

Analysis of the Presence of Small Admixtures of Heavy Elements in the Solar Plasma by Using the SAHA-S Equation of State

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The thermodynamic functions of a weakly nonideal plasma are extensively calculated for conditions typical of the depths of stars by using the SAHA-S equation of state. These calculations ensure precise analysis of the effect of the heavy-element content on adiabatic compressibility in the depths of the Sun. Comparison of model calculations with recent helioseismic data ensures more precise determination of solar-plasma composition. This comparison shows that the inclusion of additional components to the composition of heavy-element admixtures is a necessary condition for the theoretical equation of state and the results of analysis of solar oscillations to be consistent. © 2004 MAIK “Nauka/Interperiodica”.

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Extensive recent observation data on the eigenfrequencies of solar oscillations (see, e.g., [1]) provide information about physical conditions in the depths of the Sun and enable one to refine the model of the internal structure. In order to develop a model of the Sun, it is necessary to know the equation of state of the solar plasma with high accuracy. In turn, the comparison of theoretical models with helioseismic data opens a unique possibility of verifying and refining the equation of state of a weakly nonideal plasma with a high accuracy (higher than 10^{-4}). The comparison of theoretical data on the speed of sound and data reconstructed from helioseismic observations depends on the distribution of temperature, density, and chemical composition over the radius inside the Sun. Therefore, this problem is a multiparameter problem and requires the fixation of at least some of these data. In this work, we use data on the distribution of temperature and density from the model of the current Sun accepted in helioseismology [2], whereas the problem of the effect of the equation of state on the evolution and structure of the model is ignored.

Certain curves characterizing the used model data are shown in Fig. 1. Monotonically increasing solid line 1 is the density–temperature thermodynamic curve for the internal structure of the Sun. Equations of state are compared in this line. Part 3 marked by a heavy solid line corresponds to the zone of convective heat transfer, and most part of this zone is very close to the isentrope $S(T, \rho) = \text{const}$. Line 2 is the distribution of adiabatic

compressibility $\tilde{\gamma}_1 = (\partial \ln P / \partial \ln \rho)_S$. Regions of a considerable deviation of this quantity from an ideal-gas value of 5/3 correspond to the ionization of hydrogen and helium. The upper abscissa axis shows the relative radius for the corresponding temperature.

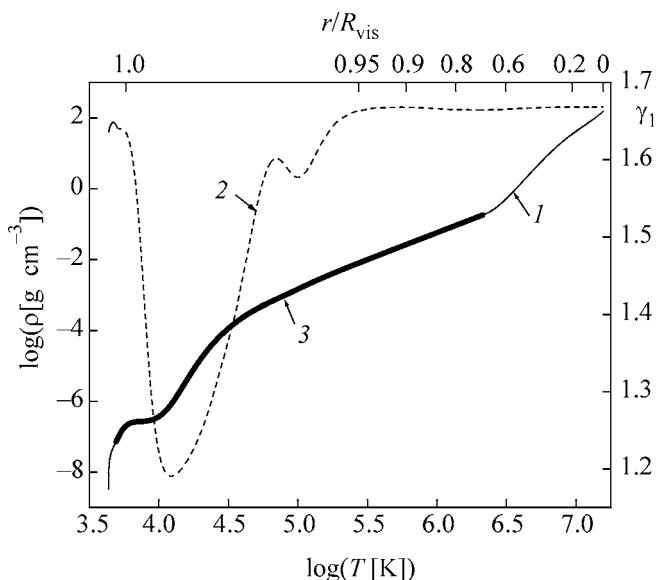


Fig. 1. Distribution of (1) density and (2) adiabatic compressibility inside the Sun according to the S model. Convective transfer zone (3) is indicated.

The equation of state and thermodynamic functions are calculated by using the “chemical” model [3]. The plasma consists of hydrogen, helium (these basic components amount to 98 wt %), and heavy admixtures of carbon, nitrogen, oxygen, and neon. The inclusion of silicon and iron is additionally studied upon the conservation of the 2% general mass fraction of admixtures. The plasma consists of more than 90 components: atoms, diatomic molecules, electrons, and ions from single-charged ions to ions with the maximum possible charge for each element.

According to [3], the free energy F of the system was represented as a sum, where the first term F_{id}^0 describes a mixture of noninteracting “ideal” particles—partially degenerate slightly relativistic electrons; classical ions; atoms and molecules; and photons (the radiation-pressure effect is quite noticeable particularly for $\tilde{\gamma}_1$). The second term ΔF_{int} describes interaction between these particles. Although the first term describes the translational degrees of freedom of particles and is decomposed into the terms for kinds of particles, chemical reactions between particles, as well as the ionization and recombination reactions, can proceed. Therefore, the resulting thermodynamic parameters differ from the ideal-gas values even in simple cases (see Fig. 1). Moreover, compound particles can have internal degrees of freedom, which are described in the below expression by the partition functions Q_j for particles of the j th kind:

$$F(\{N_i\}, V, T) = F_{\text{id}}^0 + \Delta F_{\text{int}} \\ = \sum_j N_j k_B T \left(\ln \left(\frac{n_j \lambda_j^3}{Q_j} \right) - 1 \right) + \Delta F_{\text{int}}(\{N_i\}, V, T). \quad (1)$$

Here, N_j and n_j are the number of particles of the j th kind and their density, respectively; V is volume; T is the temperature of the system; k_B is the Boltzmann constant; and λ_j is the de Broglie thermal wavelength.

Corrections in the second term ΔF_{int} are usually associated with the nonideality of the system, although they are not the main cause of deviation from the equation of state for an ideal gas under solar conditions. The main part of the correction is attributed to the Coulomb interaction in the plasma and corresponds to the Debye approximation in the grand canonical ensemble [4] with allowance for the electron degeneracy. In addition, we include corrections for the electron diffraction upon scattering. The Coulomb and diffraction corrections have a smallness of $\sim \Gamma_D$ and $\sim \Gamma_D \lambda_e \kappa_D$, respectively, where κ_D is the inverse Debye radius, $\Gamma_D = e^2 \beta \kappa_D$ is the Debye nonideality parameter, and $\beta \equiv (k_B T)^{-1}$. The SAHA-S equation of state additionally includes the exchange correction $\sim \lambda^2 e^2 \beta$. We point out the relativistic correction $\sim k_B T / m_e c^2$ that is small for cold solar-type stars, but noticeably affects adiabatic compressibility.

The OPAL equation of state [4] is currently accepted as the reference equation of state in helioseismology. It was partially obtained from the first principles in the framework of the expansion of the Helmholtz thermodynamic potential Ω in series in activities. The OPAL equation of state includes nonrelativistic degenerate electrons, classical ions, all stages of ionization and excitation, molecular hydrogen, Coulomb approximation with allowance for the electron degeneracy, electron diffraction, electron exchange, and pressure-induced ionization (contributions from ladder diagrams are also mentioned). The second virial coefficient including contributions from ring and ladder diagrams is thought to be well known since Vedenov and Larkin [5, 6] and was refined by Ebeling *et al.* [7, 8] and Kopyshchev [9]. Since the basic contribution from scattering states is proportional to $\sim \Gamma_D^2 \ln \Gamma_D$ and $\Gamma_D \sim 0.05$ in most of the internal part of the solar convective zone, these contributions in our equation of state of the solar plasma are ignored. It is considered a well-established fact [5, 7, 8] that the partition function Q_j over discrete levels is regularized by the Planck–Larkin procedure, and contribution to Ω for, e.g., nonrelativistic hydrogen atoms has the form [3]

$$\Delta \Omega_{\text{bs}} = -k_B T \zeta_e \zeta_i \lambda_{ei}^3 \\ \times \sum_{n=1}^{\infty} n^2 e^{\beta E_n} (1 - e^{-\beta E_n} - \beta E_n e^{-\beta E_n}), \quad (2)$$

where

$$E_n = \frac{Ry}{n^2}, \quad \lambda_{ei} = \sqrt{\frac{2\pi\hbar^2}{\mu k_B T}}, \quad \mu = \frac{m_e m_p}{m_e + m_p}. \quad (3)$$

Expression (2) is used in the OPAL equation of state. At the same time, the following, more rigorous expression for the contribution from bound states, which differs from the Planck–Larkin expression, was presented in [10] without the deduction that was given in [11]:

$$\Delta \Omega_{\text{bs}} = -k_B T \zeta_e \zeta_i \lambda_{ei}^3 \sum_{n=1}^{\infty} n^2 F_n(\beta) e^{\beta E_n}, \quad (4)$$

where

$$F_n(\beta) = 1 - e^{-\beta E_n} \left[4 - \frac{6}{\sqrt{\pi}} (\beta E_n)^{1/2} + \frac{4}{\sqrt{\pi}} (\beta E_n)^{3/2} \right] \\ + \frac{\Gamma\left(\frac{1}{2}, \beta E_n\right)}{\sqrt{\pi}} [3 - 4\beta E_n + 4(\beta E_n)^2]. \quad (5)$$

Asymptotically for $\beta E_n \ll 1$, Eqs. (4) and (5) yield

$$n^2 F_n(\beta) \longrightarrow 2n^2 (\beta E_n)^2, \quad (6)$$

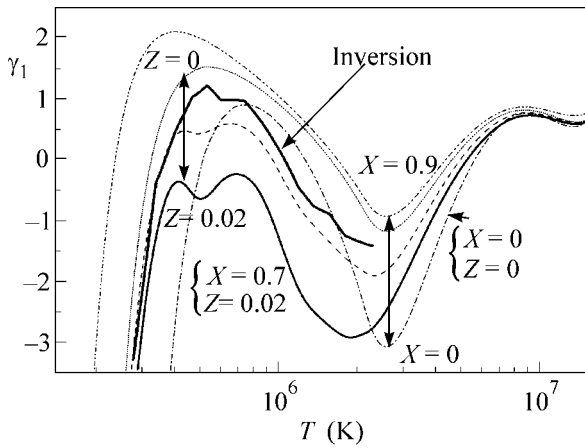


Fig. 2. Effective adiabatic index $\gamma_1 = \left[\left(\frac{d \ln P}{d \ln \rho} \right)_s - \frac{5}{3} \right] \times 10^3$ inside the Sun for various fractions of hydrogen X and heavy elements Z in comparison with the inversion-procedure results [1].

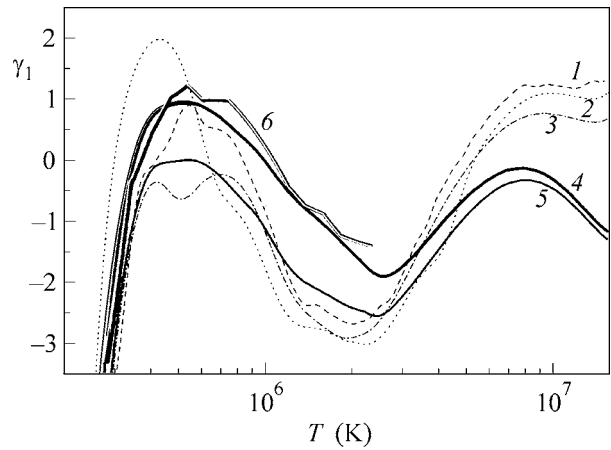


Fig. 3. Effective adiabatic index γ_1 for the equations of solar-plasma state (1) OPAL [4], (2) MHD [13], (3) SAHA-S (C-Ne), (4) SAHA-S, $Z = 0.01$ (C-Ne, Si, Fe), and (5) SAHA-S, $Z = 0.02$ (C-Ne, Si, Fe) in comparison with (6) the inversion-procedure results [1].

whereas the asymptotic behavior of Eq. (2) has the form

$$n^2 F_n^{\text{PL}}(\beta) \longrightarrow \frac{1}{2} n^2 (\beta E_n)^2. \quad (7)$$

In contrast to the OPAL equation of state [4], the SAHA-S equation of state describes the solar-plasma model [12] by using Eqs. (4) and (5). However, it is difficult to directly compare the SAHA-S and OPAL equations of state by using only the initial assumptions. We can only state that these equations are quite consistent with each other with respect to the nonideality contribution, and the difference between them lies within the uncertainty in the description of these terms, which are small for the Sun and more massive stars. The basic difference from the OPAL equation of state is the addition of silicon and iron to the mixture of heavy elements.

Using the SAHA-S code intended for calculating the thermodynamics of the nonideal plasma of astrophysical objects, we calculated the composition and thermodynamic functions of the solar plasma for densities up to 200 g/cm³ and temperatures up to 2×10^7 K. Using the calculated thermodynamic functions and procedure of calculating the solar parameters, we developed a model of the internal solar structure and its chemical evolution.

We compared the theoretical predictions of the equations of state and data of analysis of the spectrum of natural oscillations for a relatively narrow region in the depth of the Sun that covers the lower part of the convective zone. In this zone, the basic components of the solar plasma—hydrogen and helium—are already ionized, and the thermodynamic parameters are close to ideal-gas values. In particular, we focus on adiabatic compressibility, which deviates from 5/3 by no more than several thousandths in the region under consider-

ation. Moreover, the results of helioseismic inversion of adiabatic compressibility are most reliable in this region. We emphasize that the above values of the corrections responsible for various nonideality effects enable one to compare variations in pressure or internal energy. The situation is much more difficult for the effect of these factors on adiabatic compressibility. In this case, certain effects can be weakly manifested (e.g., electron degeneracy), whereas other effects can be surprisingly large (e.g., the contribution from radiation and the slight relativism of electrons).

Figures 2 and 3 show the results of calculations in comparison with the results of helioseismic inversion. We emphasize that the adiabatic index for temperatures higher than 2×10^6 K is reconstructed with much larger uncertainties, because this temperature is the bound of the convective-transfer zone with well-determined adiabatic stratification. For this reason, we focused on the interior of the convective zone.

These figures also show the results of calculations by the SAHA-S code. Figure 2 illustrates the effect of the content of helium and heavy admixture on the behavior of adiabatic compressibility. Pairs of lines connected by arrows show that an increase in the content of both helium and heavy elements reduces adiabatic compressibility. However, the “standard” chemical composition of the solar plasma corresponds to the solid line ($Z = 0.02$ and $X = 0.7$), which underestimates helioseismic data. The assumption of the low content of heavy elements ($Z = 0.00$) leads to considerable difficulties when comparing them with other astrophysical data.

Figure 3 shows the adiabatic compressibility index obtained from the inversion of helioseismic data [1] in comparison with data taken from [4, 13], SAHA-S data ($Z = 0.02$), and our calculations with the refined com-

position of heavy elements including silicon and iron. Heavy elements were 2% in mass and included 17.6% carbon, 5.2% nitrogen, 50.2% oxygen, 9.7% neon, 9.8% silicon, and 7.5% iron. Figure 3 shows that the inclusion of silicon and iron into admixtures (instead of simplified change of these elements to neon) is noticeable from the viewpoint of agreement with helioseismic data. Our model agrees well with models presented in [4, 13] in the theoretical description of nonideality in this temperature region.

Thus, the use of the SAHA-S equation of state makes it possible to calculate the internal structure and evolution of the Sun and refine (due to high and controllable accuracy of the equation of state) the composition and general content of heavy elements, which are consistent with helioseismic data.

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