General-relativistic polytropic models for non-perfect fluid neutron star

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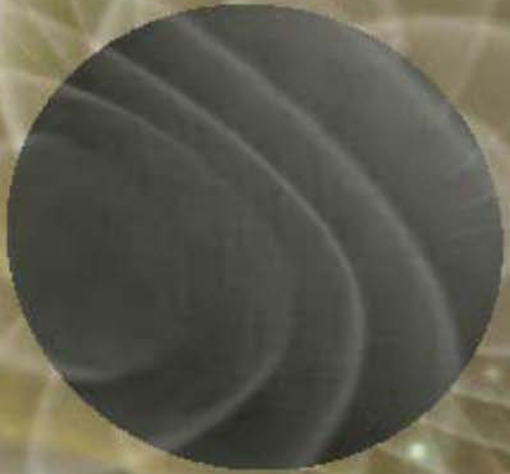


Basic problem:

- Although, the perfect-fluid approximation disregards scenarios when dissipation and energy fluxes are present, it works well for most fluid under generic conditions. However, it loses its validity when thermodynamic (i.e microscopic) time-scales are comparable to the dynamic (i.e. macroscopic) ones and thus when assumption of local thermodynamic equilibrium breaks down.
- The requisite extension of perfect-fluid description that account for dissipative terms and energy fluxes is *non-perfect fluid*.

Line element (→ metric): $ds^2 = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)$

Perfect fluid | **Non-perfect fluid**

Energy-momentum tensor: $T^{\mu\nu} = \boxed{(\epsilon + p)u^{\mu}u^{\nu} + pg^{\mu\nu}}\ \boxed{- 2\eta\sigma^{\mu\nu} - \zeta\Theta h^{\mu\nu} + u^{(\mu}q^{\nu)}}$

[pp. 98, Rezzola & Zanotti, 2013]

dynamical visc. (η), bulk visc. (ζ), thermal conductivity (q)

$u_{\mu} = (e^{\nu/2}, 0, 0, 0)$

total energy-density: $\epsilon = \rho(c^2 + \varepsilon)$

specific enthalpy: $h = \frac{p+\epsilon}{\rho} = c^2 + \varepsilon + \frac{p}{\rho} \to c^2$

In a non-relativistic regime $\epsilon \ll c^2$

Effective variables, field equations:

Effective variables:

$$\bar{\rho} = \frac{h\rho + T_{\mathrm{NPF}}{}^{0}{}_{0}}{c^2}$$

$$\bar{p} = p - T_{\mathrm{NPF}}{}^{1}{}_{1}$$

In the quasistatic regime the effective variables $(\bar{\rho}, \bar{p})$ satisfy the same Einstein's field equations as the corresponding physical variables.
→ In the quasistatic situation, effective and physical variables share the same radial dependence. [pp. 7, Herrera et al., 2002]

Einstein's field equations:

$$\frac{8\pi G}{c^2}\bar{\rho} = e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$$

$$\frac{8\pi G}{c^4}\bar{p} = e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}$$

time-dependent term

$$\frac{8\pi G}{c^4}(\bar{p} - T_{\mathrm{NPF}}{}^{1}{}_{1} + T_{\mathrm{NPF}}{}^{2}{}_{2}) = e^{-\lambda}\left(\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}\right) + \boxed{\frac{e^{-\nu}}{4c^2}\left(2\ddot{\lambda} + \dot{\lambda}(\dot{\lambda} - \dot{\nu})\right)}$$

$$\boxed{\frac{8\pi G}{c^4}T^{1}_{0} = e^{-\lambda}\frac{\dot{\lambda}}{cr}}$$

New time-dependent eq.

Mass definition:

$m' = 4\pi r^2 \rho$, if $e^{\lambda} = \left(1 - \frac{2Gm(r)}{c^2 r}\right)^{-1}$ → Regular *Schwarzschild metric*

Generalized Tolman–Oppenheimer–Volkoff equation

$$\frac{dp}{dr} = -\frac{G\rho m}{r^2}\frac{\left(1+\frac{p}{\rho c^2}\right)\left(1+\frac{4\pi r^3 p}{mc^2}\right)}{1-\frac{2Gm}{c^2 r}} + \frac{2(g_{11}T^{11}_{\mathrm{NPF}} - g_{22}T^{22}_{\mathrm{NPF}})}{r} + \frac{c^2}{16\pi G}e^{-\nu}\left(2\ddot{\lambda} + \dot{\lambda}(\dot{\lambda} - \dot{\nu})\right)$$

$$2(g_{11}T^{11}_{\mathrm{NPF}} - g_{22}T^{22}_{\mathrm{NPF}})/r = \frac{c}{2G}\eta e^{-\nu/2}\dot{\lambda}$$

Simple numerical model: two-component polytrope

$$P = \begin{cases} K_0\rho^{\Gamma_0} & \text{for } \rho < \rho_1 \\ K_1\rho^{\Gamma_1} & \text{otherwise} \end{cases}$$

where low density region is $\rho < \rho_1 = 5 \times 10^{17}\ \mathrm{kg\, m^3}$

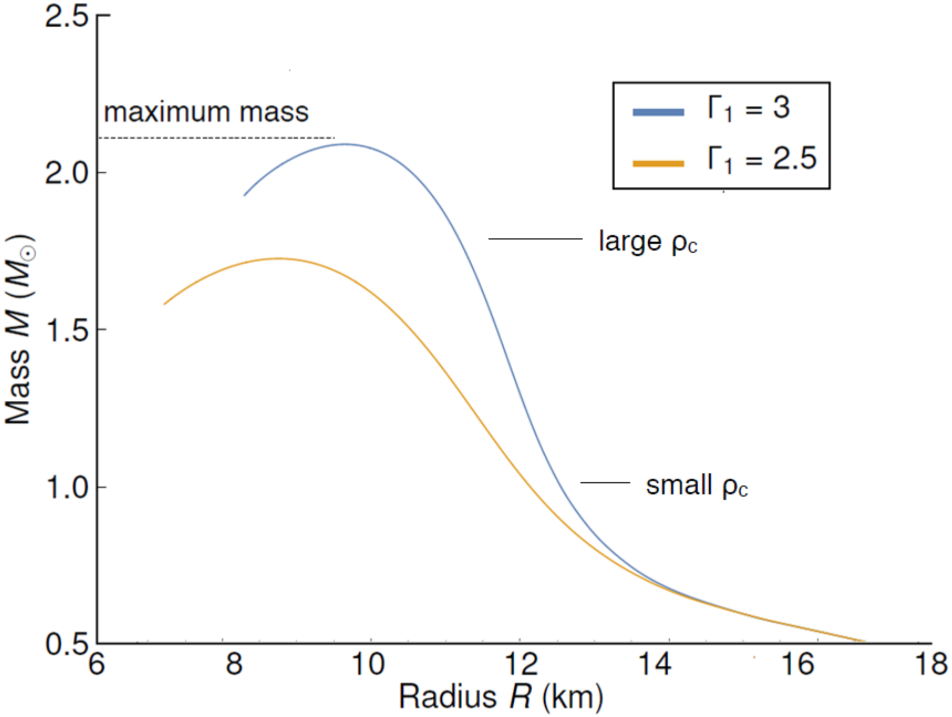
$$\Gamma_0 = \frac{5}{3} \quad \text{and} \quad K_0 = \frac{(3\pi^2)^{2/3}}{5}\frac{\hbar}{m_B^{8/3}}$$

and continuity requires

$$K_1 = K_0\rho_1^{\Gamma_0 - \Gamma_1}$$

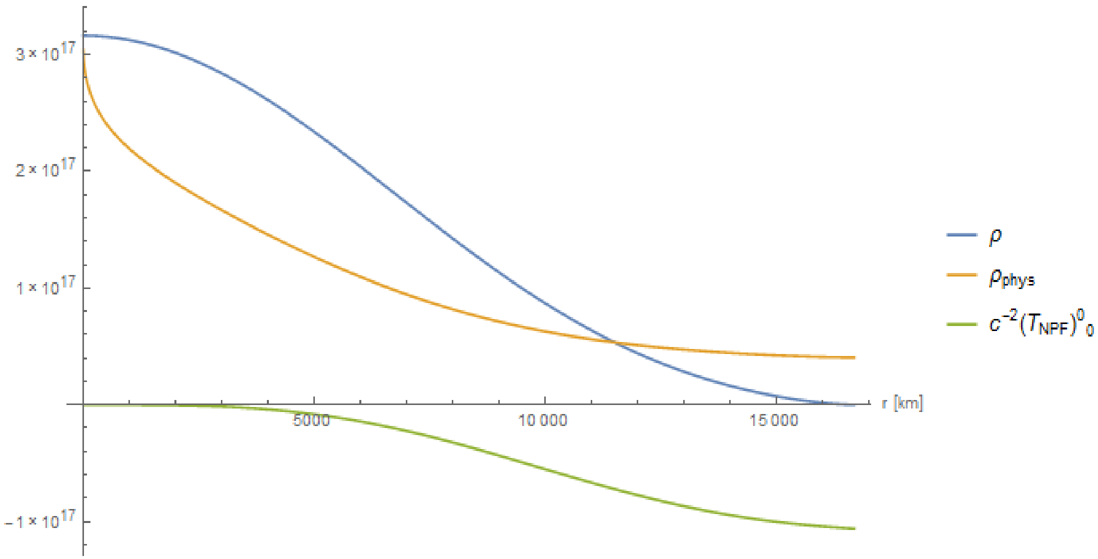
Consider soft and stiff core equations of state:

$\Gamma_1 = 2.5$ (soft) and $\Gamma_1 = 3$ (stiff)

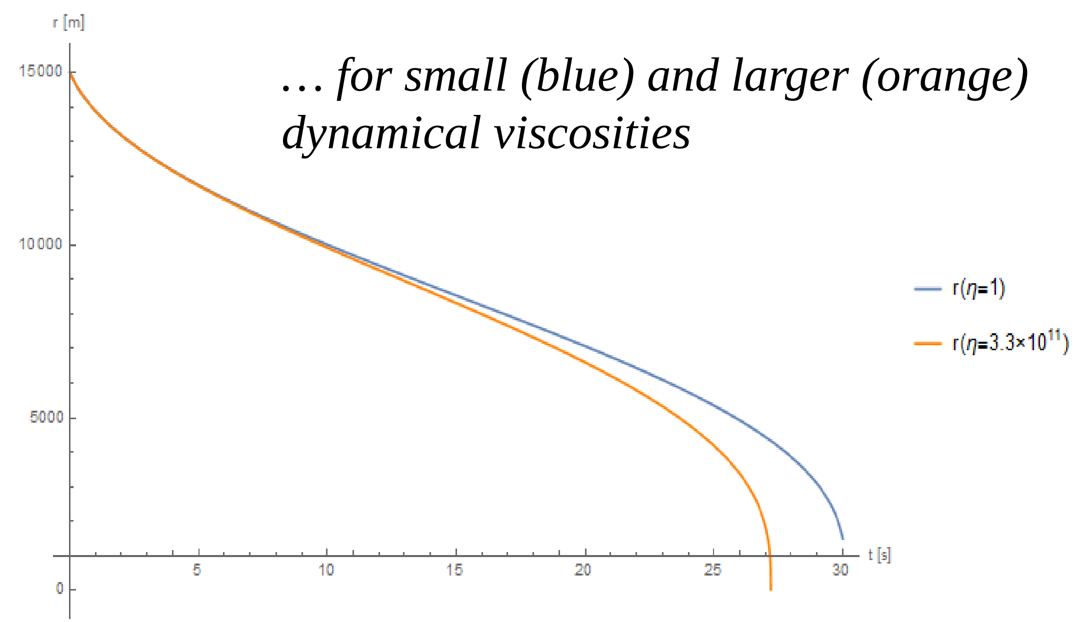


Known case: non-time dependent

Total energy-density, rest-mass density and NPF contributions



Collapse (radius-time curve):



- During their evolution, self–gravitating objects may pass through phases of intense dynamical activity, with time scales of the order of magnitude of (or even smaller than) the hydrostatic time scale, and for which the static (or the quasi–static) approximation is clearly not reliable (e.g. the collapse of very massive stars and the quick collapse phase preceding neutron star formation.
- In these cases it is mandatory to take into account terms which describe departure from equilibrium.

Future plans:

- First of all, to identify the physically valid collapse scenarios among all the numerical solutions
- More complex study of the thermodynamical properties of NS matter in terms of viscous and thermal-conducting contributions
- Add further equations of states for NS matter in addition to polytropic ones.
- Study the contributions of NS rotation to the effective variables

- Compare results with measurements

Thank you very much for your kind attention!

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