AWAKE Rb-Workshop
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Some vague proposals of Rb-vapor / plasma experiments

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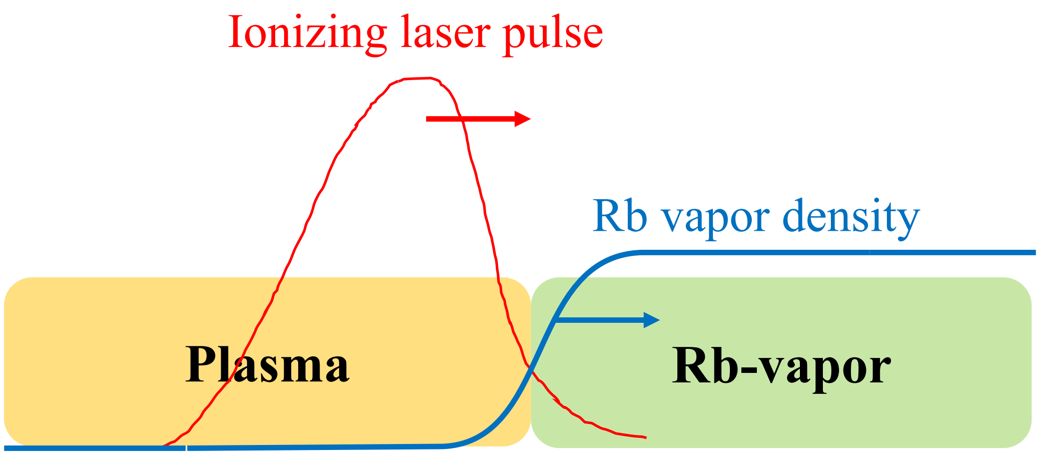
Outline – vague and conceptual

- **Longitudinal Rb-cell diagnostic**
- **RF-wave reflection from plasma with moving ionization front boundary**
- **Self-focusing / guiding / trapping next to a resonant absorption line**

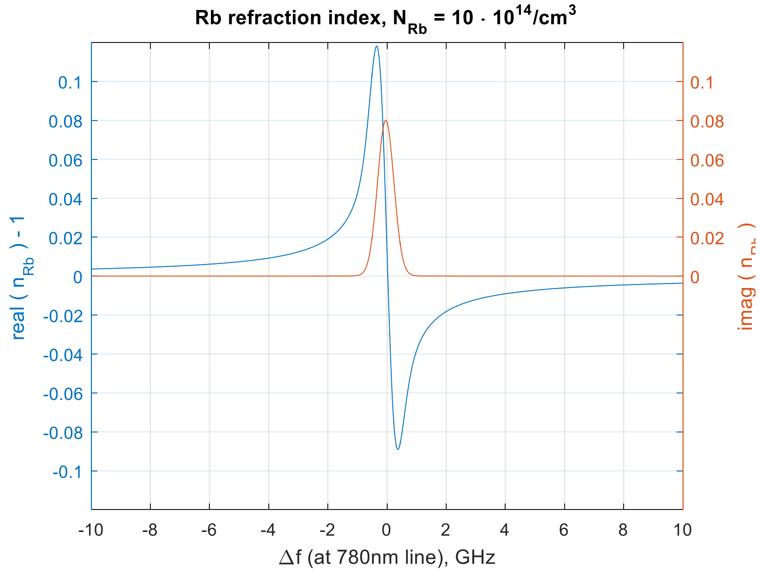
Rb vapour / plasma



phase velocity $$\frac{V_{ph}}{c} = \frac{1}{\sqrt{1-\left(\frac{f_p}{f_L}\right)^2}} \approx 1 + 2\left(\frac{f_p}{f_L}\right)^2$$

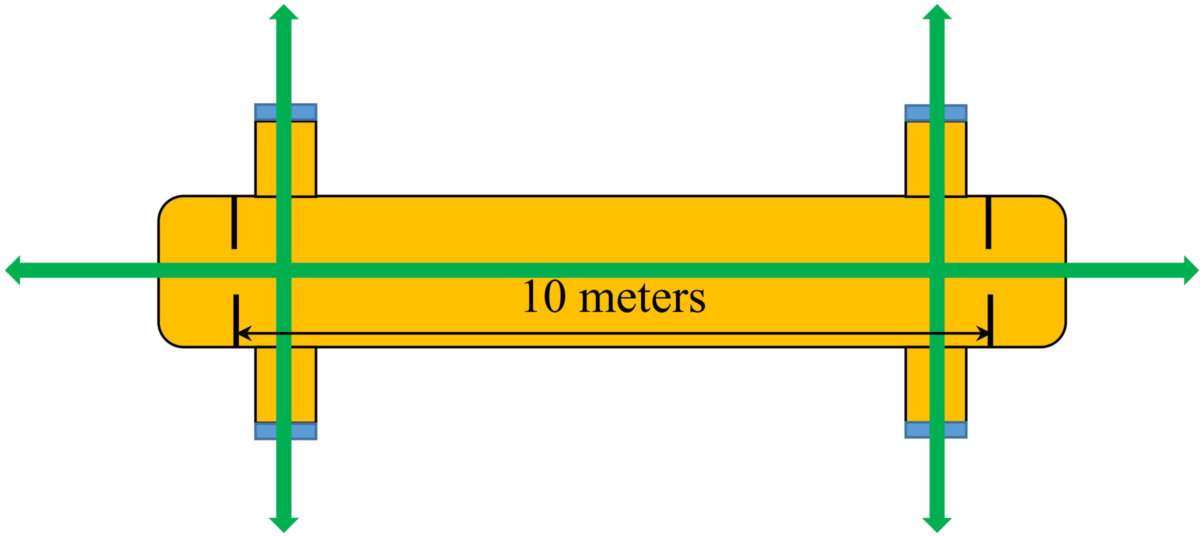
group velocity $$\frac{V_{gr}}{c} = \sqrt{1-\left(\frac{f_p}{f_L}\right)^2} \approx 1 - 2\left(\frac{f_p}{f_L}\right)^2$$

$$f_p[GHz] \approx 90\sqrt{N_{Rb}[10^{14}/cm^3]}$$



Rb vapour cell – possible probe directions

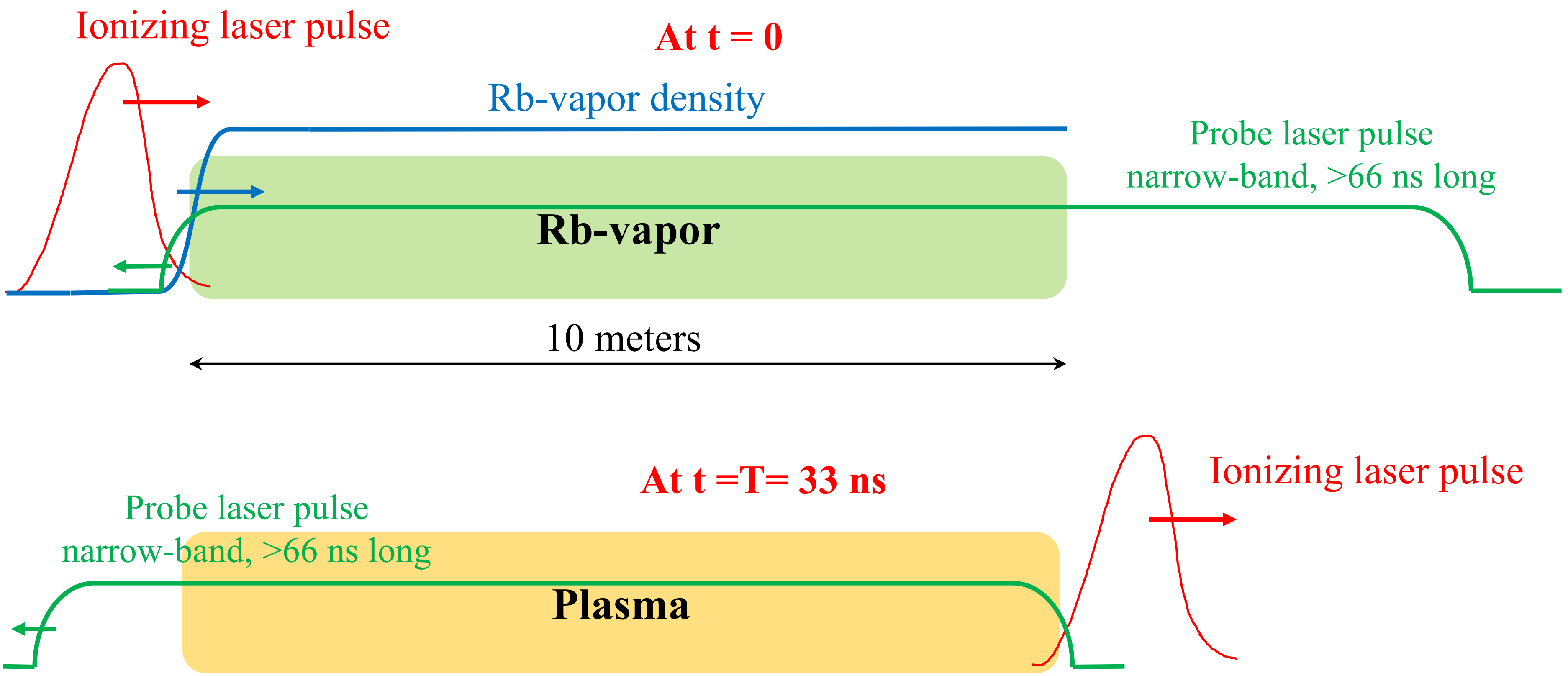
Two pairs of transverse view-ports:
Rb-density diagnostics, Schlieren imaging etc.



Longitudinal diagnostics?

- Rb vapor cell → $N_{Rb} = 1 \div 10 \cdot 10^{14}$ / cm^3
- Ti:Sa powerful laser → for ionization
- Narrow-band (<10MHz) tunable laser → for probing / diagnostics

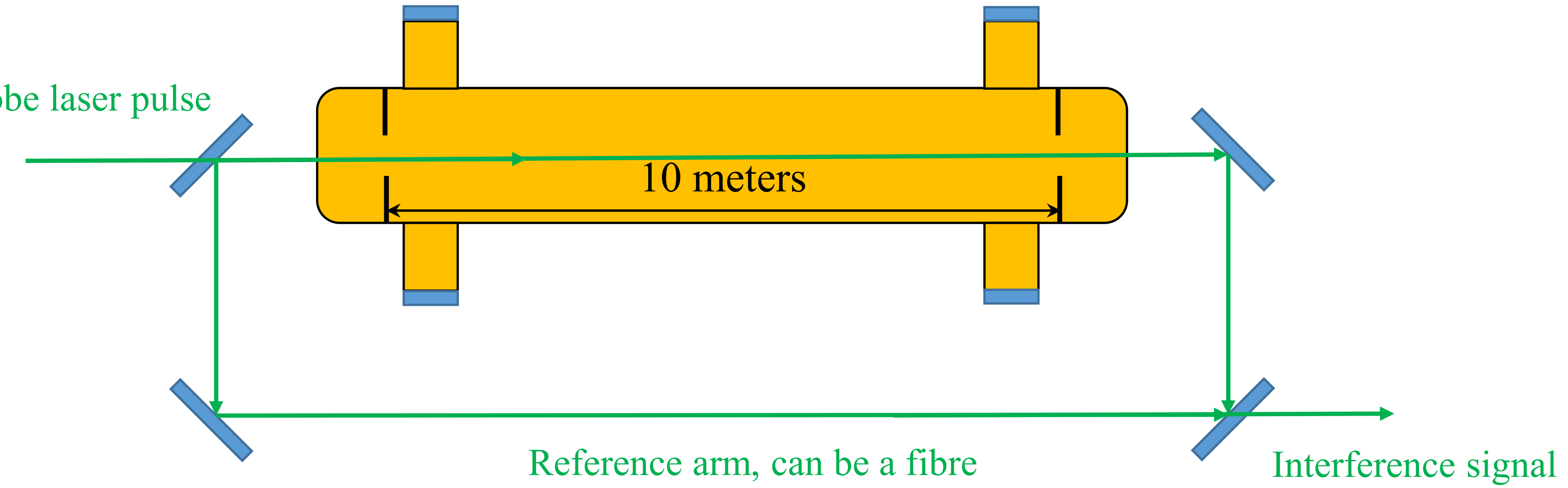
Rb cell longitudinal diagnostic



Every slice of a long probing pulse (>66ns) is moving partially in Rb vapour and partially in plasma. Depending of the delay within the probing pulse every slice experience a different phase advance Δφ(t)

$$\boxed{\Delta\varphi(t) = k_0 L(t/2T + n_{Rb}(\omega)(1-t/2T))}$$

Rb cell longitudinal diagnostic

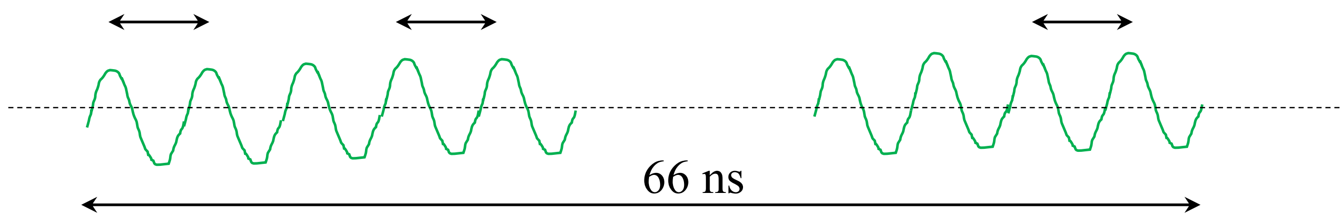


Interference signal is detected by a photo-diode on a fast scope

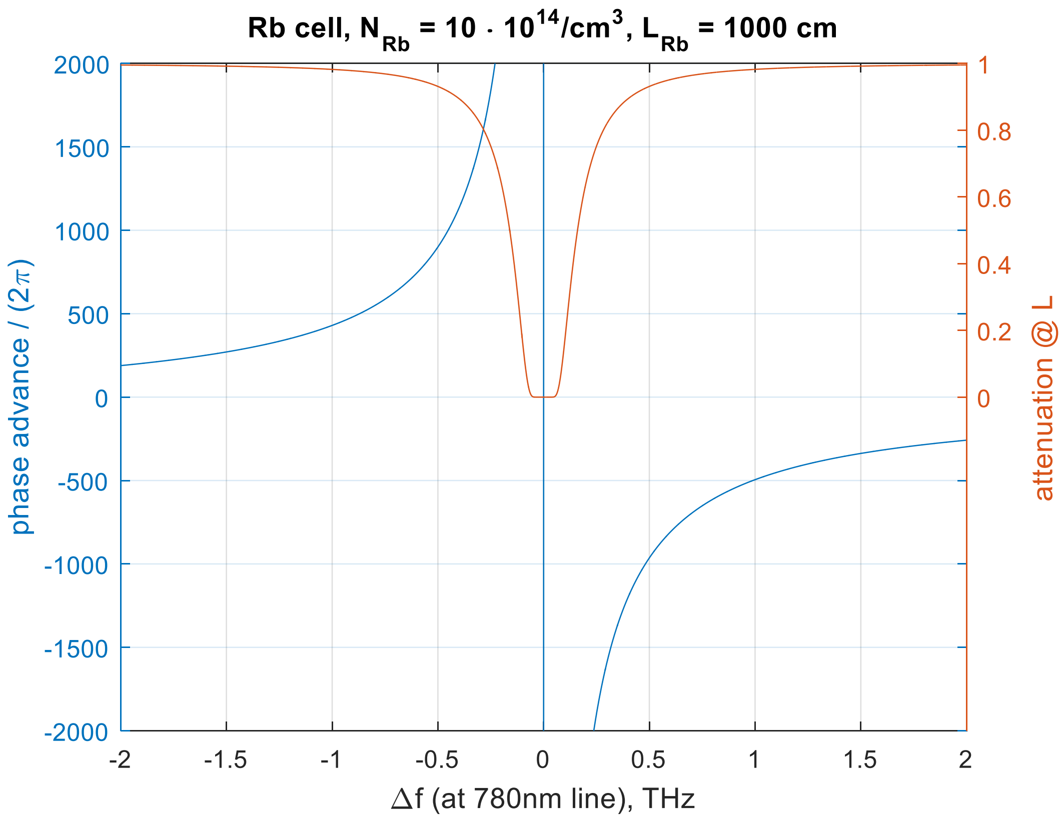
Interference signal is proportional to: $\sim \cos(k_0 L(t/2T + n_{Rb}(\omega)(1 - t/2T))$

Number of fringes depends on detuning, fringes within 66ns has to be resolved by a fast scope.

Equidistance of the fringes tells about uniformity of $n_{Rb}(z)$



Probe laser parameters for longitudinal diagnostics



Example:

Rb-density $N_{Rb} = 10^{15}$ / cm^3, L_{Rb} = 10 m

Low attenuation $\Delta f > 500$ GHz (~1 nm)

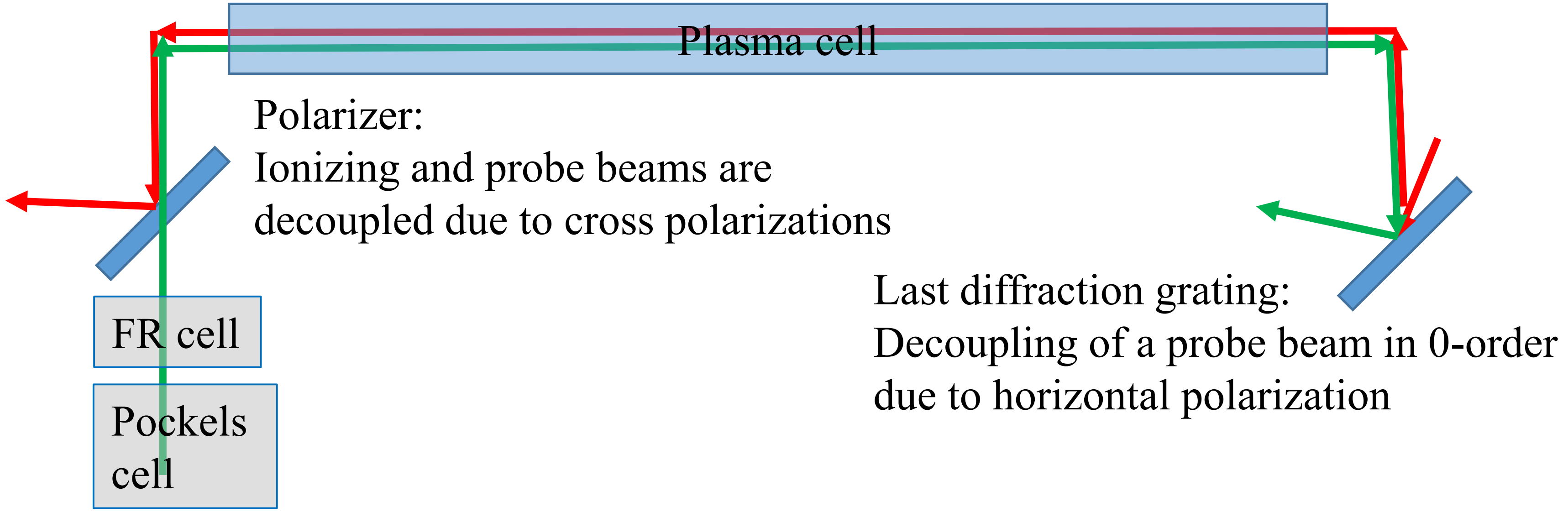
Phase advance $< 2\pi\cdot 800$

- Given typical resolution of a fast scope (20GHz/60Gsa) we can afford 200 interference fringes within 66ns, assuming 330ps/period
- So maximum phase advance is $\Delta\varphi_{max} = 2\pi\cdot 200$, and thus detuning has to be ~2 THz (~4 nm)

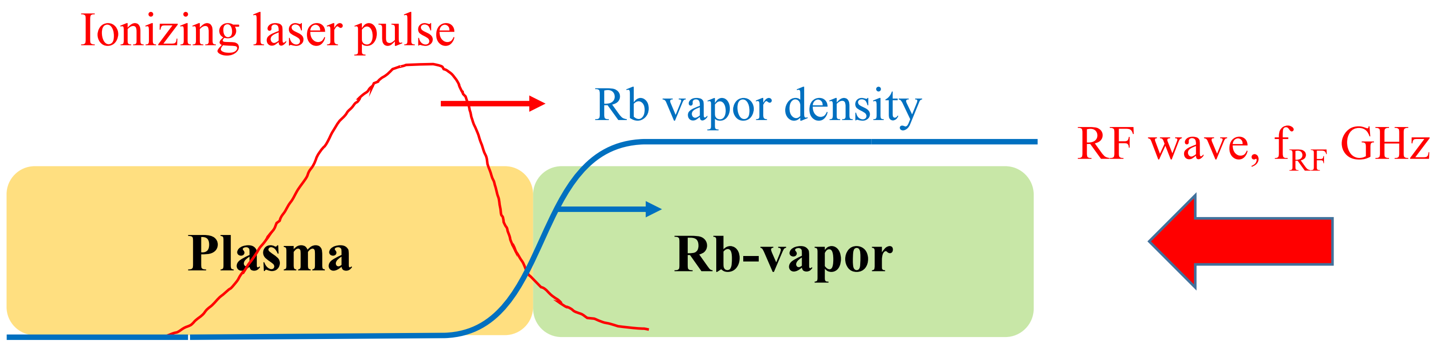
Longitudinal interferometry of Rb vapour / plasma

Problems / solutions:

- Transverse overlap of ionizing and probe lasers
  - probe pulse mode degradation
- Coupling / decoupling of lasers
  - different polarizations
  - Bruster plates / polarizers
  - Zero-order grating reflections (inside a compressor)
  - Faraday isolataors, Pockels cells etc.)



RF-wave reflection from an ionization front (a very old idea in general)



Ionization laser group velocity $\frac{V_{gr}}{c} = \sqrt{1-\left(\frac{f_p}{f_L}\right)^2} \approx 1 - 2\left(\frac{f_p}{f_L}\right)^2$ $f_p[GHz] \approx 90\sqrt{N_{Rb}[10^{14}/cm^3]}$

Ionization front velocity equal (?) to group velocity of ionizing laser:

$f_p \sim 400$ GHz ($N_{Rb} = 2\cdot 10^{15}$ / cm^3), $f_L \sim 400$ THz

$1 - V_{gr} / c \sim 2\cdot 10^{-6}$ ⇔ "equivalent Lorentz factor γ" ~ 500

What if seed an RF-wave counter propagating to an ionization front?

Boost to front frame: $f'_{RF} = f_{RF}\sqrt{\frac{1+\beta}{1-\beta}} = 1000 f_{RF}$, plasma frequency stays the same (N_e/m=inv)!

If $f'_{RF} < f_p$ (i.e. $f_{RF} < 0.4$ GHz) , then RF wave reflects from plasma boundary.

Boost back to lab frame: $f''_{RF} = f_{RF}\frac{1+\beta}{1-\beta} = 10^6 f_{RF} \sim 400\ THz$, in or example – **it is visible!**

RF-wave reflection from an ionization front. Some practical limitations

Ionization front velocity: $$1-\frac{V_{gr}}{c}=1-\beta\approx 2\left(\frac{f_p}{f_L}\right)^2$$

RF frequency in the moving front frame:
(must be less than f_p for high reflectivity) $$f'_{RF}=f_{RF}\sqrt{\frac{2}{1-\beta}}=f_{RF}\left(\frac{f_L}{f_p}\right)\quad \textit{must be} < f_p$$

Up-shifted RF frequency in lab frame: $$f''_{RF}=f_{RF}\frac{2}{1-\beta}=f_{RF}\left(\frac{f_L}{f_p}\right)^2<f_L$$

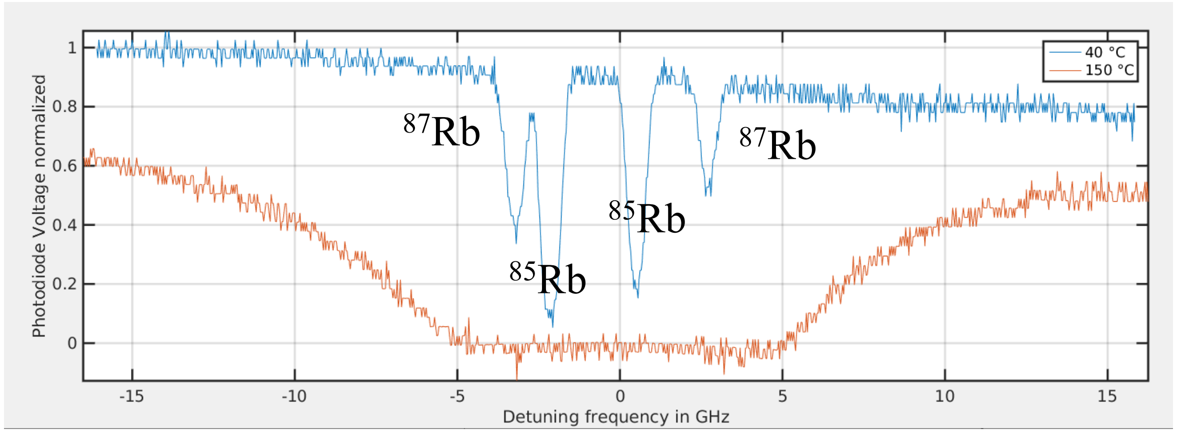
In reasonable conditions ($f'_{RF}<f_p$), up-shifted frequency always less than laser frequency!
Up-shifted frequency can be in near/far IR or THz range
Sacrificing the efficiency, $f'_{RF}<10f_p$, then $f''_{RF}<10f_L$, it is potentially deep UV.

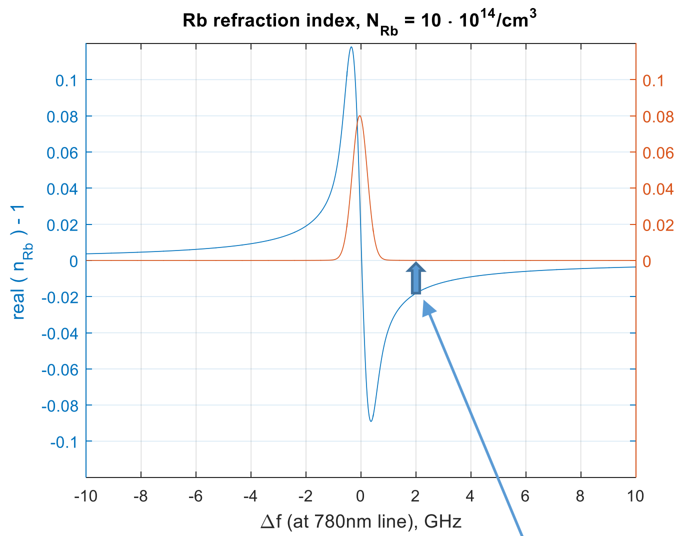
Wavelength of reflected visible light being measured by time resolving spectrometer gives information about $f_p(t = z / c)$

Self-focusing next to a resonant absorption line.

Known since 1966.

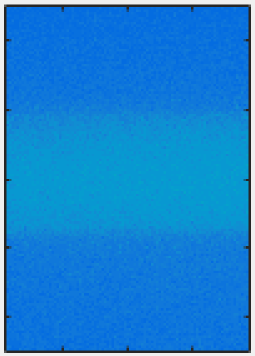
Absorption spectrum at different temperature





To the left of the line - defocusing

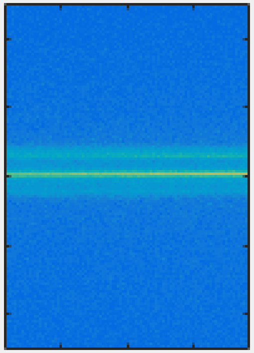
-1.9 GHz



Absolute values of detuning are not very accurate

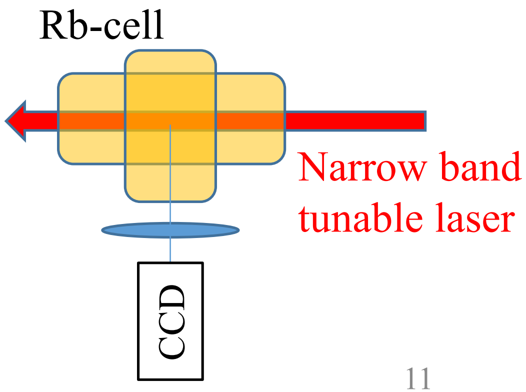
To the right of the line - focusing

4.7 GHz



Courtesy of Anna Bachmann

Laser beam being properly tuned saturate an absorption and tends to self focus at positive frequency detuning



Self-focusing: simulation of two-level system

Equations are very similar to those for laser gain (Frantz-Nodvik)

$$N_0 = N_1 + N_2 = const$$

$$\frac{\partial N_1(x,t)}{\partial t} = -(N_1 - N_2)\sigma \frac{I(x,t)}{h\nu} + \frac{N_2}{\tau}$$

$$\frac{\partial N_2(x,t)}{\partial t} = +(N_1 - N_2)\sigma \frac{I(x,t)}{h\nu} - \frac{N_2}{\tau}$$

$$\frac{\partial I(x,t)}{\partial x} + \frac{1}{c}\frac{\partial I(x,t)}{\partial t} = -(N_1 - N_2)\sigma I(x,t)$$

$$A = \sigma(N_1 - N_2)$$

$$\begin{cases} \dfrac{\partial A(z,t)}{\partial t} = -2A\dfrac{I}{E_s} \\ \dfrac{\partial I(z,t)}{\partial z} = -AI \end{cases}$$

With a solution:

$$A(z,t) = \frac{A_0(z)\exp\left(\int_{-\infty}^{z} A_0(x)dx\right)}{\exp\left(\frac{2}{E_s}\int_{-\infty}^{t} I_0(x)dx\right) + \exp\left(\int_{-\infty}^{z} A_0(x)dx\right) - 1}$$

$$I(z,t) = \frac{I_0(t)\exp\left(\frac{2}{E_s}\int_{-\infty}^{t} I_0(x)dx\right)}{\exp\left(\frac{2}{E_s}\int_{-\infty}^{t} I_0(x)dx\right) + \exp\left(\int_{-\infty}^{z} A_0(x)dx\right) - 1}$$

Self-focusing: simulation of two-level system

Split-step algorithm, for each slice dz in z-direction:

- Calculate ΔN(x,y,z) and I(x,y,t) according to Frantz-Nodvik

  - cross-section and saturation energy must be consistent with Im{χ} at given frequency

  - $\sigma = 2k_0 \,\mathrm{Im}\{\chi\}, \qquad E_{sat} = h\nu/\sigma$

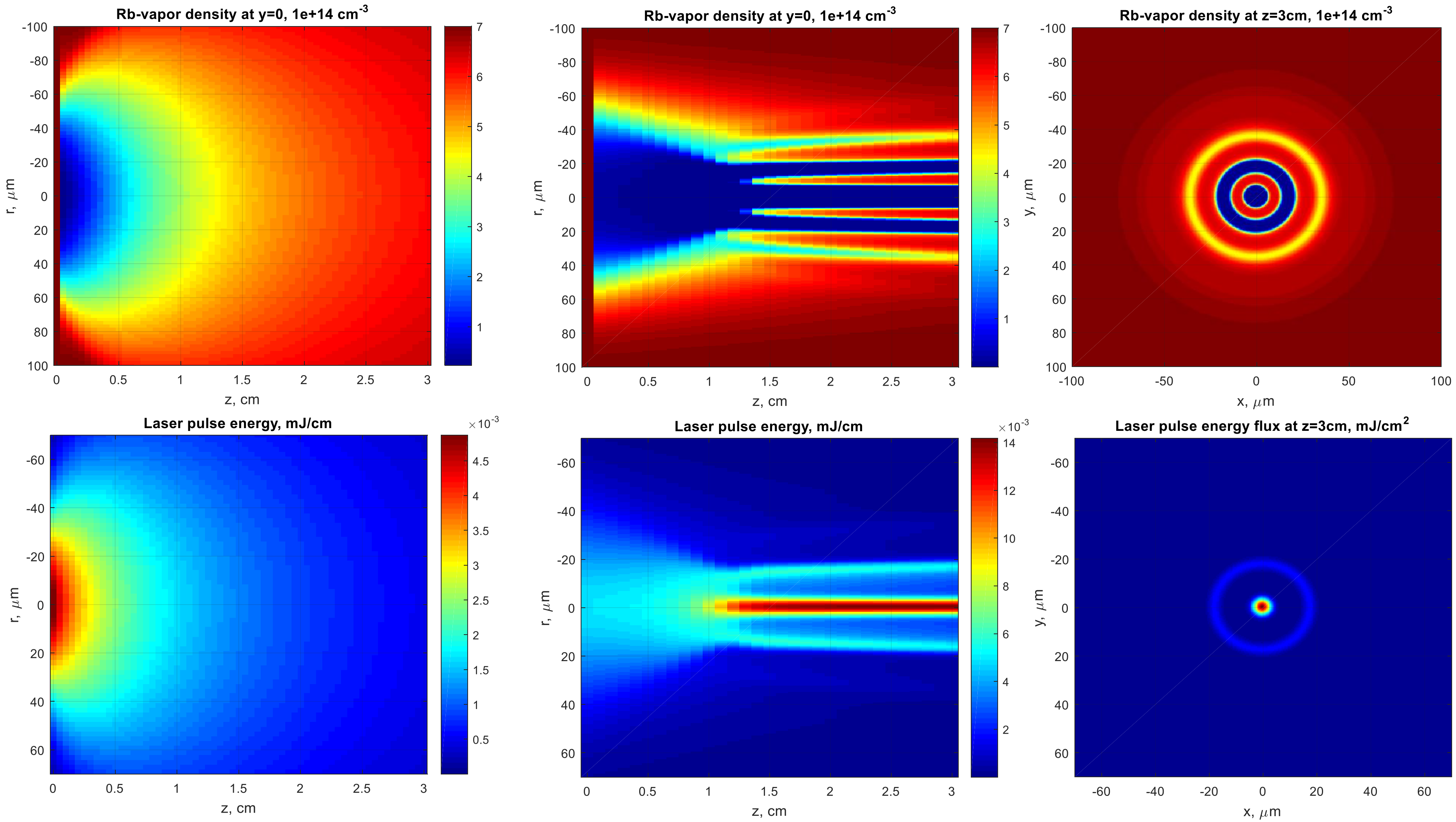
- Use ΔN to calculate the phase acquired on a z-slice: $U(x,y,t) = \sqrt{I(x,y,t)}\; e^{ik_0 dz \Delta N \,\mathrm{Re}\{\chi\}/2}$

- Do the step propagation: $U(x,y,t) = FFT^{-1}\left(FFT(U)\cdot e^{idz(k_x^2+k_y^2)/2k_0}\right)$

Self-focusing: simulation, $N_{Rb} = 7 \cdot 10^{14}$ cm^{-3}

Δf = - 5GHz
defocusing

Δf = + 5GHz
focusing



Thank you for your attention!