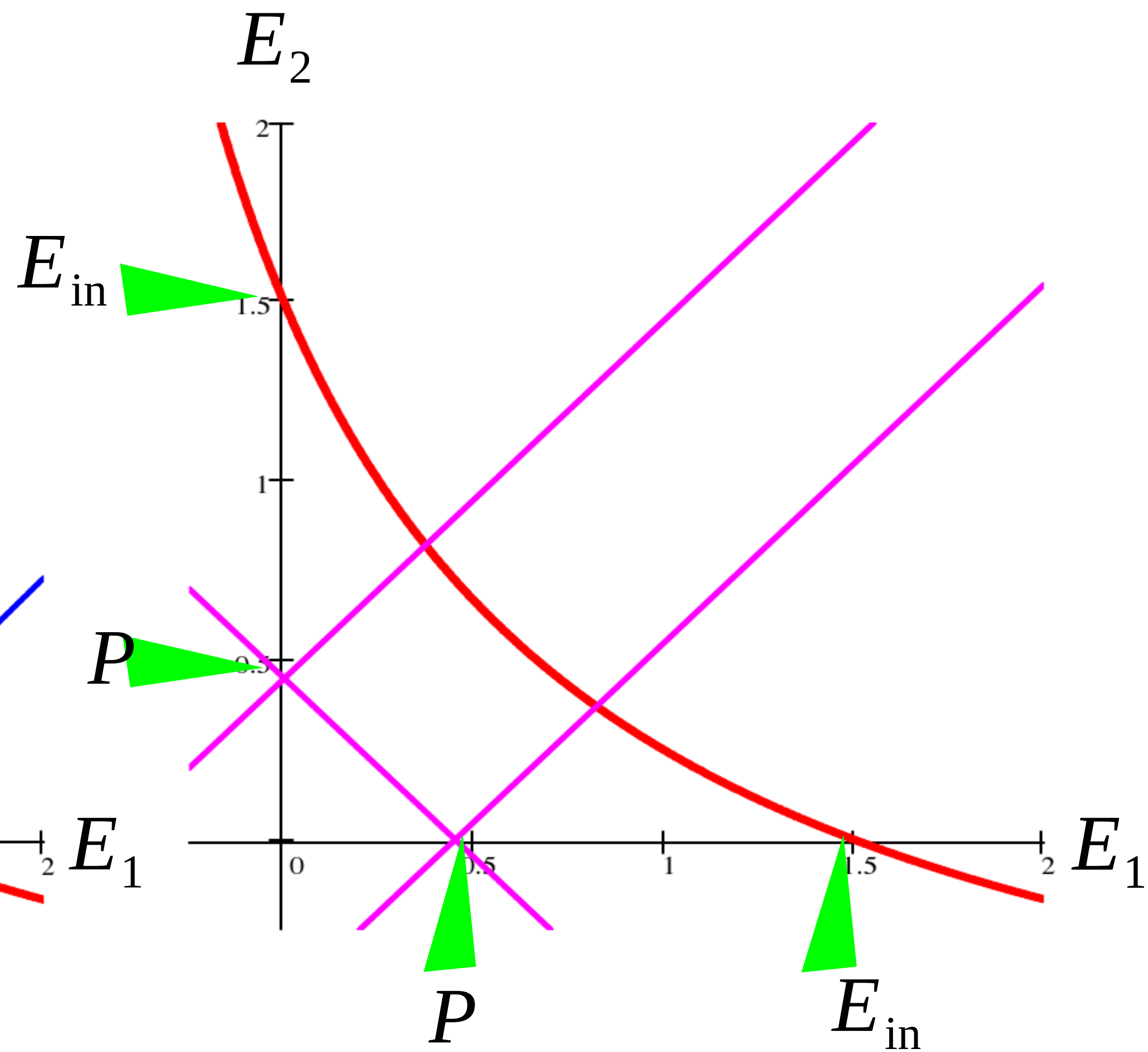
The phase space of the out going particles is

$$d^2 w = w_0\, d^3 p_1\, d^3 p_2\, \delta(\vec{p}_1 + \vec{p}_2 - \vec{P})\, \delta(E_1 \oplus E_2 - E_{\mathrm{in}})$$
$$= w_0 \frac{E_1 (E_{\mathrm{in}} - E_1)}{P(1 + a E_1)^2} dE_1\, d\phi$$

When E1 is fixed, $E_{in} = E_1 + E_2 + a E_1 E_2 \rightarrow \quad E_2 = \dfrac{E_{in} - E_1}{1 + a E_1}$

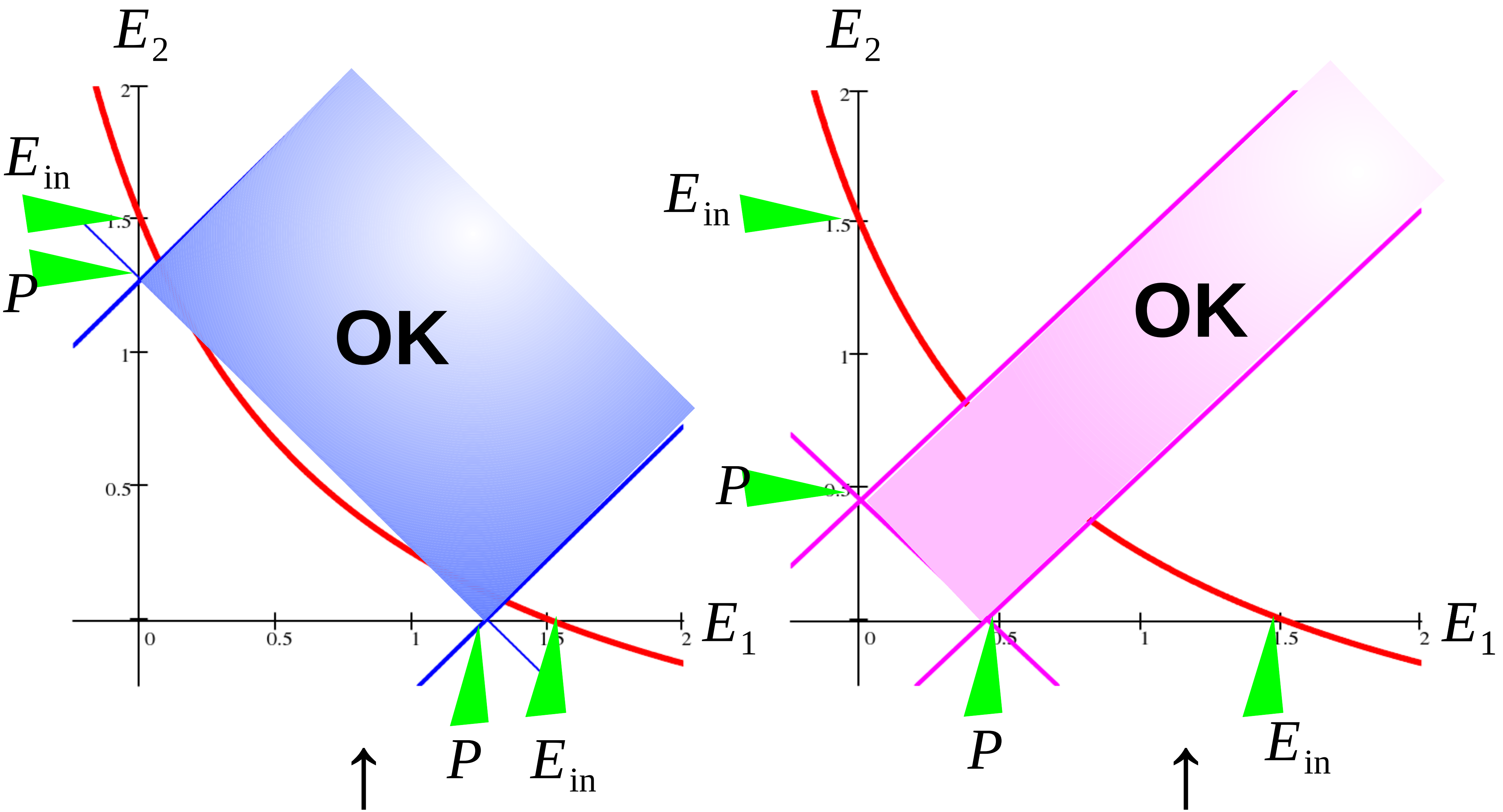


Large P



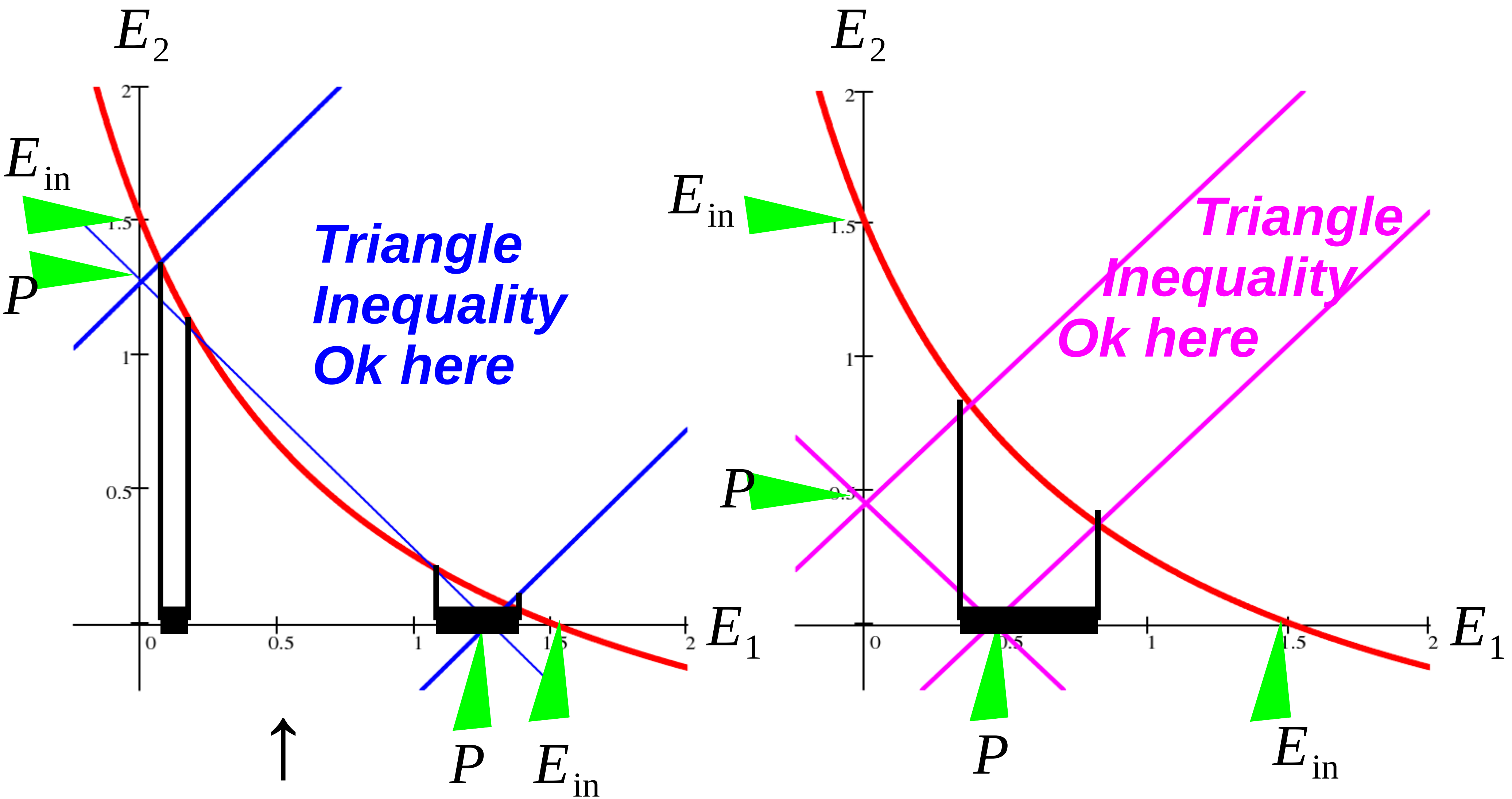
Small P

Because of the triangle inequality among $\vec{p}_1, \vec{p}_2, \vec{P}$



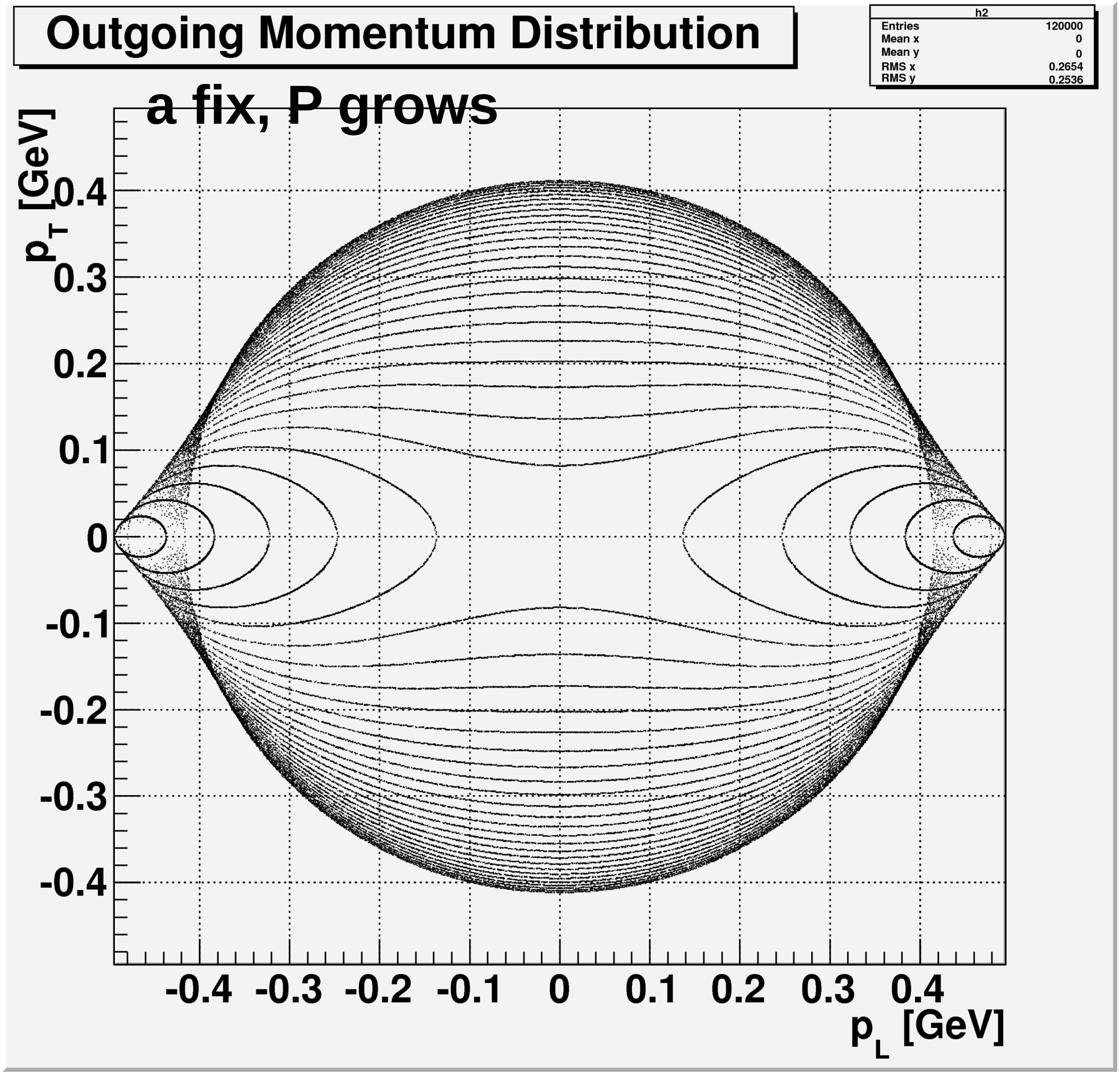
$$P \quad > \quad \frac{2}{a}\left(\sqrt{1+aE_{in}}-1\right) \quad > \quad P$$

$$E_1 \in \pm \frac{P}{2} - \frac{1}{a} + \sqrt{\left(\frac{P}{2}\right)^2 + \frac{1}{a^2} + \frac{E_{\text{in}}}{a}}$$



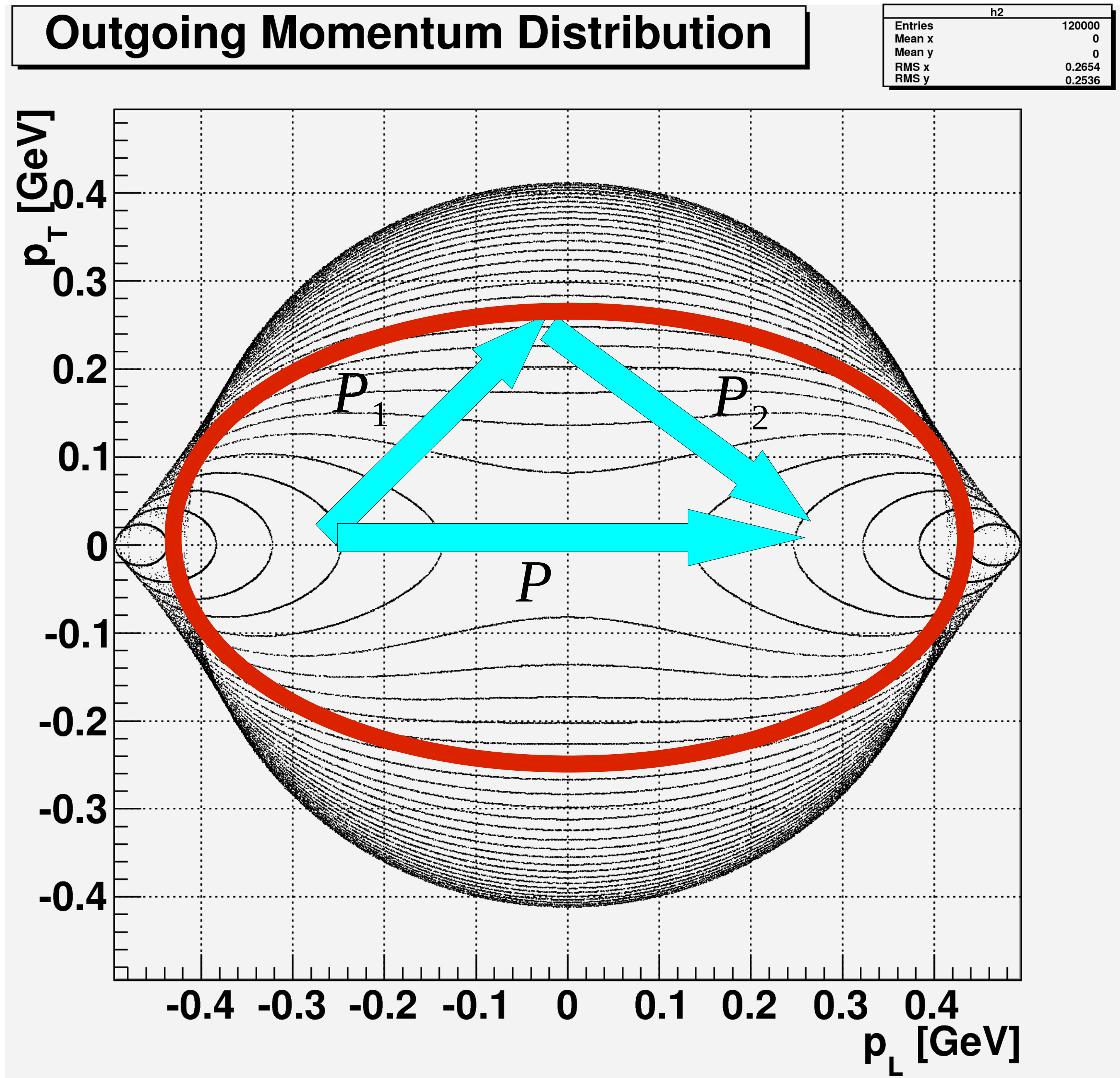
$$E_1 \notin P/2 \pm \sqrt{(P/2)^2 - (E_{\text{in}} - P)/a}$$

Elementary qq – > qq reaction



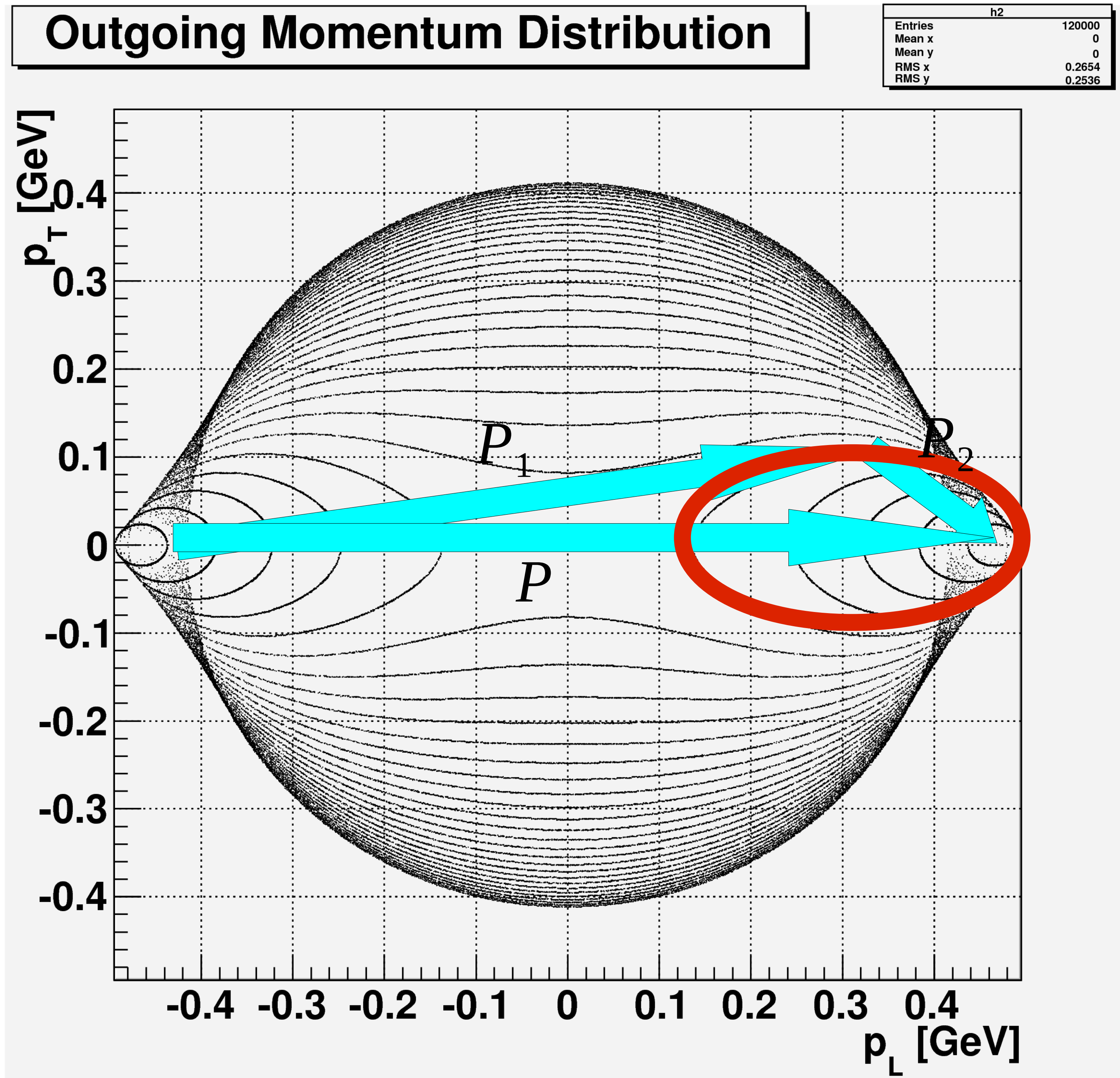
simulation using ROOT

$P < P_0$



Outgoing Momentum Distribution
h2
Entries 120000
Mean x 0
Mean y 0
RMS x 0.2654
RMS y 0.2536
p_T [GeV]
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4
p_L [GeV]
P_1
P_2
P

$P > P_0$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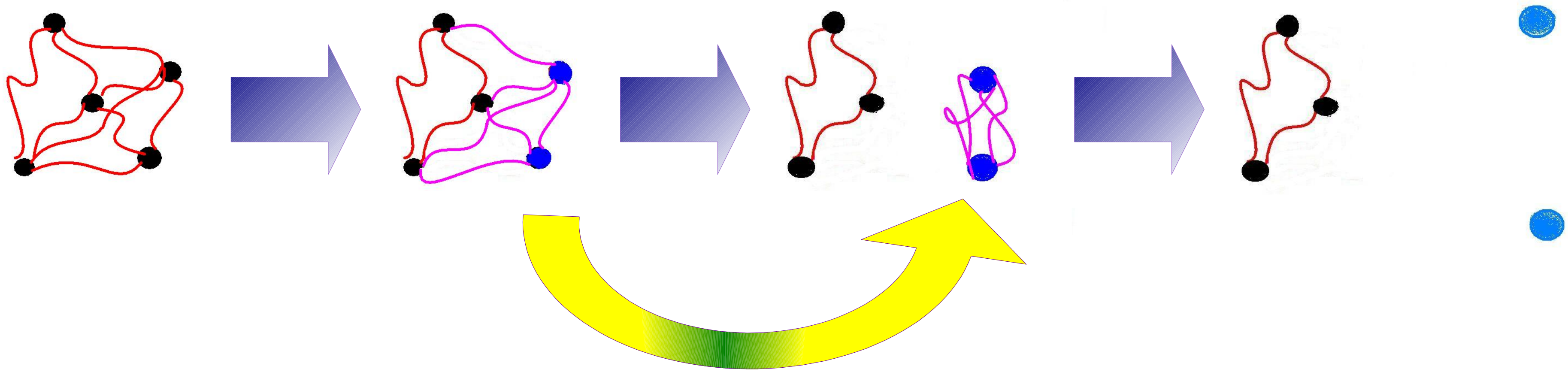
-0.2
-0.3
-0.4
-0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4
p_L [GeV]
P_1
P_2
P

Hadronisation is modelled by a $q\bar{q} \to Resonance \to \pi^{+}\pi^{-}$



$$\vec{P}_R = \vec{p}_1 + \vec{p}_2 \qquad M_R^2 = E_R^2 - \vec{P}_R^2$$

Probability of color neutrality ~1/9

$$E_R = E_N + E_{N-2}$$

Large interaction energy put into the mass of the Resonance

$$E_R \sim (1 + a\bar{\epsilon})^{N-2}(2\bar{\epsilon} + a\bar{\epsilon}^2)$$

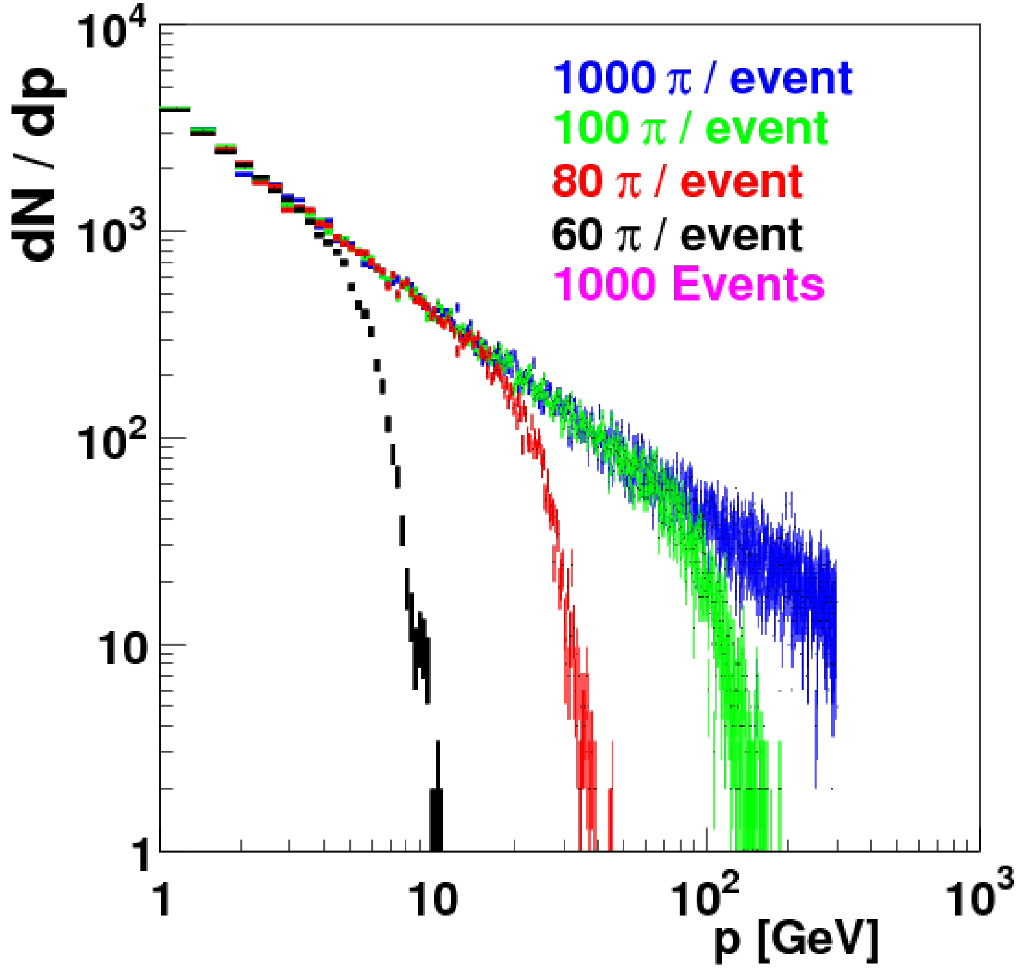
Quasi-quark Cascade Collision Simulation

Preliminary Results

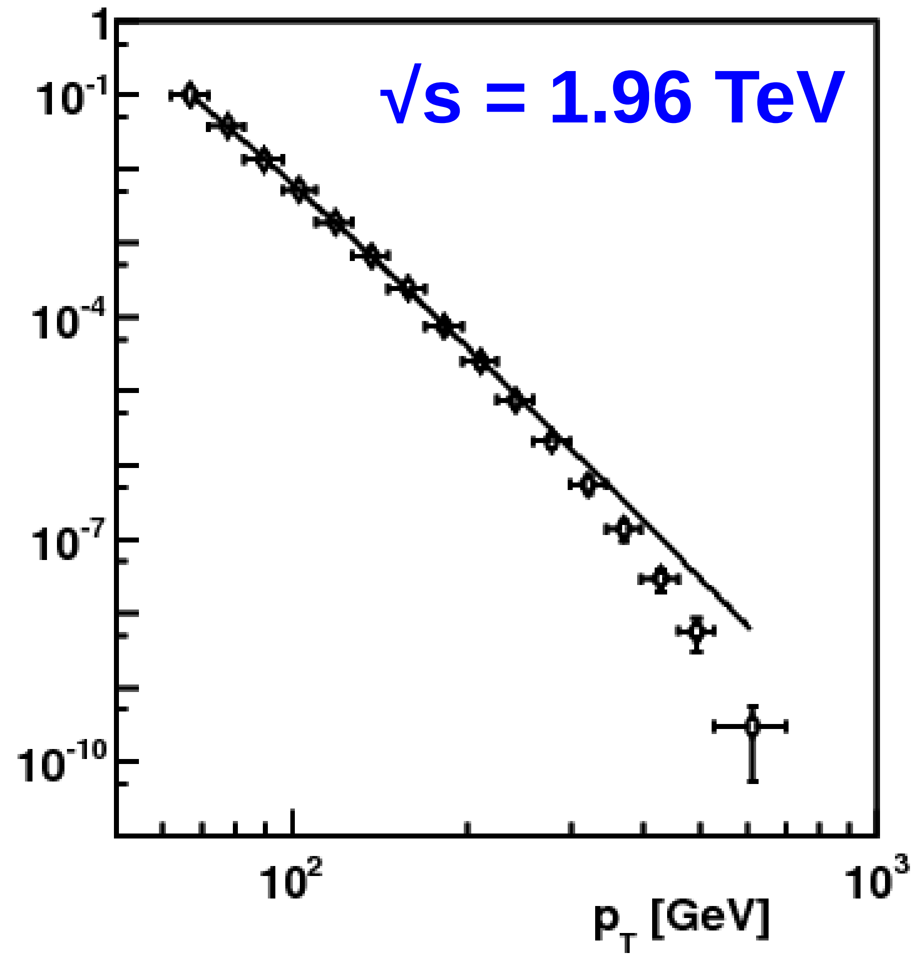
Finite *N, E* – > cut in the spectrum at high E

Interaction measure *a = 1*

Pions from parton cascade

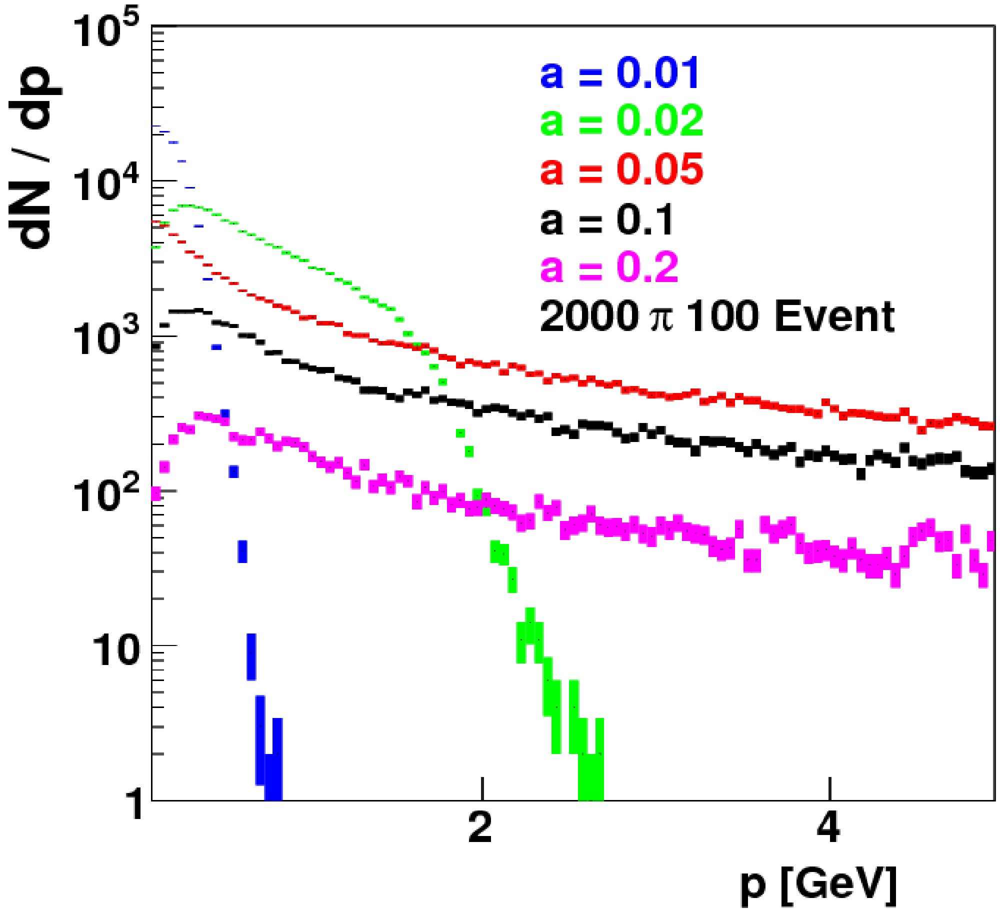


Jets at *CDF@Tevatron*

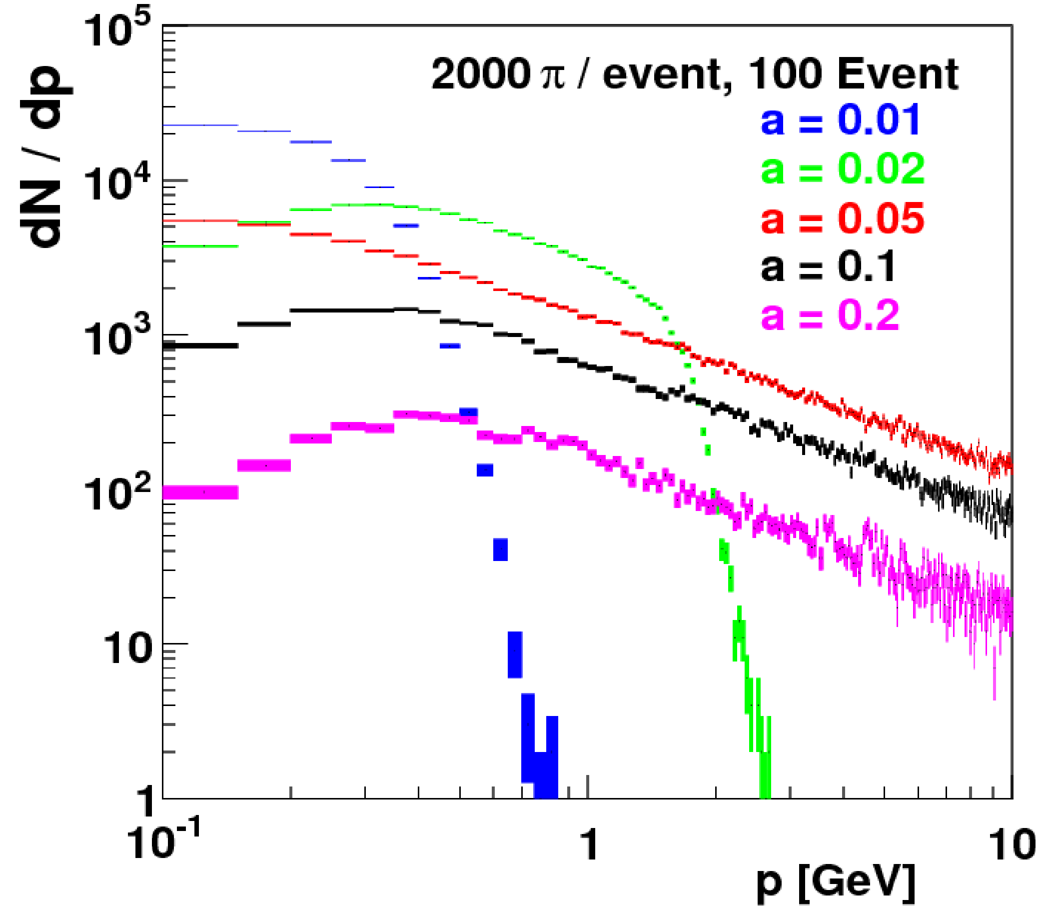


The spectrum changes from Boltzmann to Tsallis as interactions (*a*) grow

Log – Lin plot

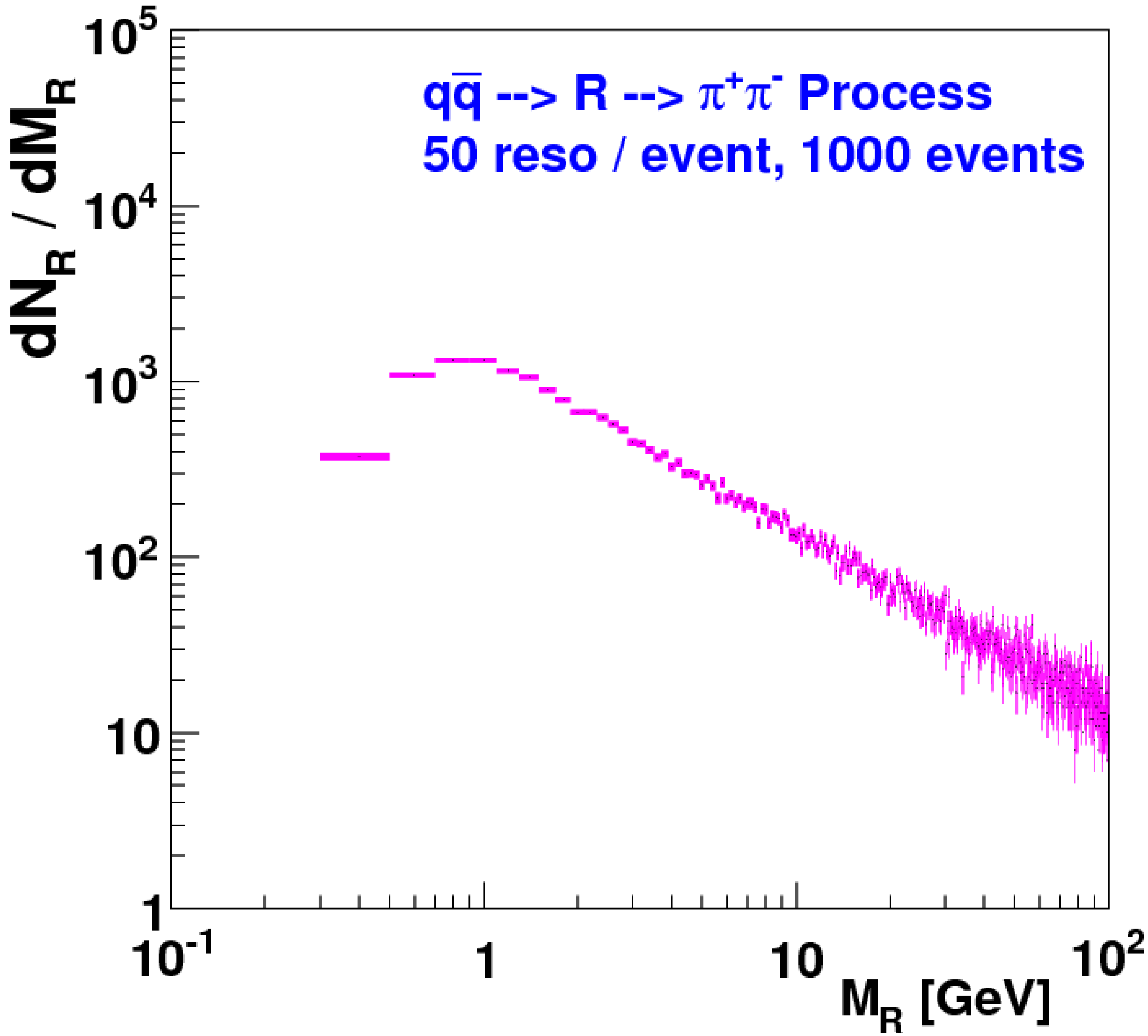


Log – Log plot



Mass Distribution of Resonances

With interaction measure *a = 1*



as usual

$$M_R^2 = E_R^2 - \vec{P}^2$$

$$E_N = E_{N-2} + E_R$$

but

$$E_N = E_1 \oplus E_2 \oplus \ldots \oplus E_N$$

Conclusions

- From ***thermal model*** **and** ***direct fits*** **to hadron spectra produced in** ***AuAu –> hX*** **and** ***pp –> hX*** **reactions we have seen that the** ***Tsallis distribution*** **is a plausibile ansatz for the newly created quasi-particles.**

- **Such distribution** ***may occure in any situation*** **that can be** ***modelled by independent, identically distributed variables*** **the sum of whose is constrained through an associative rule** $\xi_1 \oplus \ldots \oplus \xi_N = fix$.

- **The idea of** ***estimating interaction*** **energies with single particle kinetic energies (*like aE_1E_2*) gives good results when put into a quasi-quark cascade model. Simulations give a** ***pion spectrum*** **that is** ***consistent with measurements*****. The cut on the spectrum at high E (because of finite N, E) is like what is observed on jet spectra at Tevatron.**

Back-up Slides

h(x,y) as the effect of the environment

Fokker-Planck approach:

$$\partial_t f = \partial_p \left(G(E) E' + \partial_p D(E) \right) f$$

With stationary sollution

$$f(E) = \frac{A}{D(E)} \exp\left(-\beta \int^E \frac{d\tilde{E}\, G(\tilde{E})}{D(\tilde{E})} \right)$$

Hence the connection of h and D, G:

$$G(E) = \frac{\beta D(E)}{h'_2(E,0)} - D'(E)$$

h(x,y) as the effect of the environment

Boltzmann-Gibbs case:

$$h(E_{1,}E_2)=E_1+E_2\rightarrow L(E)=E$$

$$f=\exp(-\beta E)/Z$$

Tsallis case:

$$h(E_{1,}E_2)=E_1+E_2+a\,E_1\,E_2\rightarrow L(E)=\frac{1}{a}\ln(1+aE)$$

$$f=(1+aE)^{-\beta/a}/Z$$

Hyper-Tsallis case:

$$h(E_{1,}E_2)=E_1+E_2+a\,E_1\,E_2+b/2(E_1^2E_2+E_2^2E_1)$$

$$L(E)=\frac{1}{\alpha}\ln\left(\frac{2+(a-\alpha)E}{2+(a+\alpha)E}\right)\qquad f=\left(1-\frac{2\alpha E}{2+(a+\alpha)E}\right)^{\beta/\alpha}/Z$$

$$\alpha=\sqrt{a^2-2\mathrm{b}}$$