

Tsallis distribution in high-energy heavy ion collisions

Gergely Gábor Barnaföldi

KFKI RMKI of the HAS

in collaboration

T.S. Bíró, G. Kalmár, K. Ürmössy, P. Ván

Hot & Cold Baryonic Matter 2010, Budapest 15-19 August 2010

OUTLINE

- Motivation:
- High & low- p_T hadron spectra
- Can non-extensive thermodyn. resolve this?
- Notes on hadronization
- Simple joint models, tests & outlook

MOTIVATION – for the future

- Measuring jet fragmentation (PID)

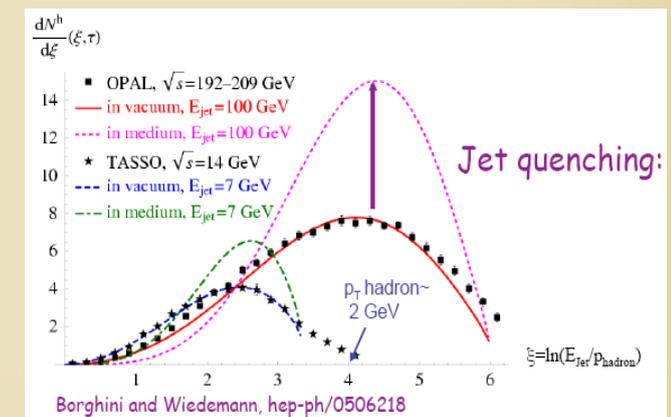
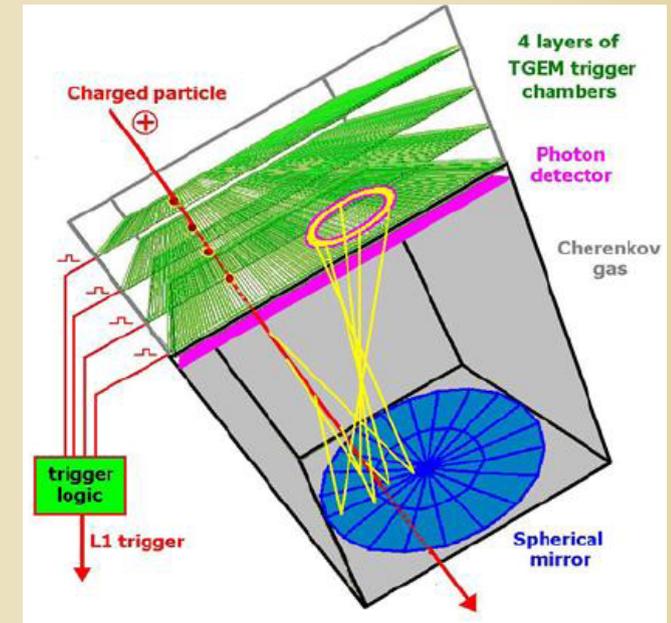
Available: PID & jet fragmentation can be measured via jet analysis, data are from: DESY, Tevatron, RHIC, LHC...
See: more on **D. Berényi's poster**

- Future PID measurement in HIC

Under development: ALICE VHMPID detectors might measure it See more on: **S. Pochybová, A. Agócs**

- Fragmentation in "matter"

Aim/dream: Measure differential jet-parameters, identifying final state effects (jet-quenching modifications).



MOTIVATION - recently

- New LHC pp data (CMS)

JHEP 1002:041(2010)

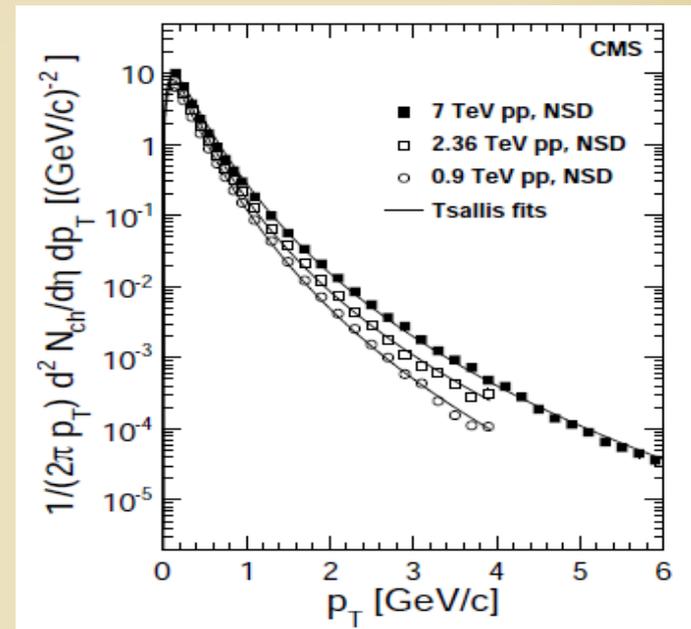
fitted Tsallis distribution for p_T spectra:

$$E \frac{d^3 N_{\text{ch}}}{dp^3} = \frac{1}{2\pi p_T} \frac{E}{p} \frac{d^2 N_{\text{ch}}}{d\eta dp_T} = C(n, T, m) \frac{dN_{\text{ch}}}{dy} \left(1 + \frac{E_T}{nT} \right)^{-n}$$

Parameters:

0.9 TeV $T = 130$ MeV, $q = 1.13$

2.36 TeV $T = 140$ MeV, $q = 1.15$



$$n := (q-1)^{-1}$$

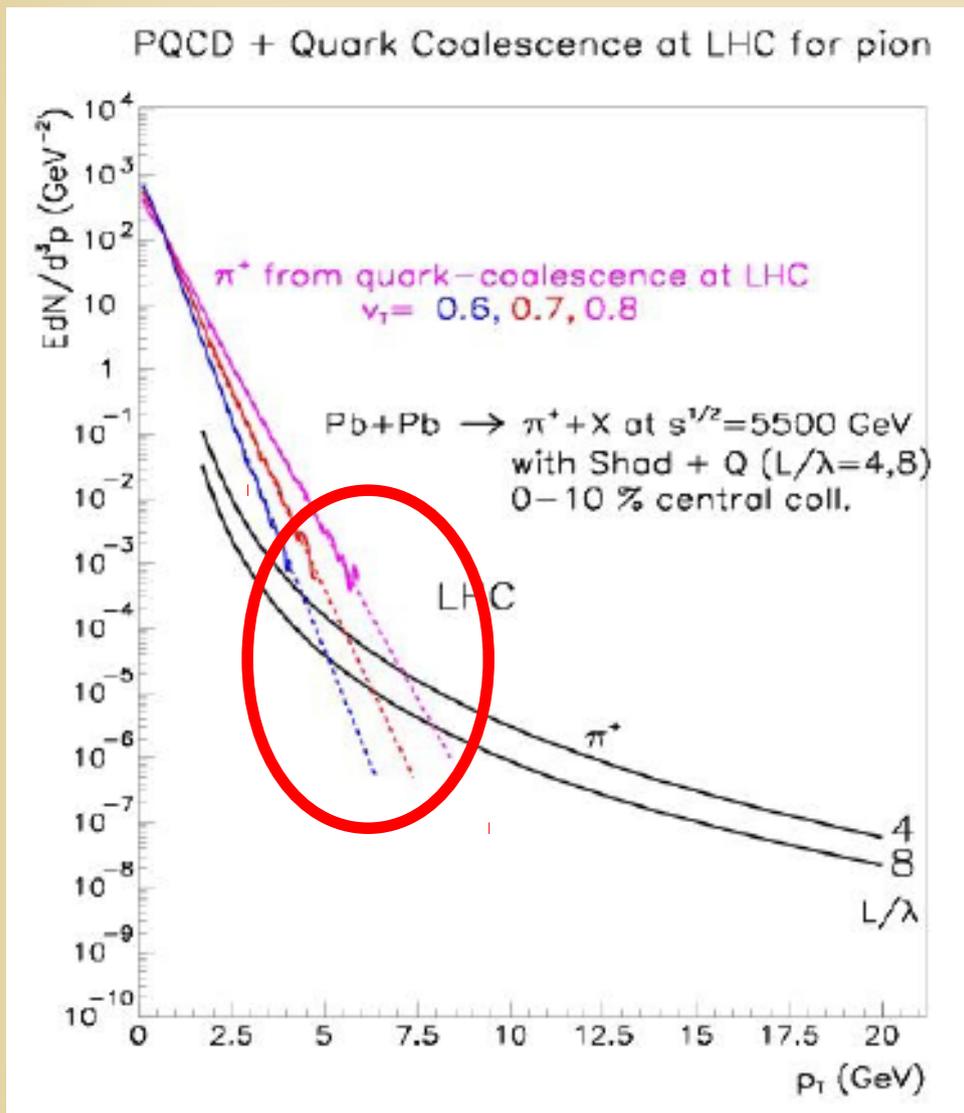
- RHIC analysis on AuAu data ($y=0$)

Cooper-Frye model: K. Ürmössy, T.S. Bíró: PL B689 14 (2010)

Parameters: $f(E) = A[1 + (q-1)E/T]^{-1/(q-1)}$

200 GeV $T = 51$ MeV, $q = 1.062$ (fit for $p_T < 6$ GeV/c)

MOTIVATION - past...

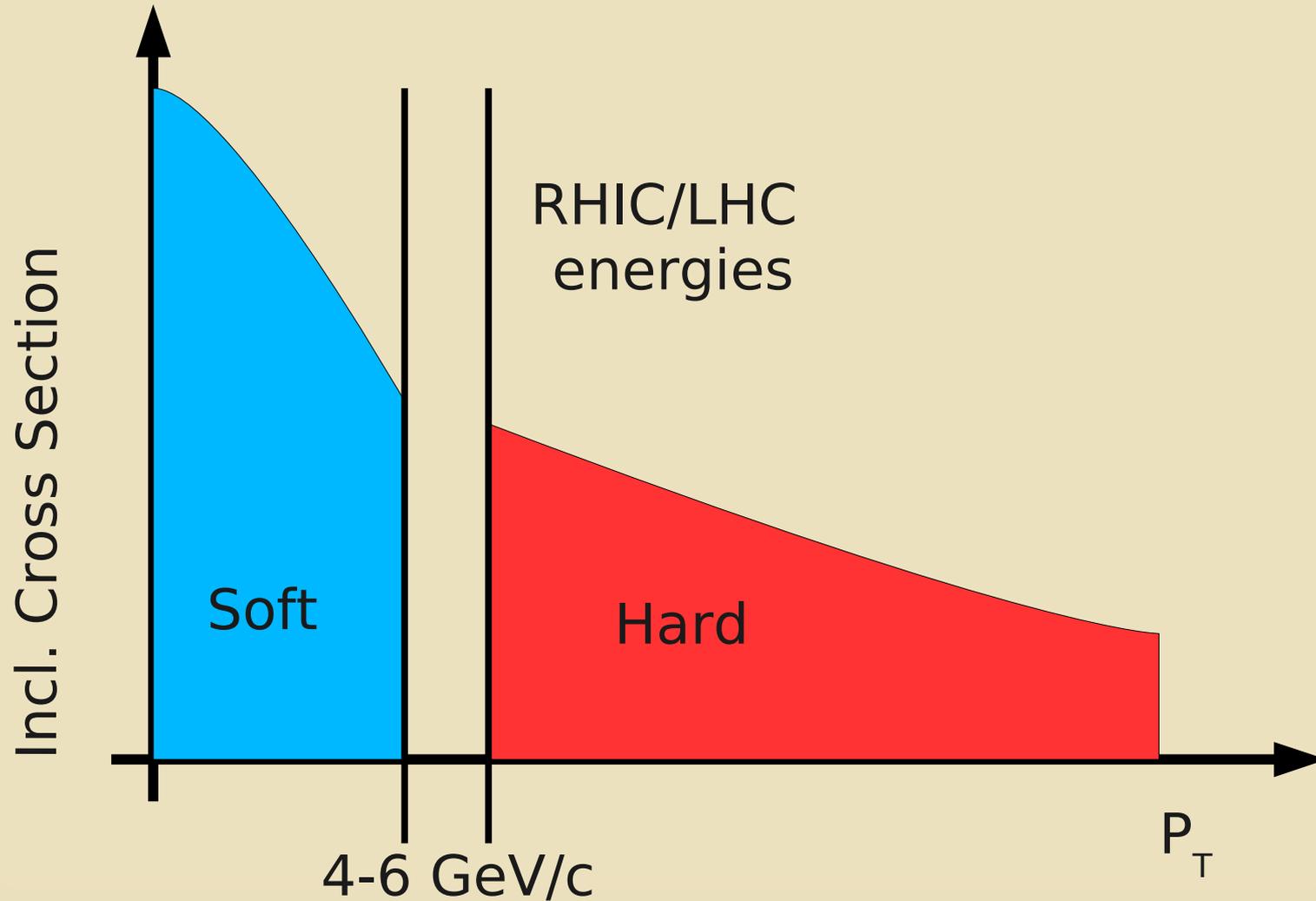


- pQCD based parton model:
 - QCD at $T \rightarrow 0$ temperature
 - power law distribution
 - strong dependence on FF
 - good for high- p_T hadrons
- Quark-coalescence model
 - Thermal, finite temperature
 - exponential distribution $e^{-m/T}$
 - parton-hadron duality
 - good for high- p_T hadrons

P. Lévai, GGB, G. Fai: JPG35, 104111 (2008)

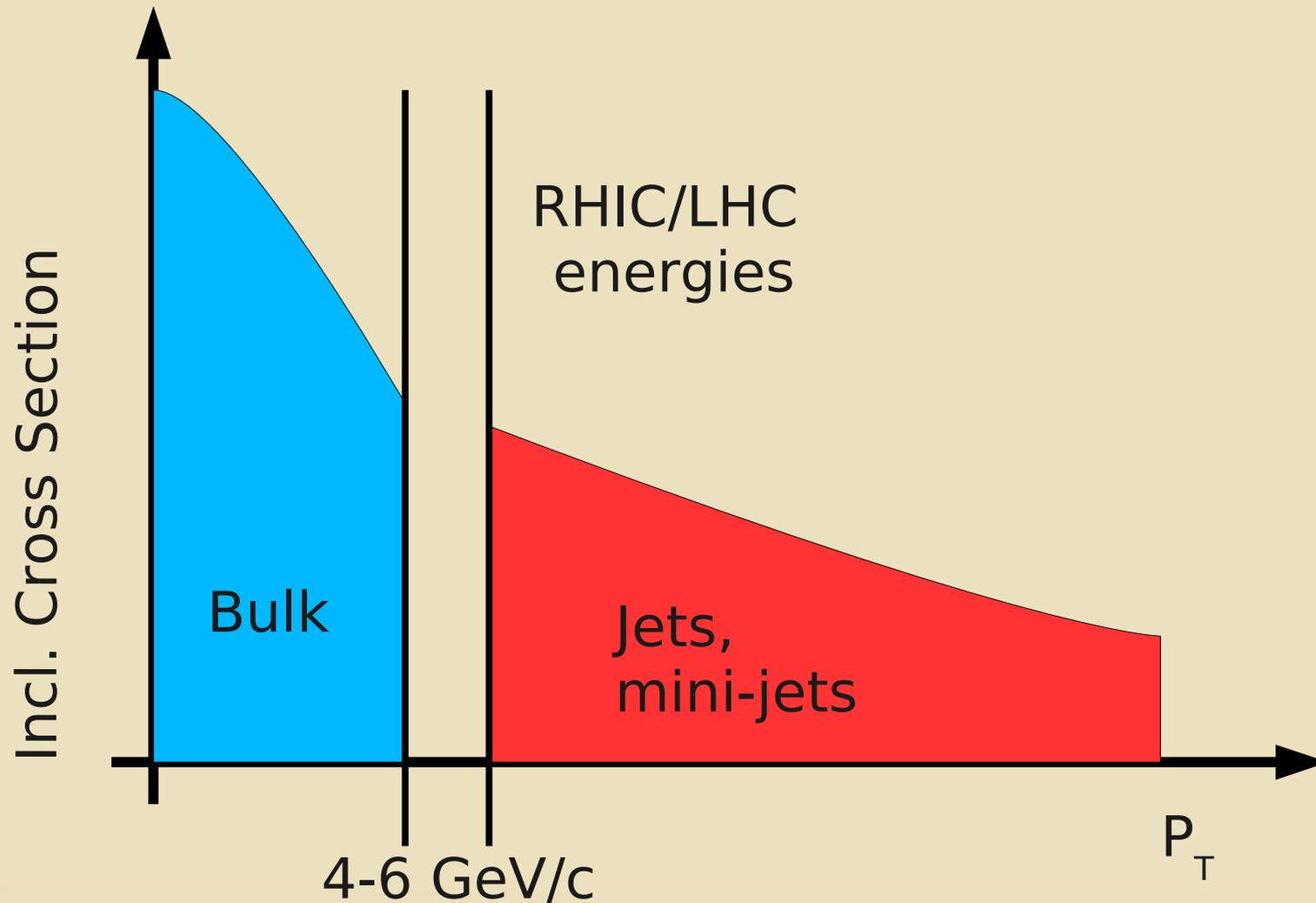
High & low p_T hadron spectra

- 1. interpretation 'soft' & 'hard' processes



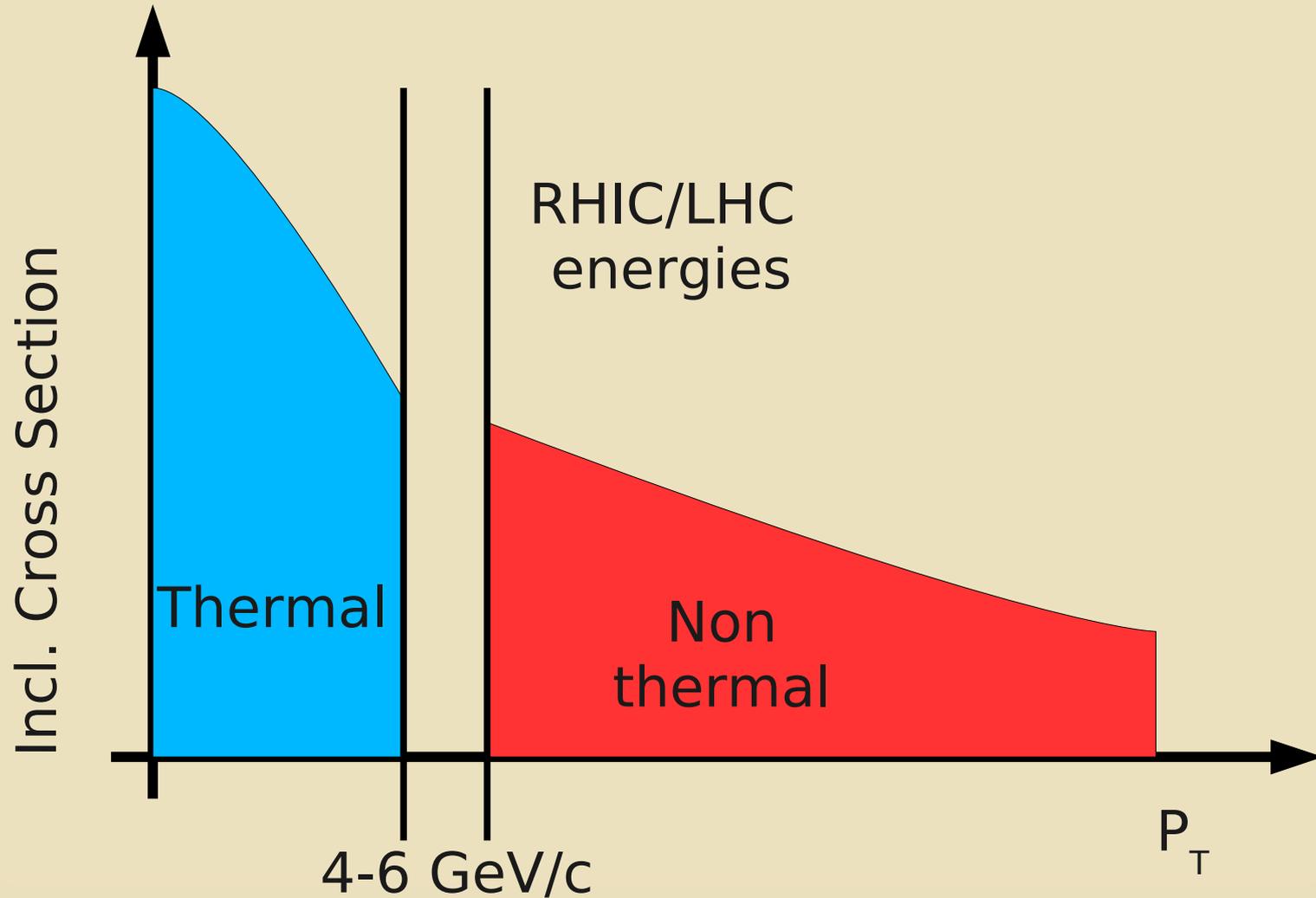
High & low p_T hadron spectra

- 2. interpretation 'bulk' & 'jet-like' processes



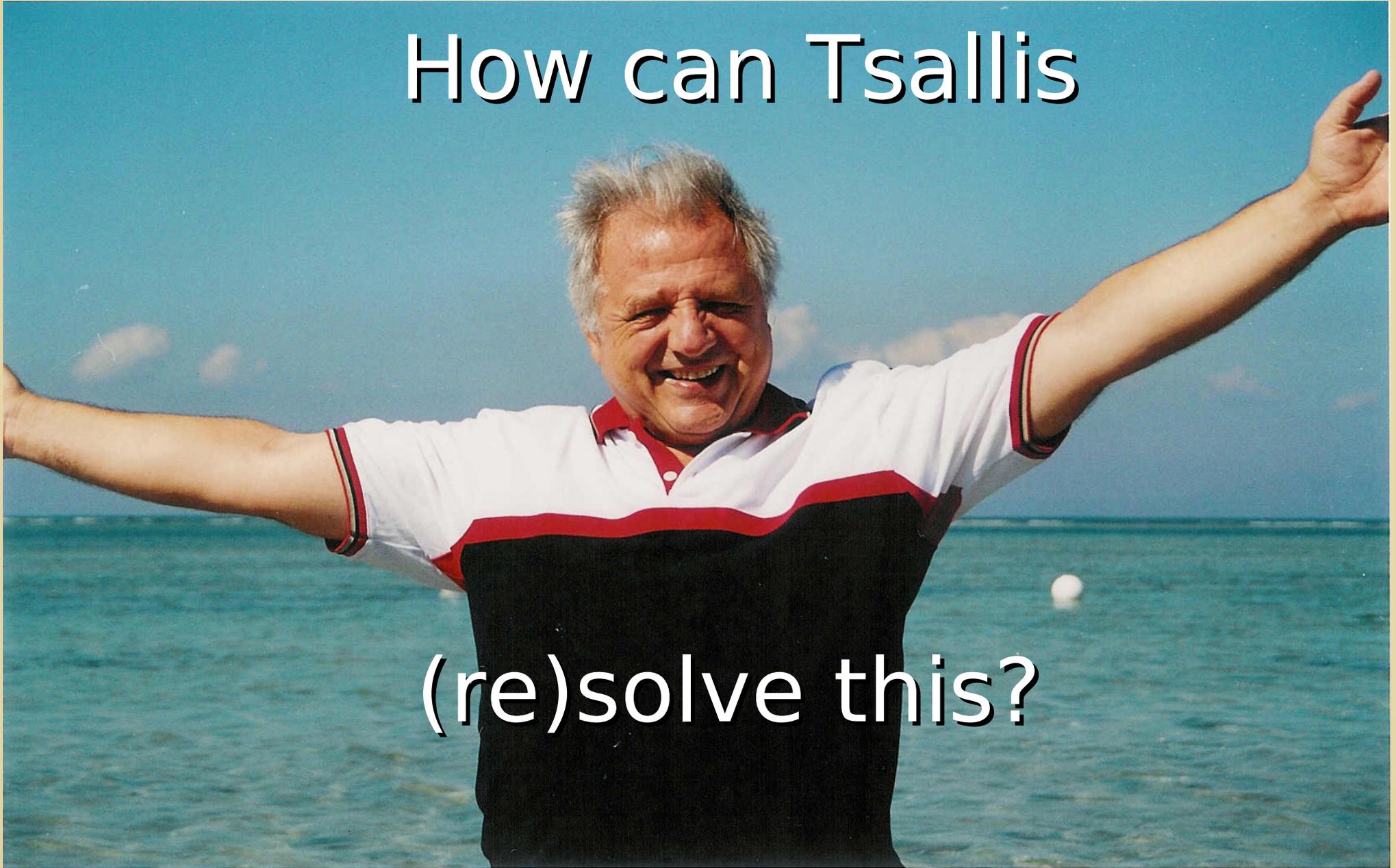
High & low p_T hadron spectra

- 3. interpretation 'thermal' & 'non-thermal' models



How can Tsallis

(re)solve this?



Basics of non-extensive thermodynamics

Non-extensive thermodynamics (Based on: T.S. Biró: EPL84, 56003,2008)
associative composition rule, (non-additive) :

$$h(h(x, y), z) = h(x, h(y, z))$$

Then should exist a strict monotonic function, $X(x)$ 'generalised logarithm'
(an entropy-like quantity), for which:

$$h(x, y) = X^{-1}(X(x) + X(y))$$

$$X(h(x, y)) = X(x) + X(y).$$

Example: (i) Classical thermodynamics:

$$f(E) = e^{-\beta E} / Z$$

$$h(x, y) = x + y.$$

(ii) Tsallis distribution

$$h(x, y) = x + y + axy$$

$$a = q - 1$$

$$f(E) = \frac{1}{Z} e^{-\frac{\beta}{a} \ln(1+aE)} = \frac{1}{Z} (1 + aE)^{-\beta/a}$$

$$S = \int f \frac{e^{-a \ln(f)} - 1}{a} = \frac{1}{a} \int (f^{1-a} - f).$$

Associative composition \Rightarrow evolution eq.

Non-extensive Gibbs, generalised

logarithm: $f(x) = \frac{1}{Z} e^{-\beta X(x)}$

Composition rule for sub-systems:

$$x_N(y) := \underbrace{h \circ \dots \circ h}_{N-1} \left(\frac{y}{N}, \dots, \frac{y}{N} \right)$$

Meanwhile satisfy: $\lim_{N \rightarrow \infty} x_N(y) < \infty$

Asymptotically, if $N_1, N_2 \rightarrow \infty$:

$$x_{N_1+N_2} = \varphi(x_{N_1}, x_{N_2})$$

recursive equation can be given:

$$x_n = h \left(x_{n-1}, \frac{y}{N} \right), \text{ where } h(x, 0) = x.$$



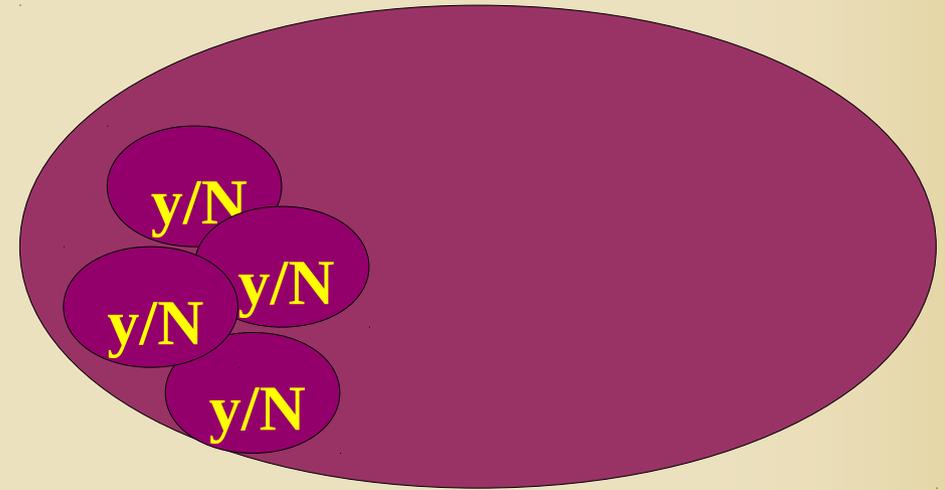
$$x_n - x_{n-1} = h \left(x_{n-1}, \frac{y}{N} \right) - h(x_{n-1}, 0)$$

Evolution equation can carry out:

$$\frac{dx}{dt} = \frac{y}{t_f} h'_2(x, 0^+)$$



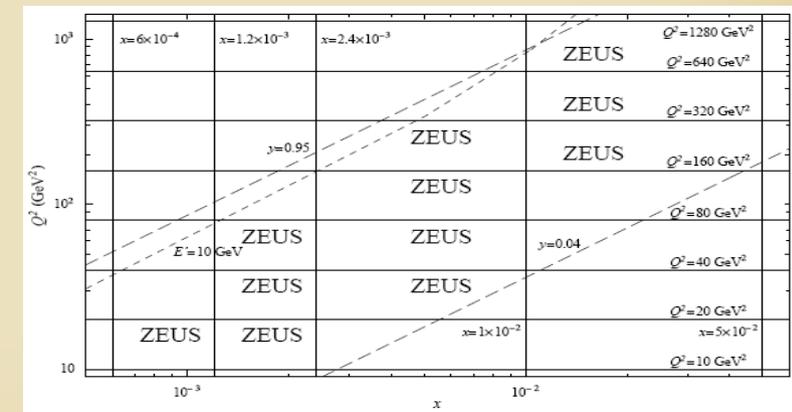
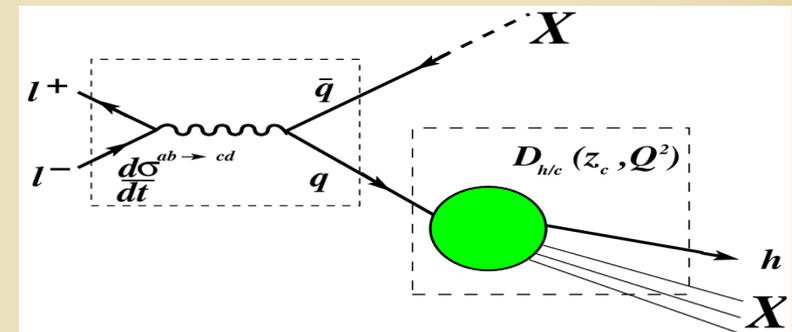
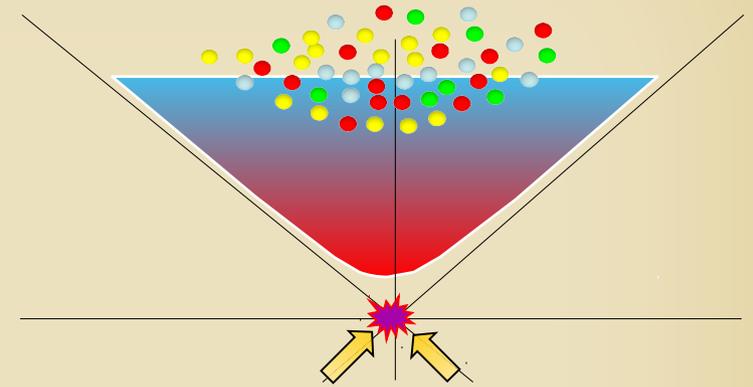
$$L(x) = \int_0^x \frac{dz}{h'_2(z, 0^+)} = y \frac{t}{t_f}$$



Notes on hadronization:
integrated vs. statistical

Hadronization processes & fragmentation

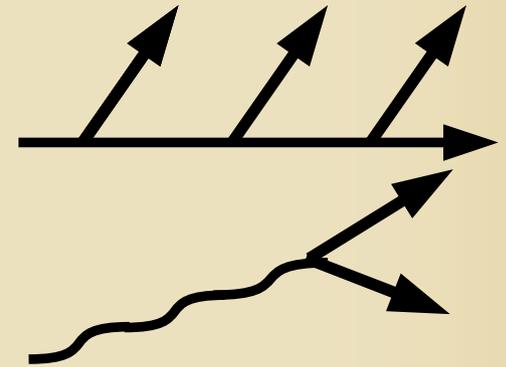
- Hadronization: requires a model, based on local parton-hadron duality (kvanturnum numbers & momenta connected to a cone around or to the leading particle.)
- Parton/hadron shower evolution comes from statistical processes (step-by-step MC evolution). → **microscopical**
- Fragmentation function (FF) carries integrated (phenomenological) information on how parton fragment into hadron. → **integrated distribution**
- Measurement lepton-antilepton annihilation, HIC, etc...



Models for fragmentation

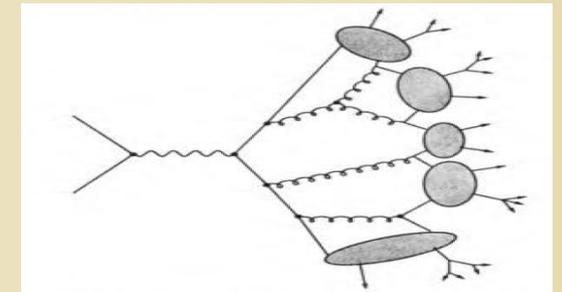
- Independent fragmentation model (Feynman - Field)

Simplest model for fragmentation by Field & Feynman : q & g channels



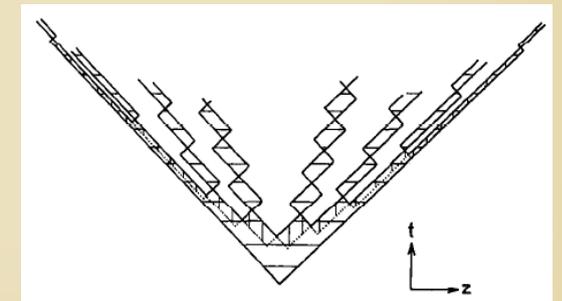
- (Quark) string model

color strings between $q\bar{q}$, breaking into quark-antiquark pair \rightarrow mesons



- Cluster model (Lund model)

Lund model: phase-space separation, forming clusters: $q\bar{q} \rightarrow M$, $qqq \rightarrow B$

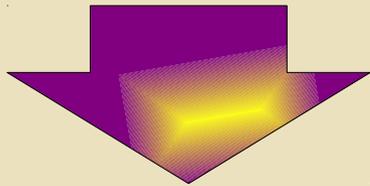


Fragmentation processes in parton model

In a pQCD based parton model, fragmentation functions (FF) gives how parton (a) fragment into a hadron (h), $D_{h/a}(z, Q^2)$.

DGLAP scale evolution:

$$\frac{\partial}{\partial \ln Q^2} D_i^h(x, Q^2) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_S}{4\pi} P_{ji}\left(\frac{x}{z}, Q^2\right) D_i^h(z, Q^2)$$

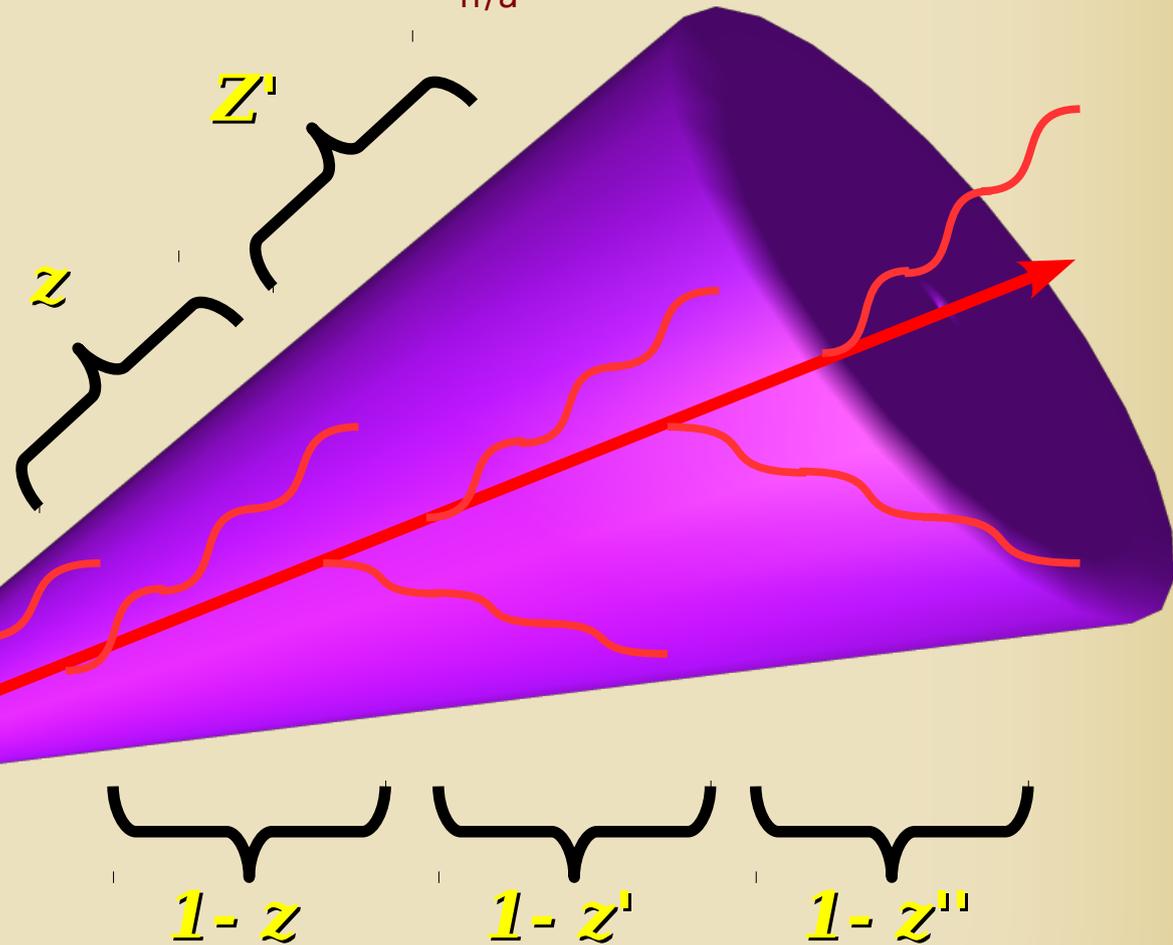
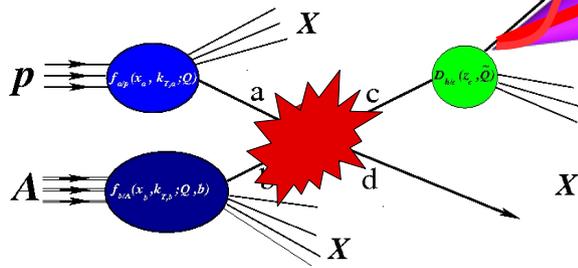


$$E_\pi \frac{d\sigma_\pi^{pA}}{d^3 p_\pi} \sim f_{a/p}(x_a, Q^2; k_T) \otimes f_{b/A}(x_b, Q^2; k_T, b) \otimes \frac{d\sigma^{ab \rightarrow cd}}{d\hat{t}} \otimes \frac{D_{\pi/c}(z_c, \hat{Q}^2)}{\pi z_c^2}$$

$f_{b/A}(x_a, Q^2; k_T, b)$: Parton Dist. Function (PDF), at scale Q^2

$D_{\pi/c}(z_c, \hat{Q}^2)$: Fragmentation Function for π (FF), at scale \hat{Q}^2

$\frac{d\sigma^{ab \rightarrow cd}}{d\hat{t}}$: Partonic cross section



Fragmentation via associative composition

Program:

1) Search and fit Tsallis distribution to data.

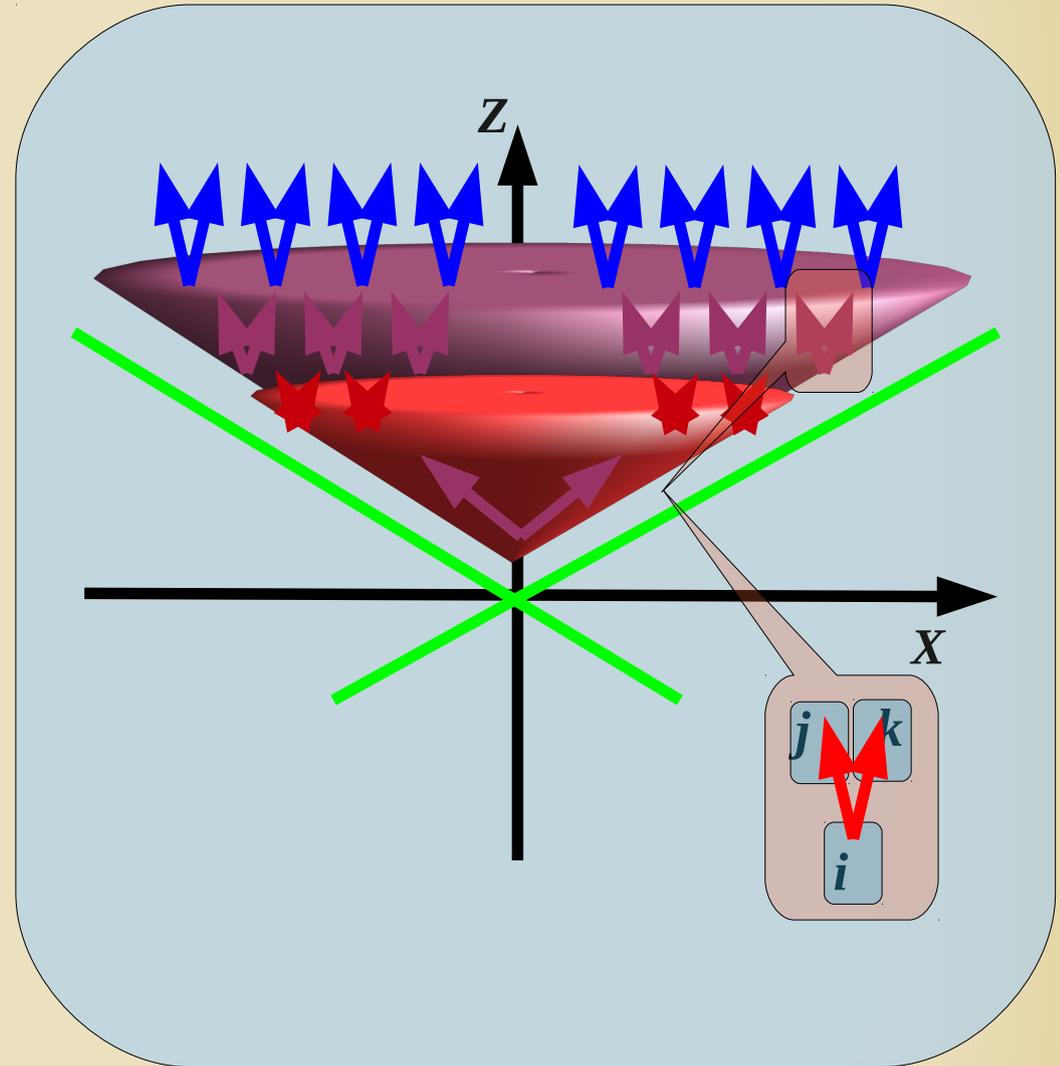
2) Search for physical meaning of T and q parameters.

3) Components of the sub-systems are e.g. 'splitting functions' P_{qg} , P_{gg}

4) Test: can a BFKL / DGLAP-like evolution equation be obtained?

$$D(x, Q^2) \sim f(E, T, q) * f(\ln(Q^2))$$

$$D(x, Q^2) \sim f(E, T(\ln(Q^2)), q(\ln(Q^2)))$$

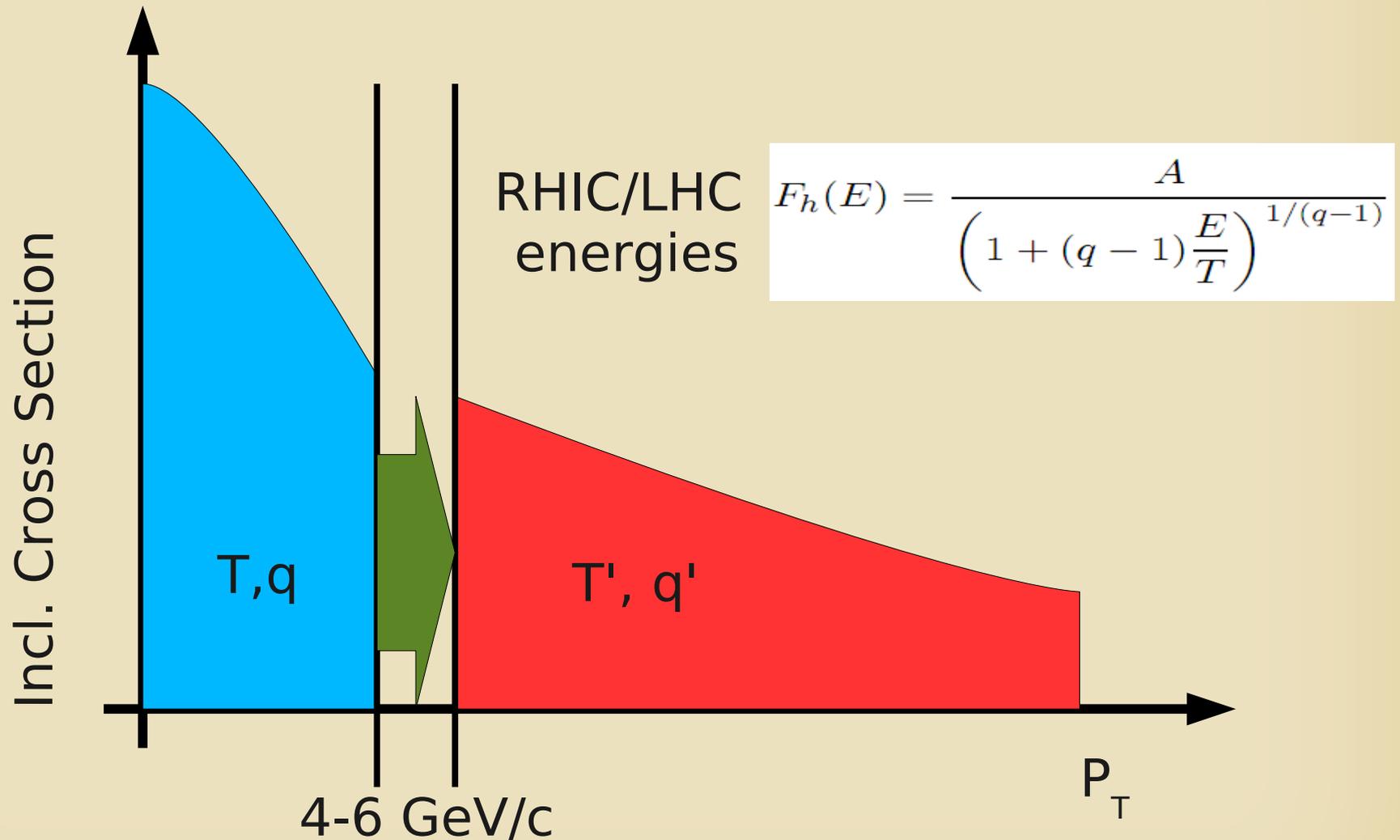


A test in two models

joint ideas...

1st way: find the distribution

- A suggested new way: Tsallis (like) distribution



1st joint model: recombination & pQCD in AA

Find a distribution for low & high momentum spectra:

$$e^{-\beta E}$$



$$F_h(E) = \frac{A}{\left(1 + (q-1)\frac{E}{T}\right)^{1/(q-1)}}$$

In AA collisions particle energy modified by the flow:

$$E = \gamma(m_t - v_{flow}p_t), \quad m_t = \sqrt{m^2 + p_t^2}$$

Slope for particle spectra can be fitted:

$$T_{slope} = -\frac{\partial E}{\partial \ln(F_h(E))} = T(1 + aE) = T + (q-1)E$$

furthermore, if $E \gg m$, then $T \rightarrow T \left[\frac{1+v}{1-v} \right]^{1/2}$

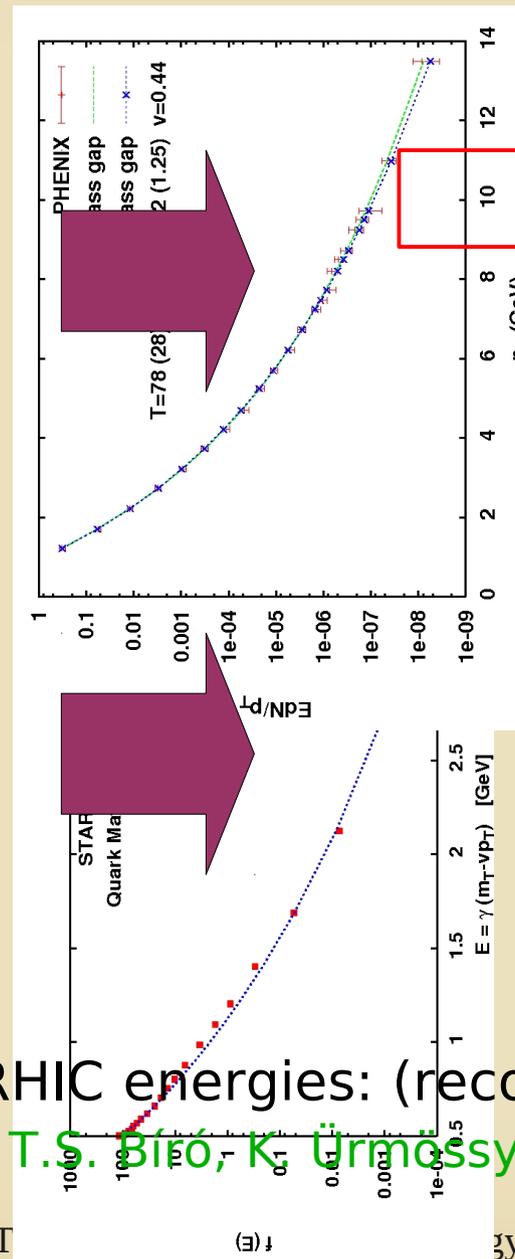
See more on details in talks by: K. Ürmösy and P. Ván

1st joint model: recombination & pQCD in AA

Pion spectrum

Kaon spectrum

Fitted T & q Tsallis parameters at RHIC energies: (recombination and NLO pQCD+AKK FF, $v=0.5$ for kaons): [T.S. Bíró, K. Ürmössy, GGB: JPG35, 04401 \(2008\)](#)



1st joint model: recombination & pQCD in AA

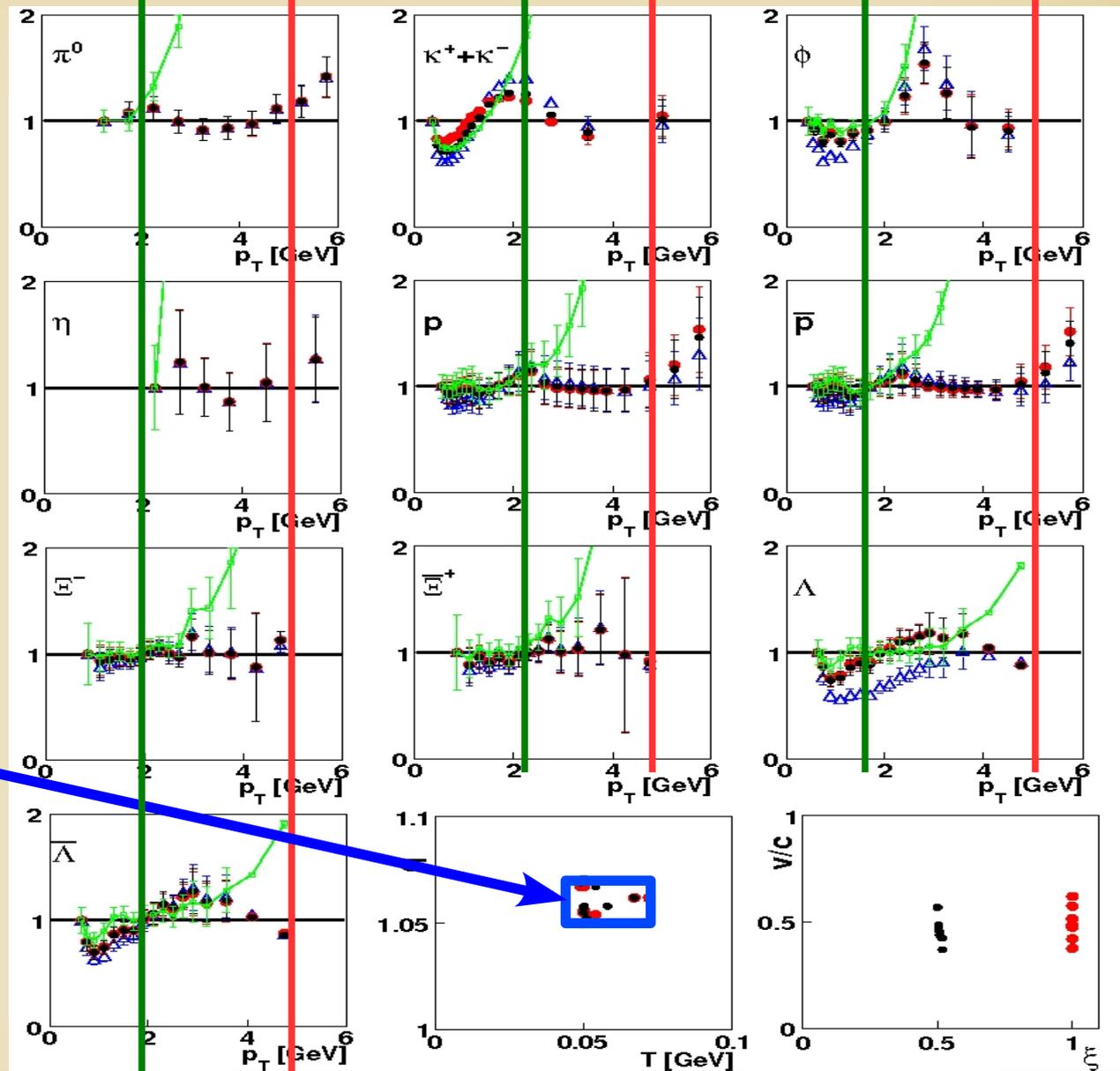
Analysing
Meson & Baryon
spectra in
AuAu collisions

- Ratio of theoretical or experimental p_T spectra in $y=0$, AuAu collisions fitted by Tsallis distribution.

$T=50-70\text{MeV}$,
 $q=1.06-1.07$

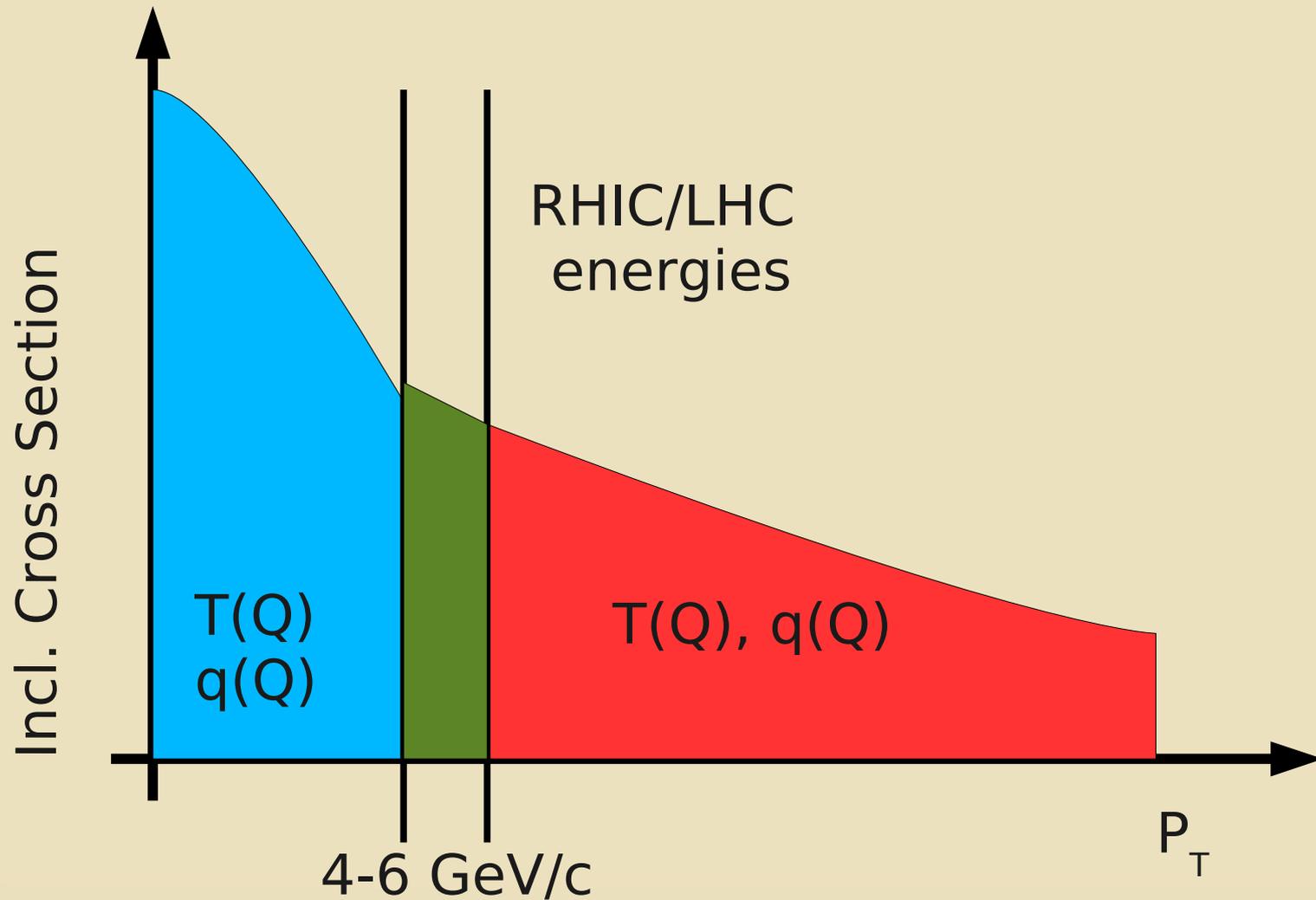
- Here the fit is only for $p_T < 6 \text{ GeV}/c$.

- K. Ürmössy, T.S. Bíró: PLB689:14 (2010)



2nd way: to resolve the gap...

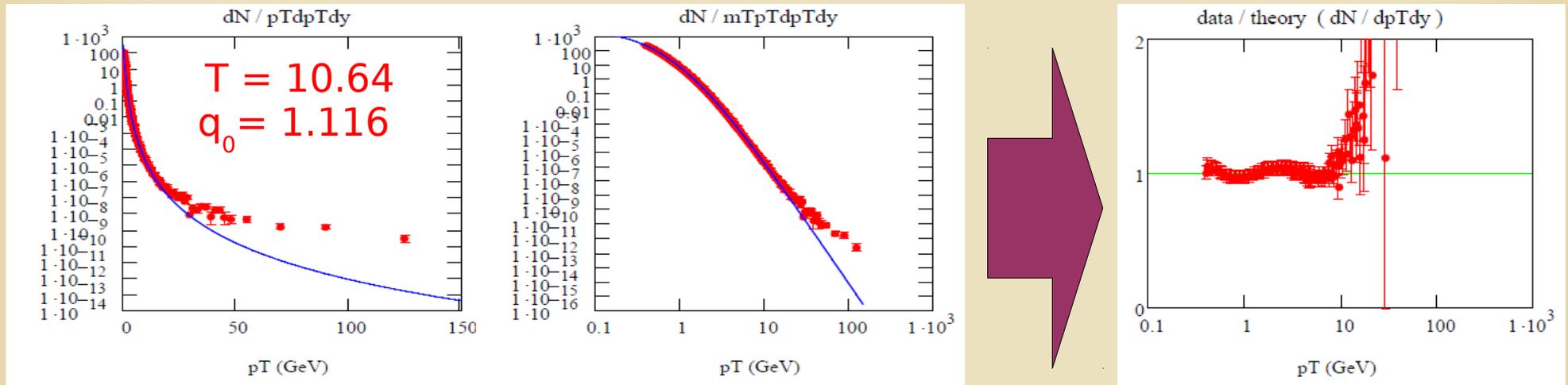
- Suggested interpretation: Tsallis + evolution



2nd joint model: Tsallis with evolution

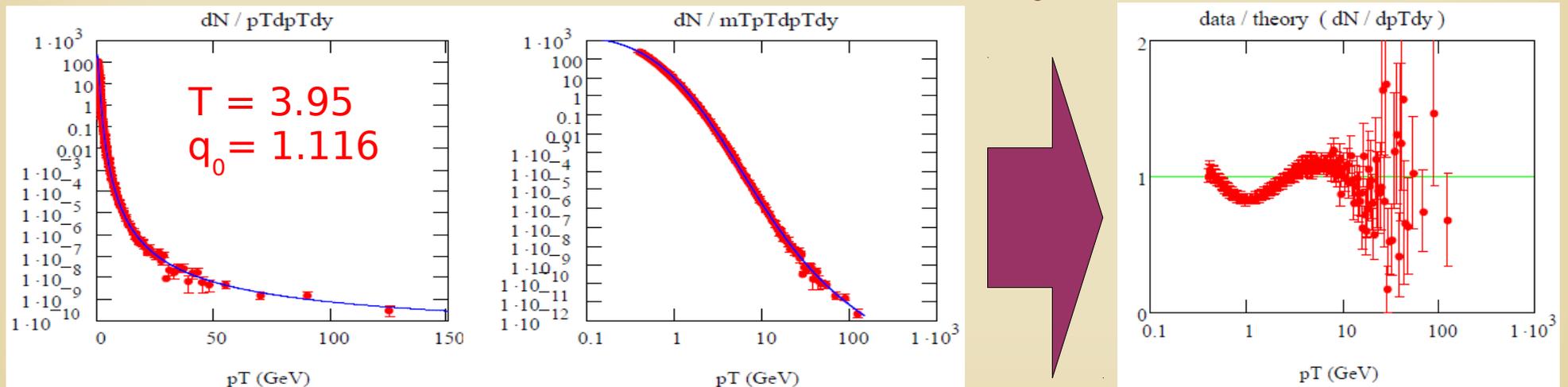
- TEST on CDF ch. hadron data in pp @ 1.96 TeV $|y| < 1$

NO evolution



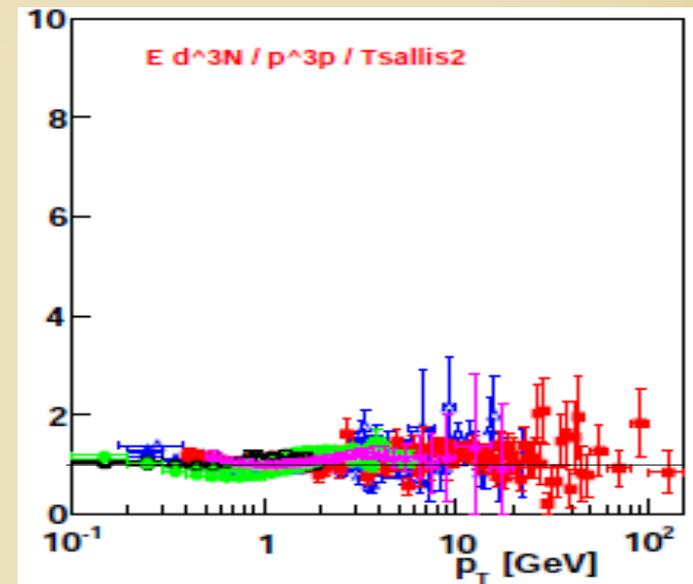
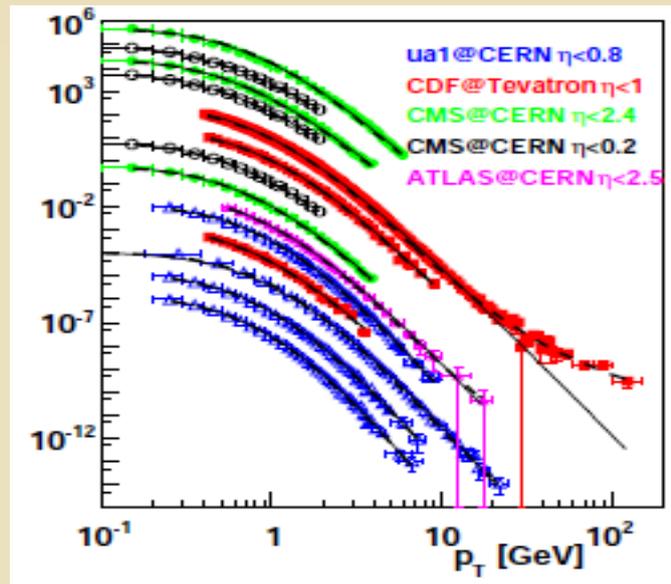
- DGLAP motivated evolution: $n = (q_0 - 1)^{-1} - 2 * \log(\log(Q))$

With evolution

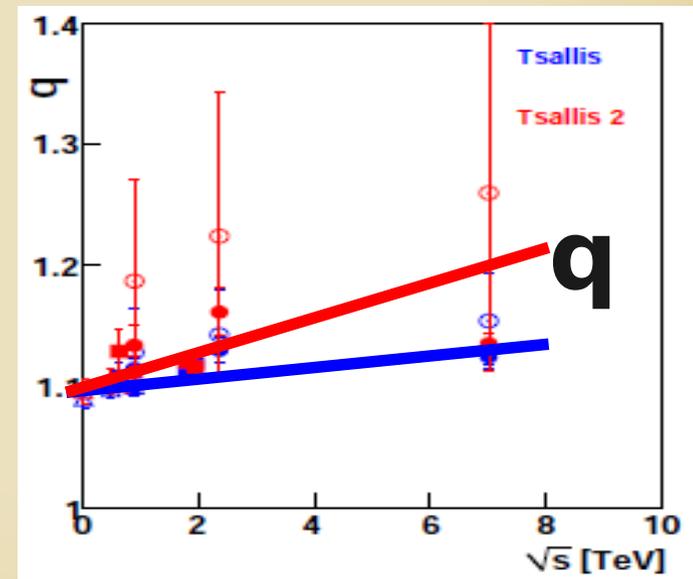
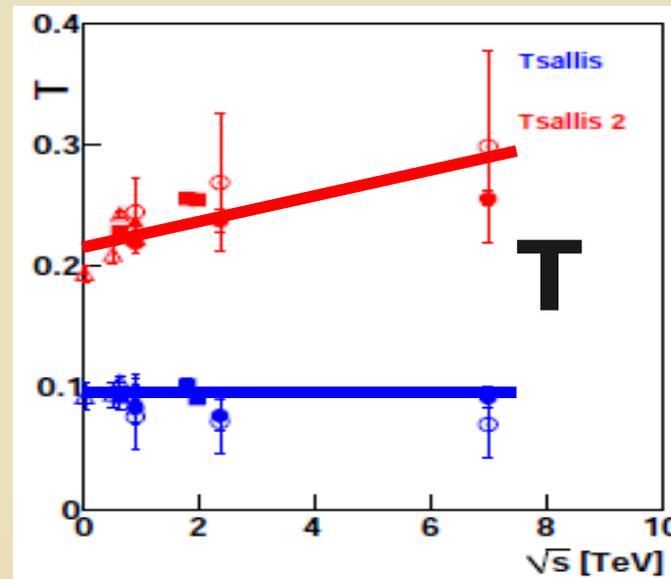


2nd joint model: Tsallis with evolution

- More TEST: 0.2 - 7 TeV midrapidity data

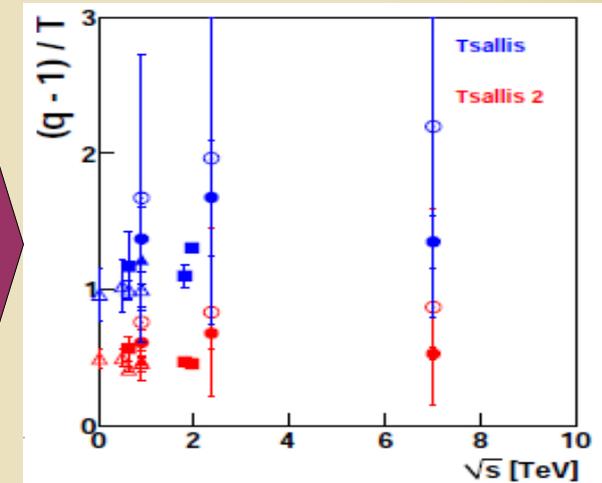
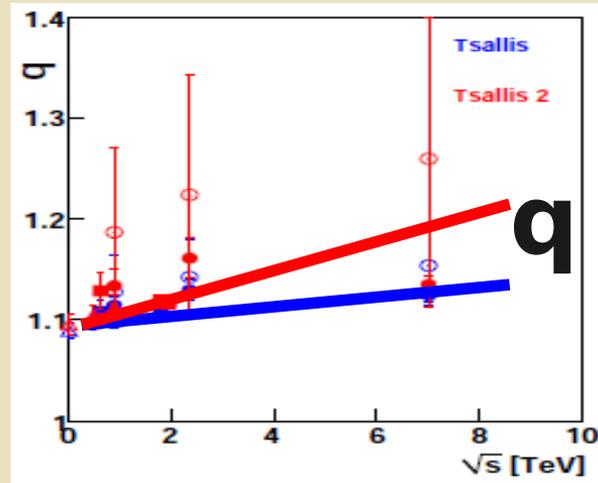
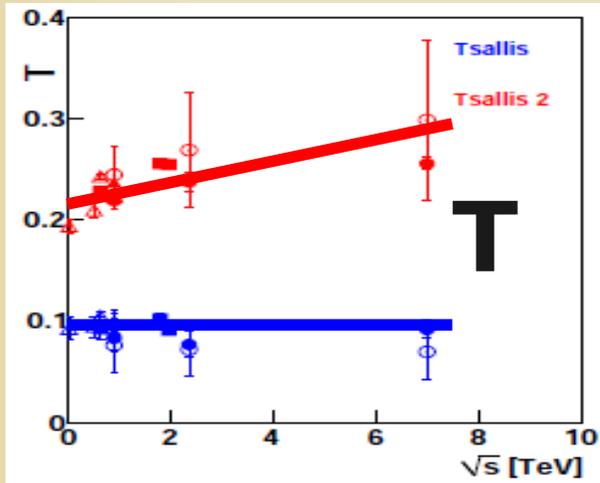


- C.m. Energy dependence of the T & q parameters

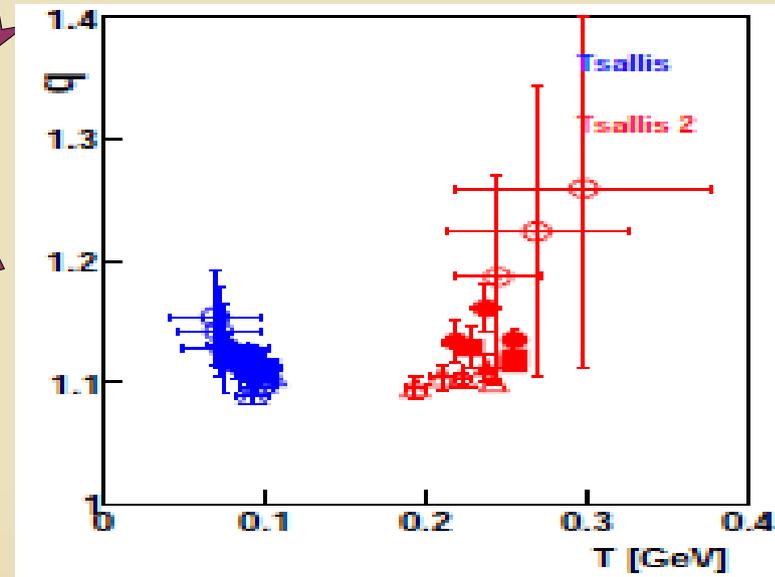


2nd joint model: Tsallis on q-T plane

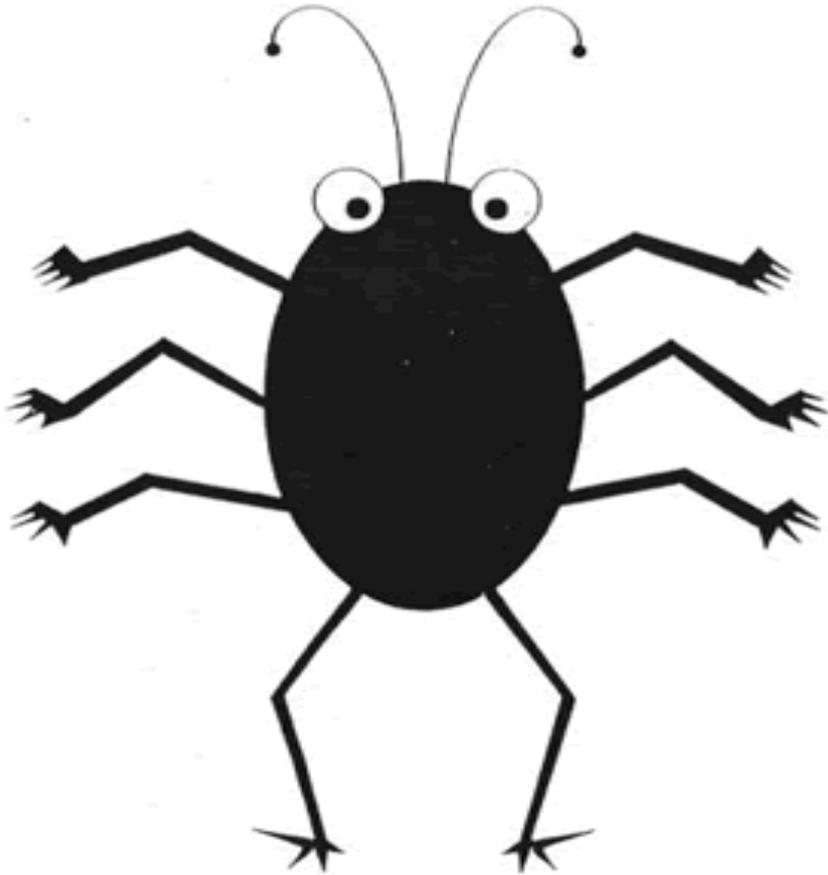
- TEST on various midrapidity data @ different cm



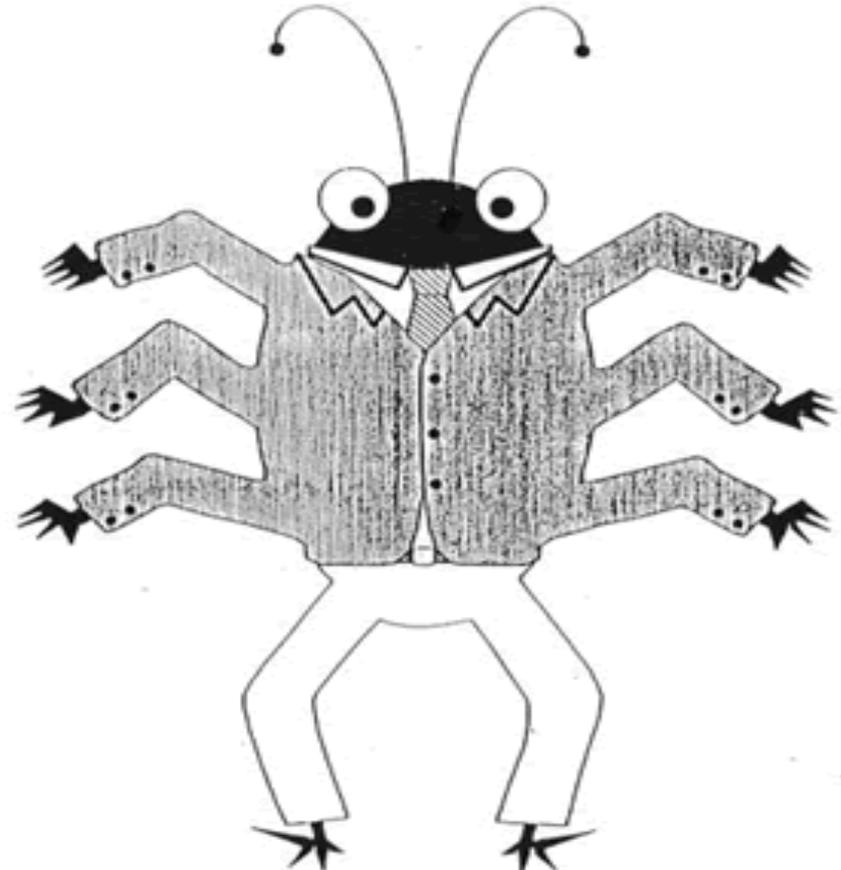
- The q-T plane



A new question on the market...



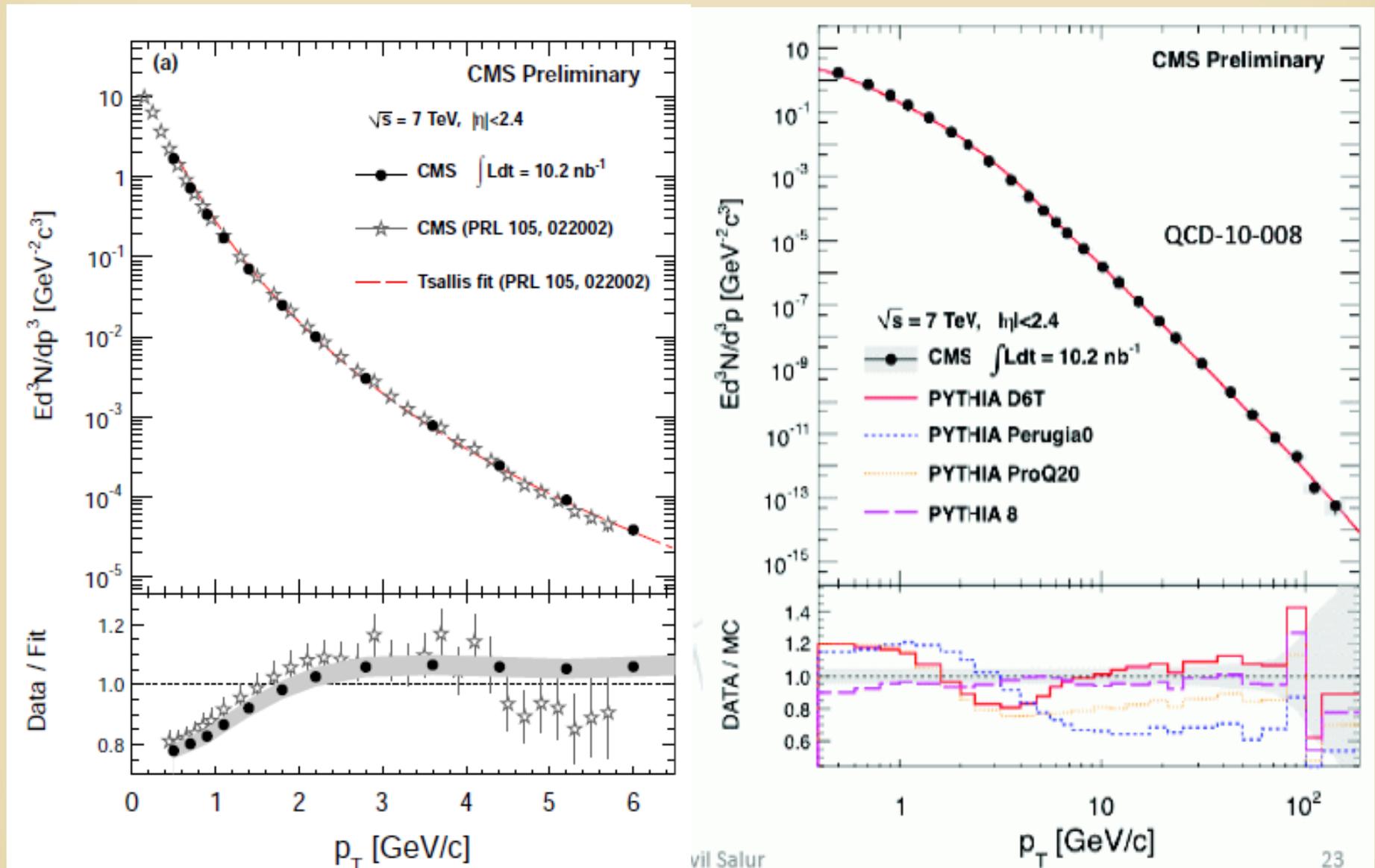
BUG



FEATURE

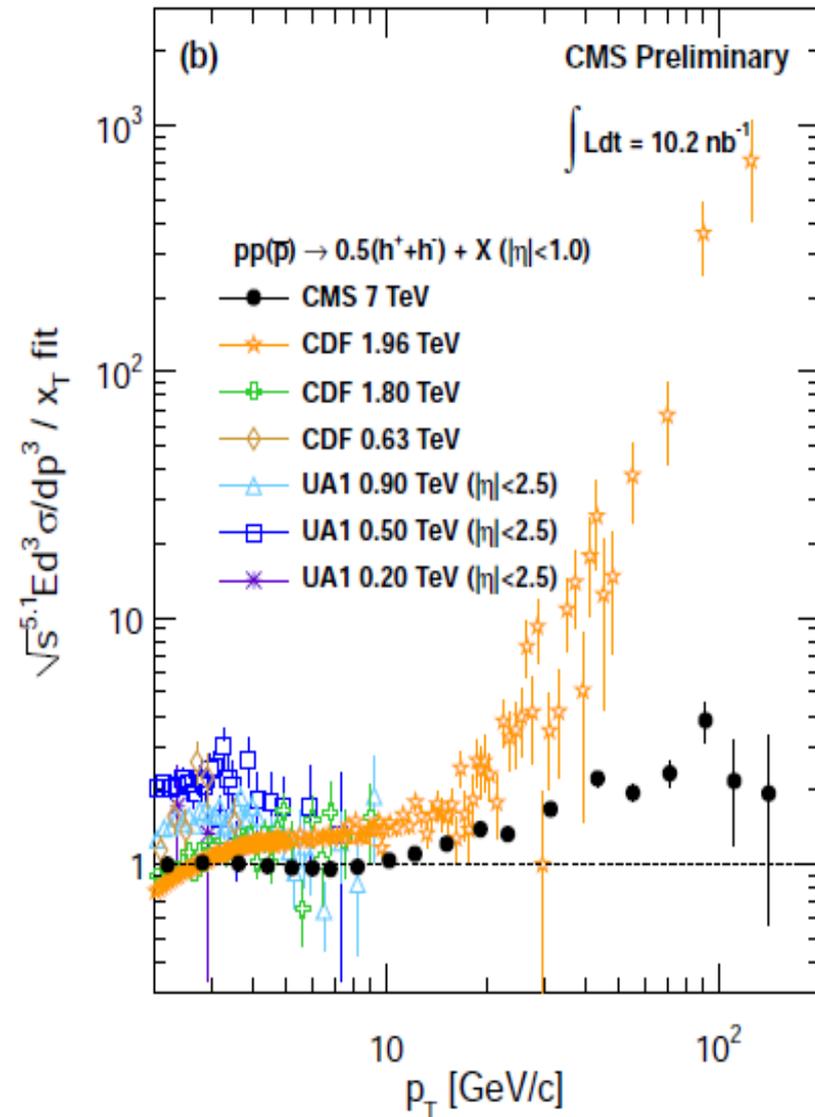
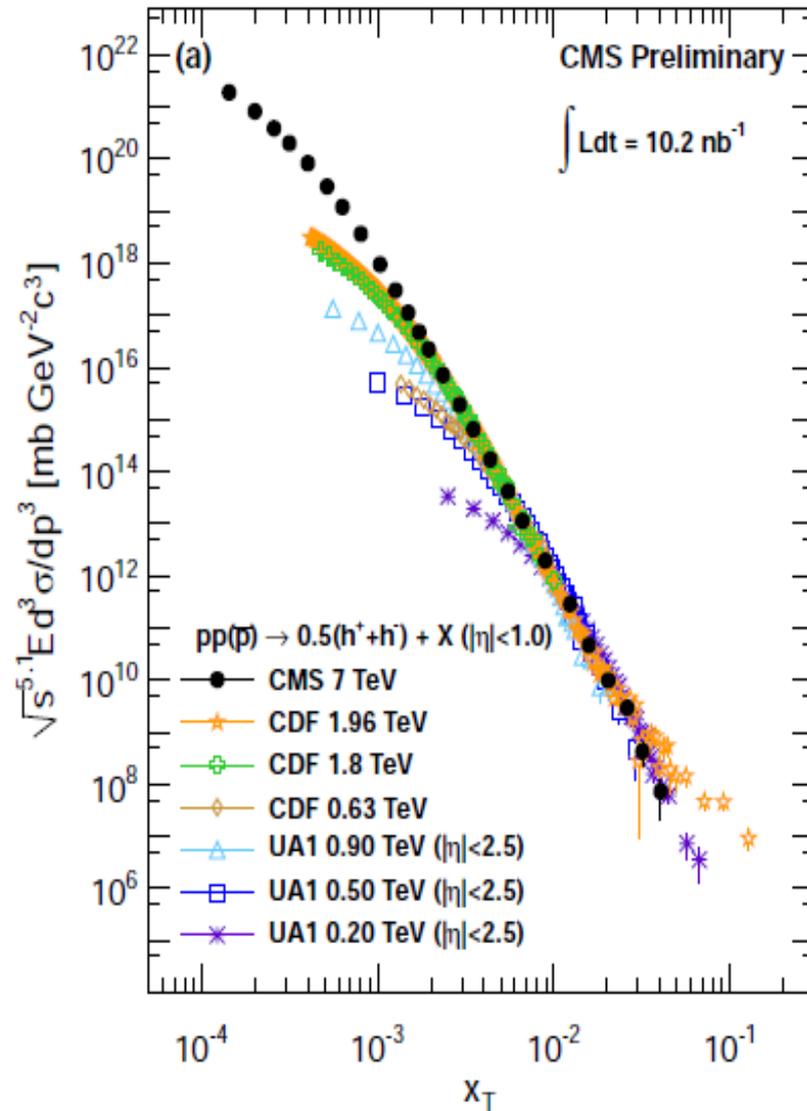
New (HOT) data at LHC energies – today

See: ALICE: Prague Jet workshop & CMS: QCD-10-008



Comparison in x_T : old/new data by Tevatron

See: CDF: PRD 79 112005 (2009) & CMS QCD-10-008



How to resolve this 'question'..

- Experimentalists' point: simply: wrong data
- Earlier signs:

F. Arleo et al.: hep-ph/1003.2963v2 LAPTH-015/10, ICCUB-10-018

- P. Lévai et al.:

See: P. Lévai's talk

- Our phenomenology:

(i) Tsallis distribution fits well, but parameters vary by e.g. energy

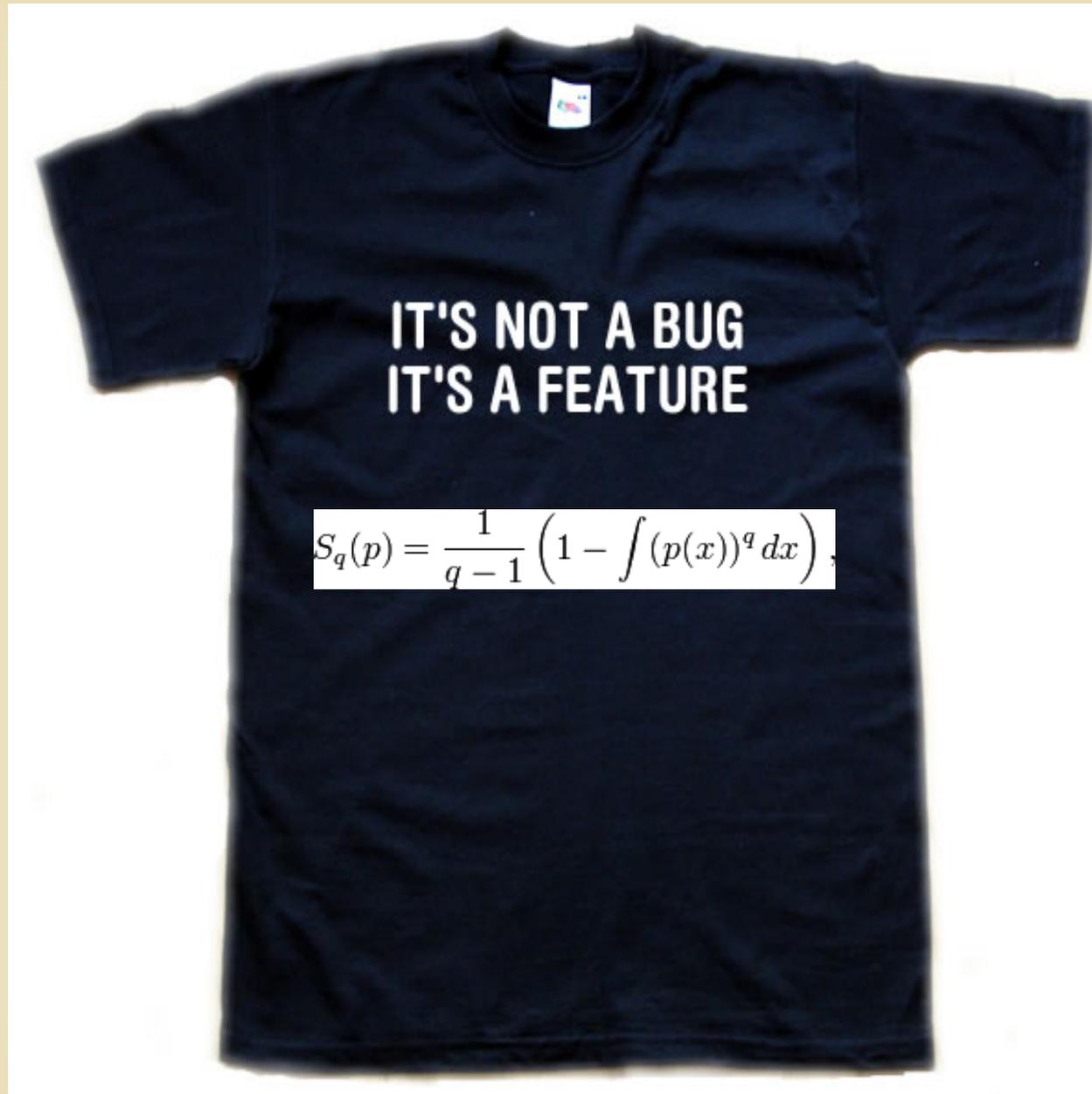
(ii) Need for evolution of the parameters: like e.g. in hadronization, (p)QCD based paron model (i.e. BFKL, DGLAP).

Note, fits are good basically for hadrons...

S U M M A R Y

- High & low p_T spectra has different distribution..
...however hadronization should not work differently.
- Non-extensive (non-equilibrium) thermodynamic
Can be applied generally. Based on composition rules, evolution eq. can be obtain even for non-thermalised case.
- Tests and models with Tsallis assumptions
Seems working for hadron production up to intermediate p_T , and extension to the highest p_T region is still question.

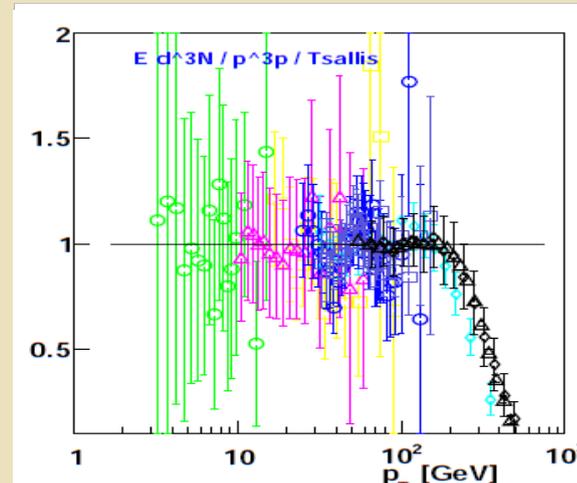
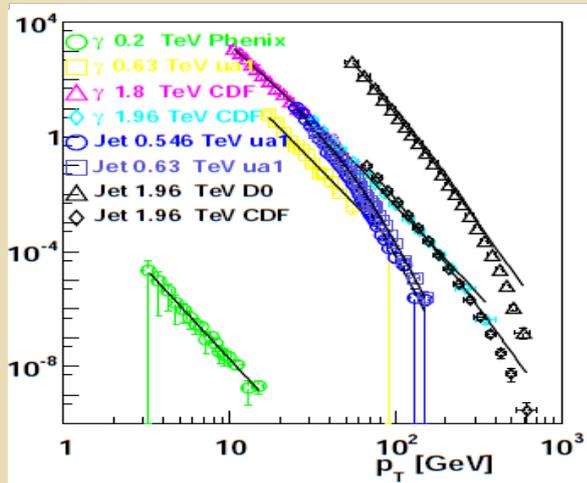
... so we hope...



B A C K U P

An extra to 2nd joint model: Tsallis on jets

- TEST on jets and direct photon production – on q only



- Fits data with irrelevant T values, but tail is described

