NONADDITIVE ENTROPY AND NONEXTENSIVE STATISTICAL MECHANICS: CONCEPTS AND APPLICATIONS

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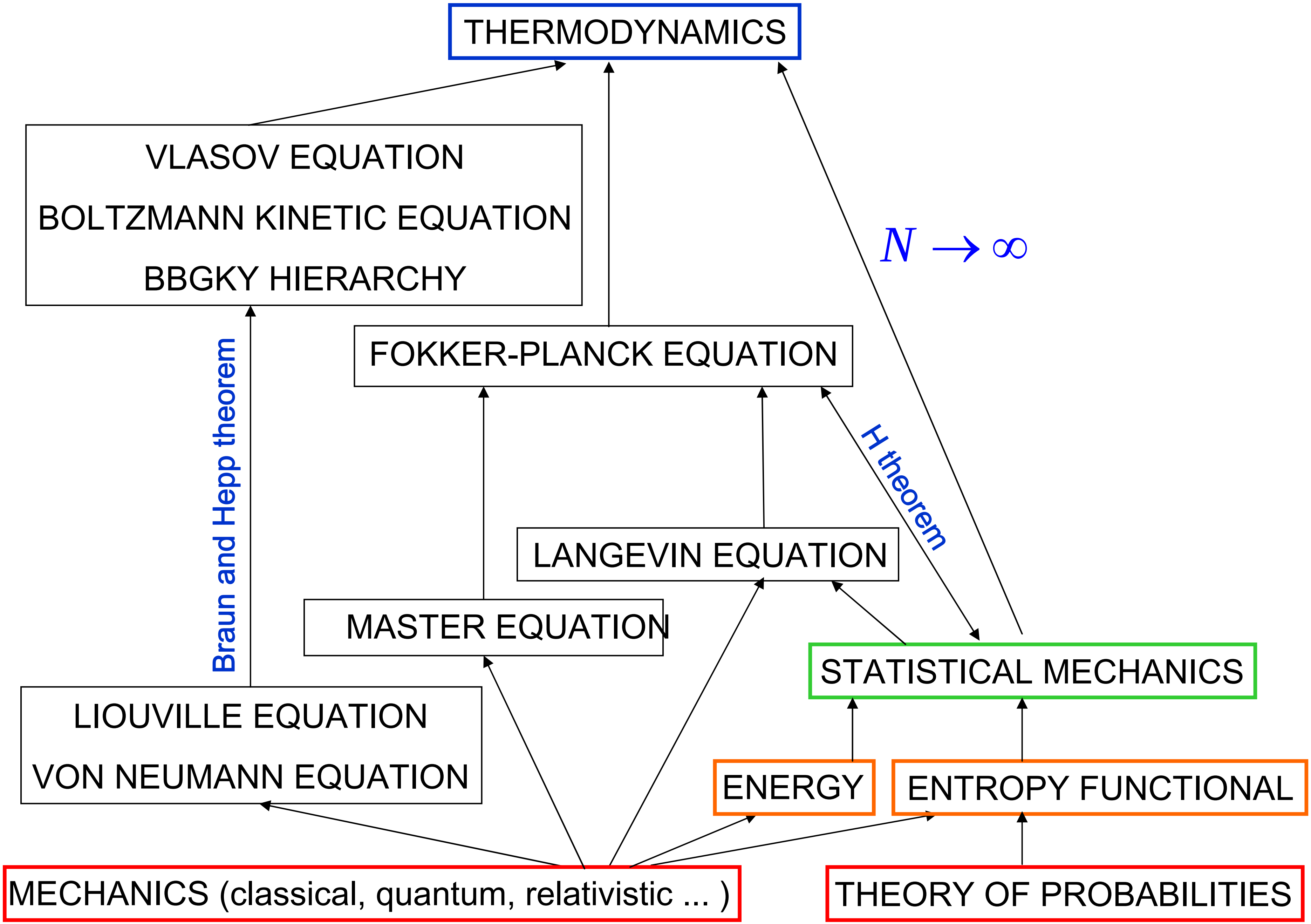
Budapest, August 2010

J.W. GIBBS

Elementary Principles in Statistical Mechanics - Developed with Especial Reference to the Rational Foundation of Thermodynamics

C. Scribner's Sons, New York, 1902; Yale University Press, New Haven, 1981), page 35

In treating of the canonical distribution, we shall always suppose the multiple integral in equation (92) [the partition function, as we call it nowadays] *to have a finite valued, as otherwise the coefficient of probability vanishes, and the law of distribution becomes illusory. This will exclude certain cases, but not such apparently, as will affect the value of our results with respect to their bearing on thermodynamics. It will exclude, for instance, cases in which the system or parts of it can be distributed in unlimited space […]. It also excludes many cases in which the energy can decrease without limit, as when the system contains material points which attract one another inversely as the squares of their distances. […]. For the purposes of a general discussion, it is sufficient to call attention to the assumption implicitly involved in the formula (92).*


THERMODYNAMICS
VLASOV EQUATION
BOLTZMANN KINETIC EQUATION
BBGKY HIERARCHY
$N \to \infty$
FOKKER-PLANCK EQUATION
Braun and Hepp theorem
H theorem
LANGEVIN EQUATION
MASTER EQUATION
STATISTICAL MECHANICS
LIOUVILLE EQUATION
VON NEUMANN EQUATION
ENERGY
ENTROPY FUNCTIONAL
MECHANICS (classical, quantum, relativistic ...)
THEORY OF PROBABILITIES

POSTULATE FOR THE ENTROPIC FUNCTIONAL

	$p_i = \frac{1}{W} \quad (\forall i)$ **equiprobability**	$\forall p_i \ (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1 \right)$
BG entropy ($q=1$)	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy S_q (q real)	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

additive

Concave

Extensive

Lesche-stable

Finite entropy production per unit time

Pesin-like identity (with largest entropy production)

Composable

Topsoe-factorizable

nonadditive (if $q \neq 1$)

Possible generalization of Boltzmann-Gibbs statistical mechanics

[C.T., J. Stat. Phys. **52**, 479 (1988)]

DEFINITION $(q-logarithm)$:

$$\ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0)$$

$$\ln_1 x = \ln x$$

Hence, the entropies can be rewritten:

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k \ln W$	$k \sum_{i=1}^{W} p_i \ln \frac{1}{p_i}$
entropy S_q $(q \in R)$	$k \ln_q W$	$k \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$

TYPICAL SIMPLE SYSTEMS:

$$\text{e.g.,}\quad W(N) \propto \mu^{N} \quad (\mu > 1)$$

Short-range space-time correlations

Markovian processes (short memory), Additive noise

Strong chaos (positive maximal Lyapunov exponent), Ergodic, Euclidean geometry

Short-range many-body interactions, weakly quantum-entangled subsystems

Linear/homogeneous Fokker-Planck equations, Gausssians

→ **Boltzmann-Gibbs entropy (additive)**

→ **Exponential dependences (Boltzmann-Gibbs weight, ...)**

TYPICAL COMPLEX SYSTEMS:

$$\text{e.g.,}\quad W(N) \propto N^{\rho} \quad (\rho > 0)$$

Long-range space-time correlations

Non-Markovian processes (long memory), Additive and multiplicative noises

Weak chaos (zero maximal Lyapunov exponent), Nonergodic, Multifractal geometry

Long-range many-body interactions, strongly quantum-entangled sybsystems

Nonlinear/inhomogeneous Fokker-Planck equations, *q*-Gaussians

→ **Entropy S_q (nonadditive)**

→ ***q*-exponential dependences (asymptotic power-laws)**

- **Additive *versus* Extensive**

- Central Limit Theorem

- Predictions, verifications and applications

ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for any two probabilistically independent systems A and B,

$$S(A+B)=S(A)+S(B)$$

Therefore, since

$$S_q(A+B)=S_q(A)+S_q(B)+(1-q)\,S_q(A)\,S_q(B)\;,$$

S_{BG} and S_q^{Renyi} $(\forall q)$ are additive, and S_q $(\forall q \neq 1)$ is nonadditive .

EXTENSIVITY:

Consider a system $\Sigma \equiv A_1 + A_2 + \ldots + A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N.

An entropy is extensive if

$$0 < \lim_{N\to\infty} \frac{S(N)}{N} < \infty \;,\; i.e.,\;\; S(N) \propto N \;\; (N \to \infty)$$

HYBRID PASCAL - LEIBNITZ TRIANGLE

(N=0) $1\times\frac{1}{1}$

(N=1) $1\times\frac{1}{2}$ $1\times\frac{1}{2}$

(N=2) $1\times\frac{1}{3}$ $2\times\frac{1}{6}$ $1\times\frac{1}{3}$

(N=3) $1\times\frac{1}{4}$ $3\times\frac{1}{12}$ $3\times\frac{1}{12}$ $1\times\frac{1}{4}$

(N=4) $1\times\frac{1}{5}$ $4\times\frac{1}{20}$ $6\times\frac{1}{30}$ $4\times\frac{1}{20}$ $1\times\frac{1}{5}$

(N=5) $1\times\frac{1}{6}$ $5\times\frac{1}{30}$ $10\times\frac{1}{60}$ $10\times\frac{1}{60}$ $5\times\frac{1}{30}$ $1\times\frac{1}{6}$

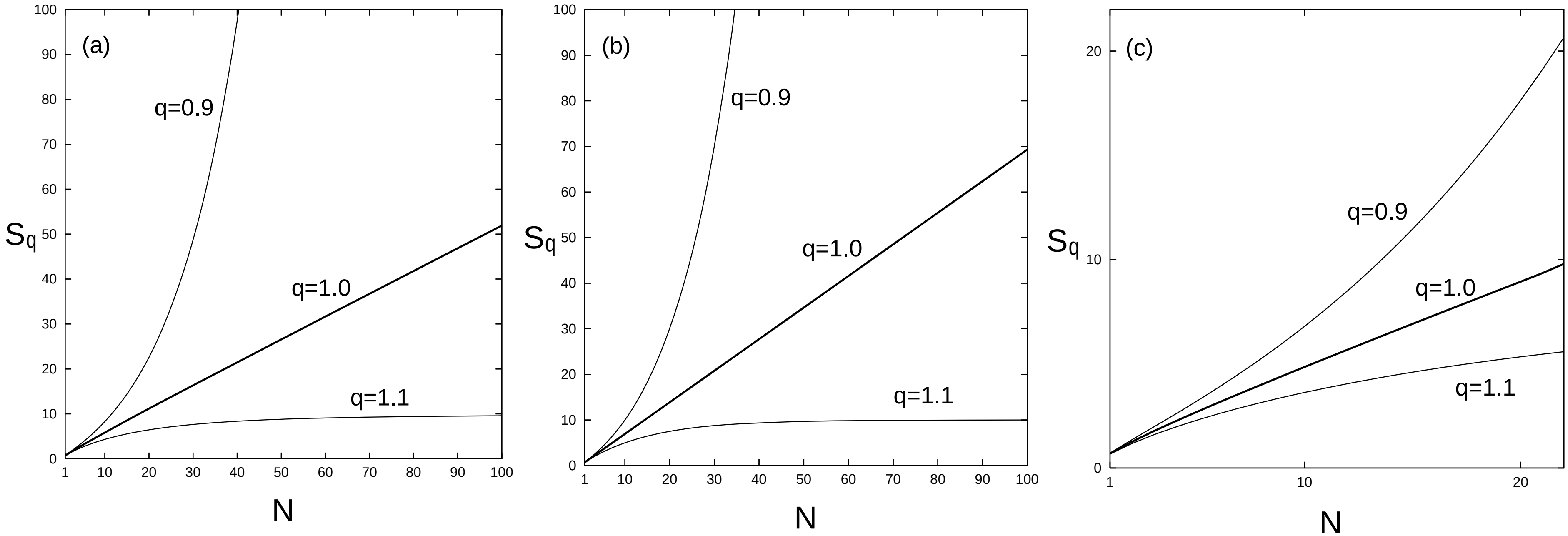
$$\sum_{n=0}^{N}\binom{N}{n} r_{N,n} = 1 \quad (\forall\ N)$$

Blaise **Pascal** (1623-1662)
Gottfried Wilhelm **Leibnitz** (1646-1716)
Daniel **Bernoulli** (1700-1782)

$q = 1$ SYSTEMS

i.e., such that $S_1(N) \propto N \quad (N \to \infty)$

> *I don't believe that atoms exist!*
>
> Ernst Mach (January 1897, Vienna)



Leibnitz triangle	*N independent coins*	*Stretched exponential*
$\left(p_{N,0} = \dfrac{1}{N+1}\right)$	$\left(p_{N,0} = p^N \quad \text{with } p = 1/2\right)$	$\left(p_{N.0} = p^{N^\alpha} \quad \text{with } p = \alpha = 1/2\right)$

(All three examples **strictly** satisfy the Leibnitz rule)

C.T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sc USA **102**, 15377 (2005)

Asymptotically scale-invariant (d=2)

$(N=0)$	1				
$(N=1)$	$1/2$	$1/2$			
$(N=2)$	$1/3$	$1/6$	$1/3$		
$(N=3)$	$3/8$	$5/48$	$5/48$	0	
$(N=4)$	$2/5$	$3/40$	$1/20$	0	0

d+1

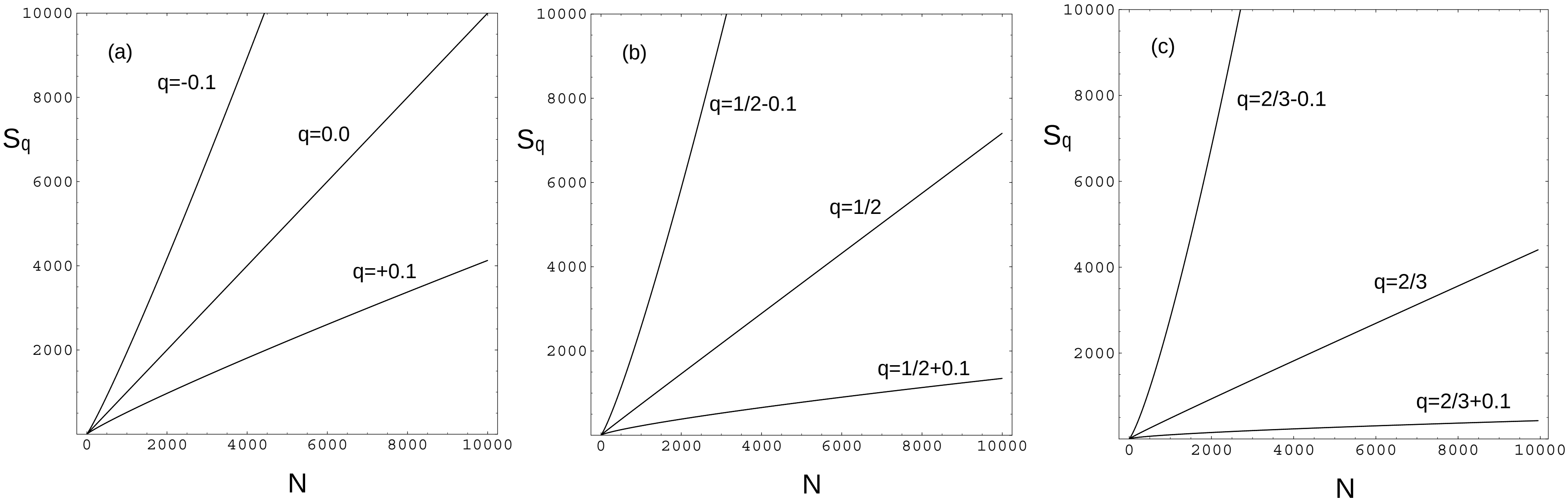
(It asymptotically satisfies the Leibnitz rule)

C.T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sc USA **102**, 15377 (2005)

$q \neq 1$ SYSTEMS

i.e., such that $S_q(N) \propto N \quad (N \to \infty)$

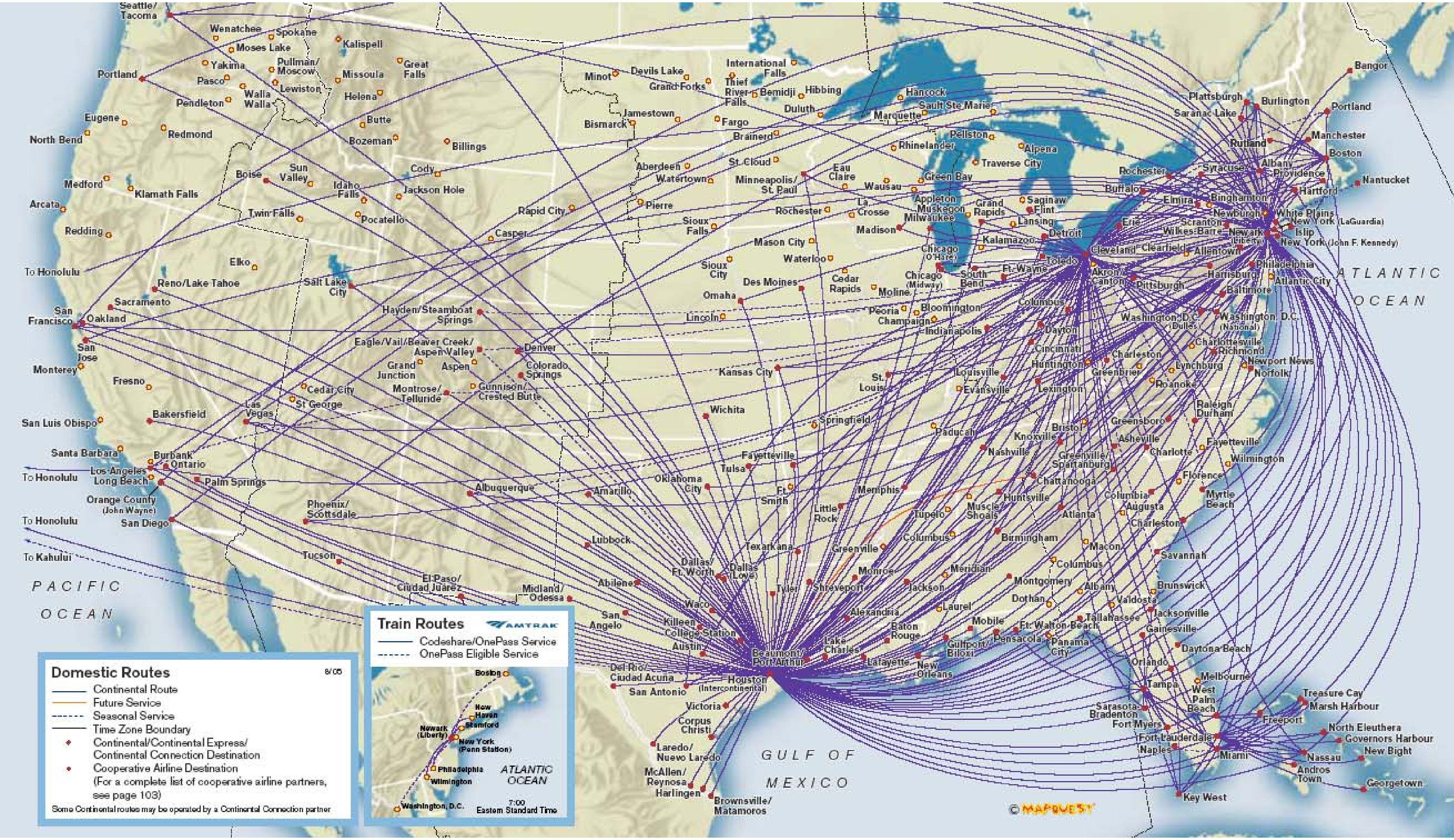
$(d=1)$ $(d=2)$ $(d=3)$



$$q = 1 - \frac{1}{d}$$

(All three examples asymptotically satisfy the Leibnitz rule)

C.T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sc USA **102**, 15377 (2005)



Continental Airlines

PHYSICAL REVIEW E **78**, 021102 (2008)

Nonadditive entropy reconciles the area law in quantum systems with classical thermodynamics

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(Received 16 March 2008; revised manuscript received 16 May 2008; published 5 August 2008)

The Boltzmann–Gibbs–von Neumann entropy of a large part (of linear size L) of some (much larger) d-dimensional quantum systems follows the so-called area law (as for black holes), i.e., it is proportional to L^{d-1}. Here we show, for $d=1,2$, that the (nonadditive) entropy S_q satisfies, for a special value of $q\neq 1$, the classical thermodynamical prescription for the entropy to be extensive, i.e., $S_q \propto L^d$. Therefore, we reconcile with classical thermodynamics the area law widespread in quantum systems. Recently, a similar behavior was exhibited in mathematical models with scale-invariant correlations [C. Tsallis, M. Gell-Mann, and Y. Sato, Proc. Natl. Acad. Sci. U.S.A. **102** 15377 (2005)]. Finally, we find that the system critical features are marked by a maximum of the special entropic index q.

SPIN ½ XY FERROMAGNET WITH TRANSVERSE MAGNETIC FIELD:

$$\hat{\mathcal{H}} = -\sum_{j=1}^{N-1} \left[(1+\gamma)\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + (1-\gamma)\hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + 2\lambda \hat{\sigma}_j^z \right]$$

$|\gamma| = 1 \quad \rightarrow \quad$ *Ising ferromagnet*

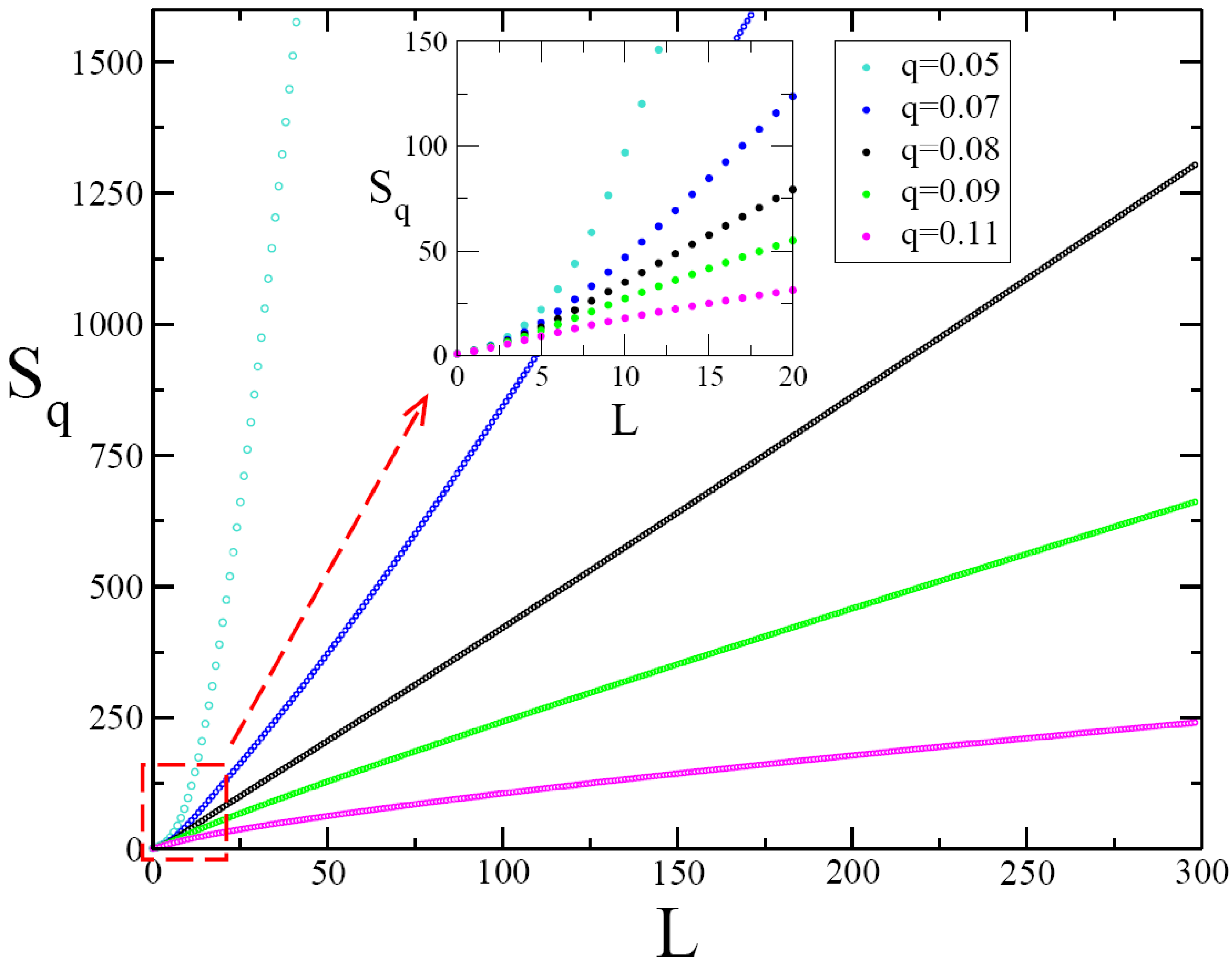
$0 < |\gamma| < 1 \quad \rightarrow \quad$ *anisotropic XY ferromagnet*

$\gamma = 0 \quad \rightarrow \quad$ *isotropic XY ferromagnet*

$\lambda \equiv$ *transverse magnetic field*

$L \equiv$ *length of a block within a* $N \rightarrow \infty$ *chain*

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)



F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

Using a Quantum Field Theory result

in P. Calabrese and J. Cardy, JSTAT P06002 (2004)

we obtain, at the critical transverse magnetic field,

$$q_{ent} = \frac{\sqrt{9+c^2}-3}{c}$$

with $c \equiv$ *central charge in conformal field theory*

Hence

Ising and anisotropic XY ferromagnets $\Rightarrow\ c = \frac{1}{2}\ \Rightarrow\ q_{ent} = \sqrt{37}-6\ \approx 0.0828$

and

Isotropic XY ferromagnet $\Rightarrow\ c = 1\ \Rightarrow\ q_{ent} = \sqrt{10}-3\ \approx 0.1623$

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

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Nonadditive entropy for random quantum spin-S chains

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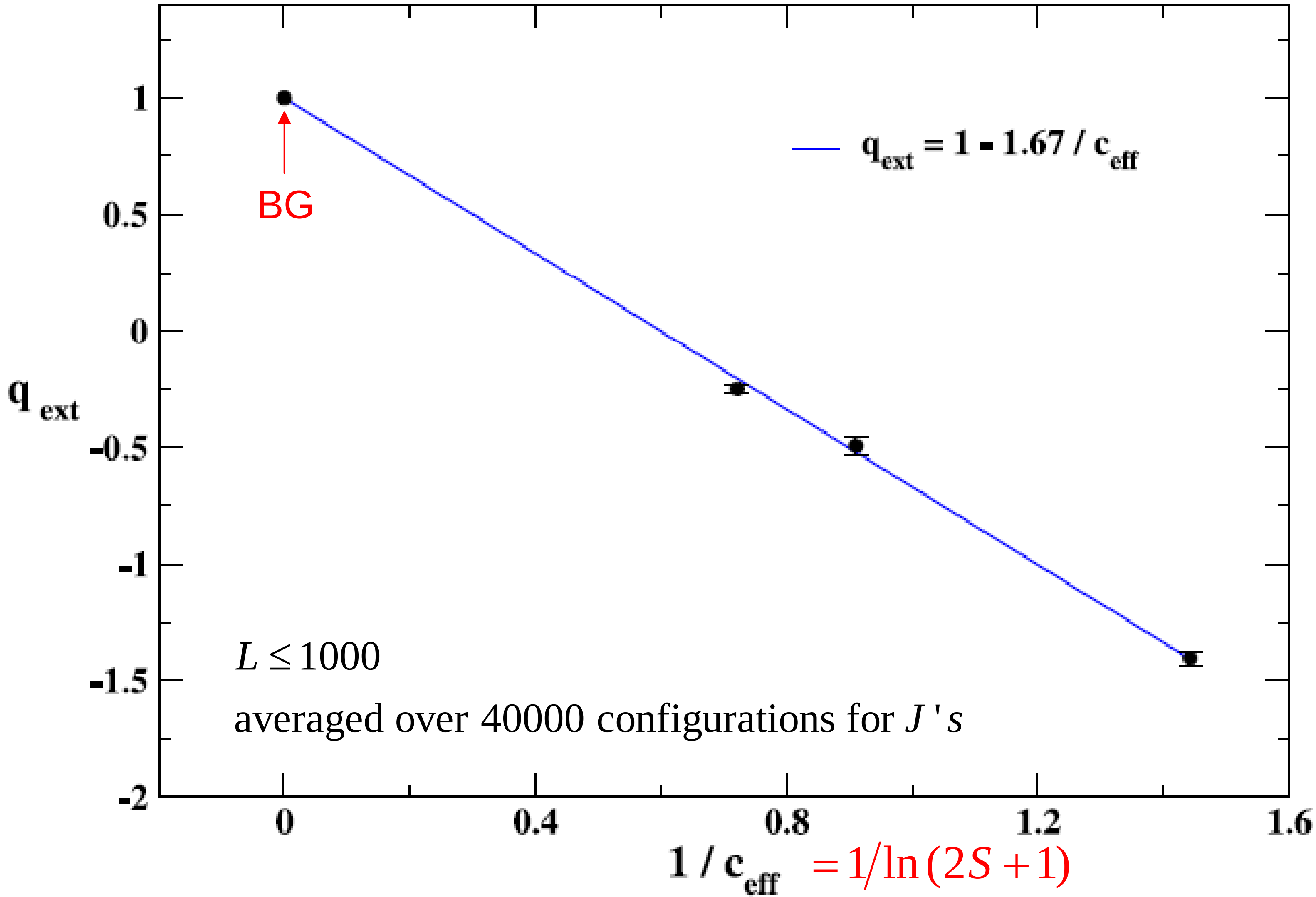
ABSTRACT

We investigate the scaling of Tsallis entropy in disordered quantum spin-S chains. We show that an extensive scaling occurs for specific values of the entropic index. Those values depend only on the magnitude S of the spins, being directly related with the effective central charge associated with the model.

$$H_{Heis} = \sum_{i=1}^{N} J_i \vec{S}_i \cdot \vec{S}_{i+1}$$

where $\{J_i\}$ are random exchange couplings obeying a probability distribution $P(J)$ and $\{\vec{S}_i\}$ are spin-S operators, with periodic boundary conditions



Also with spin-1 random-exchange biquadratic antiferromagnetic chain

$$H_{Biq} = \sum_{i=1}^{N} J_i (\vec{S}_i \cdot \vec{S}_{i+1})^2$$

Summarizing, for a wide class of quantum systems or subsystems with N elements, we know that

$$S_{BG}(N) \propto \ln L \propto \ln N \neq N \quad \text{for } d=1 \text{ quantum chains}$$

$$\propto L \propto \sqrt{N} \neq N \quad \text{for } d=2 \text{ bosonic systems}$$

$$\propto L^2 \propto N^{2/3} \neq N \quad \text{for } d=3 \text{ black hole}$$

$$\propto L^{d-1} \propto N^{(d-1)/d} \neq N \quad \text{for } d\text{-dimensional bosonic systems}$$

$$(d>1;\ \text{area law})$$

$$\propto \frac{L^{d-1}-1}{d-1} \equiv \ln_{2-d} L \neq L^d \propto N \quad (d \geq 1) \qquad \text{(NONEXTENSIVE!)}$$

For the same class of quantum systems, we expect

$$S_{q_{ent}}(N) \propto L^d \propto N \quad (d \geq 1;\ q_{ent} \neq 1) \qquad \text{(EXTENSIVE!)}$$

(analytically and/or computationally shown for $d = 1, 2$)

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

SYSTEMS	ENTROPY S_{BG} (additive)	ENTROPY S_q (q<1) (nonadditive)
Short-range interactions, weakly entangled blocks, etc	**EXTENSIVE**	**NONEXTENSIVE**
Long-range interactions (QSS), strongly entangled blocks, etc	**NONEXTENSIVE**	**EXTENSIVE**

quarks-gluons, plasma, curved space ...?

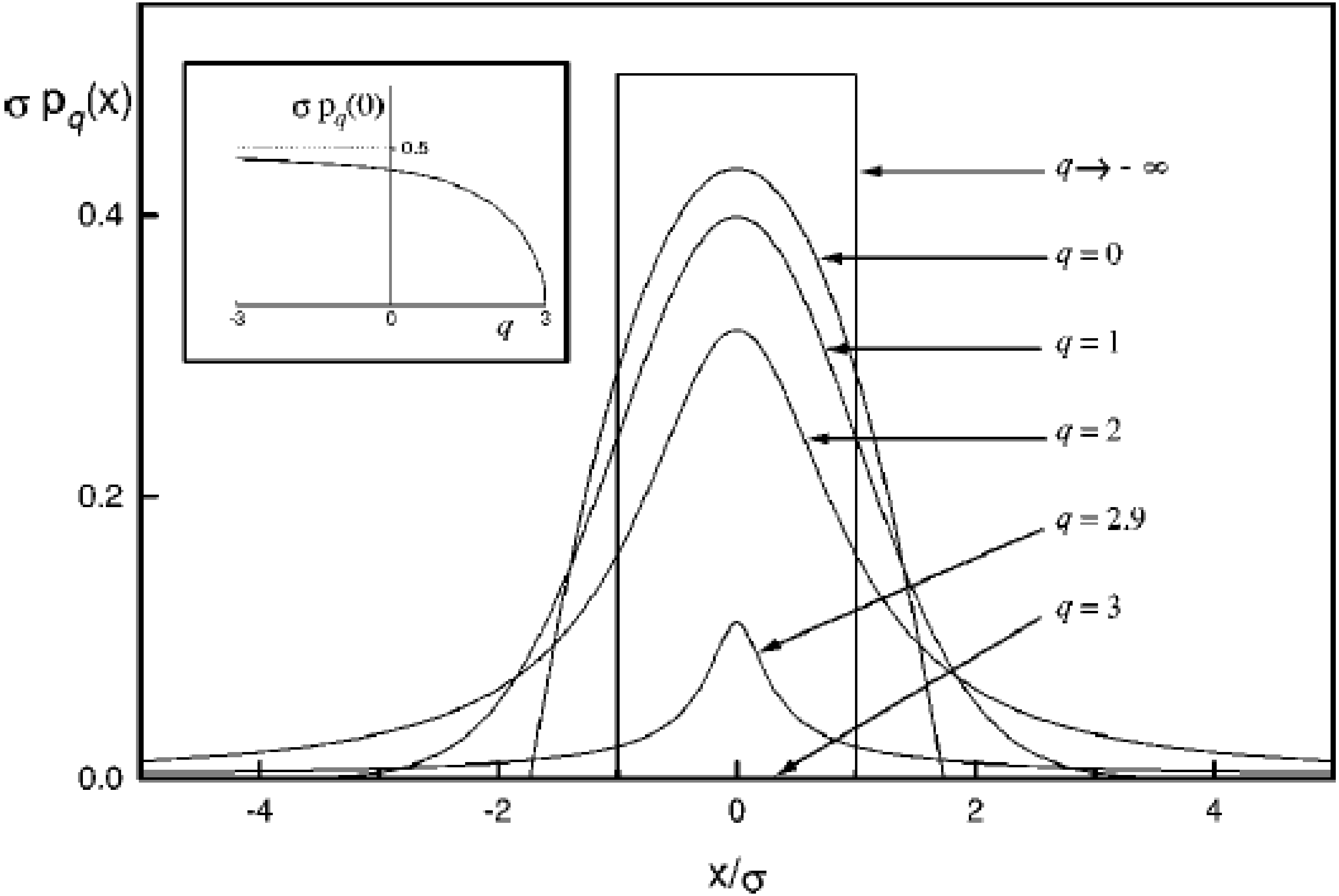
King Thutmosis III
18th Dynasty
c. 1460 B. C.

- Additive *versus* Extensive

- **Central Limit Theorem**

- Predictions, verifications and applications

q-GAUSSIANS: $p_q(x) \propto e_q^{-(x/\sigma)^2} \equiv \frac{1}{\left[1+(q-1)\,(x/\sigma)^2\right]^{\frac{1}{q-1}}}$ $(q<3)$



D. Prato and C. T., Phys Rev E **60**, 2398 (1999)

q- GENERALIZED CENTRAL LIMIT THEOREM:

S. Umarov, C.T. and S. Steinberg, Milan J Math **76,** 307 (2008)

q-Fourier transform:

$$F_q[f](\xi) \equiv \int_{-\infty}^{\infty} e_q^{ix\xi} \otimes_q f(x)\, dx = \int_{-\infty}^{\infty} e_q^{ix\xi[f(x)]^{q-1}} f(x)\, dx$$

$(q \geq 1)$

(nonlinear!)

For q<1 see K.P. Nelson and S. Umarov, Physica A **389**, 2157 (2010)

Milan j. math. 76 (2008), 307–328
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DOI 10.1007/s00032-008-0087-y

Milan Journal of Mathematics

On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics

Sabir Umarov, Constantino Tsallis and Stanly Steinberg

JOURNAL OF MATHEMATICAL PHYSICS **51**, 033502 (2010)

Generalization of symmetric α-stable Lévy distributions for $q>1$

Sabir Umarov,[1,a)] Constantino Tsallis,[2,3,b)] Murray Gell-Mann,[3,c)] and Stanly Steinberg[4,d)]
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CENTRAL LIMIT THEOREM

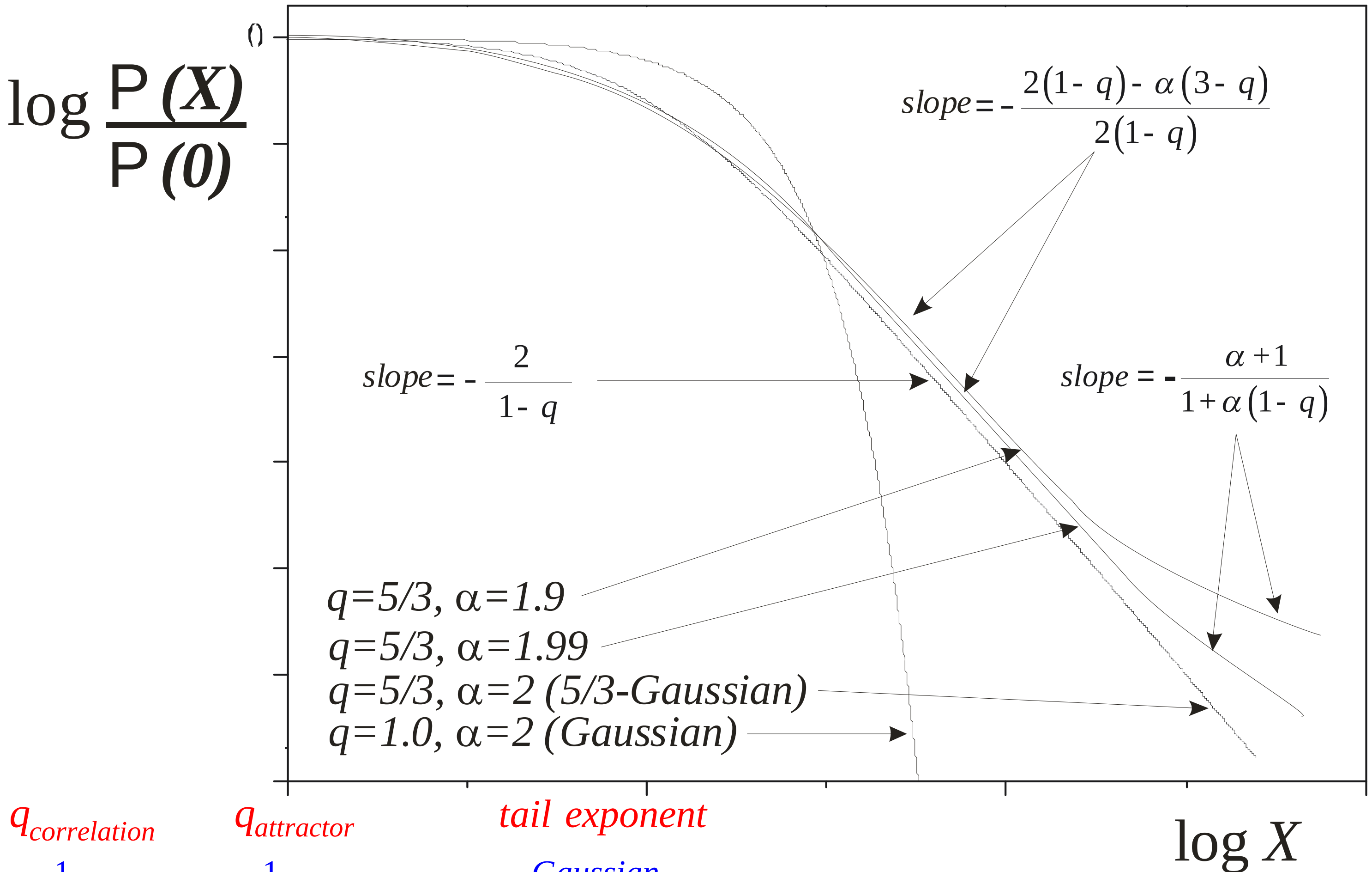
$N^{1/[\alpha(2-q)]}$ *- scaled attractor* $\mathbb{F}(x)$ *when summing* $N \to \infty$ q*-independent identical random variables*

with symmetric distribution $f(x)$ *with* $\sigma_Q \equiv \int dx\, x^2 [f(x)]^Q / \int dx\, [f(x)]^Q$ $\left(Q \equiv 2q-1,\ q_1 = \frac{1+q}{3-q} \right)$

	$q=1$ *[independent]*	$q \neq 1$ (*i.e.,* $Q \equiv 2q-1 \neq 1$) *[globally correlated]*
$\sigma_Q < \infty$ $(\alpha = 2)$	$\mathbb{F}(x) =$ *Gaussian* $G(x)$, *with same* σ_1 *of* $f(x)$ Classic CLT	$\mathbb{F}(x) = G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, *with same* σ_Q *of* $f(x)$ $G_q(x) \sim \begin{cases} G(x) & \text{if } \lvert x \rvert << x_c(q,2) \\ f(x) \sim C_q / \lvert x \rvert^{2/(q-1)} & \text{if } \lvert x \rvert >> x_c(q,2) \end{cases}$ *with* $\lim_{q \to 1} x_c(q,2) = \infty$ S. Umarov, C. T. and S. Steinberg, Milan J Math 76, 307 (2008)
$\sigma_Q \to \infty$ $(0 < \alpha < 2)$	$\mathbb{F}(x) =$ *Levy distribution* $L_\alpha(x)$, *with same* $\lvert x \rvert \to \infty$ *behavior* $L_\alpha(x) \sim \begin{cases} G(x) \\ \quad \text{if } \lvert x \rvert << x_c(1,\alpha) \\ f(x) \sim C_\alpha / \lvert x \rvert^{1+\alpha} \\ \quad \text{if } \lvert x \rvert >> x_c(1,\alpha) \end{cases}$ *with* $\lim_{\alpha \to 2} x_c(1,\alpha) = \infty$ Levy-Gnedenko CLT	$\mathbb{F}(x) = L_{q,\alpha}$, *with same* $\lvert x \rvert \to \infty$ *asymptotic behavior* $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)},\, \alpha}(x) \sim C^*_{q,\alpha} / \lvert x \rvert^{\frac{2(1-q)-\alpha(3-q)}{2(1-q)}} \\ \quad \textit{(intermediate regime)} \\ G_{\frac{2\alpha q - \alpha + 3}{\alpha+1},\, 2}(x) \sim C^L_{q,\alpha} / \lvert x \rvert^{(1+\alpha)/(1+\alpha q - \alpha)} \\ \quad \textit{(distant regime)} \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg J Math Phys 51, 033502 (2010)

q-CENTRAL LIMIT THEOREMS:

$$p(x) = L_{q,\alpha}(x) = (q\text{-}Fourier)^{-1}\left[b e_{q_1}^{-\beta|\xi|^{\alpha}} \right]$$



α	$q_{correlation}$	$q_{attractor}$	tail exponent	
2	1	1	Gaussian	
2	$\frac{q+1}{3-q}$	q	$\frac{2}{q-1}$	
α	$\frac{q+1}{3-q}$	$\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)}$	$\frac{2(1-q)-\alpha(3-q)}{2(1-q)}$	(intermediate regime)
		$\frac{2\alpha q-\alpha+3}{\alpha+1}$	$\frac{1+\alpha}{1+\alpha q-\alpha}$	(distant regime)

$$\frac{2}{(q-1)} \geq \frac{2(1-q)-\alpha(3-q)}{2(1-q)} > \frac{1+\alpha}{1+\alpha q-\alpha}$$

C. T. and S.M.D. Queiros (2007)

- Additive *versus* Extensive

- Central Limit Theorem

- **Predictions, verifications and applications**

COLD ATOMS IN DISSIPATIVE OPTICAL LATTICES:

RAPID COMMU

PHYSICAL REVIEW A **67**, 051402(R) (2003)

Anomalous diffusion and Tsallis statistics in an optical lattice

Eric Lutz
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(Received 26 February 2003; published 27 May 2003)

We point out a connection between anomalous transport in an optical lattice and Tsallis' generalized statistics. Specifically, we show that the momentum equation for the semiclassical Wigner function which describes atomic motion in the optical potential, belongs to a class of transport equations recently studied by Borland [Phys. Lett. A **245**, 67 (1998)]. The important property of these ordinary linear Fokker-Planck equations is that their stationary solutions are exactly given by Tsallis distributions. An analytical expression of the Tsallis index q in terms of the microscopic parameters of the quantum-optical problem is given and the spatial coherence of the atomic wave packets is discussed.

(i) The distribution of atomic velocities is a q-Gaussian;

(ii) $$q = 1 + \frac{44E_R}{U_0}$$ where $E_R \equiv$ recoil energy

$U_0 \equiv$ potential depth

PRL **96,** 110601 (2006) PHYSICAL REVIEW LETTERS week ending 24 MARCH 2006

Tunable Tsallis Distributions in Dissipative Optical Lattices

P. Douglas, S. Bergamini, and F. Renzoni
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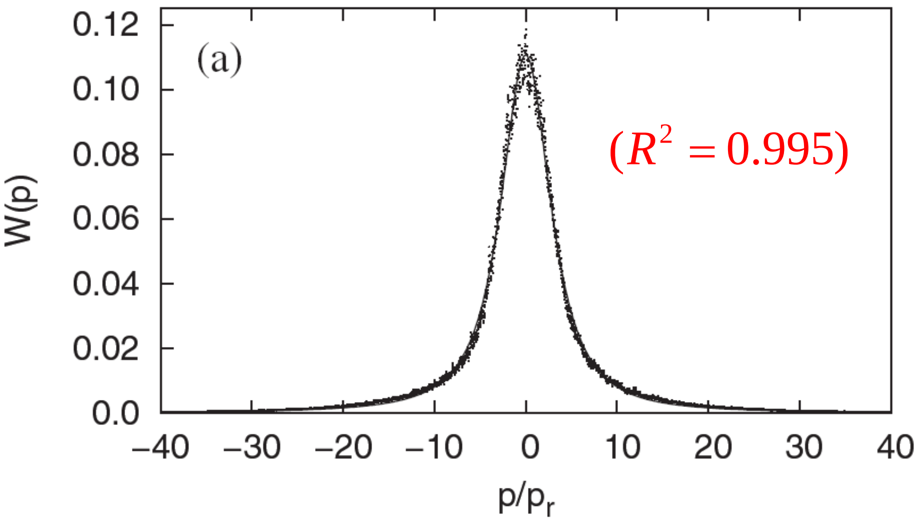
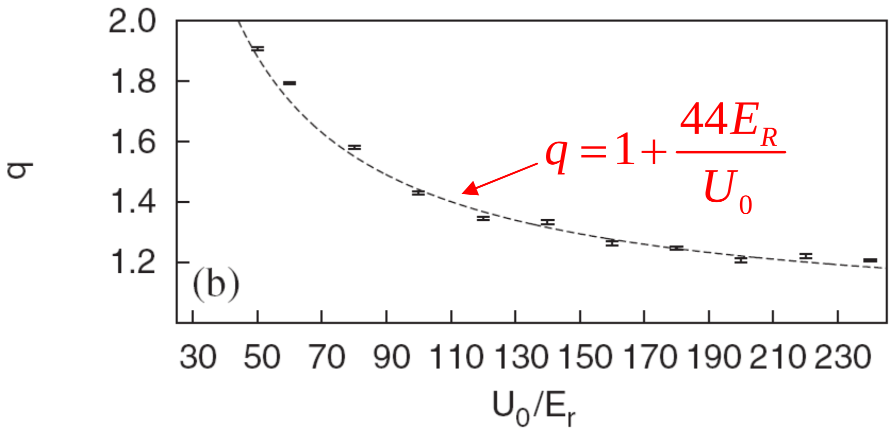
(Received 10 January 2006; published 24 March 2006)

We demonstrated experimentally that the momentum distribution of cold atoms in dissipative optical lattices is a Tsallis distribution. The parameters of the distribution can be continuously varied by changing the parameters of the optical potential. In particular, by changing the depth of the optical lattice, it is possible to change the momentum distribution from Gaussian, at deep potentials, to a power-law tail distribution at shallow optical potentials.

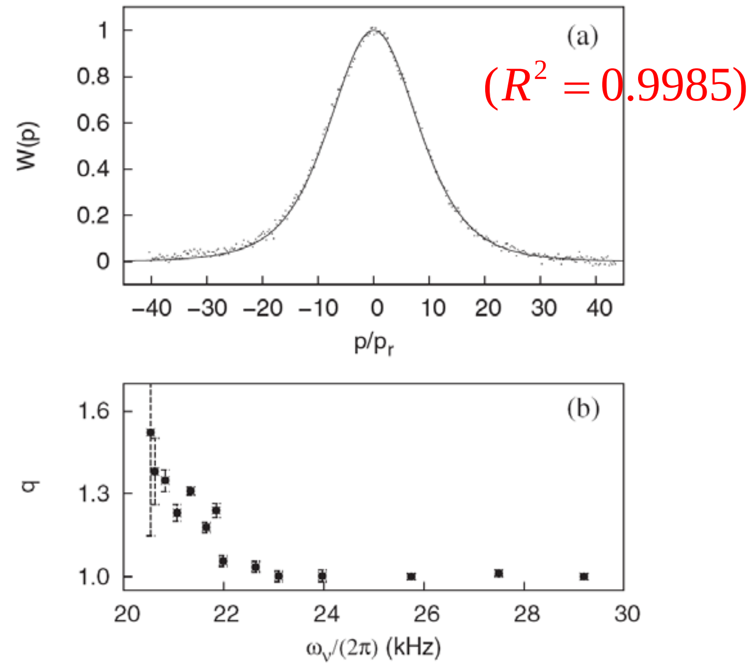
Experimental and computational verifications

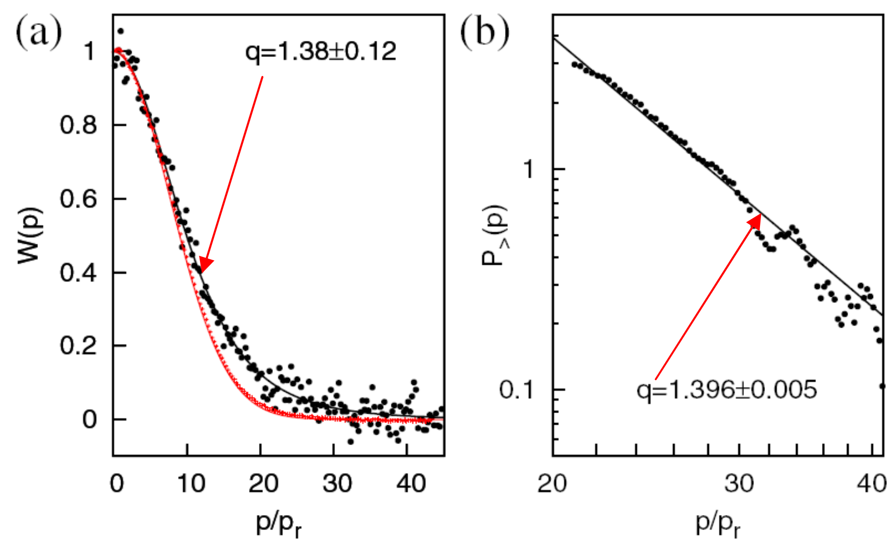
by P. Douglas, S. Bergamini and F. Renzoni, Phys Rev Lett **96**, 110601 (2006)





(Computational verification: quantum Monte Carlo simulations)





(Experimental verification: Cs atoms)

PRL 100, 055003 (2008) PHYSICAL REVIEW LETTERS week ending 8 FEBRUARY 2008

Superdiffusion and Non-Gaussian Statistics in a Driven-Dissipative 2D Dusty Plasma

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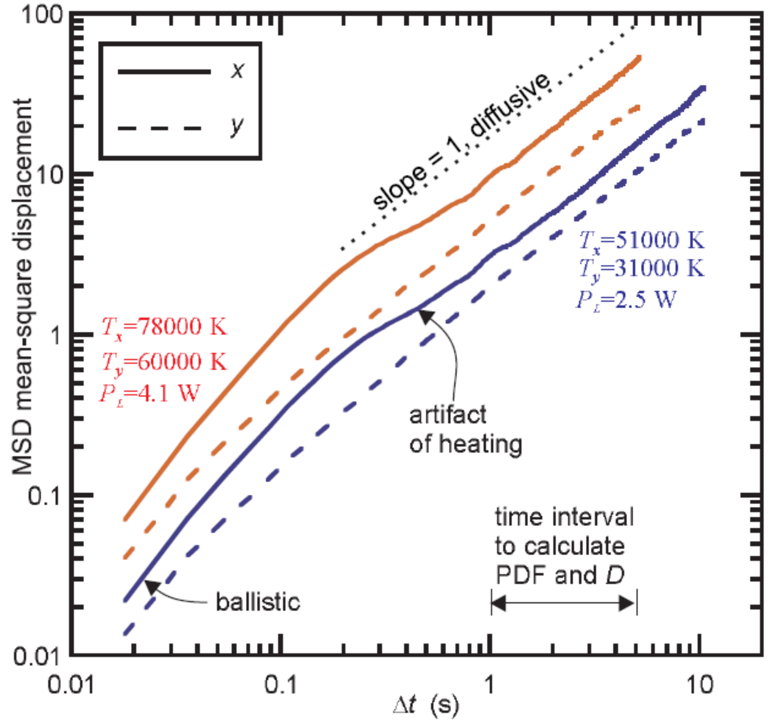
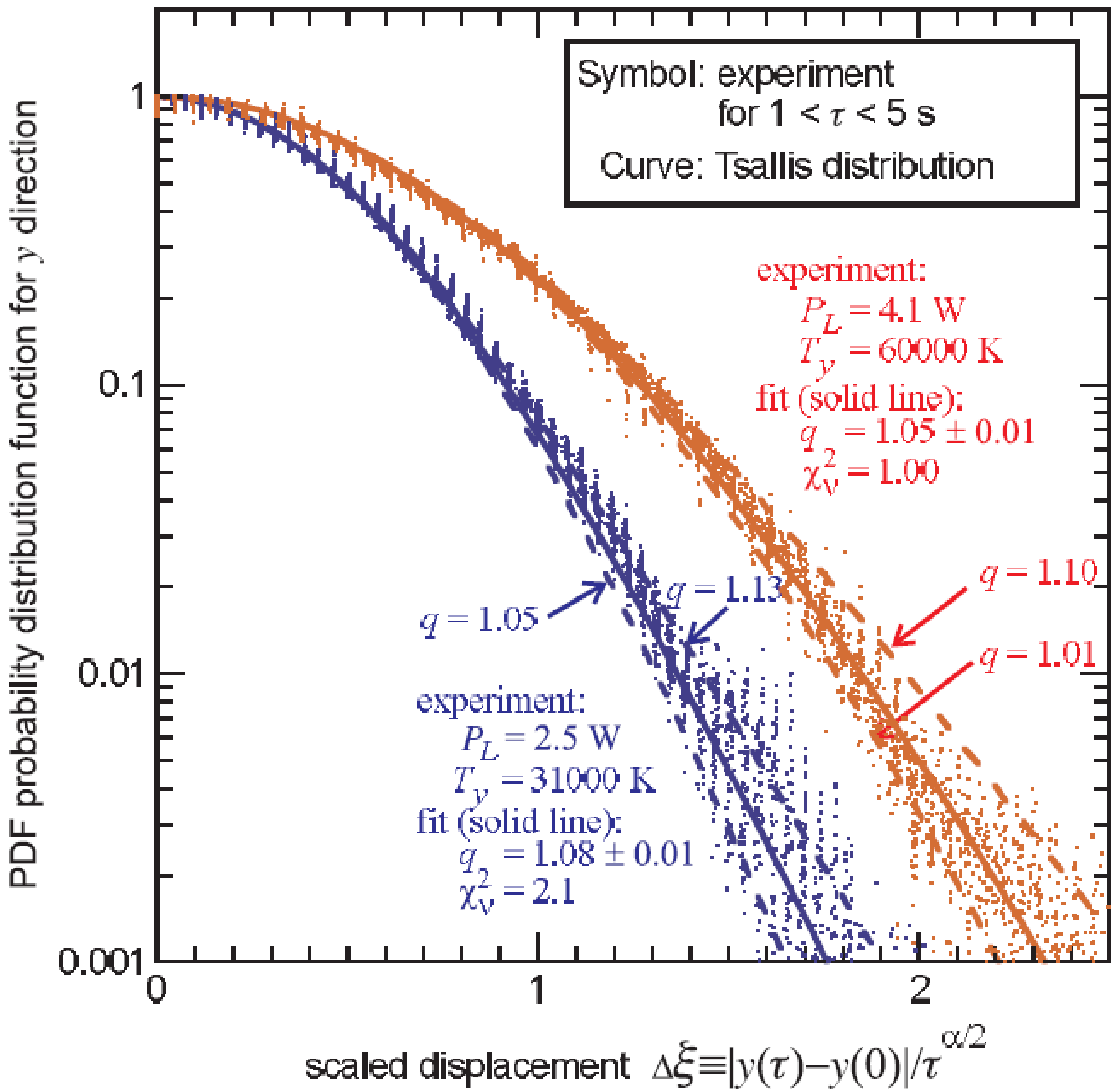
(Received 1 June 2007; published 6 February 2008)

Anomalous diffusion and non-Gaussian statistics are detected experimentally in a two-dimensional driven-dissipative system. A single-layer dusty plasma suspension with a Yukawa interaction and frictional dissipation is heated with laser radiation pressure to yield a structure with liquid ordering. Analyzing the time series for mean-square displacement, superdiffusion is detected at a low but statistically significant level over a wide range of temperatures. The probability distribution function fits a Tsallis distribution, yielding q, a measure of nonextensivity for non-Gaussian statistics.

$$< r^2 > \ \propto \ t^{\alpha}$$





B. Liu and J. Goree, Phys Rev Lett **100**, 055003 (2008)

LASER COOLING:

PRL **102,** 063001 (2009) PHYSICAL REVIEW LETTERS week ending 13 FEBRUARY 2009

Power-Law Distributions for a Trapped Ion Interacting with a Classical Buffer Gas

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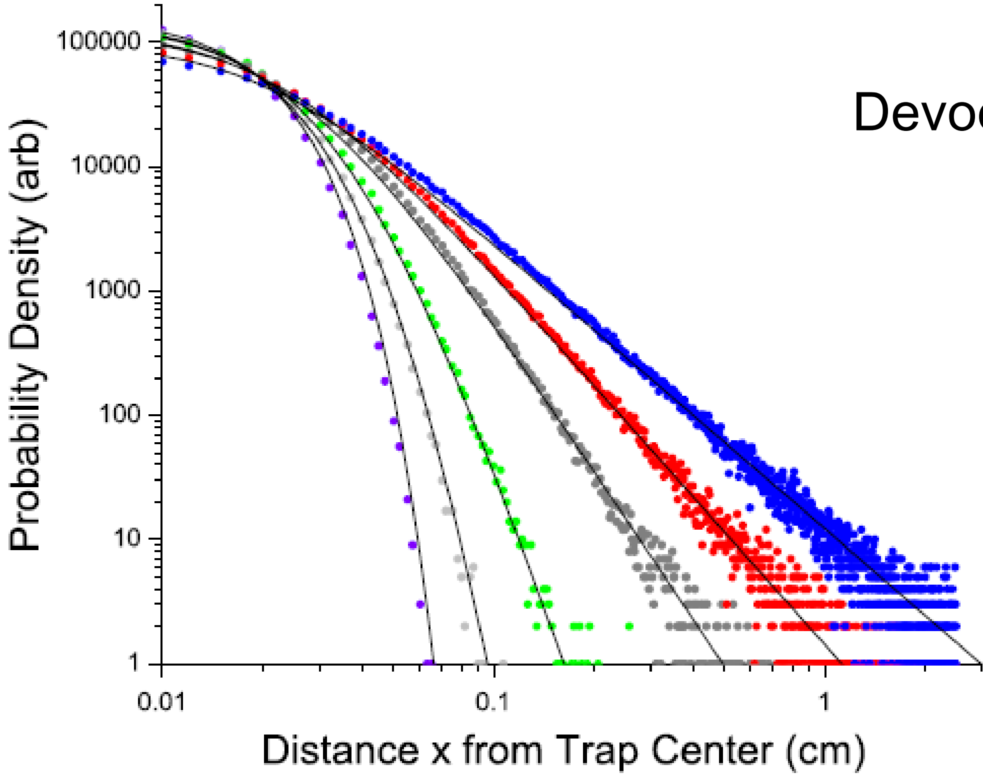
(Received 3 November 2008; published 10 February 2009)

Classical collisions with an ideal gas generate non-Maxwellian distribution functions for a single ion in a radio frequency ion trap. The distributions have power-law tails whose exponent depends on the ratio of buffer gas to ion mass. This provides a statistical explanation for the previously observed transition from cooling to heating. Monte Carlo results approximate a Tsallis distribution over a wide range of parameters and have *ab initio* agreement with experiment.



Devoe, Phys Rev Lett **102** (2009) 063001

$$T(x) = \frac{T(0)}{\left[1+(q-1)\left(\frac{x}{\sigma}\right)^2\right]^{\frac{1}{q-1}}}$$

FIG. 1 (color online). Monte Carlo distributions for a single $^{136}Ba^{+}$ ion cooled by six different buffer gases at 300 K ranging from $m_B = 4$ (left) to $m_B = 200$ (right). Note the evolution from Gaussian to power law (straight line) as the mass increases. The solid lines are Tsallis functions [Eq. (7)] with fixed $\sigma = 0.0185$ cm and the exponents of Table I.

TABLE I. Tsallis parameters n and q_T fit from Fig. 1.

Buffer gas	m_I/m_B	n	q_T
He	34.5	>60	1.03
Ar	3.40	8.2	1.12
Kr	1.70	3.8	1.26
Xe	1.0	1.98	1.51
170	0.80	1.50	1.80
200	0.68	1.15	1.87

SPIN RELAXATION IN SPIN GLASSES (NEUTRON SPIN ECHO):

PRL **102,** 097202 (2009) PHYSICAL REVIEW LETTERS week ending 6 MARCH 2009

Generalized Spin-Glass Relaxation

R. M. Pickup,[1] R. Cywinski,[2,*] C. Pappas,[3] B. Farago,[4] and P. Fouquet[4]

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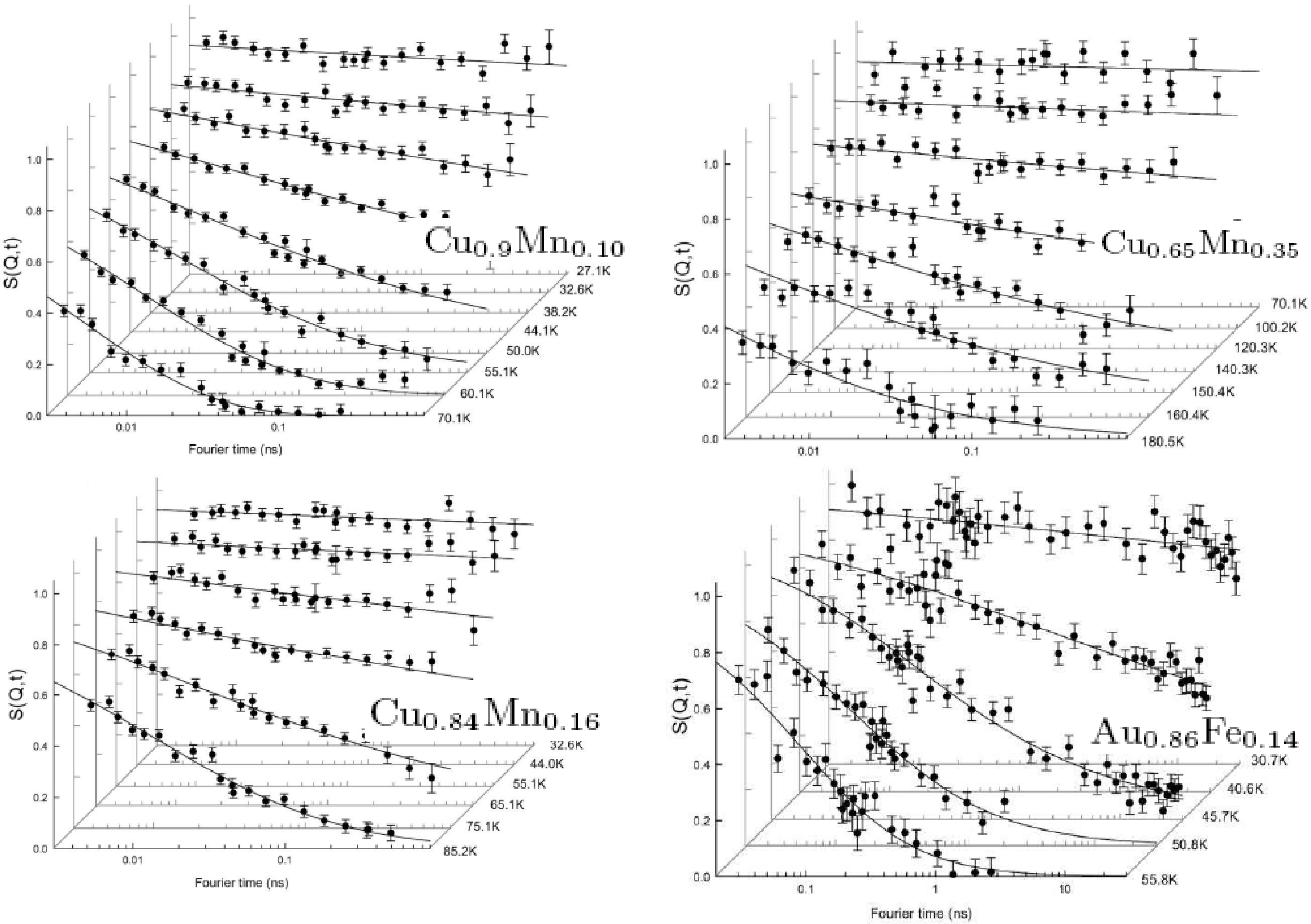
[2]*School of Applied Sciences, University of Huddersfield, Huddersfield HD1 3DH, United Kingdom*
[3]*Helmholtz Center Berlin for Materials and Energy, Glienickerstrasse 100, 14109, Berlin, Germany*
[4]*Institut Laue Langevin, 6 rue Jules Horowitz, 38000 Grenoble, France*
(Received 18 July 2008; published 4 March 2009)

Spin relaxation close to the glass temperature of CuMn and AuFe spin glasses is shown, by neutron spin echo, to follow a generalized exponential function which explicitly introduces hierarchically constrained dynamics and macroscopic interactions. The interaction parameter is directly related to the normalized Tsallis nonextensive entropy parameter q and exhibits universal scaling with reduced temperature. At the glass temperature $q = 5/3$ corresponding, within Tsallis' q statistics, to a mathematically defined critical value for the onset of strong disorder and nonlinear dynamics.



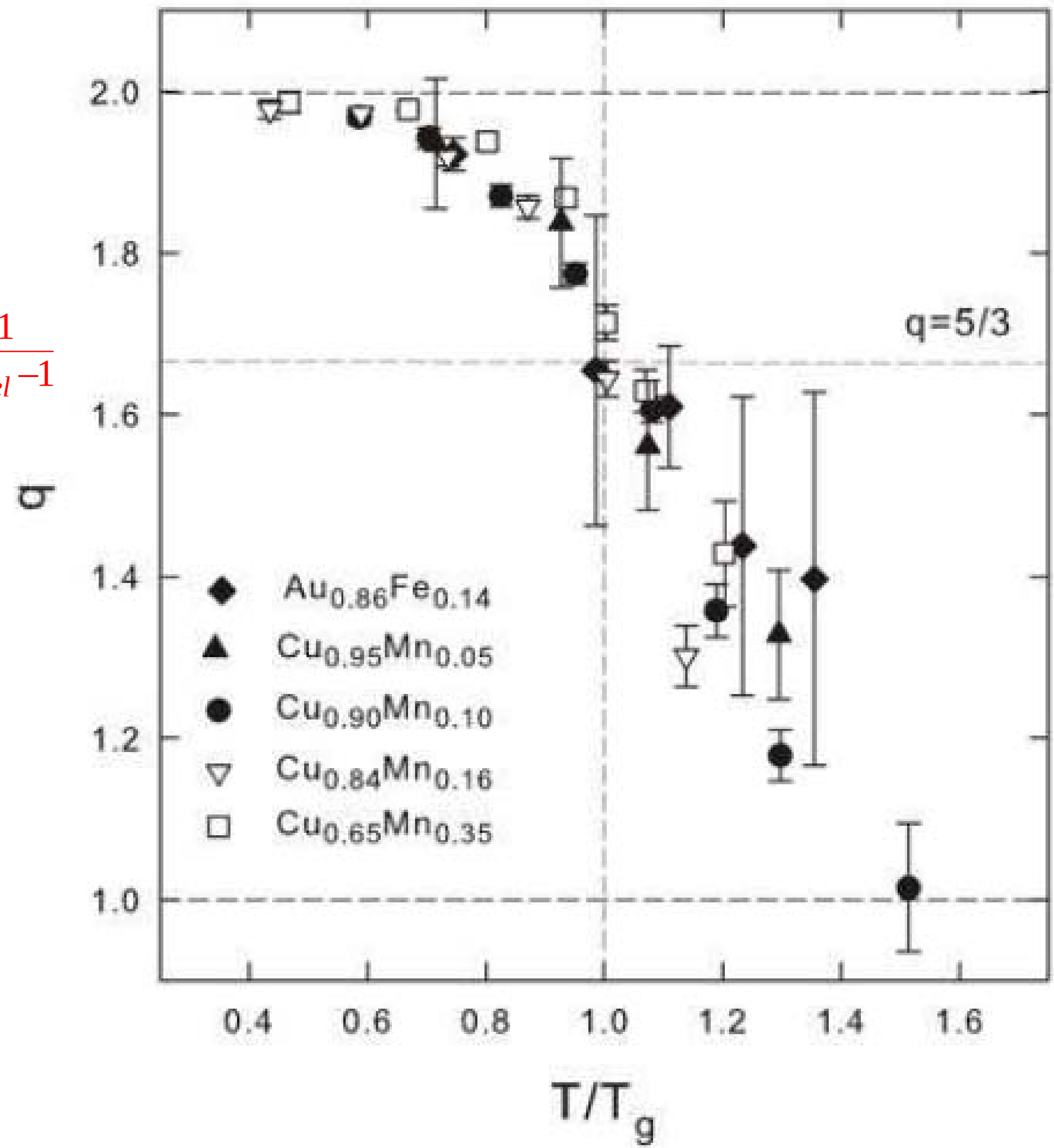
Pickup, Cywinski, Pappas, Farago, Fouquet, Phys Rev Lett **102** (2009) 097202

SPIN RELAXATION IN SPIN GLASSES (NEUTRON SPIN ECHO):

$$\phi(t) = \left[1 + \frac{q-1}{2-q}\left(\frac{t}{\tau}\right)^{\beta}\right]^{-\frac{2-q}{q-1}}$$

$$\equiv \left[1 + (q_{rel}-1)\left(\frac{t}{\tau}\right)^{\beta}\right]^{-\frac{1}{q_{rel}-1}}$$

$$q_{rel} \equiv \frac{1}{2-q}$$



Pickup, Cywinski, Pappas, Farago, Fouquet, Phys Rev Lett **102** (2009) 097202

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Physica A 356 (2005) 375–384
PHYSICA A
www.elsevier.com/locate/physa

Triangle for the entropic index *q* of non-extensive statistical mechanics observed by Voyager 1 in the distant heliosphere

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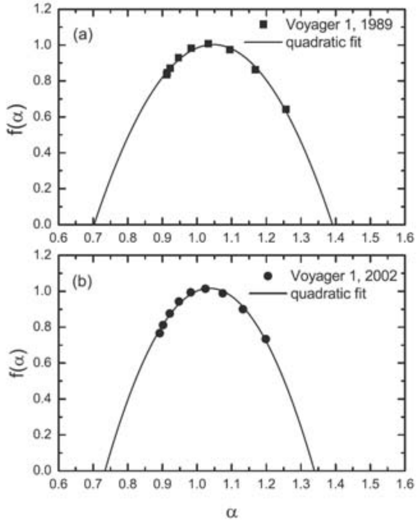
Received 10 June 2005
Available online 11 July 2005

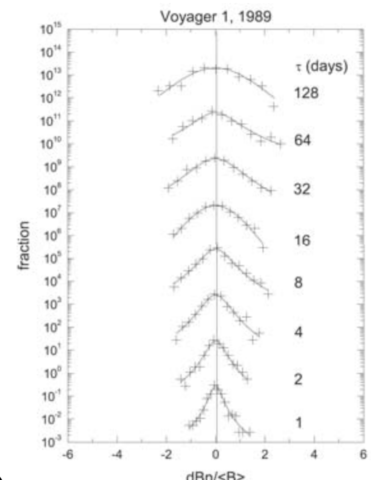
SOLAR WIND: Magnetic Field Strength

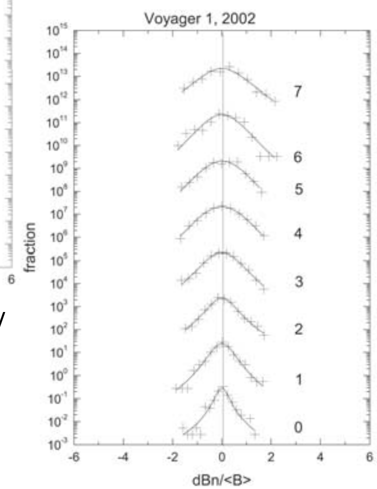
L.F. Burlaga and A. F.-Vinas (2005) / NASA Goddard Space Flight Center; Physica A **356**, 375 (2005)

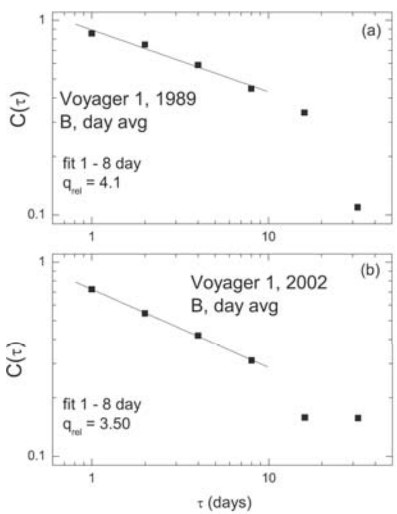
[Data: Voyager 1 spacecraft (1989 and 2002); 40 and 85 AU; daily averages]



$q_{sen} = -0.6 \pm 0.2$







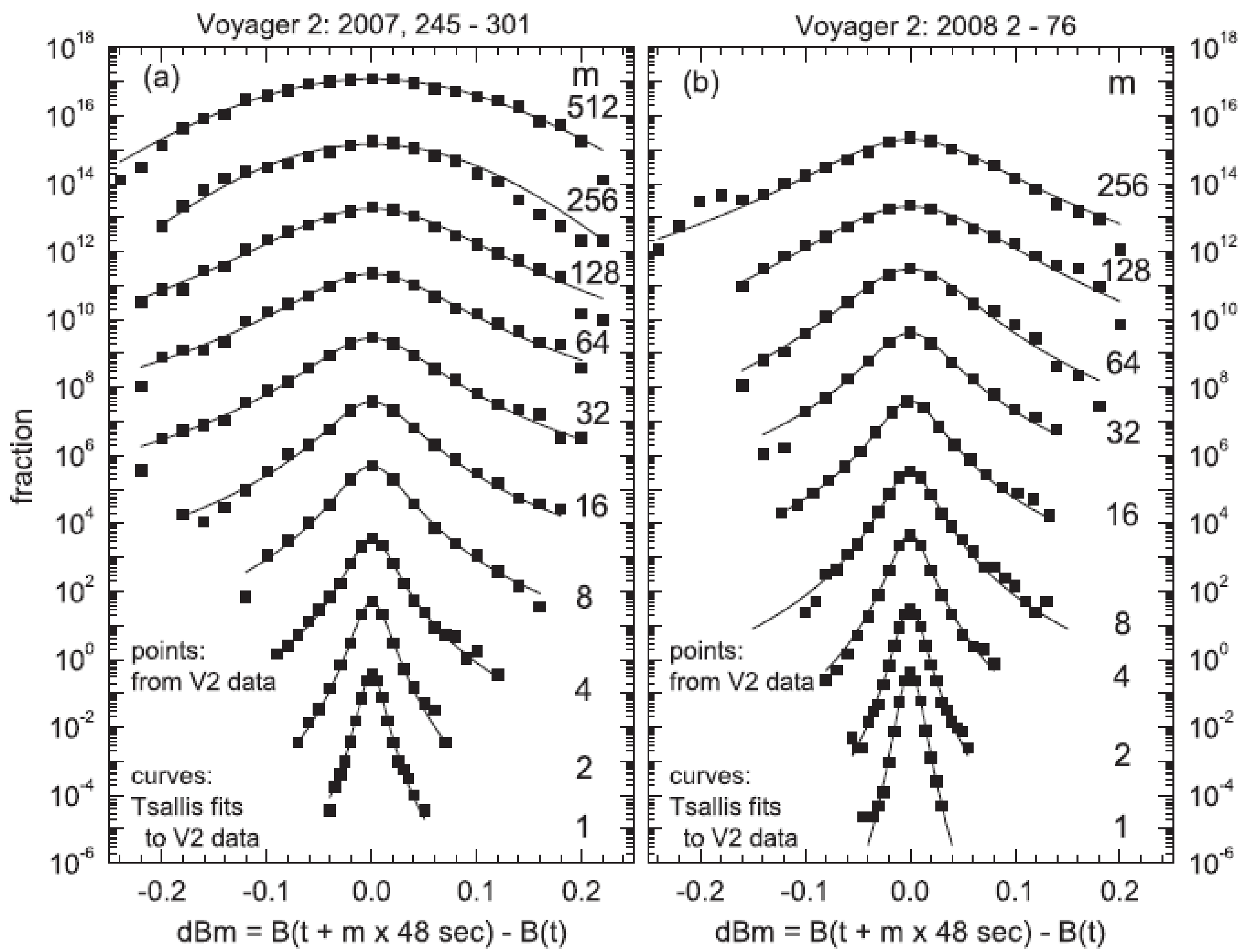
$q_{rel} = 3.8 \pm 0.3$

$q_{stat} = 1.75 \pm 0.06$

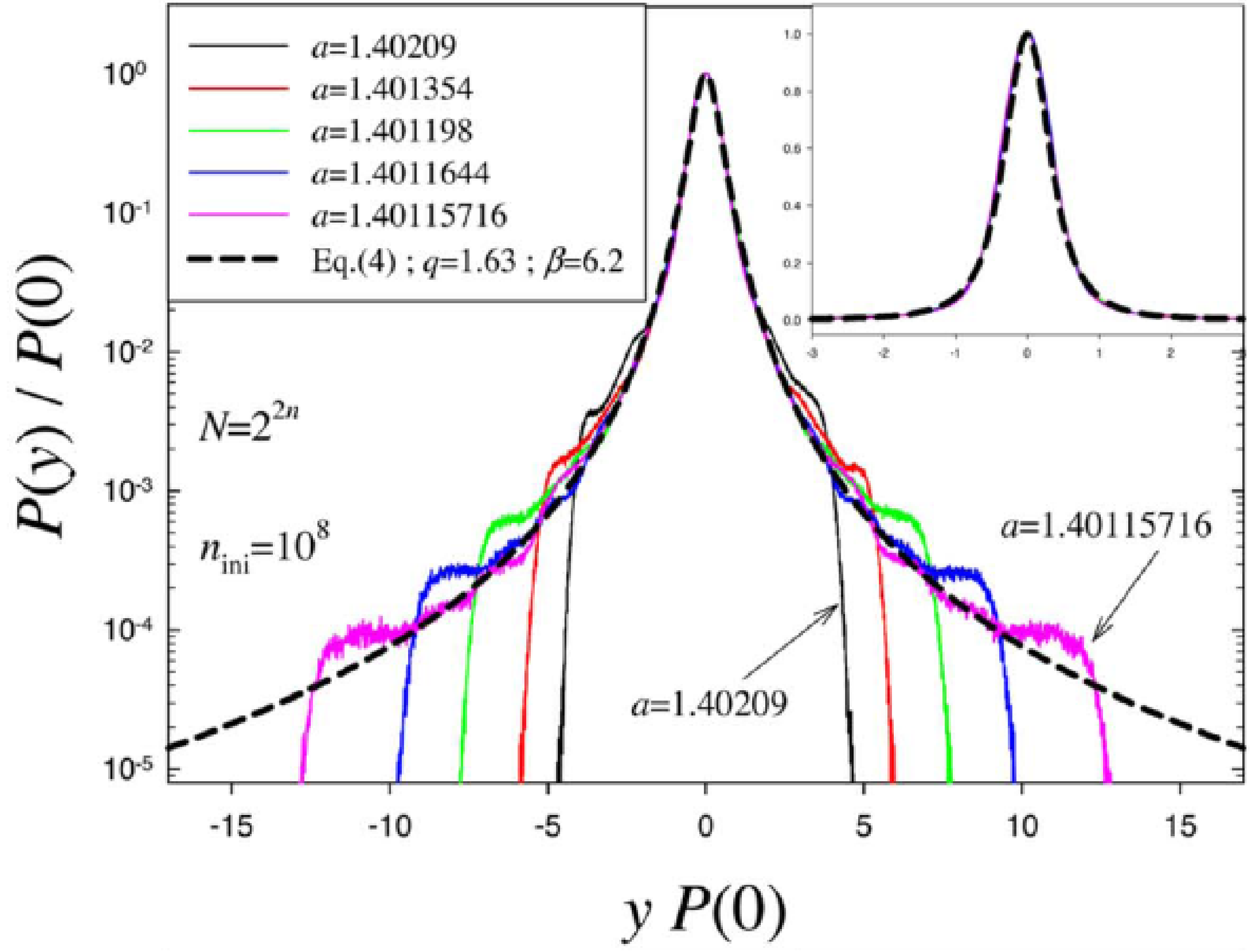
The Astrophysical Journal, 703:311–324, 2009 September 20 doi:10.1088/0004-637X/703

COMPRESSIBLE "TURBULENCE" OBSERVED IN THE HELIOSHEATH BY *VOYAGER 2*

L. F. Burlaga[1] and N. F. Ness[2]
[1] Geospace Physics Laboratory, Code 673, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA; Leonard.F.Burlaga@NASA.gov
[2] Institute for Astrophysics and Computational Sciences, Catholic University of America, Washington DC 20064, USA; nfnudel@yahoo.com
Received 2009 June 2; accepted 2009 July 22; published 2009 August 27



LOGISTIC MAP AT THE EDGE OF CHAOS:

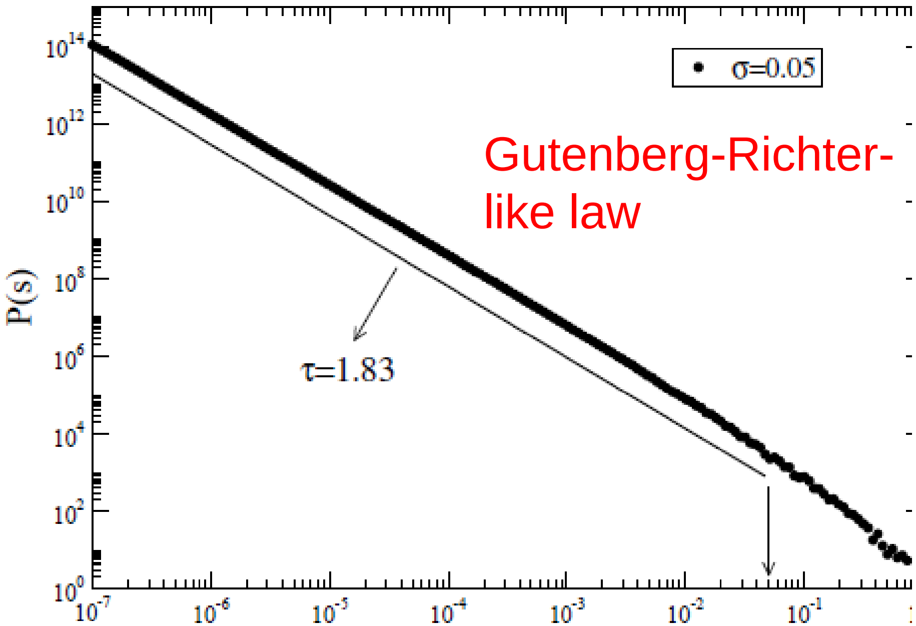


$q = 1.63$

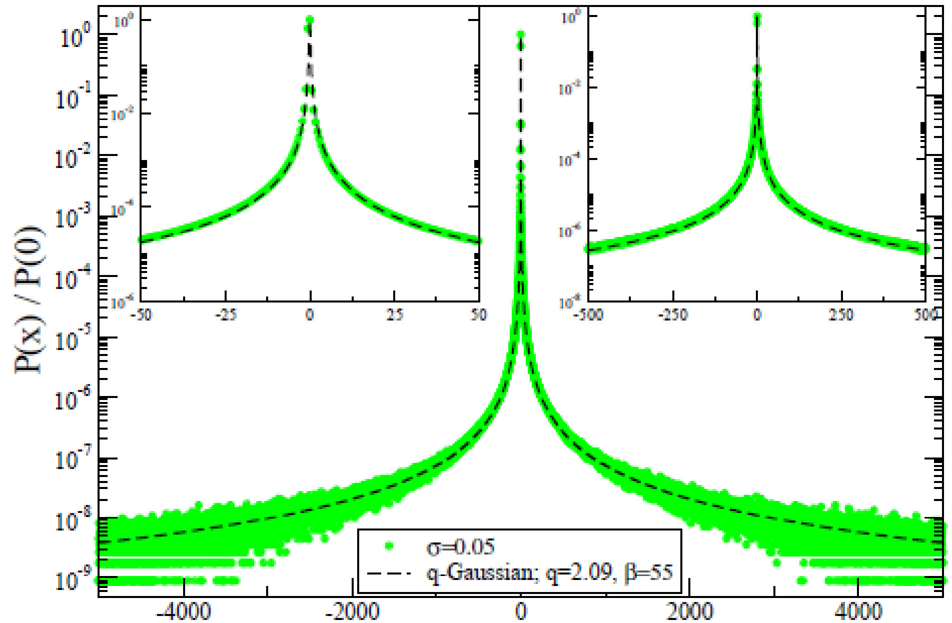
$\beta = 6.2$

U. Tirnakli, C. T. and C. Beck Phys Rev E **79** (2009) 056209

Analysis of return distributions in coherent noise model



avalanche size



avalanche return

$$q = \frac{2+\tau}{\tau} = 1 + \frac{2}{\tau} \equiv 1 + 2(q_{size} - 1) \quad \text{hence} \quad q - 1 = 2(q_{size} - 1)$$

A. Celikoglu, U. Tirnakli and S.M.D. Queiros, Phys Rev E (2010), in press

Brazilian Journal of Medical and Biological Research (2010) 43: 77-84
ISSN 0100-879X

Brain tissue segmentation using q-entropy in multiple sclerosis magnetic resonance images

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[3]Departamento de Física e Matemática, Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto, Universidade de São Paulo, Ribeirão Preto, SP, Brasil

Abstract

The loss of brain volume has been used as a marker of tissue destruction and can be used as an index of the progression of neurodegenerative diseases, such as multiple sclerosis. In the present study, we tested a new method for tissue segmentation based on pixel intensity threshold using generalized Tsallis entropy to determine a statistical segmentation parameter for each single class of brain tissue. We compared the performance of this method using a range of different q parameters and found a different optimal q parameter for white matter, gray matter, and cerebrospinal fluid. Our results support the conclusion that the differences in structural correlations and scale invariant similarities present in each tissue class can be accessed by generalized Tsallis entropy, obtaining the intensity limits for these tissue class separations. In order to test this method, we used it for analysis of brain magnetic resonance images of 43 patients and 10 healthy controls matched for gender and age. The values found for the entropic q index were 0.2 for cerebrospinal fluid, 0.1 for white matter and 1.5 for gray matter. With this algorithm, we could detect an annual loss of 0.98% for the patients, in agreement with literature data. Thus, we can conclude that the entropy of Tsallis adds advantages to the process of automatic target segmentation of tissue classes, which had not been demonstrated previously.

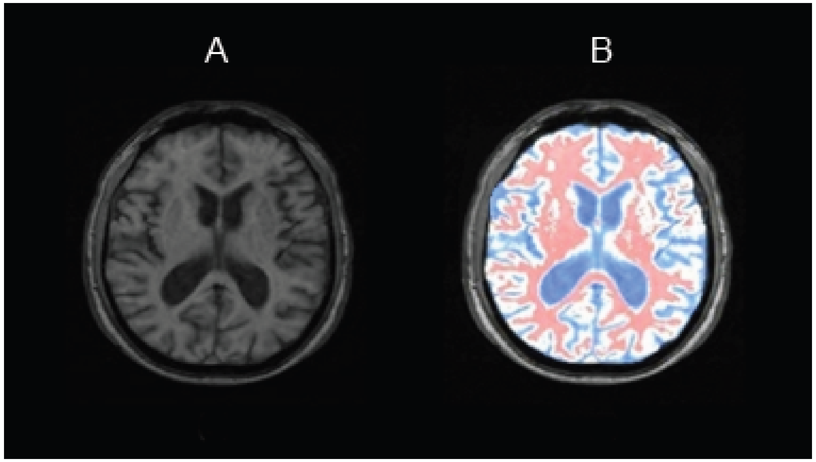

Figure 3. Maximum entropy segmentation example. *A*, Original image; *B*, image with the segmentation masks. Blue indicates cerebrospinal fluid, white indicates the gray matter, and red indicates the white matter.

The ideal q values for the segmentation of the classes are: CSF = 0.2, WM = 0.1, GM = 1.5, which have not been shown previously.

These characteristics allow its application to clinical routine.

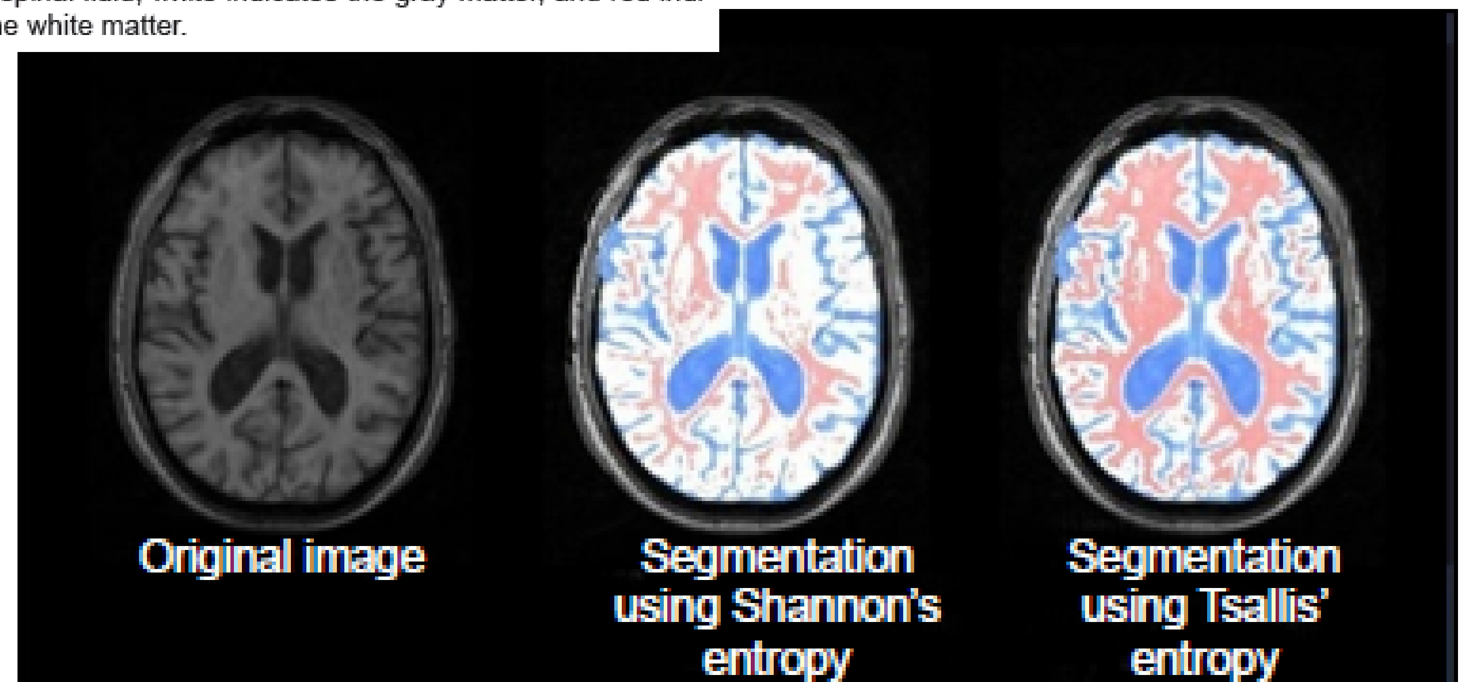

Figure 6. Segmentation using Shannon and Tsallis entropies.

PRB Diniz, LO Murta Jr., DG Brum, DB de Araujo and AC Santos
Braz. J. Med. Bio. Res. **43**, 77 (2010)

Proceedings of the 2009 IEEE
International Conference on Mechatronics and Automation
August 9 - 12, Changchun, China

Research of Automatic Medical Image Segmentation Algorithm Based on Tsallis Entropy and Improved PCNN

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Shiwl@cust.edu.cn,custmiao@126.com,
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Zhang Hongbiao

QIMING INFORMATION TECHNOLOGY CO.LTD
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Gpszhb_qm@faw.com.cn

Abstract - It needs set parameters on image segmentation based on PCNN (Pulse Coupled Neural Network) now. This paper points out the new method for medical image segmentation based on improved PCNN and Tsallis entropy. The new methods can automatically segment the medical images without selecting the PCNN parameters. It gets the best results with combining with the Tsallis entropy. The new method is very useful for PCNN application in the medical images segmentation.

(MRI)

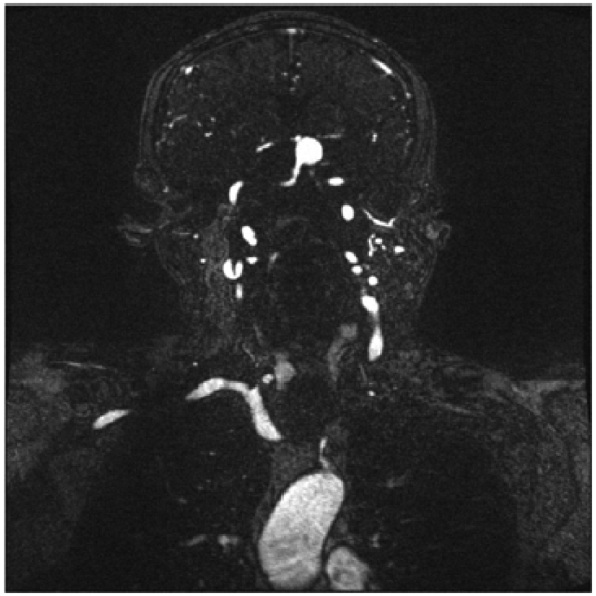

Fig.3 the human head blood vessel

(PCNN)

Fig.5 the human head vessel reconstruction result

(PCNN and q=0.8)

Fig.6 the segmentation result of human head blood vessel

(CT X - ray)

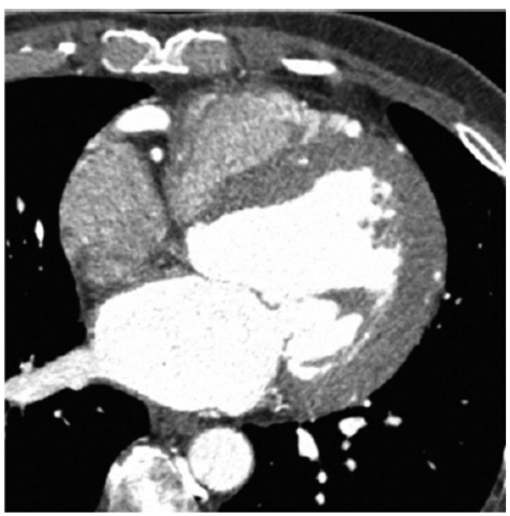

Fig.4 the human bosom

Fig.7 the human bosom reconstruction result

Fig.8 the segmentation result of bosom reconstruction

ELSEVIER

29 February 1996

PHYSICS LETTERS B

Physics Letters B 369 (1996) 308–312

Generalized statistics and solar neutrinos

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Istituto Nazionale di Fisica Nucleare, Sezioni di Cagliari e di Torino, Italy

Received 7 November 1995
Editor: R. Gatto

Abstract

The generalized Tsallis statistics produces a distribution function appropriate to describe the interior solar plasma, thought as a stellar polytrope, showing a tail depleted with respect to the Maxwell-Boltzmann distribution and reduces to zero at energies greater than about $20k_BT$. The Tsallis statistics can theoretically support the distribution suggested in the past by Clayton and collaborators, which shows also a depleted tail, to explain the solar neutrino counting rate.

HADRONIC JETS FROM ELECTRON-POSITRON ANNIHILATION:

I. Bediaga, E.M.F. Curado and J.M. de Miranda, Physica A **286** (2000) 156

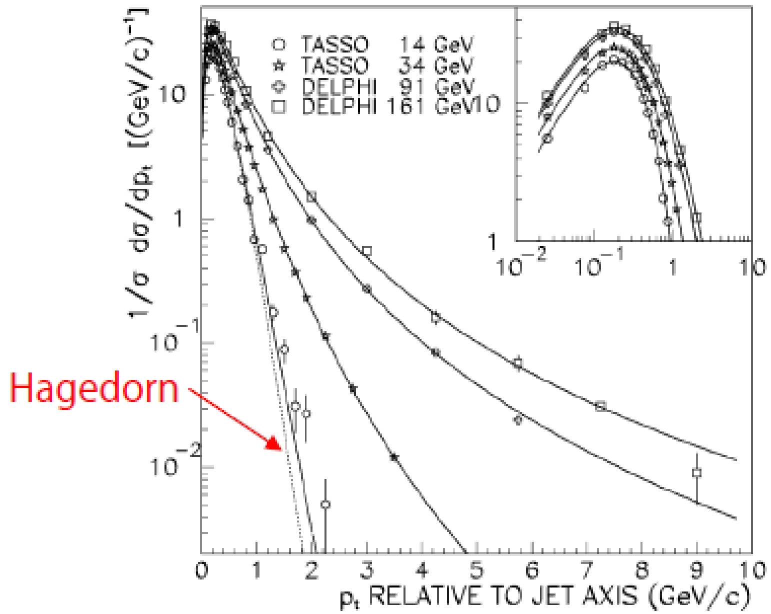
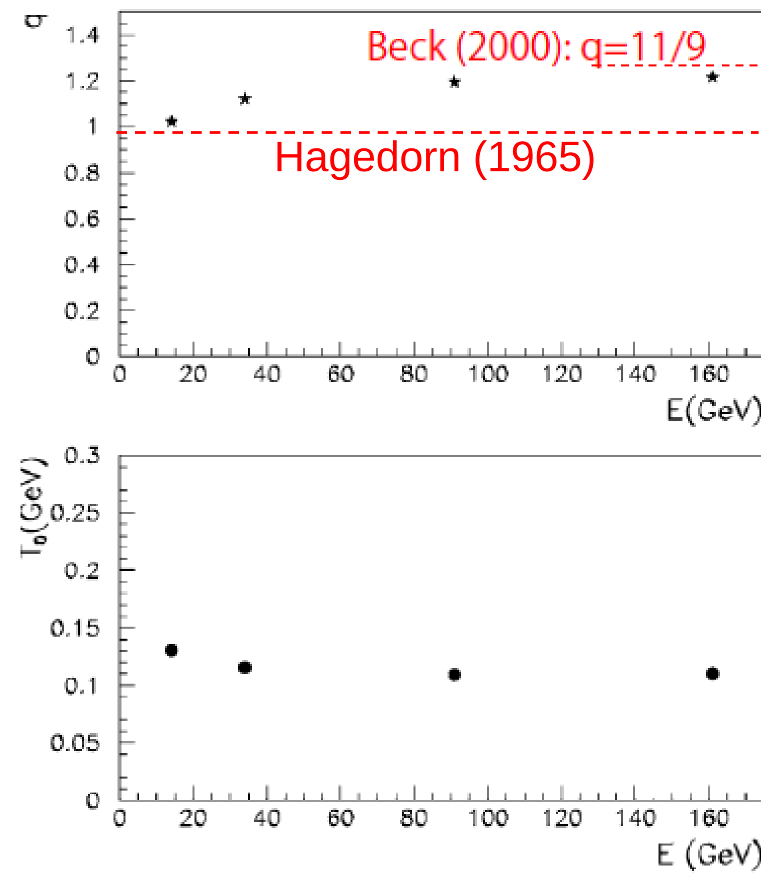


Fig. 1. Transverse momentum distribution. The distribution $(1/\sigma)\,d\sigma/dp_t$ of the transverse momentum p_t of charged hadrons with respect to jet axis (defined in these experimental results as the sphericity axis) is sketched for four different experiments, whose center-of-mass energies vary from 14 and 34 GeV (TASSO) up to 91 and 161 GeV (DELPHI). The Hagedorn predicted exponential behavior is shown by the dotted line. We can see that the deviation of the exponential behavior increases when the energy increases. The continuous lines are obtained from our Eq. (3) and agree very well with the experimental data. The inset shows the transverse momentum distribution for small values of p_t.



Eur. Phys. J. C 12, 499–506 (2000)
Digital Object Identifier (DOI) 10.1007/s100529900220

THE EUROPEAN
PHYSICAL JOURNAL C

Non-extensive statistics, fluctuations and correlations in high-energy nuclear collisions

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[2] Dipartimento di Fisica, Politecnico di Torino, C.so Duca degli Abruzzi 24, 10129 Torino, Italy
[3] Istituto Nazionale di Fisica Nucleare, Sezione di Torino
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Received: 6 June 1999 / Published online: 21 December 1999

Abstract. Starting from the experimental evidence that high-energy nucleus–nucleus collisions cannot be described in terms of superpositions of elementary nucleon–nucleon interactions, we analyze the possibility that memory effects and long-range forces imply a non-extensive statistical regime during high-energy heavy-ion collisions. The relevance of these statistical effects and their compatibility with the available experimental data are discussed. In particular, we show that theoretical estimates obtained in the framework of the generalized non-extensive thermostatistics, can reproduce the shape of the pion transverse mass spectrum and explain the different physical origin of the transverse momentum correlation function of the pions emitted during the central Pb + Pb and during the $p + p$ collisions at 158 GeV.

q = 1.038

Eur. Phys. J. A 40, 299–312 (2009)
DOI 10.1140/epja/i2009-10803-9

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Power laws in elementary and heavy-ion collisions*

A story of fluctuations and nonextensivity?

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[2] Institute of Physics, Jan Kochanowski University, Świętokrzyska 15, 25-406 Kielce, Poland

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Published online: 23 May 2009 –
Communicated by U.-G. Meißner

Abstract. We review from the point of view of nonextensive statistics the ubiquitous presence in elementary and heavy-ion collisions of power law distributions. Special emphasis is placed on the conjecture that this is just a reflection of some intrinsic fluctuations existing in the hadronic systems considered. These systems are summarily described by a single parameter q playing the role of a nonextensivity measure in the nonextensive statistical models based on Tsallis entropy.

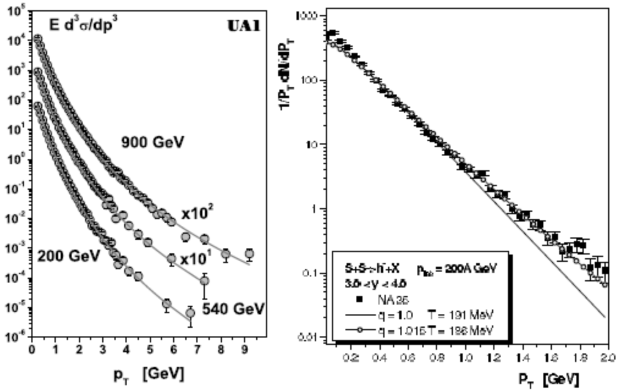

Fig. 5. Examples of applying a nonextensive approach to transverse momenta distributions. Left panel: fits to p_T spectra from the $p\bar{p}$ UA1 experiment [46] for different energies (see text for details) (reprinted from F.S. Navarra, O.V. Utyuzh, G. Wilk, Z. Włodarczyk, *Information theory approach (extensive and nonextensive) to high-energy multiparticle production processes*, Physica A **340**, 467 (2004), with kind permission of Elsevier, `http://www.elsevier.com`.). Right panel: fits to $S+S$ data from [47] (reproduced with kind permission of IOP Publishing Ltd from [15]).

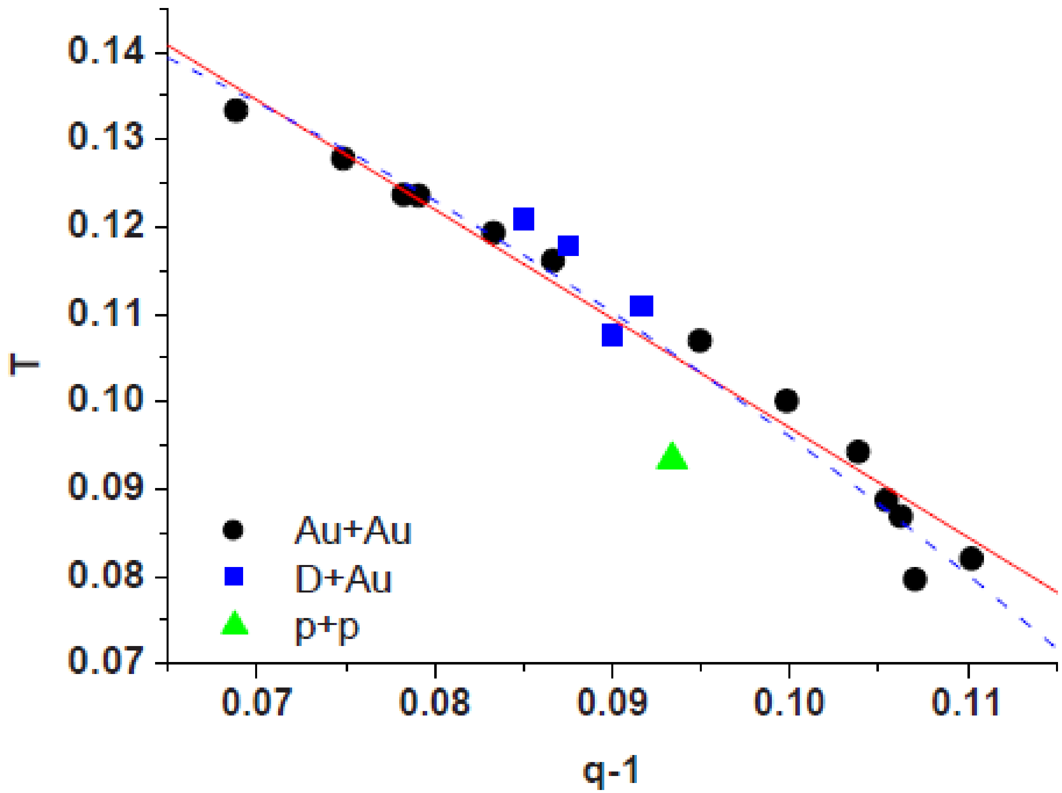

Fig. 7. Dependence of the temperature T (in GeV) on the parameter q for production of negative pions in different reactions. The solid line shows a linear fit to the obtained results: $T = 0.22$–$1.25(q-1)$ (cf. eq. (41)) and the dashed line shows the corresponding quadratic fit: $T = 0.17$–$7.5(q-1)^2$ (cf. eq. (42)).

Eur. Phys. J. A **40**, 325–340 (2009)
DOI 10.1140/epja/i2009-10806-6

THE EUROPEAN
PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Non-extensive approach to quark matter*

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Received: 31 January 2009
Published online: 27 May 2009 –
Communicated by U.-G. Meißner

Abstract. We review the idea of generating non-extensive stationary distributions based on abstract composition rules of the subsystem energies, in particular the parton cascade method, using a Boltzmann equation with relativistic kinematics and modified two-body energy composition rules. The thermodynamical behavior of such model systems is investigated. As an application hadronic spectra with power law tails are analyzed in the framework of a quark coalescence model.

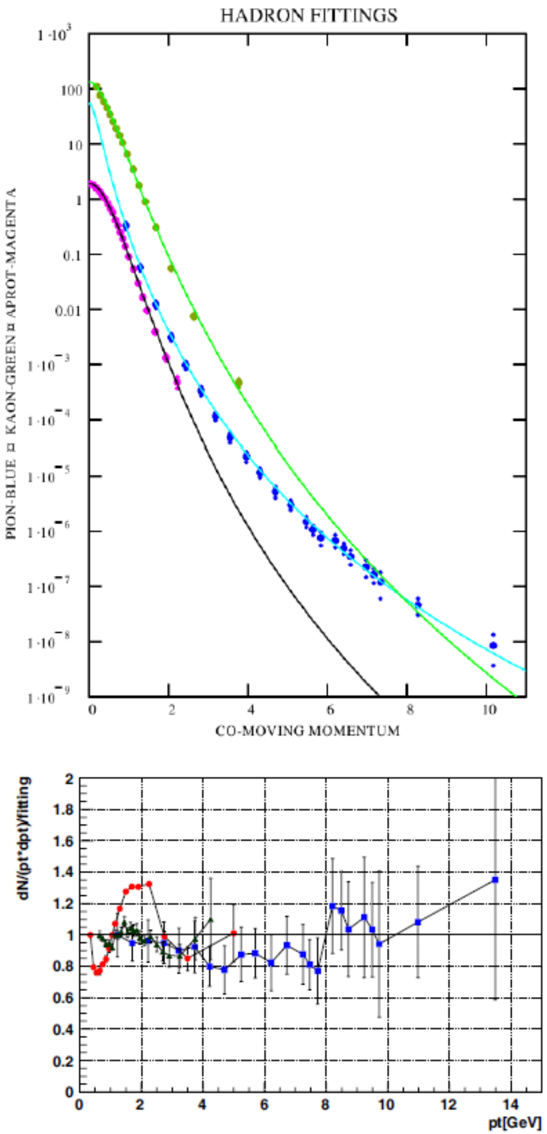

Fig. 8. General shape of p_T spectra for pions, kaons and antiprotons in relativistic heavy-ion experiments (upper panel). A fit is done by using for $X(E)$ the Tsallis-Pareto form with parameters T and a, corresponding to a common temperature of $T(m_i) = 0.160\,\mathrm{MeV}$ for the different particles and a transverse flow velocity $v_T = 0.52$. In the lower panel the ratio of the Tsallis fit to the experimental values can be inspected in a linear plot.

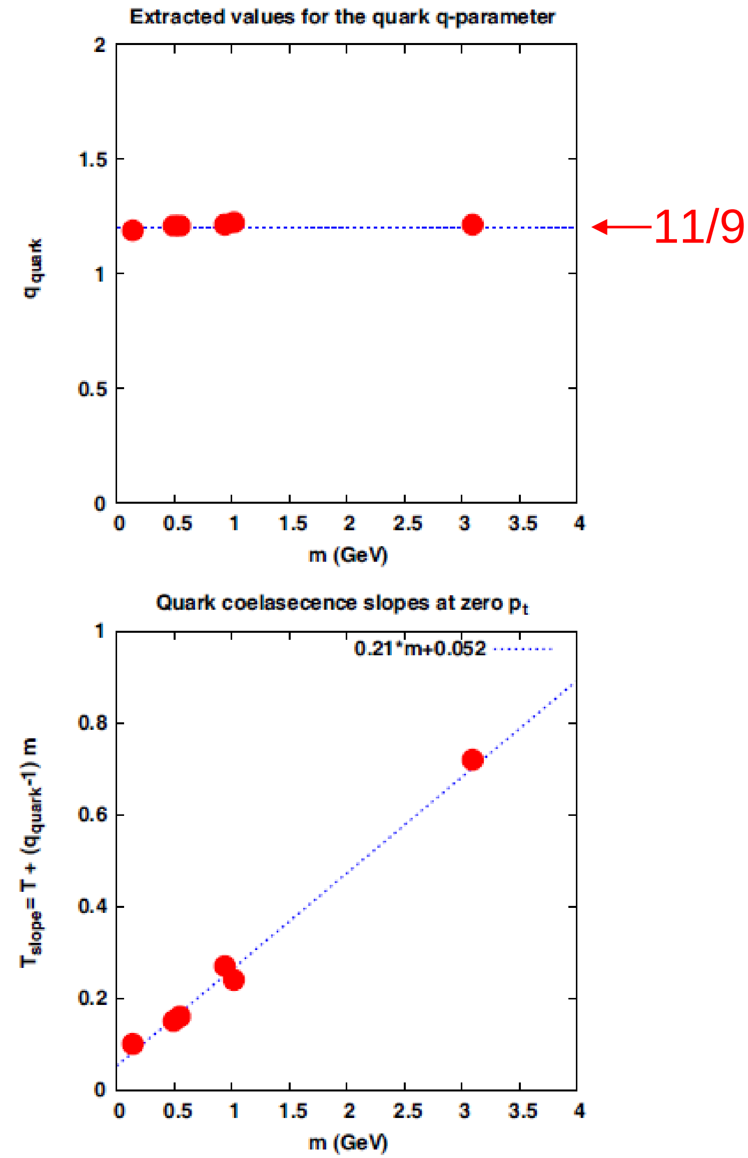

Fig. 10. The q-parameter of quark matter extracted from hadronic spectra assuming quark coalescence at a sudden hadron formation (upper panel). The spectral inverse slope as a function of the minimal energy $E_{\min} = m$ agree with the linear prediction from the coalescence scaling.

Published for SISSA by Springer

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Accepted: February 7, 2010
Published: February 10, 2010

Transverse-momentum and pseudorapidity distributions of charged hadrons in pp collisions at $\sqrt{s}$ = 0.9 and 2.36 TeV

CMS Collaboration

Abstract: Measurements of inclusive charged-hadron transverse-momentum and pseudorapidity distributions are presented for proton-proton collisions at $\sqrt{s} = 0.9$ and 2.36 TeV. The data were collected with the CMS detector during the LHC commissioning in December 2009. For non-single-diffractive interactions, the average charged-hadron transverse momentum is measured to be 0.46 ± 0.01 (stat.) $\pm$ 0.01 (syst.) GeV/c at 0.9 TeV and 0.50 ± 0.01 (stat.) $\pm$ 0.01 (syst.) GeV/c at 2.36 TeV, for pseudorapidities between -2.4 and $+2.4$. At these energies, the measured pseudorapidity densities in the central region, $dN_{\mathrm{ch}}/d\eta|_{|\eta|<0.5}$, are 3.48 ± 0.02 (stat.) $\pm$ 0.13 (syst.) and 4.47 ± 0.04 (stat.) $\pm$ 0.16 (syst.), respectively. The results at 0.9 TeV are in agreement with previous measurements and confirm the expectation of near equal hadron production in p$\bar{\text{p}}$ and pp collisions. The results at 2.36 TeV represent the highest-energy measurements at a particle collider to date.

JHEP02(2010)041

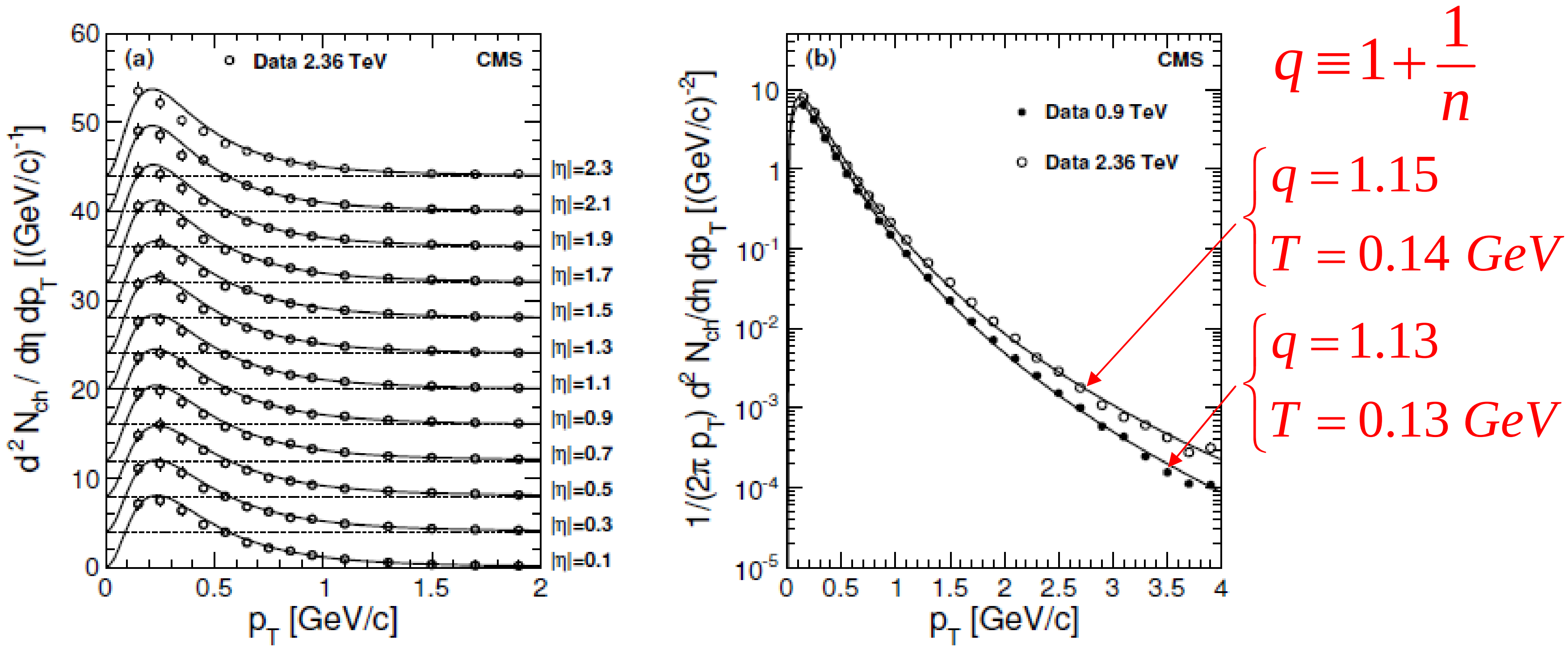


Figure 5. (a) Measured differential yield of charged hadrons in the range $|\eta| < 2.4$ in 0.2-unit-wide bins of $|\eta|$ for the 2.36 TeV data. The measured values with systematic uncertainties (symbols) and the fit functions (eq. (5.1)) are displayed. The values with increasing η are successively shifted by four units along the vertical axis. (b) Measured yield of charged hadrons for $|\eta| < 2.4$ with systematic uncertainties (symbols), fit with the empirical function (eq. (5.1)).

The yields were fit by the Tsallis function (eq. (5.1)), which empirically describes both the low-p_T exponential and the high-p_T power-law behaviours [20, 21]:

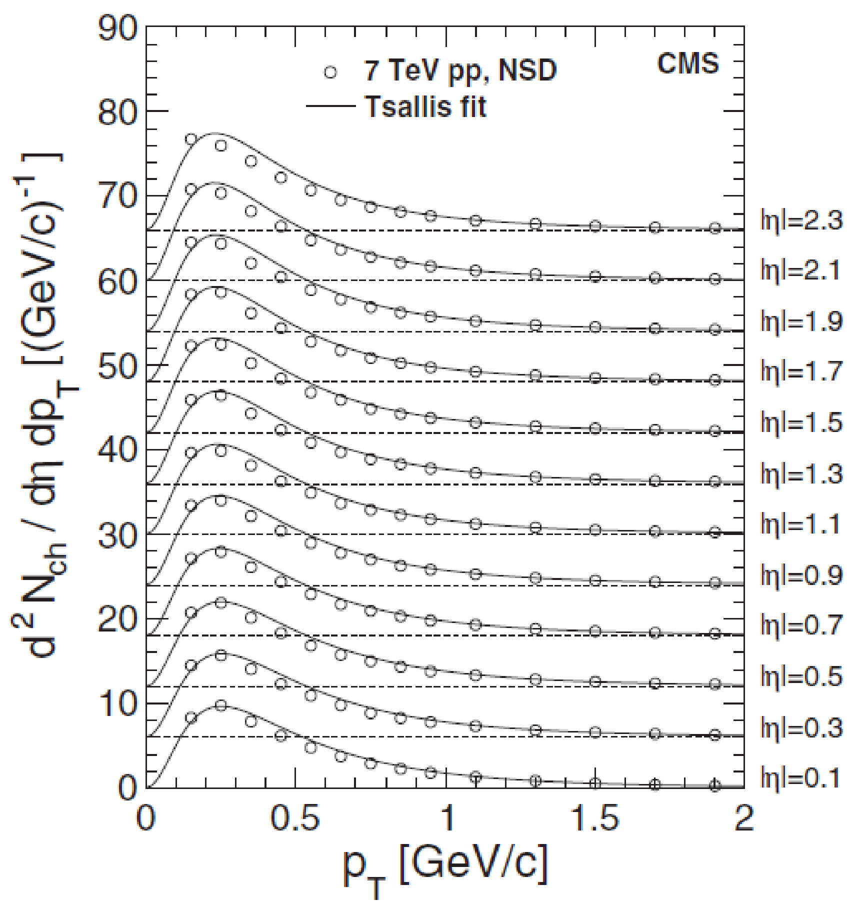
$$E\frac{d^3N_{\mathrm{ch}}}{dp^3} = \frac{1}{2\pi p_T}\frac{E}{p}\frac{d^2N_{\mathrm{ch}}}{d\eta dp_T} = C(n,T,m)\frac{dN_{\mathrm{ch}}}{dy}\left(1+\frac{E_T}{nT}\right)^{-n}, \tag{5.1}$$

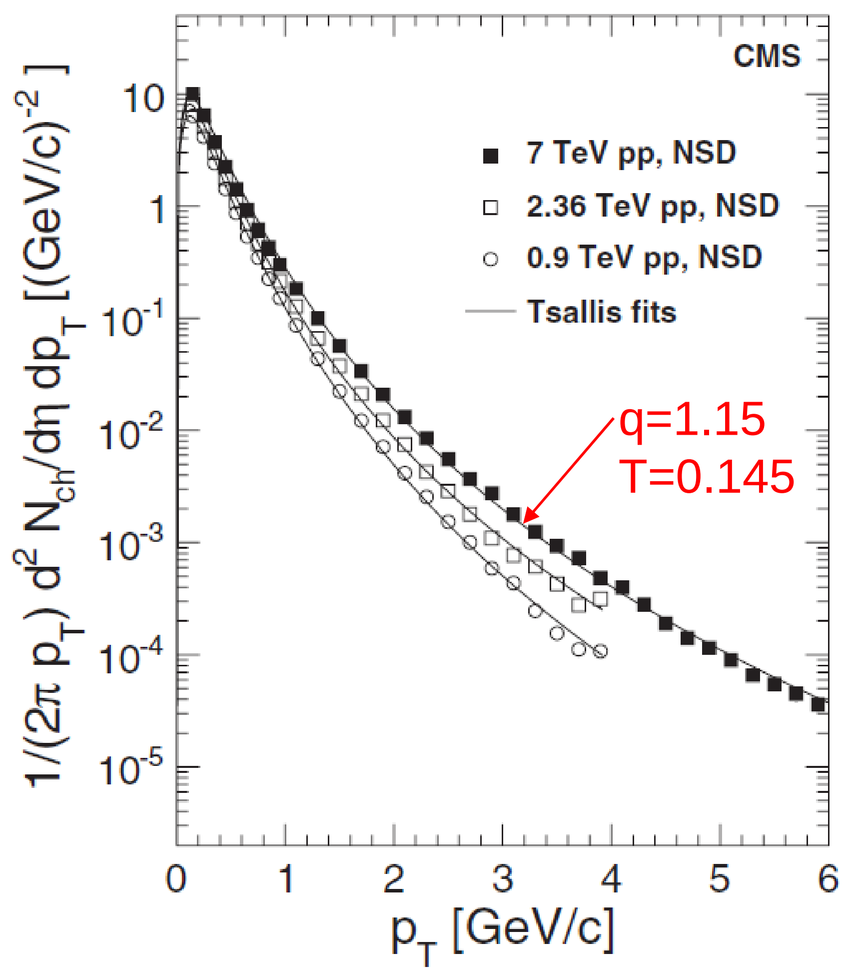
PRL **105,** 022002 (2010) PHYSICAL REVIEW LETTERS week ending 9 JULY 2010

Transverse-Momentum and Pseudorapidity Distributions of Charged Hadrons in pp Collisions at $\sqrt{s} = 7$ TeV

V. Khachatryan *et al.**
(CMS Collaboration)

(Received 18 May 2010; published 6 July 2010)



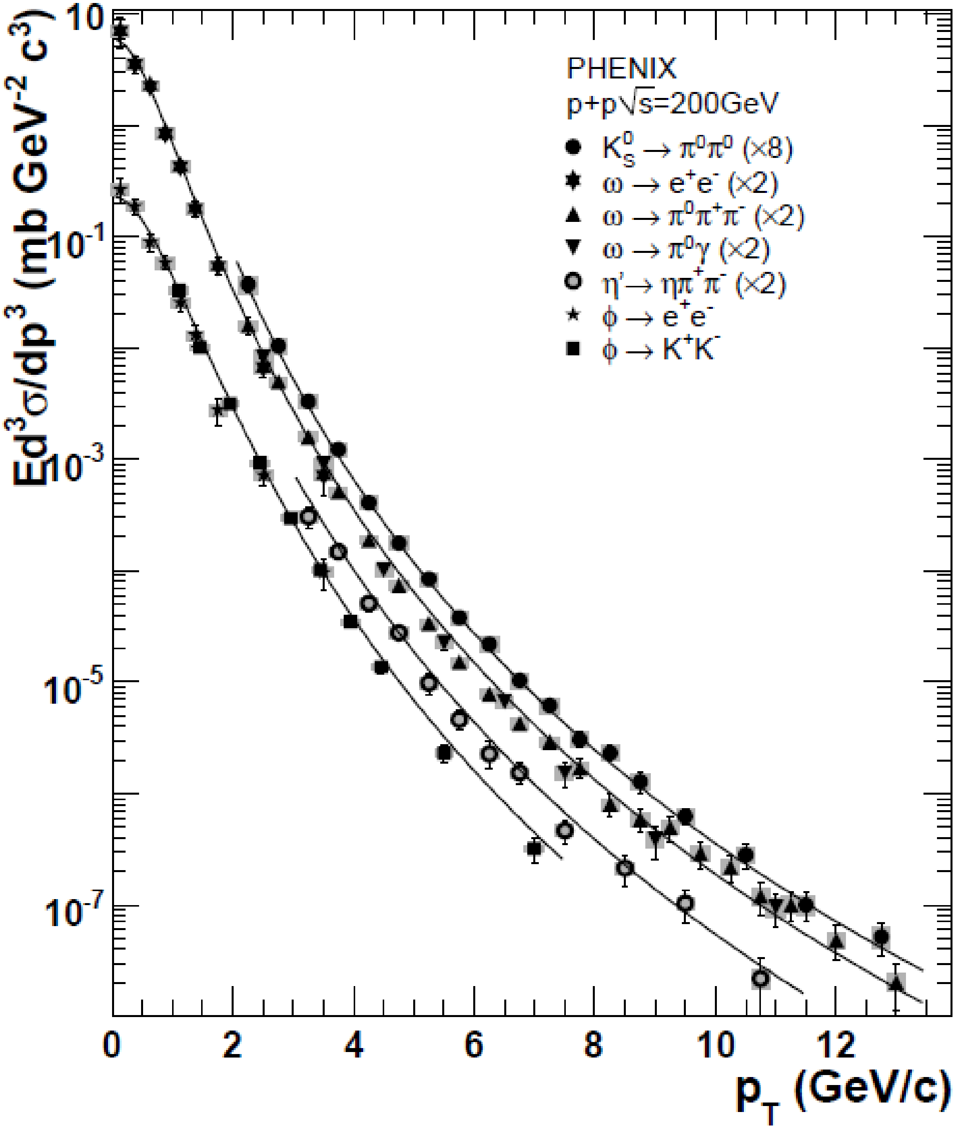


PHENIX @ RHIC

Measurement of neutral mesons in $p+p$ collisions at $\sqrt{s} = 200$ GeV and scaling properties of hadron production

A. Adare,[11] S. Afanasiev,[25] C. Aidala,[12,36] N.N. Ajitanand,[53] Y. Akiba,[47,48] H. Al-Bataineh,[42] J. Alexander,[53] K. Aoki,[30,47] L. Aphecetche,[55] R. Armendariz,[42] S.H. Aronson,[6] J. Asai,[47,48] E.T. Atomssa,[31] R. Averbeck,[54] T.C. Awes,[43] B. Azmoun,[6] V. Babintsev,[21] M. Bai,[5] G. Baksay,[17] L. Baksay,[17] A. Baldisseri,[14] K.N. Barish,[7] P.D. Barnes,[33] B. Bassalleck,[41] A.T. Basye,[1] S. Bathe,[7] S. Batsouli,[43] V. Baublis,[46] C. Baumann,[37] A. Bazilevsky,[6] S. Belikov,[6,*] R. Bennett,[54] A. Berdnikov,[50] Y. Berdnikov,[50] A.A. Bickley,[11] J.G. Boissevain,[33] H. Borel,[14] K. Boyle,[54] M.L. Brooks,[33] H. Buesching,[6] V. Bumazhnov,[21] G. Bunce,[6,48] S. Butsyk,[33,54] C.M. Camacho,[33] S. Campbell,[54] B.S. Chang,[62] W.C. Chang,[2] J.-L. Charvet,[14] S. Chernichenko,[21] J. Chiba,[26] C.Y. Chi,[12] M. Chiu,[22] I.J. Choi,[62] R.K. Choudhury,[4] T. Chujo,[58,59] P. Chung,[53] A. Churyn,[21] V. Cianciolo,[43] Z. Citron,[54] C.R. Cleven,[19] B.A. Cole,[12] M.P. Comets,[44] P. Constantin,[33] M. Csanád,[16] T. Csörgő,[27] T. Dahms,[54] S. Dairaku,[30,47] K. Das,[18] G. David,[6] M.B. Deaton,[1] K. Dehmelt,[17] H. Delagrange,[55] A. Denisov,[21] D. d'Enterria,[12,31] A. Deshpande,[48,54] E.J. Desmond,[6] O. Dietzsch,[51] A. Dion,[54] M. Donadelli,[51] O. Drapier,[31] A. Drees,[54] K.A. Drees,[5] A.K. Dubey,[61] A. Durum,[21] D. Dutta,[4] V. Dzhordzhadze,[7] Y.V. Efremenko,[43] J. Egdemir,[54] F. Ellinghaus,[11] W.S. Emam,[7] T. Engelmore,[12] A. Enokizono,[32] H. En'yo,[47,48] S. Esumi,[58] K.O. Eyser,[7] B. Fadem,[38] D.E. Fields,[41,48] M. Finger, Jr.,[8,25] M. Finger,[8,25] F. Fleuret,[31] S.L. Fokin,[29] Z. Fraenkel,[61,*] J.E. Frantz,[54] A. Franz,[6] A.D. Frawley,[18] K. Fujiwara,[47] Y. Fukao,[30,47] T. Fusayasu,[40] S. Gadrat,[34] I. Garishvili,[56] A. Glenn,[11] H. Gong,[54] M. Gonin,[31] J. Gosset,[14] Y. Goto,[47,48] R. Granier de Cassagnac,[31] N. Grau,[12,24] S.V. Greene,[59] M. Grosse Perdekamp,[22,48] T. Gunji,[10] H.-Å. Gustafsson,[35,*] T. Hachiya,[20] A. Hadj Henni,[55] C. Haegemann,[41] J.S. Haggerty,[6] H. Hamagaki,[10] R. Han,[45] H. Harada,[20] E.P. Hartouni,[32] K. Haruna,[20] E. Haslum,[35] R. Hayano,[10] M. Heffner,[32] T.K. Hemmick,[54] T. Hester,[7] X. He,[19] H. Hiejima,[22] J.C. Hill,[24] R. Hobbs,[41] M. Hohlmann,[17] W. Holzmann,[53] K. Homma,[20] B. Hong,[28] T. Horaguchi,[10,47,57] D. Hornback,[56] S. Huang,[59] T. Ichihara,[47,48] R. Ichimiya,[47] H. Iinuma,[30,47] Y. Ikeda,[58] K. Imai,[30,47] J. Imrek,[15] M. Inaba,[58] Y. Inoue,[49,47] D. Isenhower,[1] L. Isenhower,[1] M. Ishihara,[47] T. Isobe,[10] M. Issah,[53] A. Isupov,[25] D. Ivanischev,[46] B.V. Jacak,[54,†] J. Jia,[12] J. Jin,[12] O. Jinnouchi,[48] B.M. Johnson,[6] K.S. Joo,[39] D. Jouan,[44] F. Kajihara,[10] S. Kametani,[10,47,60] N. Kamihara,[47,48] J. Kamin,[54] M. Kaneta,[48] J.H. Kang,[62] H. Kanou,[47,57] J. Kapustinsky,[33] D. Kawall,[36,48] A.V. Kazantsev,[29] T. Kempel,[24] A. Khanzadeev,[46] K.M. Kijima,[20] J. Kikuchi,[60] B.I. Kim,[28] D.H. Kim,[39] D.J. Kim,[62] E. Kim,[52] S.H. Kim,[62] E. Kinney,[11] K. Kiriluk,[11] Á. Kiss,[16] E. Kistenev,[6] A. Kiyomichi,[47] J. Klay,[32] C. Klein-Boesing,[37] L. Kochenda,[46] V. Kochetkov,[21] B. Komkov,[46] M. Konno,[58] J. Koster,[22] D. Kotchetkov,[7] A. Kozlov,[61] A. Král,[13] A. Kravitz,[12] J. Kubart,[8,23] G.J. Kunde,[33] N. Kurihara,[10] K. Kurita,[49,47] M. Kurosawa,[47] M.J. Kweon,[28] Y. Kwon,[56,62] G.S. Kyle,[42] R. Lacey,[53] Y.S. Lai,[12] J.G. Lajoie,[24] D. Layton,[22] A. Lebedev,[24] D.M. Lee,[33] K.B. Lee,[28] M.K. Lee,[62] T. Lee,[52] M.J. Leitch,[33] M.A.L. Leite,[51] B. Lenzi,[51] P. Liebing,[48] T. Liška,[13] A. Litvinenko,[25] H. Liu,[42] M.X. Liu,[33] X. Li,[9] B. Love,[59] D. Lynch,[6] C.F. Maguire,[59] Y.I. Makdisi,[5] A. Malakhov,[25] M.D. Malik,[41] V.I. Manko,[29] E. Mannel,[12] Y. Mao,[45,47] L. Mašek,[8,23] H. Masui,[58] F. Matathias,[12] M. McCumber,[54] P.L. McGaughey,[33] N. Means,[54] B. Meredith,[22] Y. Miake,[58] P. Mikeš,[8,23] K. Miki,[58] T.E. Miller,[59] A. Milov,[6,54] S. Mioduszewski,[6] M. Mishra,[3] J.T. Mitchell,[6] M. Mitrovski,[53] A.K. Mohanty,[4] Y. Morino,[10] A. Morreale,[7] D.P. Morrison,[6] T.V. Moukhanova,[29] D. Mukhopadhyay,[59] J. Murata,[49,47] S. Nagamiya,[26] Y. Nagata,[58] J.L. Nagle,[11] M. Naglis,[61] M.I. Nagy,[16] I. Nakagawa,[47,48] Y. Nakamiya,[20] T. Nakamura,[20] K. Nakano,[47,57] J. Newby,[32] M. Nguyen,[54] T. Niita,[58] B.E. Norman,[33] R. Nouicer,[6] A.S. Nyanin,[29] E. O'Brien,[6] S.X. Oda,[10] C.A. Ogilvie,[24] H. Ohnishi,[47] K. Okada,[48] M. Oka,[58] O.O. Omiwade,[1] Y. Onuki,[47] A. Oskarsson,[35] M. Ouchida,[20] K. Ozawa,[10] R. Pak,[6] D. Pal,[59] A.P.T. Palounek,[33] V. Pantuev,[54] V. Papavassiliou,[42] J. Park,[52] W.J. Park,[28] S.F. Pate,[42] H. Pei,[24] J.-C. Peng,[22] H. Pereira,[14] V. Peresedov,[25] D.Yu. Peressounko,[29] C. Pinkenburg,[6] M.L. Purschke,[6] A.K. Purwar,[33] H. Qu,[19] J. Rak,[41] A. Rakotozafindrabe,[31] I. Ravinovich,[61] K.F. Read,[43,56] S. Rembeczki,[17] M. Reuter,[54] K. Reygers,[37] V. Riabov,[46] Y. Riabov,[46] D. Roach,[59] G. Roche,[34] S.D. Rolnick,[7] A. Romana,[31,*] M. Rosati,[24] S.S.E. Rosendahl,[35] P. Rosnet,[34] P. Rukoyatkin,[25] P. Ružička,[23] V.L. Rykov,[47] B. Sahlmueller,[37] N. Saito,[30,47,48] T. Sakaguchi,[6] S. Sakai,[58] K. Sakashita,[47,57] H. Sakata,[20] V. Samsonov,[46] S. Sato,[26] T. Sato,[58] S. Sawada,[26] K. Sedgwick,[7] J. Seele,[11] R. Seidl,[22] A.Yu. Semenov,[24] V. Semenov,[21] R. Seto,[7] D. Sharma,[61] I. Shein,[21] A. Shevel,[46,53] T.-A. Shibata,[47,57] K. Shigaki,[20] M. Shimomura,[58] K. Shoji,[30,47] P. Shukla,[4] A. Sickles,[6,54] C.L. Silva,[51] D. Silvermyr,[43] C. Silvestre,[14] K.S. Sim,[28] B.K. Singh,[3] C.P. Singh,[3] V. Singh,[3] S. Skutnik,[24] M. Slunečka,[8,25] A. Soldatov,[21] R.A. Soltz,[32] W.E. Sondheim,[33] S.P. Sorensen,[56] I.V. Sourikova,[6] F. Staley,[14] P.W. Stankus,[43] E. Stenlund,[35] M. Stepanov,[42] A. Ster,[27] S.P. Stoll,[6] T. Sugitate,[20] C. Suire,[44] A. Sukhanov,[6] J. Sziklai,[27] T. Tabaru,[48] S. Takagi,[58] E.M. Takagui,[51] A. Taketani,[47,48] R. Tanabe,[58]

arXiv:1005.3674v1 [hep-ex] 20 May 2010



$q \simeq 1.10$

FIG. 11: Invariant differential cross-section of neutral mesons measured in $p + p$ collisions at $\sqrt{s}$ =200 GeV in various decay modes. The lines are fits to the spectra as described further in the text.

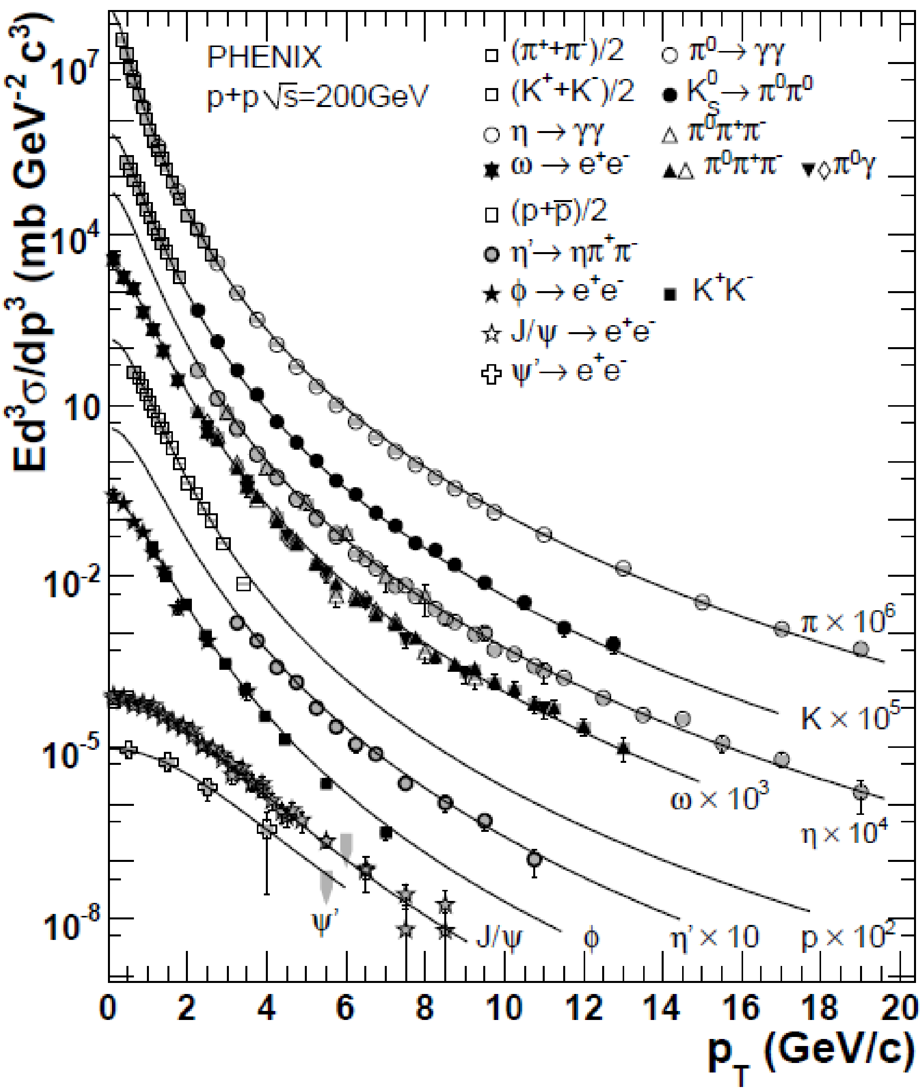


FIG. 12: Invariant differential cross sections of different particles measured in $p+p$ collisions at $\sqrt{s}$ =200 GeV in various decay modes. The spectra published in this paper are shown with closed symbols, previously published results are shown with open symbols. The curves are the fit results discussed in the text.

$q \simeq 1.10$

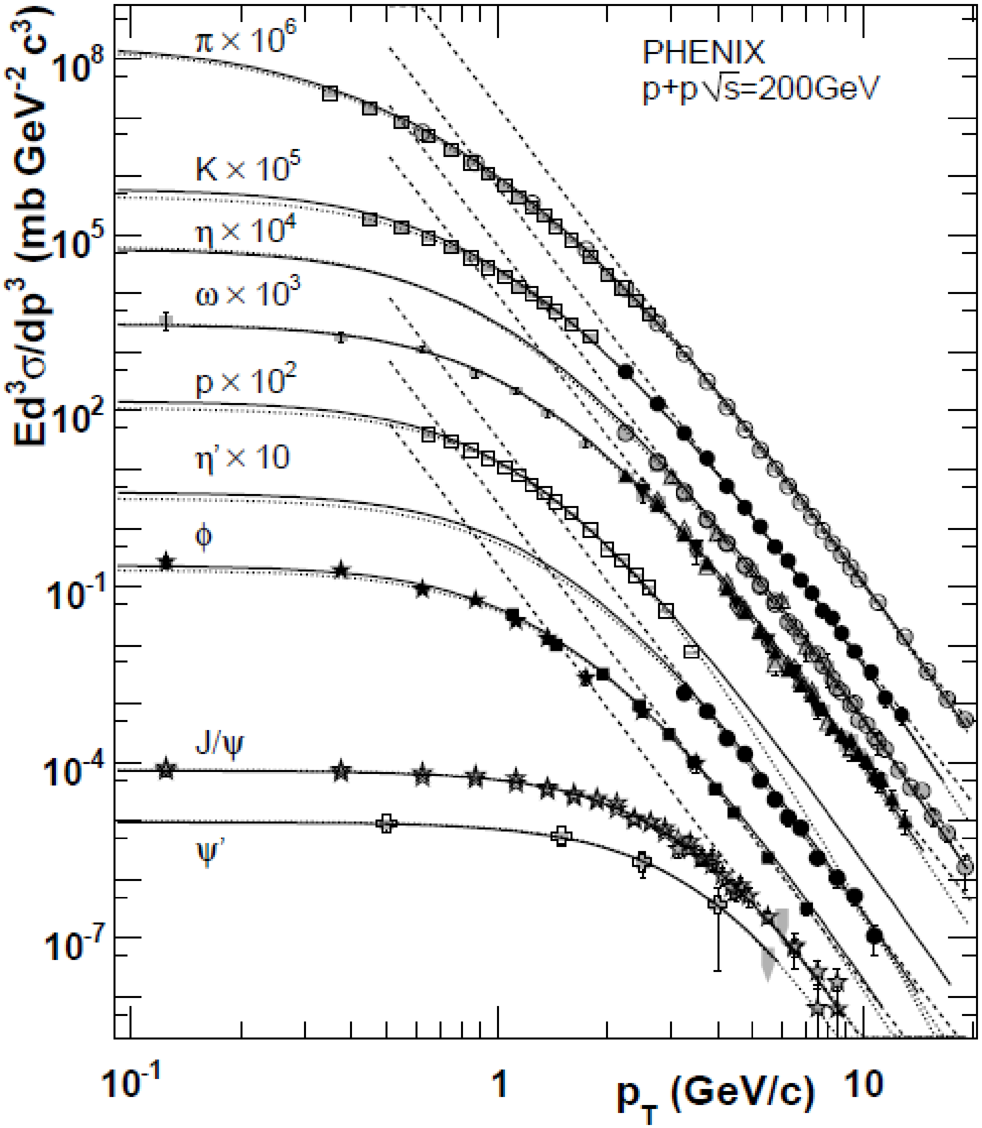
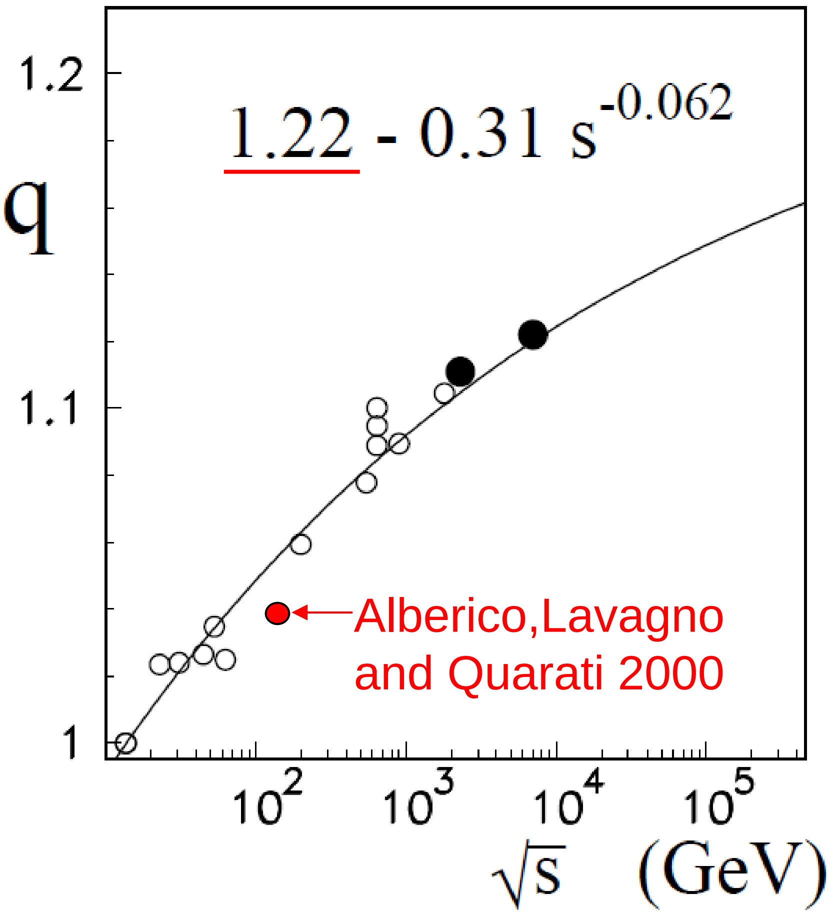
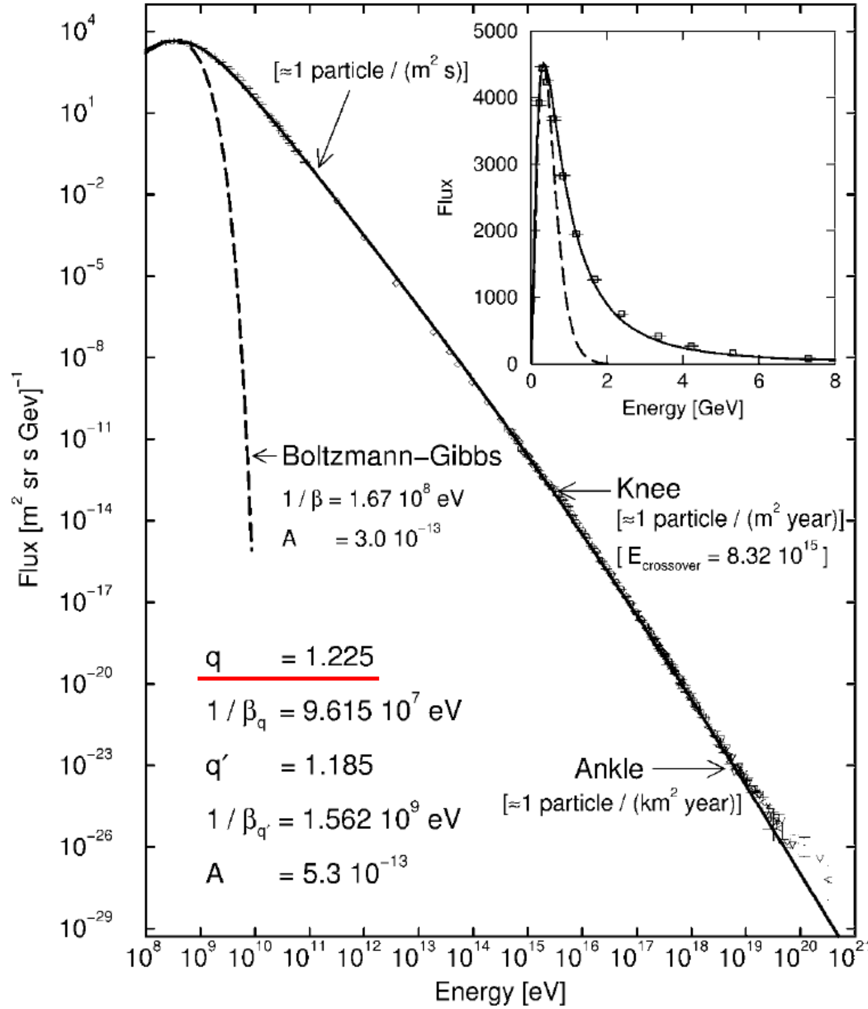


FIG. 13: The p_T spectra of various hadrons measured by PHENIX fitted to the power law (dashed lines) and Tsallis fit (solid lines). See text for more details.

$q \simeq 1.10$



Energy dependence of the non-extensivity parameter. The open symbol represents the values obtained in [7] for energies up to Tevatron. Two solid circles show values adjusted to CMS data.



T. Wibig, 1005.5652 [hep-ph]
31 May 2010

C. T., J.C. Anjos and E.P. Borges
Phys Lett A **310**, 372 (2003)

IOP PUBLISHING JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **37** (2010) 085104 (10pp) doi:10.1088/0954-3899/37/8/085104

Examination of the species and beam energy dependence of particle spectra using Tsallis statistics

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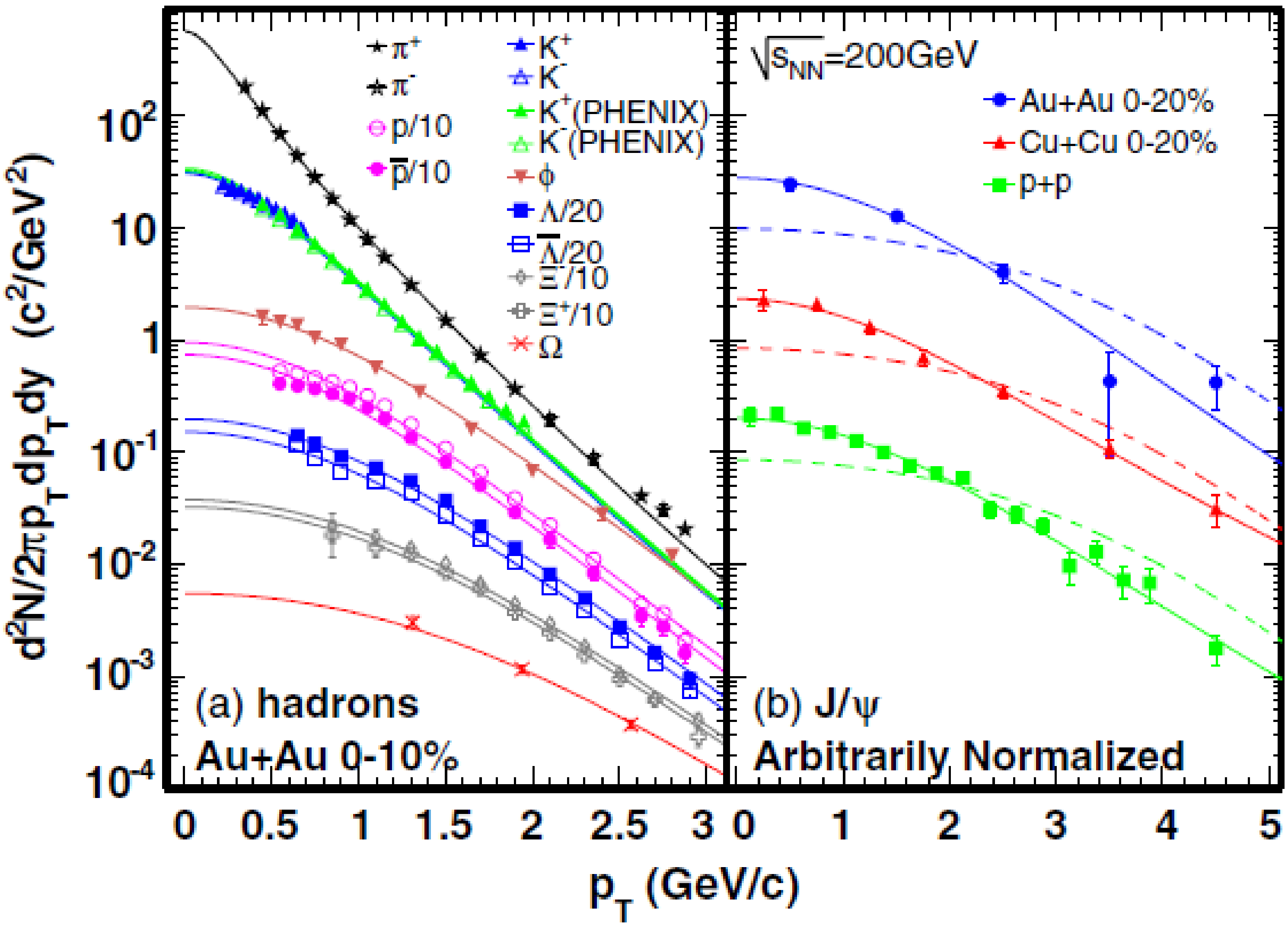
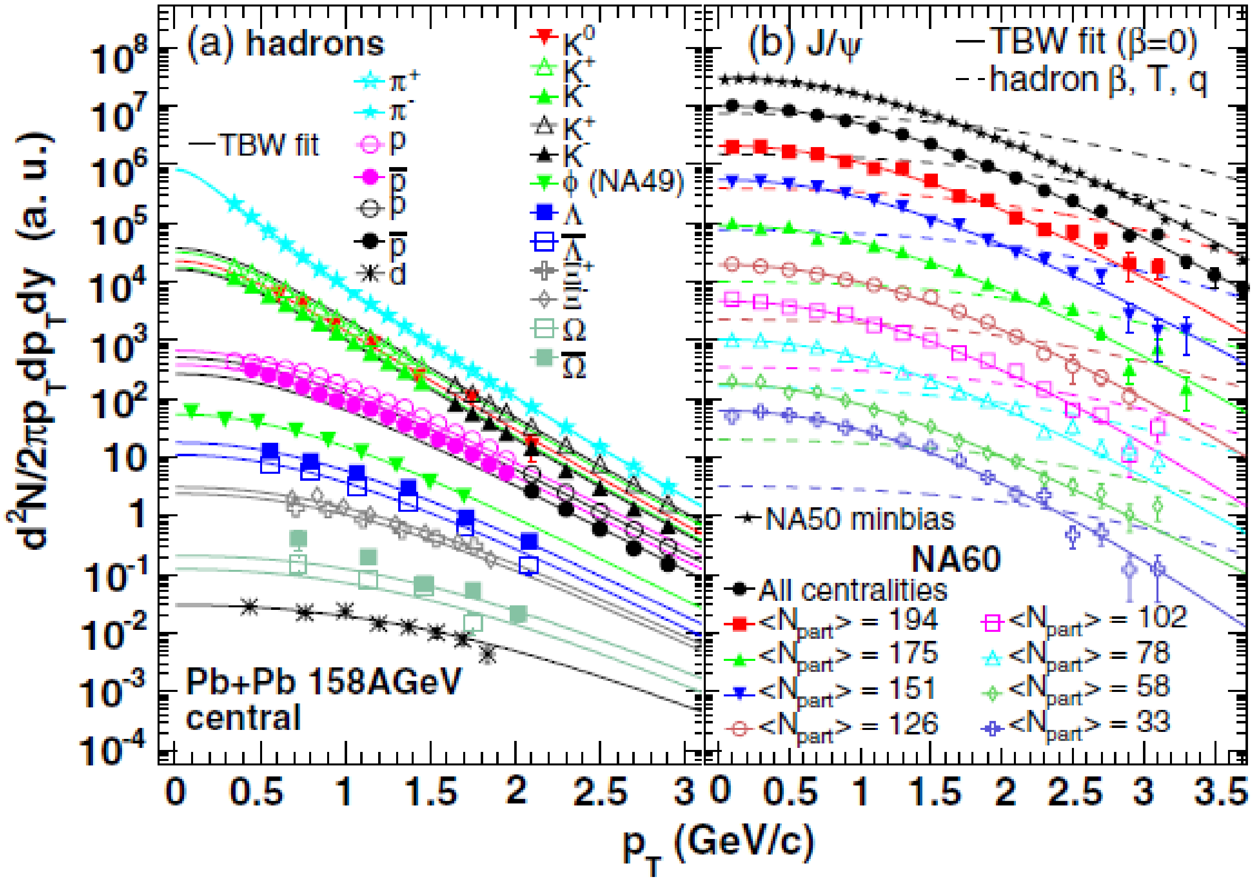
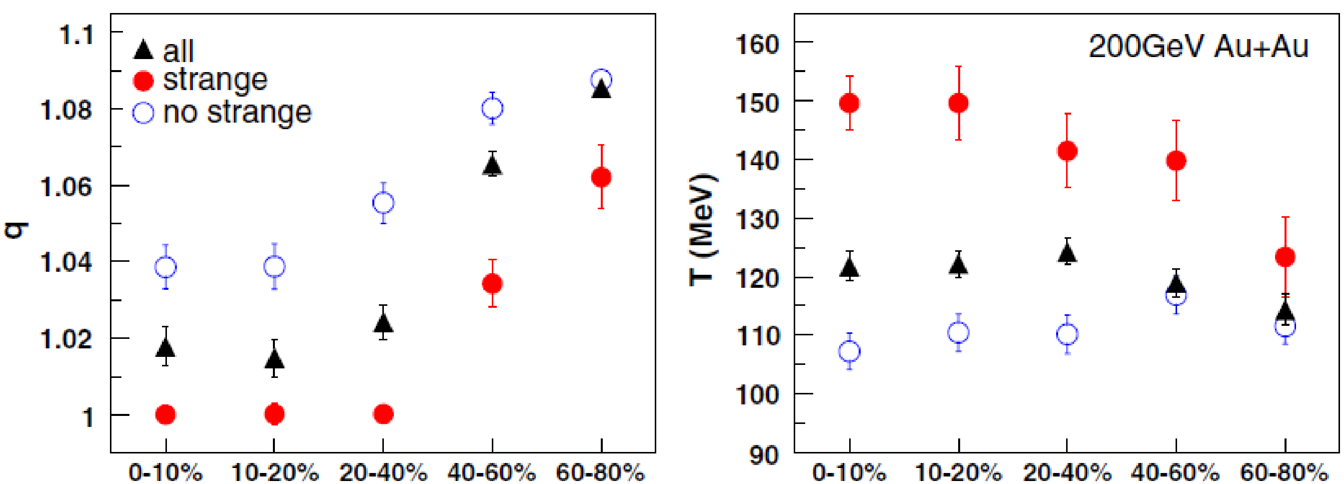
$$\frac{\mathrm{d}N}{m_{\mathrm{T}}\,\mathrm{d}m_{\mathrm{T}}} \propto m_{\mathrm{T}} \int_{-Y}^{+Y} \cosh(y)\,\mathrm{d}y \int_{-\pi}^{+\pi} \mathrm{d}\phi \int_{0}^{R} r\,\mathrm{d}r \times \left(1 + \frac{q-1}{T}(m_{\mathrm{T}}\cosh(y)\cosh(\rho) - p_{\mathrm{T}}\sinh(\rho)\cos(\phi))\right)^{-1/(q-1)}$$




(a) hadrons
(b) J/ψ
TBW fit
TBW fit (β=0)
hadron β, T, q
π+
π-
p
p̄
d
K0
K+
K-
φ (NA49)
Λ
Λ̄
Ξ+
Ξ-
Ω
Ω̄
NA50 minbias
NA60
All centralities
<Npart> = 194
<Npart> = 175
<Npart> = 151
<Npart> = 126
<Npart> = 102
<Npart> = 78
<Npart> = 58
<Npart> = 33
Pb+Pb 158AGeV
central
d2N/2πpTdpTdy (a. u.)
pT (GeV/c)


all
strange
no strange
q
200GeV Au+Au
T (MeV)
0-10%
10-20%
20-40%
40-60%
60-80%

Equilibrium Distribution of Heavy Quarks in Fokker-Planck Dynamics

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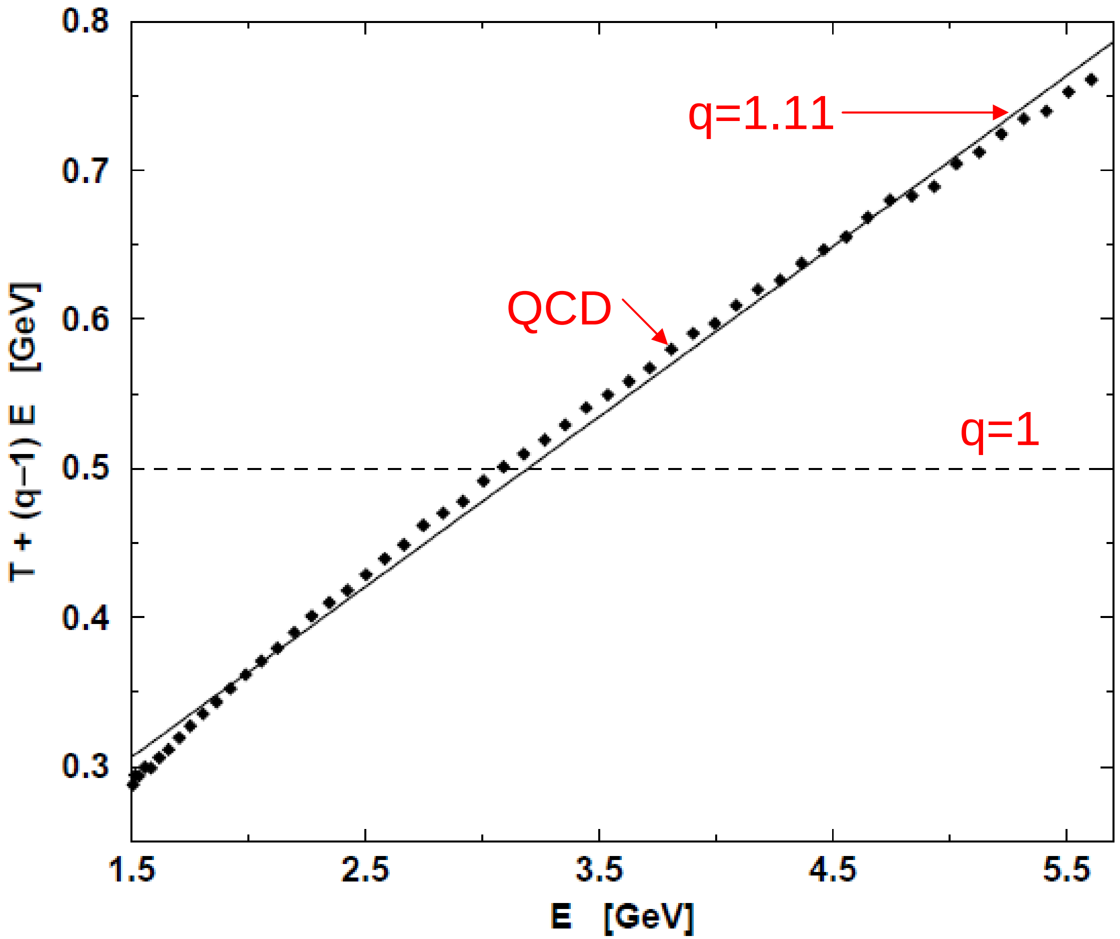
(Received 8 July 1999)

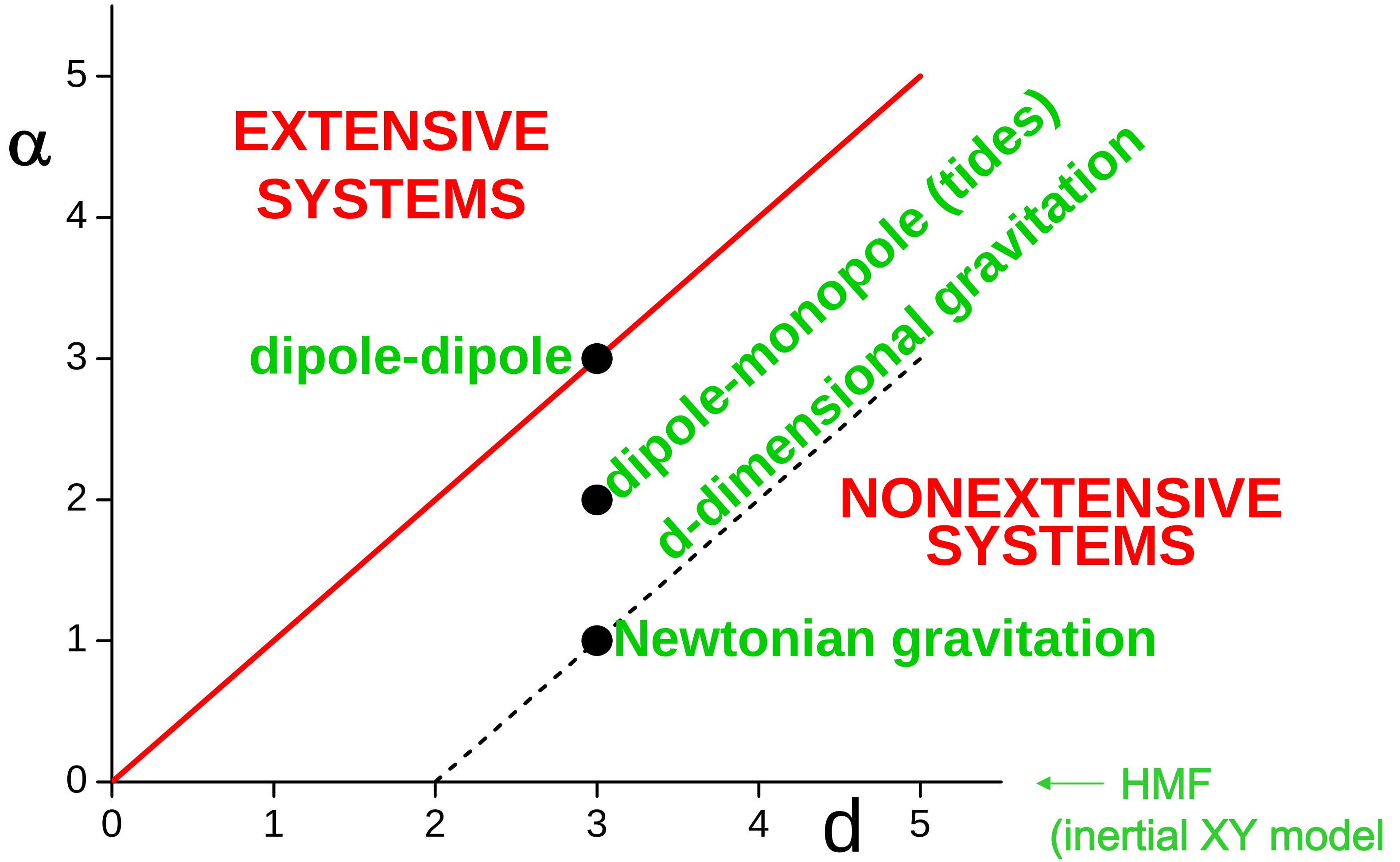


CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

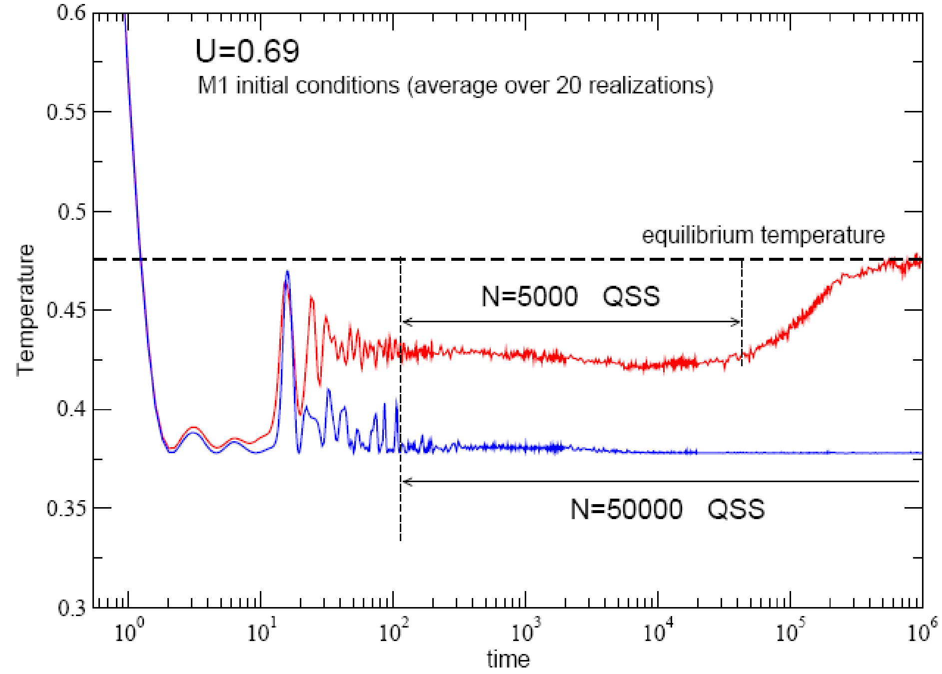
$$V(\vec{r}) \sim -\frac{A}{r^{\alpha}} \qquad (r \to \infty) \qquad (A > 0, \;\; \alpha \geq 0)$$

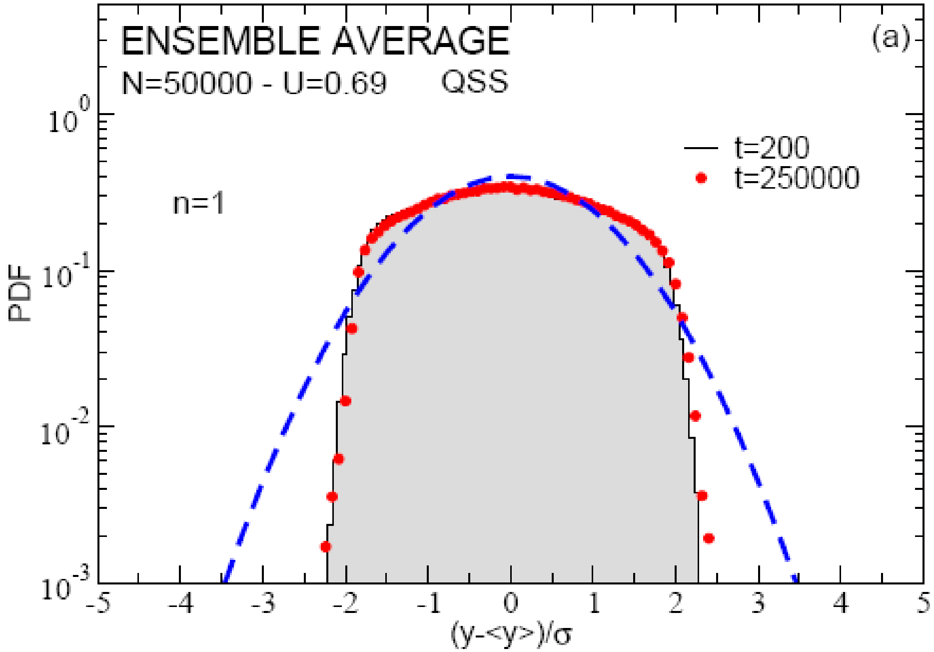
integrable if $\alpha / d > 1$ *(short-ranged)*

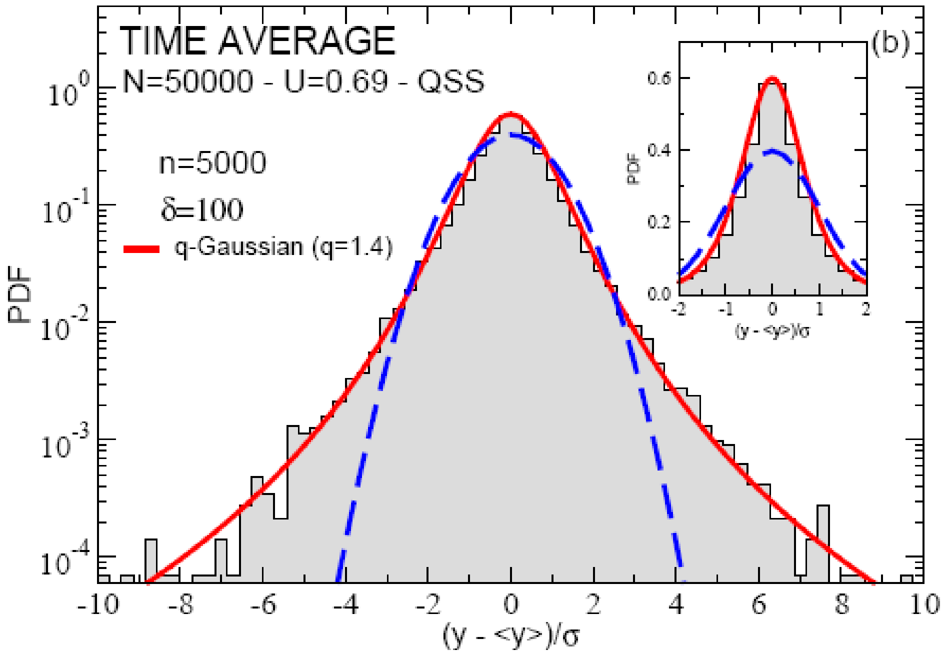
non-integrable if $0 \leq \alpha / d \leq 1$ *(long-ranged)*



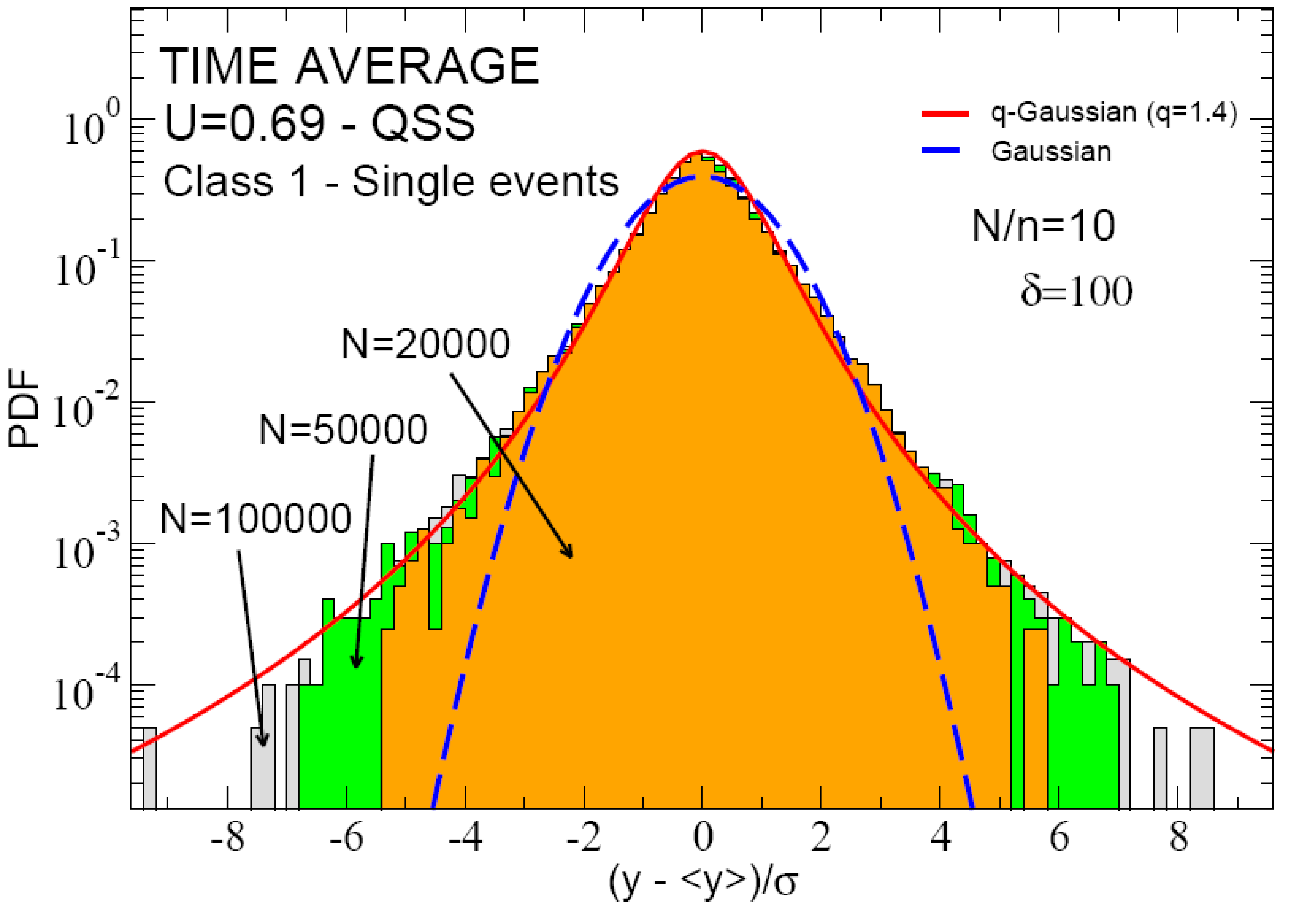
HMF MODEL







A. Pluchino, A. Rapisarda and C. T., Europhys Lett **80**, 26002 (2007)



[See A. Pluchino, A. Rapisarda and C. T., Europhys Lett **85**, 60006 (2009)]

Journal of Statistical Mechanics: Theory and Experiment
An IOP and SISSA journal

Thermostatistics in the neighbourhood of the π-mode solution for the Fermi–Pasta–Ulam β system: from weak to strong chaos

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Abstract. We consider a π-mode solution of the Fermi–Pasta–Ulam β system. By perturbing it, we study the system as a function of the energy density from a regime where the solution is stable to a regime where it is unstable, first weakly and then strongly chaotic. We introduce, as an indicator of stochasticity, the ratio ρ (when it is defined) between the second and the first moment of a given probability distribution. We will show numerically that the transition between weak and strong chaos can be interpreted as the symmetry breaking of a set of suitable dynamical variables. Moreover, we show that in the region of weak chaos there is numerical evidence that the thermostatistic is governed by the Tsallis distribution.

J. Stat. Mech. (2010) P04021

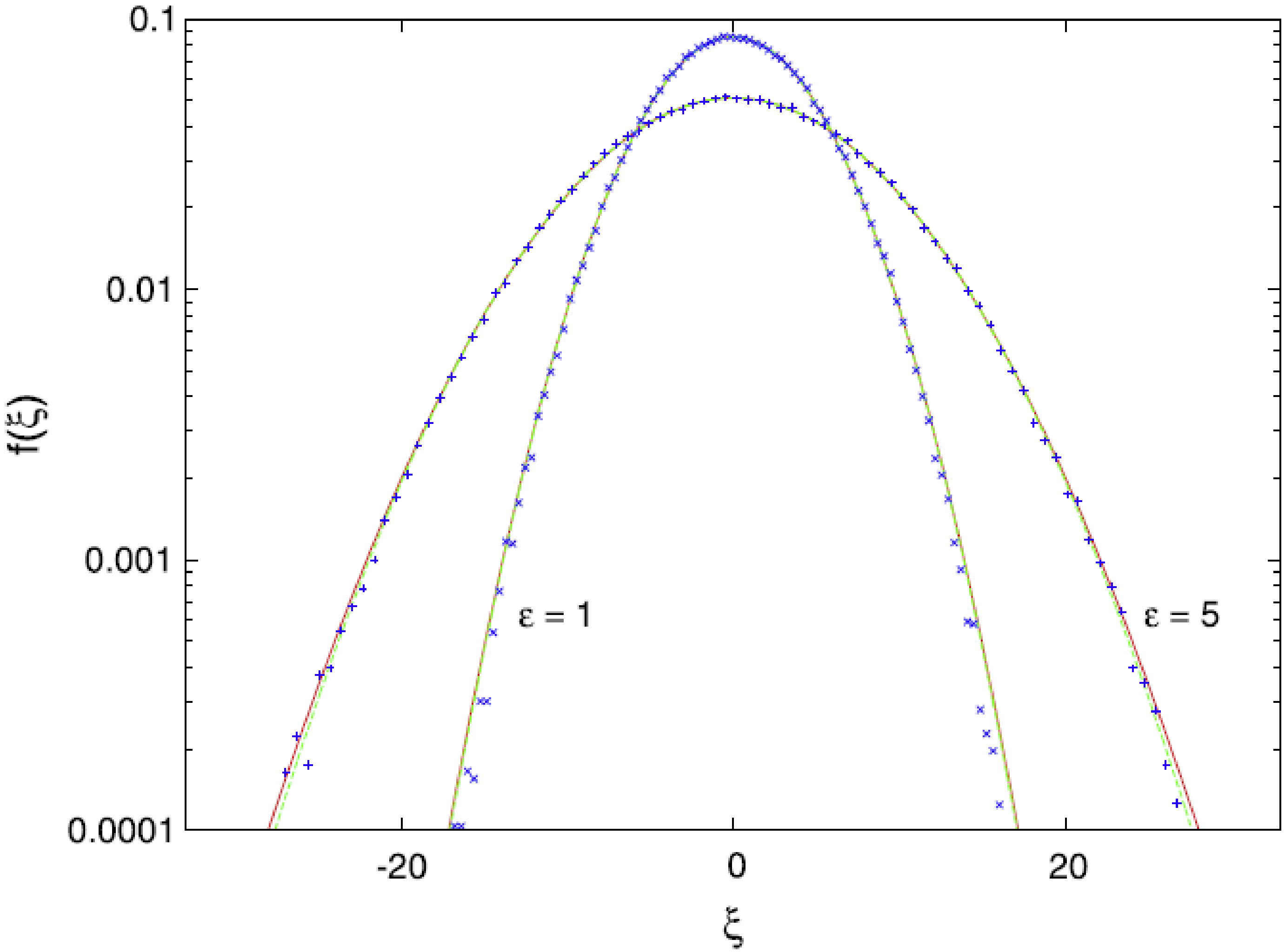


Figure 5. Plot on a linear-log scale of the numerical distribution $f(\xi)$ (blue points) fitted with a Tsallis distribution (red) and a Gauss distribution (green) for $N = 128$, $\epsilon = 1$ and 5. In both cases the Tsallis and Gaussian distributions essentially overlap.

M. Leo, R.A. Leo and P. Tempesta, J Stat Mech P04021 (2010)

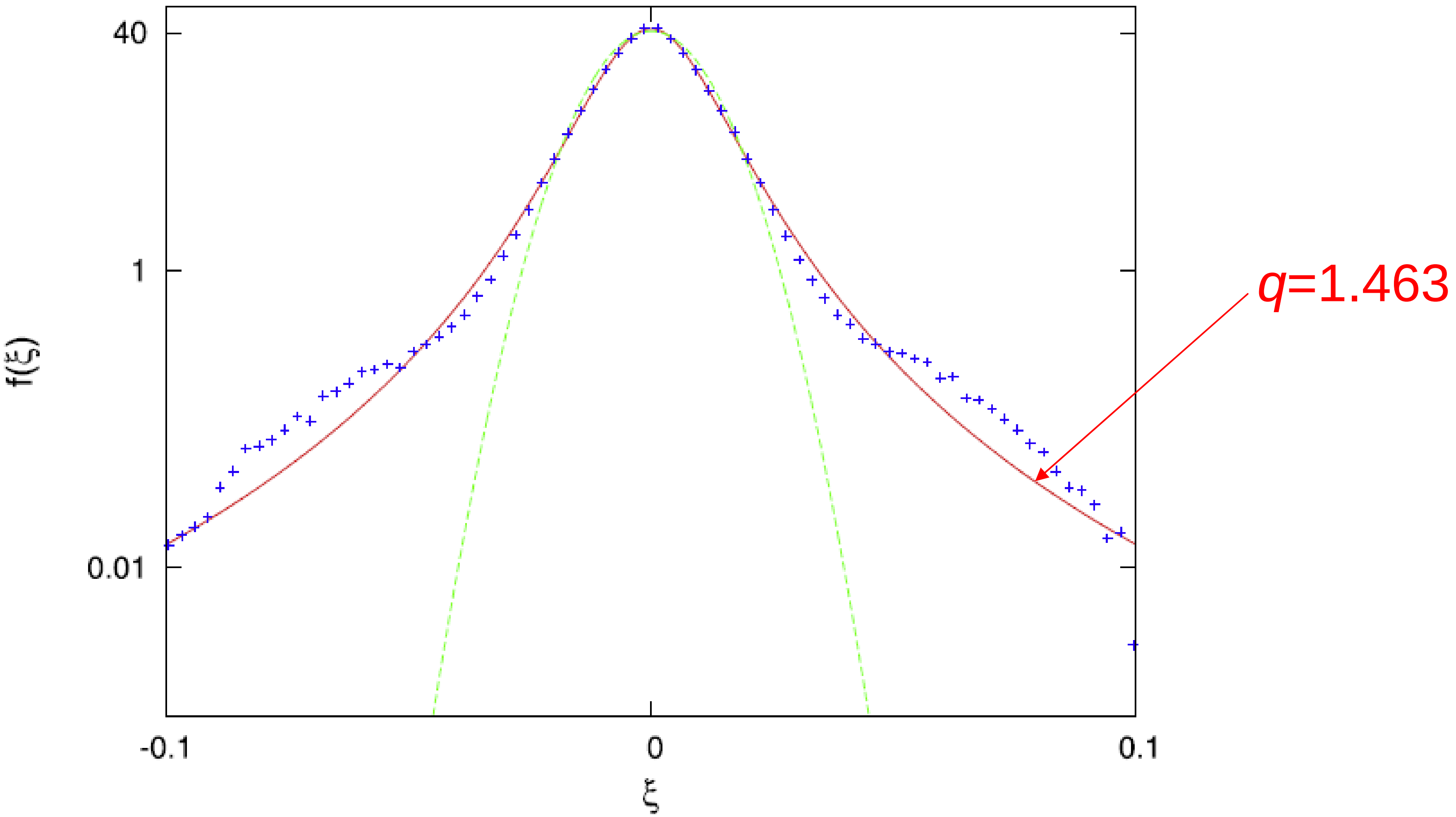
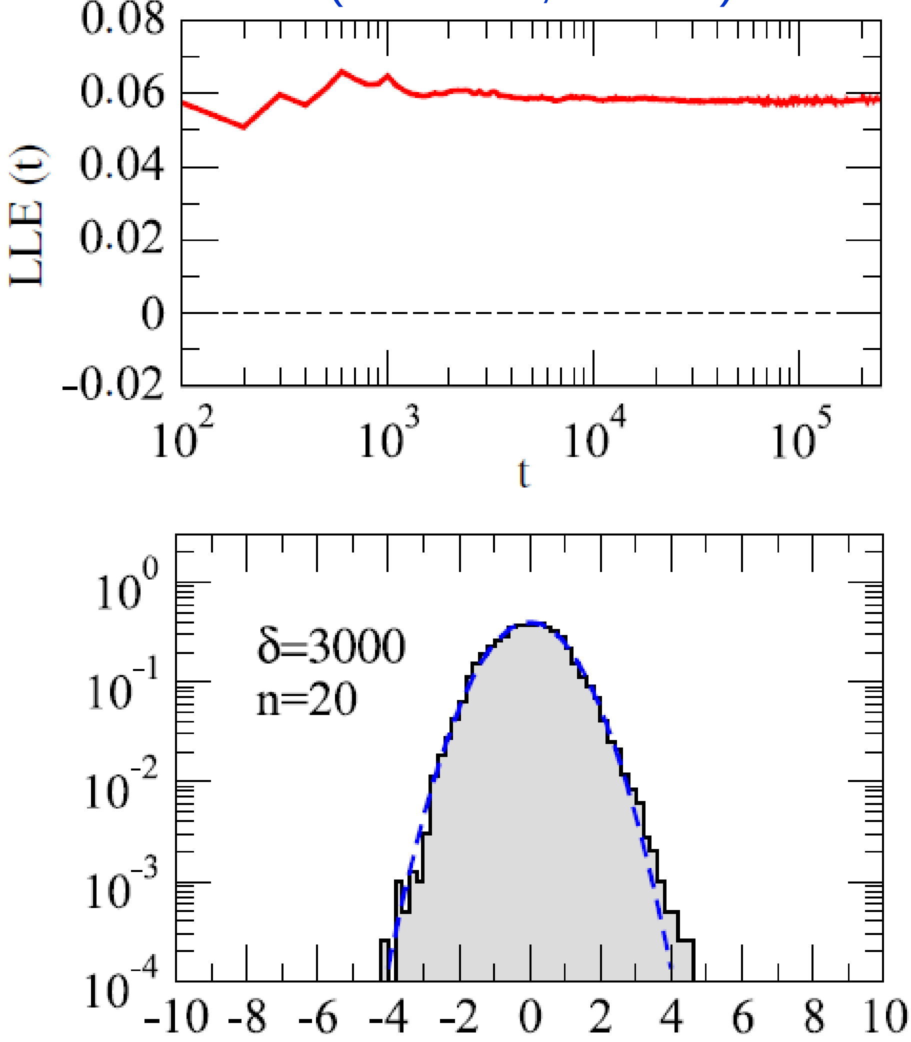


Figure 4. Plot on a linear-log scale of the numerical distribution $f(\xi)$ (blue points) fitted with a Tsallis distribution (red) and a Gauss distribution (green) for $N = 128$ and $\epsilon = 0.006$.

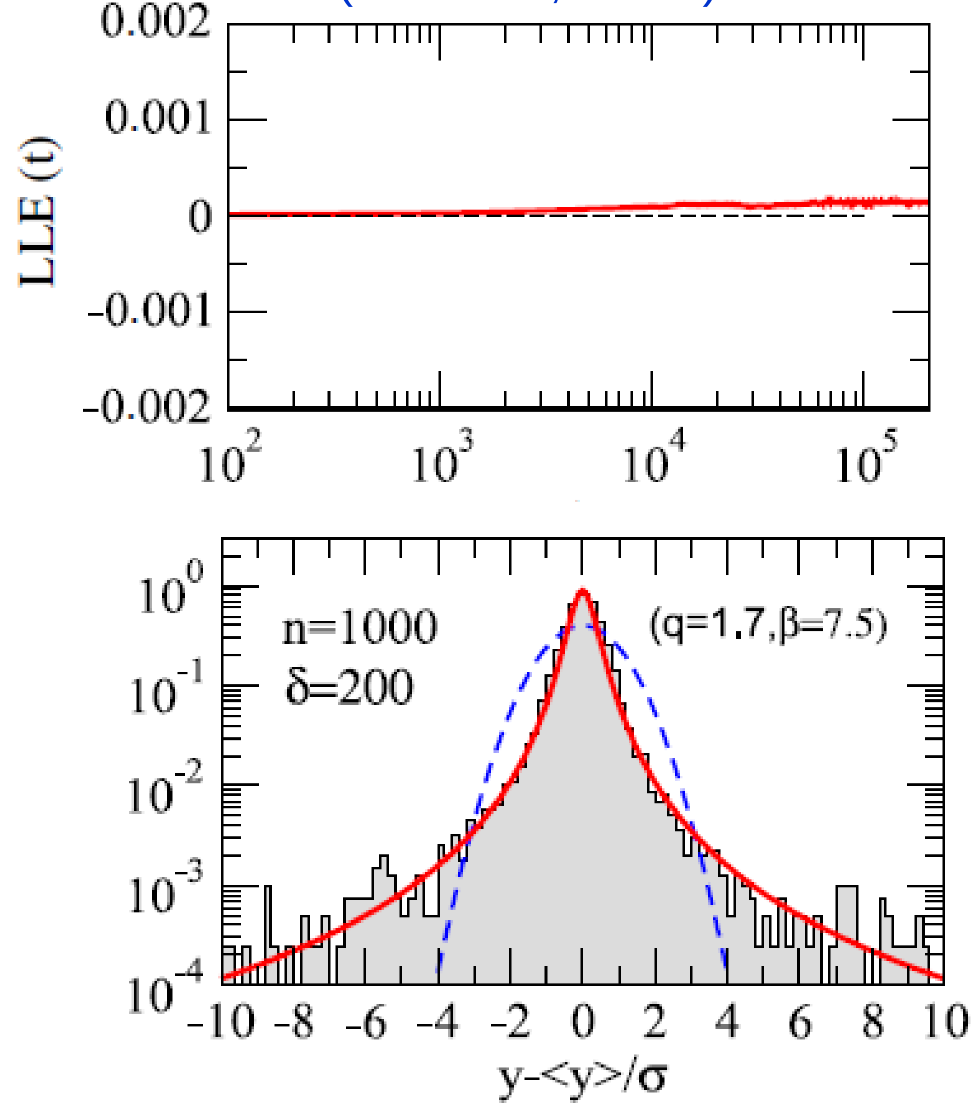
M. Leo, R.A. Leo and P. Tempesta, J Stat Mech P04021 (2010)

KURAMOTO MODEL: (N nonlinearly coupled oscillators)

(N=20000; K=2.53)



(N=20000; K=0.6)



G. Miritello, A. Pluchino and A. Rapisarda, Physica A **388**, 4818 (2009)

CONSERVATIVE MC MILLAN MAP:

G. Ruiz, T. Bountis and C. T. (2010)

$$x_{n+1} = y_n$$

$$y_{n+1} = -x_n + 2\mu \frac{y_n}{1 + y_n^2} + \varepsilon y_n$$

$\mu \neq 0 \Leftrightarrow$ nonlinear dynamics

$(\mu, \varepsilon) = (1.6, 1.2)$

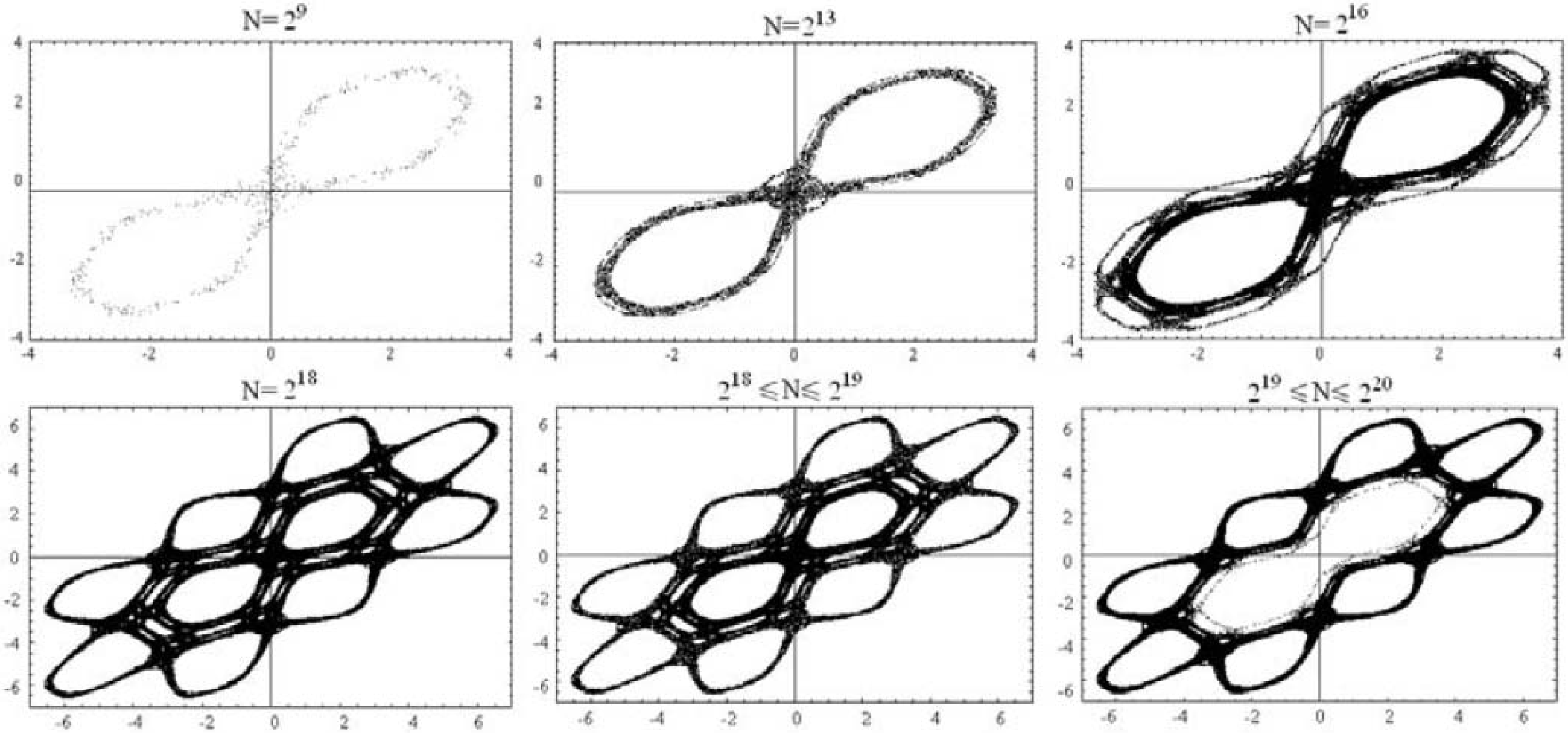
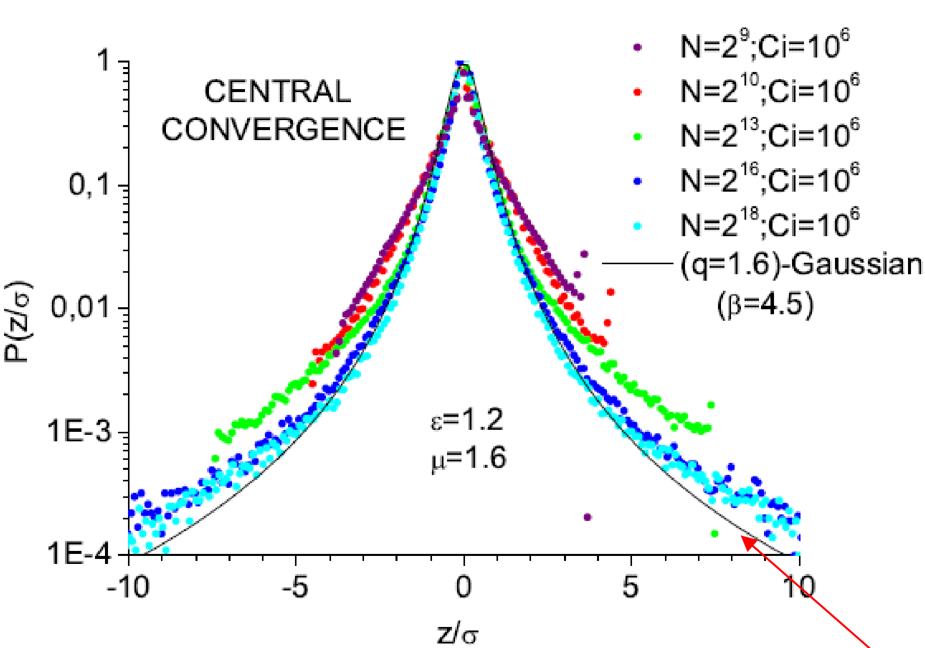
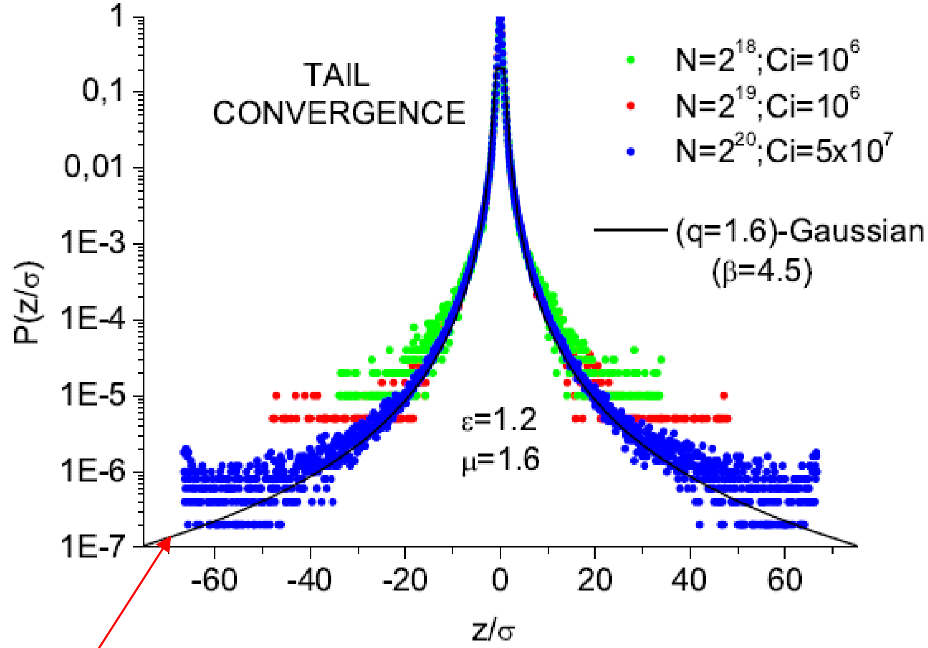


FIG. 10. Structure of phase space plot of Mc. Millan perturbed map for parameter values $\mu = 1.6$ and $\epsilon = 1.2$, starting form a randomly chosen initial condition in a square $(0, 10^{-6}) \times (0, 10^{-6})$, and for $i = 1 \ldots N$ $(N = 2^{10}, 2^{13}, N^{16}, N^{18})$ iterates.

$(\lambda_{\max} \simeq 0.05)$





$$p \propto e_q^{-\beta (z/\sigma)^2}$$

with $(q, \beta) = (1.6, 4.5)$

G. Ruiz, T. Bountis and C. T. (2010)

BROWNIAN MOTION:

PLANAR SPACE (constant curvature $R = 0$):

Metric: $ds^2 = dx^2 + dy^2$

Surface element: $dA = dxdy$

Stochastic equations: $$\frac{dx(t)}{dt} = \sqrt{2D}\,\eta_1(t) \quad (D > 0)$$

$$\frac{dy(t)}{dt} = \sqrt{2D}\,\eta_2(t)$$

$[\eta_1(t),\ \eta_2(t)$ independent Gaussian white noises$]$

Diffusion equation: $$\frac{\partial p(x,y,t)}{\partial t} = D\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)p(x,y,t) \quad (D > 0)$$

hence $p(x,y,t) \propto e^{-\frac{x^2+y^2}{2Dt}}$

HYPERBOLIC SPACE (constant curvature $R = -1$ in $y > 0$):

Metric: $$ds^2 = \frac{dx^2 + dy^2}{y^2}$$

Surface element: $$dA = \frac{dxdy}{y^2}$$

Stochastic equations (Ito): $$\frac{dx(t)}{dt} = \sqrt{2D}\, y(t)\ \eta_1(t) \quad (D > 0)$$

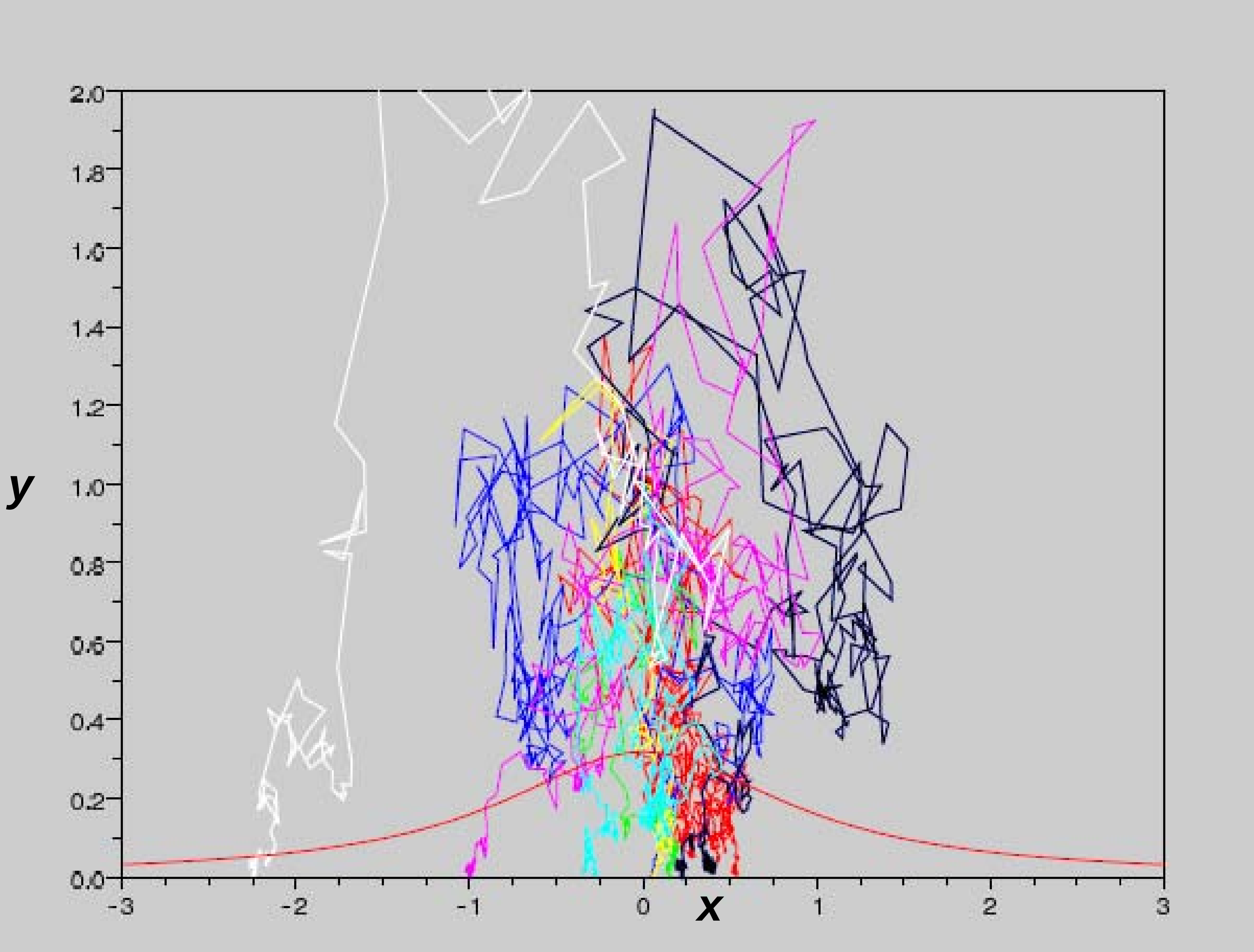
$$\frac{dy(t)}{dt} = \sqrt{2D}\, y(t)\ \eta_2(t)$$

Diffusion equation: $$\frac{\partial p(x,y,t)}{\partial t} = D\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) y^2 p(x,y,t)$$

with $\quad p(x,y,0) = \delta(x)\ \delta(y-1)$

hence $\quad p(x,y,\infty) = \delta(y-1)\ \frac{1}{\pi(1+x^2)} = \delta(y-1)\ \frac{1}{\pi} e_2^{-x^2}$

Comtet and Monthus, J Phys A **29**, 1331 (1996)



P.W. Lamberti and C. Vignat (2010)

HYPERBOLIC SPACE (constant curvature $R = -1$ in $y > 0$) with drift

Metric: $$ds^2 = \frac{dx^2 + dy^2}{y^2}$$

Surface element: $$dA = \frac{dxdy}{y^2}$$

Stochastic equations (Ito): $$\frac{dx(t)}{dt} = \sqrt{2D}\; y(t)\;\; \eta_1(t) \quad (D > 0)$$

$$\frac{dy(t)}{dt} = -2D\mu y(t) + \sqrt{2D}\; y(t)\;\; \eta_2(t)$$

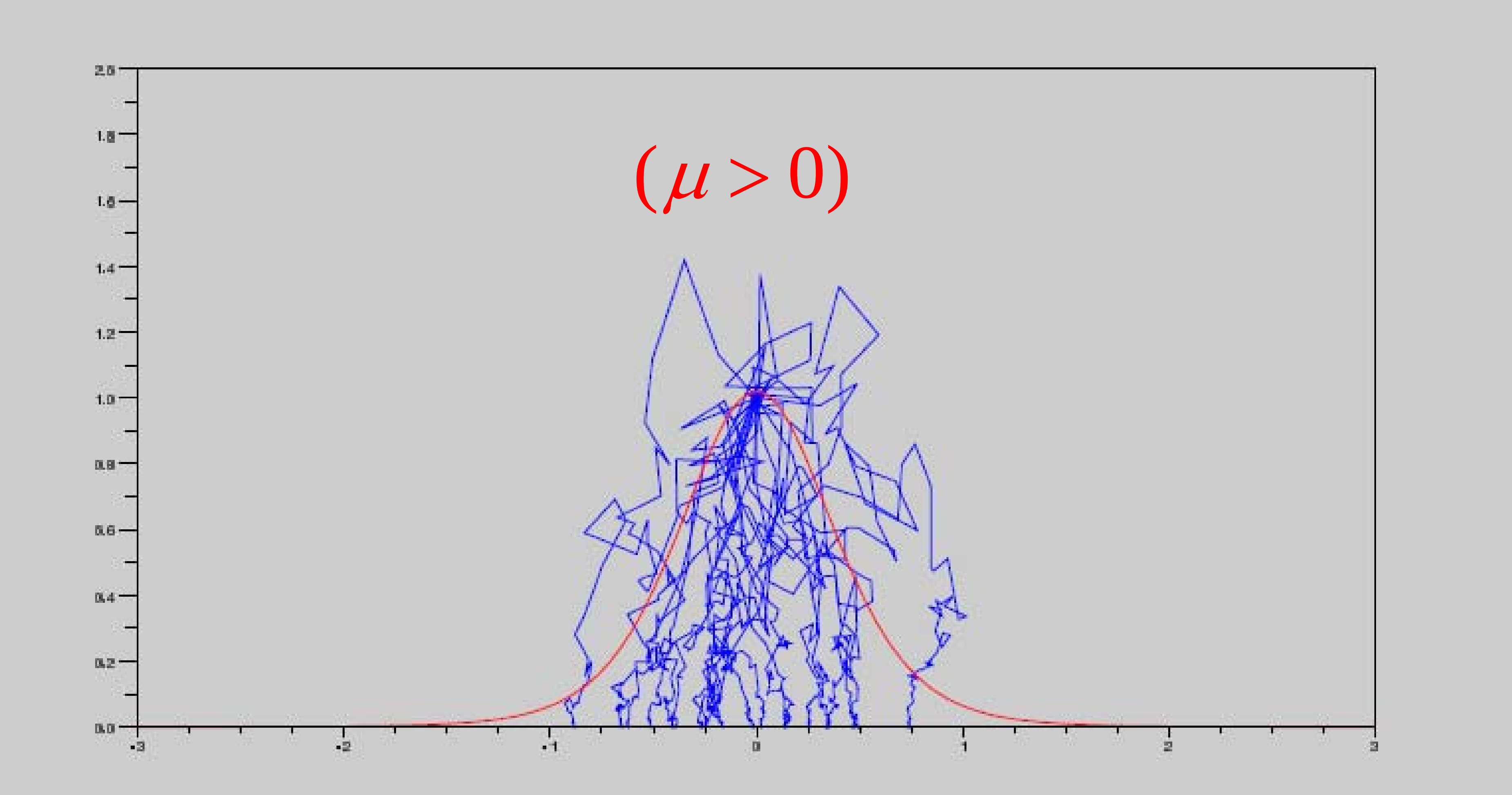
Diffusion equation: $$\frac{\partial p(x,y,t)}{\partial t} = D\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) y^2 p(x,y,t)$$

with $\quad p(x,y,0) = \delta(x)\;\delta(y-1)$

hence $$p(x,y,\infty) = \delta(y-1)\;\frac{\Gamma(\mu+1)}{\sqrt{\pi}\,\Gamma\left(\mu+\frac{1}{2}\right)}\frac{1}{\left(1+x^2\right)^{\mu+1}} \propto \delta(y-1)\; e_q^{-x^2}$$

with $$q = \frac{\mu+2}{\mu+1} \in (1,3) \text{ for } \mu \in (-1/2, \infty)$$

Comtet and Monthus, J Phys A **29**, 1331 (1996)



P.W. Lamberti and C. Vignat (2010)

q-PLANE WAVES:

1) New representation of Dirac delta:

$$\delta(x) = \frac{2-q}{2\pi}\int_{-\infty}^{\infty} dk\ e_q^{-ikx} \quad (1 \le q < 2)$$

i.e.,

$$\int_{-\infty}^{\infty} dx\ \delta(x - x_0)\, f(x) = f(x_0)$$

M. Jauregui and C. T., J Math Phys **51**, 063304 (2010)

2) New representations of π :

$$\pi = n \sum_{k=0}^{\left\lfloor \frac{n+1}{2} \right\rfloor - 1} (-1)^k \frac{\Gamma\left(n-k-\frac{1}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(2k+2)\Gamma(n-2k)}\,, \quad \forall n \in \mathbb{N}$$

$$\pi = \int_{-\infty}^{\infty} \frac{\sin(2r\arctan z)}{z\left(1+z^2\right)^r}\,dz\,, \quad \forall r \in \mathbb{R}^+$$

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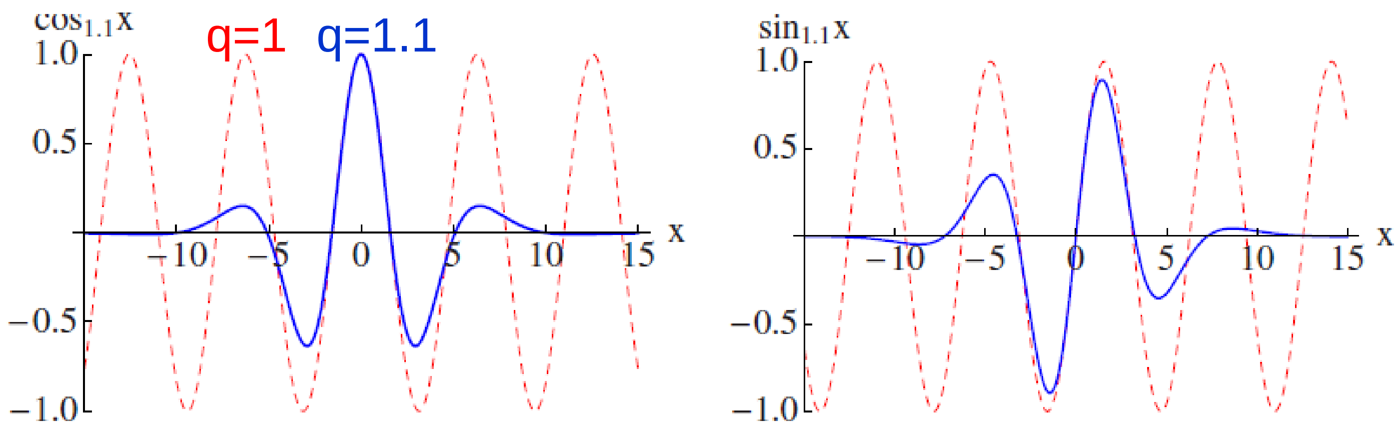
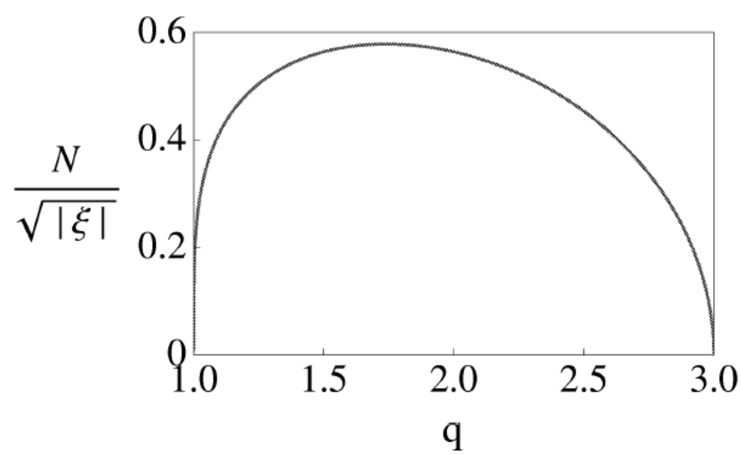
3) q-plane waves are square integrable ($0 < q < 3$):

$$\psi(x,t) = e_q^{i(kx-\omega t)} \text{ satisfies } \frac{\partial^2 \psi(x,t)}{\partial t^2} = c^2 \frac{\partial^2 \psi(x,t)}{\partial x^2} \text{ with } \omega = ck$$

$$\Psi(x) \equiv N\ e_q^{i\xi x} = N(\cos_q \xi x + i \sin_q \xi x) \text{ with } \int_{-\infty}^{\infty} dx\, |\Psi(x)|^2 = 1$$



$$\frac{N}{\sqrt{|\xi|}} = \left[\frac{(q-1)\,\Gamma\left(\frac{1}{q-1}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{3-q}{2(q-1)}\right)} \right]^{1/2}$$



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$than_q$

Introduction to Nonextensive Statistical Mechanics

APPROACHING A COMPLEX WORLD

Constantino Tsallis

Springer

Full bibliography (regularly updated):

http://tsallis.cat.cbpf.br/biblio.htm

3154 articles by 5091 scientists from 72 countries

[18 August 2010]

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ITALY	395	MEXICO	29	ROMENIA	7	INDONESIA	1
CHINA	281	UKRAINE	29	NEW ZEALAND	6	JORDAN	1
FRANCE	277	CZECK	24	NORWAY	6	PERU	1
GERMANY	263	FINLAND	24	IRELAND	5	SAUDI ARABIA	1
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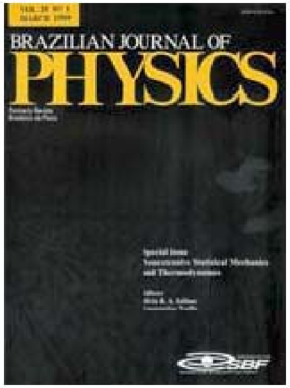

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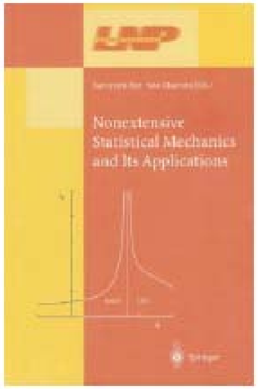

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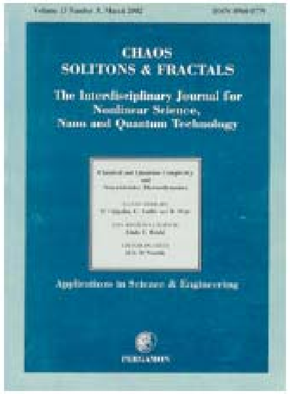

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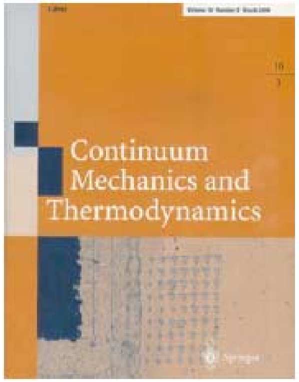

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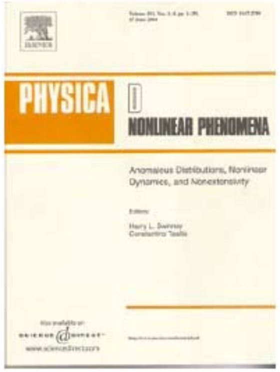

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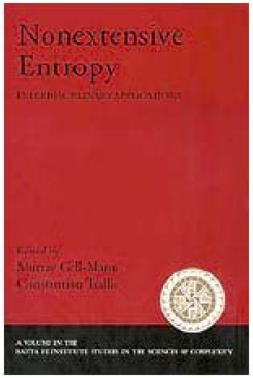

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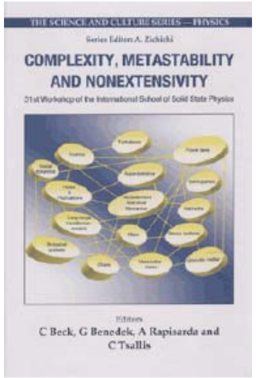

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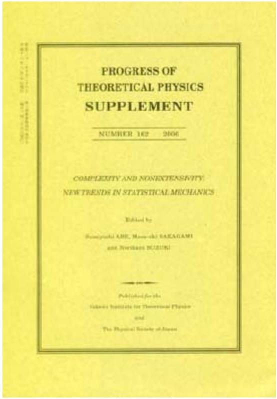

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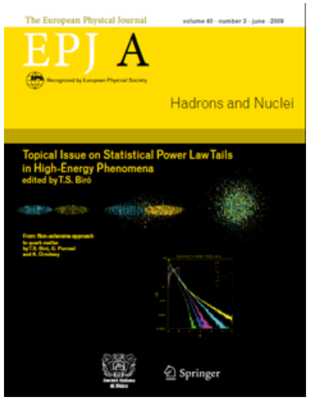

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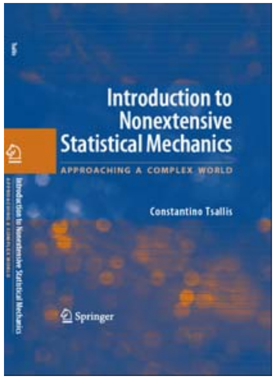

2009

The realm of Boltzmann-Gibbs statistical mechanics, based on the standard additive entropy, essentially concerns ergodic systems, Markovian-like processes, linear Fokker-Planck equations, exponential behaviors of relevant physical, geometrical and dynamical quantities, the central limit theorem. What can be done when such simplifying hypothesis are not satisfied? The nonadditive entropy Sq, and its associated nonextensive statistical mechanics, precisely address a wide class of such anomalous situations, namely whenever power-law behaviors replace the traditional exponential behaviors. A brief review will be given of the central concepts, and various applications will be exhibited, in particular those concerning high energy physics. BIBLIOGRAPHY:

(i) C. Tsallis, *Introduction to Nonextensive Statistical Mechanics - Approaching a Complex World* (Springer, New York, 2009); (ii) C. Tsallis, *Entropy*, in *Encyclopedia of Complexity and Systems Science* (Springer, Berlin, 2009); (iii) S. Umarov, C. Tsallis, M. Gell-Mann and S. Steinberg, J. Math. Phys. **51**, 033502 (2010); (iv) http://tsallis.cat.cbpf.br/biblio.htm