QCD Green's Functions and Phases of *Strongly Interacting Matter*

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Hot and Cold Baryonic Matter
Budapest, August 16, 2010

Outline

Why QCD Green's functions?

- ★ They embody confinement, DχSB, and the axial anomaly!
- ★ They provide input into hadron phenomenology:
  - Bethe–Salpeter equations for mesons
    masses, decays, reactions ...
  - Faddeev equations for baryons
    form factors, meson production, GPDs, ...
  - Phases of QCD
    high-T phase, color superconductivity, critical end point ...

Thus they would be capable of describing **hadrons** as bound states as well as **strongly interacting matter** in terms of glue and quarks.

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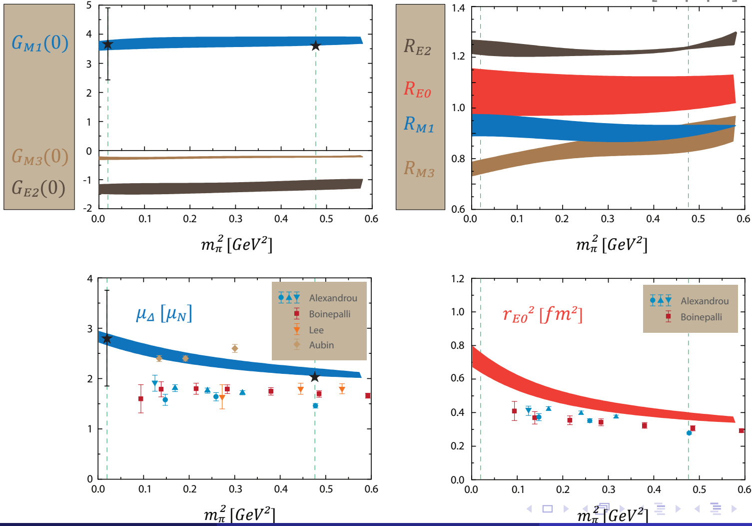
Introduction

Example for application to hadron physics:

Electromagnetic form factors of Δ and Ω

D. Nicmorus, G. Eichmann, R.A., arXiv: 1008.nnnn[hep-ph].



Functional Approaches to QCD

QCD Green's functions powerful tool,

however, infinite hierarchy of complicated equations!

??? Infrared behaviour of correlation functions . . .

Restriction to lower *n*-point functions?

E.g. in linear covariant gauges:
7 primitively divergent Green functions in QCD,
5 primitively divergent Green functions in Yang-Mills theory.

- gluon and ghost [and quark] propagators as well as
- 3-gluon, 4-gluon and gluon-ghost [and quark-gluon] vertices

R.A. *et al.*, "Confinement and Green functions in Landau-gauge QCD"
PoS(Confinement8)019 [arXiv:0812.2896 [hep-ph]]

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Functional Approaches to QCD

- **Functional Methods:**
  - (a) Dyson-Schwinger Equations (DSEs)

    Partition Function ⟶ Generating Functional ⟶ Eqs. of Motion for Green's Functions
  - (b) Functional Renormalisation Group (FRG)

    Effective Action ⟶ Energy-Momentum Cutoff ⟶ RG Flow ⟶ Wetterich Eq.
  - (c) *n*-Particle Irreducible Actions (nPI)

    Partition Function ⟶ Truncated Actions ⟶ Symmetry-pres. Eqs. for Green's Functions

  + **chiral fermions and Goldstone's theorem**
  + **analytical infrared solutions**
  + **dynamical hadronisation**
  + **chemical potential: no sign problem**

- **Lattice Gauge Theory**

  + **no truncations**
  + **manifest gauge invariance**

! — Combination of Methods — !

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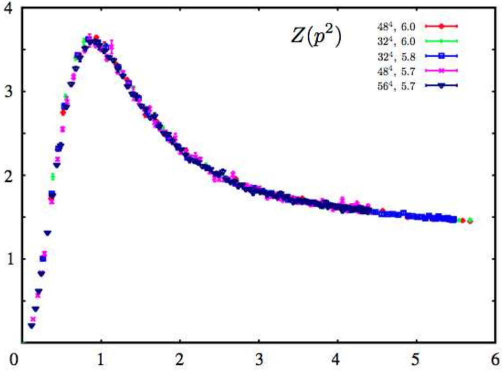
Gluon Propagator

pure Yang-Mills, $T = 0$

Landau gauge Gluon Ren. Fct. $D^{tr}_{\text{Gluon}} = \mathbf{Z(p^2)}/p^2$

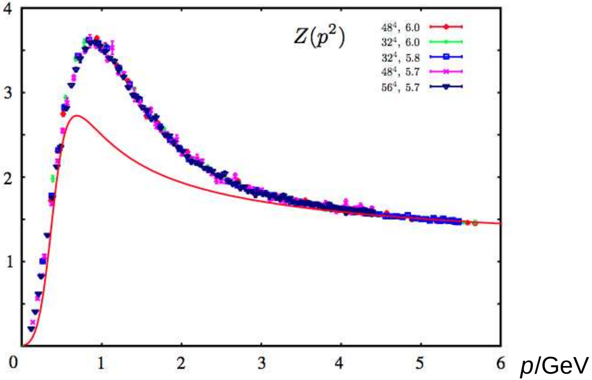
A. Sternbeck *et al.*, PoS LAT2006, 76



p/GeV

Gluon Propagator

A. Sternbeck *et al.*, PoS LAT2006, 76



— L. von Smekal, A. Hauck, R.A., Phys. Rev. Lett. 79 (1997) 3591

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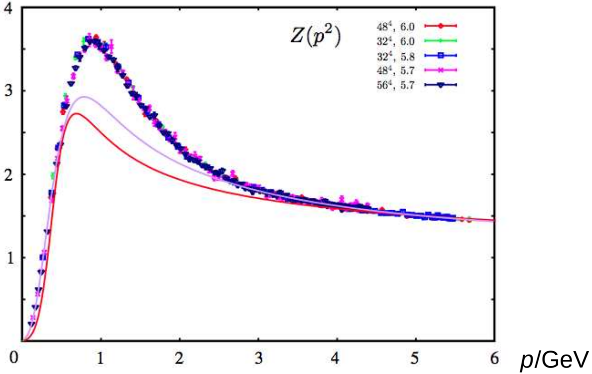

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— C. S. Fischer, R.A., Phys. Lett. **B536** (2002) 177

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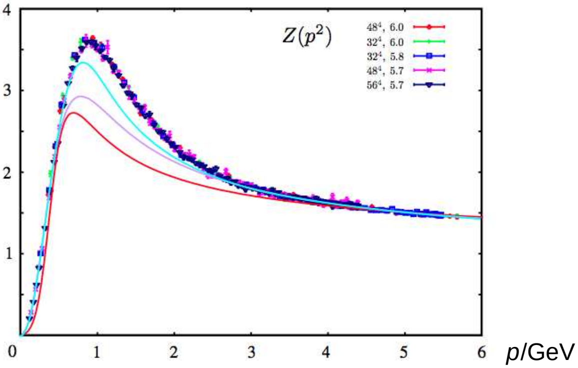

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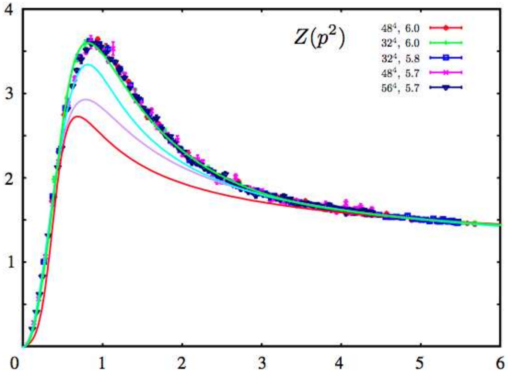

p/GeV

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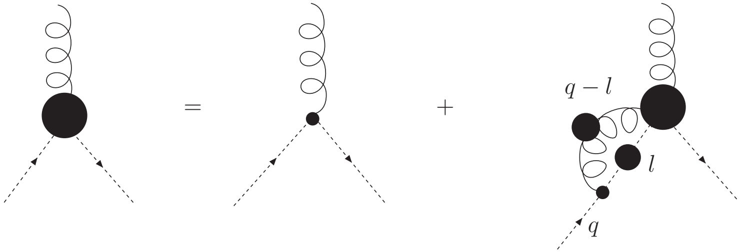
— C.S. Fischer, A. Maas, J.M. Pawlowski, Ann. Phys. **324** (2009) 2408

Infrared Structure of Landau gauge Yang-Mills theory

- Starting point in gauges with transverse gluon propagator: Ghost-Gluon-Vertex fulfills Dyson-Schwinger equation

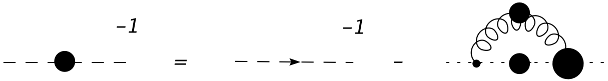


- Transversality of gluon ⇒ Bare Vertex for $q_\mu \to 0$
- No anomalous dimensions in the IR

J. C. Taylor, Nucl. Phys. B **33** (1971) 436.
C. Lerche, L. v. Smekal, PRD **65** (2002) 125006.
A. Cucchieri, T. Mendes and A. Mihara, JHEP 0412:012 (2004).
W. Schleifenbaum, A. Maas, J. Wambach and R. A., Phys.Rev.D72 (2005) 014017.

Infrared Exponents for Gluons and Ghosts

- Dyson-Schwinger equation (DSE) for the ghost-propagator:



<u>Ansatz</u> for Gluon, $Z(p^2) \sim (p^2)^\alpha$,
and Ghost Ren. Fct., $G(p^2) \sim (p^2)^\beta$.

▶ Selfconsistency $\Rightarrow \quad -\beta = \alpha + \beta =: \kappa$ i.e.

$$Z(p^2) \sim (p^2)^{2\kappa} , \quad G(p^2) \sim (p^2)^{-\kappa}$$

L. v. Smekal, A. Hauck, R. A., Phys. Rev. Lett. **79** (1997) 3591

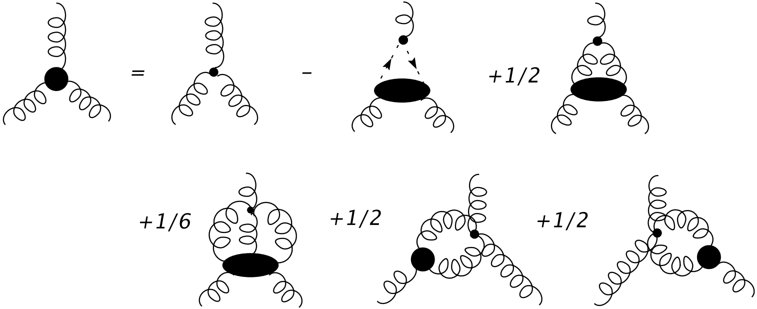
▶ IR enhanced ghost propagator: $0.5 \leq \kappa < 1$
Kugo–Ojima confinement criterion,
Oehme–Zimmermann superconvergence relation,
and Gribov–Zwanziger horizon condition fulfilled!
P. Watson and R.A., Phys. Rev. Lett. **86** (2001) 5239

Infrared Exponents for Gluons and Ghosts

R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Example: DSE for 3-gluon-vertex



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R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

MATHEMATICA:

R. A., M. Q. Huber, K. Schwenzer, Comp. Phys. Comm. **180** (2009) 965 [arXiv:0808.2939 [hep-th]]

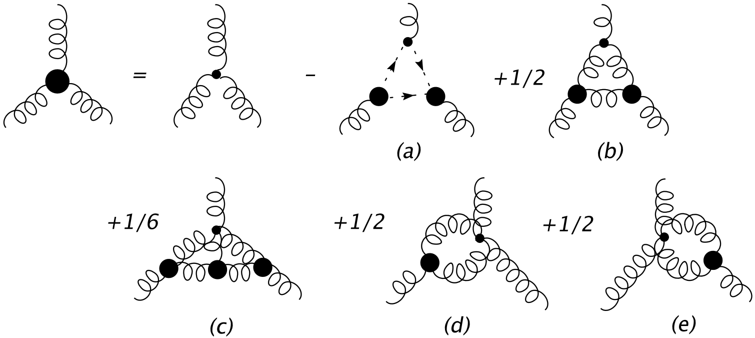
Infrared Exponents for Gluons and Ghosts

R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Skeleton expansion &

generalized formulas (neg. dim.) for Feynman integrals:

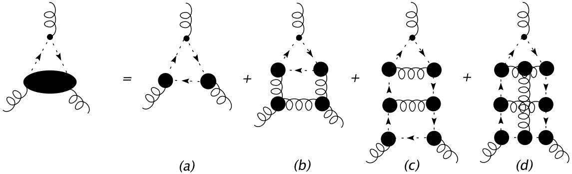


Infrared Exponents for Gluons and Ghosts

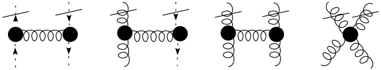
R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Three-gluon vertex: **higher order** in skeleton expansion



built from insertions



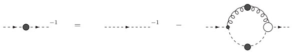
insertions have zero IR anomalous dimensions $\Rightarrow$

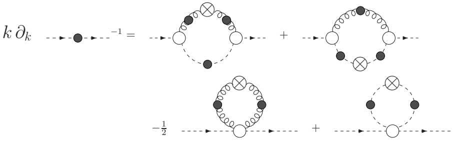
IR-analysis valid to all orders in skeleton expansion

Infrared Exponents for Gluons and Ghosts

Use DSEs **and** ERGEs:

$\longrightarrow$ Two different towers of equations for Green functions
E.g. ghost propagator





Infrared Exponents for Gluons and Ghosts

IR-Analysis of two different towers of equations $\Rightarrow$

Unique **scaling** solution

(C.S. Fischer and J.M. Pawlowski, PRD **75** (2007) 025012; PRD **80** (2009) 025023)

and a one-parameter family of solutions with IR trivial Green functions,
(the **decoupling** solutions).

C.S. Fischer, A. Maas and J.M. Pawlowski, AoP **324** (2009) 2408.

see also

A.P. Szczepaniak and E.S. Swanson, PRD **65** (2002) 025012 (! Coulomb gauge!).

A.C. Aguilar, D. Binosi , J. Papavassiliou, PRD **78** (2008) 025010 and refs. therein.

P. Boucaud *et al.*, JHEP **0806** (2008) 099 and refs. therein.

RA, M.Q. Huber, K. Schwenzer, PRD **81** (2010) 105010.
M.Q. Huber, K. Schwenzer, RA, arXiv:0904.1873, EPJC in print (published online).

Infrared Exponents for Gluons and Ghosts

Scaling solution vs. decoupling solutions:

- Most lattice calculations of gluon propagator favor decoupling sol.
- Scaling solution respects BRST,
  decoupling solution breaks BRST.
- Strong coupling lattice calculations: two different IR exponents, supports existence of scaling solution.
  (L.v. Smekal, A. Sternbeck, arXiv:0811.4300 [hep-lat];
  A. Cucchieri, T. Mendes, Phys. Rev. **D80** (2010) 016005.)

A potential resolution of the puzzle:
IR behaviour depends on non-perturbative completion of gauge.
(A. Maas, Phys. Lett. **B689** (2010) 107.)
Anyhow:
Difference of fundamental interest but phenomenologically irrelevant!

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UNI GRAZ

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General Infrared Exponents for Gluons and Ghosts

Scaling solution:

n external ghost & antighost legs and m external gluon legs
(one external scale p^2; **solves DSEs and STIs**):

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

- Ghost propagator IR divergent
- Gluon propagator IR suppressed
- Ghost-Gluon vertex IR finite
- 3- & 4- Gluon vertex IR divergent

★ IR fixed point for the coupling from each vertex

★ Conformal nature of Infrared Yang-Mills theory!

★ Ghost sector of YM-theory dominates IR!

D. Zwanziger, Phys. Rev. D **69** (2004) 016002

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D. Zwanziger, Phys. Rev. D **69** (2004) 016002

Positivity violation for the gluon propagator

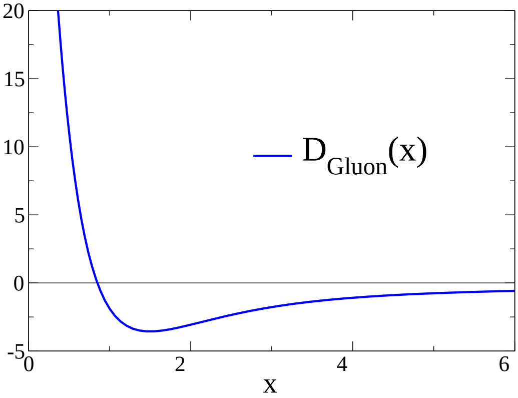
Simple argument [Zwanziger]:
IR vanishing gluon propagator implies

$$0 = D_{gluon}(k^2 = 0) = \int d^4x D_{gluon}(x)$$

$\Longrightarrow D_{gluon}(x)$ has to be negative for some values of x.

Positivity violation for the gluon propagator

Fourier transform of DSE result:

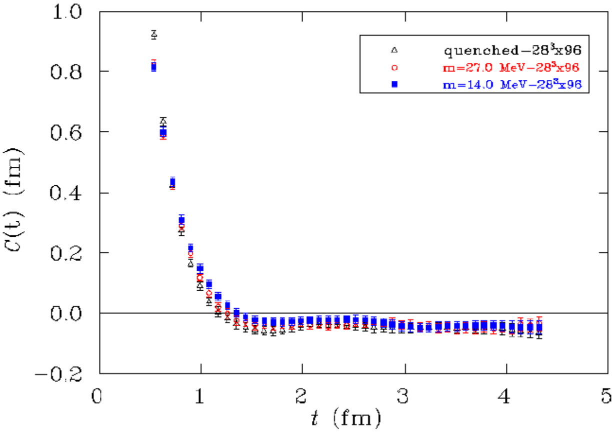


Gluons unobservable $\Longrightarrow$ **Gluon Confinement!**

R.A., W. Detmold, C.S. Fischer and P. Maris, PR**D70** (2004) 014014

Positivity violation for the gluon propagator

Lattice (P. Bowman et al., Phys.Rev.**D76** (2007) 094505):



Picturing Gluon Confinement

DSE scaling solution of Yang-Mills theory:

- Gluon propagator vanishes on the light cone, and
- *n*-point gluon vertex functions diverge on the light cone!

⇒ Attempts to kick a gluon free (*i.e.* to produce a real gluon) immediately results in production of infinitely many virtual soft gluons!

⇒ perfect color charge screening
+ positivity violation (which implies BRST quartet cancelation):

Gluon confinement!

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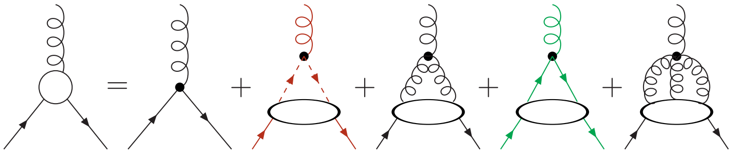
Dynamically induced scalar quark confinement

R.A., C.S. Fischer, F. Llanes-Estrada, K. Schwenzer, Annals Phys. **324** (2009) 106.

Quark-gluon vertex:



Quark diagram: Hadronic contributions ('unquenching')

Ghost diagram: Infrared leading!

Dynamically induced scalar quark confinement

Chiral symmetry dynamically or explicitely broken:
quark propagator infrared finite

$$S(p) = \frac{\not p + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \rightarrow \frac{Z_f \not p}{M^2} + \frac{Z_f}{M}$$

AND

$$\Gamma_\mu = ig \sum_{i=1}^{12} \lambda_i G^i_\mu \,, \quad G^1_\mu = \gamma_\mu \,, \; G^2_\mu = \hat{p}_\mu \,, \; G^3_\mu = \ldots$$

WITH $\lambda_{1,2,\ldots} \sim (p^2)^{-1/2-\kappa}$

INFRARED DIVERGENT

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I N F R A R E D   D I V E R G E N T

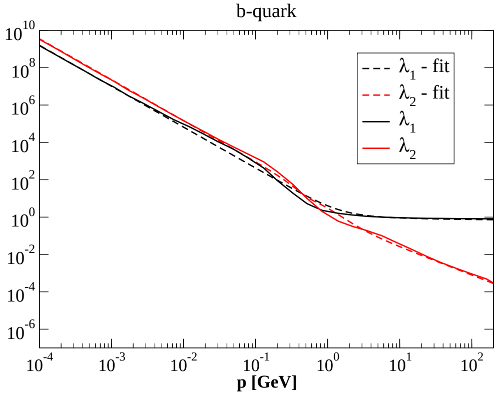
Dynamically induced scalar quark confinement

Chiral symmetry dynamically or explicitely broken:

$$\lambda_{1,2,\ldots} \sim (p^2)^{-1/2-\kappa} \text{ i.e.}$$

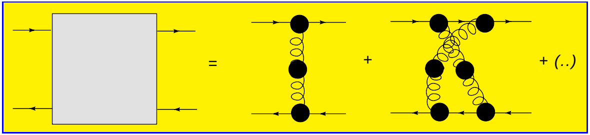
Quark-Gluon vertex IR divergent!

Scalar component λ_2 in IR even **larger** than vector component λ_1!



Dynamically induced scalar quark confinement

“Quenched´´ quark-antiquark potential



infrared divergent such that

$$V(\mathbf{r}) = \int \frac{d^3p}{(2\pi)^3} H(p^0 = 0, \mathbf{p}) e^{i\mathbf{pr}} \quad \sim \quad |\mathbf{r}|$$

i.e. linear, dominantly scalar, quark confinement!

$U_A(1)$: η' mass from IR divergent Green functions

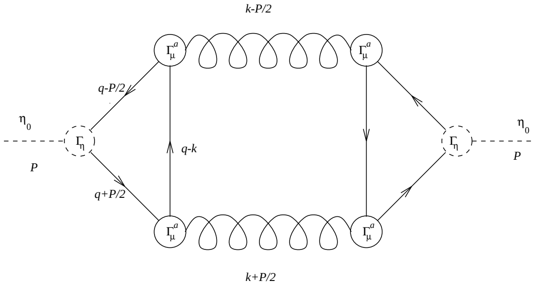
R.A., C. S. Fischer, R. Williams, Eur. Phys. J. A **38** (2008) 53.

$U_A(1)$ symmetry anomalous $\Rightarrow$ η' mass $\gg$ π mass

Where is this encoded in the Green functions?

(J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.)

E.g. in:



$$\Gamma_\mu D^{\mu\nu} \Gamma_\nu \propto 1/k^4$$

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QCD vacuum: winding number spots as, *e.g.*, instantons,

couple to chiral quark zero modes $\Rightarrow U_A(1)$ symmetry broken!

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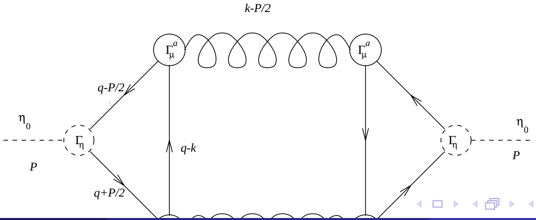

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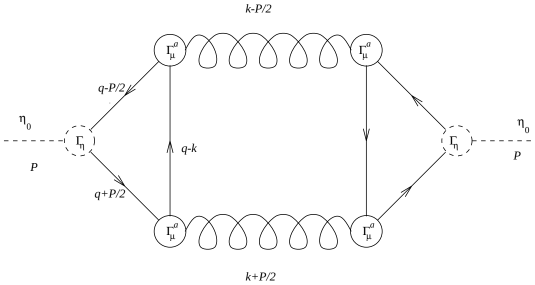

$$\Gamma_\mu D^{\mu\nu} \Gamma_\nu \propto 1/k^4$$

η' mass from IR divergent Green functions

However: Infinitely many diagrams (n-gluon exchange) contribute!

Nevertheless:
Calculate contribution from **diamond diagram only** employing DSE results for the gluon and quark propagators and quark-gluon vertex (provides correct pseudoscalar and vector meson masses):

$\chi^2 \approx (160\mathrm{MeV})^4$ vs. phenomenological value $(180\mathrm{MeV})^4$

results in: $m_\eta = 479$MeV, $m_{\eta'} = 906$MeV, $\theta = -23^0$.

Conclusion:
(Fluct.) topologically non-trivial fields $\Leftrightarrow$ IR singularities of GF!
. . . another view to generate the Witten-Venezanio mechanism . . .

Quark confinement $\Rightarrow$ $U_A(1)$anomaly!

Picturing quark confinement

Implication of YM-DSE scaling solution for quark sector:

- quark propagator IR trivial (DχSB),
- quark-gluon vertex functions including a self-consistently generated scalar quark-gluon coupling (DχSB!) diverge on the quark “mass´´ shell!

$\Rightarrow$ Attempts to kick a quark free (*i.e.* to produce a real quark) immediately results in production of infinitely many virtual soft gluons!

$\Rightarrow$ linearly rising potential
i.e., infrared slavery:

Quark confinement!

String formation? Properties of confining field configuration? ...? ...?

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Phases of strongly interacting matter

$$T \neq 0 \; (\mu = 0)$$

Landau gauge propagators:

- $D_{\mu\nu}^{\text{Gluon}} = \frac{\mathbf{Z_T(p^2)}}{p^2} P_{\mu\nu}^{T}(p) + \frac{\mathbf{Z_L(p^2)}}{p^2} P_{\mu\nu}^{L}(p)$
- $S(p) = \frac{1}{-i\vec{\gamma}\vec{p}\mathbf{A(p)} - i\gamma_4\omega_p\mathbf{C(p)} + \mathbf{B(p)}}$

$T = 0$ $Z_T = Z_L = Z$ and $C = A$, functions of p^2 only

$T \neq 0$ Six propagator functions depending on $\vec{p}^2$ and ω_p $(p = (\omega_p, \vec{p}))$
transv. to medium $\to$ chromomag. / long. to medium $\to$ chromoel.

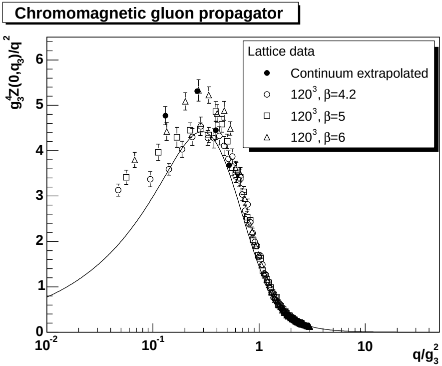
$\to \infty$ 3-dimensional Yang-Mills theory + “Higgs´´ (A_4)

Partial gluon confinement at any T

Gluon propagator, chromomagnetic part, at high T:

A. Maas, J. Wambach, RA, EPJ **C37** (2004) 335; **C42** (2005) 93.

A. Cucchieri, A. Maas and T. Mendes, PR **D75** (2007) 076003.



Gribov-Zwanziger / Kugo-Ojima scenario / positivity violation

Partial gluon confinement at any T

Gribov-Zwanziger / Kugo-Ojima scenario / positivity violation at any T:

No infrared singularities, *c.f.* Linde (1980),
because no chromomagnetic mass of type $\omega_m(\vec{k} = 0) = m_m(T)$!

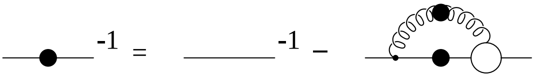
K. Lichtenegger, D. Zwanziger, Phys. Rev. D **78** (2008) 034038.

No surprise:

- three-dimensional YM theory confining
- area law for spatial Wilson loop
- Coulomb string tension $\neq 0$ at any T

Static chromomagnetic sector is never deconfined!

Ordinary chiral condensate



- input: gluon propagator and quark-gluon vertex
- antiperiodic boundary conditions for quarks
- order parameter for **chiral transition**

$$\langle \bar{q} q \rangle = Z_2 N_c \sum_{\omega_n} \int \frac{d^3p}{(2\pi)^3} \, \mathrm{tr} S$$

Dual (Gattringer) condensate

C.Gattringer, Phys. Rev. Lett. 97 (2006) 032003
F.Synatschke, A.Wipf, C.Wozar, Phys. Rev. D 75 (2007) 114003
F.Synatschke, A.Wipf, K.Langfeld, Phys. Rev. D 77 (2008) 114018

$U(1)$-valued boundary conditions for quark field in "temporal" direction:

$$q(\vec{x}, \beta = 1/T) = e^{i\varphi} q(\vec{x}, 0)$$

Matsubara frequencies: $\omega_n = 2\pi T(n + \varphi/2\pi)$

Dual (Gattringer) condensate

The condensate $\langle \bar{q}q \rangle_\varphi$ is

- the expectation value of the Dirac operator,
- expandable in a geometric series containing loops of link variables with increasing winding number.

Dual condensate $\Sigma_\nu = -\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\nu\varphi} \langle \bar{q}q \rangle_\varphi$:

- $\nu = 1$ projects out winding-#-1 loops: Dressed Polyakov Loop
- **order parameter for center symmetry breaking / confinement**
- Calculated on the lattice

  E.Bilgici *et al.*, Phys. Rev. D 77 (2008) 094007

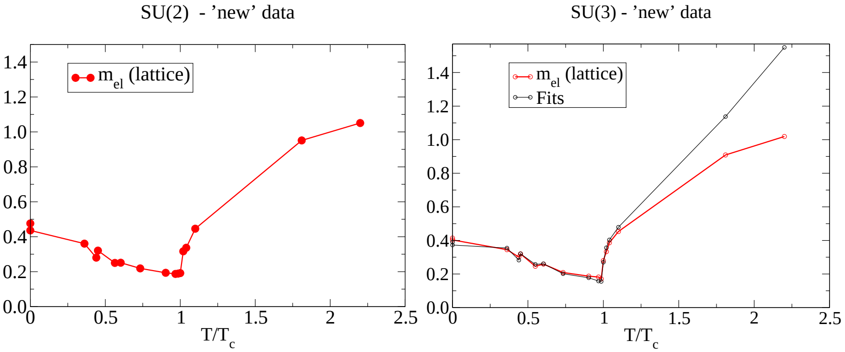
  and by functional methods

  C. S. Fischer, Phys.Rev.Lett. 103 (2009) 052003

Transition Temperature

C. S. Fischer, A. Maas, J. Müller, Eur.Phys.J. C68 (2010) 165

- T-dependent gluon propagator from lattice
- solve quark DSE with varying boundary conditions



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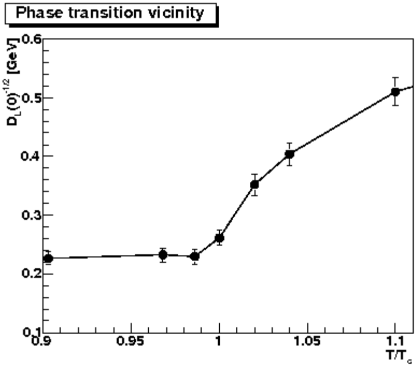

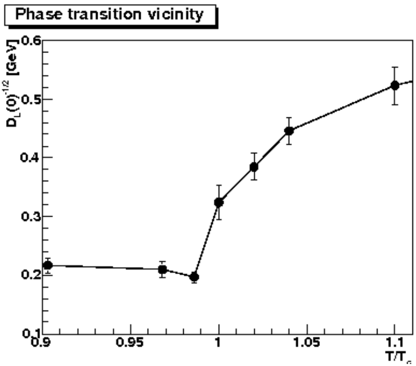

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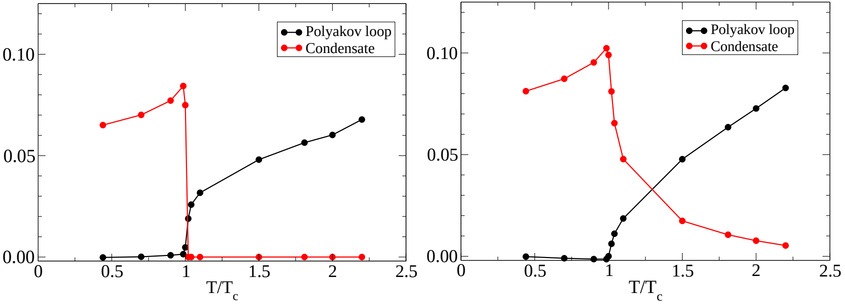


- similar transition temperatures for chiral and “deconfinement”
- SU(2): $T_c \approx$ 305 MeV; SU(3): $T_c \approx$ 270 MeV
- increasing chiral condensate due to electric screening masses

Transition Temperature

J.Braun, H.Gies, J.M.Pawlowski, Phys. Lett. B 684 (2010) 262

★ Confinement criterium from infrared exponents
via Polyakov loop potential

★ Polyakov loop potential and transition temperatures from FRG

J.Braun *et al.*, arXiv:1007.2619

★ 2nd order transition for SU(2), Ising universality class

★ 1st order transition for SU(N$\geq$3), Sp(2), E(7)

Color superconducting phase

D. Nickel, R.A., J. Wambach, Phys.Rev.**D77** (2008) 114010.

R.A., D. Horvatic, B.J. Schaefer, in preparation.

Quark Dyson–Schwinger eq. at non-vanishing chemical potential:

(No sign problem!)

- so far results for an abelian-type vertex *model*

  $$\Gamma_\mu(p,q;k) \to \gamma_\mu \alpha(k^2)/Z(k^2)$$

- add **medium modifications** of [bare] quarks to inverse vacuum gluon propagator
- use DSE $\alpha(k^2)$ (see above) or phenomenological / lattice fit

  M.S. Bhagwat *et al.*, Phys. Rev. **C68** (2003) 015203 [arXiv:nucl-th/0304003]
- impose electric and color neutrality (not decisive for main result)

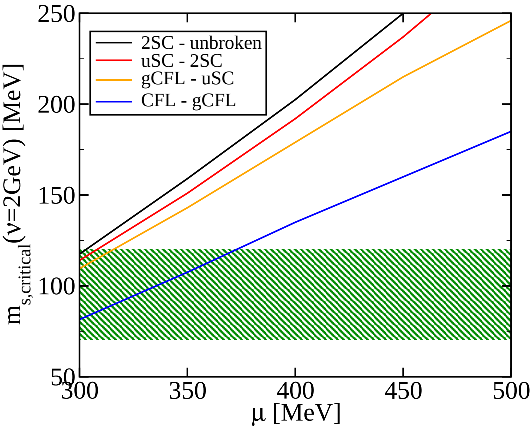
Color superconducting Phase

Huge deviations of gap functions in CFL and 2SC phase from extrapolated weak-coupling result up to $\mu \approx$ several GeV!

- weak coupling regime: gap fctns. concentrated at Fermi mom.
  strong coupling regime: no scale separation
- light quarks screen interaction also in strange quark sector, not present in NJL model calculations!
- prefered phase from pressure in CJT formalism:
  color-flavor-locked (like) phase!

Color superconducting Phase

Phase diagram: Renormalized strange mass vs. chemical potential



Translational invariance:

Only color-flavor-locked phase for realistic strange quark masses!

Summary

Landau gauge QCD Green functions

- Gluons confined by ghosts: Positivity violated!
  Gluons removed from S–matrix!
- Chiral symmetry dynamically broken! In 2- and **3-**point function!
- Quark confinement: In IR dominantly scalar!
- $U_A(1)$ anomaly: topology $\leftrightarrow$ infrared domain
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  (Kugo–Ojima Confinement,
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  in different phases

Thermodynamic observables at all T and μ!

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Thanks to all who

calculated Green functions from DSEs, FRG, lattice

either numerically or their infrared behaviour analytically:

Aguilar, Binosi, Bicudo, Bloch, Boucaud, Bogolubsky, Bornyakov, Bowman, Braun, Cucchieri, De Soto, Dudal, Fischer, Gies, Gracey, Huber, Ilgenfritz, Langfeld, Leinweber, Leroy, Litim, Llanes-Estrada, Nakamura, Natale, Nedelko, Maas, Mendes, Micheli, Mitrjushkin, Müller-Preußker, Oliveira, Papavassiliou, Pawlowski, Pene, Petreczky, Quandt, Reinhardt, Rodriguez-Quintero, Schwenzer, Silva, Skullerud, Sorella, Stamatescu, Sternbeck, Vandersickel, Verschelde, Smekal, Wambach, Williams, Zwanziger, ...

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