Nonextensive statistical mechanics: Applications to high energy physics

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Abstract. Nonextensive statistical mechanics was proposed in 1988 on the basis of the nonadditive entropy $S_q = k[1-\sum_i p_i^q]/(q-1)$ $(q \in \mathcal{R})$ which generalizes that of Boltzmann-Gibbs $S_{BG} = S_1 = -k\sum_i p_i \ln p_i$. This theory extends the applicability of standard statistical mechanics in order to also cover a wide class of anomalous systems which violate usual requirements such as ergodicity. Along the last two decades, a variety of applications have emerged in natural, artificial and social systems, including high energy phenomena. A brief review of the latter will be presented here, emphasizing some open issues.

1 Introduction

Standard statistical mechanics is based on the Boltzmann-Gibbs (BG) entropy $S_{BG} = -k\sum_{i=1}^{W} p_i \ln p_i$ $(\sum_{i=1}^{W} p_i = 1)$, where W is the number of microscopic configurations of the system. This extremely powerful theory — one of the pillars of contemporary physics — has exhibited very many successes along 140 years, in particular through its celebrated distribution for thermal equilibrium $p_i \propto e^{-\beta E_i}$, E_i being the energy of the corresponding microstate. However, as any other human intellectual construct, it has a restricted domain of validity. For nonlinear dynamical many-body systems the usual requirement is *ergodicity*, which is guaranted by strong chaos (i.e., by a *positive* maximal Lyapunov exponent for classical systems). For nonergodic systems (typically for systems whose maximal Lyapunov exponent *vanishes*), which is quite frequently the case of the so-called complex systems, there is no general reason for legitimately using the BG theory. For (some of) such anomalous systems, a generalization of the BG theory has been proposed in 1988 [1]. It is frequently referred to as *nonextensive statistical mechanics* [2–4] because the total energy of such systems typically is *nonextensive*, i.e., *not proportional* to the total number of elements of the system. This generalized theory is based on the entropy

$$S_q = k\,\frac{1-\sum_i p_i^q}{q-1} \quad (q \in \mathcal{R};\ S_1 = S_{BG}) \tag{1}$$

It can be straightforwardly verified that, if A and B are two probabilistically independent systems (i.e., if $p_{ij}^{A+B} = p_i^A p_j^B$), then

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k}\,, \tag{2}$$

which exhibits that, in contrast with S_{BG} which is additive, the entropy S_q is nonadditive for $q \neq 1$. This nonadditivity will in fact enable it to be *extensive* (i.e., proportional to the number of elements of the system) for various classes of systems (see for instance [5,6]).

2 Connection to Thermodynamics

To generalize BG statistical mechanics for the canonical ensemble (from [7]), we optimize S_q with the constraints

$$\sum_{i=1}^{W} p_i = 1 \tag{3}$$

and

$$\sum_{i=1}^{W} P_i E_i = U_q\,, \tag{4}$$

where

$$P_i \equiv \frac{p_i^q}{\sum_{j=1}^{W} p_j^q} \quad \left(\sum_{i=1}^{W} P_i = 1\right) \tag{5}$$

is the so-called *escort distribution* [8]. It follows that $p_i = \frac{P_i^{1/q}}{\sum_{j=1}^{W} P_j^{1/q}}$. There are various converging reasons for being appropriate to impose the energy constraint with the $\{P_i\}$ instead of with the original $\{p_i\}$. The full discussion of this delicate point is beyond the present scope. However, some of these intertwined reasons are explored in [2]. By imposing Eq. (4), we follow [7], which in turn reformulates the results presented in [1,9]. The passage from one to the other of the various existing formulations of the above optimization problem are discussed in detail in [7,10].

The entropy optimization yields, for the stationary state,

$$p_i = \frac{e_q^{-\beta_q(E_i-U_q)}}{\bar{Z}_q}\,, \tag{6}$$

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with

$$\beta_q \equiv \frac{\beta}{\sum_{j=1}^{W} p_j^q} , \tag{7}$$

and

$$\bar{Z}_q \equiv \sum_i^W e_q^{-\beta_q(E_i - U_q)} , \tag{8}$$

β being the Lagrange parameter associated with the constraint (4). Eq. (6) makes explicit that the probability distribution is, for fixed β_q, invariant with regard to the arbitrary choice of the zero of energies. The stationary state (or (meta)equilibrium) distribution (6) can be rewritten as follows:

$$p_i = \frac{e_q^{-\beta'_q E_i}}{Z'_q} , \tag{9}$$

with

$$Z'_q \equiv \sum_{j=1}^{W} e_q^{-\beta'_q E_j} , \tag{10}$$

and

$$\beta'_q \equiv \frac{\beta_q}{1 + (1-q)\beta_q U_q} . \tag{11}$$

The form (9) is particularly convenient for many applications where comparison with experimental or computational data is involved. Also, it makes clear that p_i asymptotically decays like $1/E_i^{1/(q-1)}$ for $q > 1$, and has a cutoff for $q < 1$, instead of the exponential decay with E_i for $q = 1$.

The connection to thermodynamics is established in what follows. It can be proved that

$$\frac{1}{T} = \frac{\partial S_q}{\partial U_q} , \tag{12}$$

with $T \equiv 1/(k\beta)$. Also we prove, for the free energy,

$$F_q \equiv U_q - TS_q = -\frac{1}{\beta} \ln_q Z_q , \tag{13}$$

where

$$\ln_q Z_q = \ln_q \bar{Z}_q - \beta U_q . \tag{14}$$

This relation takes into account the trivial fact that, in contrast with what is usually done in BG statistics, the energies $\{E_i\}$ are here referred to U_q in (6). It can also be proved

$$U_q = -\frac{\partial}{\partial \beta} \ln_q Z_q , \tag{15}$$

as well as relations such as

$$C_q \equiv T \frac{\partial S_q}{\partial T} = \frac{\partial U_q}{\partial T} = -T \frac{\partial^2 F_q}{\partial T^2} . \tag{16}$$

In fact, the entire Legendre transformation structure of thermodynamics is q-invariant, which is both remarkable and welcome.

3 Applications

3.1 In diverse systems

The nonadditive entropy S_q and its associated nonetensive statistical mechanics have been applied to a wide variety of natural, artificial and social systems. Among others we may mention (i) The velocity distribution of (cells of) *Hydra viridissima* follows a $q = 3/2$ probability distribution function (PDF) [11]; (ii) The velocity distribution of (cells of) *Dictyostelium discoideum* follows a $q = 5/3$ PDF in the vegetative state and a $q = 2$ PDF in the starved state [12]; (iii) The velocity distribution in defect turbulence [13]; (iv) The velocity distribution of cold atoms in a dissipative optical lattice [14]; (v) The velocity distribution during silo drainage [15,16]; (vi) The velocity distribution in a driven-dissipative 2D dusty plasma, with $q = 1.08 \pm 0.01$ and $q = 1.05 \pm 0.01$ at temperatures of $30000\,K$ and $61000\,K$ respectively [17]; (vii) The spatial (Monte Carlo) distributions of a trapped $^{136}Ba^+$ ion cooled by various classical buffer gases at $300\,K$ [18]; (viii) The distributions of price returns and stock volumes at the stock exchange, as well as the volatility smile [19–22]; (ix) Biological evolution [23]; (x) The distributions of returns in the Ehrenfest's dog-flea model [24,25]; (xi) The distributions of returns in the coherent noise model [26]; (xii) The distributions of returns of the avalanche sizes in the self-organized critical Olami-Feder-Christensen model, as well as in real earthquakes [27]; (xiii) The distributions of angles in the *HMF* model [28]; (xiv) Turbulence in electron plasma [29]; (xv) The relaxation in various paradigmatic spin-glass substances through neutron spin echo experiments [30]; (xvi) Various properties directly related with the time dependence of the width of the ozone layer around the Earth [31]; (xvii) Various properties for conservative and dissipative nonlinear dynamical systems [32–41]; (xviii) The degree distribution of (asymptotically) scale-free networks [42,43]; (xix) Tissue radiation response [44]; (xx) Overdamped motion of interacting particles [45]; (xxi) Rotational population in molecular spectra in plasmas [46]. The systematic study of metastable or long-living states in long-range versions of magnetic models such as the Ising [47] and Heisenberg [48] ones, or in hydrogen-like atoms [49–51] might provide further illustrations.

3.2 In high energy physics

Connections of nonextensive statistics with a specific area of solar physics, astrophysics, high energy physics, and related areas, were pioneered by Quarati and collaborators (see [52], among others), who advanced the possibility of this theory being useful in the discussion of the flux of solar neutrinos. A few years later, it was realized that the transverse momenta distribution of the hadronic jets resulting from electron-positron annihilation are well described by distributions associated with q-exponentials [53,54]: see Figs. 1 and 2. The energy distribution of cosmic rays has been satisfactorily fitted in [55,56] with distributions related to q-exponentials: see Fig. 3. The distributions of returns of magnetic field fluctuations in the solar wind plasma as observed in data from Voyager 1 [57] and from Voyager 2 [58] has provided the values associated with the so called q-triplet: see Figs. 4 e 5. Similar results have been obtained in the study of interstellar turbulence [59] (see Figs. 6 and 7), in X-ray-emitting binary systems [60] (see Fig. 8), and in the distribution of stellar rotational velocities in the Pleiades [61].

It is important to address here the fact that the distribution of transverse momenta in high-energy collisions of proton-proton, and heavy nuclei (e.g., Pb-Pb and Au-Au) have received and are receiving great attention [62–68]: see illustrative examples in Figs. 9-15. Several such data have been summarized in [69]: see Fig. 16. We realize that for such collisions the typical values of q are usually close to 1.10, apparently never above say 1.20-1.25. It remains as a challenging problem to precisely understand why (Is it a hadronization of quark matter in a sort of metastable state before attaining ergodicity?). In any case, it was shown in [71] that QCD calculations and q-statistical calculations can be consistent for $q \simeq 1.1$: see Fig. 17.

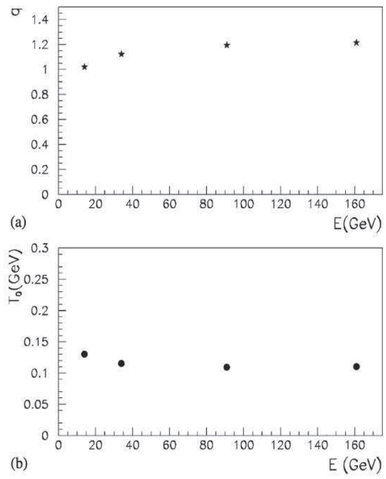

Fig. 2. Dependence of the index q (a) and the temperature T_0 (b) on the collision energies of Fig. 3. The particular case $q = 1$ corresponds to the Hagedorn 1965 theory. It is advanced in [54] the possibility that q approaches the value 11/9 in the $E \to \infty$ limit. See details in [53].

Acknowledgments

I gratefully acknowledge wonderful hospitality received in Budapest at the occasion of the HCBM meeting, very especially from Prof. T.S. Biro. Naturally, the authors of the various figures that I have reproduced in the present brief review are acknowledged as well.

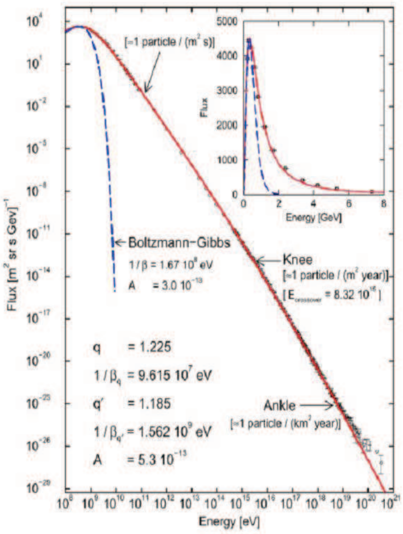

Fig. 3. Flux of cosmic rays. Curiously enough, the upper value of the index q is very close to 11/9. See details in [55,56].

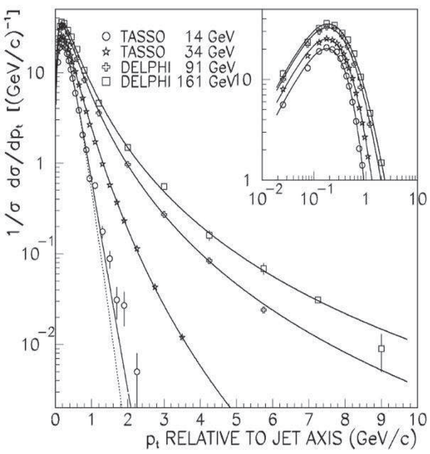

Fig. 1. Distributions of transverse momenta for four typical values of the collision energy. See details in [53].

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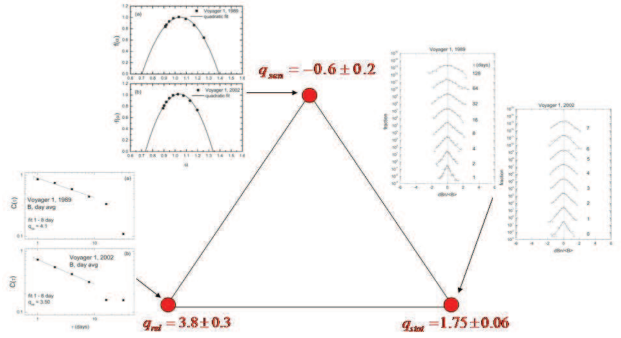

Fig. 4. The q-triplet as obtained from data of the Voyager 1. See details in [57].

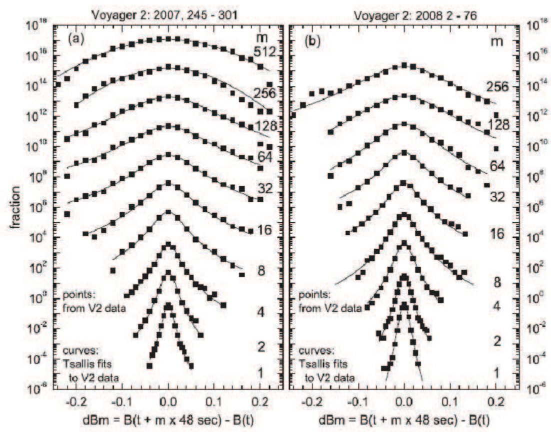

Fig. 5. Distributions from which q_{stat} is extracted, from data of the Voyager 2. See details in [58].

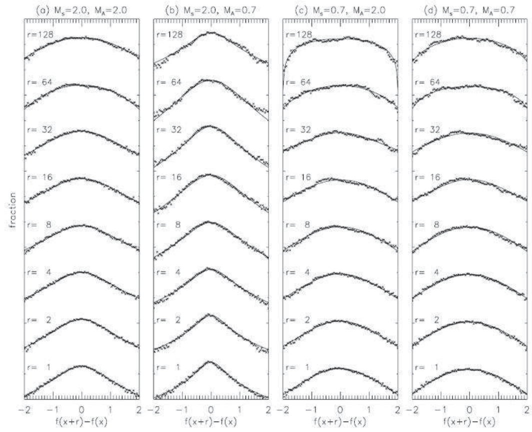

Fig. 6. Distributions of column density fluctuations for different spatial separations. See details in [59].

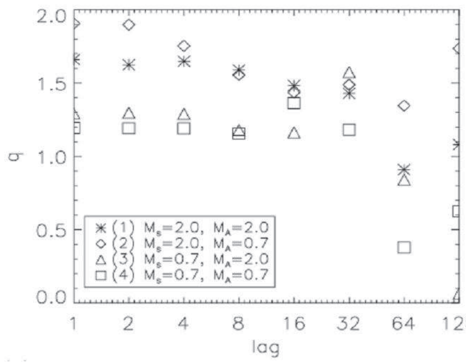

Fig. 7. Values of the index q_{stat} corresponding to Fig. 6. See details in [59].

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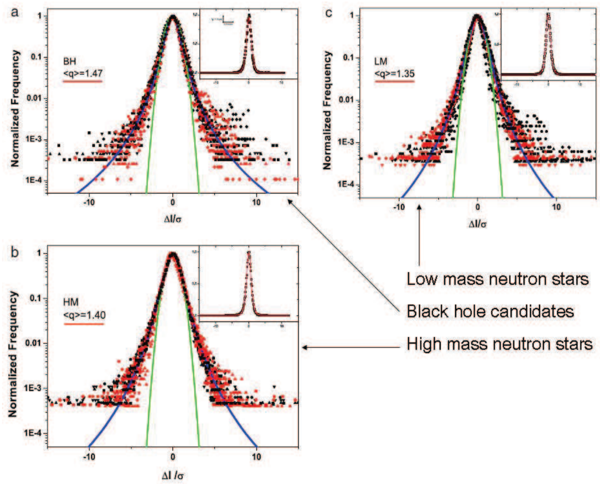

Fig. 8. Distributions of intensities of X-ray emission. See details in [60].

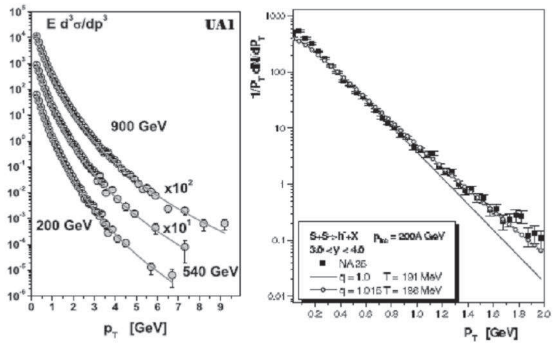

Fig. 9. Transverse momenta distributions for different energies. See details in [62].

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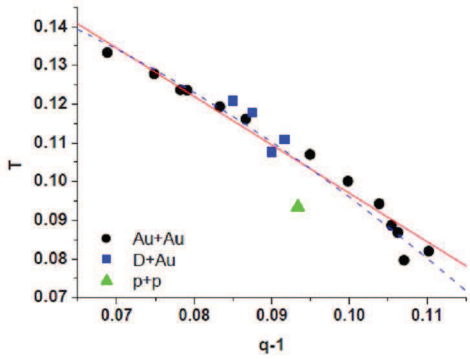

Fig. 10. Dependence of the temperature T on the index q for production of negative pions in different reactions. See details in [62].

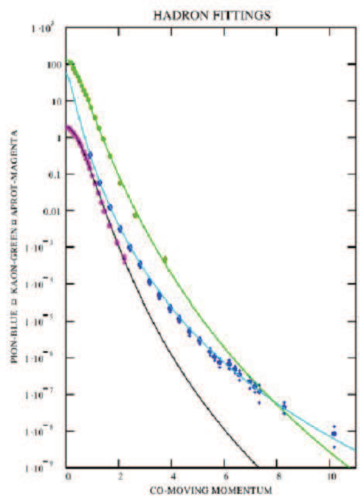

Fig. 11. Transverse momenta spectra for pions, kaons and antiprotons in relativistic heavy ions experiments. See details in [63].

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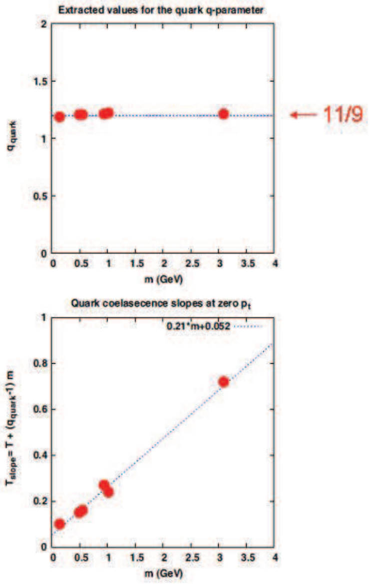

Fig. 12. The index q (top) and the temperature (bottom) extracted from hadronic spectra assuming quark coalescence at a sudden hadron formation. See details in [63].

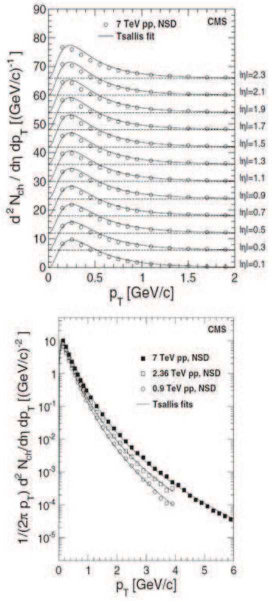

Fig. 13. Transverse momenta distributions of charged hadrons in pp collisions, as measured by the CMS Collaboration at LHC, corresponding to 0.9, 2.36 and 7 TeV. At these energies it has been obtained (q, T) = (1.13, 0.13 GeV), (1.15, 0.14 GeV), (1.15, o.145 GeV) respectively. See details in [64, 65].

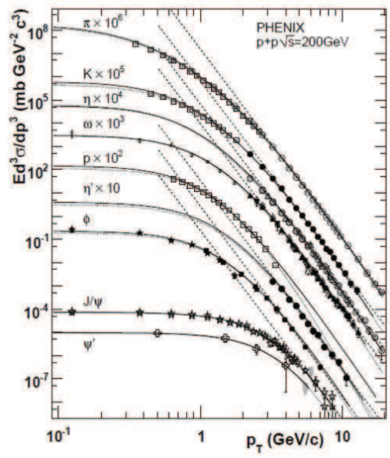

Fig. 14. Transverse momenta distributions of various hadrons in pp collisions, as measured by the PHENIX Collaboration, corresponding to 200 GeV. At this energy it has been obtained $q \simeq 1.10$. See details in [67].

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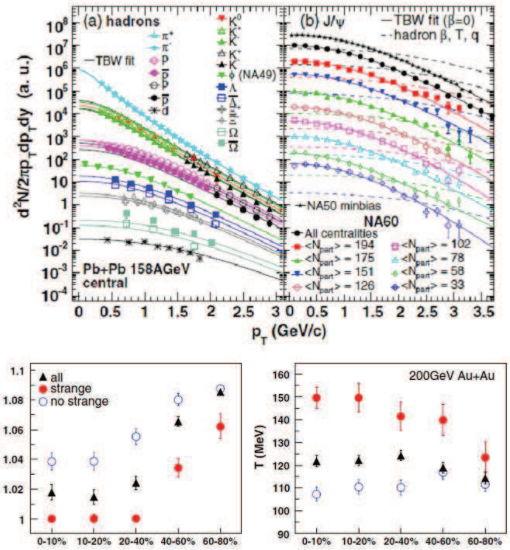

Fig. 15. Transverse momenta distributions of charged hadrons in pp and heavy ion collisions, as measured in Brookhaven. See details in [68].

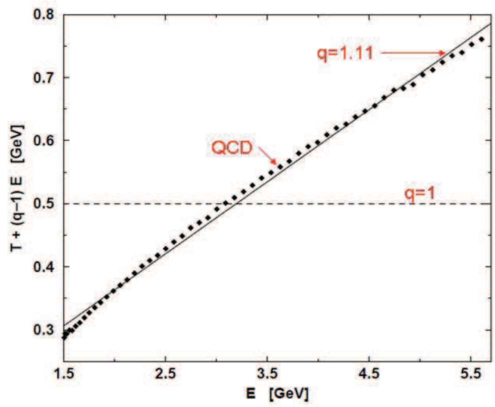

Fig. 17. Comparison of QCD diffusion calculation with its corresponding within q-statistics: they are consistent for q = 1.11. See details in [71].

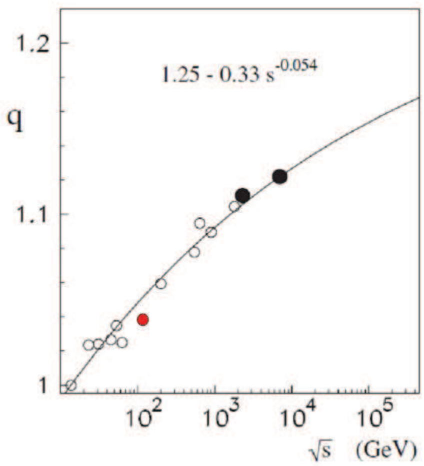

Fig. 16. Index q obtained at various energies. See details in [69]. The black dots indicate recent CMS results. The red dot indicates the value obtained in [70].

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