Nonextensive critical effects in relativistic nuclear mean field models

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HCBM 2010 – Hot & Cold Baryonic Matter 2010
International Workshop on Hot & Cold Baryonic Matter
15-20 August 2010, Budapest - Hungary

Content:

- **Motivation**
- **Example: NJL model of QCD**

  → Nonextensive NJL model: q_NJL

- **Results**
- **Summary**

Based on: J.Rożynek and G.Wilk J.Phys. G36 (2009) 125108
and Acta Phys. Polon. B41 (2010) 351.

Motivation

- The critical phenomena in strongly interaction matter are generally investigated using the mean-field model and are characterized by well defined critical exponents.

- However, such models provide only average properties of the corresponding order parameters and neglect altogether their possible fluctuations. Also the possible long range effect and correlations are neglected in the mean field approach.

- One of the possible phenomenological ways to account for such effects is to use the nonextensive approach.

- Here we investigate the critical behavior in the nonextensive version of the Nambu Jona-Lasinio model (NJL). It allows to account for such effects in a phenomenological way by means of a single parameter q, the nonextensivity parameter.

- In particular, we show how the nonextensive statistics influences the region of the critical temperature and chemical potential in the NJL mean field approach.

Motivation

- The critical phenomena in strongly interaction matter are generally investigated using the mean-field model and are characterized by well defined critical exponents.

- However, such models provide only average properties of the corresponding order parameters and neglect altogether their possible fluctuations. Also the possible long range effect and correlations are neglected in the mean field approach.

- One of the possible phenomenological ways to account for such effects is to use the nonextensive approach.

(… digression on nonextensivity and the like …)

Digression: …. nonextensive approach…. what does it mean?

The simple ***rule of thumb*** is that, if in our attempts to fit data using statistical model(s) based on exponential distributions of the type

$$\exp\left[-X/\lambda\right]$$

($X = p_T$, $m_T \cosh(y)$, $(E-\langle E\rangle)^2$, …)

we fail because data seem to follow rather power-like behaviour, we can try to fit them using the so called Tsallis distribution

$$\exp_q\left[-X/\lambda\right] = \left[1-(1-q)X/\lambda\right]^{1/(1-q)}$$

$q \neq 1$

which becomes exponential when parameter q → 1. In this way one can still use statistical language but based on the so called nonextensive statistics represented by Tsallis (for example) rather than usual Boltzman-Gibbs entropy. Parameter q summarizes action of all dynamical factors resulting in departure from the conditions needed for BG approach to be valid. When these factors are consecutively recognized and accounted for then q nears more and more to unity.

Digression: …. nonextensive approach…. what does it mean?

The nonextensive statistical mechanics proposed by Tsallis [2] generalizes the usual BG statistical mechanics in that the entropy function (we use the convention that the Boltzmann constant is set equal to unity),

$$S_{\mathrm{BG}} = -\sum_{i=1}^{W} p_i \ln p_i \Longrightarrow S_q = -\sum_{i=1}^{W} p_i^q \ln_q p_i, \tag{17}$$

$S_q \rightarrow S_{q=1} = S_{\mathrm{BG}}$ for $q \rightarrow 1$. Here, q is the nonextensive parameter and $\ln_q p = [p^{1-q} - 1]/(1-q)$. The additivity for two independent subsystems A and B (i.e., such that $p^{A\oplus B} = p^A \cdot p^B$) is now lost and takes the form:

$$S_q^{A\oplus B} = S_q^A + S_q^B + (1-q) S_q^A S_q^B, \tag{18}$$

they are called nonextensive[4].

[2] Tsallis C J 1988 *Stat. Phys.* **52** 479

Salinas S R A and Tsallis C (ed) 1999 Special issue on nonextensive statistical mechanics and thermodynamic *Braz. J. Phys.* **29**

Gell-Mann M and Tsallis C (ed) 2004 *Nonextensive Entropy: Interdisciplinary Applications* (New York: Oxford University Press)

Tsallis C 2009 *Eur. Phys. J.* A **40** 257

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they are called nonextensive[4].

[4] It is worth knowing that for subsystems with some special probability correlations, it is the BG entropy for which extensivity is not valid and is restored only for $q \neq 1$ (one refers to such systems as nonextensive [34]).

[34] Tsallis C, Gell-Mann M and Sato Y 2005 *Proc. Natl. Acad. Sci.* **120** 15377

Digression: …. nonextensive approach…. what does it mean?

This phenomenon is ubiquitous in all branches of science and very well documented. It occurs always whenether:

(*) there are long range correlations in the system (or „system is small" – like our Universe with respect to the gravitational interactions)

(*) there are memory effects of any kind

(*) the phase-space in which system operates is limited or has fractal structure

(*) there are intrinsic fluctuations in the system under consideration

(*) ………………………….

Digression: …. nonextensive approach…. what does it mean?

This phenomenon is ubiquitous in all branches of science and very well documented. It occurs always whenether:

[4] Wilk G and Włodarczyk Z 2009 *Eur. Phys. J.* A **40** 299

[5] Alberico W M and Lavagno A 2009 *Eur. Phys. J.* A **40** 313

[6] Osada T and Wilk G 2008 *Phys. Rev.* C **77** 044903
Osada T and Wilk G 2009 *Centr. Eur. J. Phys.* **7** 432

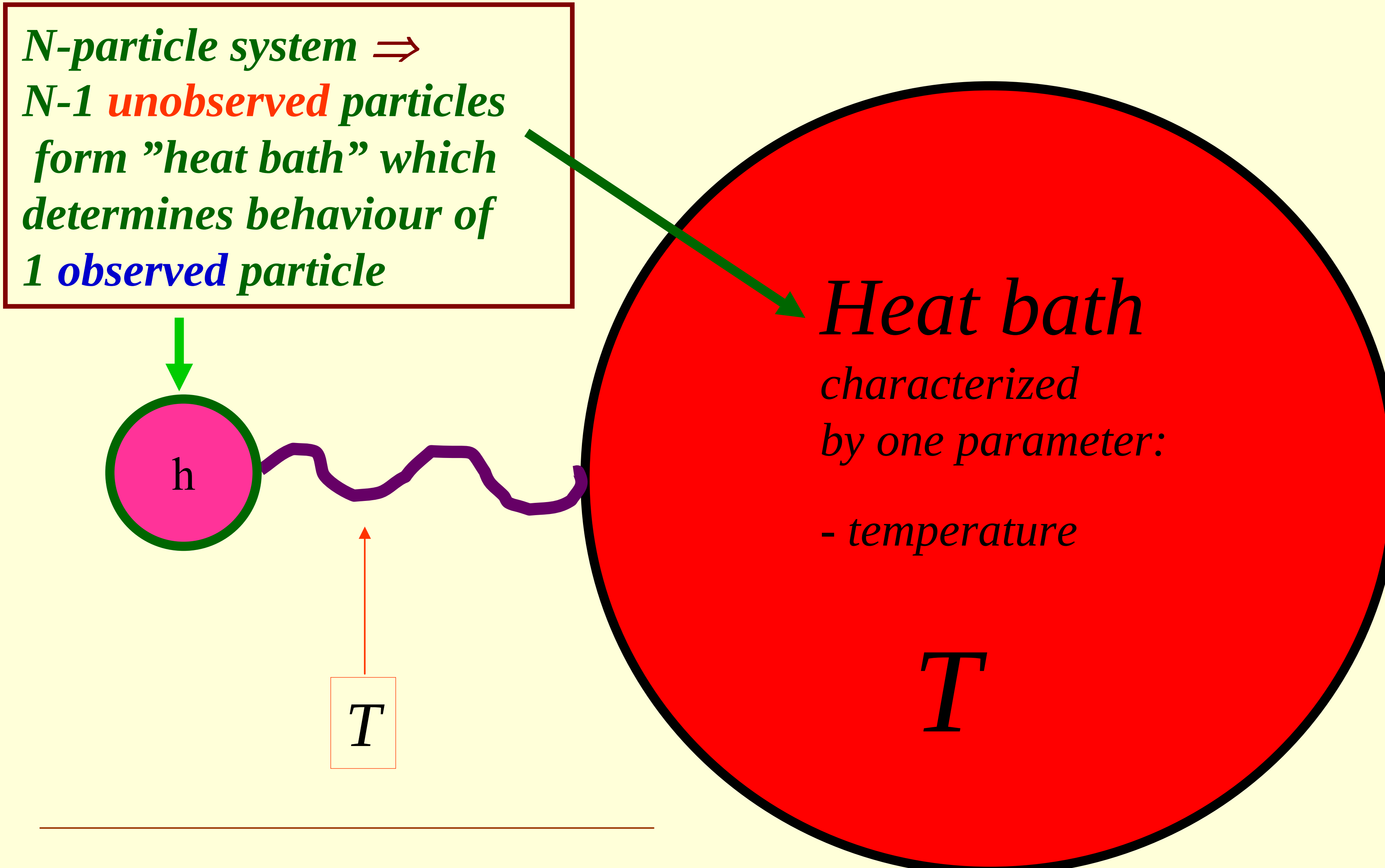
[7] Biyajima M, Mizoguchi T, Nakajima N, Suzuki N and Wilk G 2006 *Eur. Phys. J.* C **48** 597

[8] Biró T S and Purcsel G 2009 *Centr. Eur. J. Phys.* **7** 395
Biró T S, Purcsel G and Ürmošy K 2009 *Eur. Phys. J.* A **40** 325

[9] Drago A, Lavagno A and Quarati P 2004 *Physica* A **344** 472

[10] Biró T S and Purcsel G 2005 *Phys. Rev. Lett.* **95** 162302
Biró T S and Purcsel G 2008 *Phys. Lett.* A **372** 1174
Biró T S 2008 *Europhys. Lett.* **84** 56003

Digression: …. nonextensive approach…. what does it mean?
…………………………………………………………illustration

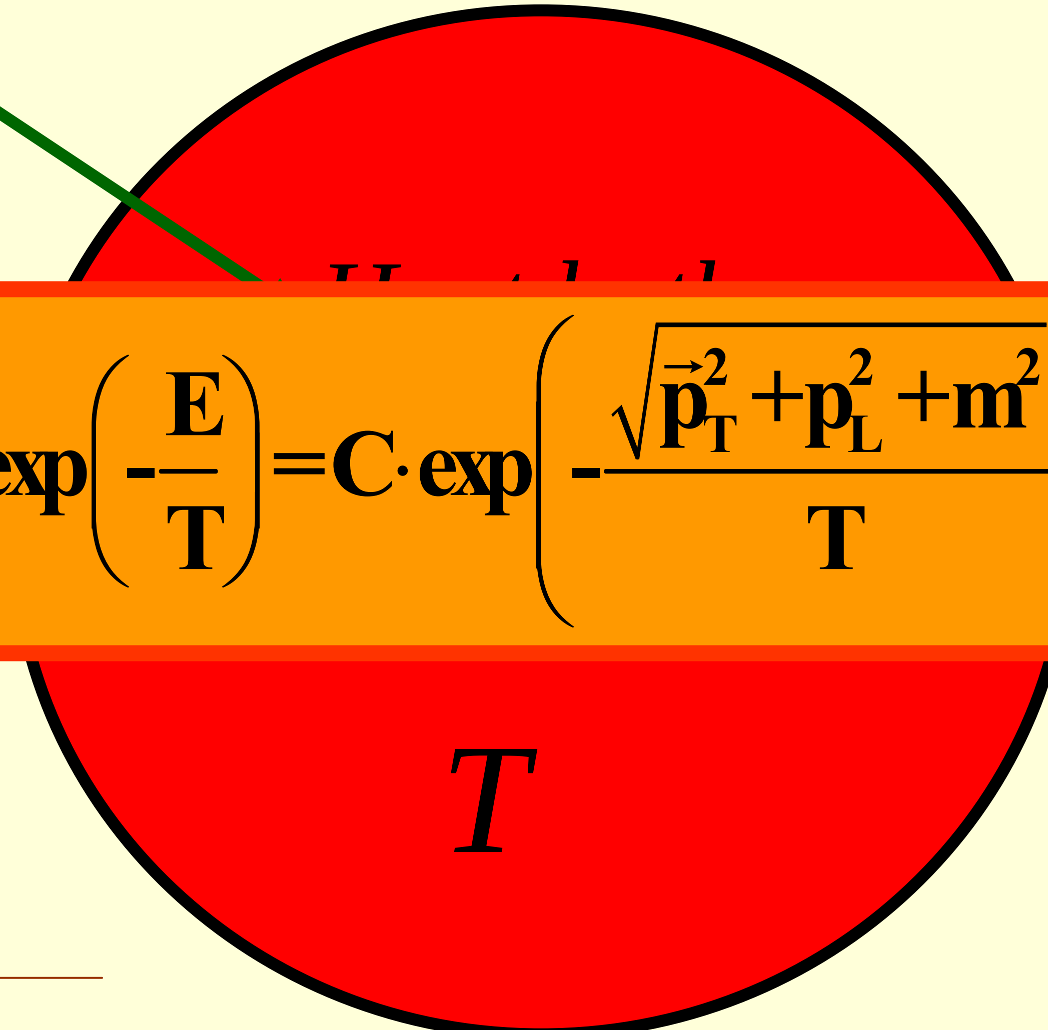


L.Van Hove, Z.Phys. C21 (1985) 93,
Z.Phys. C27 (1985) 135.

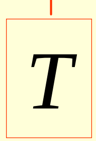
Digression: …. nonextensive approach…. what does it mean?

……………………………………………………….illustration

N-particle system $\Rightarrow$
N-1 unobserved particles form "heat bath" which determines behaviour of 1 observed particle



$$\left.\frac{d\sigma}{d^3p}\right|_{\pi^+} = f\left(\vec{p}_T, p_L\right) = C\cdot\exp\left(-\frac{E}{T}\right) = C\cdot\exp\left(-\frac{\sqrt{\vec{p}_T^{\,2}+p_L^2+m^2}}{T}\right)$$



L.Van Hove, Z.Phys. C21 (1985) 93,
Z.Phys. C27 (1985) 135.

Digression: …. nonextensive approach…. what does it mean?
……………………………………………………….illustration

But: In such "thermodynamical" approach one has to remember assumptions of infinity and homogenity made when proposing this approach - only then behaviour of the observed particle will be characterised by single parameter - the "temperature" T

In reality: This is true only approximately and in most cases we deal with system which are neither infinite and nor homogeneous

In both cases: Fluctuations occur and new parameter(s) in addition to T is(are) necessary

Can one introduce it keeping simple structure of statistical model approach?

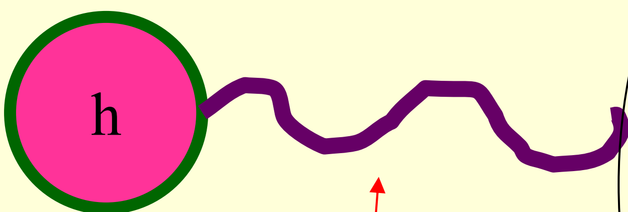
Yes, one can, by applying nonextensive statistical model.

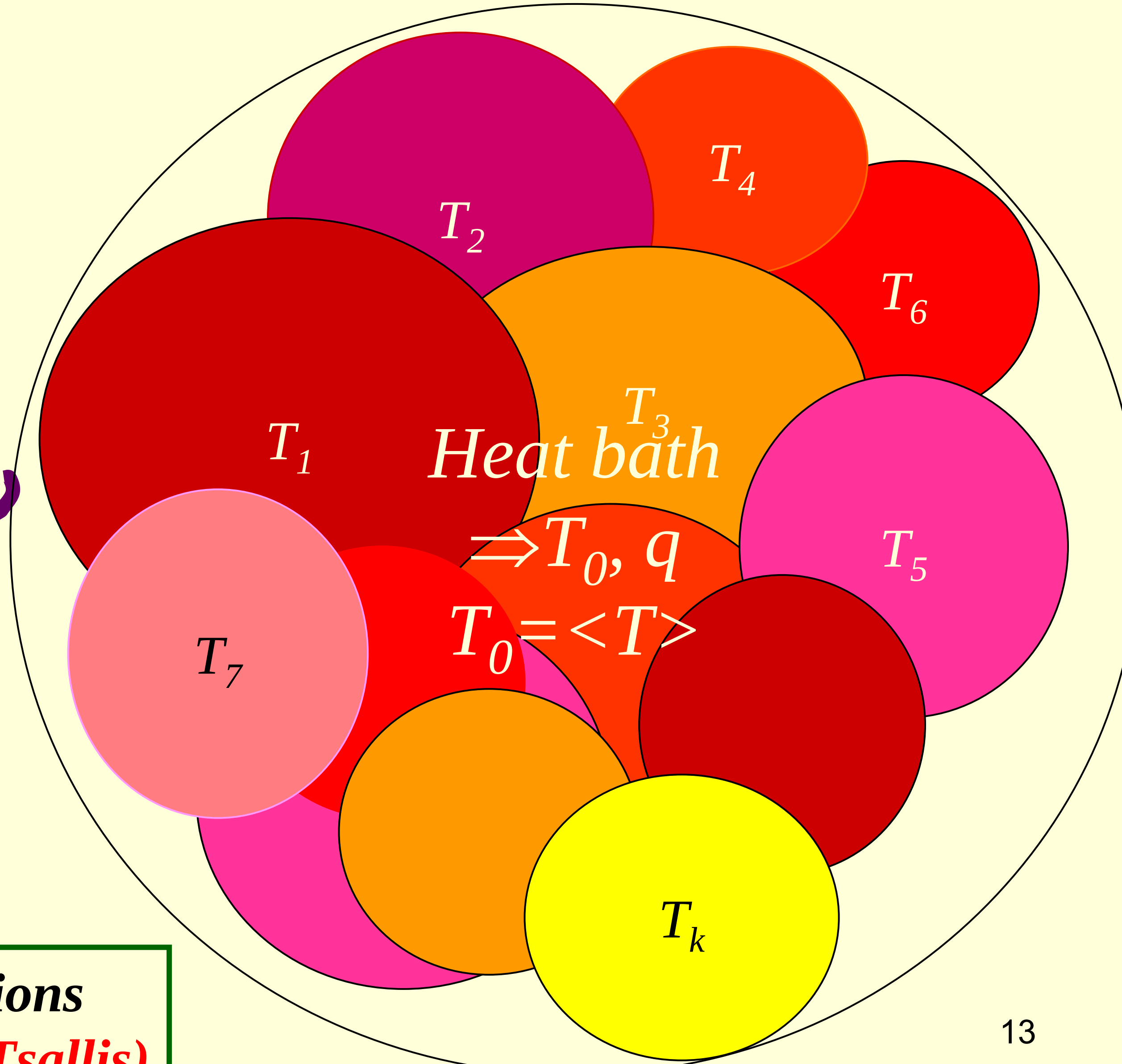
Wilk G and Włodarczyk Z 2000 *Phys. Rev. Lett.* **84** 1770
Wilk G and Włodarczyk Z 2001 *Chaos Solitons Fractals* **13** 581
Biró T S and Jakovác A 2005 *Phys. Rev. Lett.* **94** 132302

Digression: …. nonextensive approach…. what does it mean? ……………………………………………………….illustration

T* *varies $\Leftrightarrow$

fluctuations...



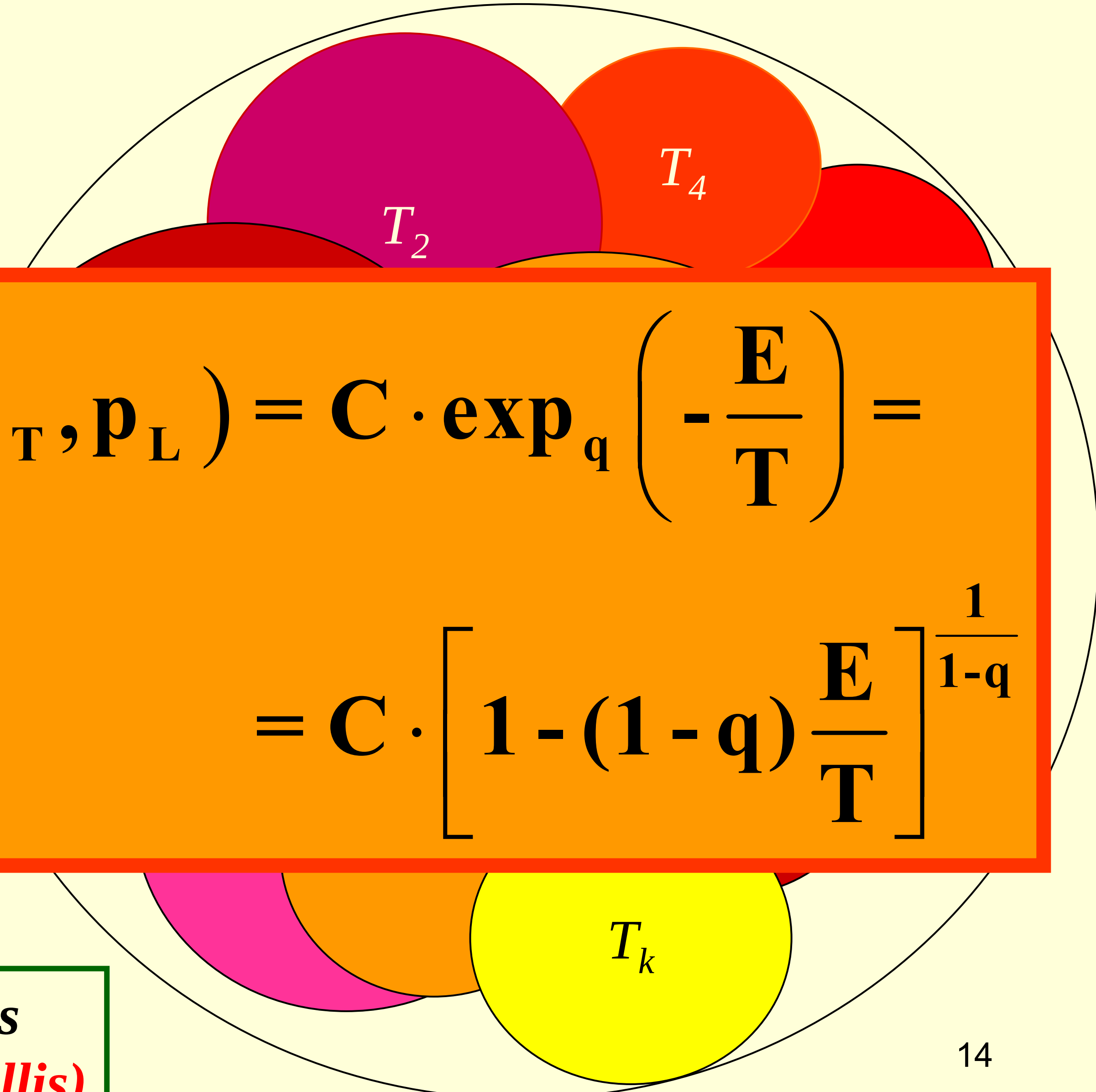


$T_0=\langle T \rangle$, q

q* - measure of fluctuations of *T ➡ ***q-statistics (Tsallis)***

Digression: …. nonextensive approach…. what does it mean? …………………………………………………………….illustration

T varies $\Leftrightarrow$ *fluctuations...*



$$\left.\frac{d\sigma}{d^3p}\right|_{\pi^+} = f_q\left(\vec{p}_T, p_L\right) = C\cdot \exp_q\left(-\frac{E}{T}\right) =$$

$$= C\cdot\left[1-(1-q)\frac{E}{T}\right]^{\frac{1}{1-q}}$$

q - measure of fluctuations of T ➡ *q-statistics (Tsallis)*

Digression: …. nonextensive approach…. what does it mean?
……………………………………………………….illustration

Generalization of the Boltzmann equation

(cf., T.S.Biro and G.Kaniadakis, EPJ B50 (2006) 3, Biro et al., EPA40(2009)325) **:**

nonextensive statistics can be obtained by
changing the corresponding collision rates being nonlinear in the one-particle densities
or – equivalently – by
using nontrivial energy composition rules in the energy conservation constraint part

Nonlinear Boltzman equation (NLBE)

Digression: …. nonextensive approach…. what does it mean?
……………………………………………………….illustration

Nonlinear Boltzman equation example for 1+2 ↔ 3+4 collision with rate of change of the one-particle phase space density $f_1 = f(\vec{p}_1)$

$$\boxed{\dot{f}_1 = \int_{234} w_{1234} \left(a_3 b_1 a_4 b_2 - a_1 b_3 a_2 b_4 \right)}$$

$a_i = a(f_i)$ - general production and $b_i = b(f_i)$ - *general* blocking factors
w_{1234} - transition probability rate factor

The standard theories are recovered for $a(f)=f$ and $b(f)=1$, or $b(f)=1 \pm f$, *respectively. In this case the stationary distribution is given by the ratio* $\kappa(f) = a(f)/b(f)$,
which becomes the traditional Boltzmann factor:

$$\boxed{\kappa(f_{eq}) = \exp\left(-\frac{E}{T}\right)}$$

This result assumes that in two-body collisions momenta and energy are composed additively:

$E_1 + E_2 = E_3 + E_4,$ $\qquad$ $p_1 + p_2 = p_3 + p_4$

Digression: …. nonextensive approach…. what does it mean?
……………………………………………………….illustration

However, one can demand that the additivity of the energy during the micro-collisions is replaced by a more general requirement:
only a given function of individual energies, physically standing for the total two-particle energy, is conserved:

$$h(E_1,E_2) = h(E_3,E_4).$$

The function h(x,y) describes a general, non-extensive energy composition rule for the two body system. If it is chosen with the property of associativity:
$h(h(x,y),z) = h(x,h(y,z))$, then its most general form is related to a strict monotonic function, $X(x)$:

$$\boxed{h(x,y) = X^{-1}\left(X(x) + X(y)\right)}$$

$X(x)$ is mapping the non-extensive composition rule to the addition rule, i.e., it is formal logarithm. It is unique up to a real multiplicative factor. The stationary solution in this case is:

$$\boxed{f_{eq} = \exp\left(-\frac{X(E)}{T}\right)}$$

which for

$$\boxed{X(x) = \frac{1}{a}\ln(1+ax)}$$

results
in Tsallis distribution with $q=1-aT$.

Digression: …. nonextensive approach…. what does it mean?

The possible meaning of the parameter q:

(*) q > 1 – action of some intrinsic fluctuations [14,15]

(*) q < 1 – action of some correlations or limitations of the phase space [16,17]

Note that in q-statistics we do not specify what is the dynamical origin of these intrinsic fluctuations or specific correlations. It is expected that every piece of new dynamical knowledge accumulated during systematic studies of the respective processes substantially lowers the values of the parameter $|q-1|$ needed to fit experimental data. This was confirmed in [7] when investigating transverse momenta distributions in heavy ion collisions, namely the gradual accounting for the intrinsic dynamical fluctuations in the hadronizing system by switching from the pure statistical approach to the modified Hagedorn formula including temperature fluctuations [7] resulted in sizeable decreasing of $q-1$. This means, therefore, that when one reaches in such a procedure the value $q=1$, it should signal that all dynamical effects spoiling the initially assumed BG approach have already been successfully accounted for.

[14] Wilk G and Włodarczyk Z 2000 *Phys. Rev. Lett.* **84** 1770
Wilk G and Włodarczyk Z 2001 *Chaos Solitons Fractals* **13** 581
[15] Biró T S and Jakovác A 2005 *Phys. Rev. Lett.* **94** 132302

[16] Kodama T, Elze H-T, Aguiar C E and Koide T 2005 *Europhys. Lett.* **70** 439
Kodama T and Koide T 2009 *Eur. Phys. J.* A **40** 289
[17] García-Morales V and Pellicer J 2006 *Physica* A **361** 161

[7] Biyajima M, Mizoguchi T, Nakajima N, Suzuki N and Wilk G 2006 *Eur. Phys. J.* C **48** 597

Motivation

- Recently, q-statistics has been applied to the Walecka many-body field theory

Pereira F I M, Silva B and Alcaniz J S 2007 *Phys. Rev.* C 76 015201

resulting (among others) in the enhancement of the scalar and vector meson fields in nuclear matter, in diminishing of the nucleon effective mass and in the hardening of the nuclear equation of state (only q>1 case was considered there).

- Here we investigate the critical behavior in the nonextensive version of the Nambu Jona-Lasinio model (NJL). It allows to account for such effects in a phenomenological way by means of a single parameter q, the nonextensivity parameter. The NJL model we modify is that presented in

Costa P, Ruivo M C and de Sousa A 2008 *Phys. Rev.* D 77 096001

In particular, we show how the nonextensive statistics influences the region of the critical temperature and chemical potential in the NJL mean field approach.

Example of NJL model of QCD

We start by recollecting some basic formulae concerning the NJL model used in [23]. They used the usual Lagrangian of the NJL model, invariant (except of the current quarks mass term) under the chiral $SU_L(3) \otimes SU_R(3)$ transformations (described by coupling constant g_S) and containing a term breaking the $U_A(1)$ symmetry, which reflects the axial anomaly in QCD (described by coupling constant g_D). When put in a form suitable for the bosonization procedure (with four quark interaction only) it results in the following effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \bar{q}(\mathrm{i}\gamma^\mu \partial_\mu - \hat{m})q + S_{ab}[(\bar{q}\lambda^a q)(\bar{q}\lambda^b q)] + P_{ab}[(\bar{q}\mathrm{i}\gamma_5\lambda^a q)(\bar{q}\mathrm{i}\gamma_5\lambda^b q)], \tag{1}$$

where $\hat{m} = \operatorname{diag}(m_{\mathrm{u}}, m_{\mathrm{d}}, m_{\mathrm{s}})$ and S_{ab} and P_{ab} are projectors,

$$S_{ab} = g_{\mathrm{S}}\delta_{ab} + g_{\mathrm{D}} D_{abc}\langle\bar{q}\lambda^c q\rangle, \tag{2}$$

$$P_{ab} = g_{\mathrm{S}}\delta_{ab} - g_{\mathrm{D}} D_{abc}\langle\bar{q}\lambda^c q\rangle \tag{3}$$

with D_{abc} being the $SU(3)$ structure constants d_{abc} for $a, b, c = (1, 2, \ldots, 8)$ whereas $D_{0ab} = -\delta_{ab}/\sqrt{6}$ and $D_{000} = \sqrt{2/3}$. We work with $q = (u, d, s)$ quark fields with three flavors, $N_f = 3$, and three colors, $N_c = 3$, λ^a are the Gell-Mann matrices, $a = 0, 1, \ldots, 8$ and $\lambda^0 = \sqrt{\frac{2}{3}}\,\mathbf{I}$. Integrating over the quark fields in the functional integral with $\mathcal{L}_{\text{eff}}$ one gets an effective action expressed by the natural degrees of freedom of low-energy QCD in the mesonic sector, namely σ and φ (the notation Tr stands for taking trace over indices N_f and N_c and integrating over momentum):

$$\begin{aligned} W_{\text{eff}}[\varphi, \sigma] = &-\tfrac{1}{2}(\sigma^a S_{ab}^{-1}\sigma^b) - \tfrac{1}{2}(\varphi^a P_{ab}^{-1}\varphi^b) \\ &-\mathrm{i}\,\mathrm{Tr}\ln[\mathrm{i}\gamma^\mu\partial_\mu - \hat{m} + \sigma_a\lambda^a + (\mathrm{i}\gamma_5)(\varphi_a\lambda^a)]. \end{aligned} \tag{4}$$

Example of NJL model of QCD

$$W_{\mathrm{eff}}[\varphi,\sigma] = -\tfrac{1}{2}\left(\sigma^a S_{ab}^{-1}\sigma^b\right) - \tfrac{1}{2}\left(\varphi^a P_{ab}^{-1}\varphi^b\right) - \mathrm{i}\,\mathrm{Tr}\ln[\mathrm{i}\gamma^\mu\partial_\mu - \hat{m} + \sigma_a\lambda^a + (\mathrm{i}\gamma_5)(\varphi_a\lambda^a)]. \tag{4}$$

The first variation of W_{eff} leads to the gap equations for the constituent quark masses M_i:

$$M_i = m_i - 2g_{\mathrm{S}}\langle\bar{q}_i q_i\rangle - 2g_{\mathrm{D}}\langle\bar{q}_j q_j\rangle\langle\bar{q}_k q_k\rangle, \tag{5}$$

with cyclic permutation of $i, j, k = u, d, s$ and with the quark condensates given by $\langle\bar{q}_i q_i\rangle = -\mathrm{i}\mathrm{Tr}[S_i(p)]$ ($S_i(p)$ is the quark Green function); m_i denotes the current mass of quark of flavor i (note that nonzero g_{D} introduces mixing between different flavors).

The baryonic thermodynamical potential of the grand canonical ensemble,

$\Omega(T,V,\mu_i)$

is also obtained directly from the effectve action W_{eff} above.

Example of NJL model of QCD

Let us consider a system of volume V, temperature T and the i th quark chemical potential μ_i characterized by the baryonic thermodynamic potential of the grand canonical ensemble (with quark density equal to $\rho_i = N_i / V$, the baryonic chemical potential $\mu_B = \frac{1}{3}(\mu_\mathrm{u}+\mu_\mathrm{d}+\mu_\mathrm{s})$ and the baryonic matter density as $\rho_B = \frac{1}{3}(\rho_\mathrm{u} + \rho_\mathrm{d} + \rho_\mathrm{s})$),

$$\boxed{\Omega(T, V, \mu_i) = E - TS - \sum_{i=u,d,s} \mu_i N_i.} \tag{6}$$

The internal energy, E, the entropy, S, and the particle number, N_i, are given by (here $E_i = \sqrt{M_i^2 + p^2}$)

$$\begin{aligned} E = & -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \left[\int p^2 \mathrm{d}p \frac{p^2 + m_i M_i}{E_i} (1 - n_i - \bar{n}_i) \right] \\ & - g_S V \sum_{i=u,d,s} (\langle \bar{q}_i q_i \rangle)^2 - 2 g_D V \langle \bar{u} u \rangle \langle \bar{d} d \rangle \langle \bar{s} s \rangle, \end{aligned} \tag{7}$$

$$S = -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \int p^2 \mathrm{d}p \cdot \tilde{S},$$

$$\text{where} \quad \tilde{S} = [n_i \ln n_i + (1 - n_i) \ln(1 - n_i)] + [n_i \to 1 - \bar{n}_i], \tag{8}$$

$$N_i = \frac{N_c}{\pi^2} V \int p^2 \mathrm{d}p (n_i - \bar{n}_i). \tag{9}$$

Example of NJL model of QCD

The quark and antiquark occupation numbers, n_i and $\bar{n}_i$, are

$$n_i = \frac{1}{\exp[\beta(E_i - \mu_i)] + 1}, \qquad \bar{n}_i = \frac{1}{\exp[(\beta(E_i + \mu_i)] + 1}. \tag{10}$$

With these occupation numbers one can now calculate values of the quark condensates present in equation (5),

$$\boxed{\langle \bar{q}_i q_i \rangle = -\frac{N_c}{\pi^2} \sum_{i=u,d,s} \left[\int \frac{p^2 M_i}{E_i} (1 - n_i - \bar{n}_i) \right] \mathrm{d}p.} \tag{11}$$

Equations (5) and (11) form a self-consistent set of equations from which one gets the effective quark masses M_i and the values of the corresponding quark condensates (once a temperature and chemical potential are given).

The values of the pressure, P, and the energy density, ϵ,

$$P(\mu_i, T) = -\frac{\Omega(\mu_i, T)}{V}, \qquad \epsilon(\mu_i, T) = \frac{E(\mu_i, T)}{V} \tag{12}$$

are defined such that $P(0, 0) = \epsilon(0, 0) = 0$.

Example of NJL model of QCD

To summarize: The gap equations for the constituent quark masses M_i

$$M_i = m_i - 2g_S \langle \bar{q}_i q_i \rangle - 2g_D \langle \bar{q}_j q_j \rangle \langle \bar{q}_k q_k \rangle$$

where

$$\langle \bar{q}_i q_i \rangle = -\frac{N_c}{\pi^2} \sum_{i=u,d,s} \left[\int \frac{p^2 M_i}{E_i} (1 - n_i - \bar{n}_i) \right] dp$$

form a self consistent set of equations from which one gets the effective masses M_i and values of the corresponding quark condensates.

Example of NJL model of QCD

Only π^0 and σ mesons were considered as illustration. Their effective masses are obtained from the effective action W_{eff} (eq. (4)) by expanding it over meson fields and calculating the respective propagators. In the case of a σ meson one must account fot its matrix structure in isospin space.

Parameters used:

[3] For numerical calculations we use the same parameter set as that in [23]: $m_u = m_d = 5.5$ MeV, $m_s = 140.7$ MeV, $g_S\Lambda^2 = 3.67$, $g_D\Lambda^5 = -12.36$ and $\Lambda = 602.3$ MeV. It has been determined by fixing the values $M_\pi = 135.0$ MeV, $M_K = 497.7$ MeV, $f_\pi = 92.4$ MeV, and $M_{\eta'} = 960.8$ MeV. For the quark condensates at $T = 0$ we obtain $\langle \bar{q}_u q_u \rangle = \langle \bar{q}_d q_d \rangle = -(241.9\ \text{MeV})^3$ and $\langle \bar{q}_s q_s \rangle = -(257.7\ \text{MeV})^3$, and for the constituent quark masses $M_u = M_d = 367.7$ MeV and $M_s = 549.5$ MeV.

Nonextensive NJL model: q-NJL
Motivation

- The NJL model is formulated in the grand canonical ensemble and assumes the additivity of some thermodynamical properties, especialy entropy.

- This is a very strong approximation for the system under the phase transition where long-range correlations or fluctuations are very important.

- One could, alternatively, consider equilibrium statistics using microscopic ensembles of Hamiltonian systems, whereas canonical ensembles fail in the most interesting, mostly inhomogeneous, situations like phase separations or away from the thermodynamical limit [31].

- The alternative way to describe the nonadditivity of interacting systems which have long-range correlations is to use q-statistics [32].

[31] Gross D H E 2004 *Physica* A **340** 76
Gross D H E 2006 *Lecture Notes in Physics* vol 602 (Berlin: Springer) p 23
Gross D H E 2006 *Physica* A **365** 138
see also Gross D H E 2001 *Microcanonical Thermodynamics, Phase transitions in 'Small Systems' (WS Lecture Notes in Physics* vol 66*)* (Singapore: World Scientific)
[32] Tsallis C, Rapisarda A, Latora V and Baldovin F 2002 *Lecture Notes in Physics* vol 602 (Berlin: Springer), p 140

Nonextensive NJL model: q-NJL
Motivation

Let us illustrate this with two examples. First note that in the NJL model (which in the mean field approximation is given by the effective Lagrangian (1)) one introduces a strong attractive interaction between a quark and an antiquark represented by couplings g_S and g_D; usually assumed to be independent of the temperature T. However, this interaction induces the instability of the Fock vacuum of the massless quarks which, in turn, results in the non-perturbative ground state with nonzero $(q\bar{q})$ condensates and in the breaking of chiral symmetry endowing constituent quarks with finite masses. This effect takes place in some range of temperatures so, to control it, one allows in some cases for a temperature-dependent coupling constant g_D as, for example, [27]. It was assumed there that g_D, corresponding to breaking of axial symmetry $U_A(1)$, is given by

$$\boxed{g_D(T) = g_D(T=0)\exp\left[-\left(\frac{T}{T_0}\right)^2\right],} \tag{16}$$

where T_0 is a parameter ($T_0 = 100$ MeV in [27]). As shown in [27], depending on the assumed value of T_0, chiral symmetry starts earlier. The retardation effect introduced by equation (16) violates the simple extensivity of the system, therefore it calls for an effective nonextensive description provided by q-statistics [2].

[27] Hatsuda T and Kunihiro T 1994 *Phys. Rep.* **247** 221 and references therein
See also Bernard V, Meissner U-G and Zahed I 1987 *Phys. Rev.* D 36 819

Nonextensive NJL model: q-NJL
Motivation

The second example concerns the description of the spinodal region in the NJL models in which one observes a coexistence of two phases: (a) the phase with broken chiral symmetry and with massive quarks ($m \sim 300$ MeV) and large negative $q\bar{q}$ condensates which constitute the physical vacuum, it develops for small density; (b) for high density the $q\bar{q}$ condensates disappear and quarks are almost massless ($m \sim 5$ MeV). The highest point on the temperature scale of the coexistence curve, T_{crit}, is the critical point. Of special importance is the fact that, within the q-statistic, one can discuss the occurrence of negative specific heat in a nonextensive system which has an equilibrium second-order phase transition [33]. According to this analysis, the specific heat is negative in a transient regime and corresponds to metastable states. Exactly such metastable states are observed in the NJL model during the spinodal phase transition below the critical temperature [23].

[33] Rapisarda A and Latora V 2002 Negative specific heat in out-of-equilibrium nonextensive systems arXiv:nucl-th/0202075
Gross D H E 2006 *Physica* A **365** 138 (and references therein)

[23] Costa P, Ruivo M C and de Sousa A 2008 *Phys. Rev.* D **77** 096001

Nonextensive NJL model: q-NJL
Motivation

- **Finally,**

let us note that the NJL model does not contain color and therefore does not produce confinement.

- **Therefore,**

resigning from the assumption of additivity in this case and introducing a description based on the nonextensive approach, which, according to [8], can be understood as containing some residual interactions between considered objects (here quarks) seems to be an interesting and promising possibility.

[8] Biró T S and Purcsel G 2009 *Centr. Eur. J. Phys.* 7 395
Biró T S, Purcsel G and Ürmoššy K 2009 *Eur. Phys. J.* A 40 325

Nonextensive NJL model: q-NJL

Technicalities:

$$n_i = \frac{1}{\exp[\beta(E_i - \mu_i)] + 1}, \qquad \bar{n}_i = \frac{1}{\exp[(\beta(E_i + \mu_i)] + 1}. \tag{10}$$

$$n_{qi} = \frac{1}{\tilde{e}_q(\beta(E_i - \mu_i)) \pm 1}, \tag{19}$$

$$\tilde{e}_q(x) = \begin{cases} [1 + (q-1)x]^{\frac{1}{q-1}} & \text{if} \quad x > 0 \\ [1 + (1-q)x]^{\frac{1}{1-q}} & \text{if} \quad x \leqslant 0 \end{cases}$$

for q>1 **x=β(E-μ)**

$$\tilde{e}_q(x) = \begin{cases} [1 + (q-1)x]^{\frac{1}{q-1}} & \text{if} \quad x \leqslant 0 \\ [1 + (1-q)x]^{\frac{1}{1-q}} & \text{if} \quad x > 0. \end{cases}$$

for q<1

(notice symmetry for q and 1-q)

Teweldeberhan A M, Plastino A R and Miller H C 2005 *Phys. Lett.* A 343 71

Nonextensive NJL model: q-NJL

This is because only then can one treat consistently on the same footing (and for all values of x) quarks and antiquarks, which should show the particle–hole symmetry observed in the q-Fermi distribution in plasma containing both particles and antiparticles, namely that

$$n_q(E, \beta, \mu, q) = 1 - n_{2-q}(-E, \beta, -\mu). \tag{22}$$

This means, therefore, that in a system containing both particles and antiparticles (as in our case) both q and $2 - q$ occur (or, when expressed by a single q only, that one can encounter both $q > 1$ and $q < 1$ at the same time). These dual possibilities warn us that not only $q > 1$ but also $q < 1$ (or $(2 - q) > 1$ have physical meaning in the systems we are considering. This distinguishes our q-NJL model from the q-version of the QHD-I model of [22].

Notice that
(*) for **q→1** one recovers the standarf FD distribution;
(*) for **T→0** one **always** gets $\mathbf{n_q(\mu,T)\rightarrow n(\mu,T)}$, irrespectice of the value **q**

[22] Pereira F I M, Silva B and Alcaniz J S 2007 *Phys. Rev.* C 76 015201

Nonextensive NJL model: q-NJL

$$E = -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \left[\int p^2 \mathrm{d}p \frac{p^2 + m_i M_i}{E_i} (1 - n_i - \bar{n}_i) \right] - g_S V \sum_{i=u,d,s} (\langle \bar{q}_i q_i \rangle)^2 - 2 g_D V \langle \bar{u} u \rangle \langle \bar{d} d \rangle \langle \bar{s} s \rangle, \qquad (7)$$

$$E_q = -\frac{N_c}{\pi^2} V \sum_{i=\mathrm{u,d,s}} \left[\int p^2 \,\mathrm{d}p \frac{p^2 + m_i M_i}{E_i} (1 - n_{qi}^q - \bar{n}_{qi}^q) \right] - g_S V \sum_{i=\mathrm{u,d,s}} (\langle \bar{q}_i q_i \rangle_q)^2 - 2 g_D V \langle \bar{u} u \rangle_q \langle \bar{d} d \rangle_q \langle \bar{s} s \rangle_q, \qquad (23)$$

$$\langle \bar{q}_i q_i \rangle = -\frac{N_c}{\pi^2} \sum_{i=u,d,s} \left[\int \frac{p^2 M_i}{E_i} (1 - n_i - \bar{n}_i) \right] \mathrm{d}p. \qquad (11)$$

$$\langle \bar{q}_i q_i \rangle_q = -\frac{N_c}{\pi^2} \sum_{i=\mathrm{u,d,s}} \left[\int \frac{p^2 M_i}{E_i} (1 - n_{qi}^q - \bar{n}_{qi}^q) \right] \mathrm{d}p. \qquad (24)$$

Nonextensive NJL model: q-NJL

$$E = -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \left[\int p^2 \mathrm{d}p \frac{p^2 + m_i M_i}{E_i} (1 - n_i - \bar{n}_i) \right] - g_S V \sum_{i=u,d,s} (\langle \bar{q}_i q_i \rangle)^2 - 2 g_D V \langle \bar{u} u \rangle \langle \bar{d} d \rangle \langle \bar{s} s \rangle, \quad (7)$$

$$E_q = -\frac{N_c}{\pi^2} V \sum_{i=\mathrm{u,d,s}} \left[\int p^2 \, \mathrm{d}p \frac{p^2 + m_i M_i}{E_i} (1 - n_{qi}^q - \bar{n}_{qi}^q) \right] - g_S V \sum_{i=\mathrm{u,d,s}} (\langle \bar{q}_i q_i \rangle_q)^2 - 2 g_D V \langle \bar{u} u \rangle_q \langle \bar{d} d \rangle_q \langle \bar{s} s \rangle_q, \quad (23)$$

$$\langle \bar{q}_i q_i \rangle = -\frac{N_c}{\pi^2} \sum_{i=u,d,s} \left[\int \frac{p^2 M_i}{E_i} (1 - n_i - \bar{n}_i) \right] \mathrm{d}p. \quad (11)$$

$$\langle \bar{q}_i q_i \rangle_q = -\frac{N_c}{\pi^2} \sum_{i=\mathrm{u,d,s}} \left[\int \frac{p^2 M_i}{E_i} (1 - n_{qi}^q - \bar{n}_{qi}^q) \right] \mathrm{d}p. \quad (24)$$

Alberico W M and Lavagno A 2009 *Eur. Phys. J.* A 40 313

Nonextensive NJL model: q-NJL

$$S = -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \int p^2 \mathrm{d}p \cdot \tilde{S},$$

$$\text{where} \quad \tilde{S} = [n_i \ln n_i + (1-n_i)\ln(1-n_i)] + [n_i \to 1-\bar{n}_i], \tag{8}$$

$$\tilde{S}_q = \left[n_{qi}^q \ln_q n_{qi} + (1-n_{qi})^q \ln_q(1-n_{qi})\right] + \{n_{qi} \to 1-\bar{n}_{qi}\}. \tag{25}$$

whereas form of N_i remains the same but now with $n_i \to n_{qi}$:

$$N_i = \frac{N_c}{\pi^2} V \int p^2 \mathrm{d}p (n_i - \bar{n}_i). \tag{9}$$

Results

We concentrate on such features of the q-NJL model:

(1) Chiral symmetry restoration in the q-NJL („Results-chiral")

(2) Spinodial decomposition in the q-NJL („Results-spinodial")

(3) Critical effects in the q-NJL („Results-critical effects")

As our goal was to demonstrate the sensitivity to the nonextensive effects represented by |q-1| ≠0, we do not reproduce here the whole wealth of results provided in

Costa P, Ruivo M C and de Sousa A 2008 *Phys. Rev.* D 77 096001

but concentrate on the most representative results.

Results-chiral

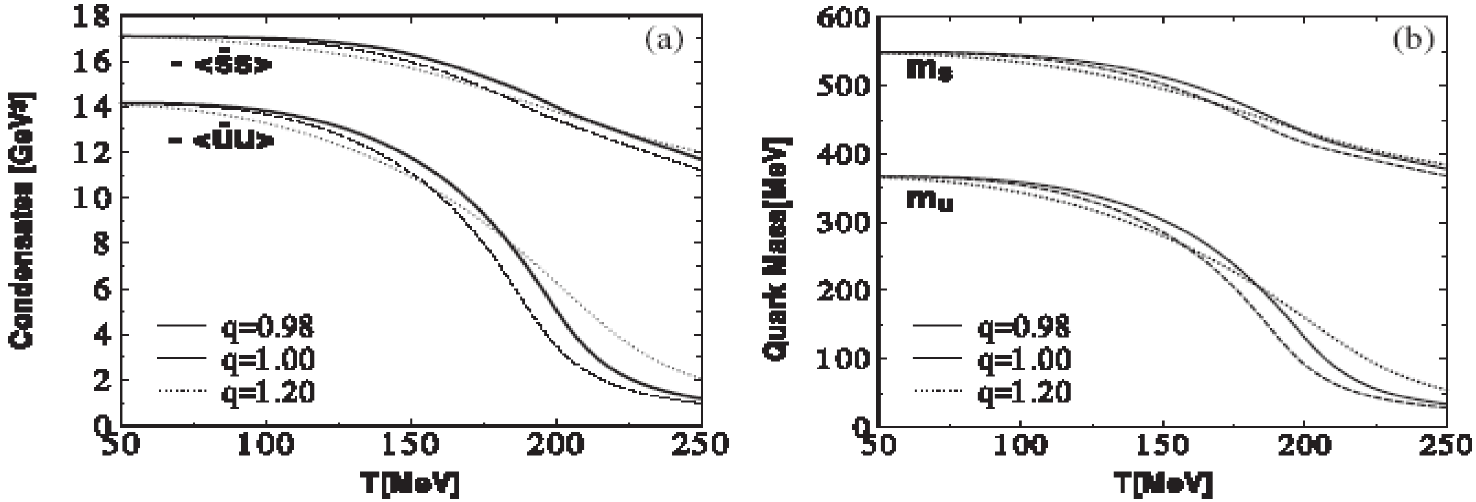


Figure 1. (a) Quark condensates and (b) effective quark masses as functions of the temperature for different values of the nonextensive parameter q ($q = 1$ corresponds to Boltzmann–Gibbs statistics).

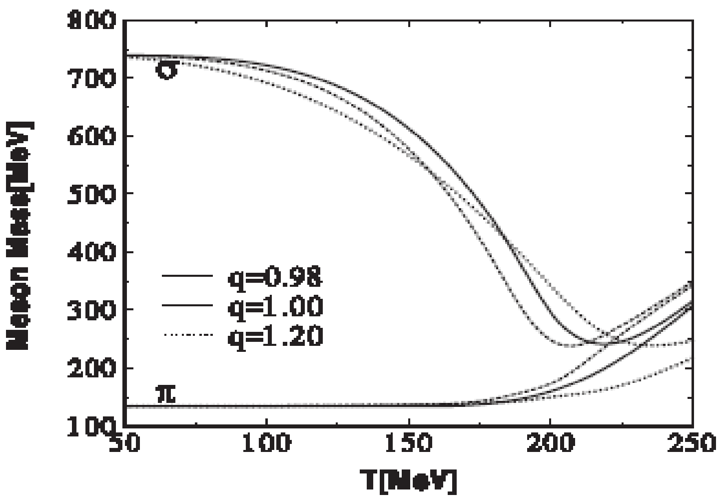


Figure 2. Masses of π and σ mesons as functions of the temperature for different values of the nonextensive parameter q ($q = 1$ corresponds to BG statistics).

Results-chiral

They were calculated assuming zero chemical potentials and solving numerically the q-version of gap equation

$$M_i = m_i - 2g_{\mathrm{s}} \langle \bar{q}_i q_i \rangle - 2g_{\mathrm{D}} \langle \bar{q}_j q_j \rangle \langle \bar{q}_k q_k \rangle,$$

with $\langle \bar{q}_i q_i \rangle \rightarrow \langle \bar{q}_i q_i \rangle_q$ given by:

$$\langle \bar{q}_i q_i \rangle_q = -\frac{N_c}{\pi^2} \sum_{i=\mathrm{u,d,s}} \left[\int \frac{p^2 M_i}{E_i} \left(1 - n^q_{qi} - \bar{n}^q_{qi}\right) \right] \mathrm{d}p.$$

In what concerns temperature dependence:

There is still an ongoing discussion on the meaning of the temperature in nonextensive systems. However, in our case the small values of the parameter q deduced from data allow us to argue that, to first approximation, $T_q = T$ used here . In high energy physics it is just the hadronizing temperature (and instead of the state of equilibrium one deals there with some kind of stationary state). For a thorough discussion of the temperature of nonextensive systems, see Abe S 2006 *Physica* A 368 430

Results-chiral

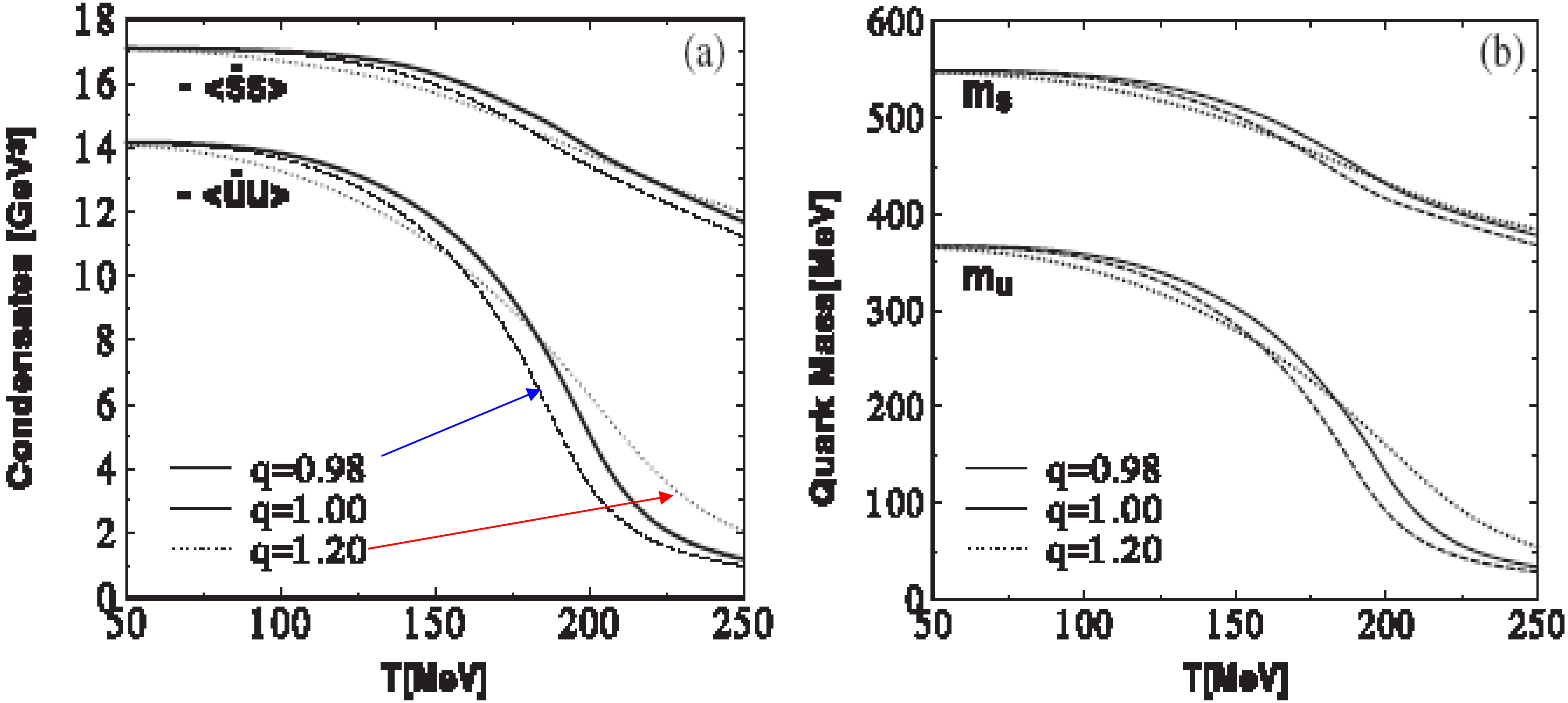


Figure 1. (a) Quark condensates and (b) effective quark masses as functions of the temperature for different values of the nonextensive parameter q ($q = 1$ corresponds to Boltzmann–Gibbs statistics).

(*) Notice the difference between q<1 and q>1 cases.
(*) The effects caused by nonextensivity are practically invisible for heavier quarks.

Results-chiral

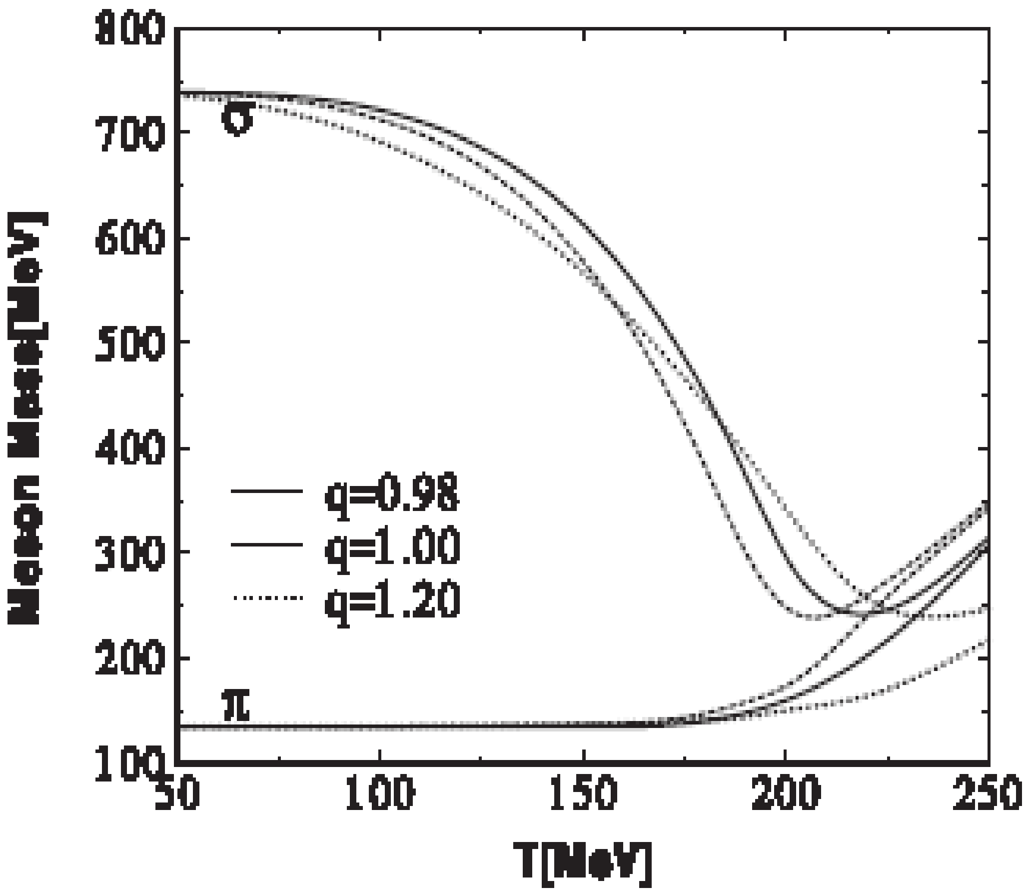


Figure 2. Masses of π and σ mesons as functions of the temperature for different values of the nonextensive parameter q ($q = 1$ corresponds to BG statistics).

Results-chiral

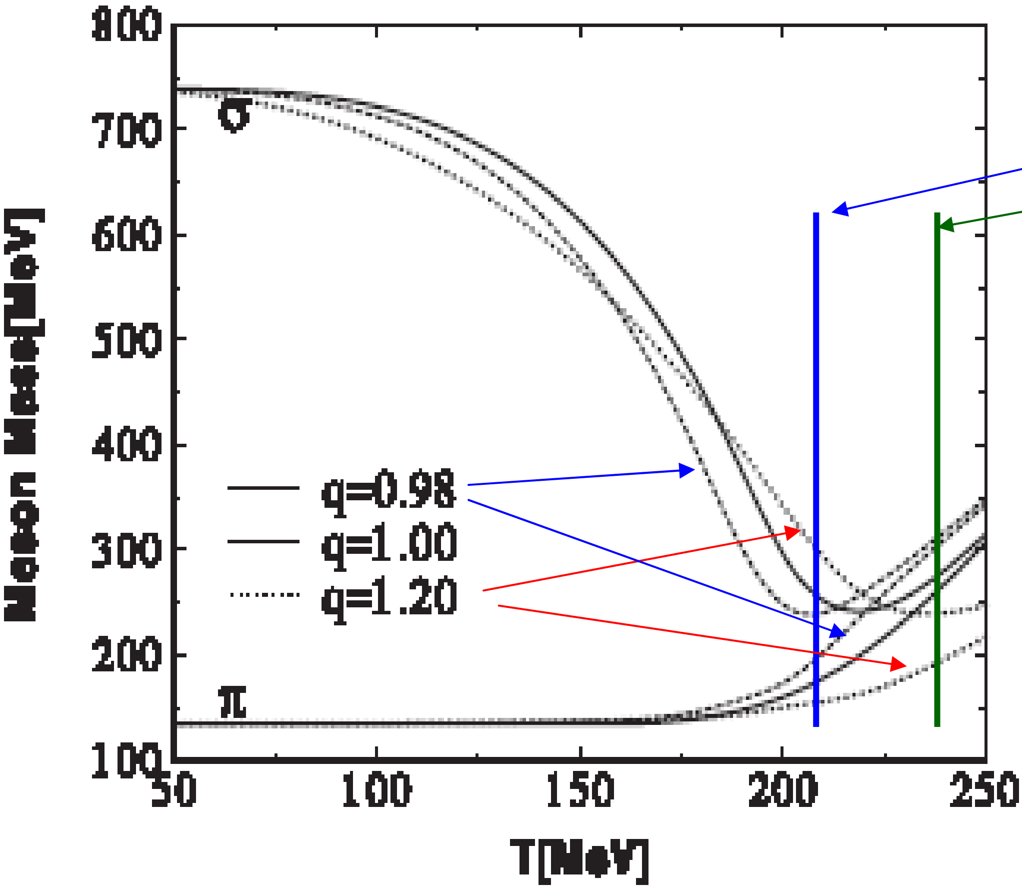


Figure 2. Masses of π and σ mesons as functions of the nonextensive parameter q ($q = 1$ corresponds to BG statis

(*) Temperature for which σ mass reaches minimum is smaller for q<1 and larger for q>1

(by amount ~|q-1|).

(*) Final values of masses is larger for q<1 and smaller for q>1 (by amount ~|q-1|).

(*) Fluctuations (q>1) dilute the region where the chiral phase transition takes place.

(*) Correlations (q<1) only shift the condensates, quark masses and and meson masses towards smaller temperatures.

(*) They refer to quarks, not hadrons as in q-Walecka model (where only q>1, i.e., fluctuations were considered with similar effect).

Results-chiral

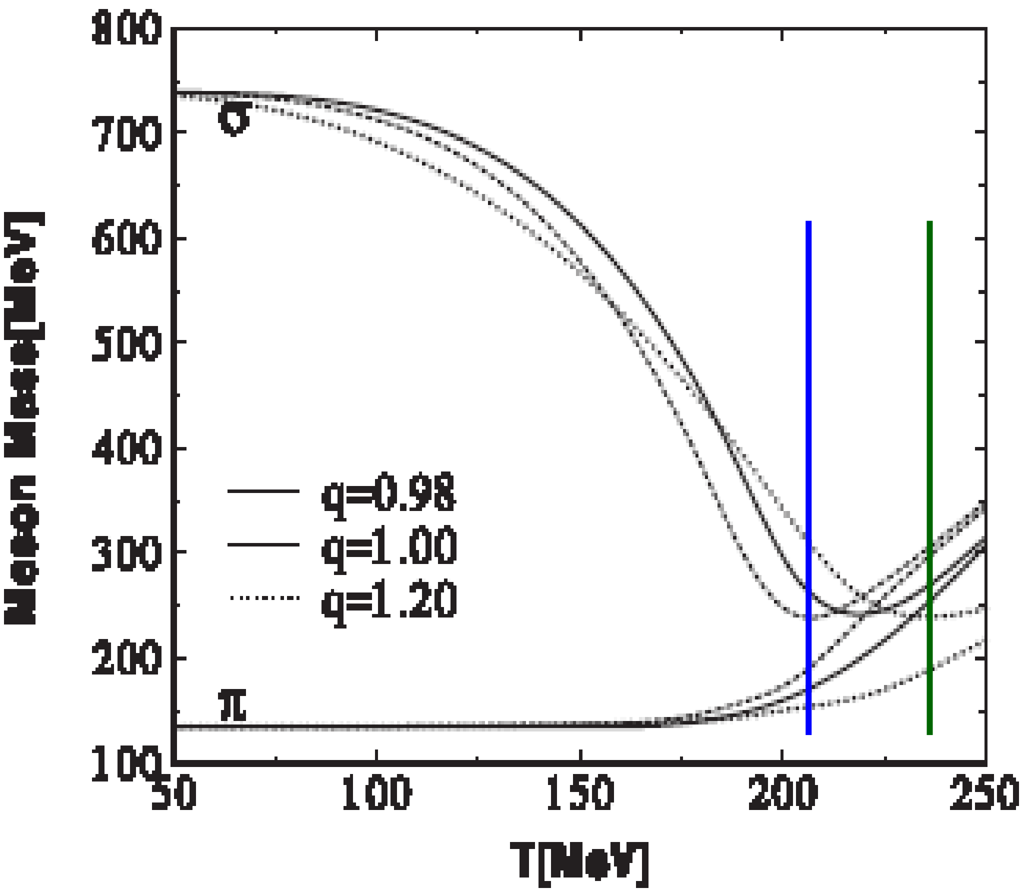


Figure 2. Masses of π and σ mesons as functions of the temperature for different values of the nonextensive parameter q ($q = 1$ corresponds to BG statistics).

Note: we are not specifying here what the actual dynamical mechanisms behind such fluctuations/correlations are, we just model them by the parameter q. An example of such a dynamical effect is the temperature dependence of the respective coupling constants, as mentione before:

$$g_D(T) = g_D(T=0)\exp\left[-\left(\frac{T}{T_0}\right)^2\right]$$

Results-spinodial

(*) The spinodial phase transition occurs (in general) for finite densities and for the $T < T_{cr}$. Above it we do not observe the phase transition of the first order but rather a smooth corossover.

(*) To address the influence of q-statistics on the spinodial phase transition discussed in Costa P, Ruivo M C and de Sousa A 2008 *Phys. Rev.* D 77 096001 one has to proceed to finite density calculations assuming chemical equilibrium in the form

$$\mu_u = \mu_d = \mu_s = \mu$$

allowing for nonzero baryon density $\rho = (1/3) \sum_i N_i/V$.

(*) The first observation is that details of the spinodial phase transition are now much more sensitive to (q-1) than it was observed before.

(*) In general: for q<1 the pressure decreases and energy increases
for q>1 the tendency is opposite.

This is a direct consequence of the (q-1) term in the S_q:

$$S_q^{A \oplus B} = S_q^A + S_q^B + (1 - q) S_q^A S_q^B$$

Results-spinodial

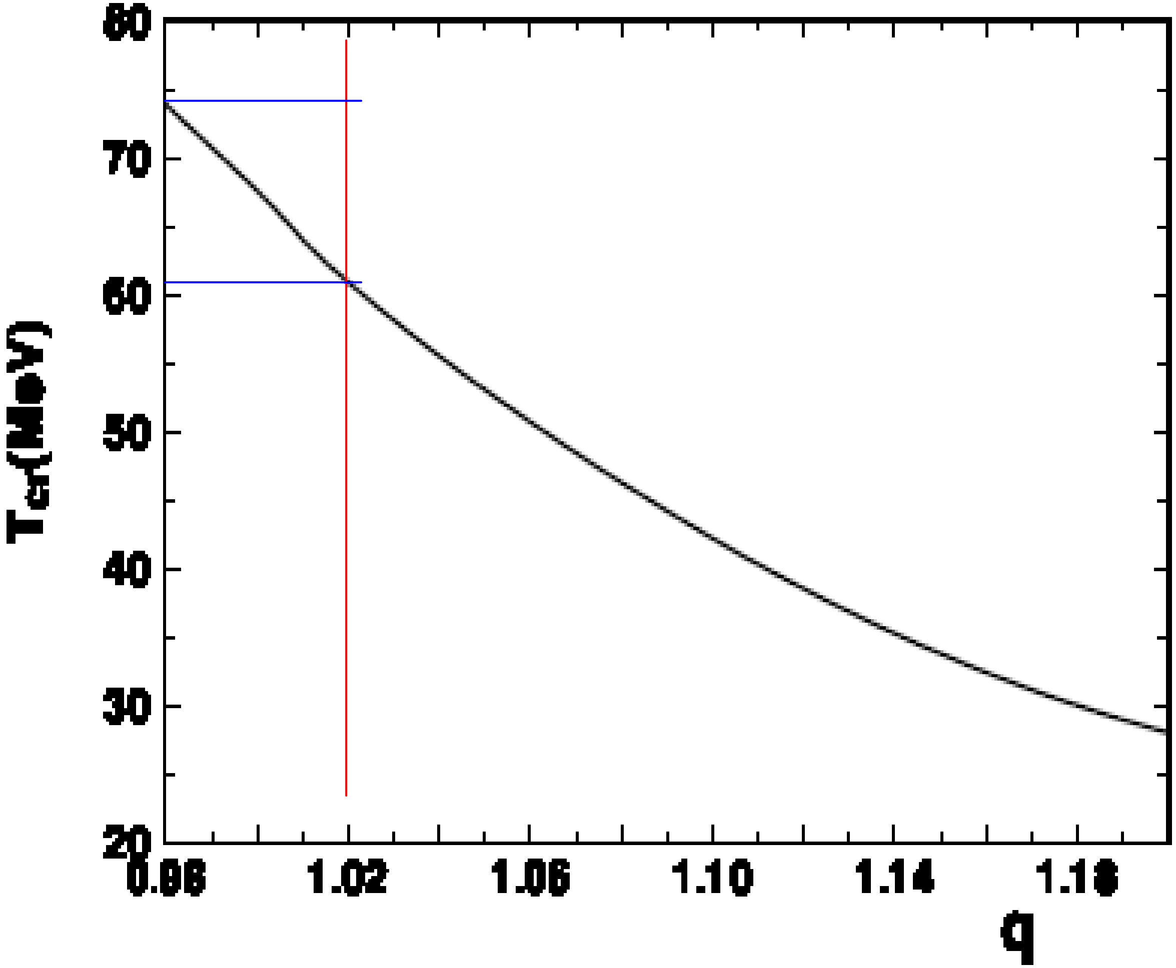
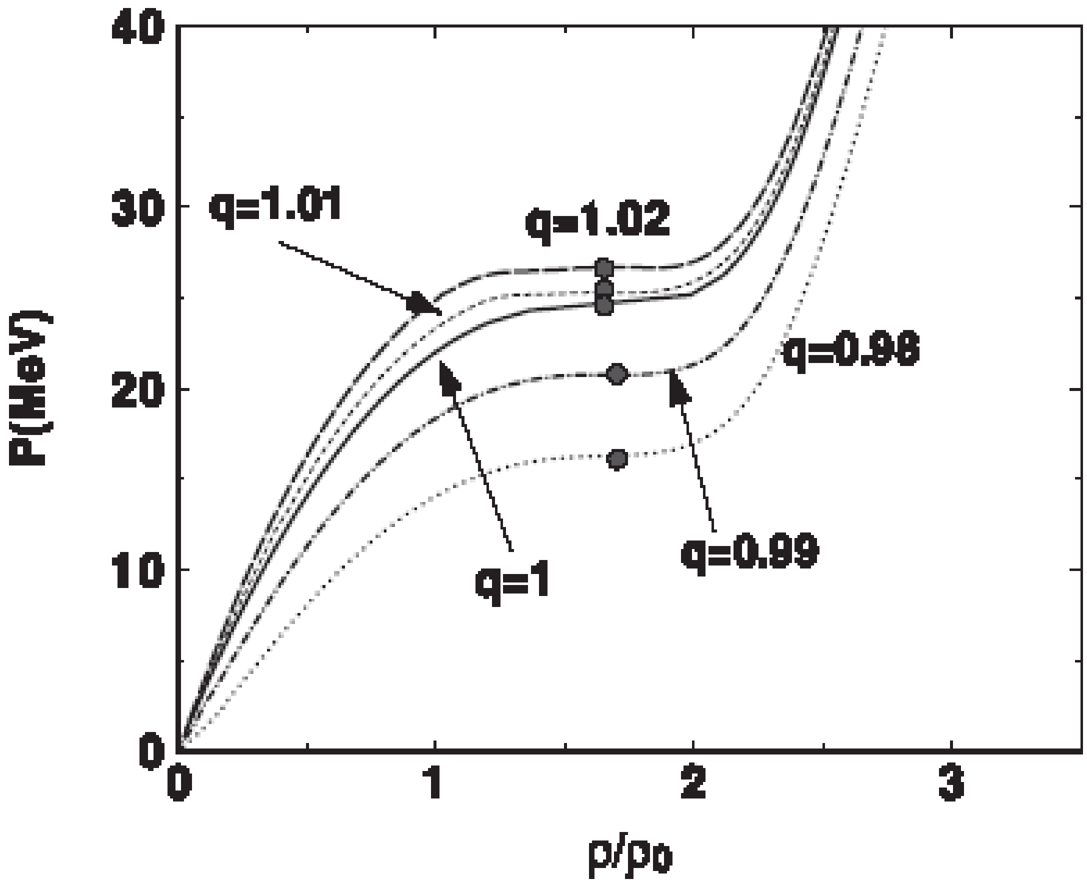


Figure 3. Critical temperature T_{cr} as a function of the nonextensivity parameter q (in the range of q considered here).

Results-spinodial



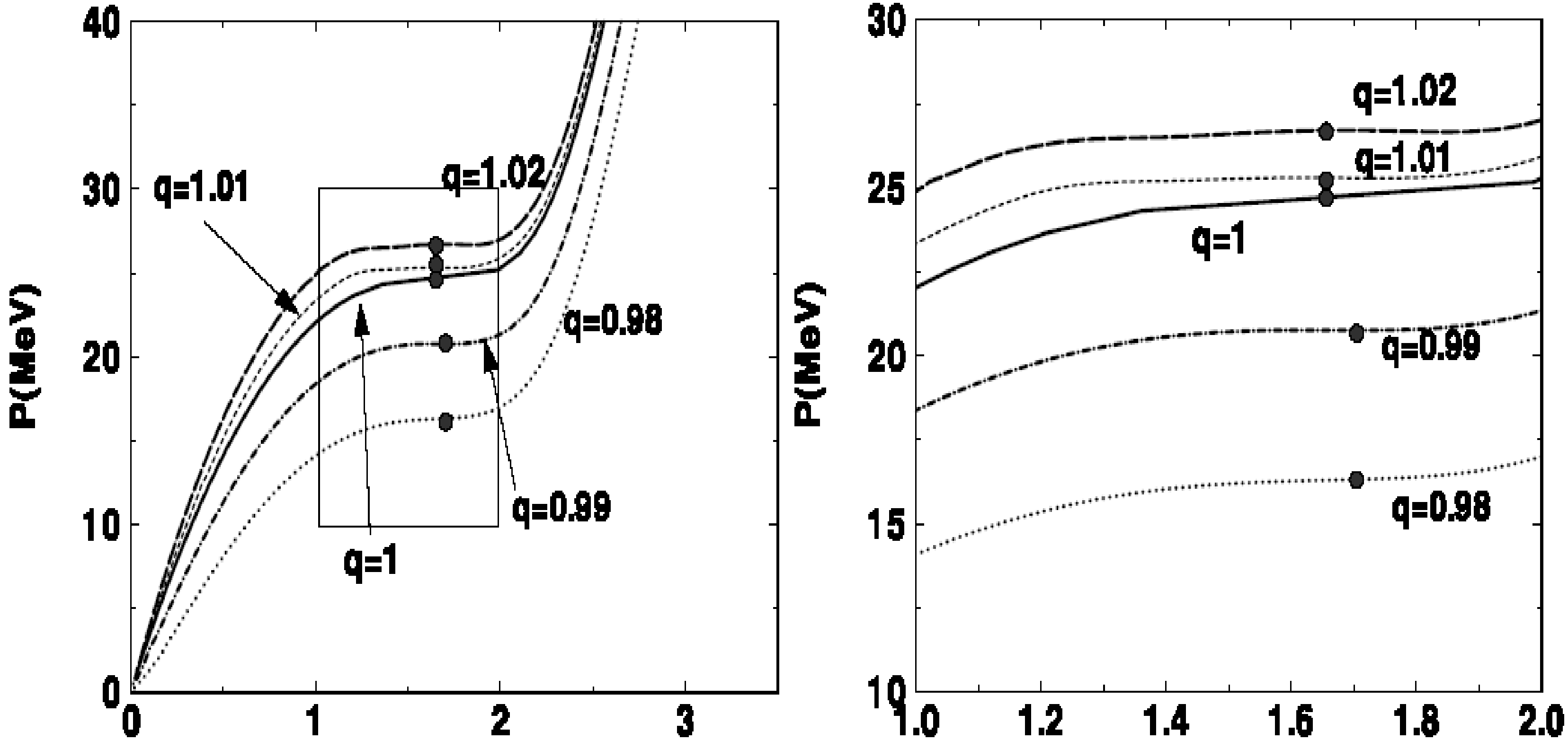
(ρ_0 =0.17fm^{-3})

Note that effect is stronger for q<1 and, essentially, the saddle point remains at the same value of compression.
For q<1 the critical pressure is smaller and for q>1 is bigger than the critical pressure for BG.
Note that position of inflection points (dots) are practically q-independent.

Figure 4. The pressure at critical temperature T_{cr} as a function of the compression ρ/ρ_0 calculated for different values of the nonextensivity parameter q. The dots indicate positions of the inflection points for which the first derivative of pressure in compression vanishes. As in [23] for $q = 1$ the corresponding compression is $\rho/\rho_0 = 1.67$ (and this leads to $\mu = 318$ MeV); it remains the same for $q > 1$ considered here (but now $\mu = 321$ MeV for $q = 1.01$ and $\mu = 325$ MeV for $q = 1.02$), whereas it is shifted to $\rho/\rho = 1.72$ for $q < 1$ (resulting in $\mu = 313$ MeV for $q = 0.99$ and $\mu = 307$ MeV for $q = 0.98$).

[23] Costa P, Ruivo M C and de Sousa A 2008 *Phys. Rev.* D 77 096001

Results-spinodial



(ρ_0 =0.17fm^{-3})

Note that effect is stronger for q<1 and, essentially, the saddle point remains at the same value of compression.

For q<1 the critical pressure is smaller and for q>1 is bigger than the critical pressure for BG.

Note that position of inflection points (dots) are practically q-independent.

Results-spinodial

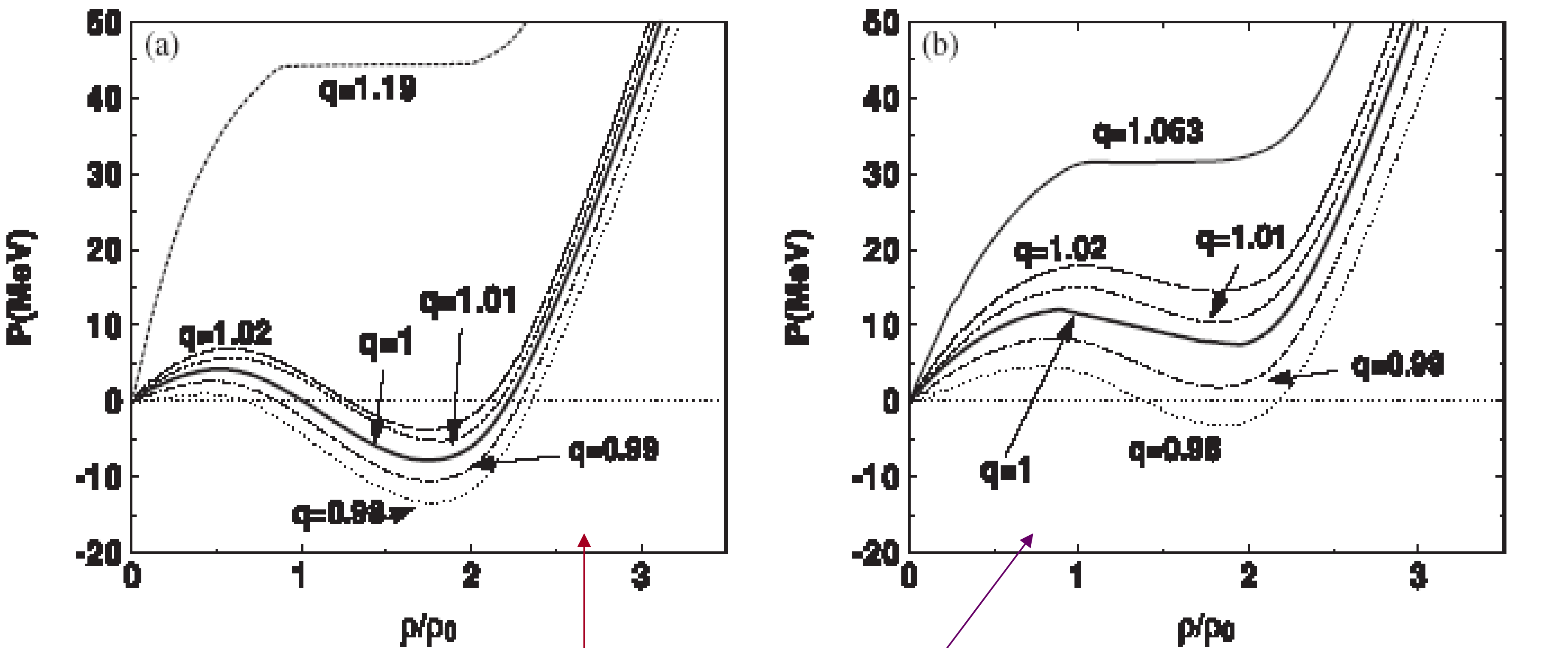
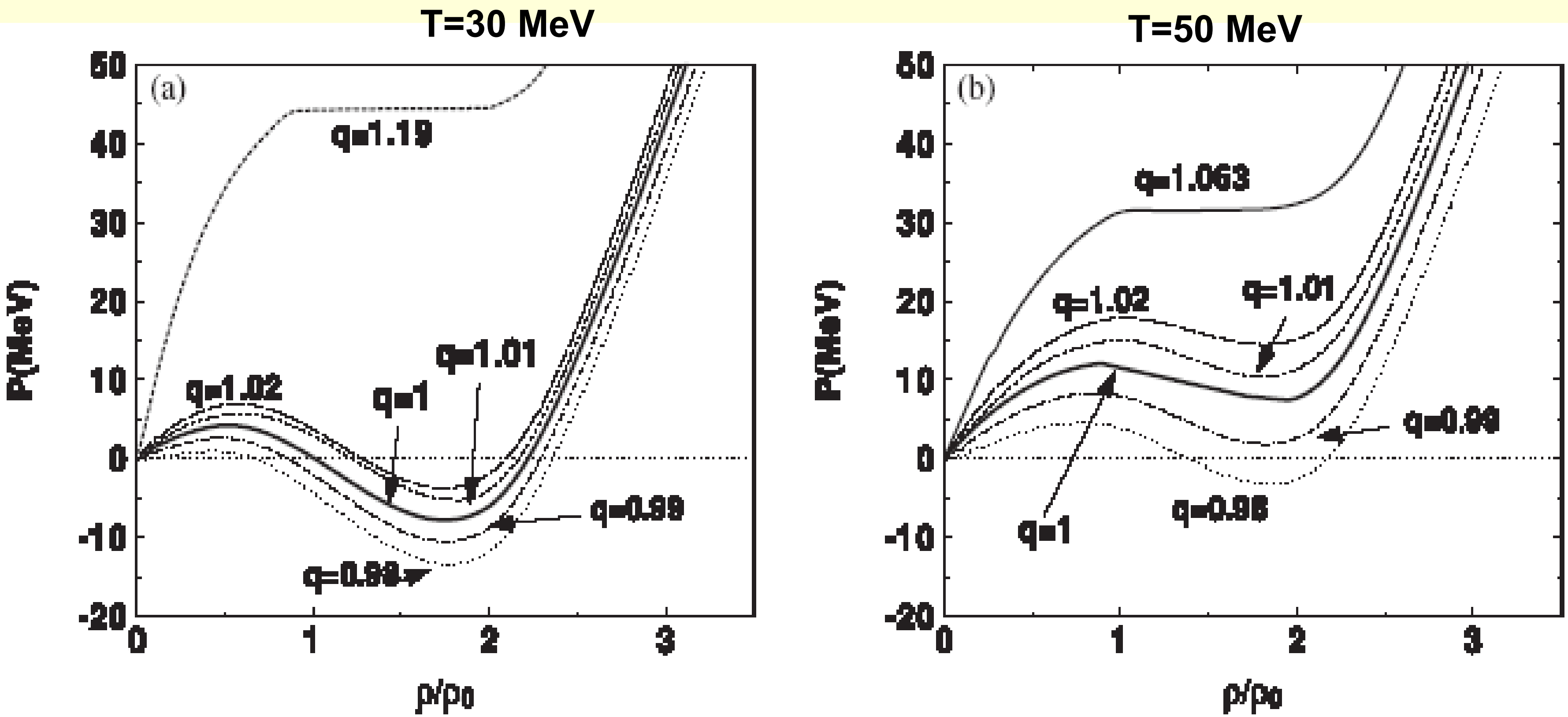


Figure 5. The pressure calculated for different values of the nonextensivity parameter q for temperatures $\boxed{T = 30\ \text{MeV}}$ (a) and $\boxed{T = 50\ \text{MeV}}$ (b) as a function of the compression ρ/ρ_0. The curves for q for which the temperature considered is the critical temperature are also shown, they correspond to $q = 1.19$ for $T = 30$ MeV and $q = 1.063$ for $T = 50$ MeV.

Results-spinodial



(*) Note the occurence of the typical spinodial structure, which is more pronounced for lower T whereas its sensitivity to the q parameter gets stronger with increasing T.

(*) Note the curves for q's for which the $T=T_{crit}$:

$q=q_{crit}=1.19$ for $T=30$ MeV and $q=q_{crit}=1.063$ for $T=50$ MeV,

i.e., the corresponding values of q decrease with T (as expected from previous Fig. 3).

(*) It means that for each T a $q=q_{crit}>1$ exists for which there is no more mixed phase and for which spinodial effect vanishes..

(*) In contrast, effects leading to $q<1$ work towards an increase of T_{crit} and make the spinodial effect more pronounced.

Results-spinodial

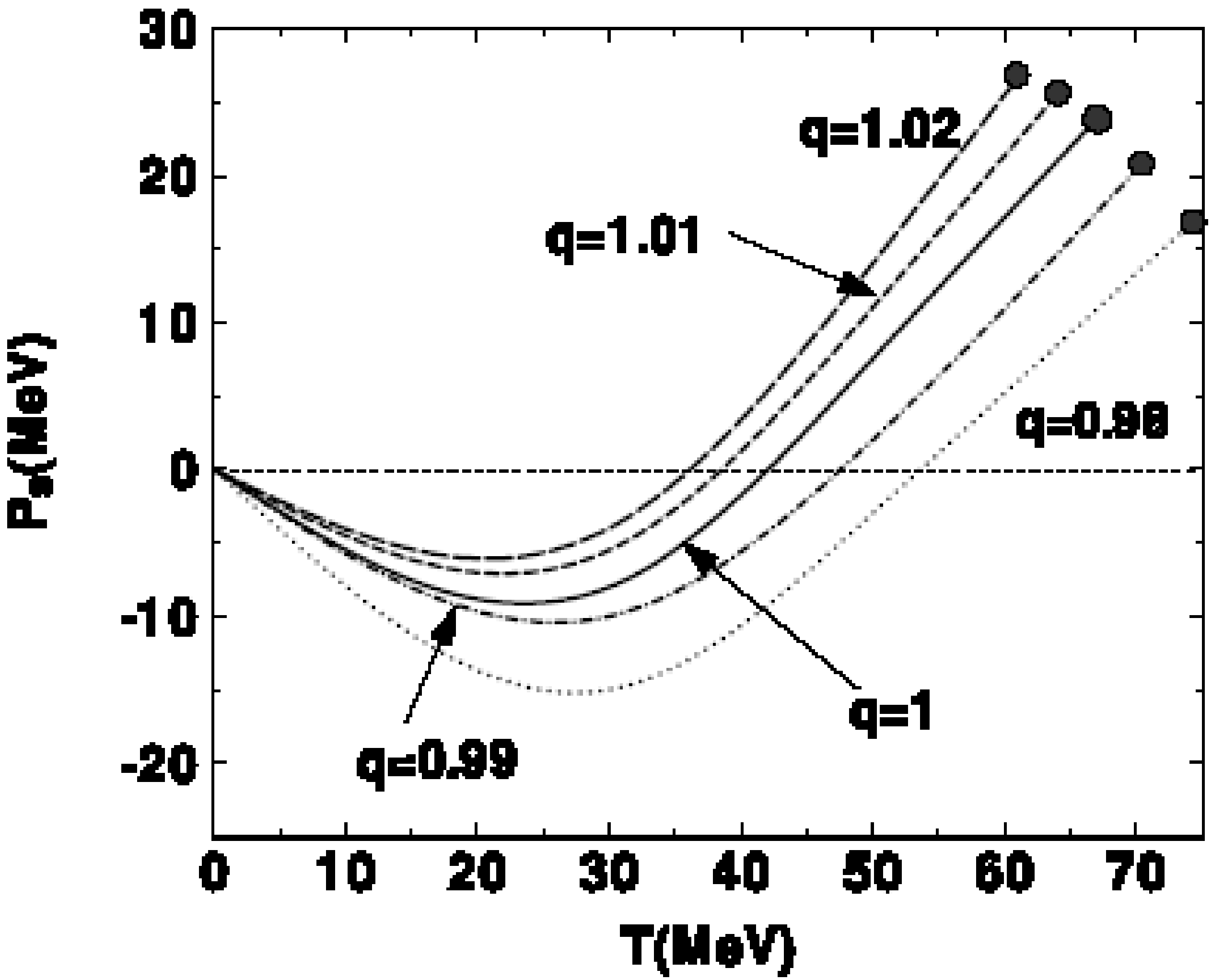
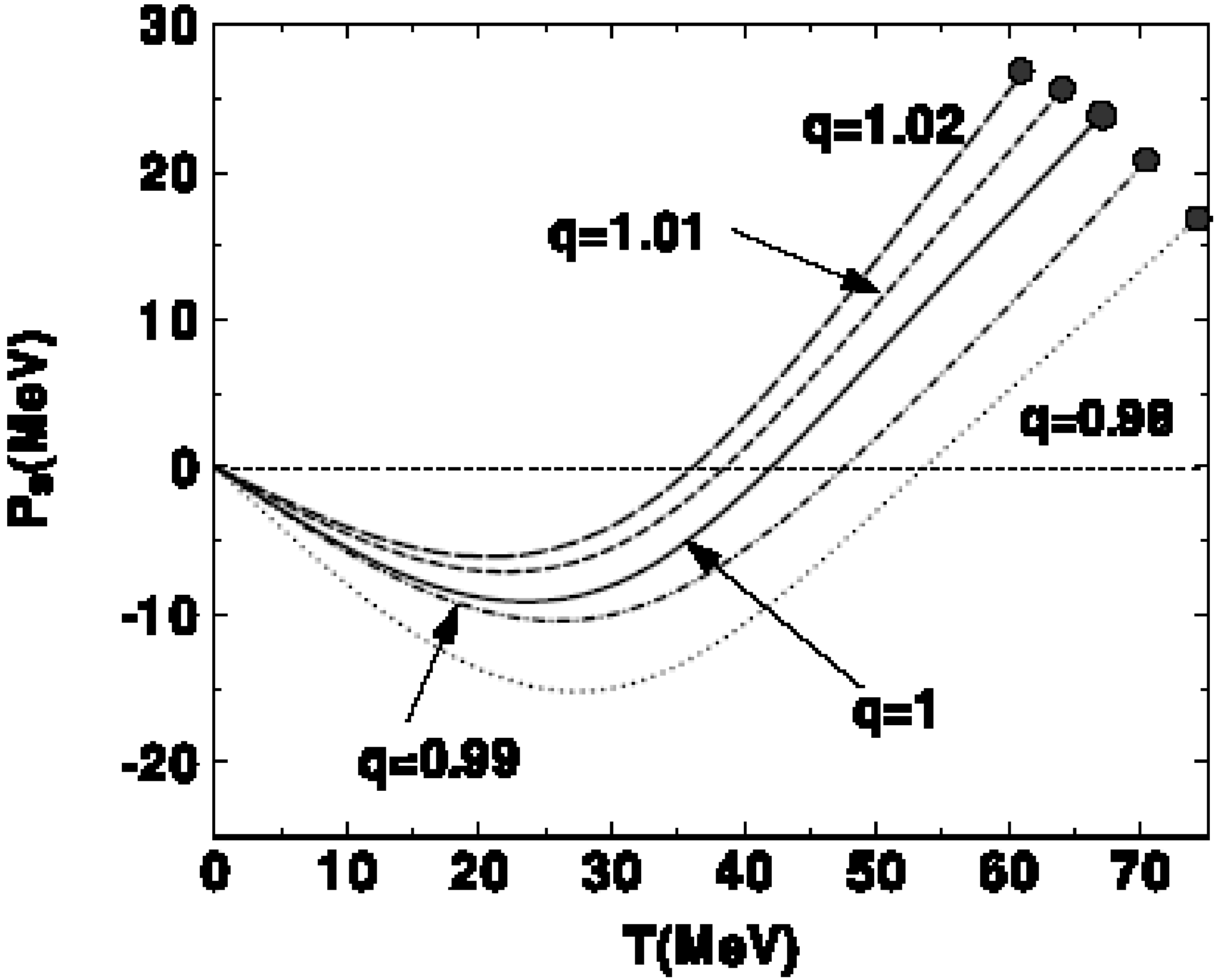


Figure 6. Pressure P_s corresponding to the local minimum in figures 4 and 5 versus temperature T calculated for some selected values of the nonextensivity parameter q. The curves end at the critical points.

Results-spinodial



(*) Temperature at which P_s starts to be positive is shifted towards smaller values with increasing q.

(*) P_s gets deeper into negative values with decreasing q.

Figure 6. Pressure P_s corresponding to the local minimum in figures 4 and 5 versus temperature T calculated for some selected values of the nonextensivity parameter q. The curves end at the critical points.

Results-spinodial

The effect of negative pressure can be best understood by invoking the bag model picture of the nucleon [29]. Generally, in the phase of hadron gas we observe a decrease of the pressure and the critical temperature T_{cr} with the increase of fluctuations given by q. That phenomenon resembles to some extent the behavior of the bag constant for the nucleon in the medium where the bag pressure decreases with the increase of the chemical potential μ in order to get a proper equation of state [37]. In the nuclear thermodynamical models this bag constant is modified because the vacuum, in which hadrons are embedded, is modified by the residual interaction present in the nuclear medium (acting toward the Wigner realization of the chiral symmetry in which masses of π and σ are degenerated). Here such density dependence corrections are introduced by nonextensive effects inside the nuclear medium. In that way the nuclear vacuum for the temperatures below the critical temperature and critical densities, the usual area of the spinodal phase transition, can be properly described effectively by the nonextensive statistics.

[29] Buballa M 2005 *Phys. Rep.* 407 205

[37] Leonidov A, Redlich K, Satz H, Suhonen E and Weber G 1994 *Phys. Rev.* D 50 4657
Patra B K and Singh C P 1996 *Phys. Rev.* D 54 3551

Results-spinodial

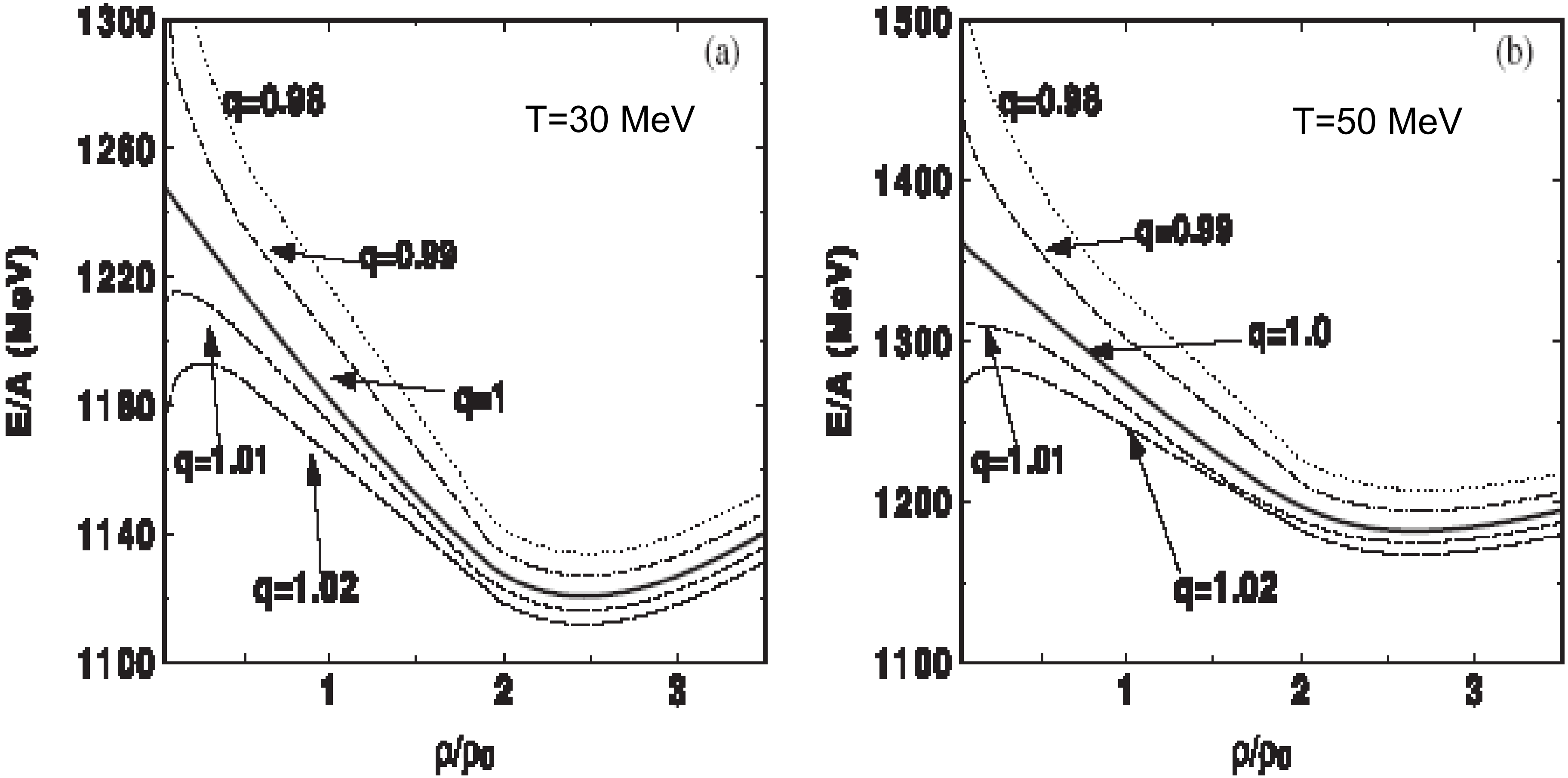
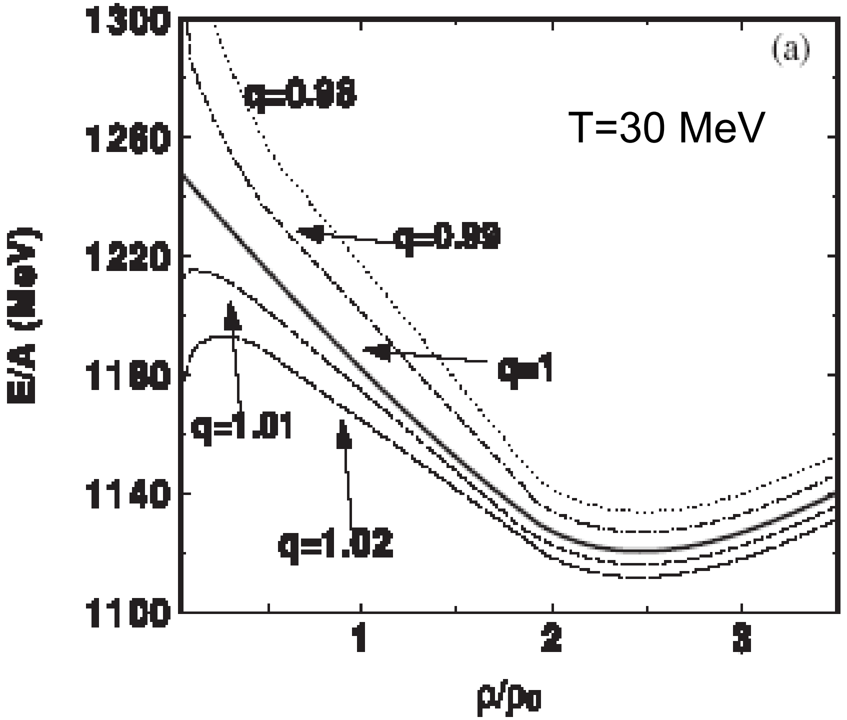
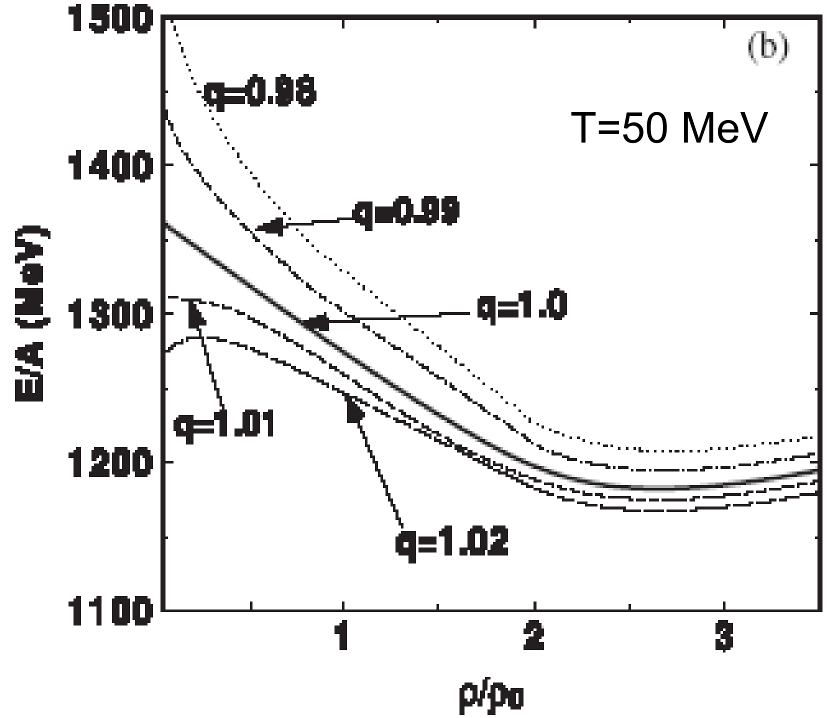


Figure 7. The energy per particle at the temperatures $T = 30$ and $T = 50$ MeV as a function of the compression ρ/ρ_0 calculated for different values of the nonextensivity parameter q.

Results-spinodial





(*) Energy exceeds the usual one for q<1 and gets smaller for q>1 (especially for compression smaller than two. This is an opposite trend to that observed for the corresponding behavior of the pressure.

(*) The absolute minimum of energy for the given T does not depend on the parameter q. For example, at T=30 MeV it is at ρ/ρ_0 =2.45 and to obtain the stable state here (i.e., P=0) one has to choose q=0.97. In such a way, the final droplets of quarks [38] in the mixed phase can appear at finite temperatures.

[38] Buballa M and Oertel M 1998 *Nucl. Phys.* A **642** 39 1999 *Phys. Lett.* B **457** 251

Results-spinodial (summary)

(*) Nonextensive dynamics enter the NJL calculations through which are symmetrical for q and 1-q.

$$n_{qi} = \frac{1}{\tilde{e}_q(\beta(E_i - \mu_i)) \pm 1},$$

(*) The differences between q<1 and q>1 cases are due to our way of defining the energy and entropy where following

Alberico W M and Lavagno A 2009 *Eur. Phys. J.* A 40 313 Drago A, Lavagno A and Quarati P 2004 *Physica* A 344 472

we are using $[n_{qi}]^q$ and $[\bar{n}_{qi}]^q$ instead of n_{qi} and $\bar{n}_{qi}$ (**).

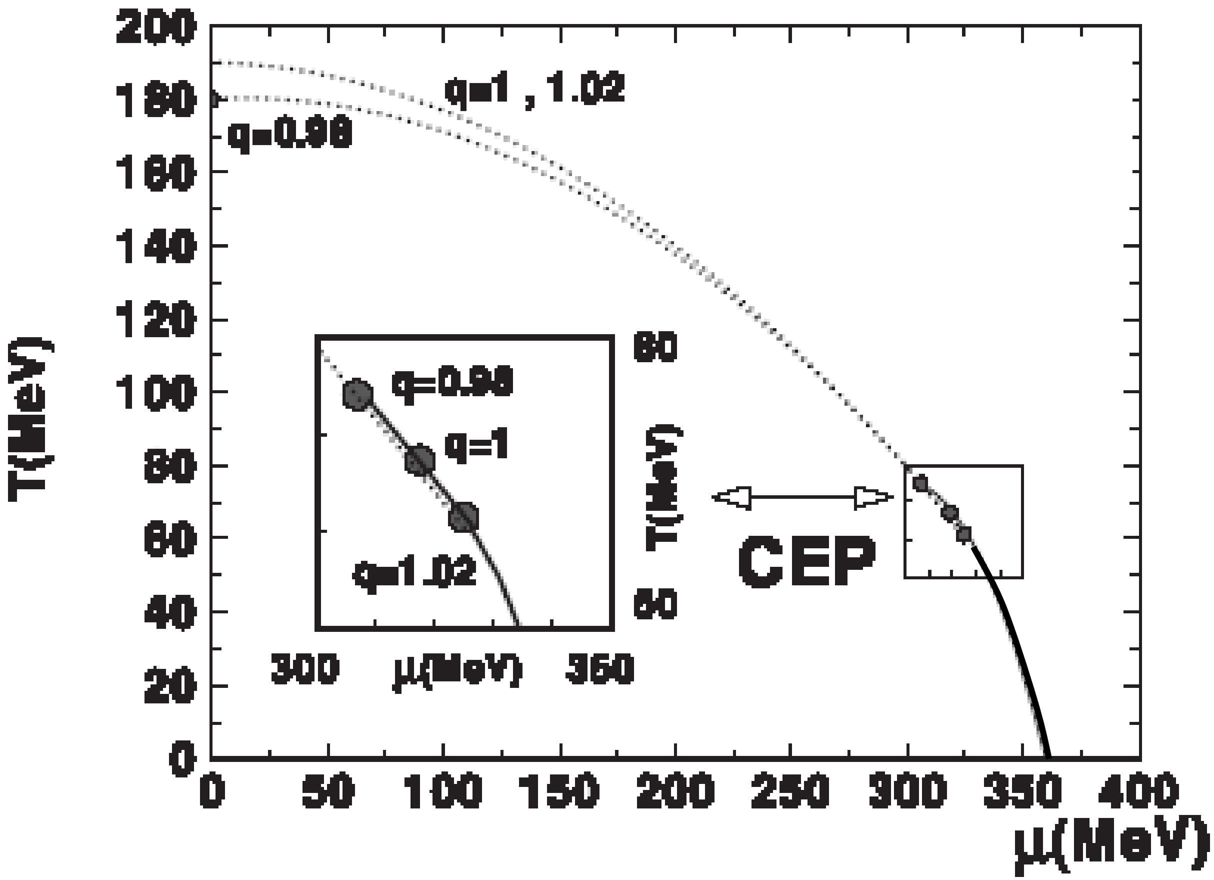
(*) q<1 increases then the effective occupancy and make the absolute values of quark condensates begin to decrease for q=0.98 at lower T than for q=1. The corresponding energy is bigger, i.e., the residual attractive correlations increase the energy and lead to hadronization occur at lower T.

(*) Fluctuations introduced by q>1 decrease the effective occupations and the energy and smear out the chiral phase transition.

(**) It is worth noting that in [22], which considers only the $q > 1$ case and uses number distributions without powers of q, the significant effects were obtained only for much bigger values of the nonextensive parameter $q = 1.2$.

[22] Pereira F I M, Silva B and Alcaniz J S 2007 *Phys. Rev.* C 76 015201

Results-critical effects



Notice: the overlap of curves observed indicate that the critical point is smeared to a kind of critical area.

Figure 8. Phase diagram in the q-NJL model in the T–μ plane for values of q considered before: $q = 0.98, 1.0, 1.02$. Solid and dashed lines denote, respectively, the first-order and crossover phase transitions. The results are presented for three different values of the nonextensivity parameter q with the vicinity of the (q-dependent) critical end points (CEP) enlarged in the inset. The crossover phase transition for $q = 0.98$ and for $\mu \to 0$ takes place for smaller temperature T.

Results-critical effects

(*) The transition between confined and deconfined phases and/or chiral phase transition can be seen by measuring, event-by-event, the difference in the magnitude of the local fluctuation of the net baryon number in heavy ion collisions.

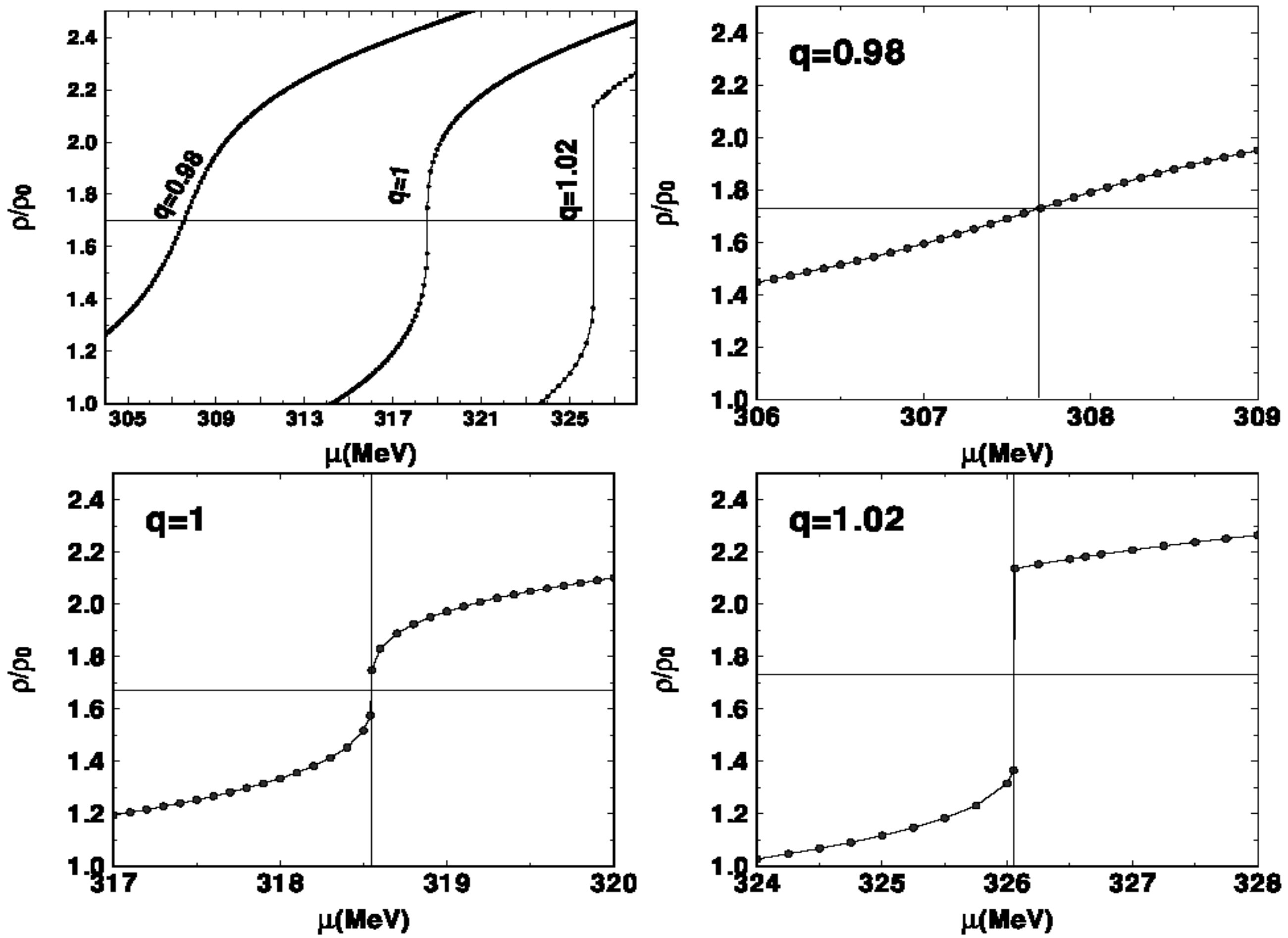
(*) They are initiated and driven mainly by the quark number fluctuation, described by

$$\chi_B = \frac{1}{V}\sum_{i=u,d,s}\left(\frac{\partial \rho_i}{\partial \mu_B}\right)_T = -\frac{1}{V}\sum_{i=u,d,s}\frac{\partial^2 \Omega}{\partial^2 \mu_B}\bigg|_T$$

and can survive through the freezout.

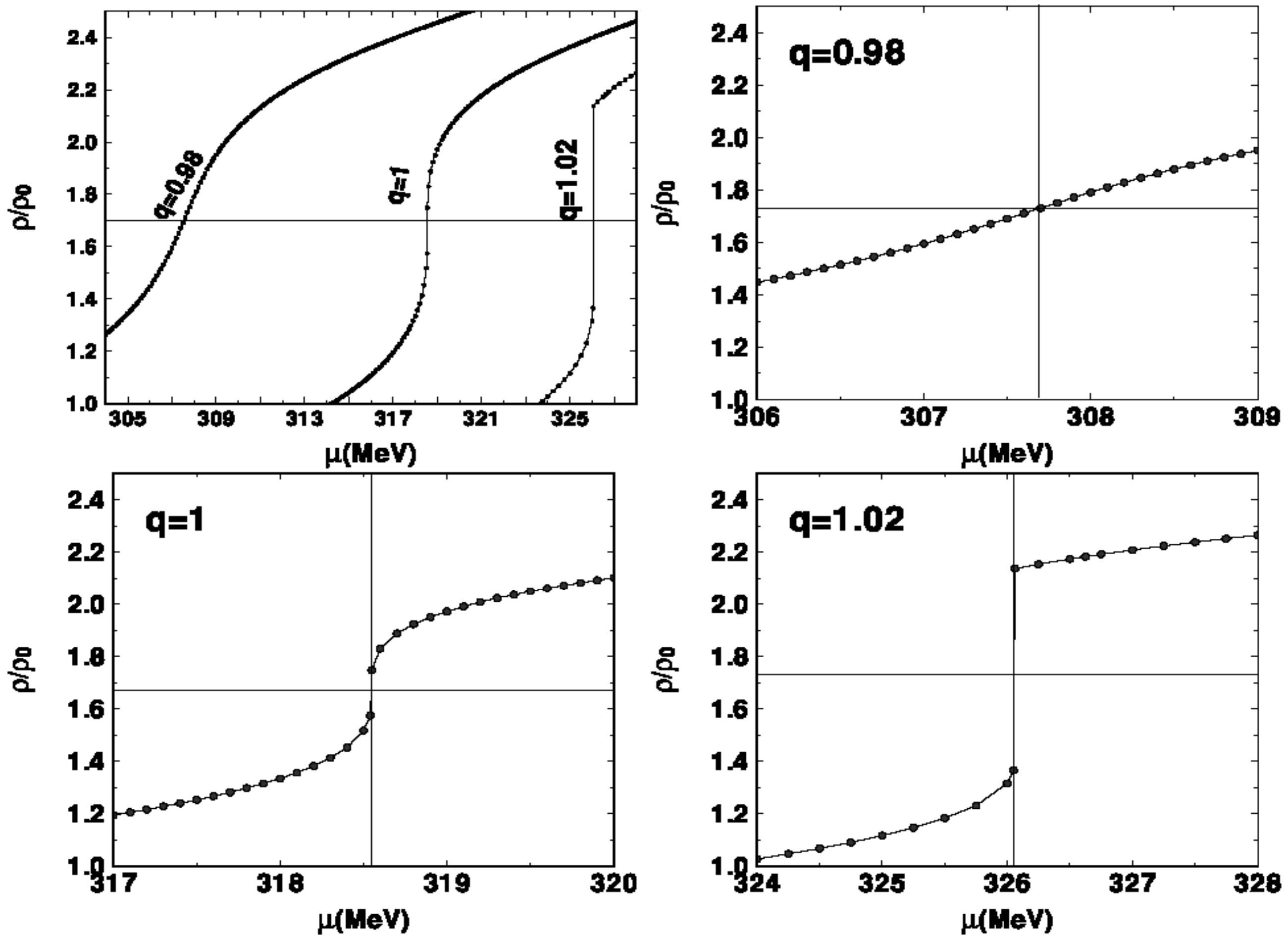
(*) Consequently, our q-NJL model allows to make the fine tuning for the magnitude of baryon number fluctuations and to find the characteristic for this system value of the parameter q. However, it does not allow to differentiate between possible dynamical mechanisms of baryon fluctuation.

Results-critical effects



The baryon compression ρ/ρ_0 (calculated in the vicinity of the critical values of temperature and density indicated by the corresponding dotted lines) as function of the chemical potential μ for different values of the nonextensivity parameter, $q = 0.98, 1.00, 1.02$. The summary presented in the top-left panel is detailed in the three consecutive panels.

Results-critical effects



Notice the remarkable difference for the density derivative at the critical point: from the smooth transition through the critical point for q<1 to a big jump in the density for critical value of chemical potential for q>1. It reflects the infinite values of the baryon number susceptibility

$$\chi_B = \frac{1}{V}\sum_{i=u,d,s}\left(\frac{\partial \rho_i}{\partial \mu_B}\right)_T = -\frac{1}{V}\sum_{i=u,d,s}\frac{\partial^2 \Omega}{\partial^2 \mu_B}\bigg|_T$$

Results-critical effects

(*) The transition between confined and deconfined phases and/or chiral phase transition can be seen by measuring, event-by-event, the difference in the magnitude of the local fluctuation of the net baryon number in heavy ion collisions.

(*) They are initiated and driven mainly by the quark number fluctuation, described by

$$\chi_B = \frac{1}{V}\sum_{i=u,d,s}\left(\frac{\partial \rho_i}{\partial \mu_B}\right)_T = -\frac{1}{V}\sum_{i=u,d,s}\frac{\partial^2 \Omega}{\partial^2 \mu_B}\bigg|_T$$

and can survive through the freezout.

(*) Consequently, our q-NJL model allows to make the fine tuning for the magnitude of baryon number fluctuations and to find the characteristic for this system value of the parameter q. However, it does not allow to differentiate between possible dynamical mechanisms of baryon fluctuation.

(*) Using q-dependent χ_B results in q-dependent ε of the critical exponents which describe the behavior of baryon number susceptibilities near the critical point:

- in the mean field universality class ε=ε'=2/3,

- our preliminary result using q_NJL show:

  - smaller value of this parameter for q > 1: ε ~ 0.6 for q=1.02;
  - greater value of this parameter for q < 1: ε ~ 0.8 for q=0.98.

Summary

● We have investigated the sensitivity of the mean field theory presented by the NJL model to the departure from the conditions required by the application of the BG approach. This was done using Tsallis nonextensive statistical mechanics and both $q>1$ and $q<1$ cases were considered (believed to be connected with fluctuations/correlations). The observed effects depend on T and tend to vanish for $T \rightarrow 0$.

● For $q<1$ we observe decreasing of P, which reaches negative values for a broad (q-dependent) range of T, and increasing of T_{crit}.

● For $q>1$ we observe decreasing of T_{crit} , therefore in the limit of large q we do not have a mixed phase but rather a quark gas in the deconfined phase above the critical line (but the compresion ot at T_{crit} does not depend on q). The resulting EoS is stiffer (in the sense that for a given density we get larger pressure with increasing q).

● The nonextensive statistics dilutes the border between the crossover and the first order transition.

● It also changes the behavior of baryon number susceptibilities near the critical point.

Summary

Finally, we would like to bring ones attention to the mot recent investigations of the type presented here, see:

arXiv:1005.4643 :

Nonextensive statistical effects in the hadron to quark-gluon phase transition

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We investigate the relativistic equation of state of hadronic matter and quark-gluon plasma at finite temperature and baryon density in the framework of the nonextensive statistical mechanics, characterized by power-law quantum distributions. We study the phase transition from hadronic matter to quark-gluon plasma by requiring the Gibbs conditions on the global conservation of baryon number and electric charge fraction. We show that nonextensive statistical effects play a crucial role in the equation of state and in the formation of mixed phase also for small deviations from the standard Boltzmann-Gibbs statistics.

Summary

Physics Letters A 373 (2009) 4214–4218

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Nonextensive effects on the phase structure of quantum hadrodynamics

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ARTICLE INFO

Article history:
Received 16 August 2009
Received in revised form 13 September 2009
Accepted 15 September 2009
Available online 19 September 2009
Communicated by A.R. Bishop

ABSTRACT

In this Letter, we investigate nonextensive effects on phase transition in nuclear matter in the context Walecka many-body field theory. A difference is observed when the results calculated for the nuclear matter at vanishing baryon density is compared to those obtained through the standard Fermi–Dirac distribution. It is observed a dependence between the nonextensive parameter q and the coupling constants C_S^2 of the phase transition. A numerical relation for this thermodynamical dependence is also proposed.

Summary

epl A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

January 2010

EPL, **89** (2010) 10004 www.epljournal.org
doi: 10.1209/0295-5075/89/10004

Nonextensive quantum H-theorem

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received 4 August 2009; accepted in final form 7 December 2009
published online 18 January 2010

PACS 05.30.-d – Quantum statistical mechanics
PACS 03.65.Ta – Foundations of quantum mechanics; measurement theory
PACS 05.70.Ln – Nonequilibrium and irreversible thermodynamics

Abstract – A proof of the quantum H-theorem taking into account nonextensive effects on the quantum entropy S_q^Q is shown. The positiveness of the time variation of S_q^Q combined with a duality transformation implies that the nonextensive parameter q lies in the interval [0,2]. It is also shown that the stationary states are described by quantum q-power law extensions of the Fermi-Dirac and Bose-Einstein distributions. Such results reduce to the standard ones in the extensive limit, thereby showing that the nonextensive entropic framework can be harmonized with the quantum distributions contained in the quantum statistics theory.

Summary

arXiv:1006.3963

Thermodynamic Consistency of the q-Deformed Fermi-Dirac Distribution in Nonextensive Thermostatics

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(Dated: June 22, 2010)

The q-deformed statistics for fermions arising within the non-extensive thermostatistical formalism has been applied to the study of various quantum many-body systems recently. The aim of the present note is to point out some subtle difficulties presented by this approach in connection with the problem of thermodynamic consistency. Different possible ways to apply the q-deformed quantum distributions in a thermodynamically consistent way are considered.

E N D

How to get Tsallis distribution from Tsallis entropy

$$\mathbf{S_{q=1} = -\sum p_i \ln p_i \quad = \quad -\langle \ln p_i \rangle_{q=1}}$$

$$\Updownarrow$$

$$\mathbf{S_q = -\frac{1}{1-q}\sum\left[1 - p_i^q\right] \quad = \quad -\langle \ln_q p_i \rangle_q = -\langle \ln_{2-q} p_i \rangle_{q=1}}$$

where

$$\mathbf{\langle A \rangle_q = \sum p_i^q A_i \qquad \left(\mathit{with} \quad \sum p_i = 1\right)}$$

How to get Tsallis distribution from Tsallis entropy

$$\langle x \rangle_1 \Rightarrow \quad \mathbf{f(x)} = \frac{\mathbf{q}}{\left[\mathbf{1} + \left(\mathbf{1} - \mathbf{q}\right)\mathbf{x}\right]^{\mathbf{1}/\left(\mathbf{1}-\mathbf{q}\right)}}$$

$$\xrightarrow{\mathbf{q'=2-q}} \left(\mathbf{2} - \mathbf{q'}\right)\left[\mathbf{1} + \left(\mathbf{q'} - \mathbf{1}\right)\mathbf{x}\right]^{\mathbf{1}/\left(\mathbf{1}-\mathbf{q'}\right)}$$

$$\langle x \rangle_q \Rightarrow \quad \mathbf{f(x)} = \left(\mathbf{2} - \mathbf{q}\right)\left[\mathbf{1} + \left(\mathbf{q} - \mathbf{1}\right)\mathbf{x}\right]^{\mathbf{1}/\left(\mathbf{1}-\mathbf{q}\right)}$$

Similar (but more involved) results one gets when using escort probabilities:

$$\mathbf{F}\left(\mathbf{x}\right) = \frac{\mathbf{f}^{\mathbf{q}}\left(\mathbf{x}\right)}{\int \mathbf{f}^{\mathbf{q}}\left(\mathbf{x}\right)\mathbf{dx}}$$