Modeling the interaction of ionizing laser pulses with rubidium atoms for the AWAKE project

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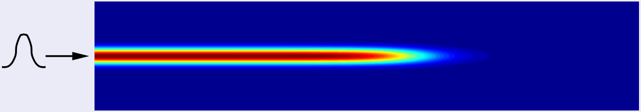
Outline

1. Introduction
   - The problem
   - The concept based on RNO
   - The „minimal” model
2. Developing the model
   - A systematic approach
   - The problems of ionization
3. Some results
   - Testing the improved model
   - Propagation of chirped pulses
   - Outlook

The AWAKE problem: a ∼40 TW pulse propagating in Rb vapor



atomic physics: $\Psi(r, t)$
ionization, resonance

$\Longleftrightarrow$

optics:
$E(r, t)$, propagation

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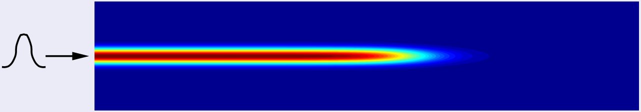

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Maxwell for E:

$$\nabla^2 E(r,t) - \frac{1}{c^2}\frac{\partial^2 E(r,t)}{\partial t^2} = \mu_0 \frac{\partial^2 \textcolor{red}{P}}{\partial t^2}$$

$\Rightarrow$

Schrödinger for atoms:

$|\Psi\rangle = \sum \alpha_i |i\rangle$, $\hat{H}_I = -\hat{d}E$
phenom. loss terms

$\Swarrow$

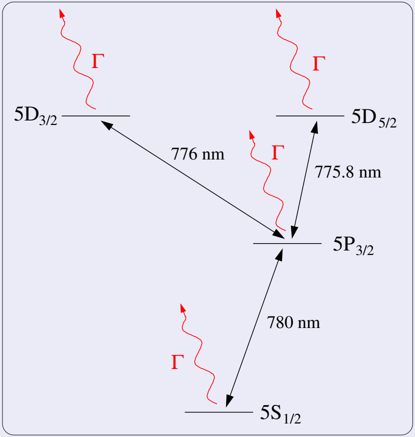
$\Nwarrow$

Material optical response:

$$P(r,t) = \mathcal{N}\langle\Psi|\hat{d}|\Psi\rangle + \text{ionization loss}$$

The „minimal” atomic model + 1D propagation

- geared for 780 nm and 100 fs pulses
- 4 essential atomic levels: $5S_{1/2}$, $5P_{3/2}$, $5D_{3/2}$, $5D_{5/2}$
- Γ-s: PPT rates + experimental ionization cross-sections



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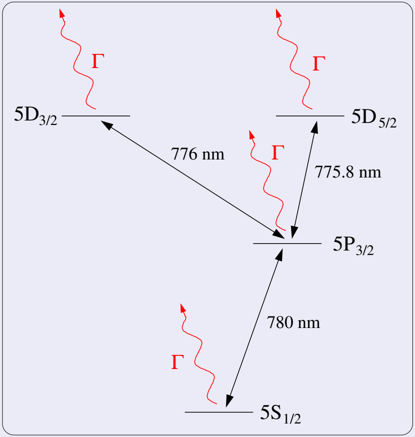

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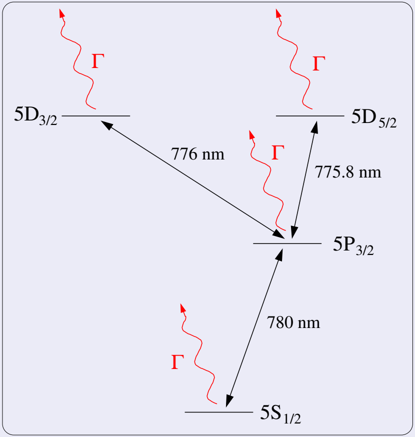

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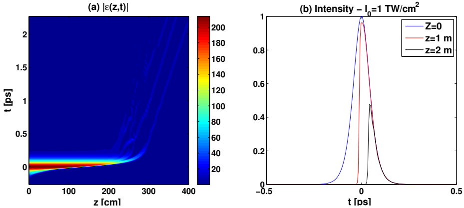

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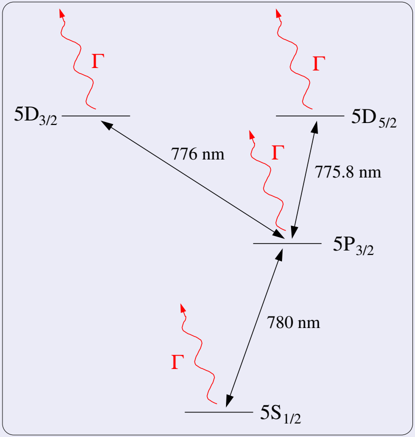

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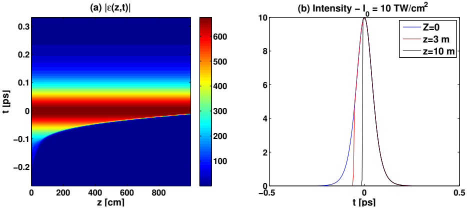

Steepening of leading edge

A more systematic approach

Relevant for different wavelengths ($\lambda = 780 - 810\ \mathrm{nm}$) and bandwidths ($\tau_p \gtrsim 40\ \mathrm{fs}$).

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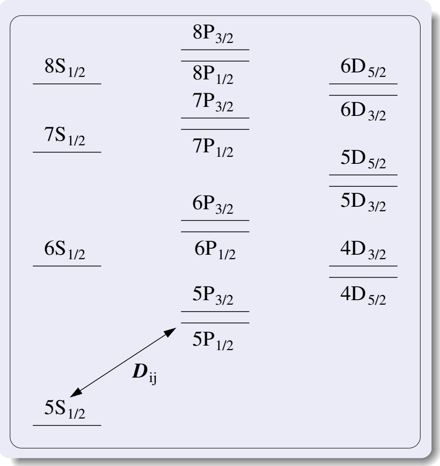

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    - max state probabilites
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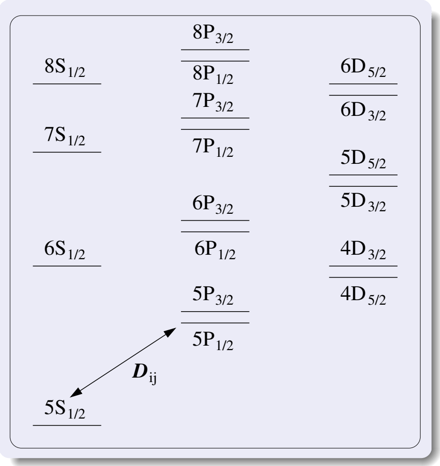

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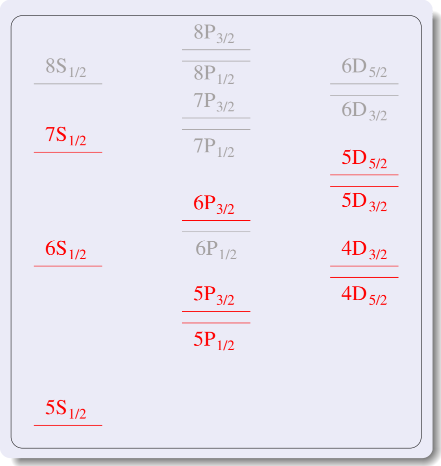
4. Drop „unnecessary" states (say, $P_{max} < 0.01$, $P_{ion} < 0.01$).

⇒ 10 level model



Coupled atom-field equations summarized:

atoms: probability amplitudes α_i, $i \in \{5\mathrm{S}, 5\mathrm{P}_{1/2}, \ldots\}$ (RWA, $\Delta_{i,j} = \omega_L - \omega_{ij}$ detunings, $\Omega = \mathcal{E}(t) e a_0/\hbar$ field strength, $\mathcal{D}_{i,j}$ matrix elements.)

$$\begin{aligned}
\partial_t \alpha_{5S} =& \frac{i}{2}\Omega^* \exp(i\Delta_{5S,5P_{1/2}} t)\mathcal{D}_{5S,5P_{1/2}} \alpha_{5P_{1/2}} \\
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& \frac{i}{2}\Omega^* \exp(i\Delta_{5S,6P_{3/2}} t)\mathcal{D}_{5S,6P_{3/2}} \alpha_{6P_{3/2}} - \frac{\Gamma_{5S}}{2}\alpha_{5S} \\
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field: (1D, moving frame, Slowly Varying Envelope Approx. SVEA: $E = \mathcal{E}e^{i(kz-\omega t)} + c.c.,\quad \partial_t\mathcal{E} \ll \omega\mathcal{E}, \partial_z\mathcal{E} \ll k\mathcal{E},\quad \kappa = k(ea_0)^2\mathcal{N}/\hbar\varepsilon_0$)

$$\partial_z\mathcal{E} = i\kappa\sum_{k<l}\alpha_k^*\mathcal{D}_{k,l}\alpha_l\exp(i\Delta_{l,k}t) - \frac{\kappa}{\mathcal{E}}\sum_k \bar{n}_k\Gamma_k|\alpha_k|^2$$

The ionization problem ... on PPT rates...

Above-threshold ionization

$$w = \sum_{n>\kappa} w_n$$

$$\bar{n} = \frac{\sum_{n>\kappa} n w_n}{w}$$

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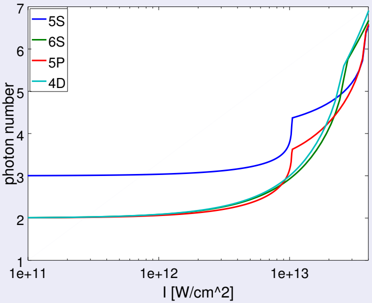

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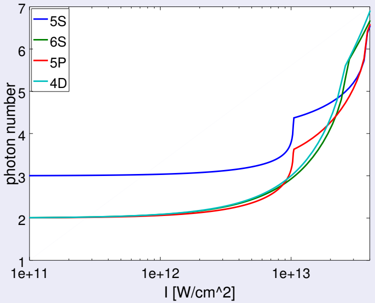
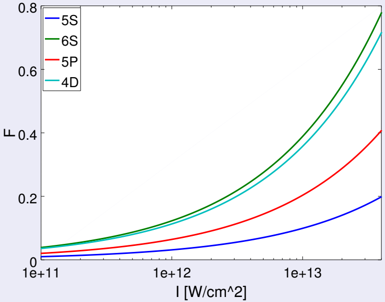


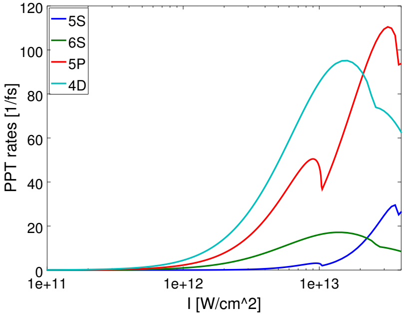
Limits of reliability

PPT rates assume $F = E/E_0 \ll 1$
Reported unreliable above $F \gtrsim 0.1$



$F = 0.1$ limits:
$\rightarrow$ 5S $I \sim 10^{13}\mathrm{W/cm^2}$
$\rightarrow$ 5P $I \sim 3 - 4 \times 10^{12}\mathrm{W/cm^2}$
$\rightarrow$ 6S, 4D $I \sim 10^{12}\mathrm{W/cm^2}$

Some further warnings...



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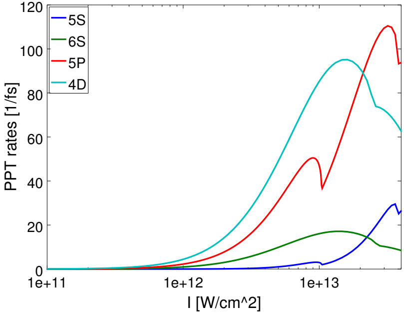

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Ab initio calculation for potassium: → stabilization! (totally outside PPT)

Morales etal., PNAS **108** 16906 (2011)

(no resonance, OBI?)

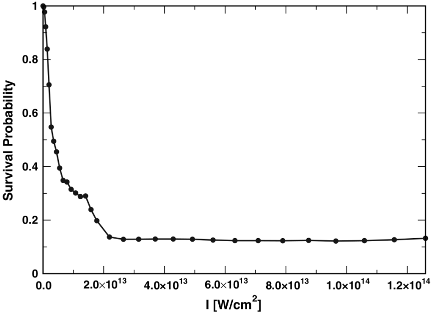
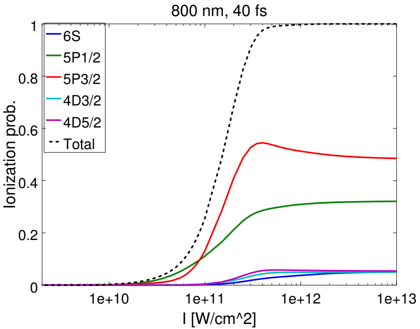
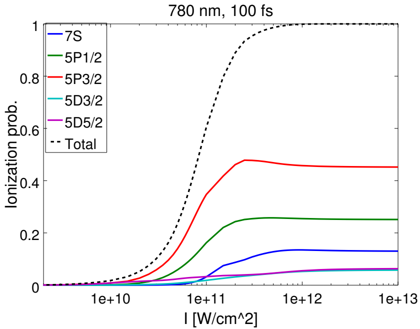


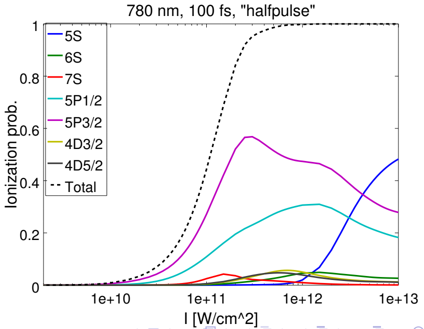
Fig. 3. Stabilization of the potassium atom in a superatomic field: survival probability vs. laser intensity for a 800 nm, 65-fs laser pulse.

Some solutions with 10 levels

- Corroborate dominant ionization via 5P $\rightarrow$ model usable to $\sim 3-4\times 10^{12}\mathrm{W/cm}^2$
- Cannot do with less states for a flexible model...

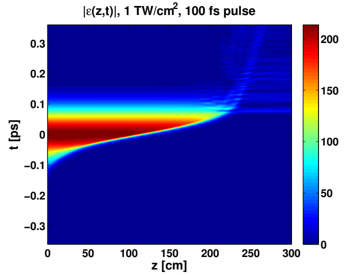

800 nm, 40 fs

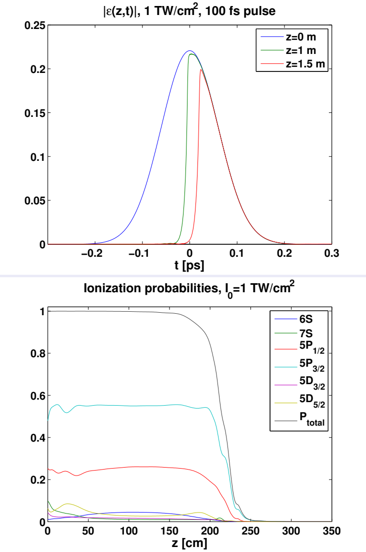

780 nm, 100 fs


780 nm, 100 fs, "halfpulse"

1D propagation calculation

780 nm, 100 fs pulse, $I_0 = 1\mathrm{TW}/\mathrm{cm}^2$:





Results:

- pulse front steepening
- ionization from 5P level

So ... the model has its merits ... but results unreliable for 40 TW pulses. Can we still use it to say something interesting?

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Idea: increase pulse energy via duration instead of peak intensity.

Gaussian pulse:
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increase τ_p
$\tau_0 \to \tau$ but
keep bandwidth!

$\Rightarrow$ chirped pulse:
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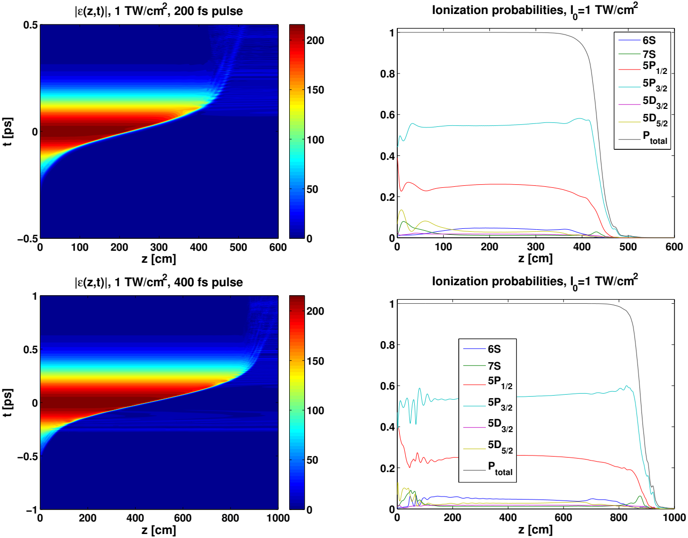
$|\varepsilon(z,t)|$, 1 TW/cm^2, 200 fs pulse

Ionization probabilities, I_0=1 TW/cm^2

$|\varepsilon(z,t)|$, 1 TW/cm^2, 400 fs pulse

Ionization probabilities, I_0=1 TW/cm^2

How to proceed

1. Better description of ionization seems essential!
   - more accurate rates
   - back-action on light
   - check the story with stabilization
2. Relax SVEA, nonlinear envelope equations.
3. 2D / 3D propagation, transversal structures.

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Thank you for your attention