Zero temperature properties of mesons in a vector meson extended linear sigma model

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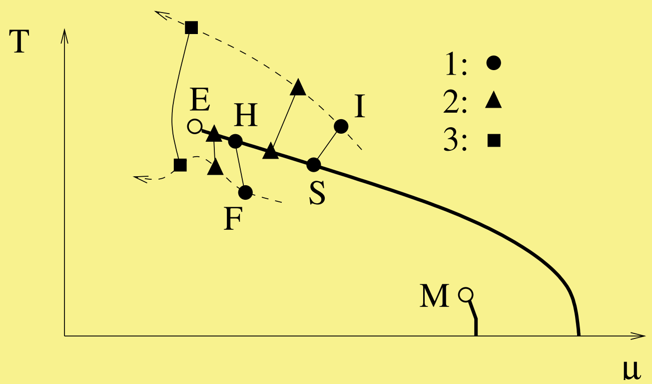
- Motivation for building an effective model
- Sigma model without vectormesons
- Previous results at finite temperature/chemical potentials
- Sigma model with vectormesons
- Mixing, parameterization
- Conclusions

Collaborators: D. Rischke, D. Parganlija, F. Giacosa, Gy. Wolf

Investigation of the critical endpoint (CEP)



- the CEP is experimentally accessible
- $\mu_B, \mu_I \neq 0$ in heavy ion collision experiments
- μ_B is tunable $\rightarrow$ beam energy, centrality
- μ_I is tunable $\rightarrow$ different isotopes of an element
- In some experiment even μ_Y plays role
- Focusing effect: if CEP exist it cannot be missed

Brookhaven AGS exp. Si+Au collision: at $\mu_B = 540$ MeV $\rightarrow \mu_Y \approx 150$ MeV

CERN SPS exp. Pb+Pb collision:
at $\mu_B = 233 - 266$ MeV $\rightarrow \mu_Y \approx 70 - 80$ MeV, $\mu_I \approx 12 - 13$ MeV

CBM exp. at FAIR will explore QCD phase diagram at high μ_B

Analogy to the QCD CEP $\rightarrow$ liquid-gas phase transition which is easy to hit

lattice simulations at finite μ is very difficult
$\Longrightarrow$ not all the methods predict/find the CEP

CEP found at: $(T, \mu_B)_{\text{CEP}} = (162 \pm 2, 360 \pm 40)$ MeV, volume: 12×4^3 and $m_\pi = m_\pi^{\text{phys}}$

Z. Fodor, S. D. Katz, JHEP 0404:050,2004

it is important to study the CEP and its μ_I, μ_Y dependence in effective models

Investigation of the pion condensation

- Quark matter can exist in neutron stars $\longrightarrow$ at very large bariochemical potential ($\mu_{\mathrm{B}} \approx 1$ GeV)

  If the isospin chemical potential is also different from zero $\longrightarrow$ possibility of pion condensation

- In heavy ion collisions μ_I is tunable with different isotopes of an element

Neutrino emission from pion condensed quark matter $\rightarrow$ direct Urca processes:

$$d \rightarrow u + e^- + \bar{\nu}$$

$$u + e^- \rightarrow d + \nu$$

$\Longrightarrow$ It might be investigated experimentaly

Sigma model without vectormesons

$$\mathcal{L} = \mathrm{Tr}(\partial_\mu\Phi^\dagger\partial^\mu\Phi - m_0^2\Phi^\dagger\Phi) - \lambda_1\left(\mathrm{Tr}(\Phi^\dagger\Phi)\right)^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2$$
$$+ c\left(\det(\Phi)+\det(\Phi^\dagger)\right) + \mathrm{Tr}(H(\Phi+\Phi^\dagger)) + \bar\psi\,(i\partial\!\!\!/ - g_F\Phi_5)\,\psi.$$

$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i$, $\Phi_5 = \sum_{i=0}^{8}(\sigma_i + i\gamma_5\pi_i)T_i$, $H = \sum_{i=0}^{8} h_i T_i$

pseudo(scalar) fields: π_i, σ_i, constituent quark field: $\bar\psi = (u, d, s)$ (optional)

$U(3)$ generators: $T_0 := \frac{1}{\sqrt{6}}\mathbf{1}$, $T_i = \frac{\lambda_i}{2}$ $i = 1\ldots 8.$

determinant breaks $U_A(1)$ symmetry
explicit symmetry breaking: external fields $h_0, h_8 \neq 0 \Longleftrightarrow m_u = m_d \neq 0, m_s \neq 0$
or $h_0, h_3, h_8 \neq 0 \Longleftrightarrow m_u \neq m_d \neq 0, m_s \neq 0$

broken symmetry phase: non-zero condensates $(\langle\sigma_0\rangle, \langle\sigma_8\rangle) \longleftrightarrow (\Phi_N, \Phi_S)$ or
$(\langle\sigma_0\rangle, \langle\sigma_8\rangle), \langle\sigma_3\rangle \longleftrightarrow (\Phi_N, \Phi_S), \Phi_I$

Φ_N: non-strange, Φ_S: strange

technical difficulty: mixing in the $0, 8$ or $0, 3, 8$ sector

parameters determined from the $T = 0$ mass spectrum

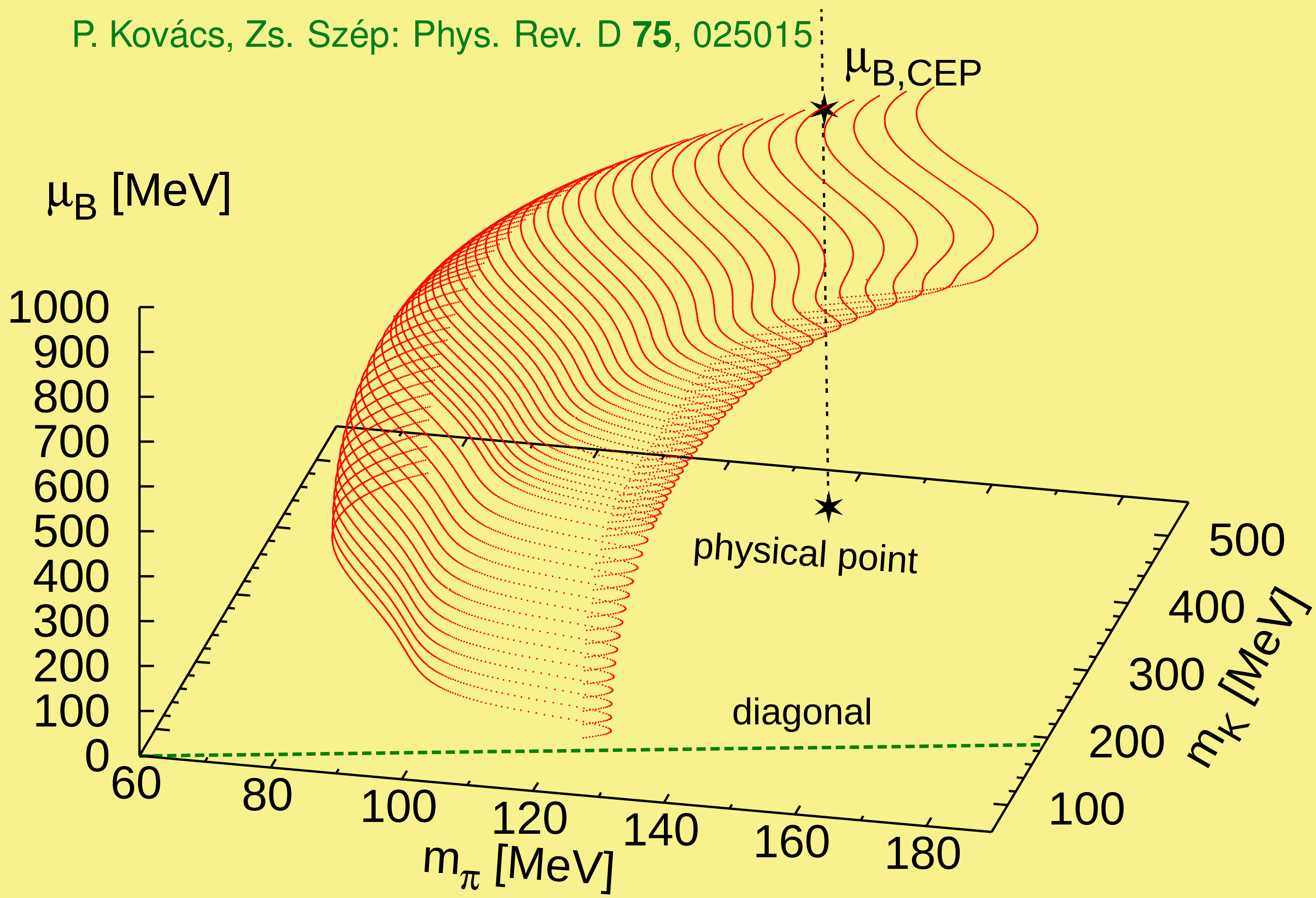
The optimized perturbation theory (OPT)

At finite temperature tree level mass squares can become negative $\rightarrow$ some sort of resummation is needed

Using the optimized perturbation theory (OPT):

- a temperature dependent mass term introduced in the Lagrangian
- the difference is treated as a higher order counterterm
- the new mass parameter determined by the FAC criterion ($m^{1\text{–loop}} = m^{\text{tree}}$) $\rightarrow$ can be transformed to an equation for a resummed particle mass
- conserves Ward–identities

Results at zero μ_I, μ_Y: critical surface and CEP



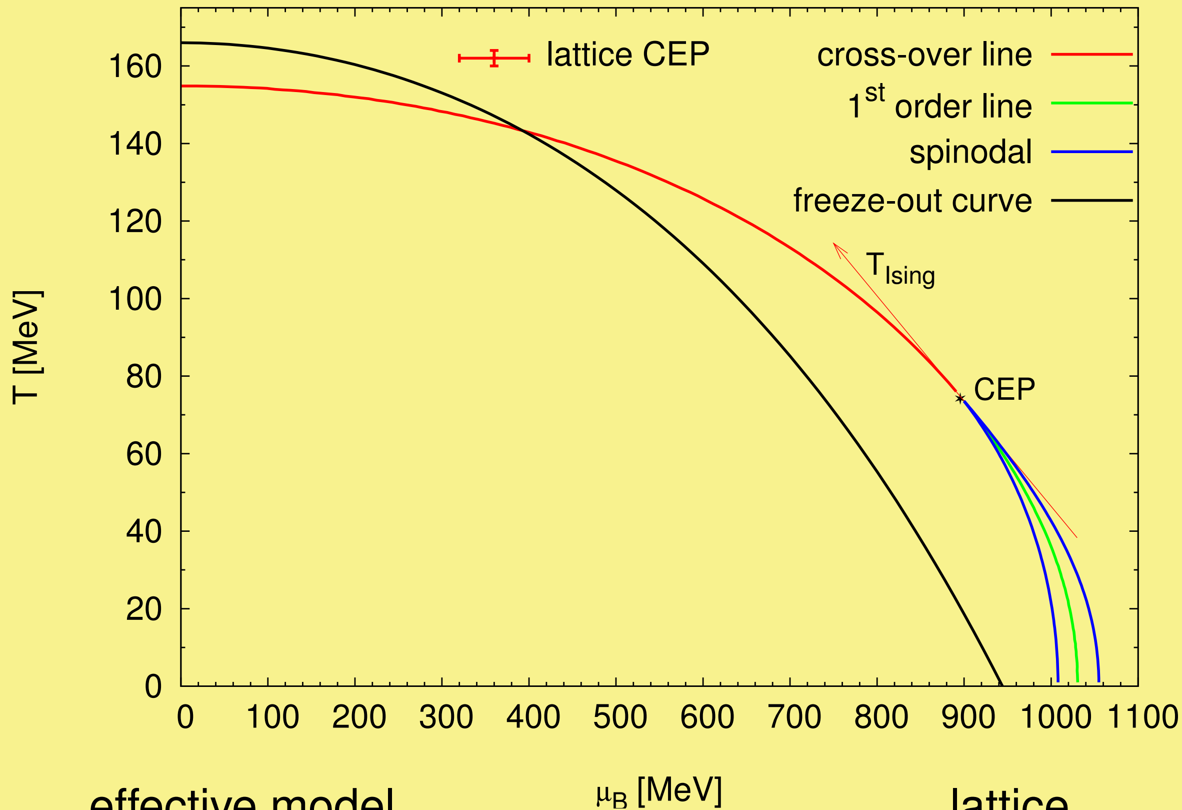
The second order surface bends towards the physical point

$\Longrightarrow$ The CEP must exist

The continuation is reliable up to $m_K \approx 500$ MeV and above the diagonal

The CEP at the physical point of the mass plane

P. Kovács, Zs. Szép: Phys. Rev. D **75**, 025015



effective model

- $T_c(\mu_B = 0) = 154.84$ MeV
  $\Delta T_c(x\chi) = 15.5$ MeV

- $T_{CEP} = 74.83$ MeV
  $\mu_{B,CEP} = 895.38$ MeV

- $T_c \frac{d^2 T_c}{d\mu_B^2}\Big|_{\mu_B=0} = -0.09$

lattice

- $T_c(\mu_B = 0) = 151(3)$ MeV
  $\Delta T_c(\chi_{\bar{\psi}\psi}) = 28(5)$ MeV

  Y. Aoki, *et al.*, PLB **643**, 46 (2006)

- $T_{CEP} = 162(2)$ MeV
  $\mu_{B,CEP} = 360(40)$ MeV

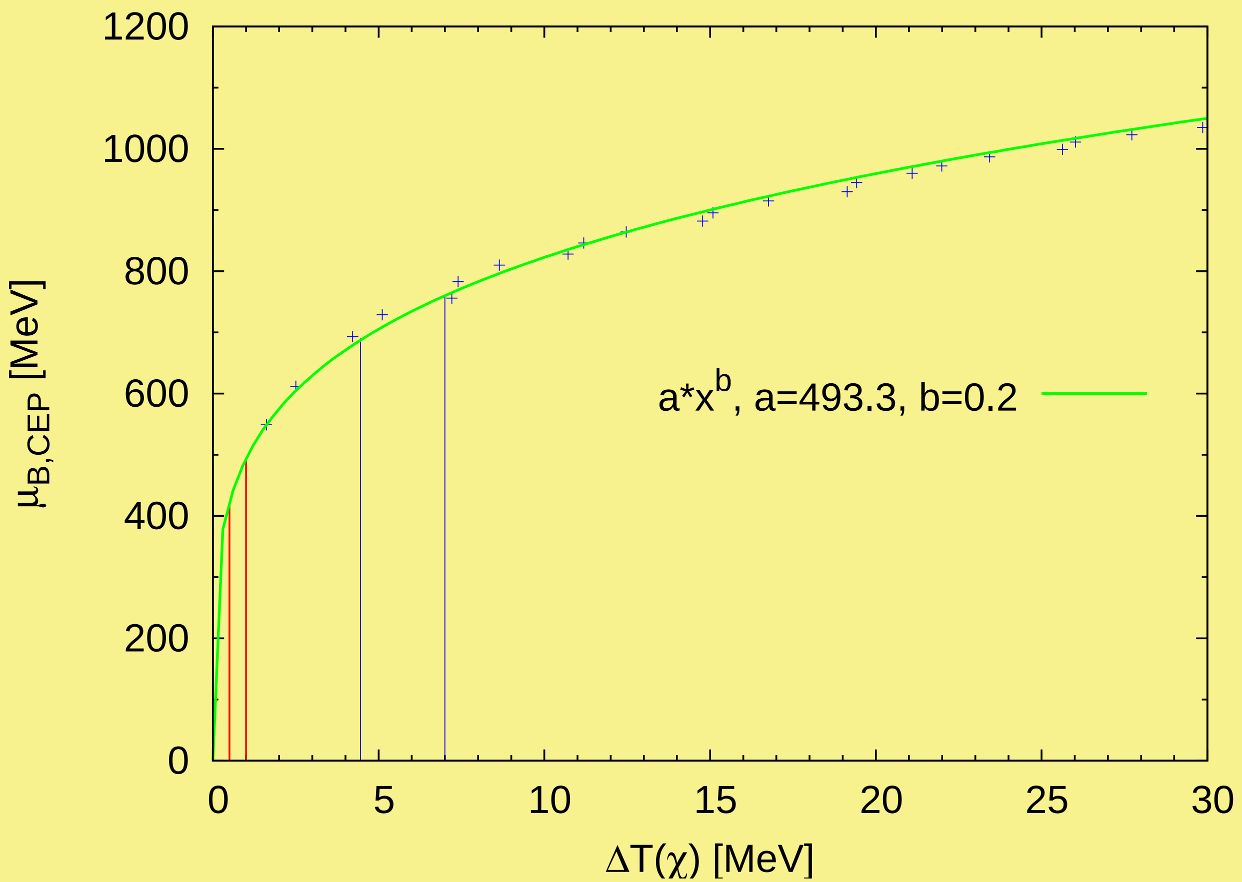
- $-0.058(2)$

  Z. Fodor, *et al.*, JHEP 0404 (2004) 050

Dependence of the $\mu_{B,CEP}$ on the width of the susceptibility

$\mu_{\mathrm{B,CEP}} = 725(35)$ MeV $\longrightarrow$ non-physical quark mass

$\mu_{\mathrm{B,CEP}} = 360(40)$ MeV $\longrightarrow$ physical quark mass



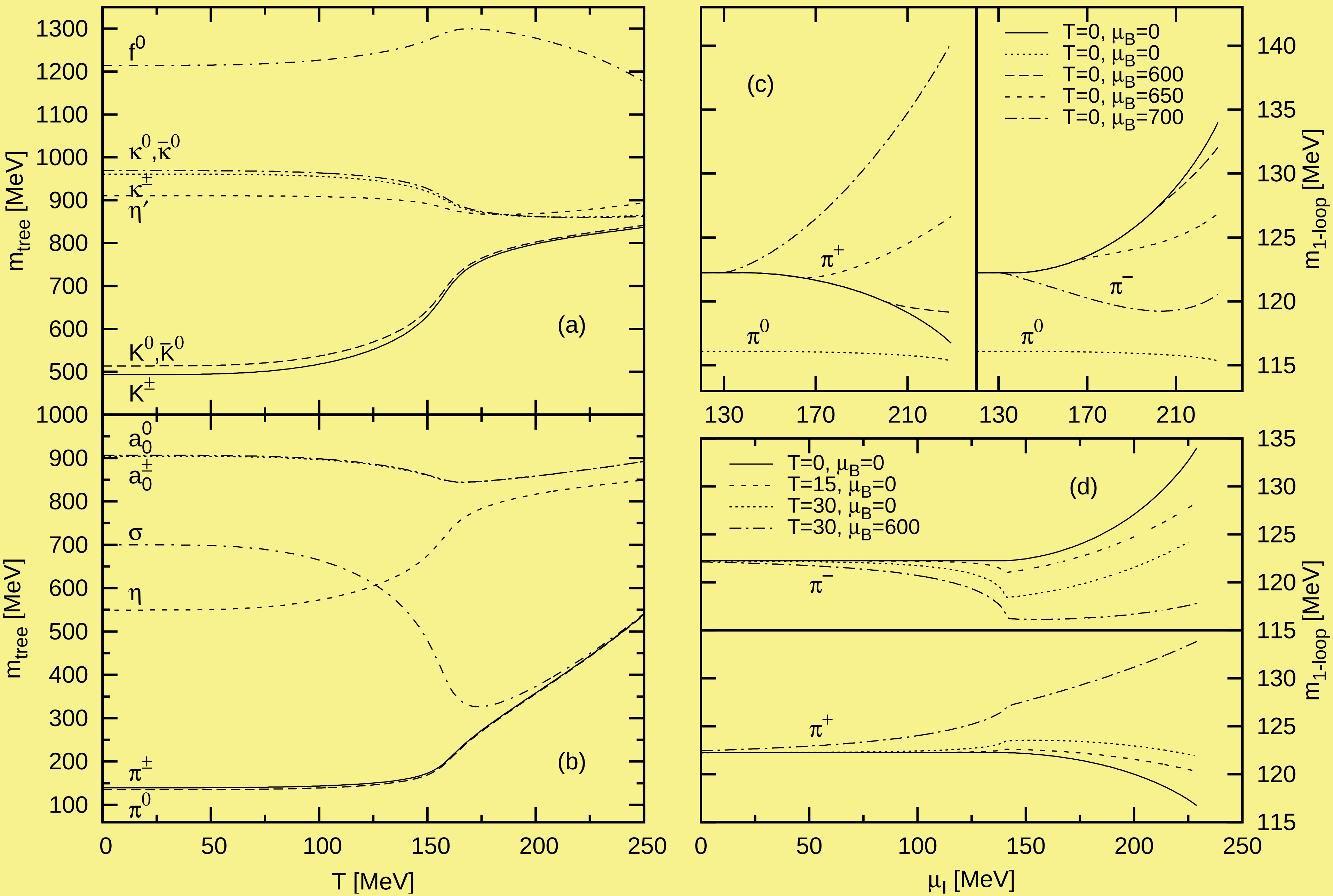
Preliminary lattice estimation by S. Katz: $\Delta T_c(\chi_{\bar\psi\psi}) \approx 0.5 - 1$ MeV

$\Delta T_c(\chi_{\bar\psi\psi}) \approx 2 - 4$ MeV

Since $\Delta T_c(\chi_{\bar\psi\psi}) \approx 28$ MeV at the physical point $\longrightarrow$ higher $\mu_{B,CEP}$ expected

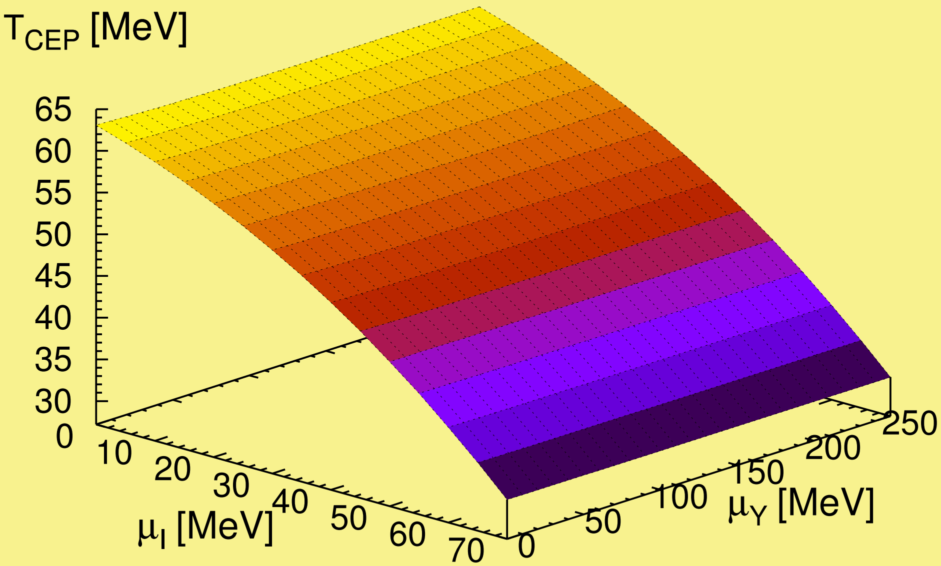
Tree-level and 1-loop pole masses

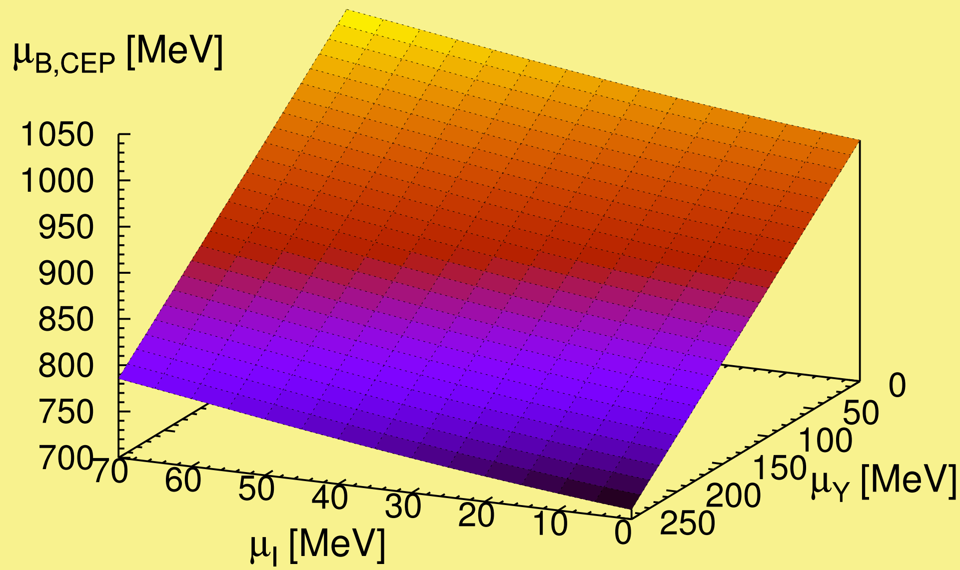
P. Kovács, Zs. Szép: Phys. Rev. D **77**, 065016



Dependence of the CEP on μ_I, μ_Y

P. Kovács, Zs. Szép: Phys. Rev. D **77**, 065016





T_{CEP} is almost independent of μ_Y, but significantly depend on μ_I

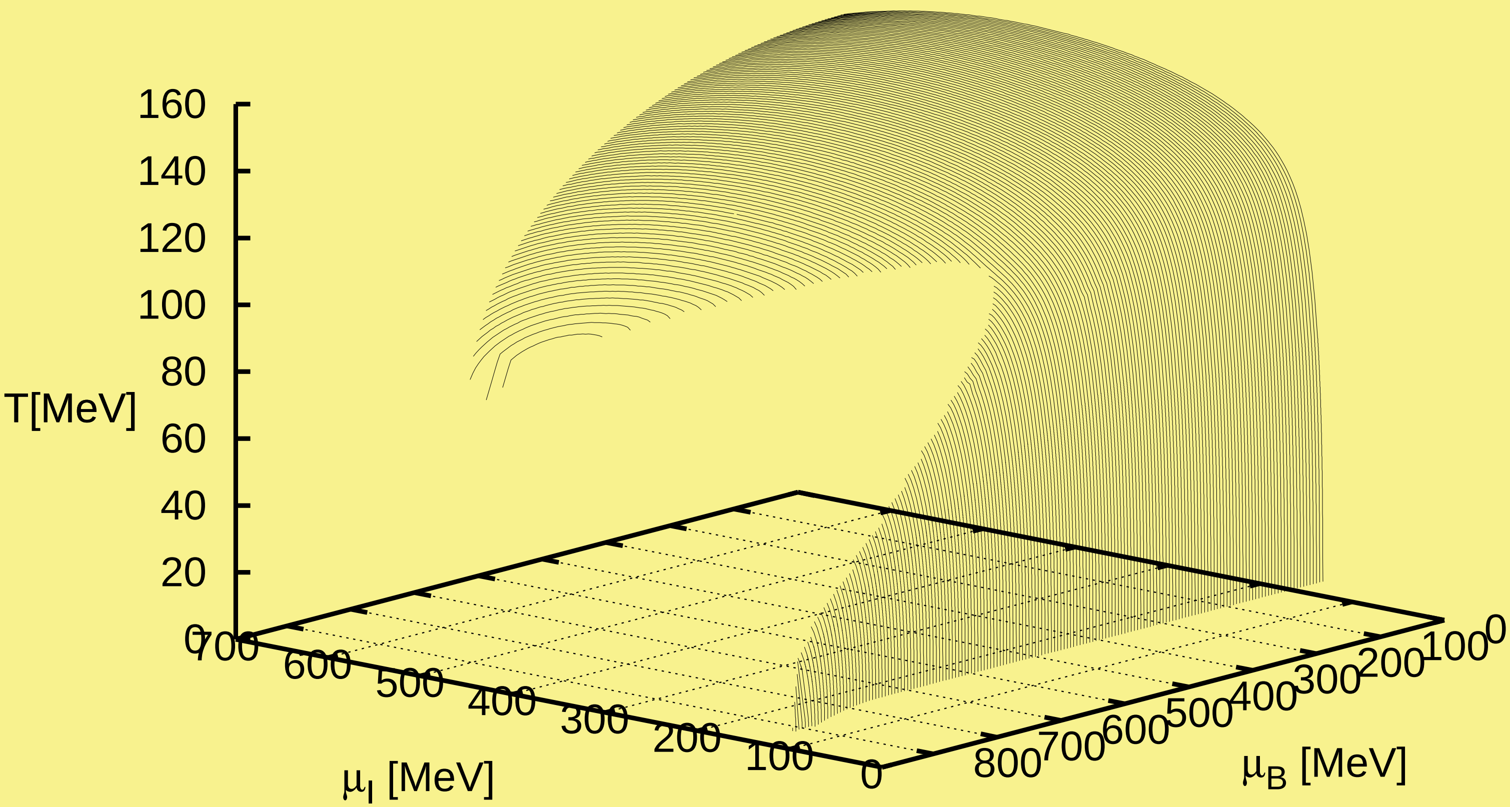
$\mu_{B,\mathrm{CEP}}$ has an almost linear dependence on both other chemical potential

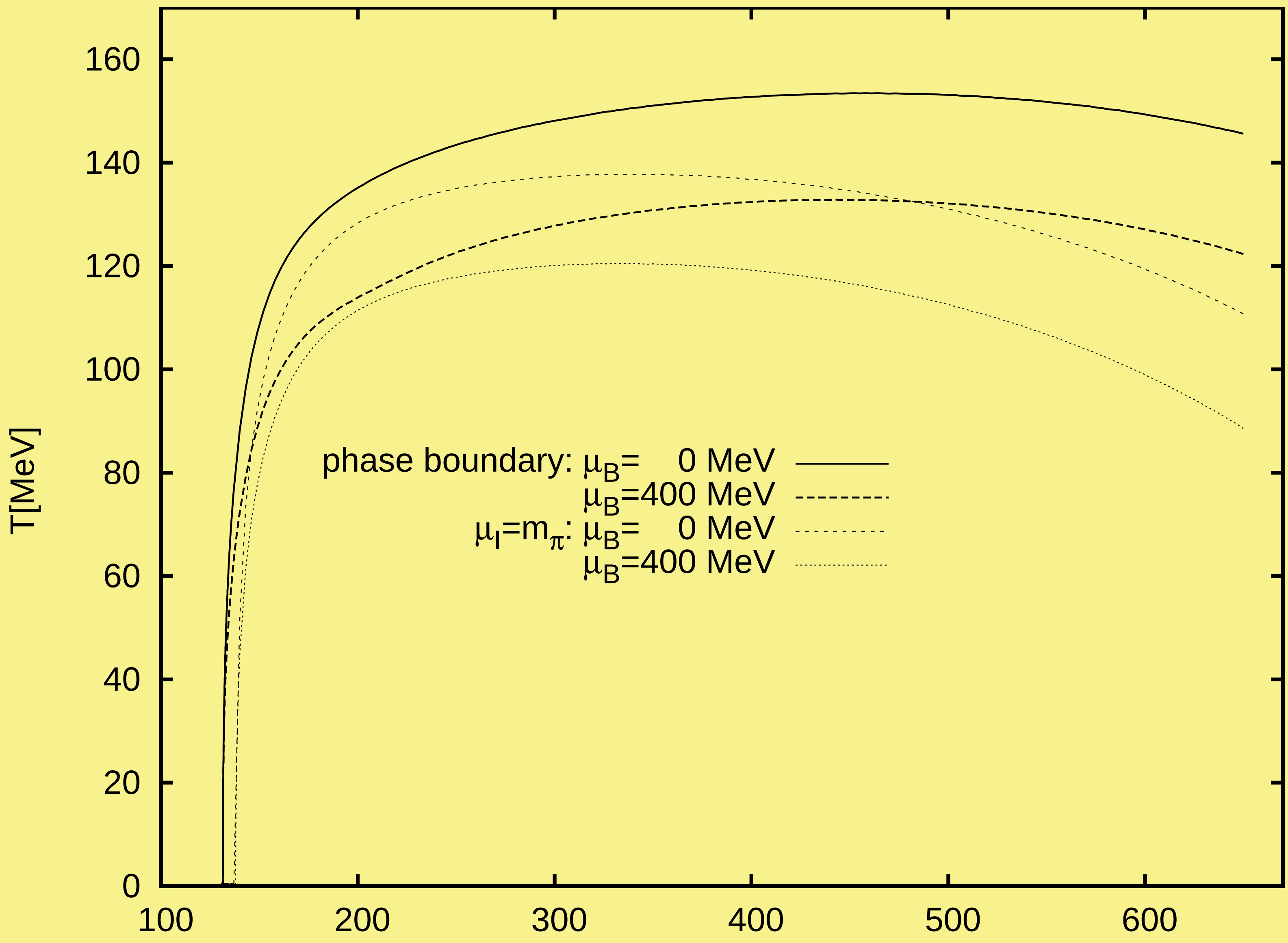
As μ_Y is increased the phase transition at $T = 0$ becomes stronger

Critical surface of pion condensation

The second order boundary for the occurrence of the pion condensation in the $\mu_\mathrm{I} - \mu_\mathrm{B} - T$ space from the two flavoured sigma model:

T. Herpay, P. Kovács: Phys. Rev. D **78**,116008





condensation starts at $\mu_I = 131$ MeV at zero temperature

Sigma model with vectormesons

$$\mathcal{L} = \mathrm{Tr}\left[(D^{\mu}\Phi)^{\dagger}(D^{\mu}\Phi)\right] - m_0^2\mathrm{Tr}(\Phi^{\dagger}\Phi) - \lambda_1\left[\mathrm{Tr}(\Phi^{\dagger}\Phi)\right]^2 - \lambda_2\mathrm{Tr}\left[(\Phi^{\dagger}\Phi)^2\right]$$
$$+\mathrm{Tr}\left[H(\Phi+\Phi^{\dagger})\right] + c(\det\Phi + \det\Phi^{\dagger}) - \frac{1}{4}\mathrm{Tr}\left[(L^{\mu\nu})^2 + (R^{\mu\nu})^2\right]$$
$$+\frac{m_1^2}{2}\mathrm{Tr}\left[(L^{\mu})^2 + (R^{\mu})^2\right] - 2ig_2(\mathrm{Tr}\{L_{\mu\nu}[L^{\mu},L^{\nu}]\} + \mathrm{Tr}\{R_{\mu\nu}[R^{\mu},R^{\nu}]\})$$
$$-2g_3\left[\mathrm{Tr}\left(\{\partial_{\mu}L_{\nu} - ieA_{\mu}[T_3,L_{\nu}] + \partial_{\nu}L_{\mu} - ieA_{\nu}[T_3,L_{\mu}]\}\{L^{\mu},L^{\nu}\}\right)\right.$$
$$\left. + \mathrm{Tr}\left(\{\partial_{\mu}R_{\nu} - ieA_{\mu}[T_3,R_{\nu}] + \partial_{\nu}R_{\mu} - ieA_{\nu}[T_3,R_{\mu}]\}\{R^{\mu},R^{\nu}\}\right)\right]$$
$$+\frac{\xi_1}{2}\mathrm{Tr}(\Phi^{\dagger}\Phi)\mathrm{Tr}[(L^{\mu})^2 + (R^{\mu})^2] + \xi_2\mathrm{Tr}[(\Phi R^{\mu})^2 + (L^{\mu}\Phi)^2] + 2\xi_3\mathrm{Tr}(\Phi R_{\mu}\Phi^{\dagger}L^{\mu}).$$
$$+g_4\{\mathrm{Tr}[L^{\mu}L^{\nu}L_{\mu}L_{\nu}] + \mathrm{Tr}[R^{\mu}R^{\nu}R_{\mu}R_{\nu}]\} + g_5\{\mathrm{Tr}[L^{\mu}L_{\mu}L^{\nu}L_{\nu}] + \mathrm{Tr}[R^{\mu}R_{\mu}R^{\nu}R_{\nu}]\}$$
$$+g_6\mathrm{Tr}[R^{\mu}R_{\mu}]\,\mathrm{Tr}[L^{\nu}L_{\nu}] + g_7\{\mathrm{Tr}[L^{\mu}L_{\mu}]\,\mathrm{Tr}[L^{\nu}L_{\nu}] + \mathrm{Tr}[R^{\mu}R_{\mu}]\,\mathrm{Tr}[R^{\nu}R_{\nu}]\}\ ,$$

where

$$D^{\mu}\Phi = \partial^{\mu}\Phi - ig_1(L^{\mu}\Phi - \Phi R^{\mu}) - ieA^{\mu}[T_3,\Phi]$$

$$R^\mu = \sum_{i=0}^{8} (\rho_i^\mu - b_i^\mu) T_i$$

$$L^\mu = \sum_{i=0}^{8} (\rho_i^\mu + b_i^\mu) T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA^\nu [T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA^\nu [T_3, R^\mu]\}$$

particles:

- scalars: $a_0(980)$, $K_s(1425)$, $f_0(1370)$, $f_0(1710)$ (or $f_0(600)$, $f_0(980)$)
- pseudoscalars: $\pi(138)$, $K(496)$, $\eta(548)$, $\eta'(958)$
- vectormesons: $\rho(775)$, $K^\star(893)$, $\omega(783)$, $\Phi(1020)$
- axialvector-mesons: $a_1(1230)$, $K_1(1272)$, $f_1(1170)$, $f_1(1426)$

$\longrightarrow$ All the mesons up to ~ 1 GeV are taken into account

Spontaneous symmetry breaking and field mixing

$\langle\Phi\rangle = T_i v_i \Longrightarrow \sigma_i \to \sigma_i + v_i$, only v_0 and v_8 are nonzero
($v_3 = 0 \to$ isospin symmetry)

Quadratic terms after shifting σ_i:

$$
\begin{aligned}
\mathcal{L}^{quad} = & -\frac{1}{2}\sigma_a(\partial^2\delta_{ab} + (m_\sigma^2)_{ab})\sigma_b - \frac{1}{2}\pi_a(\partial^2\delta_{ab} + (m_\pi^2)_{ab})\pi_b \\
& - \frac{1}{2}\rho_a^\mu\left[(-g_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu)\delta_{ab} - g_{\mu\nu}(m_\rho^2)_{ab}\right]\rho_b^\nu \\
& - \frac{1}{2}b_a^\mu\left[(-g_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu)\delta_{ab} - g_{\mu\nu}(m_b^2)_{ab}\right]b_b^\nu \\
& - \frac{1}{2}\rho_a^\mu(g_1 f_{abc}v_c\partial_\mu)\sigma_b - \frac{1}{2}\sigma_a(g_1 f_{abc}v_c\partial_\mu)\rho_b^\mu \\
& - \frac{1}{2}b_a^\mu(g_1 d_{abc}v_c\partial_\mu)\pi_b + \frac{1}{2}\pi_a(g_1 d_{abc}v_c\partial_\mu)b_b^\mu
\end{aligned}
$$

Mixing in the $0-8$ sector for every field and mixing between $\rho_a^\mu \leftrightarrow \sigma$ and $b_a^\mu \leftrightarrow \pi$
$\Longrightarrow$

For instance the pseudoscalar-axialvector mixing matrix

$$\begin{pmatrix} d_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & d_{08} \\ 0 & 0 & d_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & d_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & d_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & d_4 & 0 & 0 \\ d_{08} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & d_8 \end{pmatrix}$$

where $d_0 = \frac{\sqrt{6}}{3} v_0$, $d_1 = \frac{\sqrt{6}}{3} v_0 + \frac{\sqrt{3}}{3} v_8$, $d_4 = \frac{\sqrt{6}}{3} v_0 - \frac{\sqrt{3}}{6} v_8$, $d_8 = \frac{\sqrt{6}}{3} v_0 - \frac{\sqrt{3}}{3} v_8$, $d_{08} = \frac{\sqrt{6}}{3} v_8$

The mixing terms can be written in a much simpler form if the base is switched from $0 - 8$ to the non-strange–strange base

$\longrightarrow$ With linear transformations of the fields all the mixing can be resolved

$\longrightarrow$ This will result in the appearence of wave function renormalization constants: $Z_\pi, Z_\eta, Z_{\eta'}, Z_K, Z_{K_s}$

Parameterization

Unknown parameters of the model are: $m_0^2, m_1^2, c, g_1, \lambda_1, \lambda_2, \xi_1, \xi_2, \xi_3$
and the two condensates: $\Phi_N, \Phi_S \longrightarrow$ 11 parameters

$\longrightarrow$ With these parameters all the tree level masses as well as every wave function renormalization constants can be expressed

For instance:

$$m_\pi^2 = Z_\pi^2 \left[m_0^2 + \left(\lambda_1 + \frac{\lambda_2}{2} \right) \Phi_N^2 + \lambda_1 \Phi_S^2 - \frac{c}{\sqrt{2}} \Phi_S \right]$$

$$Z_\pi = \frac{m_{a_1}}{\sqrt{m_{a_1}^2 - g_1^2 \Phi_N^2}}$$

There is 14 different mass from 16 $\longrightarrow$ it can be choosen 11 to determine the parameters (with multiparametric minimalization)

Conclusions and outlook

- Lots of intresting phenomena/physical quantity can be investigated with the linear sigma model, like particle spectra, temperature and chemical potential dependence of the masses, the CEP, pion/kaon condensates, decay width, scattering lengthes, etc.

- The vectormeson extended linear sigma model contains every diquark degrees of freedom up to ~ 1 GeV

- The particle mixing can be resolved, which introduce wave function renormalization to some of the mesons

- Low lying scalar particle is problematic $\longrightarrow$ it may be not a $q\bar{q}$ state $\longrightarrow$ further investigation is needed

- In the future: finite temperature/chemical potential investigations with the vectormeson extended sigma model