FRG Approach to Nuclear Matter in Extreme Conditions

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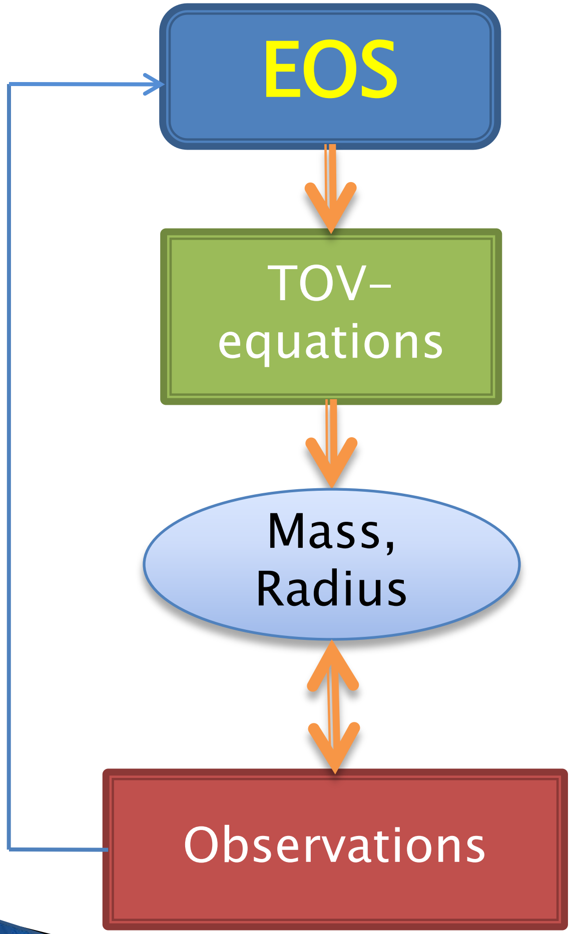
Outline

1. Introduction to FRG
2. Fermi-gas model at finite temperature
3. Solving the Wetterich-equation at finite chemical potential
4. Comparison between FRG results and other methods

Motivation for Using FRG



What are the effects of quantum fluctuations on the Equaiton of State (EOS) ?

What is the difference between the same parameters in mean field and quantum flucuations included ?

- Compressibility (important for neutron star mass!)
- Binding energy
- Surface tension of nuclear matter

FRG is a general method to take quantum fluctuations into account.

Functional Renormalization Group (FRG)

- General non-perturbative method to determine the effective action of a system.
  - **Scale dependent effective action (k scale parameter)**

$$\boxed{\partial_k \Gamma_k = \frac{1}{2}\int dp^D \, STr\left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k}\right]}$$

Wetterich equation

k=Λ

Classical action

Integration

k=0

Quantum fluctuations included

Functional Renormalization Group (FRG)

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$$\boxed{\partial_k \Gamma_k = \frac{1}{2}\int dp^D \; STr\left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k}\right]}$$

Need an ansatz for the integration

Not necessarily perturbative ansatz!

$$\Gamma_k = \sum_{l=1}^{l=N} \frac{g_l(k)}{l!}\,\hat{O}_l$$

Scale dependent coupling

Functional Renormalization Group (FRG)

- General non-perturbative method to determine the effective action of a system.
  - **Scale dependent effective action (k scale parameter)**

$$\boxed{\partial_k \Gamma_k = \frac{1}{2}\int dp^D \; STr\left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k}\right]}$$

Regulator:

- determines the modes present on scale *k*
- physics is regulator independent

Interacting Fermi-gas model

Ansatz for the effective action:

$$\boxed{\Gamma_k\left[\varphi,\psi\right] = \int d^4x \left[\bar{\psi}\left(i\partial\!\!\!/ - g\varphi\right)\psi + \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - U_k(\varphi)\right]}$$

Fermions : *m=0,* **Yukawa-coupling** generates mass

Bosons: the **potential** contains self interaction terms

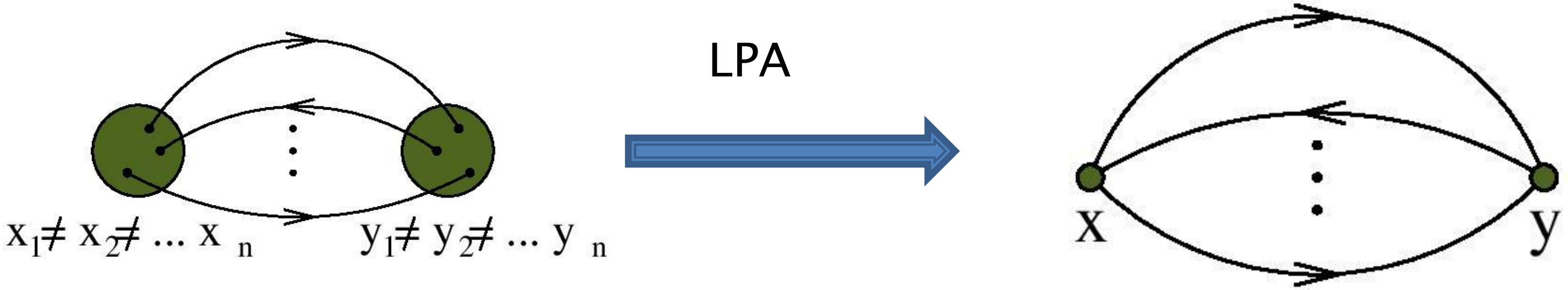
We study the scale dependence of the potential only!!

Local Potential Approximation (LPA)

What does the ansatz exactly mean ?

LPA is based on the assumption that the contribution of these two diagrams are close.

(momentum dependence of the vertices is suppressed)



This implies the following ansatz for the effective action:

$$\Gamma_k[\psi] = \int d^4x \left[\frac{1}{2}\psi_i K_{k,ij}\psi_j + U_k(\psi)\right]$$

Interacting Fermi-gas at finite temperature

Ansatz for the effective action:

$$\boxed{\Gamma_k\left[\varphi,\psi\right] = \int d^4x \left[\bar{\psi}\left(i\partial\!\!\!/ - g\varphi\right)\psi + \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - U_k(\varphi)\right]}$$

Wetterich –equation

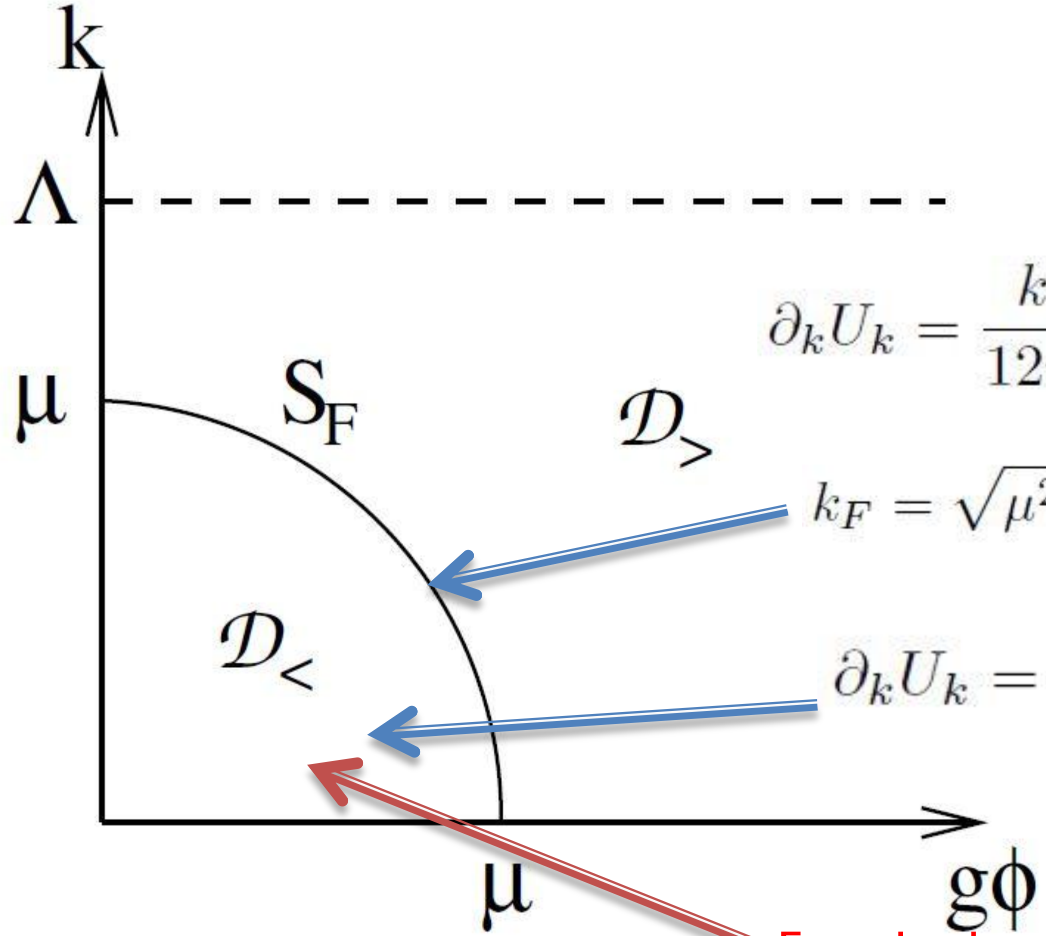
$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\underbrace{\frac{1+2n_B(\omega_B)}{\omega_B}}_{\text{Bosonic part}} + 4\underbrace{\frac{-1+n_F(\omega_F-\mu)+n_F(\omega_F+\mu)}{\omega_F}}_{\text{Fermionic part}}\right]$$

$$U_\Lambda(\varphi) = \frac{m_0^2}{2}\varphi^2 + \frac{\lambda_0}{24}\varphi^4 \qquad \omega_F^2 = k^2 + g^2\varphi^2 \qquad \omega_B^2 = k^2 + \partial_\varphi^2 U \qquad n_{B/F}(\omega) = \frac{1}{1\mp e^{-\beta\omega}}$$

Interacting Fermi-gas at zero temperature

$T=0\,,\ \mu\neq 0 \Longrightarrow n_F(\omega) \to \Theta(-\omega)$

We have two equations for the two values of the step function each valid on different domain



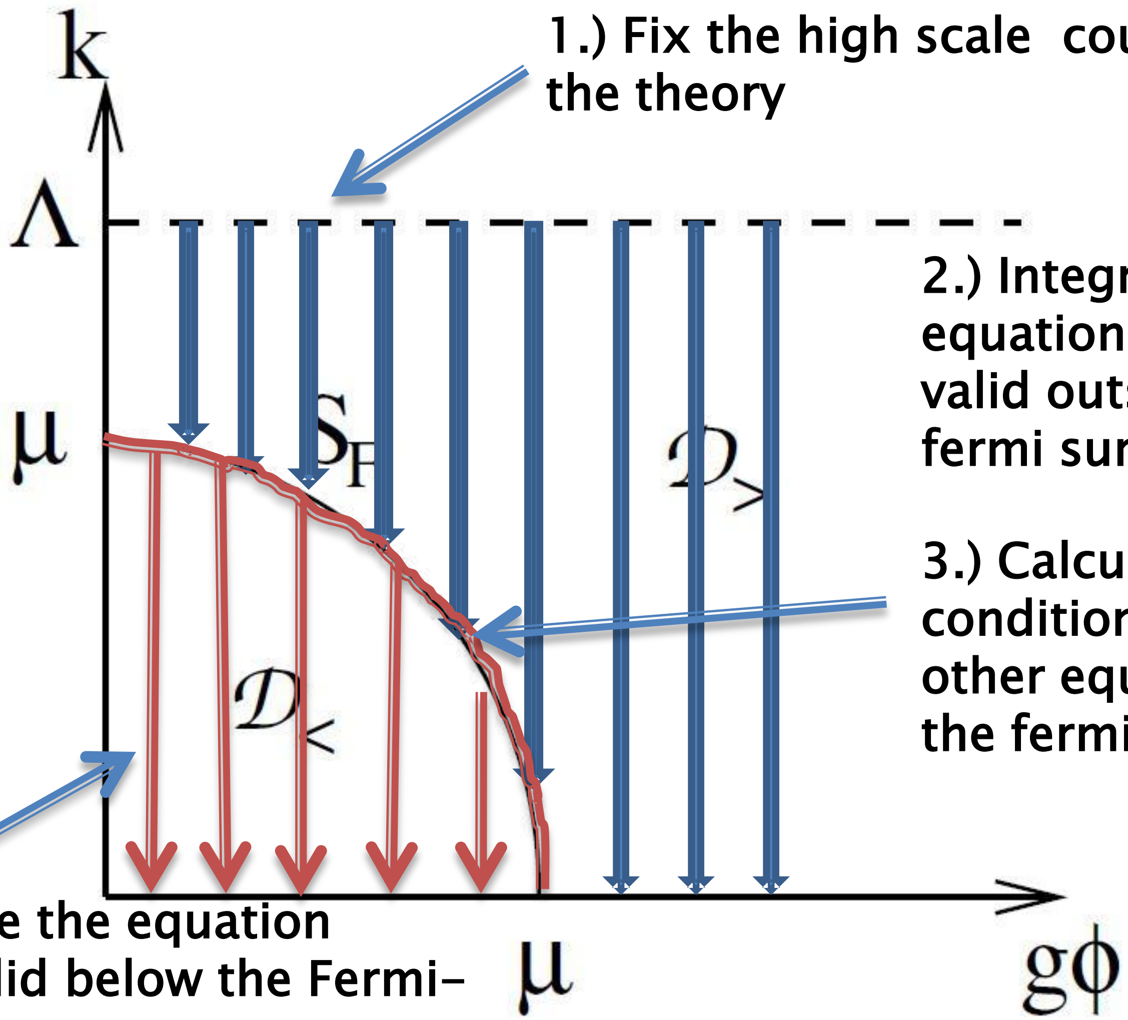
$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_B} - \frac{4}{\omega_F}\right]$$

$$k_F = \sqrt{\mu^2 - g^2\varphi^2},$$ Fermi-surface

$$\partial_k U_k = \frac{k^4}{12\pi^2}\frac{1}{\omega_B}$$

Fermionic vacuum fluctuations and thermodynamic fluctuations cancel

Integration of the Wetterich-equaiton



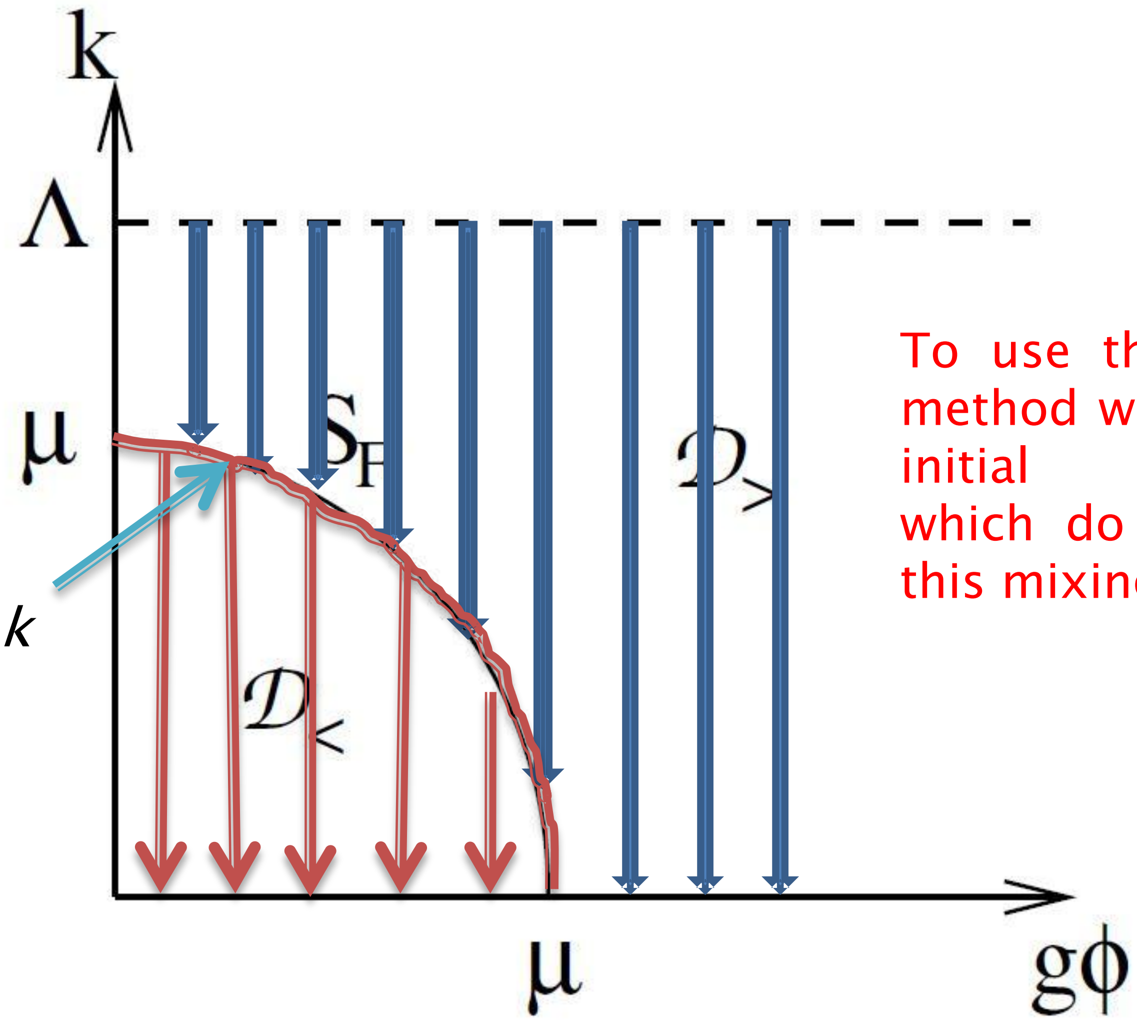
1.) Fix the high scale couplings in the theory

2.) Integrate the equation which is valid outside of the fermi surface

3.) Calculate the initial conditions for the other equation inside the fermi surface

4.) Integrate the equation which is valid below the Fermi-surface

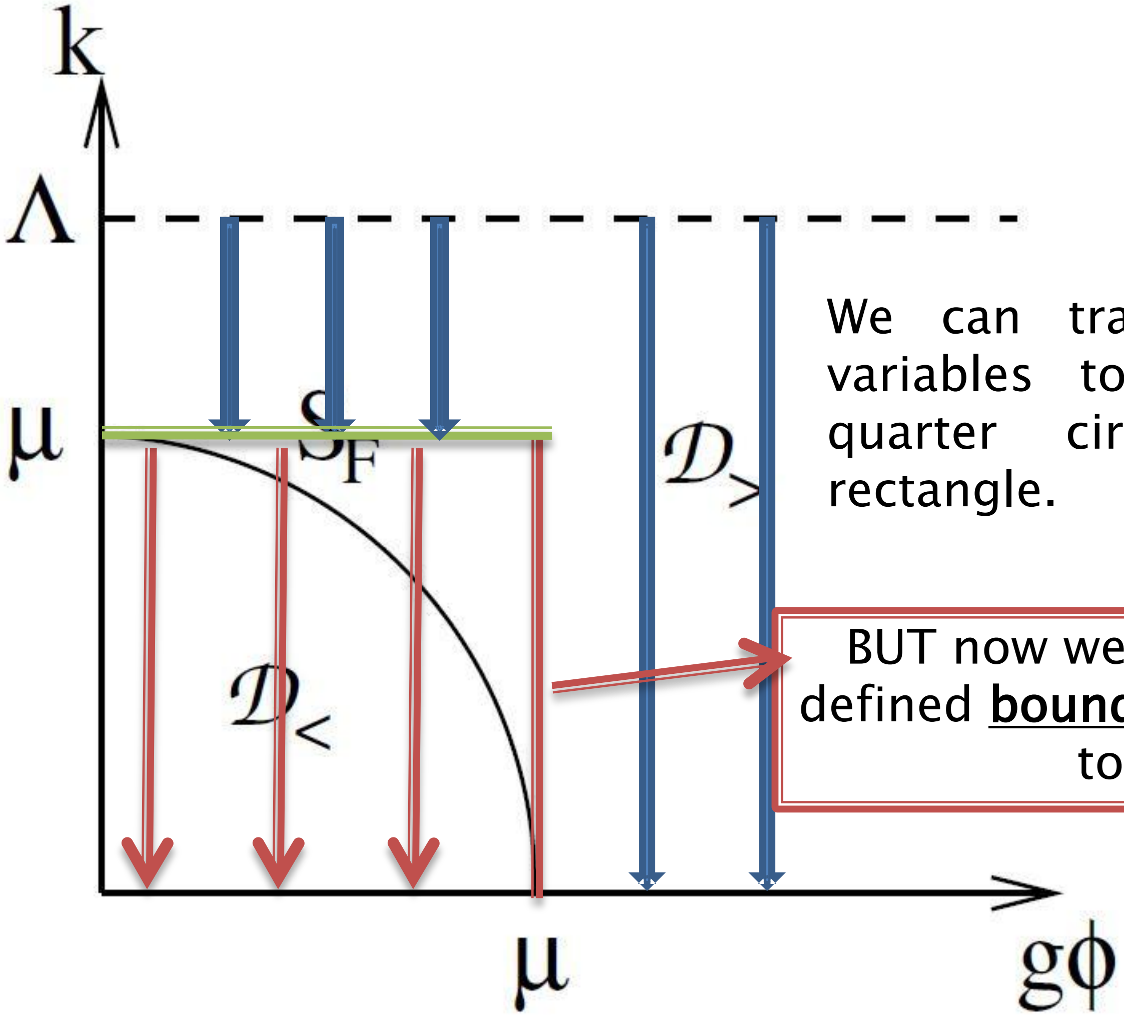
BUT…



The boundary condition mix k and $g\phi$

To use the orginal method we need an initial condition which do not have this mixing

Transform the variables



We can transform the variables to make the quarter cirle into a rectangle.

BUT now we have a well defined **boundary condition** too!

The transformed equation

- Circle-rectangle transformation: $(k,\varphi)\mapsto(x,y)\quad x=\varphi_F(k),\quad y=\frac{\varphi}{x}$
- Transformation of the potential: $\tilde U(x,y)=V_0(x)+\tilde u(x,y)$

Boundary condition at Fermi-surface

- The transformed Wetterich-equation:

$$x\partial_x\tilde u=-xV_0'+y\partial_y\tilde u-\frac{g^2(kx)^3}{12\pi^2}\frac{1}{\sqrt{(kx)^2+\partial_y^2\tilde u}},$$

- And the new boundary conditions:

$$\tilde u(x=0,y)=\tilde u(x,y=\pm 1)=0.$$

Solution by orthogonal system

- Solution is expanded in an **orthogonal basis** to accomodate the strict boundary conditoin in the trasformed area

$$\tilde{u}(x,y) = \sum_{n=0}^{\infty} c_n(x) h_n(y) \quad h_n(1) = 0 \quad \int_0^1 dy\, h_n(y) h_m(y) = \delta_{nm}$$

- The **square root** in the Wetterich-equation is also expanded:

$$x c'_n(x) = \int_0^1 dy\, h_n(y) \left[-xV'_0 + y\partial_y \tilde{u} - \frac{g^2 (kx)^3}{12\pi^2} \underbrace{\sum_{p=0}^{\infty} \binom{-1/2}{p} \frac{(\partial_y^2 \tilde{u} - M^2)^p}{\omega^{2p+1}}}_{\text{Expanded square root}} \right]$$

Where: $\omega^2 = (kx)^2 + M^2$

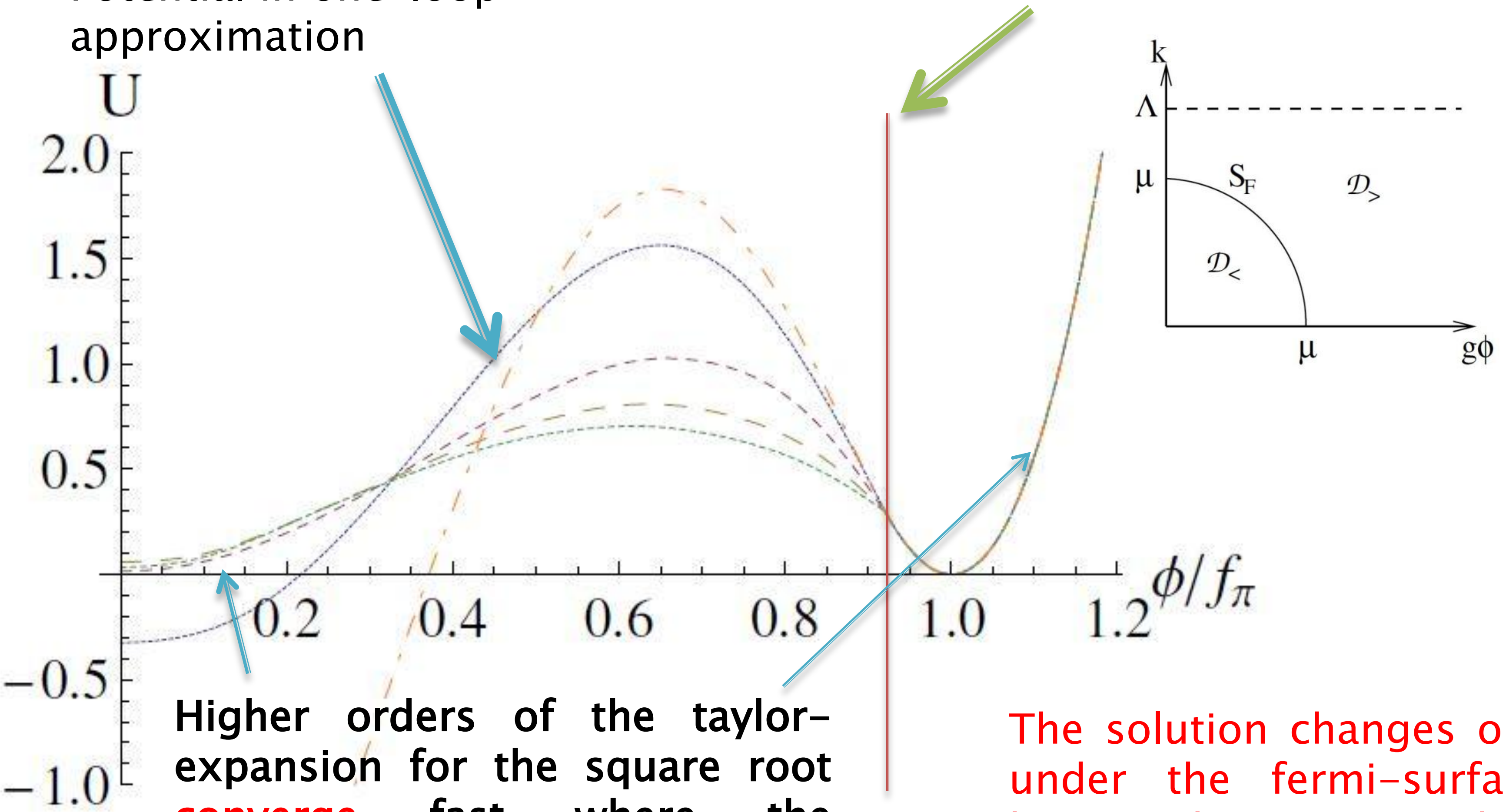
We use harmonic base

$$h_n(y) = \sqrt{2} \cos q_n y, \qquad q_n = (2n+1)\frac{\pi}{2}$$

Results-I

Potential in one-loop approximation

Fermi-surface in the field variable



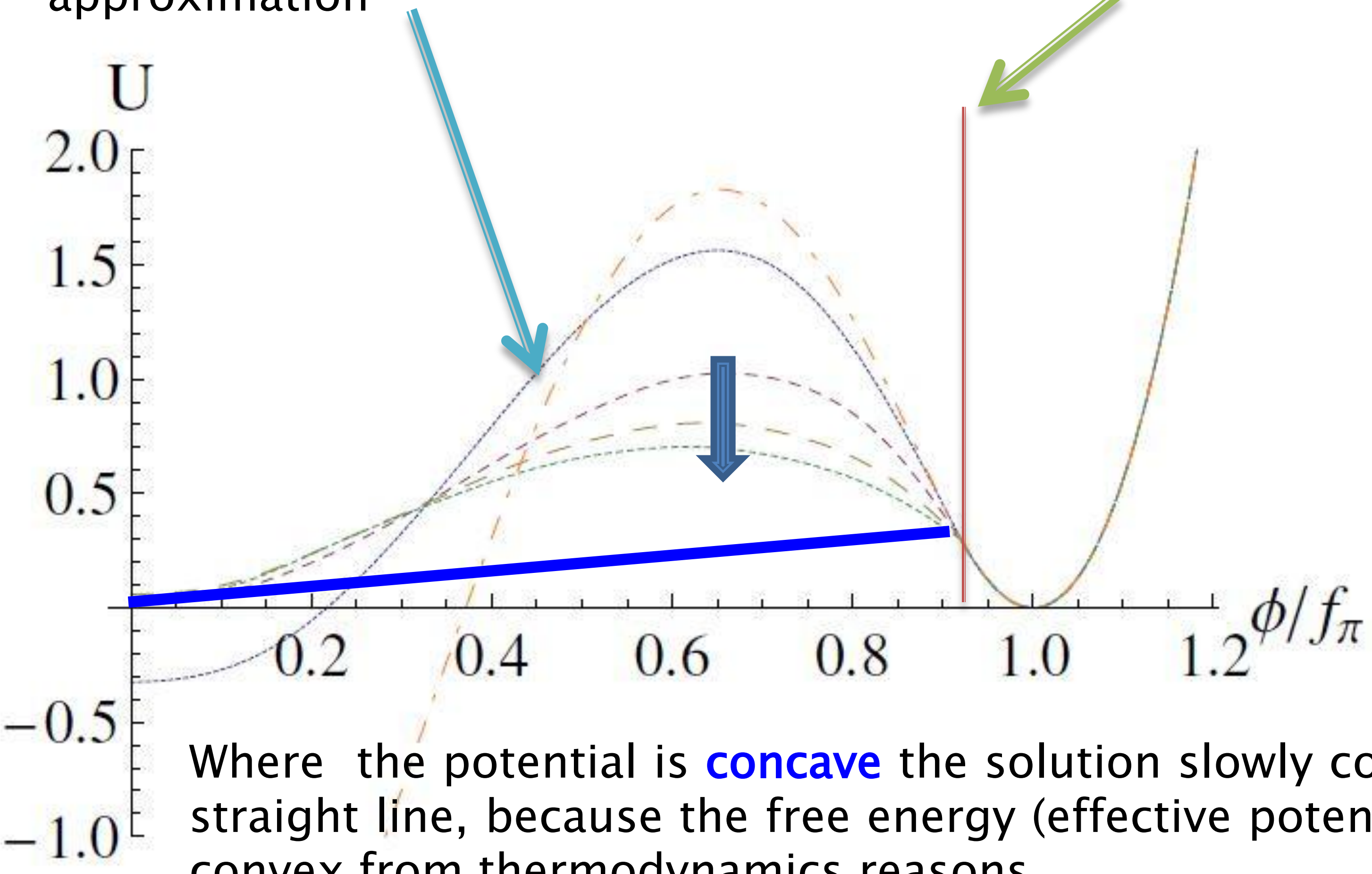
Higher orders of the taylor-expansion for the square root converge fast where the potential is convex

The solution changes only under the fermi-surface, because here we switch to the other equation

Results-I

Potential in one-loop approximation

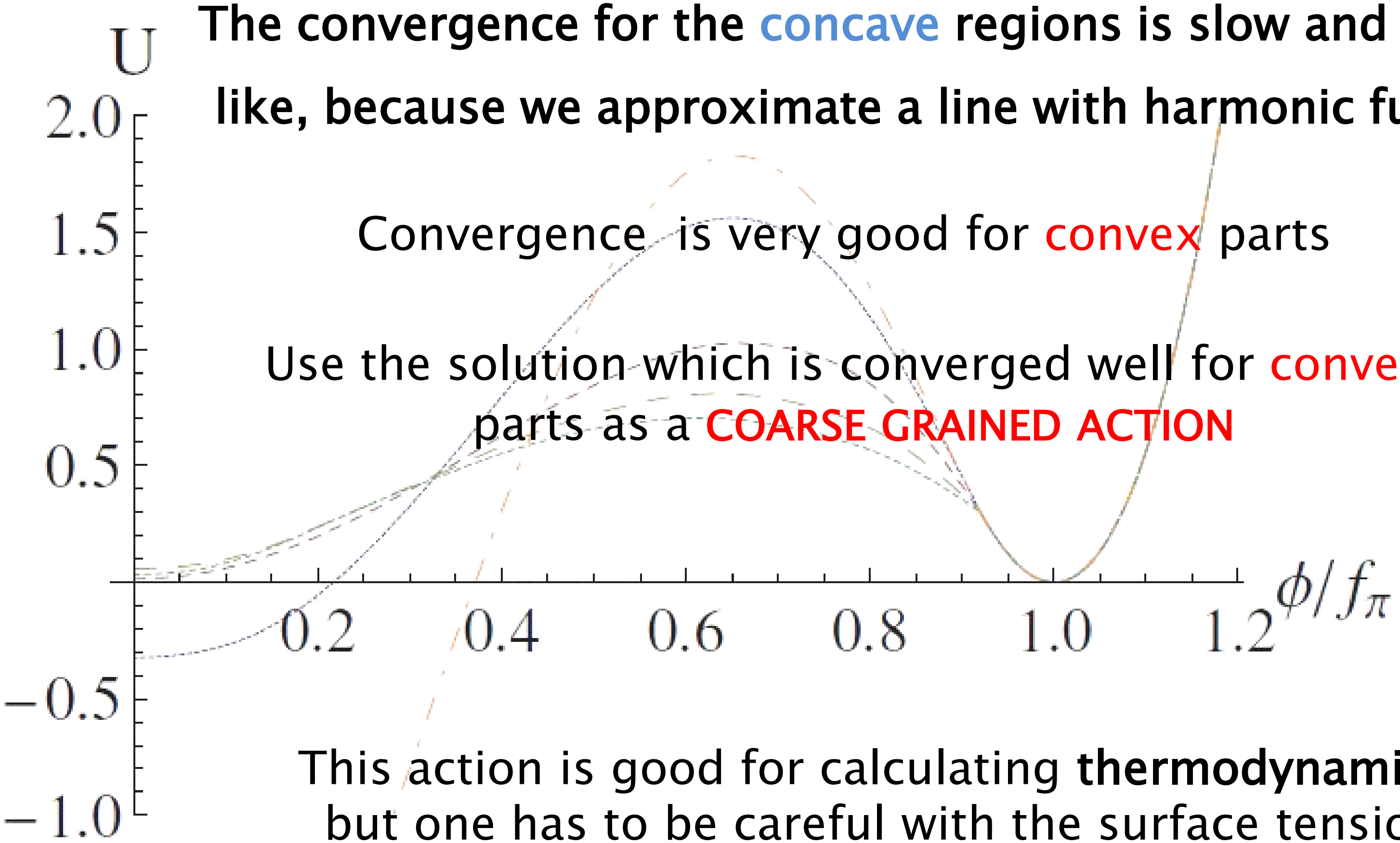
Fermi-surface in the field variable



Where the potential is concave the solution slowly converges to a straight line, because the free energy (effective potential) must be convex from thermodynamics reasons.

This is the **Maxwell construction.**

Results-I



The convergence for the concave regions is slow and not point like, because we approximate a line with harmonic functions.

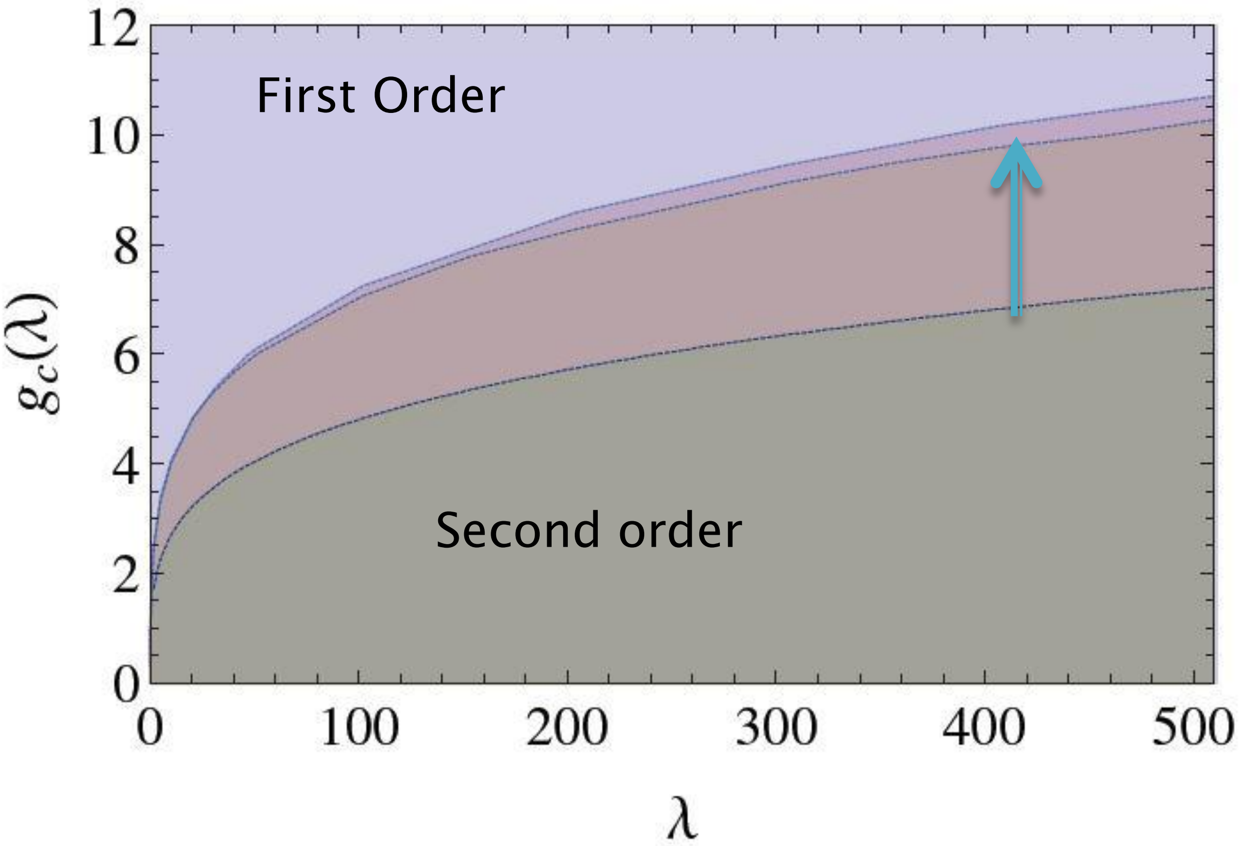
Convergence is very good for convex parts

Use the solution which is converged well for convex parts as a COARSE GRAINED ACTION

This action is good for calculating **thermodynamics**, but one has to be careful with the surface tension

Results–II

Phase structure of the interacting Fermi–gas model



- FRG
- One loop

- Mean field

Yukawa coupling

Scalar self interaction

Application

Effective model for the chiral phase transition

We can study the hardly accesible parts of the QCD phase diagram.

- High density
- High chemical potential
- Low temperature

Neutron star equation of state

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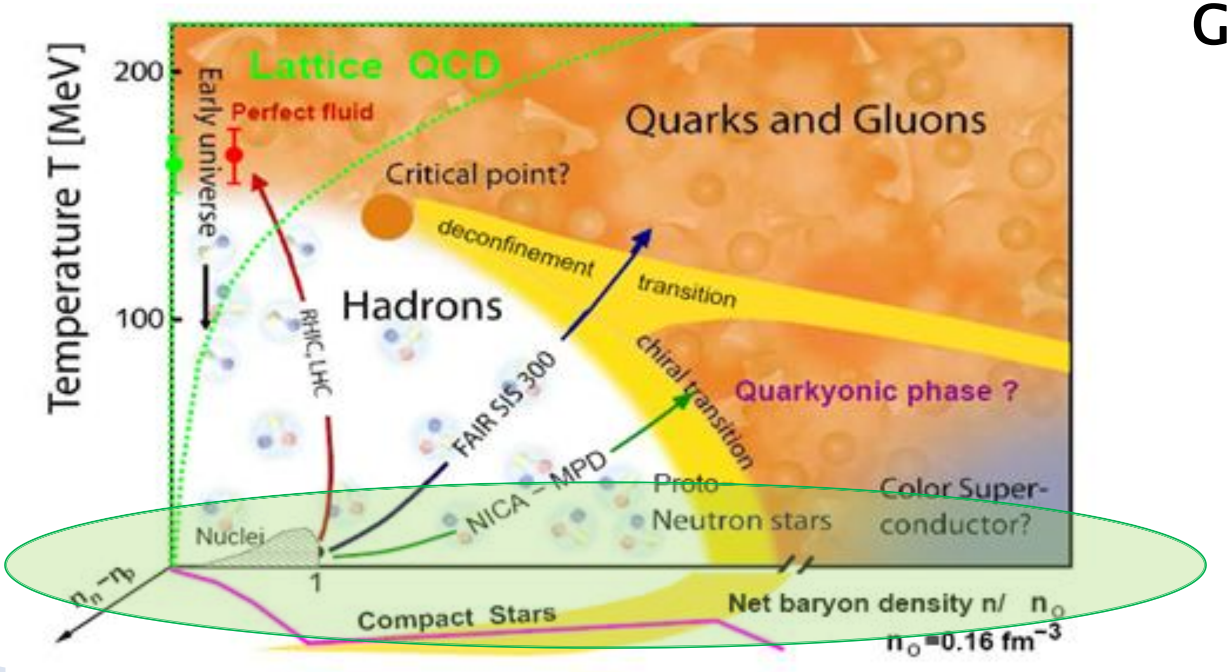
Gravity –wave sources!



Thank you for the attention !

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