Recent Theoretical Developments in the QCD Phase Diagram

HCBM 2010
Budapest, Aug. 16-19, 2010

Jochen Wambach, TU Darmstadt and GSI

outline:

1. the quark-hadron transition at vanishing μ
2. a 'quarkyonic phase' at large μ and small T?
3. inhomogeneous phases and the CEP

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The quark-hadron transition at vanishing μ

3-flavor PQM model

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PQM model contains essential features of QCD (symmetries, anomalies,..)
same universality class as QCD

model Lagrangian:

$$\mathcal{L}_{PQM} = \mathcal{L}_{quark} + \mathcal{L}_{meson} + \mathcal{L}_{pol}$$

quark part:

$$\mathcal{L}_{quark} = \bar{q} \left(i\partial\!\!\!/ - G \frac{\lambda_a}{2} \left(\sigma_a + i\gamma_5 \pi_a \right) \right) q$$

meson part:

$$\begin{aligned} \mathcal{L}_{meson} &= \mathrm{Tr}(\partial_\mu M^\dagger \partial^\mu M) - m^2 \mathrm{Tr}(M^\dagger M) - \lambda_1 [\mathrm{Tr}(M^\dagger M)]^2 - \lambda_2 \mathrm{Tr}(M^\dagger M)^2 \\ &\quad + c \left(\det(M) + \det(M^\dagger) \right) + \mathrm{Tr}[H(M + M^\dagger)] \end{aligned}$$

with

$$M = \sum_a \frac{\lambda_a}{2} \left(\sigma_a + i\pi_a \right) ; \;\; H = \sum_a \frac{\lambda_a}{2} h_a$$

Polyakov loop:

$$\mathcal{L}_{pol} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\ell, \bar{\ell}); \qquad \ell = \frac{1}{N_c} \mathrm{Tr} P \exp[i \int_0^\beta d\tau \, A_0(\mathbf{x}, \tau)]$$

The quark-hadron transition at vanishing μ

thermodynamic potential in MFT

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grand canonical potential:

$$\Omega(T,\mu;\sigma_x,\sigma_y,\ell,\bar{\ell}) = U\left(\sigma_x,\sigma_y\right) + \Omega_{\bar{q}q}\left(\sigma_x,\sigma_y,\ell,\bar{\ell}\right) + \mathcal{U}\left(\ell,\bar{\ell}\right)$$

fermionic part:

$$\Omega_{\bar{q}q}(\sigma_x,\sigma_y,\ell,\bar{\ell}) =$$

$$-2T\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\left\{\ln\left[1+3\ell e^{-(E_{p,f}-\mu_f)/T}+3\bar{\ell}e^{-2(E_{p,f}-\mu_f)/T}+e^{-3(E_{q,f}-\mu_f)/T}\right]\right.$$

$$\left.-2T\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\left\{\ln\left[1+3\ell e^{-(E_{p,f}+\mu_f)/T}+3\bar{\ell}e^{-2(E_{p,f}+\mu_f)/T}+e^{-3(E_{q,f}+\mu_f)/T}\right]\right.\right.$$

in the hadronic phase **single** quarks and **di**quarks suppressed!

phase diagram from

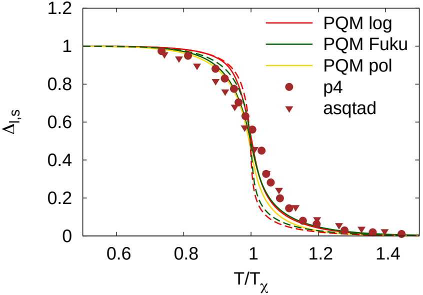
$$\frac{\partial\Omega}{\partial\sigma_x} = \frac{\partial\Omega}{\partial\sigma_y} = \frac{\partial\Omega}{\partial\ell} = \left.\frac{\partial\Omega}{\partial\bar{\ell}}\right|_{\min} = 0$$

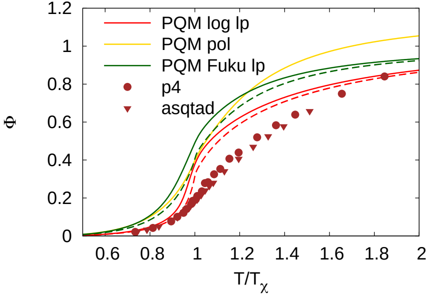
QCD Thermodynamics $N_f = 2 + 1$

lattice comparison @ $\mu = 0$

(pseudo) order parameters: B.-J. Schaefer et al. (2010)

lattice data: Bazavov et al. (2009) $m_\pi \sim 220$ MeV





$$\Delta_{l,s} = \frac{\langle \bar{l}l \rangle(T) - \frac{\hat{m}_l}{\hat{m}_s}\langle \bar{s}s \rangle(T)}{\langle \bar{l}l \rangle(T=0) - \frac{\hat{m}_l}{\hat{m}_s}\langle \bar{s}s \rangle(T=0)}$$

- solid lines: PQM with lattice masses (HOTQCD)
- dashed lines: PQM with realistic masses

QCD Thermodynamics $N_f = 2 + 1$

EoS B.-J. Schaefer et al. (2010)

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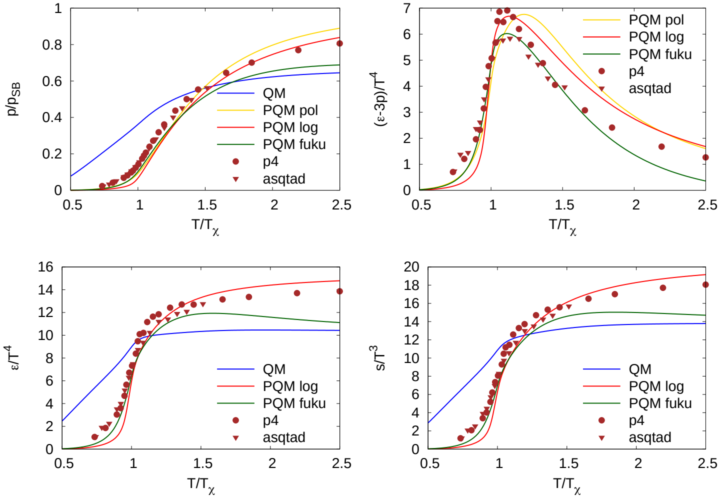

QCD Thermodynamics $N_f = 2 + 1$

susceptibilities **B.-J. Schaefer et al. (2010)**

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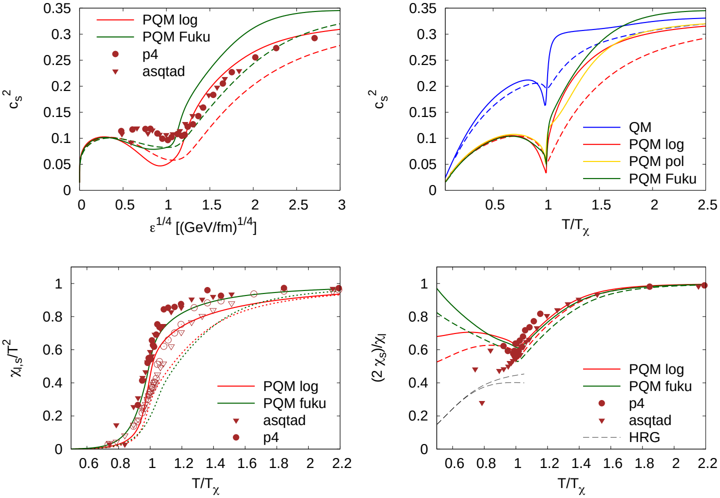

Including quantum fluctuations

FRG approach **C. Wetterich (1993), B.-J. Schaefer et al. (2005)**

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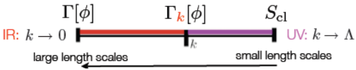
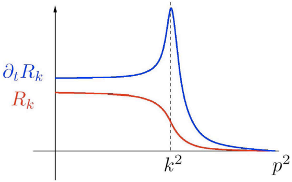


start at classical action and include quantum fluctuations successively by lowering k

$$k\partial_k \Gamma_k \equiv \partial_t \Gamma_k = \frac{1}{2}\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}; \qquad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

regulator R_k suppresses particle propagation with momenta lower than k



$$\Omega(T,\mu) = \lim_{k\to 0}\left(\Omega_k(T,\mu) = (T/V)\Gamma_k\right)$$

FRG for the PQM model
flow equations **B.-J. Schaefer at al. (2007) E. Nakano et al. (2010)**

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flow equation for Ω with quarks coupled to classical background gluon field

$$\partial_k \Omega_k = \frac{k^4}{12\pi^2}\left[\frac{3}{E_\pi}(1+2n_B(E_\pi)) + \frac{1}{E_\sigma}(1+2n_B(E_\sigma)) - \frac{N_c N_f}{E_q}\left(1 - n_q(\ell,\bar\ell) - n_{\bar q}(\ell,\bar\ell)\right)\right]$$

with

$$\begin{aligned} E_\pi^2 &= 1 + 2\Omega_k'/k^2 \quad E_\sigma^2 = 1 + 2\Omega_k'/k^2 + 4\phi^2\Omega_k''/k^2 \\ E_q^2 &= 1 + G\phi^2/k^2 \quad \Omega_k' = \partial\Omega_k/\partial\phi \quad etc \quad \phi = \langle\sigma\rangle \end{aligned}$$

quark densities modified by gluon field

$$n_q(\ell,\bar\ell) = \frac{1 - 2\bar\ell\exp((E_q-\mu)/T) + \ell\exp(2(E_q-\mu)/T)}{1 + 3\ell\exp(2(E_q-\mu)/T) + 3\bar\ell\exp(2(E_q-\mu)/T) + \exp(3(E_q-\mu)/T)}$$

FRG flow equations solved with

$$\frac{\partial}{\partial\ell}\Omega(T,\mu;\ell,\bar\ell) = 0 \qquad \frac{\partial}{\partial\bar\ell}\Omega(T,\mu;\ell,\bar\ell) = 0 \qquad \Omega(T,\mu;\ell,\bar\ell) = \Omega_{k\to 0}(T,\mu;\ell,\bar\ell) - \mathcal{U}(\ell,\bar\ell)$$

FRG for the PQM model:

results: V. Skokov et al. (2010), T.K. Herbst et al. (2010)

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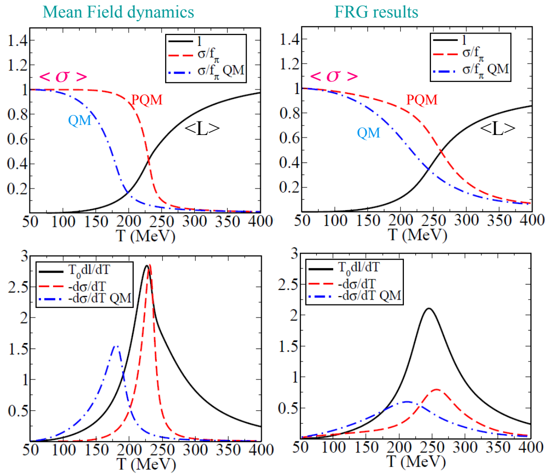

quantum fluctuations make transition smoother!

$1/N_c$ - expansion

M. Oertel et al. (2000)

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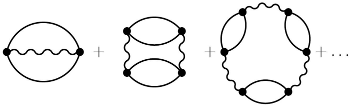
the $1/N_c$ - expansion provides physical picture for smoothening of the transition

$$\Omega = \Omega_{MF} + \delta\Omega$$



$1/N_c$-diagrams can be summed to all order ('ring sum')

$$\delta\Omega = \sum_M \Omega_M; \qquad \Omega_M = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{T}{2} \sum_{i\omega_q} \ln(1 - 2G\Pi_M(i\omega_q, \vec{q}))$$

$$\Omega_M = -\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} (1 + 2n_B(\omega))\, \phi_M; \quad \phi_M = \frac{1}{2i} \ln \frac{1 - 2G\Pi_M(\omega - i\eta, \vec{q})}{1 - 2G\Pi_M(\omega + i\eta, \vec{q})}$$

$1/N_c$ - expansion

A. Radzhabov et al. (2008), T. Hell et al. (2009)

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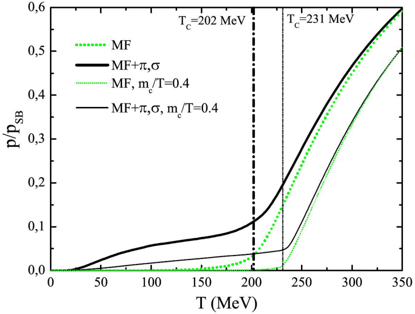
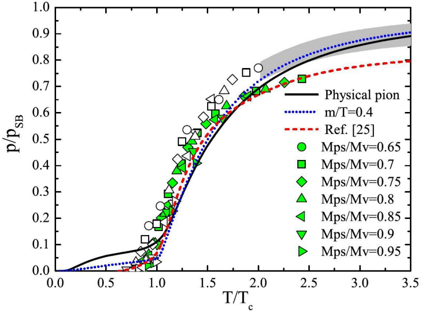
N_F = 2 results

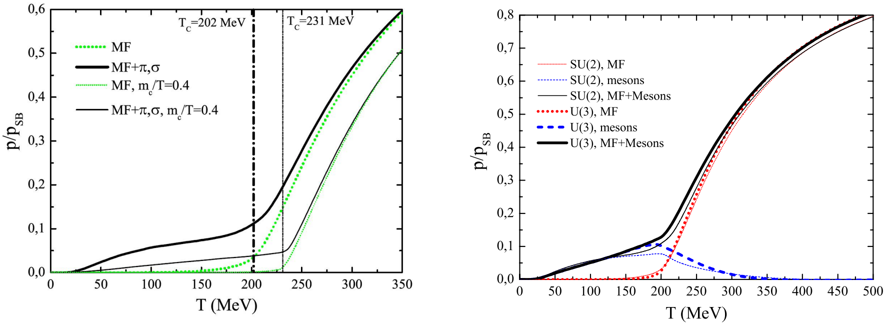




- mesonic fluctuations contribute to pressure in the hadronic phase
- thus contribute to the chiral condensate ($\langle \bar{q}q \rangle = \partial\Omega/\partial m_q$)
- Polyakov loop couples dynamically to the chiral condensate

Resonance gas?

A. Radzhabov (2009)



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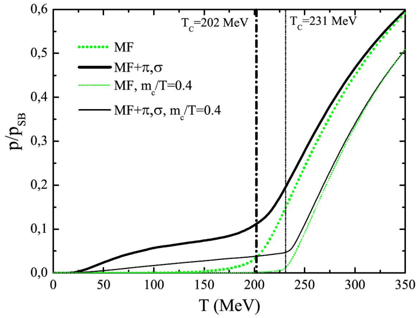

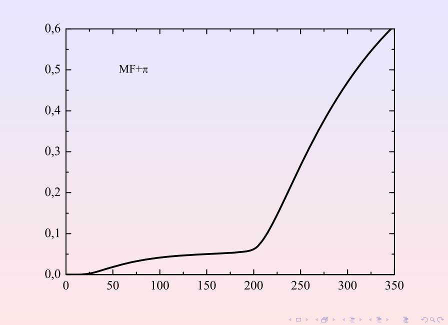

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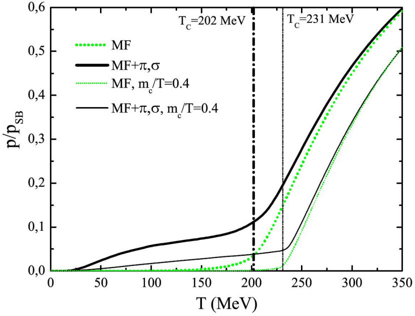

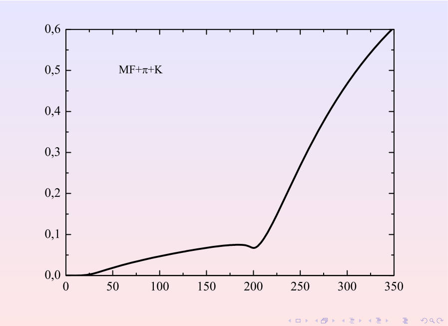

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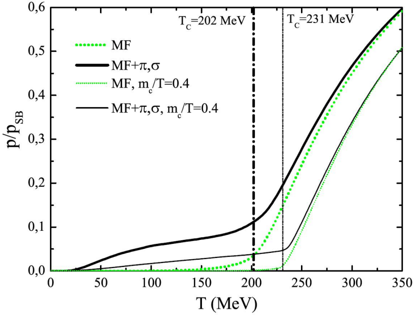

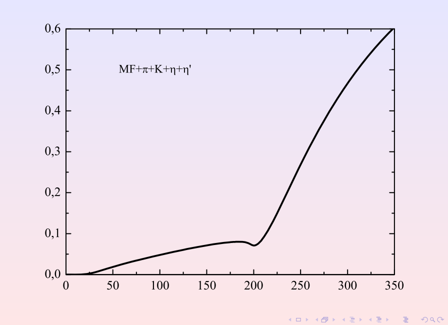

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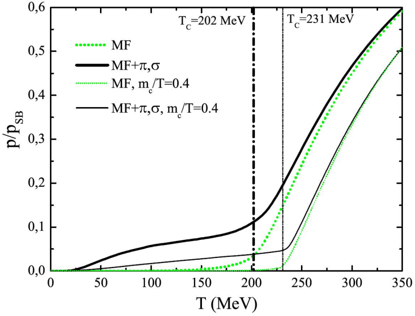

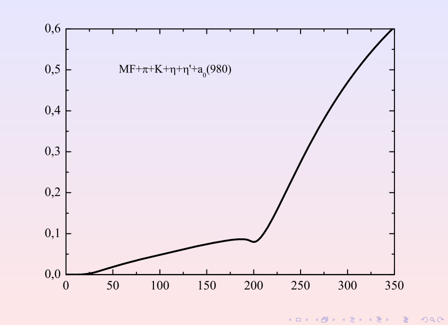

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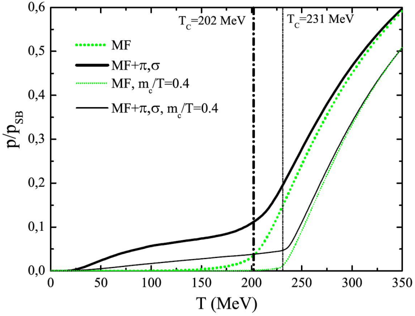

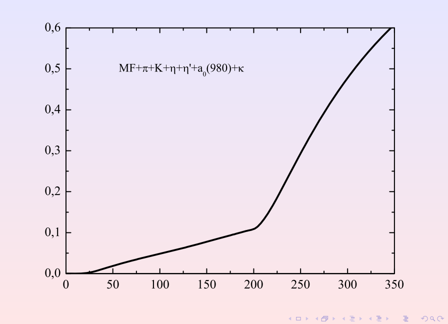

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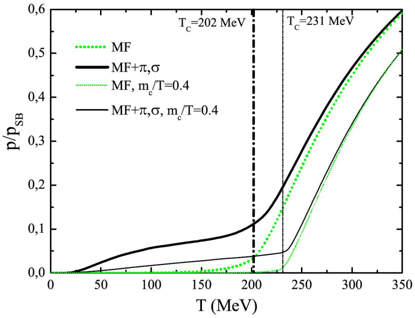

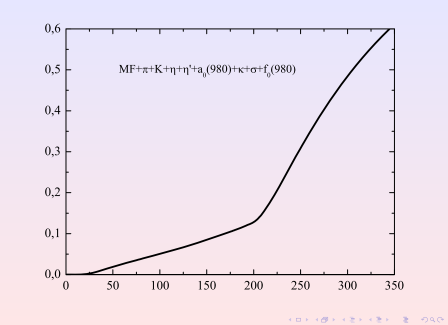

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Conclusions and outlook
Quark-hadron transition at $\mu = 0$

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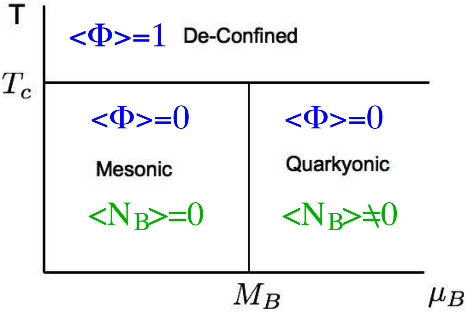
- QCD-inspired models give good account of the $N_f = 2 + 1$ lattice EoS and susceptibilities
- quantum fluctuations smooth out chiral- and deconfinement transition
- understood through additional pressure in the hadronic phase
- 'Hagedorn singularity' avoided through 'melting' of resonances above T_c
- is confinement properly accounted for by the Polyakov loop?
- transport coefficients?

’Quarkyonic’ phase at low T and high μ?

L. McLerran et al. (2007)

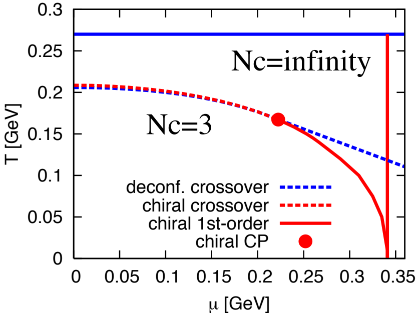
L. McLerran et al. (2007)

large N_c ’conjecture’



L. McLerran et al. (2009)

PNJL study



- ’quarkyonic’ phase is confining but chirally restored ($\rightarrow$ parity-doubled hadrons)
- ’triple point’ in the phase diagram
- excludes BCS-like ’color superconductivity’ at moderate density

’Quarkyonic’ phase at low T and high μ?

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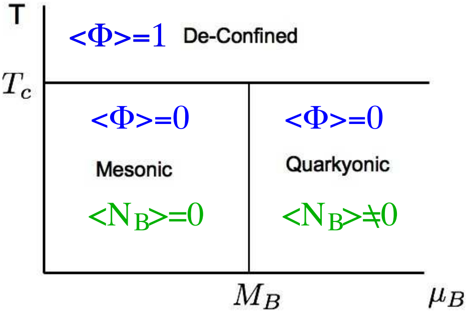
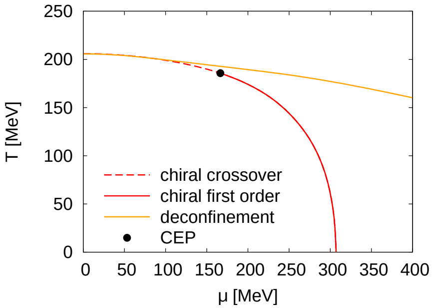


B.-J. Schaefer et al. (2010)

PQM study



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'Quarkyonic' phase at low T and high μ?

L. McLerran et al. (2007)

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width of the quarkyonic phase crucially depends on Polyakov loop dynamics!

polynomial potential: Polyakov (1978), Meisinger (1996), Pisarski (2000)

$$\frac{\mathcal{U}(\ell,\bar{\ell})}{T^4} = -\frac{b_2(T,T_0)}{2}\ell\bar{\ell} - \frac{b_3}{6}\left(\ell^3+\bar{\ell}^3\right) + \frac{b_4}{16}\left(\ell\bar{\ell}\right)^2$$

with

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

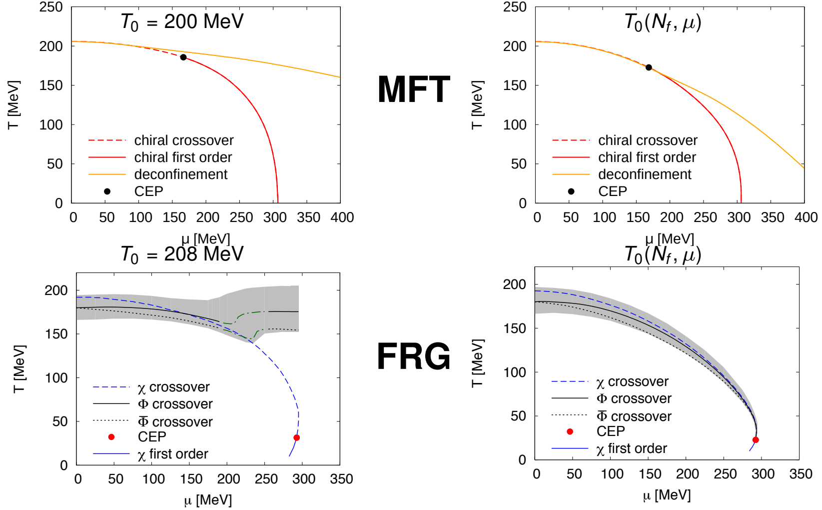
- T_0 adjusted to precise lattice data for the 'pure gauge' transition (T_0 = 270 MeV)
- in the presence of dynamial quarks $T_0 = T_0(N_f, \mu)$ B.-J. Schaefer et al. (2007)

N_f	0	1	2	2+1	3
T_0 [MeV]	270	240	208	187	178

based on 'one loop' running of QCD β-function

'Quarkyonic' phase at low T and high μ?

B.-J. Schaefer et al. (2010), T.K. Herbst et al. (2010)



'Quarkyonic' phase at low T and high μ?

Lessons from the 'statistical model' K. Fukushima (2010)

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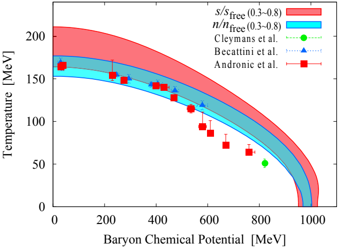
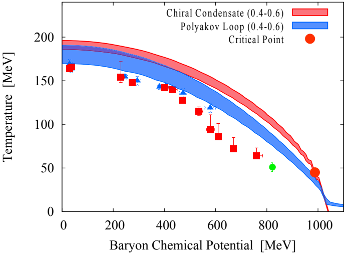
results from THERMUS

S. Wheaton et al. (2009)



PNJL consistent with stat. model

K. Fukushima (2010)



$$T_f(\mu_B) = a - b\mu_b^2 - c\mu_b^4$$

$a = 166(2)$ MeV $b = 1.39(16) \times 10^{-4}$ MeV^{-1}

$c = 5.3(21) \times 10^{-11}$ MeV^{-3}

$$T_0(\mu)/T_0(0) = 1 - (bT_0)(\mu_b/T_0)^2$$

$bT_0 = 2.78 \times 10^{-2}$ MeV

$bT_0 = 2.1 \times 10^{-2}$ MeV (one − loop)

lattice : $T_c(\mu_B)/T_c(0) = 1 - 6.8 \times 10^{-3}(\mu_b/T_0)^2$ curvature $\sim 1/3!$?

Conclusions and outlook
Quarkyonic phase?

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- do the chiral and deconfinement transition coincide at all T and μ?
- possibly yes $\rightarrow$ little room for 'quarkyonic phase'
- depends crucially on $T_0(N_f, \mu)$
- consistency with the statistical model?
  $\rightarrow$ likely the chemical freeze-out line
  is **not** the phase boundary @ all T and μ
- role of baryons at small T and large μ
- inclusion of quark-diquark correlations
- inhomogeneous phases?

Inhomogeneous phases of QCD matter

phase diagram (chiral limit: $m_q = 0$)

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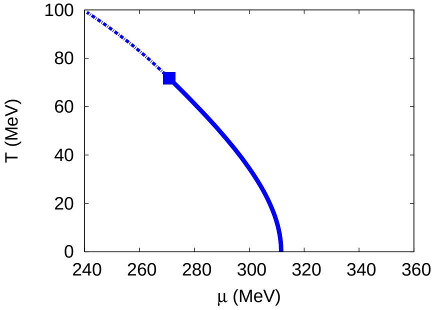
chiral transition (conventional)



- homogeneous:
- first-order chiral phase transition
- critical end point

is the homogeneous phase stable near the CEP?

Is the homogeneous phase stable near the CEP?

GL analysis (D. Nickel 2009)

GL functional near the CEP: (chiral limit)

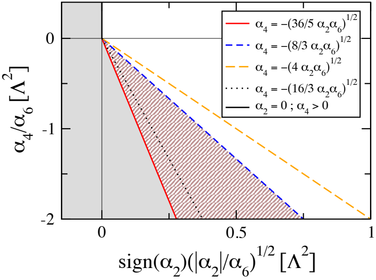
$$\Omega_{GL}(T,\mu;M(\mathbf{x})) = \frac{\alpha_2}{2}M(\mathbf{x})^2 + \frac{\alpha_4}{4}\left(M(\mathbf{x})^4 + (\nabla M(\mathbf{x}))^2\right)$$
$$\frac{\alpha_6}{6}\left(M(\mathbf{x})^6 + 5(\nabla M(\mathbf{x}))^2 M(\mathbf{x})^2 + \frac{1}{2}(\Delta M(\mathbf{x}))^2\right)$$

CEP: $\alpha_2 = \alpha_4 = 0, \alpha_6 > 0$ for inhomogeneous phases $\alpha_4 < 0$

1D inhomogeneities:

exact mass function

$$M(\mathbf{x}) = M(z) = \sqrt{\nu} q \,\mathrm{sn}(qz,\nu)$$



Phase structure in 1+1 dimensions

M.Thies et al. (2004)

exact solutions of 1+1 D fermionic theories for $N \to \infty$, $Ng^2 = const$
(Gross-Neveu, NJL, 't Hooft)

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m_0\right)\psi + \frac{g^2}{2}\left(\bar{\psi}\psi\right)^2$$

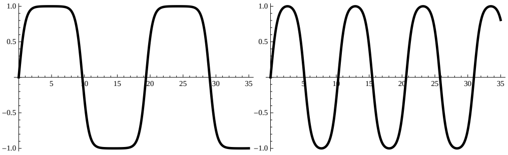
- $U(N)$ flavor symmetry
- Z_2 chiral symmetry
  $\psi \to \gamma_5\psi \quad \bar{\psi}\psi \to -\bar{\psi}\psi$
- for $N \to \infty$ MF exact

$$\left(-i\gamma_5\partial_z + \gamma^0 M(z)\right) = \epsilon_\alpha\psi_\alpha$$

$$M - m_0 = -Ng^2\sum_\alpha n_\alpha\bar{\psi}_\alpha\psi_\alpha$$

mass function: $(m_0 = 0)$

$$\to M(z) = \sqrt{\nu}\,q\,\mathrm{sn}(qz,\nu)$$



Phase diagram

O. Schnetz et al. (2006)

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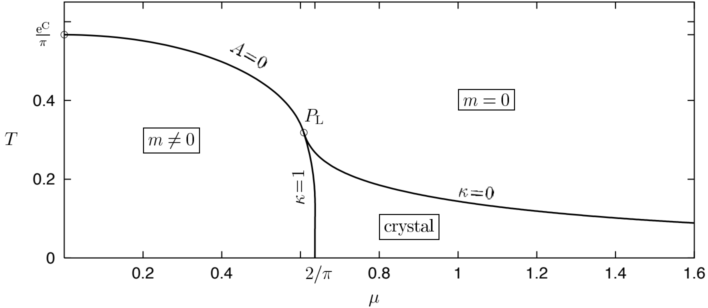
thermodynamic potential:

$$\Omega(T,\mu) = -T\,\mathrm{Tr}\log\left(S^{-1}\right)\Big) + \frac{1}{2Ng^2\lambda}\int_0^\lambda dz\; M(z)^2$$



NJL Model in 3+1 dimensions

D. Nickel (2009)

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- start from the $N_f = 2$ NJL Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ - m_q\right)q + G_s\left((\bar{q}q)^2 + \left(\bar{q}i\gamma^5\tau^a q\right)^2\right)$$

- allow for spatially inhomogeneous condensates

$$\langle\bar{q}q\rangle = S(\vec{x}) \qquad \langle\bar{q}i\gamma^5\tau^a q\rangle = P_a(\vec{x})$$

- in mean-field approximation

$$\mathcal{L}_{MF}(\mathbf{x}) = \bar{q}\left(i\partial\!\!\!/ - m_q + 2G_s\left(S(\mathbf{x}) + i\gamma^5\tau_3 P(\mathbf{x})\right)\right)q - G_s\left(S(\mathbf{x})^2 + P(\mathbf{x})^2\right)$$

$$M(\mathbf{x}) = m_q - 2G_s\left(S(\mathbf{x} + iP(\mathbf{x})\right)$$

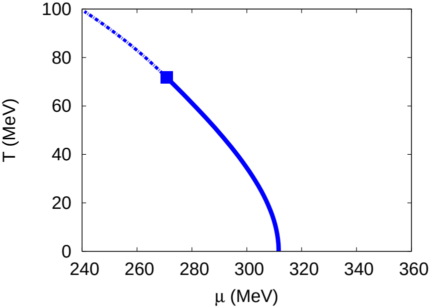
- thermodynamic potential: (1D modulations)

$$\Omega(T,\mu) = -\frac{2T}{V}\sum_{\alpha}\int_{p_\perp}\ln\left(2\cosh\left(\frac{\alpha\sqrt{1+\mathbf{p}_\perp^2/\alpha^2}-\mu}{2T}\right)\right) + \int_V \frac{|M(\mathbf{x})-m_q|^2}{4G_s V}$$

Phase diagram

chiral limit: $m_q = 0$

chiral transition (conventional)

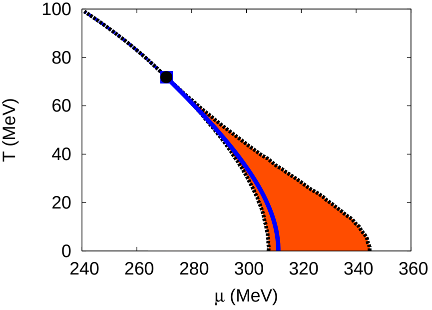


- homogeneous:
- first-order chiral phase transition
- critical end point

Phase diagram

chiral limit: $m_q = 0$

chiral transition (new scenario)



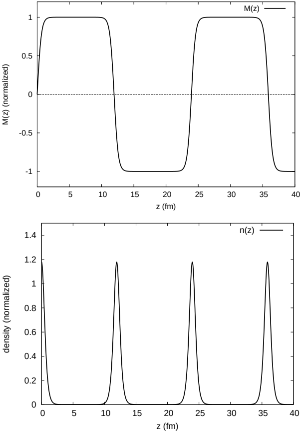
D. Nickel (2009)

- inhomogeneous:
- inhomogeneous region bounded by 2nd-order transition lines
- first-order transition line completely covered by inhomogeneous region
- critical end point $\rightarrow$ 'Lifschitz' point

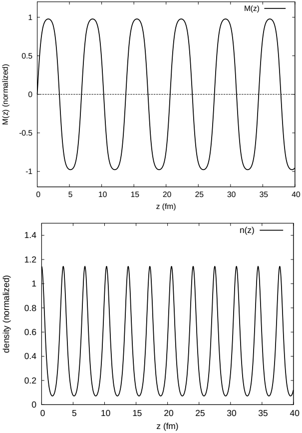
similar conclusions hold in the QM model

mass and density modulations (T=0)

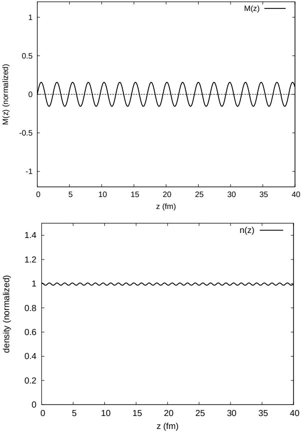
μ = 307 MeV



μ = 308 MeV



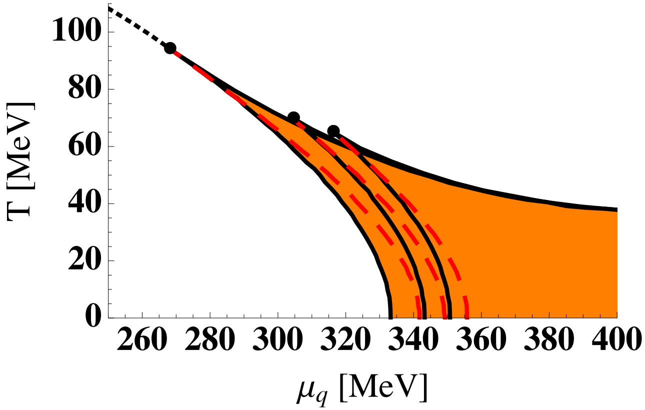
μ = 342 MeV



Phase diagram

finite quark mass

two different choices of bare quark mass (m_q = 5, 10 MeV)



NJL with vector interactions

S. Carignaro et al. (2010)

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- extended $N_f = 2$ NJL Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ - m_q\right) q + G_s\left((\bar{q}q)^2 + \left(\bar{q}i\gamma^5\tau^a q\right)^2\right) - G_v\left(\bar{q}\gamma^\mu q\right)^2$$

- spatially varying chemical potential: $\tilde{\mu}$

$$\tilde{\mu}(\mathbf{x}) = \mu - 2G_\mathrm{V}\, n(\mathbf{x})$$

- sacrifice complete self-consistency: pick $\tilde{\mu} \equiv \langle\tilde{\mu}\rangle_z$ instead of $\tilde{\mu}(z)$

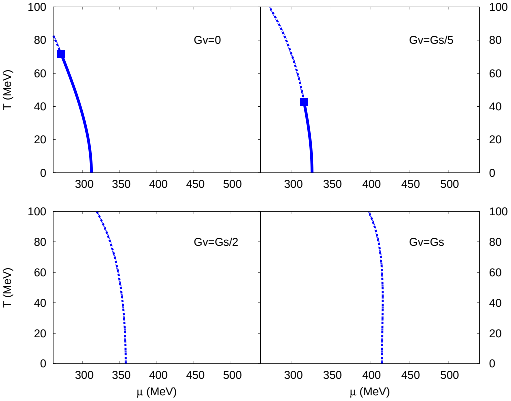
$$\Omega(T,\mu) \to \Omega(T,\tilde{\mu}) - \frac{(\tilde{\mu}-\mu)^2}{4G_\mathrm{V}}$$

- determine $\tilde{\mu}$ from

$$\frac{\delta\Omega}{\delta\tilde{\mu}} = 0$$

Phase diagram with vector interactions
$m_q = 0$

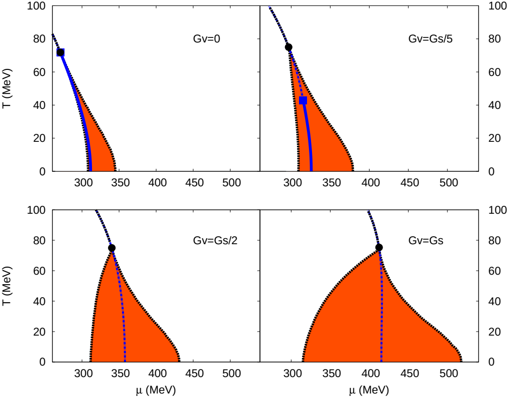
homogeneous



- stretch towards higher μ
- critical end point moves towards lower T and higher μ

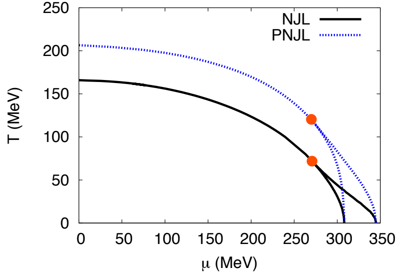
Phase diagram with vector interactions
$m_q = 0$

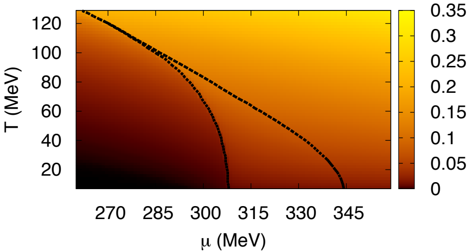
inhomogeneous



- stretch towards higher μ
- Lifschitz point only moves in μ
- Lifschitz- and CEP split

Phase diagram with vector interactions including the Polyakov loop





stretching in the T - direction

well known from homogeneous case

ℓ ($\bar{\ell}$) rather small in the inhomogeneous region

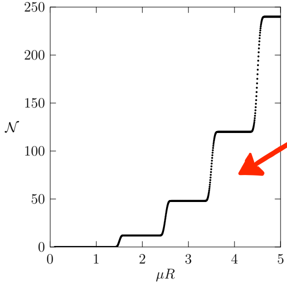
Conclusions and outlook
Inhomogeneous phases

- self-consistent lower-dimensional spatial modulations can be studied by relying on analytical results from the Gross-Neveu model
- inhomogeneous 1D phases favored in a region of the QCD phase diagram
- 'solitonic' region bounded by 2nd order phase transitions
- inclusion of vector interactions $\rightarrow$ shift to higher μ
  $\rightarrow$ split of CEP and Lifschitz point
- 1D phases unstable against thermal fluctuations!
- higher-dimensional inhomogeneities?
  $\rightarrow$ complicated 'band structure' calculations
- interplay between chiral- und superconducting phases
- inhomogeneous Polyakov loop?

QCD_2 lattice results

J. Myers et al. (2010)

QCD_2 on a hypersphere $S^3 \times S^1$ with $R \ll \Lambda_{QCD}$
'Two-color Attoworld' (S. Hands)



Quark density rises stepwise as finite "shells" are filled with occupancy

$$\mathcal{N}_L = N_c N_f \sum_{\ell=1}^{L} 2\ell(\ell+1)$$

Polyakov line indicates alternating ranges of confinement and deconfinement $\Leftrightarrow$ partially-filled shell