

Multiplicity and flow scaling in rapidity in weakly and strongly
coupled systems

(Ie, how to rule out hydro by drawing lines with a ruler and
waving your arms around, based on 0911.5479)

Giorgio Torrieri



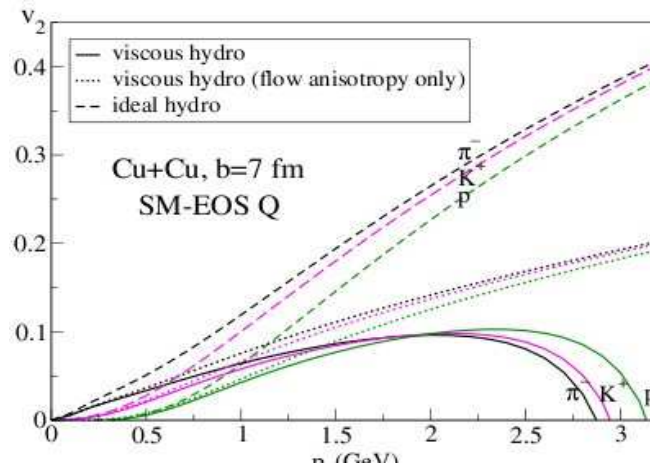
Flavoring the quark–gluon plasma with charm

After thinking a bit, and consulting with the organizing committee, I decided a different talk is more interesting to this audience. To those who wanted this talk:

- Apologies
- Ask me about it

Hydro: The current situation

v_2 200 GeV fits well, f.o. more important than previously thought if $\eta/s > 0$



U. Heinz and
H.Song
0712.3715

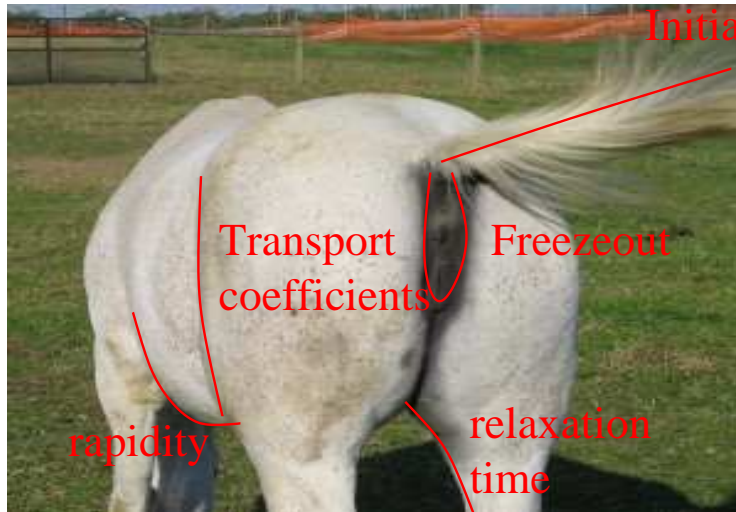
Early contribution of viscosity (quenching of flow gradients) as important as late one (effect of $\Pi_{\mu\nu}$ in Cooper-Frye). Can not ignore late hadronic stage even for v_2 (If you dont believe Cooper-Frye, 50% systematic error)

v_2 **with rapidity** OK with ideal hydro, if Glauber initial conditions.
Interplay between viscosity and initial conditions important here

HBT radii could fit OK if pre-thermal flow included in initial condition.
Introduces ambiguity: How much transverse and elliptic flow via pre-thermal processes (no dependence on $EoS, \eta/s$) and how much to hydro

Freeze-out Not really understood: Cooper-Frye, afterburner, coalescence...?

We are all waiting for 3D viscous hydro to investigate interplay between: $EoS, \eta/s, \tau_{II}$, transverse initial conditions, longitudinal initial conditions, pre-existing flow, freeze-out dynamics.



Initial flow

Transport
coefficients

Freezeout

rapidity

relaxation
time

And therein lies
the danger!

We understand the equation of state and hopefully might understand the viscosity from first principles. But initial conditions and their dependence in energy are a problem: Even when you are trying to fit lots of data simultaneously, a model with many correlated parameters can describe nearly *any* physical system to some degree.

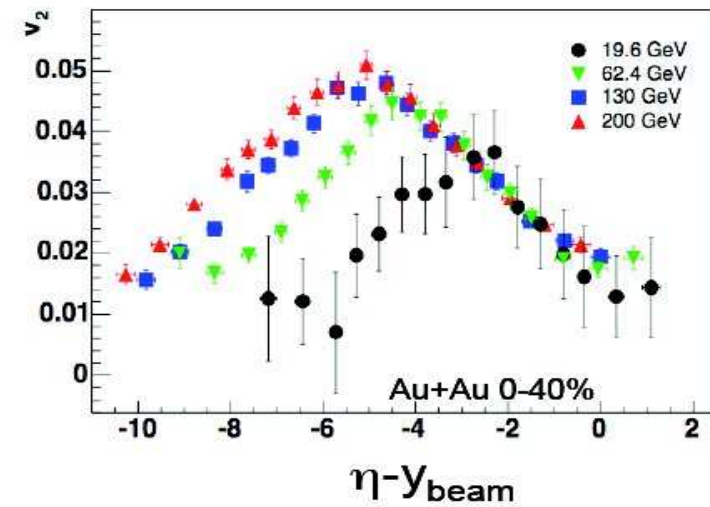
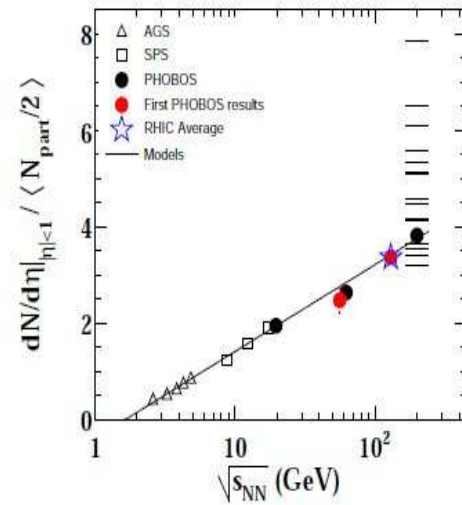
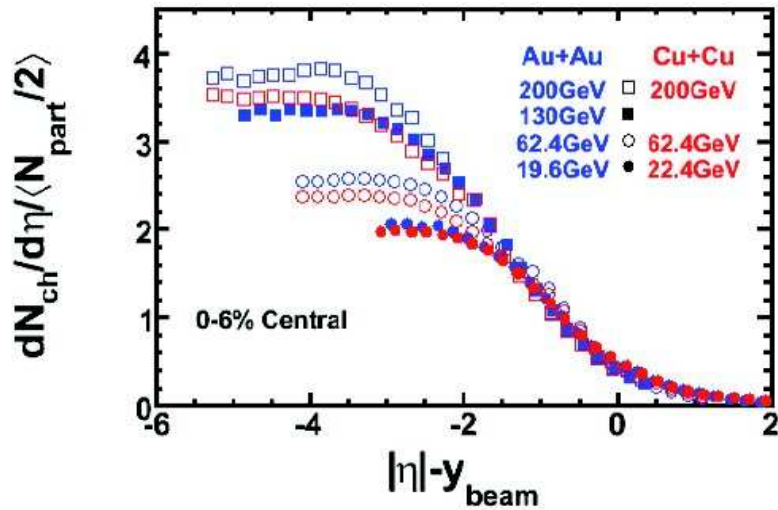
The answer: Naturalness!



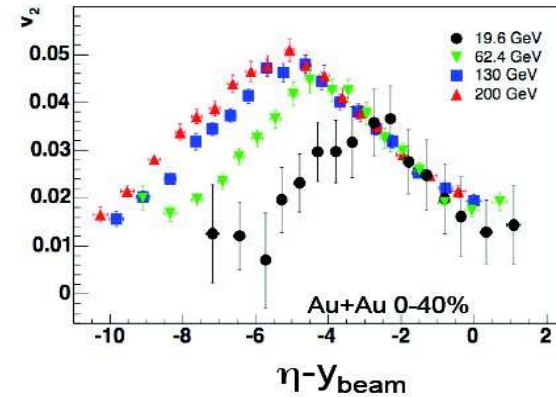
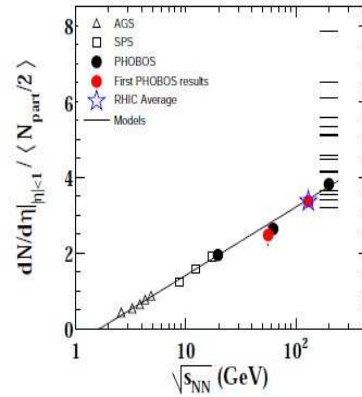
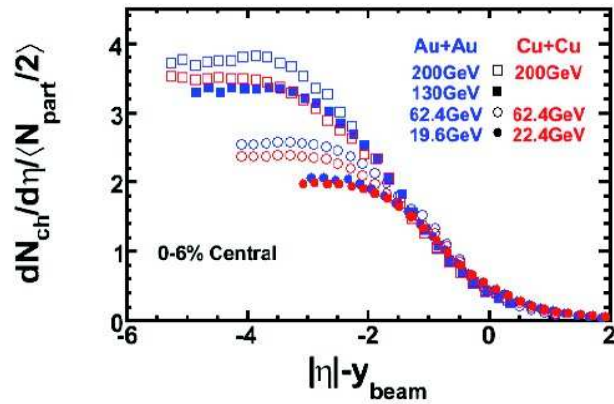
We have collected an impressive array of data. We can bin it in $y, N_{part}, A, \sqrt{s}, \text{species}, \dots$. In a physically correct model, parameters will scale in a way that is both intuitive and expected from theory. Things which should not be related will not be, and things that should be will be. A model with no natural scaling, generally, can not be fixed by adding parameters!

The rules (borrowed from quantum field theory)

- All dimensionless parameters $\mathcal{O}(1)$, unless a good reason is given
- A theory has one or more identifiable **relevant scaling variables (RSV)**
In the first case, one expects the observable to depend on only one scaling parameter, no matter how the scaling parameter was arrived at (eg, by changing energy or system size). **In the second, one does not!**
And complicating model without adding "symmetries" wont work!
- A theory with dimensionful parameters in the Lagrangian is in the second group (eg $T_c/\sqrt{s}, T_c^2 \frac{1}{S} \frac{dN}{dy}$. Furthermore, this parameter T_c^d combined with parameters scanned by experiment (eg $\frac{1}{S} \frac{dN}{dy}$ should produce dimensionless relevant scaling variables (say α_i . When these are small, above consideration quantitatively rigorous \equiv **Taylor expansion**
 $\langle observable \rangle \sim A_0 + A_1 \alpha_1 + B_2 \alpha_2 + \mathcal{O}(\alpha_1 \alpha_2) + \dots$



And something funny has certainly been found when scaling in both energy and rapidity

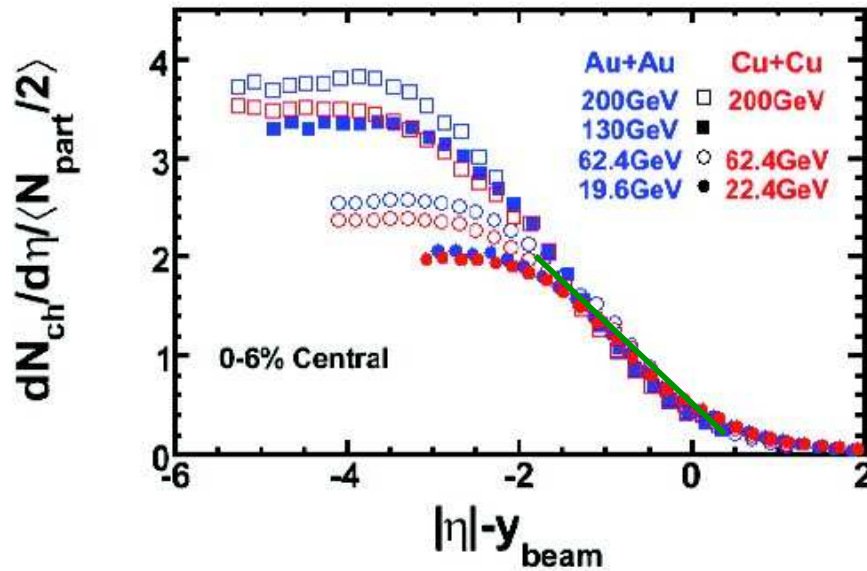


I plan to show you that...

The first scaling follows naturally from QCD-inspired initial conditions

The second scaling Can be accommodated by a not-unreasonable modification of these

The third scaling is very tricky (impossible?) to model within hydro, but arises naturally in **weakly coupled systems!** ($Kn \geq 1$)

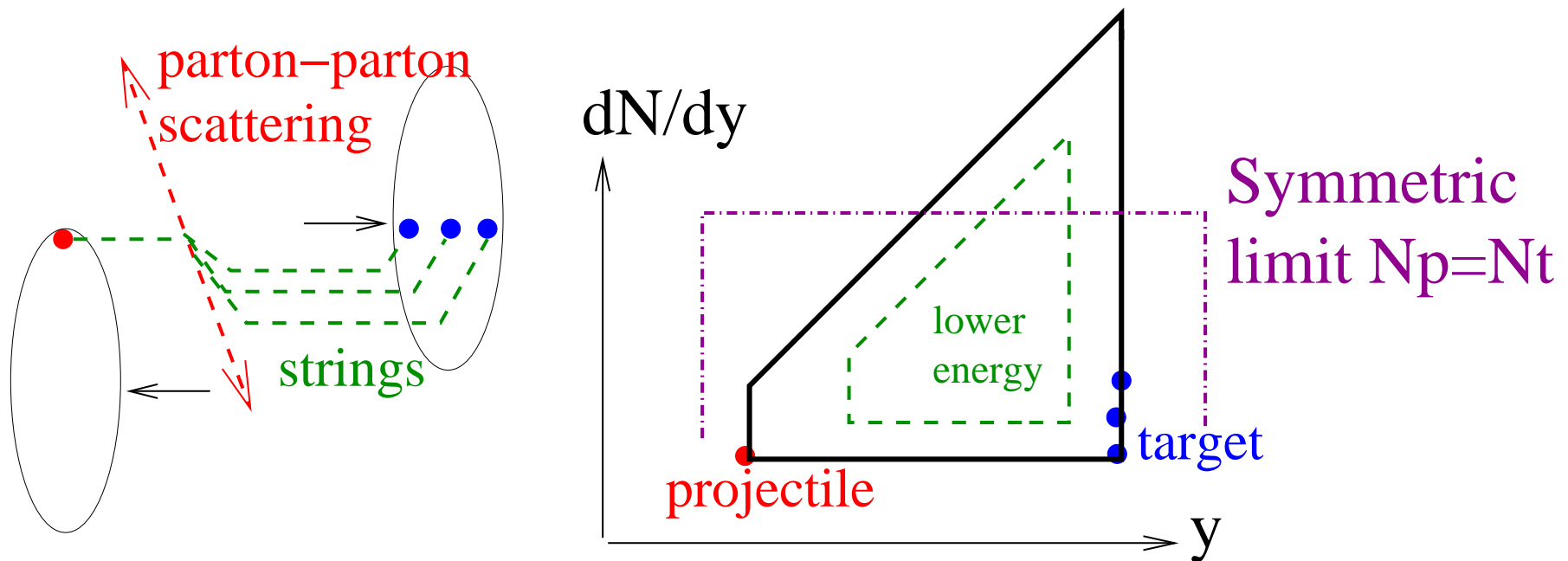


Universal fragmentation
Slope of spectators
independent of
energy

It is easy to see that, by kinematics, $y_{lim} \sim \log \sqrt{s}$
Universal fragmentation is more involved, but ultimately understandable
within QCD phenomenology

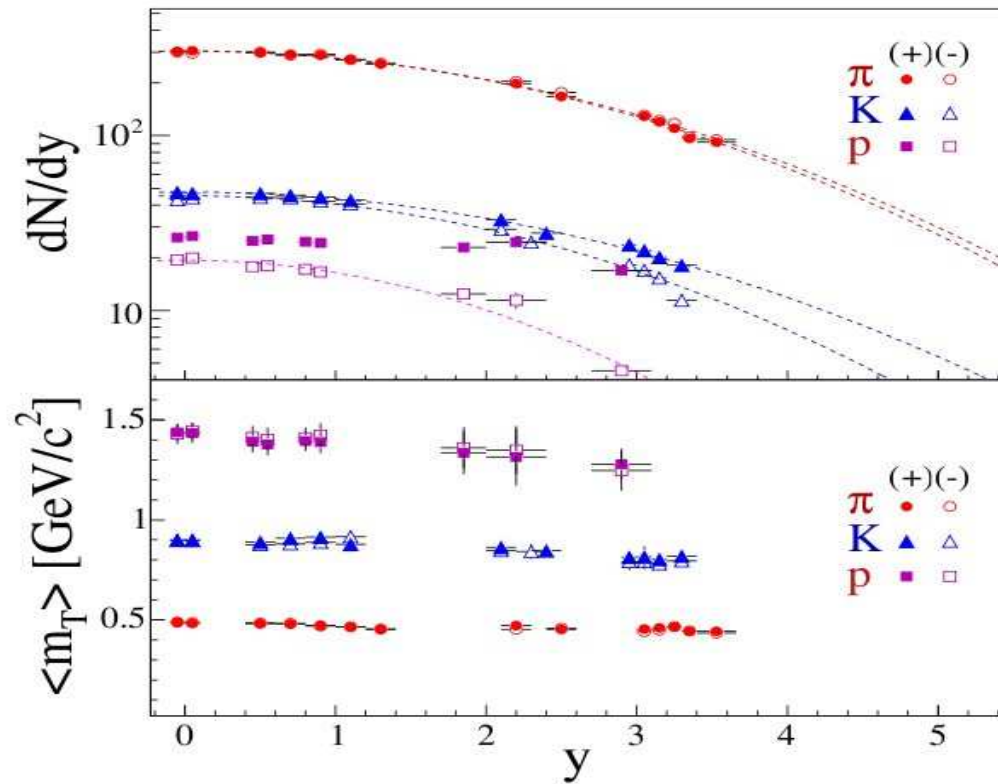
A generic intuitive explanation: Brodsky-Gunion-Kuhn (BGK)!

- Each target-projectile collision produces parton at y^* , uniformly distributed between y_{lim}^T and y_{lim}^P
 - Each Target/Projectile (T/P) wounded nucleon produces a string disintegrating between $y_{lim}^{T,P}$ and y^* .
 - Total multiplicity $\sim \sum$ independent string fragmentations
 - Number of strings at projectile/target $\sim N_{part}^{P,T}$ of projectile/target (Universal fragmentation for different \sqrt{s}/y_{lim} , same N_{part})
 - Density linearly interpolates between them away from limiting rapidities ("Triangle" seen experimentally in $(dN/d\eta)_{AA}/(dN/d\eta)_{pp}$)
- Initial Bjorken flow** ($y = \eta$) but no boost-invariance except for symmetric systems



Comparing same system $A - B$ with different energies, we see that as long as “interpolation” weakly dependent on energy (it is in string picture), universal fragmentation easily follows. NB: In symmetric limit, system Boost-invariant. **HIJING** based on this. **CGC** based on this picture+transverse dynamics

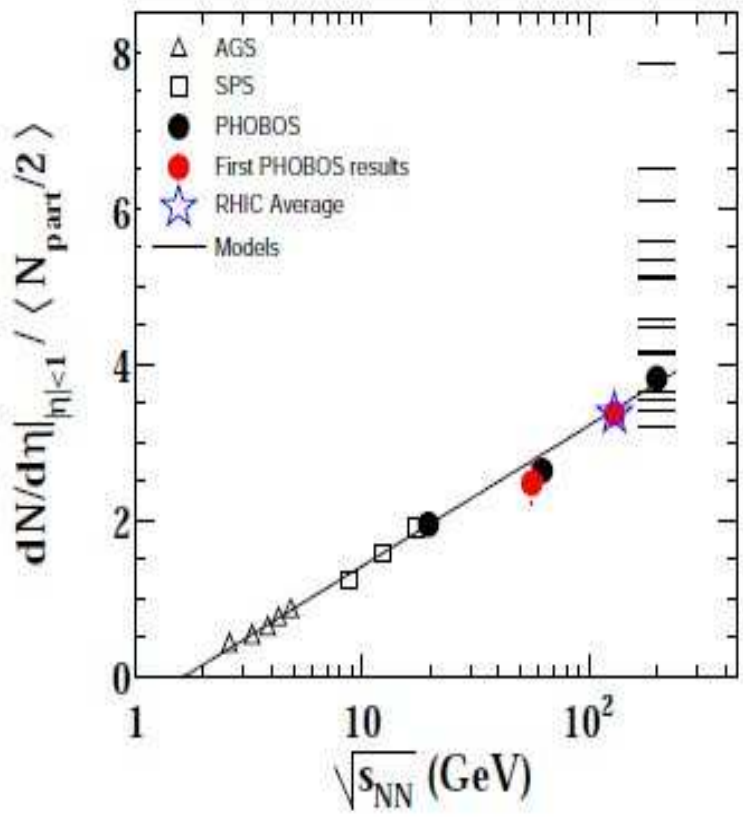
But this picture has a problem I...



Brahms white paper

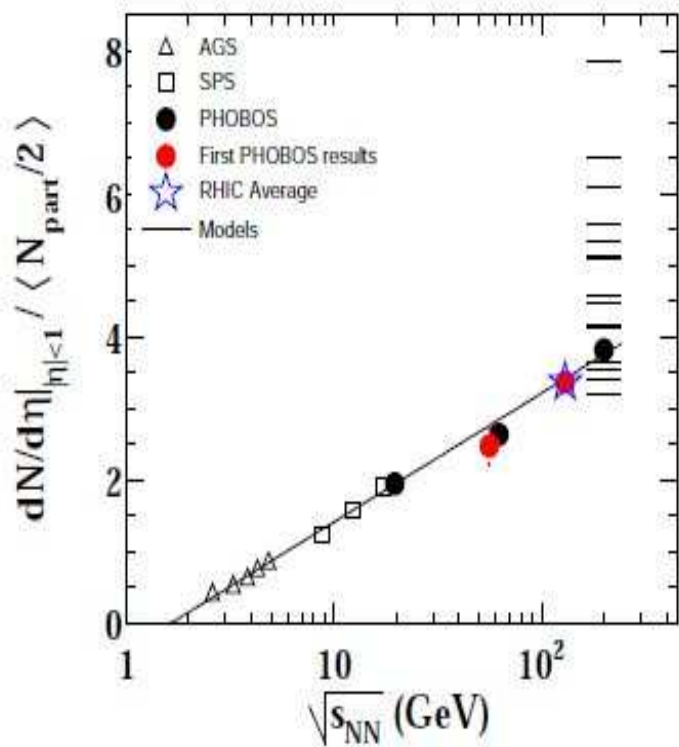
Even at RHIC top centrality there is no boost-invariance!

But this picture has a (related?) problem...



The multiplicity
rapidity density at $y=0$
also scales with
 $\ln(\sqrt{s})$ at all \sqrt{s}

$$\frac{dN}{dy} \sim N_{part} \ln(\sqrt{s})$$



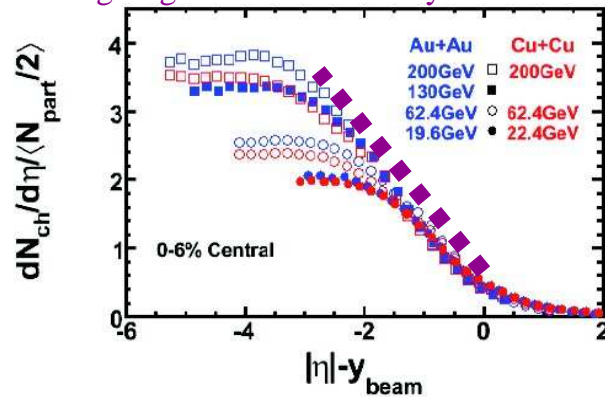
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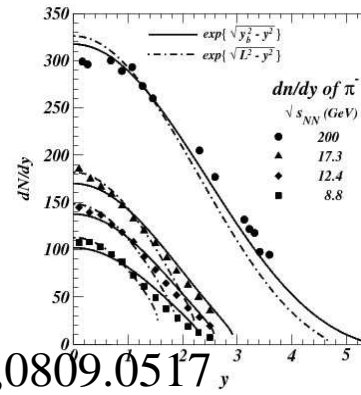
NOT Feynmann scaling! He predicted, from local Boost-invariance and dimensional analysis, $dN/dp_z \sim 1/Q$, that $\langle N_{tot} \rangle \sim \ln \sqrt{s}$. It appears its $\langle N_{tot} \rangle \sim (\ln \sqrt{s})^2$. Does mid-rapidity know about limiting fragmentation?

Not Landau either!

Limiting fragmentation of dN/dy



vs

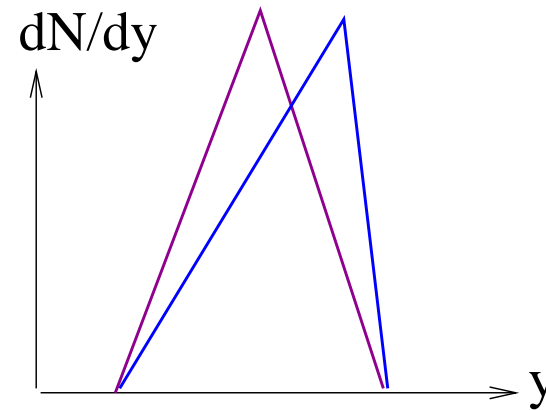
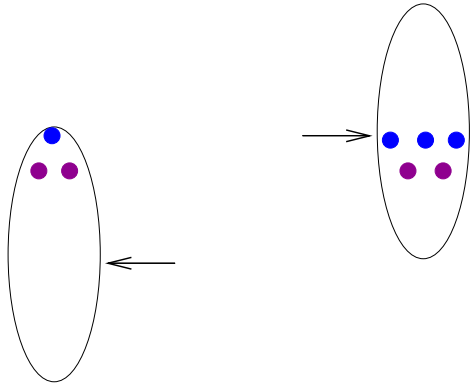


Scaleless EoS
no transverse
flow

Wong, 0809.0517

Landau becomes Bjorken after a few $T_{initial}^{-1} \sim \mathcal{O}(1) / \sqrt{s}$
 That means approximate limiting framgnetaiton well away from mid-rapidity
 (Not perfect, even with ideal EoS, inapplicable in cross-over/hadronic), but
 not to mid-rapidity, **Which is why Landau $dN/dy \neq \ln(\sqrt{s})$**
 Need initial Boost-invariance ($y = \eta$) for limiting fragmentation up to
 mid-rapidity, but large stopping in the middle to account for dN/dy

A simple explanation: limiting fragmentation up to $y = 0 \rightarrow$ triangles!

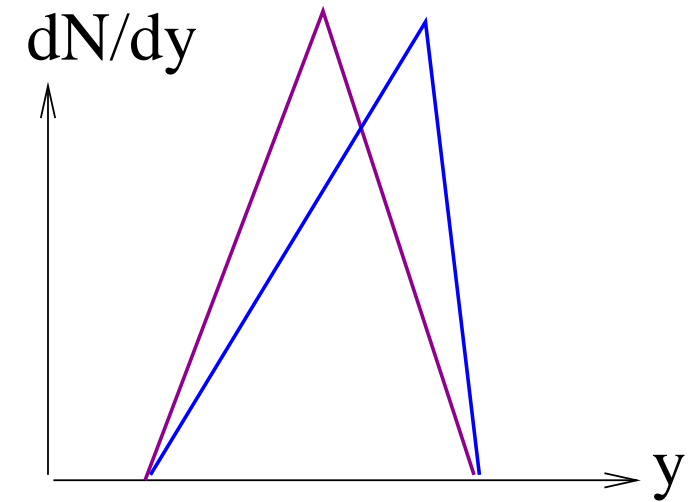
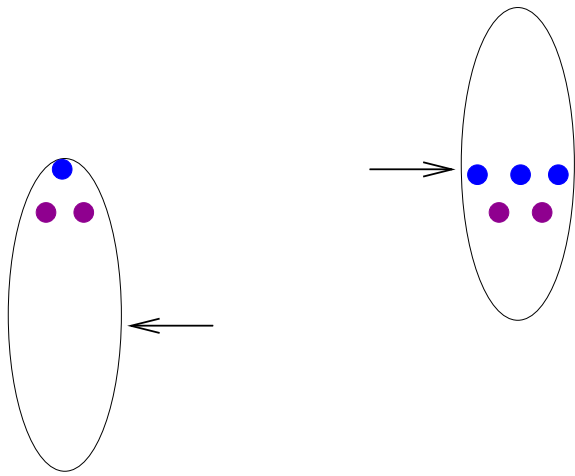


- Slope $\sim N_{Part}^{P,T}$, independent of \sqrt{s}
- x-intercept $\sim y_{lim}^{P,T} \sim \ln \sqrt{s}$

So intersection at maximum, also $\sim (N_{part}^P + N_{part}^T) \ln \sqrt{s}$

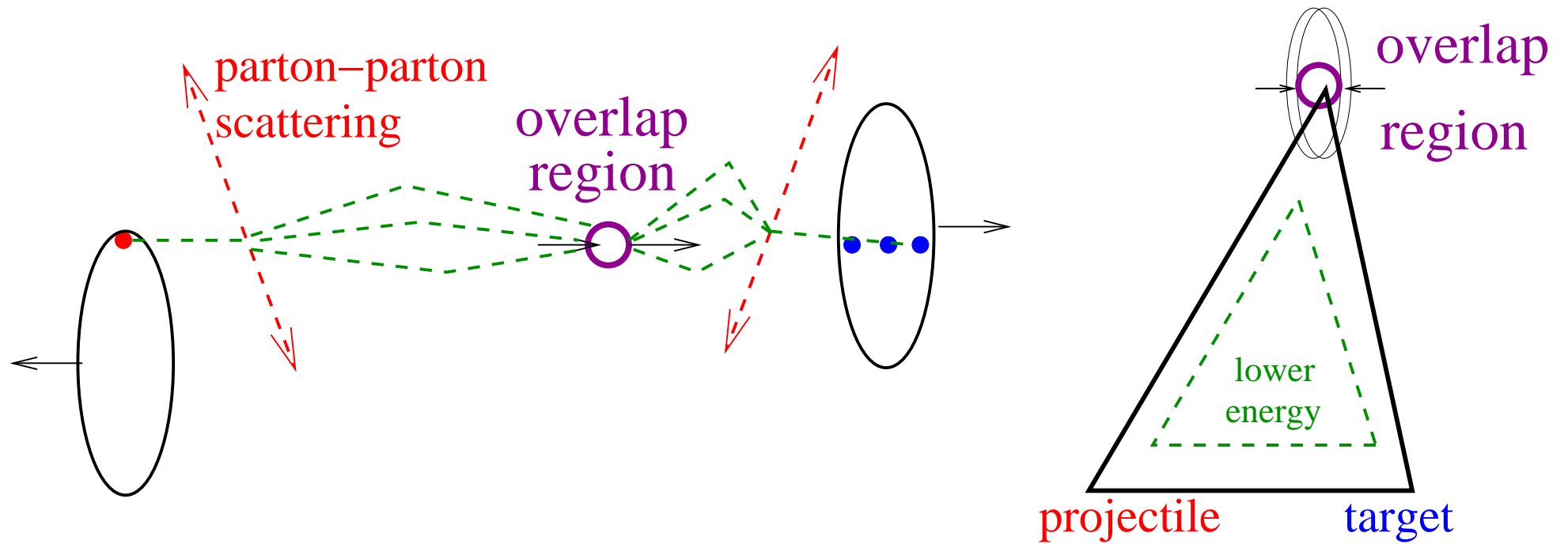
Boost invariance, even in symmetric collisions, goes away **like in data** !

Asymmetric systems (eg p-A, A-A at large r_{\perp}) \rightarrow **BGK** as C of M at large y



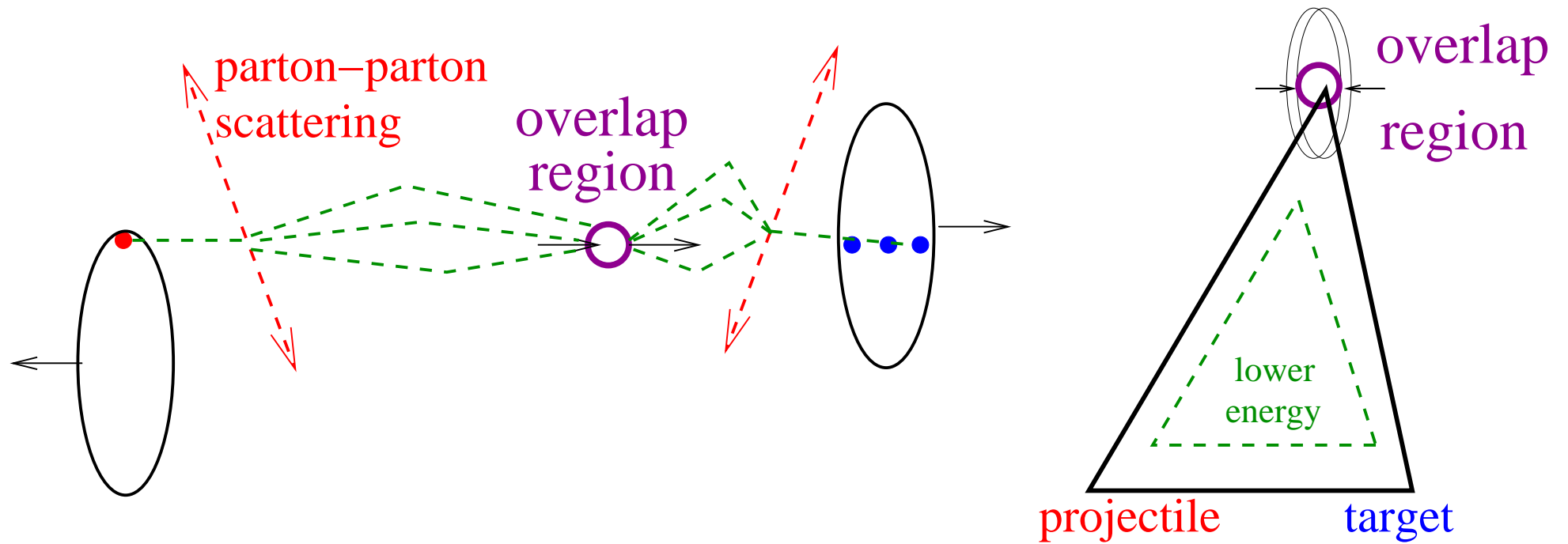
NB: Breakdown in $dN/dy \sim \ln \sqrt{s} \Leftrightarrow$ No limiting fragmentation. **LHC?**

How to reconcile this with QCD...



Keep BGK picture but assume strings originate in 3 points: (A minimum) at projectile, (a minimum) at target, and a maximum at the point in spacetime of intersection, moving with net momentum!

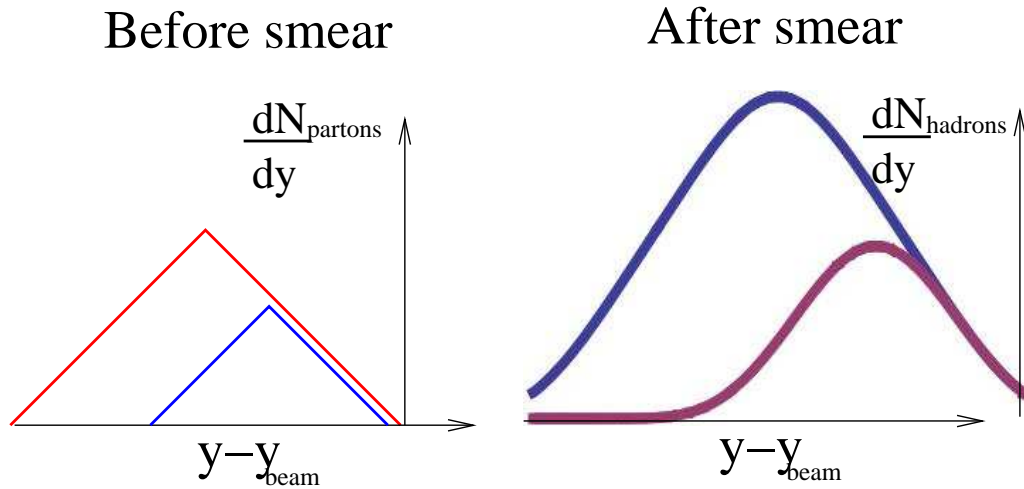
How to reconcile this with QCD...



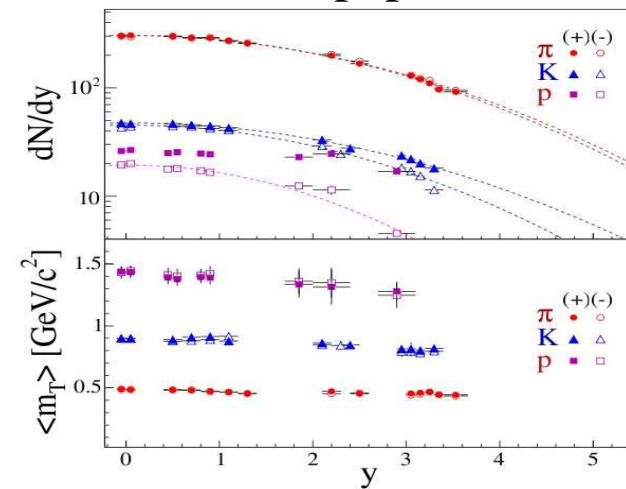
NOT weiszaker-Williams or partons! But not unreasonable given many-body physics (strong coupling? Plasma instabilities?) in the overlap region.

Universal fragmentation survives thermal smearing...

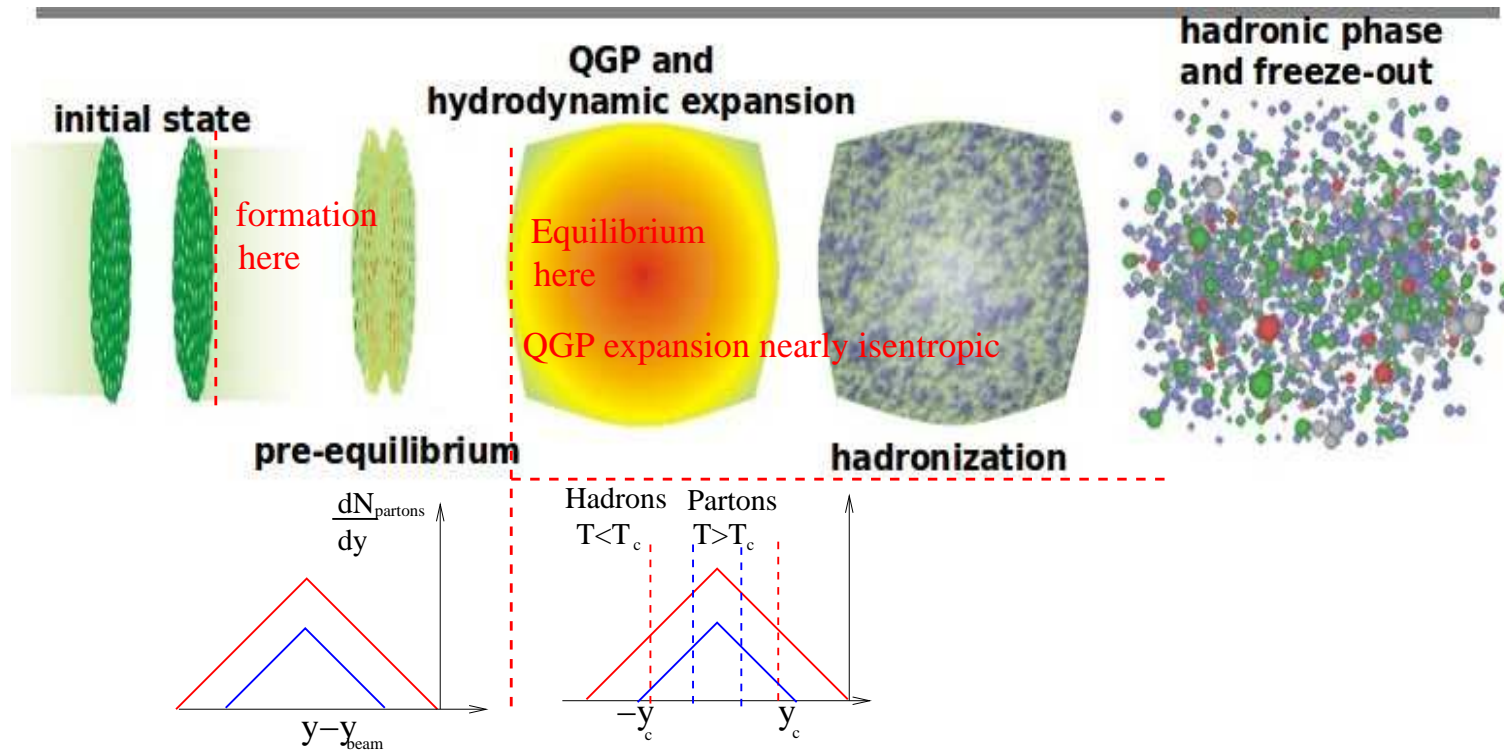
$$f(y, y_L) = y_L \int_{-y_L}^{y_L} dy' \left(1 - \frac{|y'|}{y_L} \right) \exp [-\cosh (y' - y_L)]$$



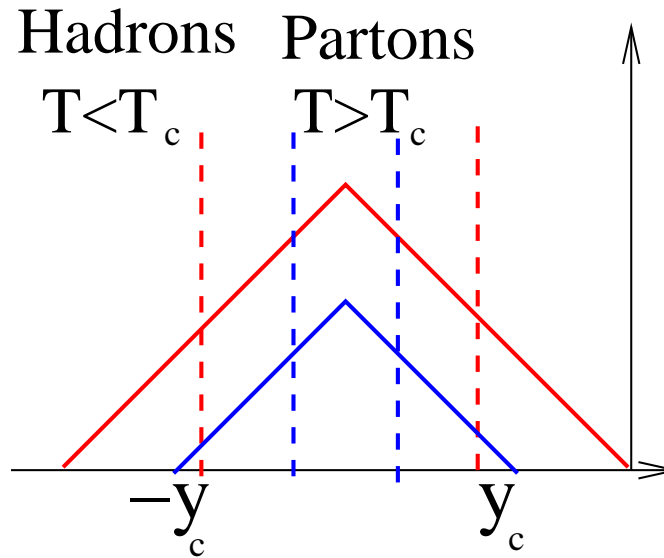
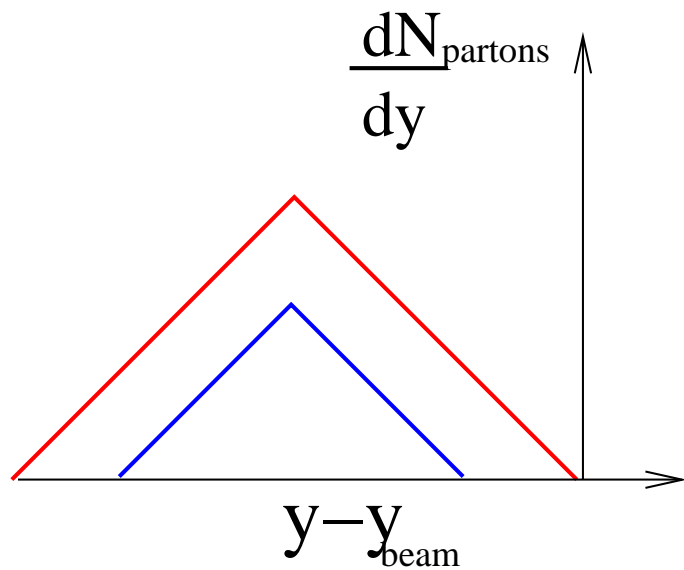
Brahms white paper



Symmetric limit Bjorken-flowing ($\eta = y$) but not Boost-invariant...

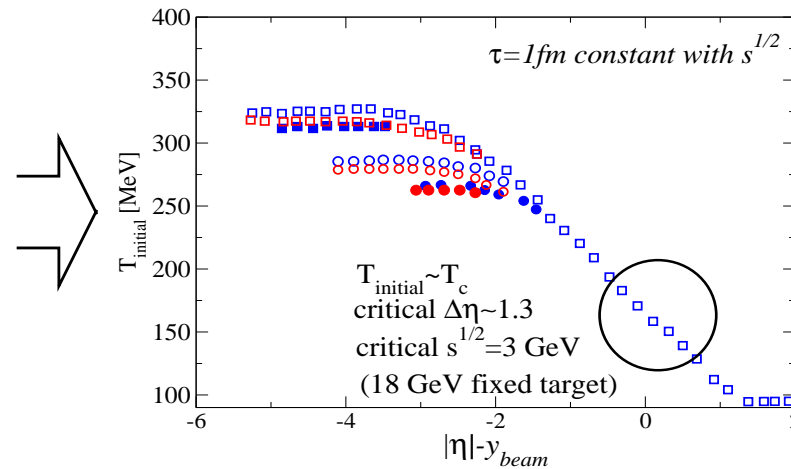
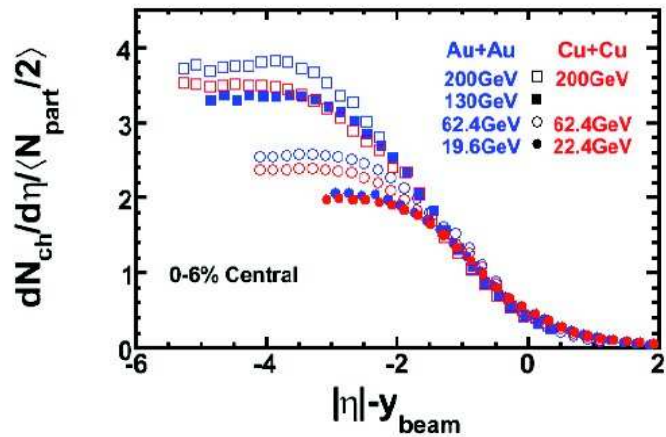


So far everything I told you related to the "formation time", $\sim Q_s^{-1}$ (Or some such scale). At this time, system is partonic everywhere. For system to "know" if its QGP or HG, its pressure gradient and η/s , one has to wait until the later equilibration time $\sim \mathcal{O}(1) \times R \times Kn$ So....



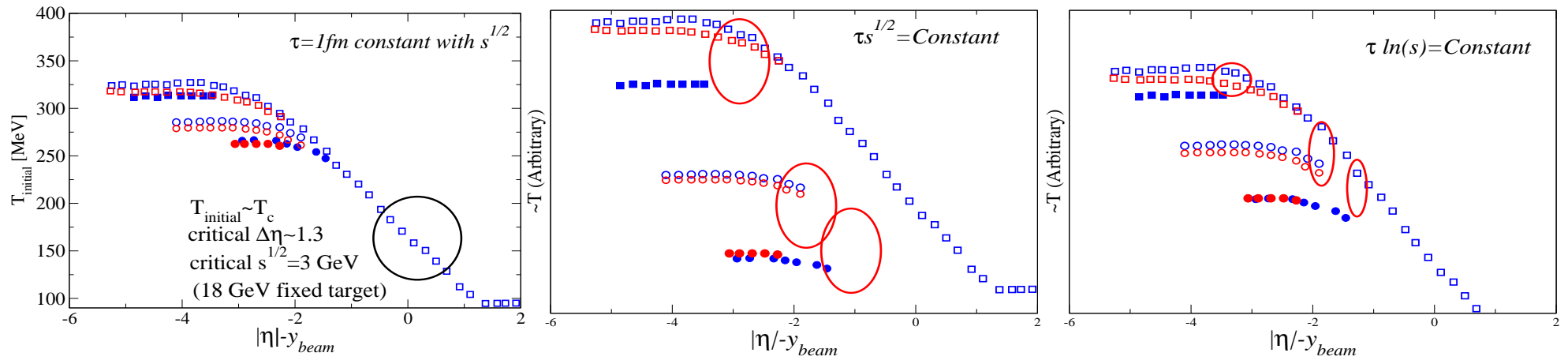
Remember That we have a phase transition!

So, if there is Bjorken flow (distinct slices not talking to each-other), there will be slices dominated by partons and others by hadrons *at equilibrium*



We can estimate this critical rapidity *very roughly*, by just plugging the experimental $\frac{2}{N_{part}} \frac{dN}{d\eta}$ distribution into the back-of-the-envelope entropy formula, $\frac{ds}{dy} = 4 \frac{dN}{d\eta} \sim N_{part} \tau f m^2 g T^3$. We get

- The "critical" y should be well within detection (critical \sqrt{s} @ low energy SPS)

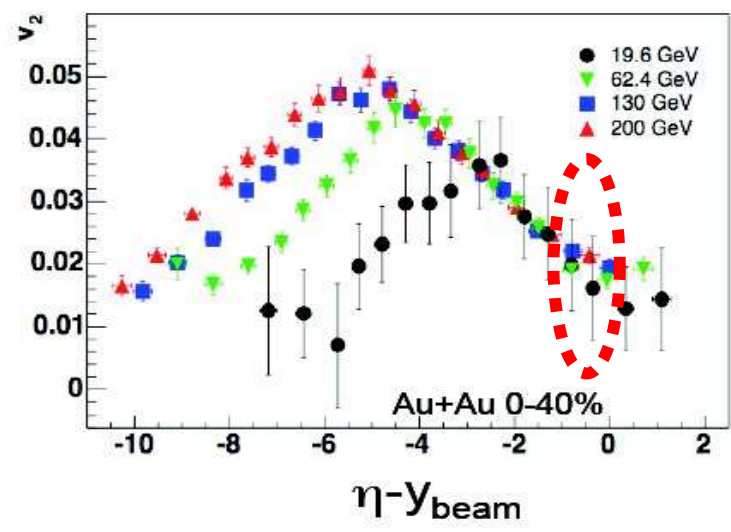
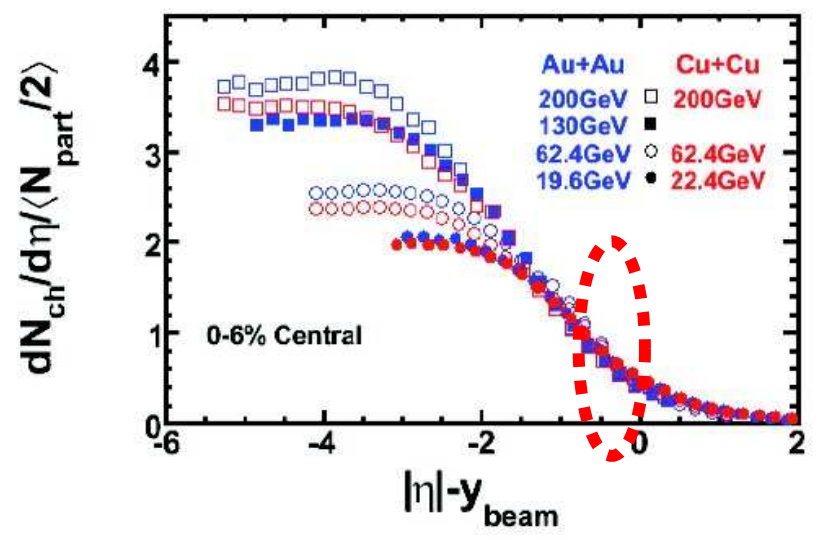
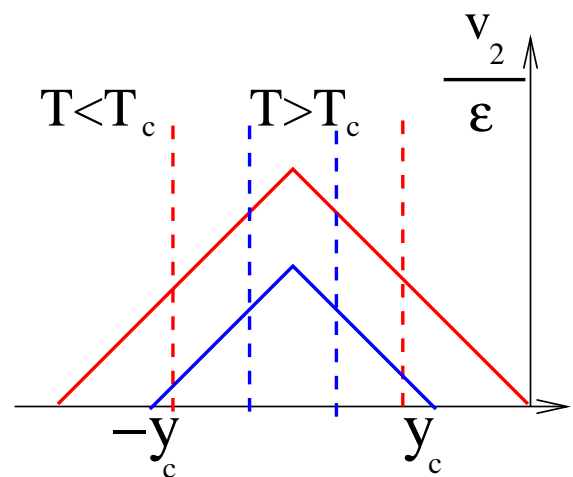
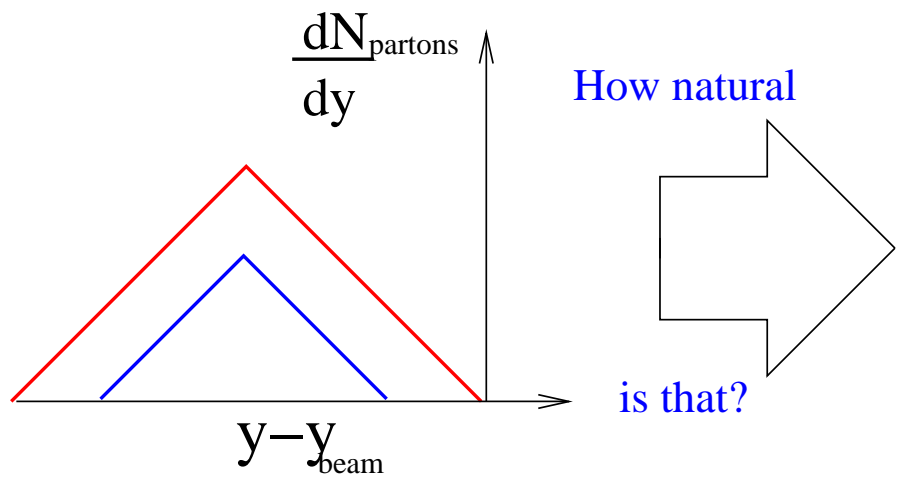


- Making the model more "physical" ($\tau(\sqrt{s})$) would require unnatural cancellations to reproduce observed scaling (Red ellipses show breaking of universal fragmentation *in temperature*.)
No apparent natural way to balance τ and T !
Is this a problem? not really!!!

What can these considerations tell us about hydro and phase transitions?
Perhaps very much...

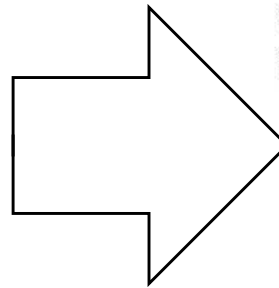
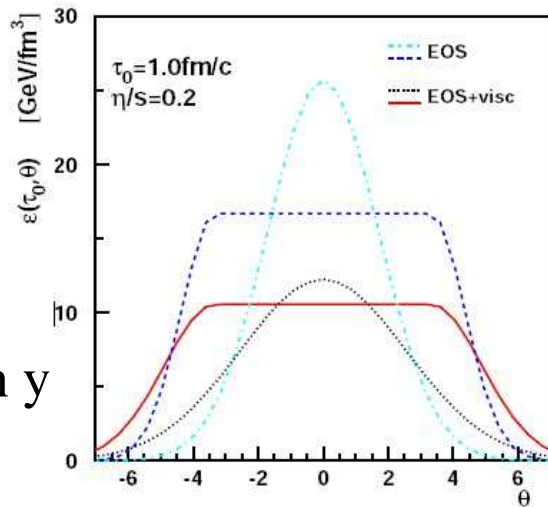
- Initially (“formation time”) the system is partonic
- But *at equilibrium* its partonic at $y < y_c (T > T_c)$ and hadronic otherwise
- System “probably” nearly ideal as a QGP $y < y_c$, a lousy liquid ($Kn \sim 1$) at central rapidity, a lousy hadron gas away
- But both free streaming and ideal liquid conserve entropy, so in those two limits not much should change with dN/dy .

So perhaps very little... but v_2 is a different story!

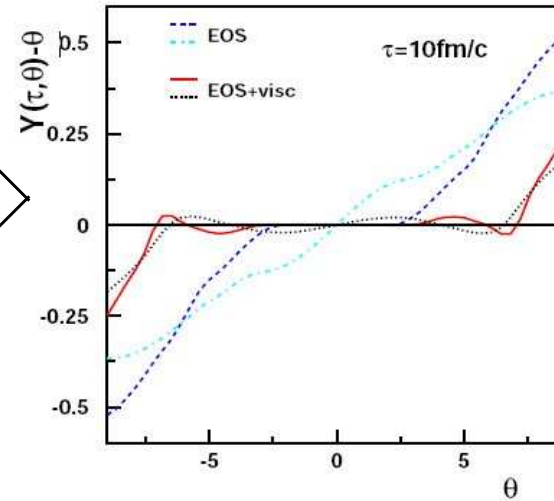


First problem: Rarefaction wave in y would destroy scaling!

initial
 $y = \theta$
 but
 density
 varies in y



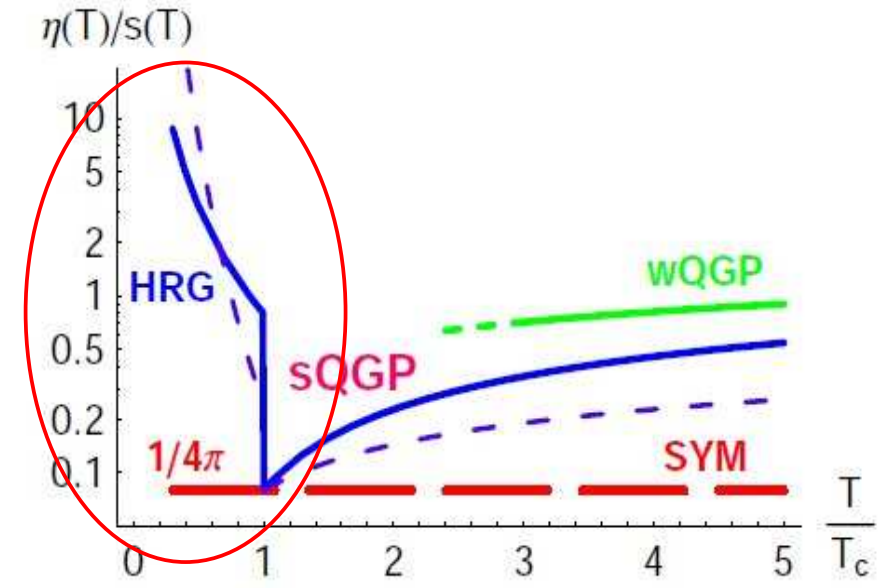
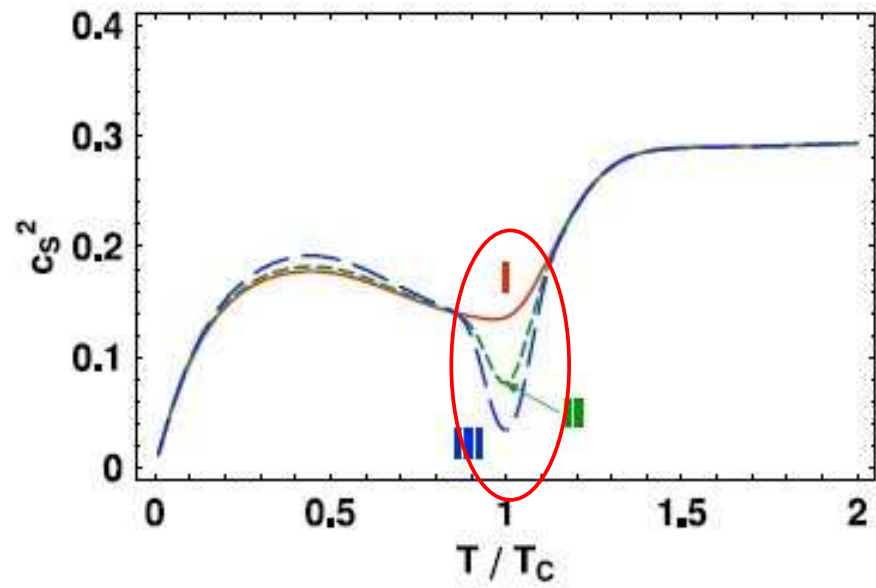
10 fm
 of hydro



P.Bozek
 0803.4447

Bjorken flow \neq boost-invariance, but under hydro a non-boost invariant initial condition destroys Bjorken flow!

Second problem: Both EoS and η/s should have a scale, T_c



At T_c (mixed phase) speed of sound experiences a dip (not to 0, as its a cross-over, but a dip). Above T_c , $\eta/s \sim N_c^0$, below T_c , $\eta/s \sim N_c^2$.

What does v_2 depend on? follow Gombeaud+Borghini+Ollitraut

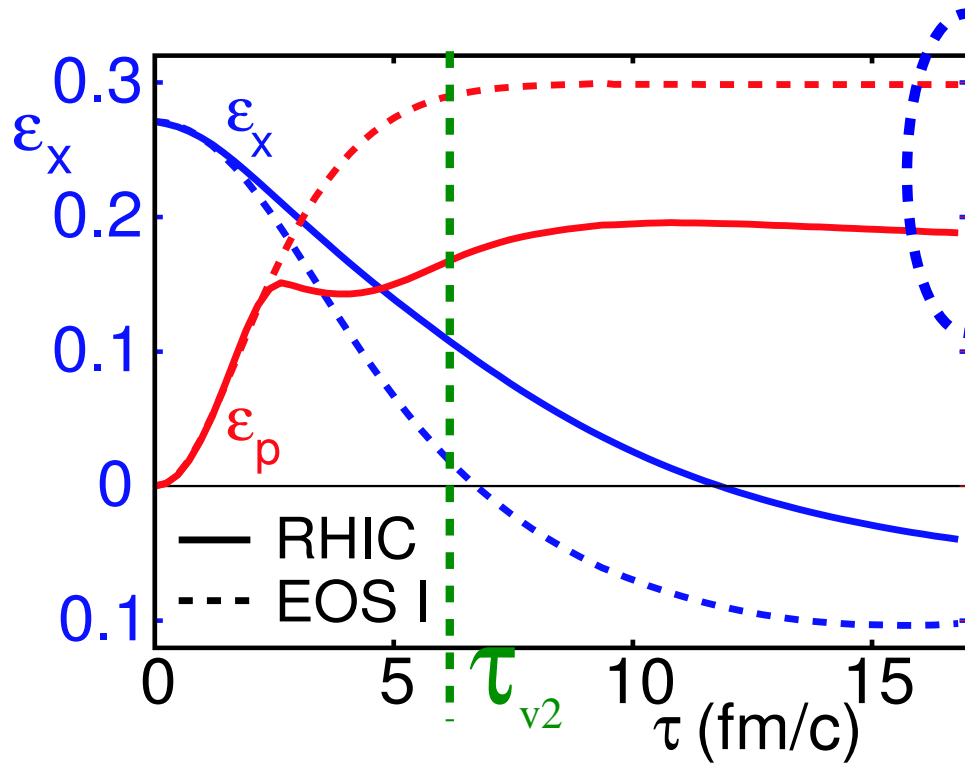
Eccentricity $v_2|_{ideal} \propto \epsilon + \mathcal{O}(\epsilon^2)$ since ϵ small and dimensionless

Knudsen number $\frac{v_2}{\epsilon} = \frac{v_2}{\epsilon}|_{ideal} (1 - \mathcal{O}(1) Kn) \sim \frac{v_2}{\epsilon}|_{ideal} (1 - \mathcal{O}(1) \frac{\eta c_s}{s TR})$

speed of sound From what we know of shock-wave expansion

$\frac{v_2}{\epsilon}|_{ideal, \tau \rightarrow \infty} \sim c_s$ and $\tau \rightarrow \infty$ is an OK approximation since anisotropy in flow saturates quickly wrt lifetime of system

Beyond linearity... v_2 saturates!, on a scale τ_{v2}



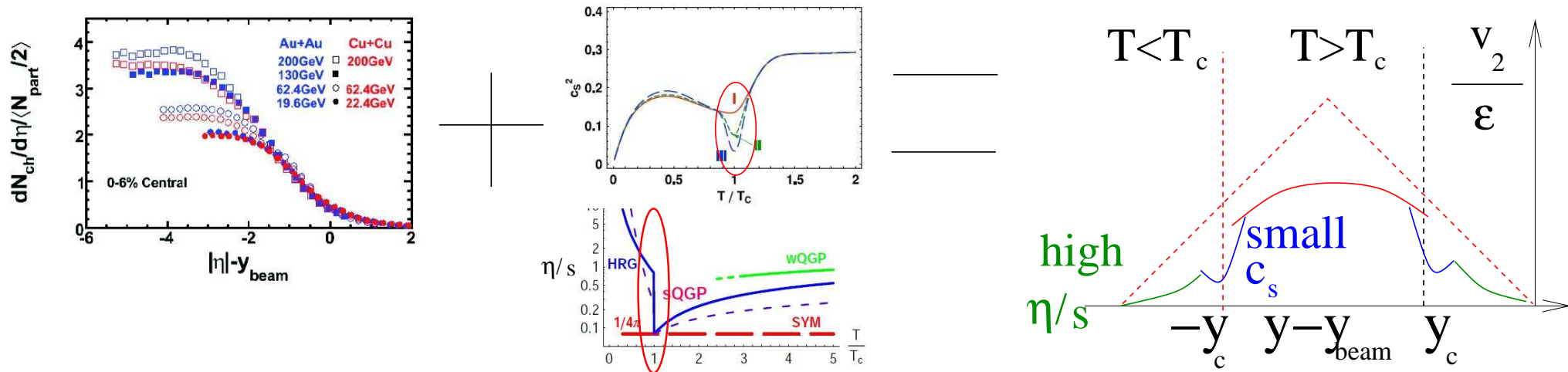
+Freezeout
 ϵ_p
effects

U. Heinz

P.Kolb

nucl-th/0305084

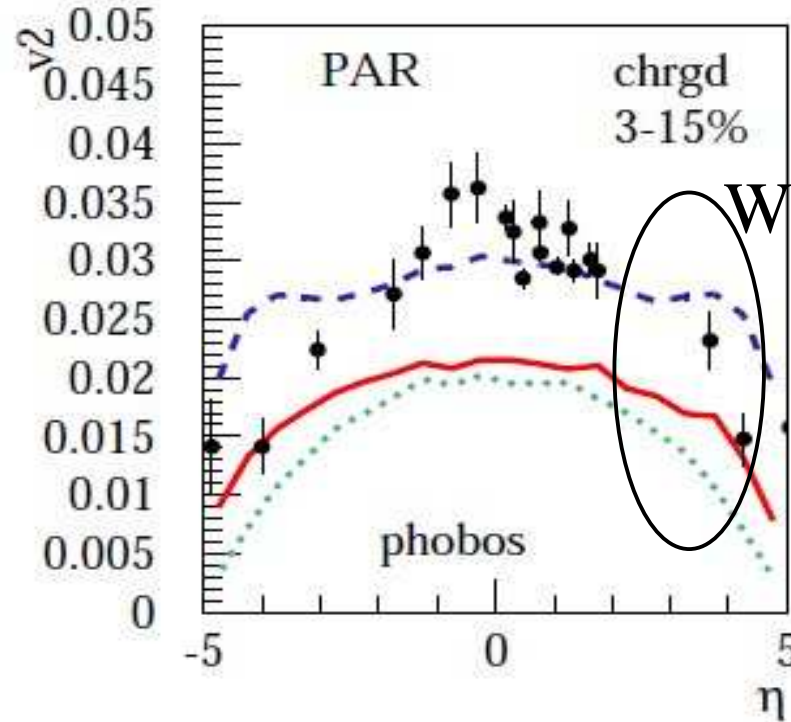
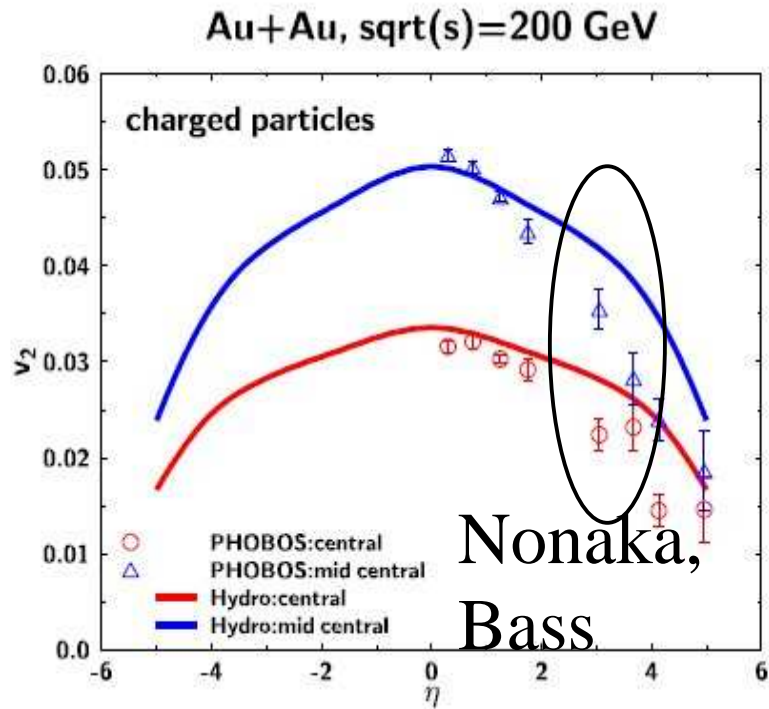
If you dont change η/s but increase lifetime, you generally get same v_2/ϵ .
Putting everything together...



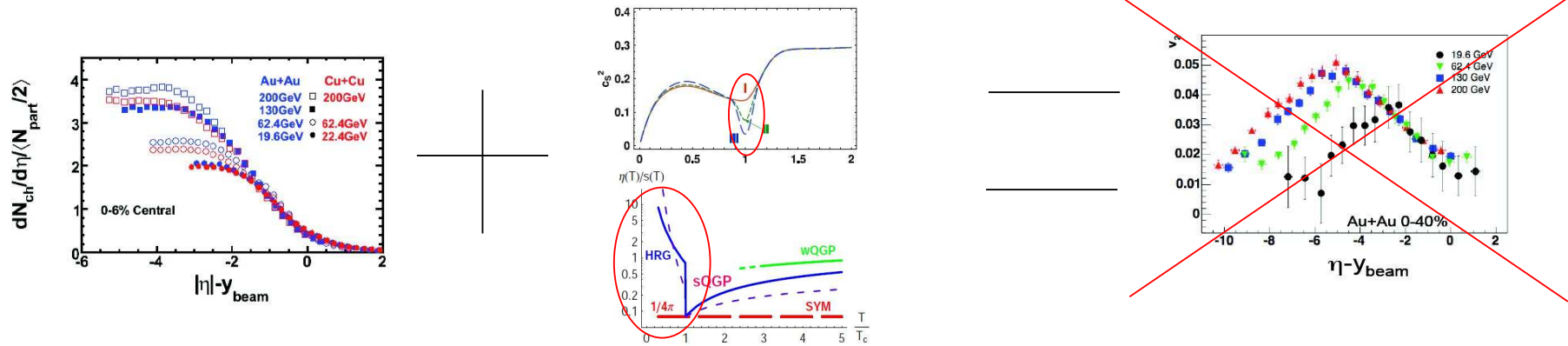
$$\frac{v_2}{\epsilon} \sim \underbrace{c_s}_{\text{Dips@}T_c} \left(1 - \mathcal{O}(1) \frac{\eta}{c_s s} \frac{1}{TR} \right) \underbrace{\tanh\left(\frac{\tau}{\tau_{v2}}\right)}_{\text{saturation}}$$

To describe universal fragmentation in $dN/d\eta$, T changes smoothly with η , R independent of it. This destroys universal fragmentation of v_2/ϵ !

Beyond arms-waving...

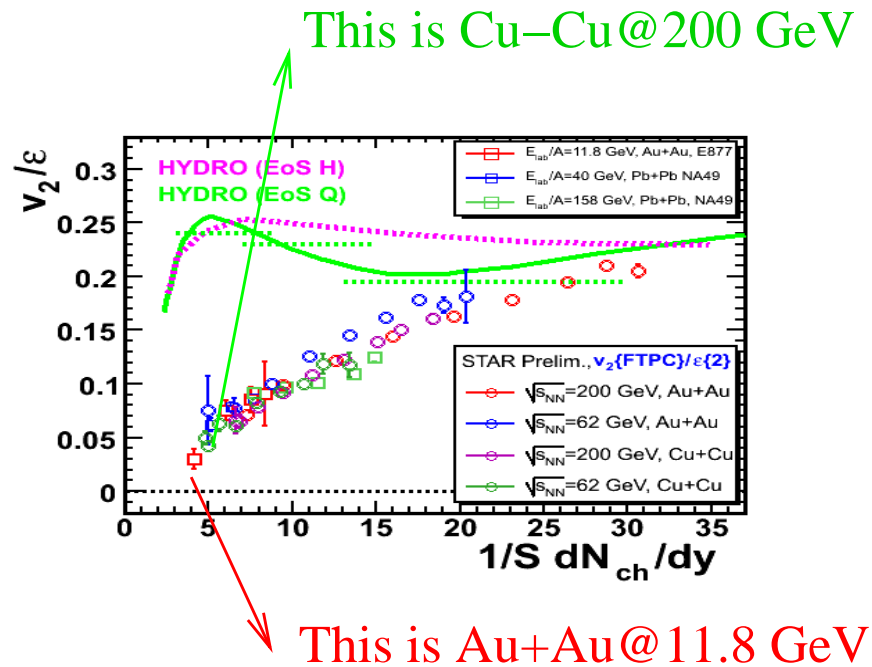


3D Ideal hydro typically gives a change in gradient at T_c from sound speed alone. Introducing η/s bound to increase this effect. At one energy not so noticeable since data has error bars, but scaling is the key!



It is difficult to see how any initial condition describing universal fragmentation in $dN/d\eta$ with an an EoS and set of transport coefficients containing T_c can also describe universal fragmentation in v_2/ϵ . For this, One would have to have non-scaling in initial conditions where the effects of longitudinal flow and entropy production at high η/s would "miraculously" cancel out. This is unnatural (see earlier def.)

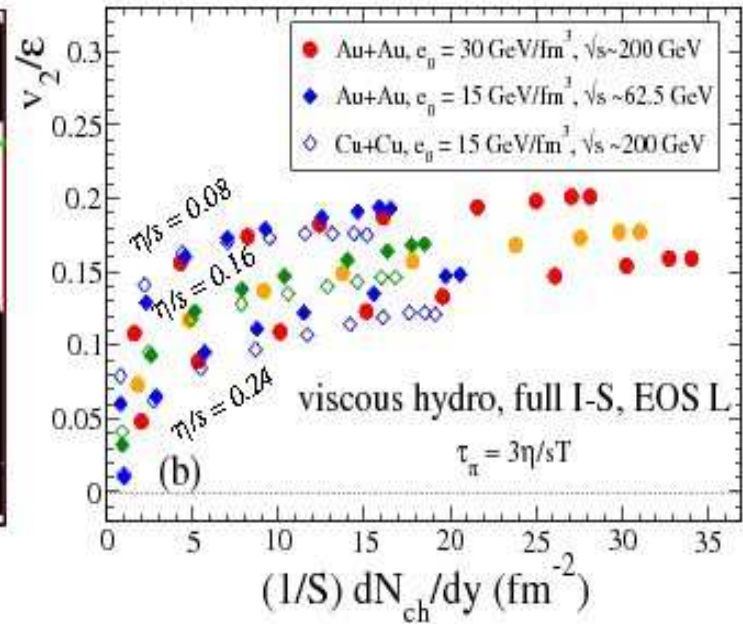
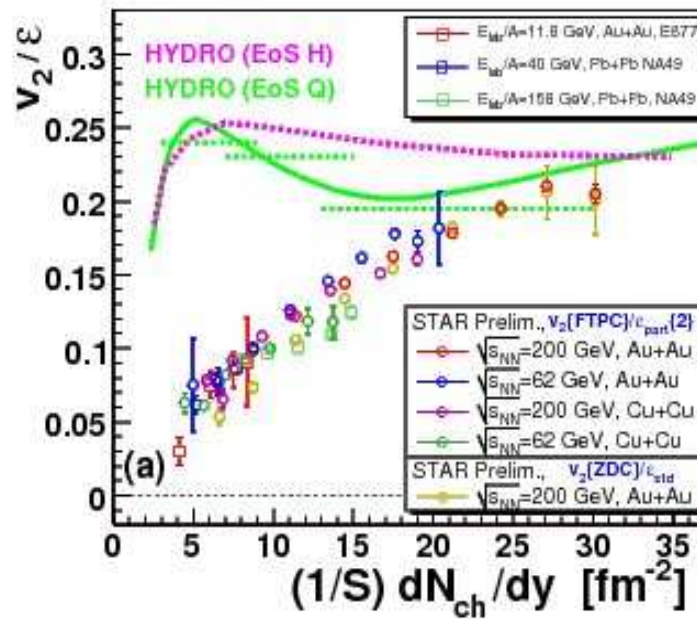
For lower energies... Integrating over all rapidity...



v_2/ϵ is the same for a given $\frac{1}{S} \frac{dN}{dy}$, even if the energy is very different!!!!

Expected from $v_2 \sim \epsilon \frac{dN}{dy}$ + universal fragmentation, but...

simulation
by
U.Heinz
H.Song



same η/s + Bjorken-type Initial conditions at from AGS to RHIC or initial conditions and η/s evolution precisely cancel out? **Latter possible if...**

The minimal resolution : Triangle initial conditions, and

$$\frac{\eta}{s} \sim A \left(\frac{\Lambda_{QCD}^3}{s} \right) \sim \left(\frac{1}{S} \frac{dN}{dy} \right)^{-1}$$

where A doesnot change at all at T_c (constant over hadron and quark phase). but...

- A, α constant@ the cross-over? In $SU(N_c)$ $\eta/s \sim N_c^0 @ T > T_c$, but $\sim N_c^2 @ T < T_c$
- "triangular partonic" initial conditions independent of \sqrt{s} , *even* at lower energies where $T \leq T_c$ at mid-rapidity according to the Bjorken formula
- Since ideal hydro reached at RHIC, and v_2 saturates, v_2/ϵ **at most same** (violation of scaling) at LHC!

Bottom line: The unnaturalness of the scaling, in my opinion, is a good motivation to look for radically different explanations of v_2 where scaling is more natural. **Need a model where $\frac{v_2}{\epsilon} \sim \frac{1}{S} \frac{dN}{dy}$ with weak dependence on anything else**

Some suspects...

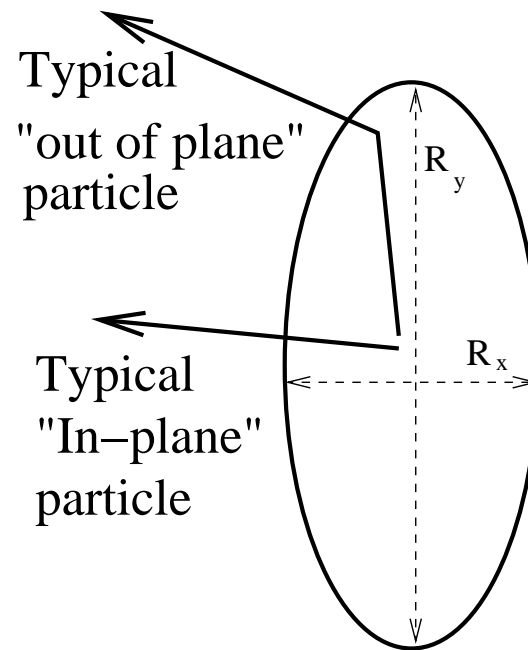
- Weakly coupled partons in mean field
- Rotational motion

The observed experimental scaling is unnatural enough in the strongly coupled limit that, I think, alternative origins of v_2 have to be reconsidered. Eg...**weak coupling**

- At weak coupling+Bjorken initial conditions, rapidity slices do not know about each other: Free streaming Bjorken remains Bjorken! (No shock waves)
- At weak coupling, system not in chemical equilibrium, partonic all the way!!!
- At weak coupling, scaling of v_2 with multiplicity natural!

$\frac{v_2}{\epsilon} \propto \frac{1}{S} \frac{dN}{dy}$ arises naturally in $Kn \geq 1$

H. Heiselberg and A. M. Levy, PRC **59**, 2716 (1999) nucl-th/9812034



$l_{mfp} \sim R \quad 1 \quad \text{Interaction/particle/lifetime}$

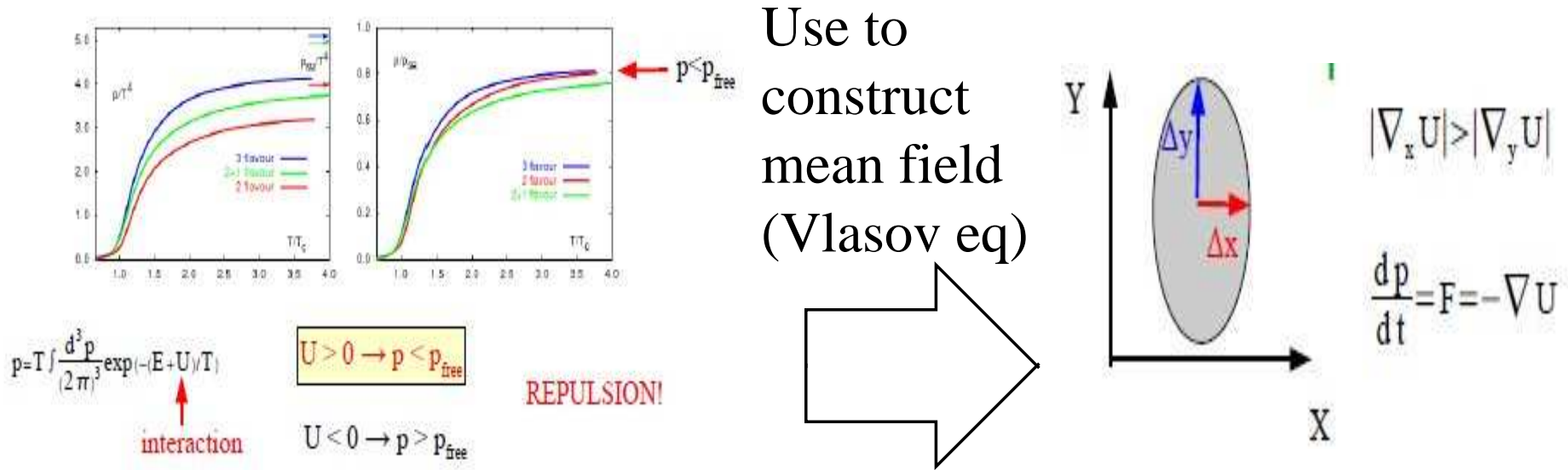
Boltzmann equation with these initial conditions...

$$\frac{v_2}{\epsilon} \propto \frac{\langle \sigma_{ij} v_{ij} \rangle dN}{R_x R_y dy}$$

A break of this scaling would have signalled a sudden change in $\langle \sigma_{ij} v \rangle$, driven perhaps by the transition from a weakly coupled hadron gas to a weakly coupled quark-gluon gas. But in absence of chemical equilibrium, no reason for transition!

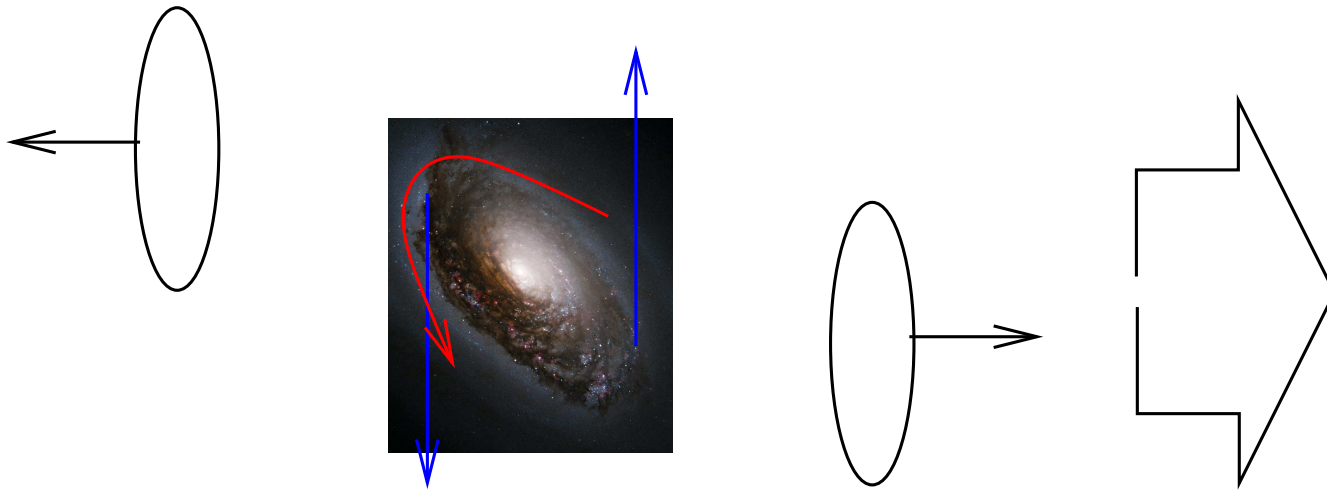
Scaling of v_2 works, but v_2 too small!

As we know, however, $Kn \geq 1$ can not describe $y = 0$ value of v_2 .
 One way out... mean field model (V.Koch, QM2009)

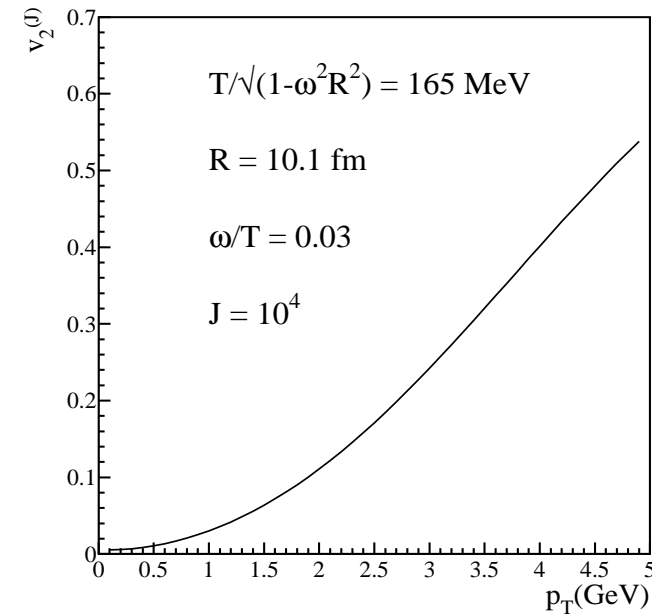


Same considerations of previous slides apply, but v_2 can be large

Another idea: v_2 actually caused by Rotational motion



Becattini, Piccinini,
Rizzo 0711.1253



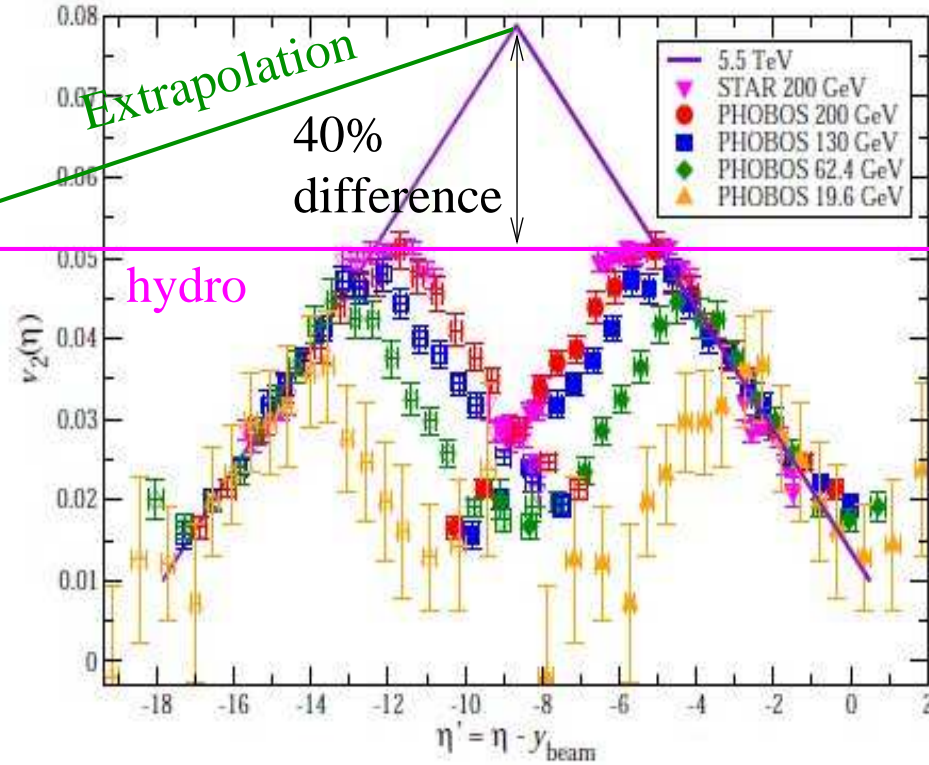
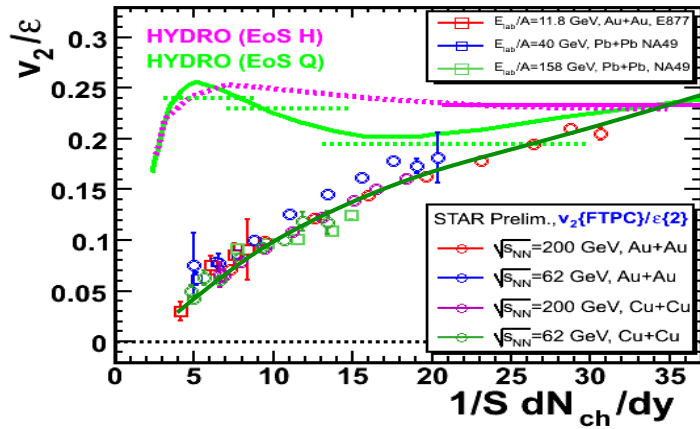
Scaling follows from relation between angular momentum, centrality, multiplicity, **100% transparency** (\sim Landau) assumed in this calculation...

but...

- Result larger by $\sim 10^2$
- No rapidity scaling
- Experimental polarization limits

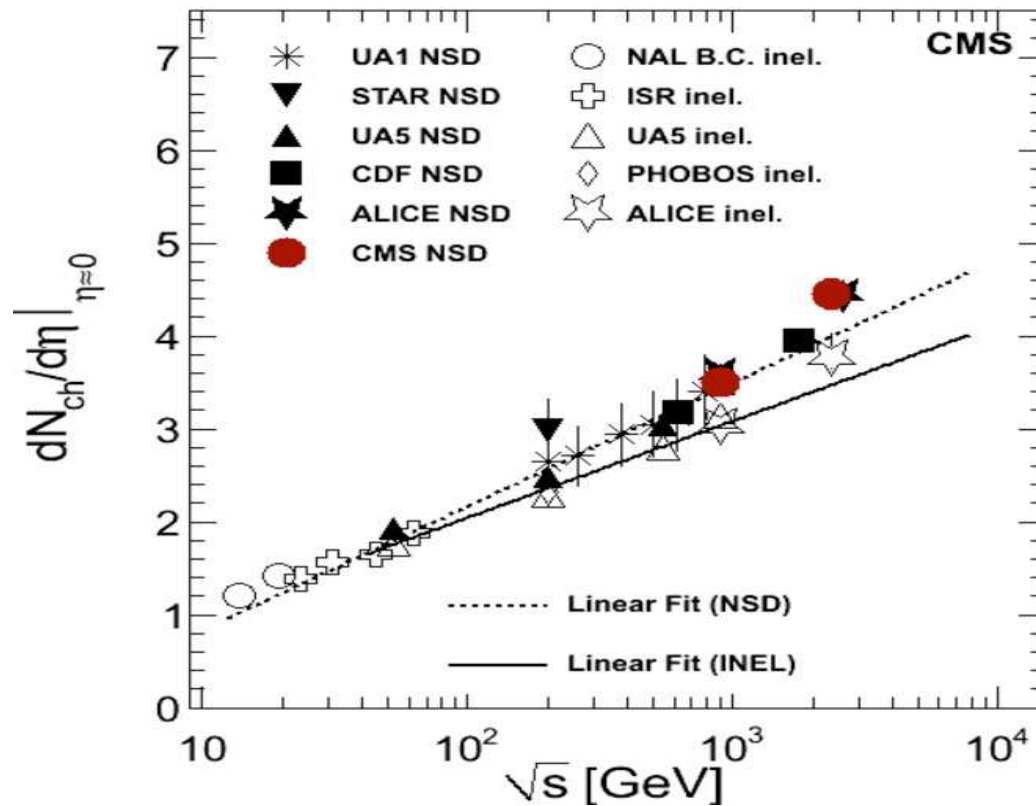
All three might be overcome by a nearly (to $\sim 10^{-2}$) Boost-invariant initial condition, but need a quantitative model

STAR,nucl-ex/0701038



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0707.0564
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[PHOBOS]
0907.4719

At LHC, hydrodynamics predicts a large breakdown of the scaling! we will know!



W.Busza
Saturation workshop
BNL

Data from
JHEP 02(2010)041
(CMS collaboration)

In fact, CMS claims $dN/dy \sim \ln \sqrt{s}$ scaling broken already. If the considerations here are correct, so should limiting fragmentation!

But lower energy scans can help as well!

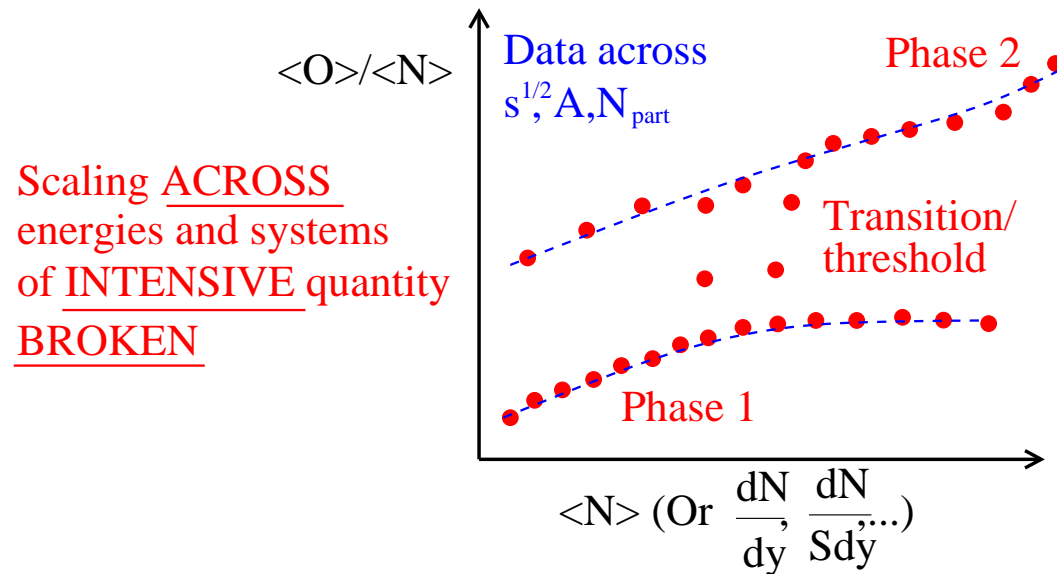
Experimental: Do observables dependent on flow know about $EoS, \eta/s$, or do they just universally fragment?

- $\langle p_T \rangle$ ("NA49 step" in rapidity?)
- HBT $R_{o,s}$ any softening in EoS?
- Particle species (No limiting fragmentation for baryons. Is appearance of scaling connected to "horn" baryon/meson anomaly?)

Theory: Hydrodynamic assessment of scaling with non-boost invariant initial conditions: How serious are the effects elucidated here

- Scaling naturalness should be demanded of any model, especially "complicated" ones

(very few!!!) conclusions



This would be the ideal QGP signature... and we are not there yet! There are good reasons to fear that such a signature is unrealistic. Certainly, jet suppression and elliptic flow do not qualify. But the scaling suggests they might at some point, if we find where/how it breaks

(very few!!!) conclusions

- Simple scalings have been found to hold for $\frac{dN}{dy}, \frac{dN}{dy}\Big|_{y=0}, v_2$

$\frac{dN}{dy}$ natural within our understanding of QCD

$\frac{dN}{dy}\Big|_{y=0}$ is also natural, provided interesting dynamics happens in the overlap region (non-pQCD)

v_2 unnatural within hydrodynamics, alternatives need to be looked into (scaling more natural in a weakly coupled system)

- Experimental measurements of limiting fragmentation in other soft observables ($\langle p_T \rangle, R_{out,side}$) could help clarify the situation.
- 3D viscous hydro needed to make these statements more quantitative