

# Zero temperature properties of mesons in a vector meson extended linear sigma model

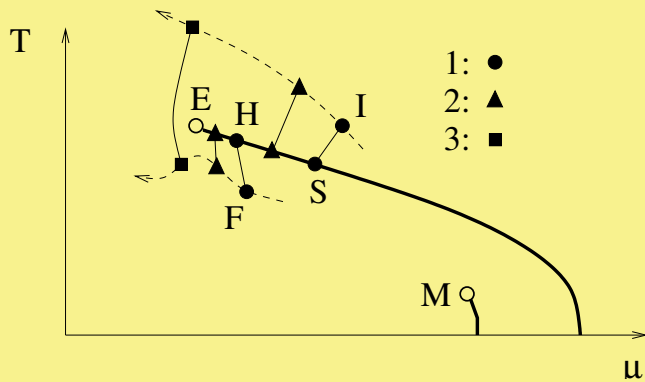
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- Motivation for building an effective model
- Sigma model without vectormesons
- Previous results at finite temperature/chemical potentials
- Sigma model with vectormesons
- Mixing, parameterization
- Conclusions

# Investigation of the critical endpoint (CEP)



- the CEP is experimentally accessible
- $\mu_B, \mu_I \neq 0$  in heavy ion collision experiments
- $\mu_B$  is tunable  $\rightarrow$  beam energy, centrality
- $\mu_I$  is tunable  $\rightarrow$  different isotopes of an element
- In some experiment even  $\mu_Y$  plays role
- Focusing effect: if CEP exist it cannot be missed

Brookhaven AGS exp. Si+Au collision: at  $\mu_B = 540$  MeV  $\rightarrow \mu_Y \approx 150$  MeV

CERN SPS exp. Pb+Pb collision:  
at  $\mu_B = 233 - 266$  MeV  $\rightarrow \mu_Y \approx 70 - 80$  MeV,  $\mu_I \approx 12 - 13$  MeV

CMB exp. at FAIR will explore QCD phase diagram at high  $\mu_B$

Analogy to the QCD CEP  $\rightarrow$  liquid-gas phase transition which is easy to hit

lattice simulations at finite  $\mu$  is very difficult

$\implies$  not all the methods predict/find the CEP

CEP found at:  $(T, \mu_B)_{\text{CEP}} = (162 \pm 2, 360 \pm 40)$  MeV, volume:  $12 \times 4^3$  and  $m_\pi = m_\pi^{\text{phys}}$

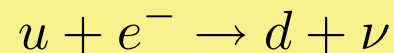
Z. Fodor, S. D. Katz, JHEP 0404:050,2004

it is important to study the CEP and its  $\mu_I, \mu_Y$  dependence in effective models

# Investigation of the pion condensation

- Quark matter can exist in neutron stars  $\longrightarrow$  at very large bariochemical potential ( $\mu_B \approx 1 \text{ GeV}$ )  
If the isospin chemical potential is also different from zero  $\longrightarrow$  possibility of pion condensation
- In heavy ion collisions  $\mu_I$  is tunable with different isotopes of an element

Neutrino emission from pion condensed quark matter  $\rightarrow$  direct **Urca processes**:



$\implies$  It might be investigated experimentally

# Sigma model without vectormesons

$$\mathcal{L} = \text{Tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m_0^2 \Phi^\dagger \Phi) - \lambda_1 (\text{Tr}(\Phi^\dagger \Phi))^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2$$

$$+ c (\det(\Phi) + \det(\Phi^\dagger)) + \text{Tr}(H(\Phi + \Phi^\dagger)) + \bar{\psi} (i\partial - g_F \Phi_5) \psi.$$

$$\Phi = \sum_{i=0}^8 (\sigma_i + i\pi_i) T_i, \quad \Phi_5 = \sum_{i=0}^8 (\sigma_i + i\gamma_5 \pi_i) T_i, \quad H = \sum_{i=0}^8 h_i T_i$$

pseudo(scalar) fields:  $\pi_i, \sigma_i$ , constituent quark field:  $\bar{\psi} = (u, d, s)$  (optional)

$$U(3) \text{ generators: } T_0 := \frac{1}{\sqrt{6}} \mathbf{1}, \quad T_i = \frac{\lambda_i}{2} \quad i = 1 \dots 8.$$

**determinant** breaks  $U_A(1)$  symmetry

**explicit symmetry breaking:** external fields  $h_0, h_8 \neq 0 \iff m_u = m_d \neq 0, m_s \neq 0$   
 or  $h_0, h_3, h_8 \neq 0 \iff m_u \neq m_d \neq 0, m_s \neq 0$

broken symmetry phase: non-zero condensates  $(\langle \sigma_0 \rangle, \langle \sigma_8 \rangle) \longleftrightarrow (\Phi_N, \Phi_S)$  or  
 $(\langle \sigma_0 \rangle, \langle \sigma_8 \rangle), \langle \sigma_3 \rangle \longleftrightarrow (\Phi_N, \Phi_S), \Phi_I$

$\Phi_N$ : non-strange,  $\Phi_S$ : strange

technical difficulty: mixing in the 0, 8 or 0, 3, 8 sector

parameters determined from the  $T = 0$  mass spectrum

# The optimized perturbation theory (OPT)

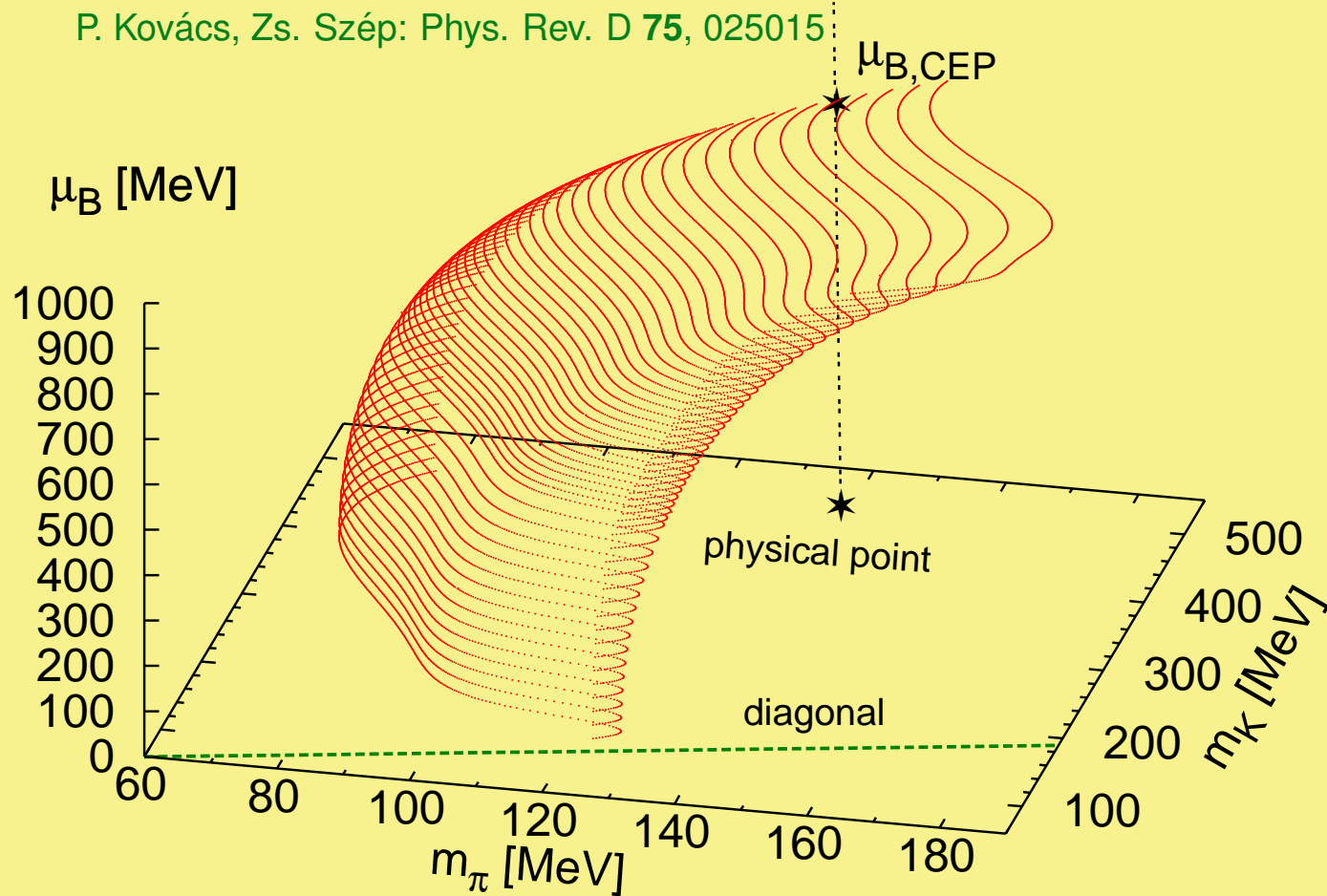
At finite temperature tree level mass squares can become negative  $\rightarrow$  some sort of resummation is needed

Using the optimized perturbation theory (OPT):

- a temperature dependent mass term introduced in the Lagrangian
- the difference is treated as a higher order counterterm
- the new mass parameter determined by the FAC criterion ( $m^{1\text{-loop}} = m^{\text{tree}}$ )  $\rightarrow$  can be transformed to an equation for a resummed particle mass
- conserves Ward-identities

# Results at zero $\mu_I, \mu_Y$ : critical surface and CEP

P. Kovács, Zs. Szép: Phys. Rev. D **75**, 025015

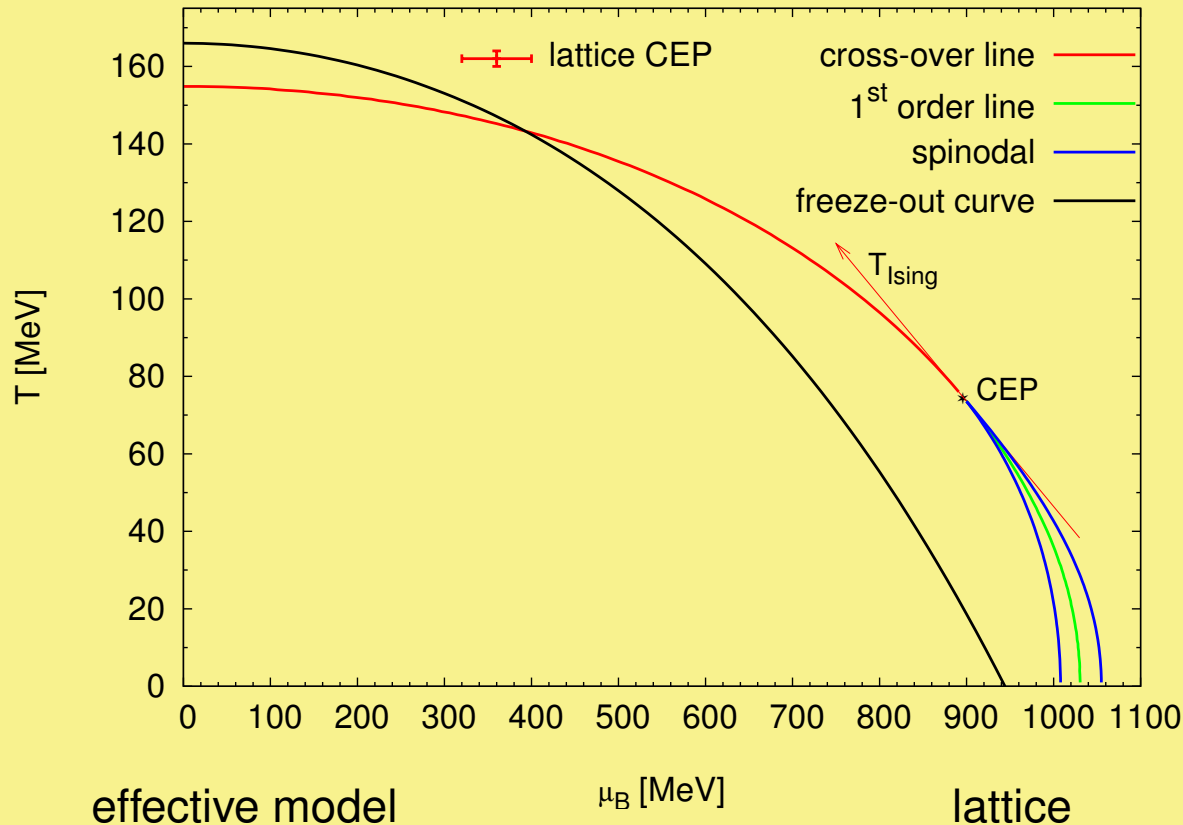


The second order surface bends towards the physical point  
 $\implies$  **The CEP must exist**

The continuation is reliable up to  $m_K \approx 500$  MeV and above the diagonal

# The CEP at the physical point of the mass plane

P. Kovács, Zs. Szép: Phys. Rev. D **75**, 025015



- $T_c(\mu_B = 0) = 154.84 \text{ MeV}$   
 $\Delta T_c(x\chi) = 15.5 \text{ MeV}$

- $T_{CEP} = 74.83 \text{ MeV}$   
 $\mu_{B,CEP} = 895.38 \text{ MeV}$

- $T_c \left. \frac{d^2 T_c}{d\mu_B^2} \right|_{\mu_B=0} = -0.09$

- $T_c(\mu_B = 0) = 151(3) \text{ MeV}$   
 $\Delta T_c(\chi_{\bar{\psi}\psi}) = 28(5) \text{ MeV}$

Y. Aoki, *et al.*, PLB **643**, 46 (2006)

- $T_{CEP} = 162(2) \text{ MeV}$   
 $\mu_{B,CEP} = 360(40) \text{ MeV}$

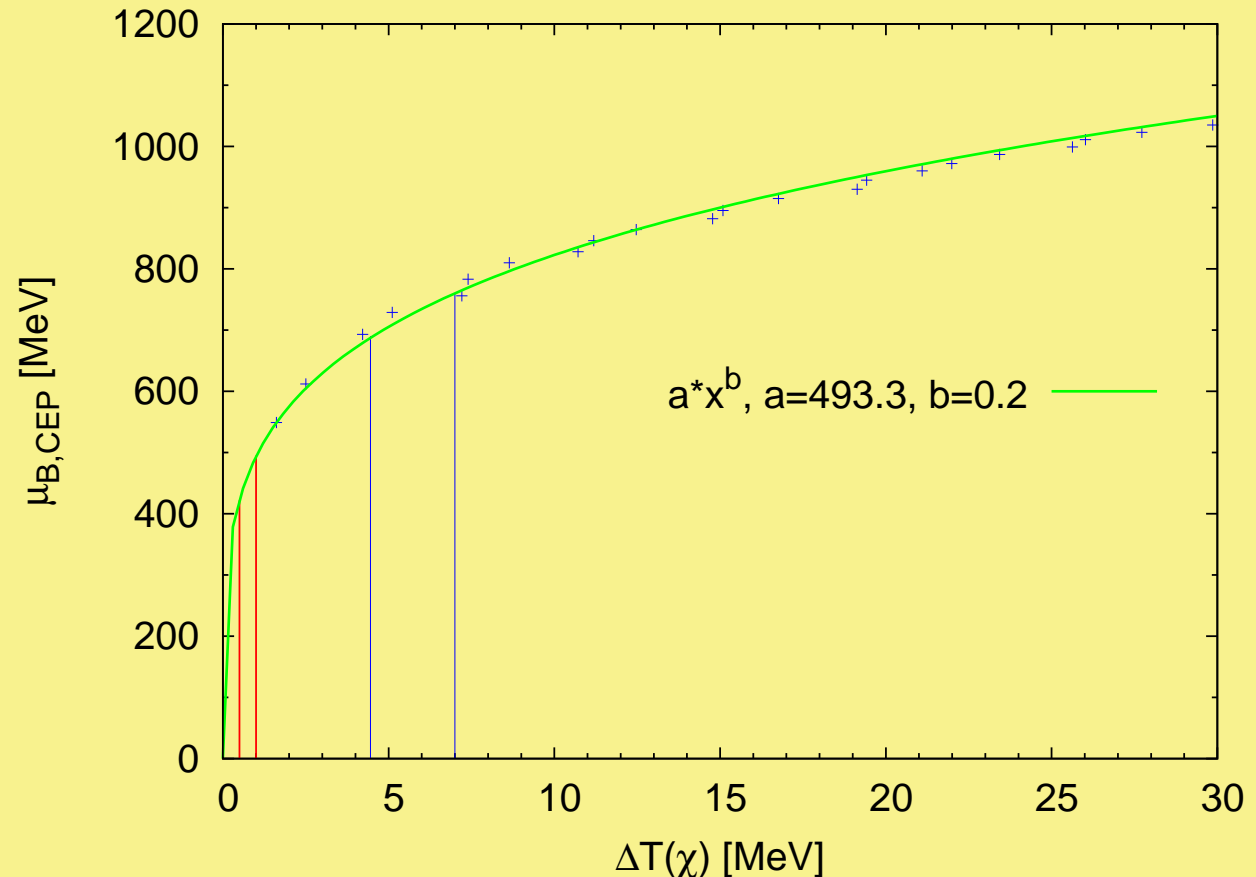
- $-0.058(2)$

Z. Fodor, *et al.*, JHEP 0404 (2004) 050

# Dependence of the $\mu_{B,CEP}$ on the width of the susceptibility

$\mu_{B,CEP} = 725(35) \text{ MeV} \rightarrow$   
non-physical quark mass

$\mu_{B,CEP} = 360(40) \text{ MeV} \rightarrow$   
physical quark mass



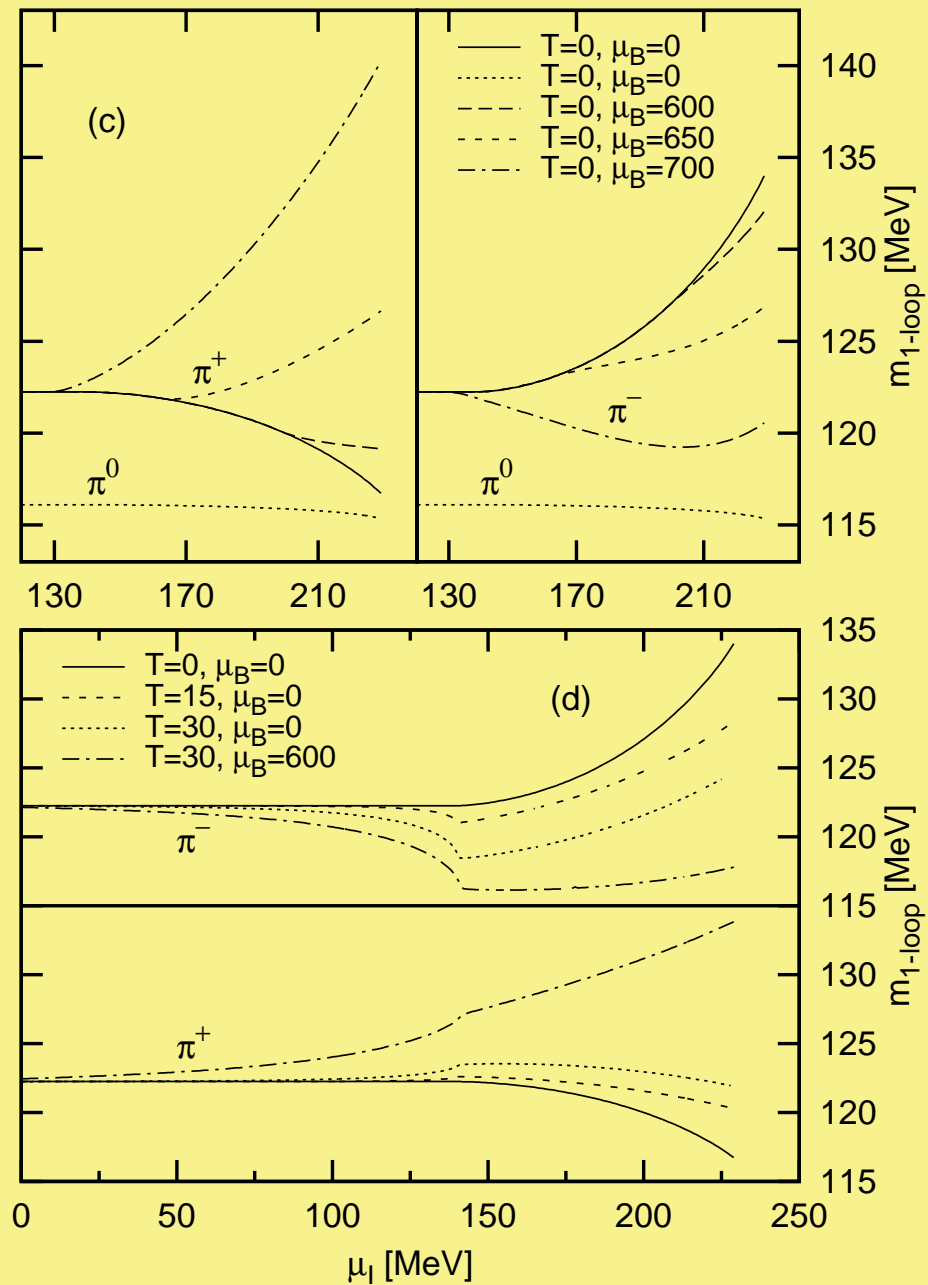
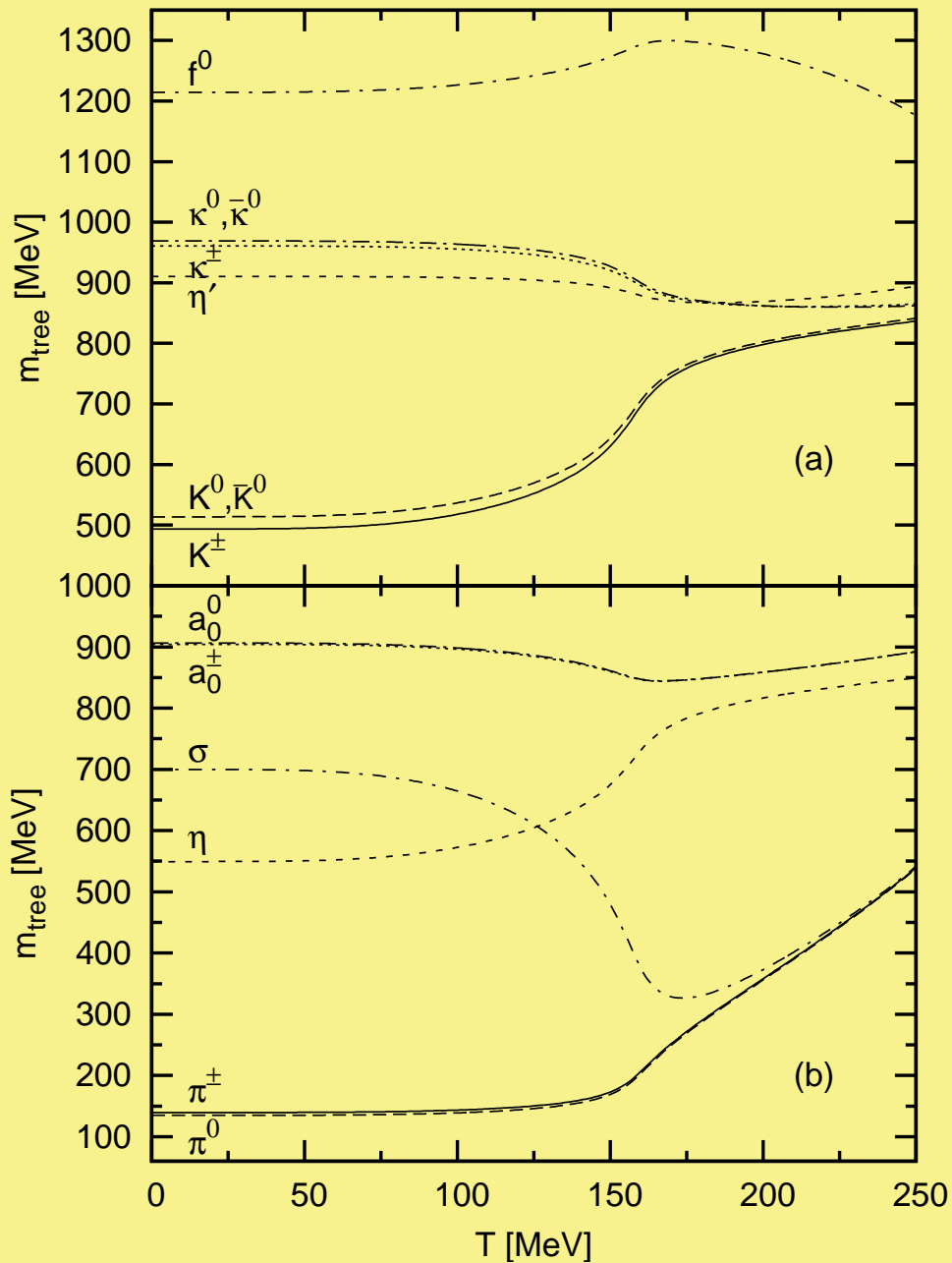
Preliminary lattice estimation by S. Katz:  $\Delta T_c(\chi_{\bar{\psi}\psi}) \approx 0.5 - 1 \text{ MeV}$   
 $\Delta T_c(\chi_{\bar{\psi}\psi}) \approx 2 - 4 \text{ MeV}$

Since  $\Delta T_c(\chi_{\bar{\psi}\psi}) \approx 28 \text{ MeV}$  at the physical point  $\longrightarrow$  higher  $\mu_{B,CEP}$  expected



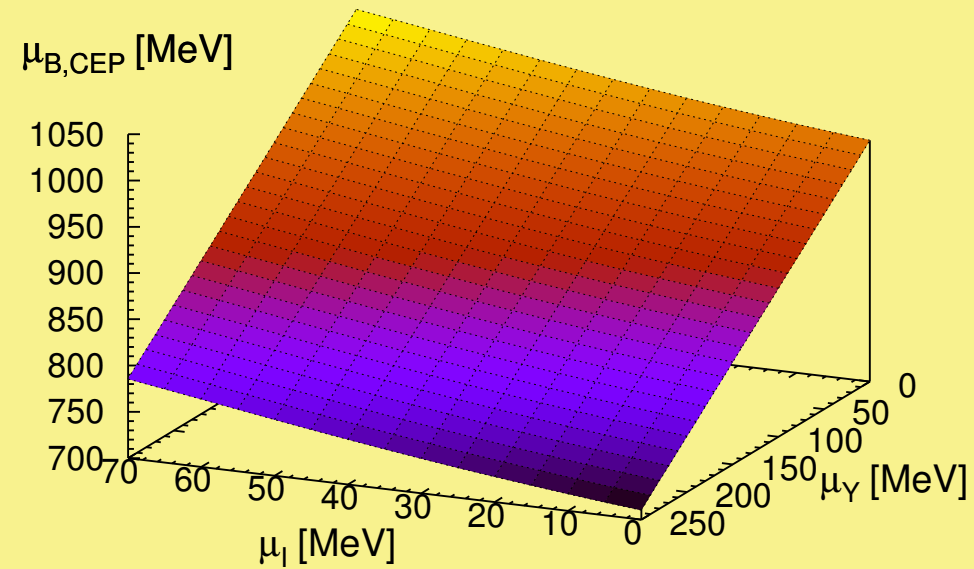
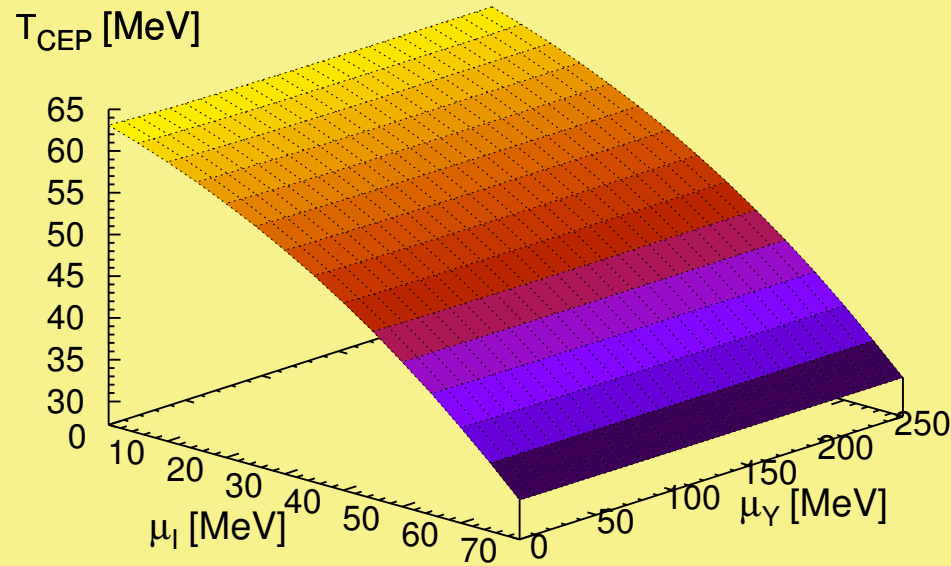
# Tree-level and 1-loop pole masses

P. Kovács, Zs. Szép: Phys. Rev. D 77, 065016



# Dependence of the CEP on $\mu_I, \mu_Y$

P. Kovács, Zs. Szép: Phys. Rev. D **77**, 065016



$T_{\text{CEP}}$  is almost independent of  $\mu_Y$ , but significantly depend on  $\mu_I$

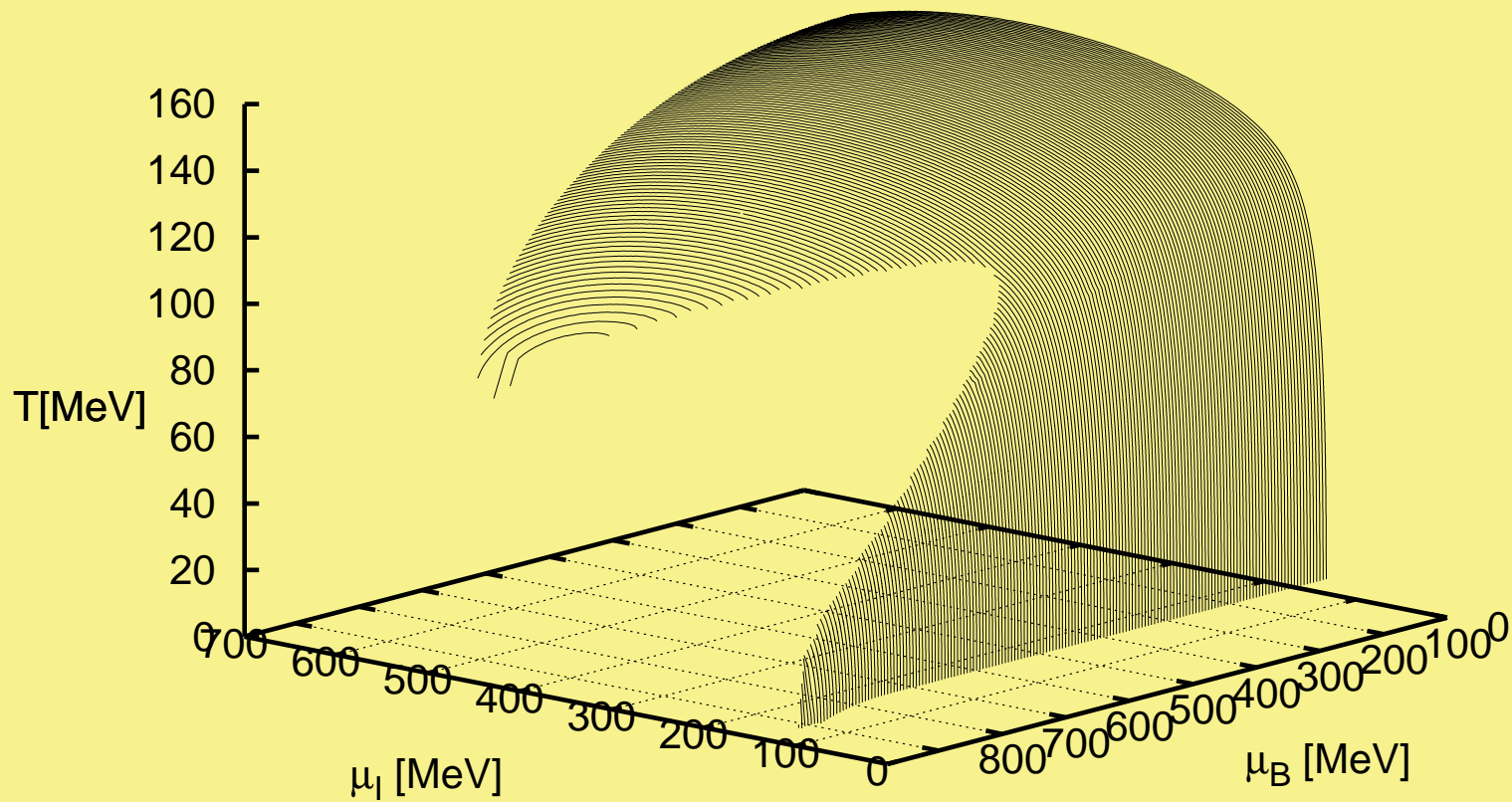
$\mu_{B,\text{CEP}}$  has an almost linear dependence on both other chemical potential

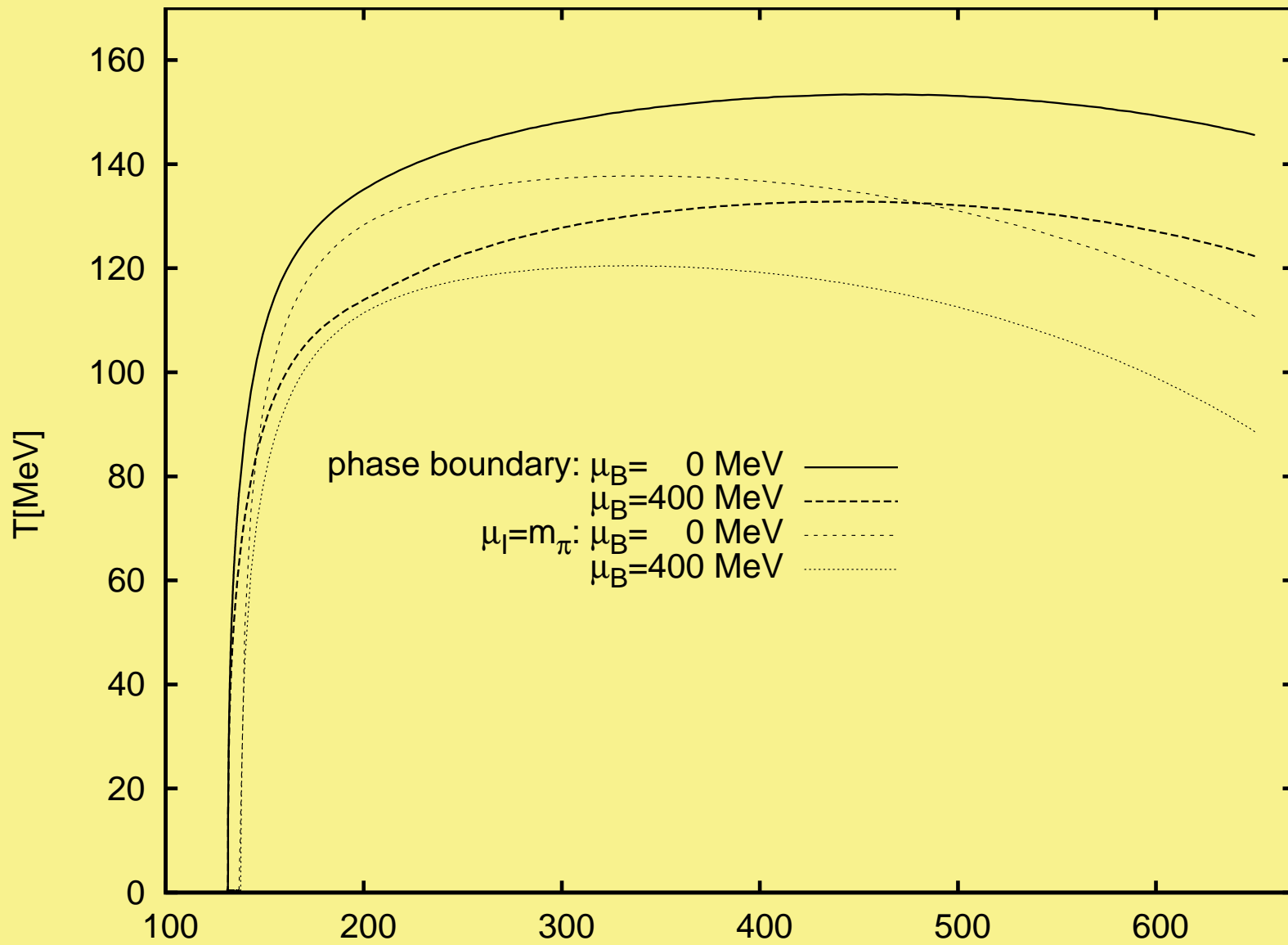
As  $\mu_Y$  is increased the phase transition at  $T = 0$  becomes stronger

# Critical surface of pion condensation

The second order boundary for the occurrence of the pion condensation in the  $\mu_I - \mu_B - T$  space from the two flavoured sigma model:

T. Herpay, P. Kovács: Phys. Rev. D **78**,116008





condensation starts at  $\mu_I = 131$  MeV at zero temperature

## Sigma model with vectormesons

$$\begin{aligned}
 \mathcal{L} = & \text{Tr} [(D^\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr} [(\Phi^\dagger \Phi)^2] \\
 & + \text{Tr} [H(\Phi + \Phi^\dagger)] + c(\det \Phi + \det \Phi^\dagger) - \frac{1}{4} \text{Tr} [(L^{\mu\nu})^2 + (R^{\mu\nu})^2] \\
 & + \frac{m_1^2}{2} \text{Tr} [(L^\mu)^2 + (R^\mu)^2] - 2ig_2 (\text{Tr}\{L_{\mu\nu} [L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu} [R^\mu, R^\nu]\}) \\
 & - 2g_3 [\text{Tr} (\{\partial_\mu L_\nu - ieA_\mu [T_3, L_\nu] + \partial_\nu L_\mu - ieA_\nu [T_3, L_\mu]\} \{L^\mu, L^\nu\}) \\
 & \quad + \text{Tr} (\{\partial_\mu R_\nu - ieA_\mu [T_3, R_\nu] + \partial_\nu R_\mu - ieA_\nu [T_3, R_\mu]\} \{R^\mu, R^\nu\})] \\
 & + \frac{\xi_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}[(L^\mu)^2 + (R^\mu)^2] + \xi_2 \text{Tr}[(\Phi R^\mu)^2 + (L^\mu \Phi)^2] + 2\xi_3 \text{Tr}(\Phi R_\mu \Phi^\dagger L^\mu). \\
 & + g_4 \{ \text{Tr} [L^\mu L^\nu L_\mu L_\nu] + \text{Tr} [R^\mu R^\nu R_\mu R_\nu] \} + g_5 \{ \text{Tr} [L^\mu L_\mu L^\nu L_\nu] + \text{Tr} [R^\mu R_\mu R^\nu R_\nu] \} \\
 & + g_6 \text{Tr} [R^\mu R_\mu] \text{Tr} [L^\nu L_\nu] + g_7 \{ \text{Tr}[L^\mu L_\mu] \text{Tr}[L^\nu L_\nu] + \text{Tr}[R^\mu R_\mu] \text{Tr}[R^\nu R_\nu] \} ,
 \end{aligned}$$

where

$$D^\mu \Phi = \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu) - ieA^\mu [T_3, \Phi]$$

$$R^\mu = \sum_{i=0}^8 (\rho_i^\mu - b_i^\mu) T_i$$

$$L^\mu = \sum_{i=0}^8 (\rho_i^\mu + b_i^\mu) T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA^\nu [T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA^\nu [T_3, R^\mu]\}$$

particles:

- scalars:  $a_0(980)$ ,  $K_s(1425)$ ,  $f_0(1370)$ ,  $f_0(1710)$  (or  $f_0(600)$ ,  $f_0(980)$ )
- pseudoscalars:  $\pi(138)$ ,  $K(496)$ ,  $\eta(548)$ ,  $\eta'(958)$
- vectormesons:  $\rho(775)$ ,  $K^*(893)$ ,  $\omega(783)$ ,  $\Phi(1020)$
- axialvector-mesons:  $a_1(1230)$ ,  $K_1(1272)$ ,  $f_1(1170)$ ,  $f_1(1426)$

→ All the mesons up to  $\sim 1$  GeV are taken into account

# Spontaneous symmetry breaking and field mixing

$\langle \Phi \rangle = T_i v_i \implies \sigma_i \rightarrow \sigma_i + v_i$ , only  $v_0$  and  $v_8$  are nonzero  
 ( $v_3 = 0 \rightarrow$  isospin symmetry)

Quadratic terms after shifting  $\sigma_i$ :

$$\begin{aligned}
 \mathcal{L}^{quad} = & -\frac{1}{2}\sigma_a(\partial^2\delta_{ab} + (m_\sigma^2)_{ab})\sigma_b - \frac{1}{2}\pi_a(\partial^2\delta_{ab} + (m_\pi^2)_{ab})\pi_b \\
 & - \frac{1}{2}\rho_a^\mu [(-g_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu)\delta_{ab} - g_{\mu\nu}(m_\rho^2)_{ab}] \rho_b^\nu \\
 & - \frac{1}{2}b_a^\mu [(-g_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu)\delta_{ab} - g_{\mu\nu}(m_b^2)_{ab}] b_b^\nu \\
 & - \frac{1}{2}\rho_a^\mu (g_1 f_{abc} v_c \partial_\mu) \sigma_b - \frac{1}{2}\sigma_a (g_1 f_{abc} v_c \partial_\mu) \rho_b^\mu \\
 & - \frac{1}{2}b_a^\mu (g_1 d_{abc} v_c \partial_\mu) \pi_b + \frac{1}{2}\pi_a (g_1 d_{abc} v_c \partial_\mu) b_b^\mu
 \end{aligned}$$

Mixing in the 0 – 8 sector for every field and mixing between  $\rho_a^\mu \leftrightarrow \sigma$  and  $b_a^\mu \leftrightarrow \pi$   
 $\implies$

For instance the pseudoscalar-axialvector mixing matrix

$$\begin{pmatrix} d_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & d_{08} \\ 0 & 0 & d_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & d_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & d_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & d_4 & 0 & 0 \\ d_{08} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & d_8 \end{pmatrix}$$

where  $d_0 = \frac{\sqrt{6}}{3}v_0$ ,  $d_1 = \frac{\sqrt{6}}{3}v_0 + \frac{\sqrt{3}}{3}v_8$ ,  $d_4 = \frac{\sqrt{6}}{3}v_0 - \frac{\sqrt{3}}{6}v_8$ ,  $d_8 = \frac{\sqrt{6}}{3}v_0 - \frac{\sqrt{3}}{3}v_8$ ,  
 $d_{08} = \frac{\sqrt{6}}{3}v_8$

The mixing terms can be written in a much simpler form if the base is switched from 0 – 8 to the non-strange–strange base

→ With linear transformations of the fields all the mixing can be resolved

→ This will result in the appearance of **wave function renormalization constants**:  $Z_\pi, Z_\eta, Z_{\eta'}, Z_K, Z_{K_s}$



# Parameterization

Unknown parameters of the model are:  $m_0^2, m_1^2, c, g_1, \lambda_1, \lambda_2, \xi_1, \xi_2, \xi_3$   
and the two condensates:  $\Phi_N, \Phi_S \longrightarrow$  **11 parameters**

$\longrightarrow$  With these parameters all the tree level masses as well as every wave function renormalization constants can be expressed

For instance:

$$m_\pi^2 = Z_\pi^2 \left[ m_0^2 + \left( \lambda_1 + \frac{\lambda_2}{2} \right) \Phi_N^2 + \lambda_1 \Phi_S^2 - \frac{c}{\sqrt{2}} \Phi_S \right]$$
$$Z_\pi = \frac{m_{a_1}}{\sqrt{m_{a_1}^2 - g_1^2 \Phi_N^2}}$$

There is **14 different mass** from 16  $\longrightarrow$  it **can be chosen 11** to determine the parameters (with multiparametric minimalization)

## Conclusions and outlook

- Lots of interesting phenomena/physical quantity can be investigated with the linear sigma model, like particle spectra, temperature and chemical potential dependence of the masses, the CEP, pion/kaon condensates, decay width, scattering lengths, etc.
- The vectormeson extended linear sigma model contains every diquark degrees of freedom up to  $\sim 1$  GeV
- The particle mixing can be resolved, which introduce wave function renormalization to some of the mesons
- Low lying scalar particle is problematic  $\longrightarrow$  it may be not a  $q\bar{q}$  state  $\longrightarrow$  further investigation is needed
- In the future: finite temperature/chemical potential investigations with the vectormeson extended sigma model