

SAHA-S model: Equation of State and Thermodynamic Functions of Solar Plasma

V.K.Gryaznov¹, S.V.Ayukov², V.A.Baturin², I.L.Iosilevskiy³,
A.N.Starostin⁴, V.E.Fortov¹

¹*Institute of Problems of Chemical Physics RAS, Chernogolovka, Russia* ²*Sternberg Astronomical Institute, Moscow, Russia* ³*Moscow Institute of Physics and Technology, Dolgoprudnyi, Russia* ⁴*Troitsk Institute for Innovation and Fusion Research, Troitsk, Russia*
Abstract. A thermodynamic model SAHA-S for solar plasma is presented. Effects of Coulomb interaction, exchange and diffraction effects, free electron degeneracy, relativistic corrections, radiation pressure contributions are taken into account. Contribution of bound states of atoms and ions is corrected in comparison with Plank-Brilluin-Larkin formula. Calculations of equation of state of solar plasma with different element composition are carried out. Contribution of various plasma effects and chemical element abundance to thermodynamic functions and in particular Γ_1 is discussed.

INTRODUCTION

A great amount of high precision observation data on eigenfrequencies of solar oscillations obtained during the last decade [1, 2] provides detailed information on physical conditions in the Sun interior and permits to develop more precise models of its inner structure. To build an adequate model of the Sun one needs a very accurate equation of state (EOS hereafter) of solar plasma together with other microphysical parameters (opacity, nuclear reaction rate et al.). A comparison of computed model frequencies with observational data [1, 2] gives us an opportunity to improve and refine the EOS of weakly coupled plasmas on the high level of accuracy – better then 10^{-4} . Modern EOS models of solar plasma now in use [3, 4] reproduce sound velocity with high accuracy over the whole temperature range, but there are some questions which have to be clarified. Among them is the level of accuracy of theoretical estimations for sound speed and adiabatic exponent Γ_1 over solar model profile, and especially in the lower part of the convection zone. Also, what is possible effect in EOS of nonideal plasma corrections on the profile of Γ_1 in the solar conditions? These questions are actual in relation with real discrepancy between observation and model frequencies caused by sound speed deviations in the lower part of the convection zone.

This paper presents a thermodynamic model of weakly coupled solar plasma named SAHA-S. The SAHA-S EOS inherits basic plasma physical effects, which have traditionally been concerned with solar EOS, but also proposes generalizations of some of them. We are intended to discuss the adiabatic exponent profile inside the Sun, using comparison of the values for different EOS-models, which successively include different plasma effects

THERMODYNAMIC MODEL

The SAHA-S EOS is based on so-called “chemical picture” of plasmas [5, 6, 7] which starts from the representation of the free energy as a sum of the zero approximation term $F^{(id)}$, corresponding to the “ideal-gaseous” mixture with varying composition of a wide spectrum of simple and complex particles – (electrons and ions, atoms and molecules and so on), and of further contributions stemming from interaction between these particles. Historically, the free energy formulation in astrophysical EOS procedure has been written in [8]. Generally complex particles have internal degrees of freedom - excited states, being in thermodynamic equilibrium with the system as a whole, i.e. with its translational degrees of freedom.

$$F(\{N_i\}, V, T) = \sum_i F_i^{(id)} + F_e^{(id)} + F^{(rad)} + \Delta F_{ii,ee,ie}^{(int)}(\{N_i\}, V, T) \quad (1)$$

Here the first term on the right-hand-part side is the contribution of the ideal-gas of “heavy” particles, atoms, ions and molecules. The second term is the ideal-gas contribution of electrons (which may be partially degenerate). The third term represents contribution of radiation and the forth term is responsible for the inter-particle interactions between all particles and includes contribution of Coulomb interactions between charged particles, neutral-neutral interactions, and also charge-neutral interactions.

We suppose that the electrically neutral system (multi-component plasma) consists of L components (atoms, molecules, atomic and molecular ions and electrons) within a fixed volume V and at a temperature T . All particles of the system are constructed of K sorts of nuclei and electrons. It is well known [9] that at fixed V , T , and number of nuclei of each species $\{N_k^0\}, k = 1, 2, \dots, K$ the free Helmholtz energy is minimal in this volume. The minimum necessary condition gives the equations of chemical and ionization equilibrium:

$$\mu_j = \sum_{k=1}^{K+1} \nu_j^k \mu_k, \quad j = K+2, \dots, L; \quad \mu_j = \frac{\partial F(\{N_i\}, V, T)}{\partial N_j} \quad (2)$$

where μ_j is the chemical potential of the j -th component, ν_j^k is the number of nuclei of species k in a particle of species j .

To complete the equilibrium system of equations one has to add to the equations (2) the equations

$$\frac{\sum_{j=1}^L \nu_j^k n_j}{\sum_{l=1}^K \sum_{i=1}^L \nu_i^k n_i} = c_k^0, \quad k = 1, 2, \dots, K \quad (3)$$

of chemical proportions, the equations

$$\sum_{p=1}^L n_p Z_p^{(+)} = \sum_{m=1}^L n_m Z_m^{(-)} \quad (4)$$

of electrical neutrality and the equation

$$\sum_{j=1}^L m_j n_j = \rho_0 \quad (5)$$

of mass conservation.

Here $n_j = N_j / V$ is the particle density of species j , $Z_i^{(+)}$ and $Z_i^{(-)}$ are the charges of particles with positive and negative charges respectively, m_j is the mass of a particle of species j . The abundance of the chemical element of species k , c_k^0 and mass density ρ_0 are given constants. So we have $L+1$ equations (5-8) with respect to L variables n_j . Notice that due to the condition

$$\sum_{k=1}^K c_k^0 = 1$$

only $K-1$ equations of the equations (3) are linearly independent. Solving the system of equations (2-5) we can calculate the component composition $\{n_j\}$.

The first term of the Helmholtz free energy (1) represents the Boltzmann ideal gas expression for a mixture of particles of various species

$$F_i^{(id)} = \sum_{j=1}^L N_j k_B T \left(\ln \frac{n_j \tilde{\lambda}_j^3}{Q_j} - 1 \right) \quad (6)$$

Here k_B is Boltzmann constant, $\tilde{\lambda}_j = \sqrt{\frac{2\pi\hbar^2}{m_j k_B T}}$ is the thermal De Broglie wavelength, Q_j is the partition function for a particle of species j .

$$Q(\{n_j\}, \tau, T) = \sum_i \omega_i(\{n_j\}, T) g_i e^{-\epsilon_i / k_B T}. \quad (7)$$

Here ϵ_i , g_i are the excited energy levels and statistical weights for an isolated particle. For atoms and ions we use data [10, 11], for diatomic molecules the partition functions correspond to the approximation of the non-rigid rotator – the non-harmonic oscillator with the data from [12,13]. The factor ω in general is dependent on particle numbers and temperature, but in the present work only the dependence on temperature is taken into account. For molecules

$$\omega_i(T) = \begin{cases} 1, & \epsilon_i \leq D - k_B T \\ 0, & \epsilon_i > D - k_B T \end{cases} \quad (8)$$

where D is the dissociation energy of diatomic molecule.

For atoms and ions two versions are used. One of them is the Planck- Larkin partition function [14],

$$\omega_i^{PL}(T) = 1 - (1 + \tilde{E}_i) e^{-\tilde{E}_i}; \quad \tilde{E}_i = \frac{\epsilon_i}{k_B T} \quad (9)$$

another one uses a more rigorous [15] expression (for the contribution due to the bound states only) than that given by the Planck-Larkin formula [14] representing in fact some mixture of contributions of bound and scattering states. In this case, the factor ω is given by the expression [15]

$$\omega_i^{SR}(T) = 1 - e^{-\tilde{E}_i} \left[4 - \frac{6}{\sqrt{\pi}} (\tilde{E}_i)^{1/2} + \frac{4}{\sqrt{\pi}} (\tilde{E}_i)^{3/2} \right] + \frac{\Gamma(1/2, \tilde{E}_i)}{\sqrt{\pi}} [3 - 4\tilde{E}_i + 4\tilde{E}_i^2] \quad (10)$$

Notice that from (9) and (10) and at $\varepsilon_i/k_B T \ll 1$, i.e in the very high temperature limit, we have

$$\omega_i^{SR}(T) \Rightarrow 4\omega_i^{PL}(T) \quad (11)$$

In Fig.1 the dependence of the ratio $\omega_i^{SR}(T)/\omega_i^{PL}(T)$ on temperature is represented. It can be seen that for low temperature the difference between two expressions for the partition function is negligible, but at the maximum solar temperature $\approx 1000 \text{ eV}$ is close to the asymptotic limit (11).

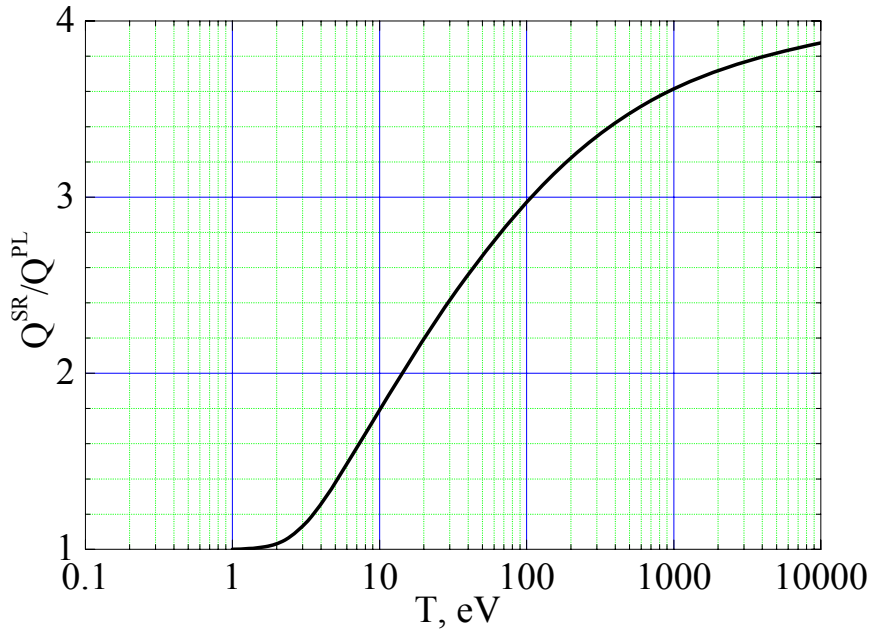


FIGURE 1. Ratio of Q^{SR} to Q^{PL} at various values of temperature for hydrogen atoms.

The contribution of the electronic ideal gas to the Helmholtz free energy expression (1) is represented by the second term $F_e^{(id)}$. The ideal gas of free electrons is considered as a partially degenerate Fermi gas. In accordance with [9]

$$F_e^{id} = N_e k_B T \left[\alpha - \frac{4}{3\sqrt{\pi}} \frac{2}{n_e \tilde{\lambda}_e^3} \int_0^\infty \frac{x^{3/2}}{1 + \exp(x - \alpha)} dx \right]; \quad \alpha = \frac{\mu_e}{k_B T}, \quad (12)$$

where the electronic particle density n_e and electronic chemical potential are related by the expression

$$\frac{n_e \tilde{\lambda}_e^3}{2} = 2\pi^{-1/2} I_{1/2}(\alpha); \quad I_t(\alpha) = \int_0^\infty \frac{y^t dy}{1 + \exp(y - \alpha)} \quad (13)$$

From (12) expressions for pressure, internal energy and chemical potential can be obtained by simple differentiation. The interpolation formulas for the Fermi-Dirac

functions $I_l(\alpha)$ [16] are used, they have the relative accuracy not worse than 10^{-6} over the whole range of degeneracy parameter $n_e \tilde{\lambda}_e^3$.

The third term of the Helmholtz free energy (1) is responsible for inter-particle interaction. In this work the main inter-particle interaction for solar plasma is the Coulomb interaction between charged particles in the framework of the Debye approximation in the grand canonical ensemble, generalized for the case of multi-stage ionization (DGCE)[17].

$$-\frac{\Omega^{(Coul)}}{Vk_B T} \equiv -\frac{F^{(Coul)} - \sum_{i=1}^L N_i \mu_i^{(Coul)}}{Vk_B T} \equiv \frac{P^{(Coul)}}{k_B T} = \sum_{i=1}^L n_i - \frac{\tilde{\Gamma}_D^3}{24\pi f^3}. \quad (14)$$

Here $\Omega^{(Coul)}$, $F^{(Coul)}$, $\mu^{(Coul)}$, $P^{(Coul)}$ are the thermodynamic potential, the Helmholtz free energy, the chemical potential and the pressure of charged particles, $f = e^2/k_B T$ is the Coulomb scattering amplitude and $\tilde{\Gamma}_D$ is a real root of the equation

$$\tilde{\Gamma}_D^2 = 4\pi f^3 \sum_{i=1}^L \frac{Z_i^2 n_i}{1 + Z_i^2 \frac{\tilde{\Gamma}_D}{2}}, \quad (15)$$

and Z_i is the particle charge number. In (14,15) the summation is extended over the charged particles only. Due to (14) the dimensionless correction to the ideal gas pressure and energy are

$$\frac{\Delta P^{(Coul)}}{n_e k_B T} = \frac{\Delta E^{(Coul)}}{3N_e k_B T} = -\frac{\tilde{\Gamma}^3}{24\pi n_e f^3}, \quad (16)$$

and for the chemical potential one has:

$$\frac{\Delta \mu_i^{diff}}{k_B T} = -\ln \left(1 + Z_i^2 \frac{\tilde{\Gamma}}{2} \right). \quad (17)$$

Certainly, the properties of the DGCE approximation differ from the well known Debye-Hückel (DH) one. In the DH approximation the correction to the ideal gas pressure has the form

$$\frac{\Delta P^{(DH)}}{\sum_i Z_i^2 n_i k_B T} = -\frac{\Gamma_D}{6}, \quad (18)$$

where Γ_D is the Debye coupling parameter

$$\Gamma_D = f \sqrt{4\pi f \sum_i Z_i^2 n_i}. \quad (19)$$

Notice that the DH pressure loses stability along the isotherm at $\Gamma_D=4$ and becomes negative at $\Gamma_D=6$ so that

$$\frac{\Delta P^{(DH)}}{\sum_i Z_i^2 n_i k_B T} \Rightarrow -\infty \quad \text{when } \Gamma_D \Rightarrow \infty,$$

while the DGCE is thermodynamically stable at any value of the coupling parameter and

$$\frac{\Delta P^{(DGCE)}}{\sum_i n_i k_B T} \Rightarrow -\frac{1}{3} \quad \text{when } \Gamma_D \Rightarrow \infty.$$

To make the EOS of the solar plasma more precise, the electron exchange and diffraction corrections to the Coulomb interaction are taken into account. In accordance with [18], the Coulomb correction to the ideal gas pressure (thermodynamic potential Ω) with the exchange and diffraction effects has the form

$$\begin{aligned} \frac{\Delta P^{(CoulExDif)}}{n_e k_B T} = & -\frac{\tilde{\Gamma}^3}{24\pi m_e f^3} - \frac{1}{4} \zeta_e \tilde{\lambda}_e^2 f \frac{\zeta_e}{n_e} + \frac{\tilde{\lambda}_e \zeta_e f^2}{4} \frac{\tilde{\Gamma}^2}{4n_e f^3} + \\ & + \frac{\pi \tilde{\lambda}_e f^2 \zeta_e}{4} \frac{\zeta_e}{n_e} \left\{ \frac{(1-\sqrt{2})}{\sqrt{2}} + \frac{1}{2} \sum_{i=1}^L \sum_{k=1}^L Z_i^2 Z_k^2 \frac{\zeta_i}{\zeta_e} \frac{\zeta_k}{\zeta_e} \frac{\tilde{\lambda}_{ik}}{\tilde{\lambda}_e} \right\}, \end{aligned} \quad (20)$$

where $\tilde{\lambda}_e, \zeta_e$ are the thermal De Broglie wavelength and activity ($\zeta_e = \exp(\mu_e / k_B T) / \tilde{\lambda}_e^3$) for the electrons and the same $\tilde{\lambda}_i, \zeta_i$ for the ions, $\tilde{\lambda}_{ik}$ being the thermal De Broglie wavelength with the reduced mass $m_{ik} = m_i m_k / (m_i + m_k)$, the summation in (20) is over ions only. In (20) all activities have to be expressed in terms of the particle densities using equations [18]

$$\begin{aligned} \zeta_e = & \frac{n_e}{1 + \frac{\tilde{\Gamma}}{2} + \frac{1}{4} \zeta_e \tilde{\lambda}_e^2 f - \frac{\tilde{\lambda}_e \tilde{\Gamma}^2}{f 16} - \frac{\pi \tilde{\lambda}_e f^2 \zeta_e}{4} [\sqrt{2} - 1]} \\ \zeta_i = & \frac{n_i}{1 + Z_i^2 \frac{\tilde{\Gamma}}{2} - \frac{\pi \tilde{\lambda}_e f^2 \zeta_e Z_i^2}{4} \left(1 + \sum_{k=2}^L Z_k^2 \frac{\zeta_k}{\zeta_e} \frac{\tilde{\lambda}_{ik}}{\tilde{\lambda}_e} \right)} \end{aligned} \quad (21)$$

By virtue of (21) the corrections to the ideal gas chemical potentials of electrons and ions have the form

$$\begin{aligned} \frac{\Delta \mu_e^{diff}}{k_B T} = & -\ln \left(1 + \frac{\tilde{\Gamma}}{2} + \frac{1}{4} \zeta_e \tilde{\lambda}_e^2 f - \frac{\tilde{\lambda}_e \tilde{\Gamma}^2}{f 16} - \frac{\pi \tilde{\lambda}_e f^2 \zeta_e}{4} [\sqrt{2} - 1] \right) \\ \frac{\Delta \mu_i^{diff}}{k_B T} = & -\ln \left(1 + Z_i^2 \frac{\tilde{\Gamma}}{2} - \frac{\pi \tilde{\lambda}_e f^2 \zeta_e Z_i^2}{4} \left(1 + \sum_{k=2}^L Z_k^2 \frac{\zeta_k}{\zeta_e} \frac{\tilde{\lambda}_{ik}}{\tilde{\lambda}_e} \right) \right) \end{aligned} \quad (22)$$

and the same for the internal energy:

$$\begin{aligned} \frac{\Delta E^{(CoulExDif)}}{N_e k_B T} = & -\frac{\tilde{\Gamma}^3}{8\pi m_e f^3} - \frac{1}{2} \zeta_e \tilde{\lambda}_e^2 f \frac{\zeta_e}{n_e} + \frac{5}{2} \frac{\tilde{\lambda}_e f^2 \zeta_e}{4} \frac{\tilde{\Gamma}^2}{4 n_e f^3} + \\ & + \frac{5}{2} \frac{\pi \tilde{\lambda}_e f^2 \zeta_e}{4} \frac{\zeta_e}{n_e} \left[\left(\frac{1}{\sqrt{2}} - 1 \right) + \frac{1}{2} \sum_{i=2}^I \sum_{k=2}^I Z_i^2 Z_k^2 \frac{\zeta_i}{\zeta_e} \frac{\zeta_k}{\zeta_e} \frac{\tilde{\lambda}_{ik}}{\tilde{\lambda}_e} \right] \end{aligned} \quad (23)$$

Due to the high temperature values of plasma in the inner part of the Sun, the relativistic effects in the electron motion and radiation pressure can affect thermodynamics of the solar plasma. To understand how important these effects are, the relativistic corrections [9] were accounted for. It has to be noticed that the first order correction to the pressure vanishes exactly,

$$\frac{\Delta P_e^{rel}}{k_B T} = 0, \quad (24)$$

but there are corrections to the internal energy of the same order:

$$\frac{\Delta E^{rel}}{k_B T} = \zeta_e \frac{15 k_B T}{8 m_e c^2}, \quad (25)$$

and the electronic chemical potential

$$\frac{\Delta \mu_e^{rel}}{k_B T} = -\ln \left[1 + \frac{15 k_B T}{8 m_e c^2} \right], \quad (26)$$

where c is the light velocity.

The radiation pressure and the internal energy contributions are taken in the form [9]

$$\Delta P^{(Rad)} = \frac{\Delta E^{(Rad)}}{3V} = \frac{4\sigma_{SB}}{3c} T^4; \quad \sigma_{SB} = \frac{\pi^2 k_B^4}{60 \hbar^3 c^2}. \quad (27)$$

In the context of the structure of the Helmholtz free energy expression described above, all corrections to the ideal gas thermodynamic values, the expression for the chemical potential of any particle can be split into two parts. They are Boltzmann ideal gas contribution and the correction to its value

$$\frac{\mu_j}{k_B T} = \ln \frac{n_j \tilde{\lambda}_j^3}{Q_j} + \Delta \mu_j^{(Corr)}(\{n_i\}, T). \quad (28)$$

Notice that if the second term in (28) is put equal to zero, the partition function is reduced to the ground state statistical weight, equations (2) are transformed to the Saha equations [19] for the of ionization equilibrium.

RESULTS AND DISCUSSION

On the basis of the chemical picture and using the model (1-28), massive computations of composition and thermodynamic properties of the solar plasma have been carried out. All the calculations were performed by specially designed computer code SAHA-S. This code continues the SAHA code line [6, 20, 21, 22, 13] used to

compute the EOS and thermodynamic functions of multi-component plasmas with strong inter-particle interactions. Two types of computations were performed. The first one is the calculation of comprehensive tables for different helium abundance Y , fixed heavy elements abundance Z (2%), densities ranging from 10^{-8} g/cm³ to 10^4 g/cm³, and temperatures ranging from 10^3 to 10^8 K. On the basis of these tables a full model of the Sun was thus constructed and its results for the calculated values of Γ_I were compared to the data obtained from other solar models.

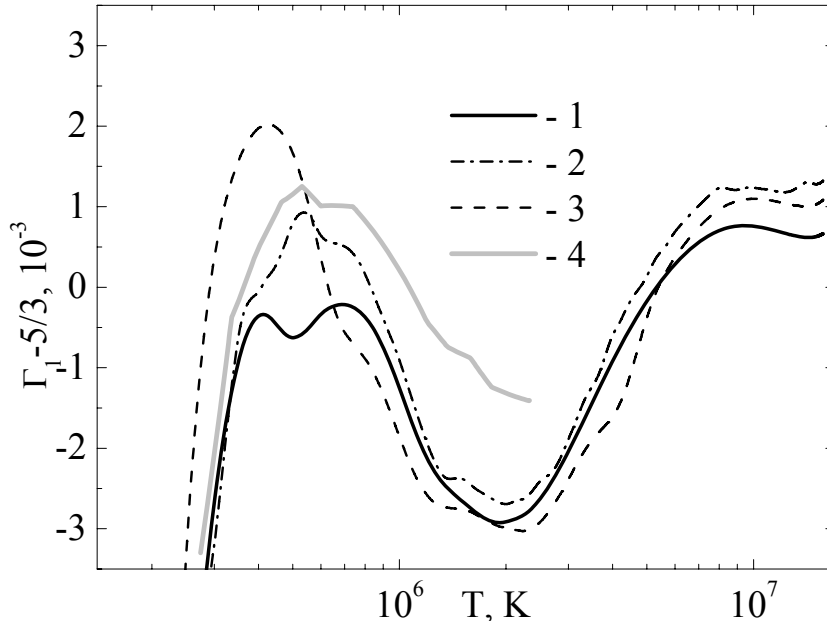


FIGURE 2. Adiabatic exponent along the solar trajectory, 1 - present work, 2 – OPAL[3], 3 – MHD [4], 4 – the result of helioseismic inversion [1].

The “adiabatic exponent” (AE) Γ_I

$$\Gamma_I = \left(\frac{\partial \ln P}{\partial \ln \rho} \right)_s, \quad (29)$$

which, strictly speaking, has to be called the isentropic exponent, one of the key thermodynamic value in helioseismology. This object will be paid the main attention.

In these calculations the heavy element composition (in mass) was the following: carbon fraction was 0.1906614, nitrogen - 0.0558489, oxygen - 0.5429784, neon - 0.2105114. In these calculations plasma consisted of 54 components, including electrons, atoms, all ions for each chemical element, and the diatomic molecules. From the model described above the following elements of the approach were taken into account. Corrections due to the Coulomb interactions of charged particles were included in the framework of the DGCE (14-17), the effects of degeneracy of free electrons were described according to (12,13), and the partition functions were calculated in the form (10-11), effects of the radiation pressure by (27). At the first step in the solution of equations (2-5), we obtained the component composition, then the thermodynamic functions, the dimensionless parameters, and then the differential

characteristics such as heat capacities, sound velocity, adiabatic compressibility were calculated. One can see that the model SAHA-S demonstrates a good agreement with other [3, 4] solar models. Nevertheless it is seen that the SAHA-S and the other models demonstrate growing discrepancy with the results of helioseismic inversion [1] to temperatures, corresponding to the bottom of the convection zone. Understanding of possible reasons of this discrepancy is impossible without answering the question on the influence of various plasma effects on the behavior of thermodynamic values along the solar trajectory.

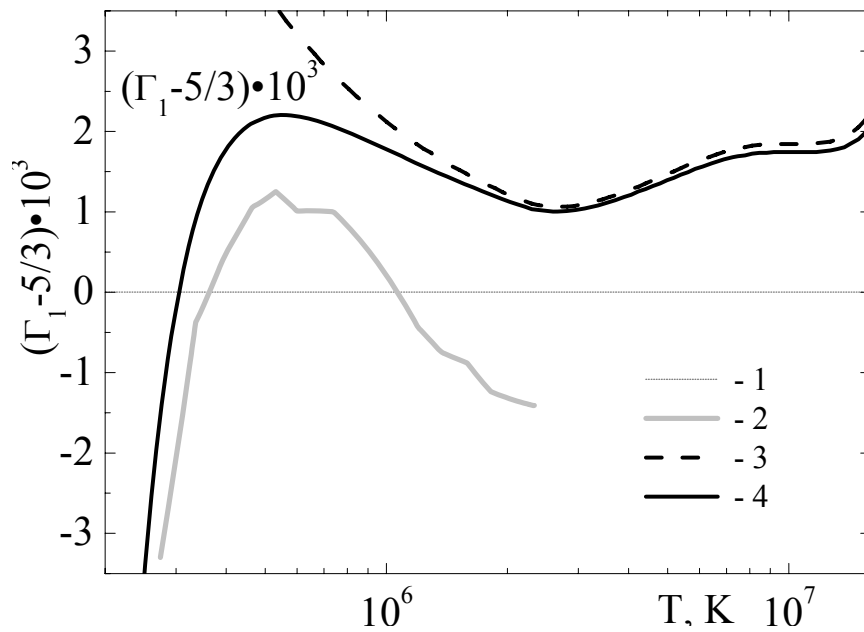


FIGURE 3. Adiabatic exponent of ideal plasma of protons and alpha-particles (curve 1) is the present calculation, the results of inversion [1] -2, 3 is the fully ionized plasma of hydrogen and helium with Coulomb interaction in the framework of the DGCE (14-18), 4 – the contribution of bound states of H and He to AE.

From this point of view of the second type of computations of thermodynamics of solar plasmas was carried out. The idea of these calculations consisted in successive switching of the plasma effects to the EOS on to analyze the dependence of the thermodynamic functions on these effects. In this case we used data of the model S for the solar trajectory, namely the dependence of the density on temperature, and the helium and heavy elements abundance. The first step was the simplest one.

The fully ionized plasma of hydrogen and helium was considered as the ideal Boltzmann gas (no corrections of (7-28) were taken into account), X corresponded to its value in the model-S. In this case Γ_1 has to be equal to 5/3 exactly, that is natural for the ideal Boltzmann gas of particles without any internal structure. In the next step the fully ionized plasma of hydrogen and helium was considered with the Coulomb interactions in accordance to the DGCE (14-17). Fig. 3 demonstrates the shift of Γ_1 to the higher values due to the contribution of the charged particle interactions.

Larger shift at lower temperatures corresponds to higher values of the Coulomb coupling parameter (19). It is seen in Fig.4, where the Coulomb coupling parameter and degeneracy parameter vs. the solar radius are represented.

In the next step the calculations of plasma properties were carried out in which together with the Coulomb interactions the recombination of free charges was allowed. The contribution of bound states using the Planck-Larkin [14] formula to calculate the partition functions was considered. The results of such calculations are demonstrated also in Fig.3. It is seen that at region of lower temperature where the contribution of bound states is noticeable, the recombination effects lead AE to a steep falling down. This effect is important up to the temperature of the convection zone bottom, where the helium ionization goes to finish. Addition of degeneracy effects of free electrons does not change this picture, since the adiabatic exponent of the ideal electron gas is exactly equal to five thirds, which is the strict result for the ideal Fermi gas at any value of the degeneracy parameter [9].

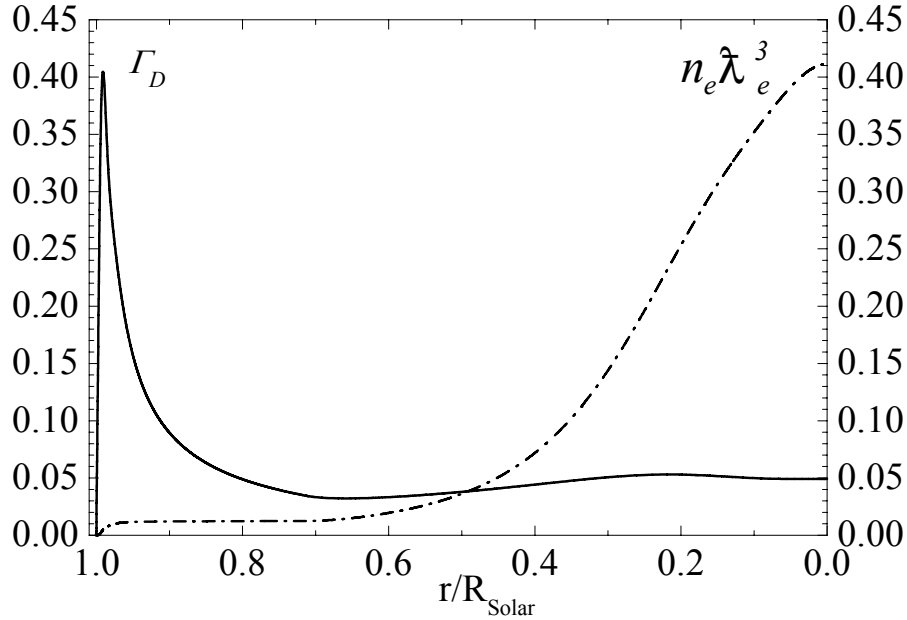


FIGURE 4. Coulomb coupling parameter and degeneracy parameter along the solar radius.

The next step in our calculations consisted in the inclusion of heavy elements. The abundance of heavy elements corresponded to the model S and the distribution of mass fractions was the same, as in the calculations represented in Fig.2 (C - 0.1906614, N - 0.0558489, O - 0.5429784, Ne - 0.2105114). All degrees of ionization for each chemical element were taken into account so as the diatomic molecules H_2 , H_2^+ , C_2 , N_2 , O_2 , CH , CN , CO , NH , NO . The partition functions of excited states of atoms were calculated using the Planck-Larkin formula; the partition functions of molecules were calculated in the approximation of the non-rigid rotator - non-harmonic oscillator with the cutoff procedure (8). Results of these calculations are presented in Fig.5. The maxima of AE in the temperature range 400-700 kK are due to the ionization of inner electronic shells of heavy elements. The difference between the

values of AE calculated with the Planck-Larkin (9) and Starostin - More - Rörich (10) formulae for the partition functions of atoms and ions can be seen in the same Figure.

It is clear from (11) and Fig.1 that the maximal difference between the calculation results with two partition functions is of factor 4 at very high temperature, but for the solar plasma the maximal effect occurs at the temperatures of about 500kK, which is due to the contribution of the abundance of bound states in this temperature region.

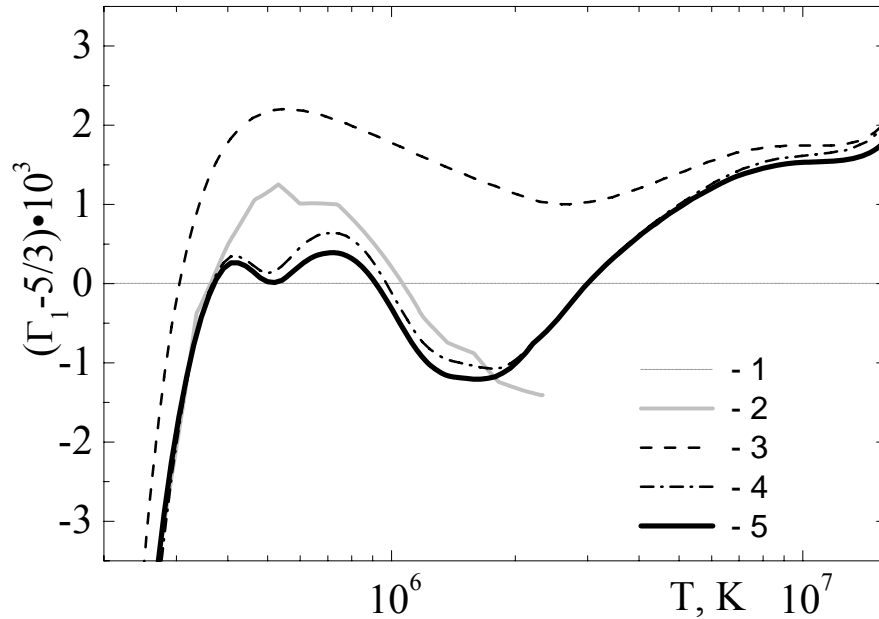


FIGURE 5. Contribution of heavy elements and bound states with the Plank-Larkin (4) and Starostin-More-Rörich (5) partition functions; 1, 2, 3 as 1,2,4 in Fig.4.

Whereas in the center of the Sun where temperature reaches maximum, this difference is not so high due to the full ionization of the main chemical elements of the solar plasma, H and He. The contribution of the radiation pressure and energy to AE can be seen in Fig.6. The difference between AE with and without the radiation effects is practically constant beginning from the temperature of about 1000kK, which can be explained by the increase of the solar plasma density in the direction to the center of the Sun and, respectively, the growth of the kinetic pressure and energy. A smaller effect is connected with additional quantum corrections (exchange and diffraction), those are given by (24-27), where the Coulomb corrections in the framework of the DGCE combine with the exchange and diffraction corrections. The contribution of these corrections is represented by the curve 5 in Fig.6. The last effect under consideration was the relativistic one. As it is seen from (28-30), the first order pressure relativistic correction equals zero exactly, but nevertheless and due to a very high temperature of the solar plasma in the zone deeper than the bottom of the convection zone, these effects are quite noticeable. The curve 6 in Fig.6 for AE represents the calculation with all plasma effects considered above including the Coulomb interaction effects with quantum corrections, degeneracy of free electrons, radiation effects, relativistic effects and the partition functions of atoms and ions according to the Starostin-More-Rörich taken into account.

The last numerical experiment in the present work consisted of the variation of the composition heavy elements, namely it was enriched with iron and silicon. The variation of fractions consisted mainly in the addition of small fractions of Fe and Si and corresponding decreasing of the Ne fraction.

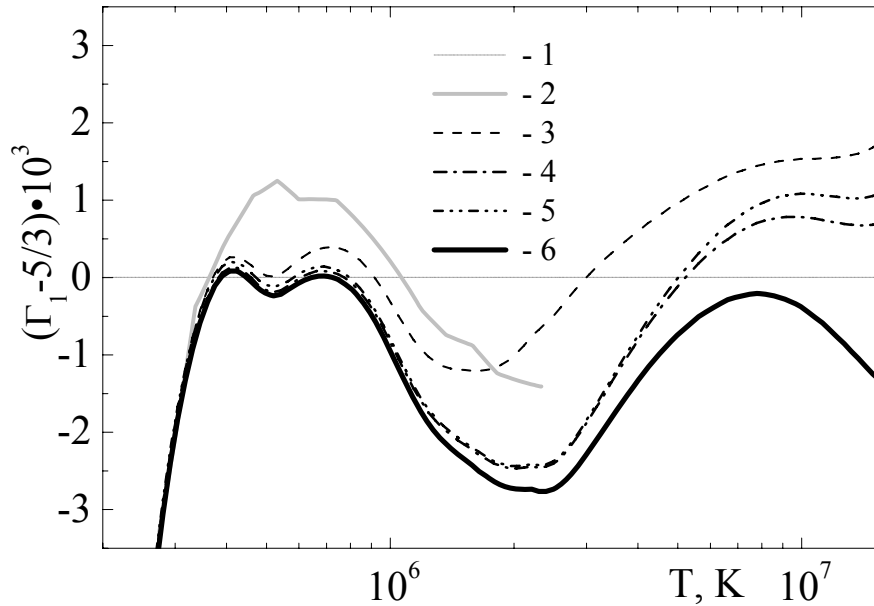


FIGURE 6. Contribution of radiation effects – curve (4), exchange and diffraction effects (5) and relativistic effects (6) to AE. 1, 2, 3 are the same as 1, 2, 5 in Fig.5.

The element composition of heavy elements (in the limits of the abundance of heavy elements within the model S) was the following: C – 0.1762, N – 0.0516, O – 0.5018, Ne – 0.0967, Fe – 0.0982, Si – 0.0755. Other parameters of calculation coincided with those demonstrated in Fig.6, that is all plasma effects were taken into account.

The results of such calculations are represented in Fig.7, where the comparison with the same calculations but with usual (more poor) element composition is carried out. It is seen that the enriched composition of heavy elements does not lead to the radical change in the AE behavior and becomes apparent by removing the two peaks structure in the 400-700kK temperature range.

The last step in this type of calculations was the variation of the heavy element abundance. In this case the heavy element abundance was equal to $Z/2$, where Z was that corresponding to the model S value. The results of these calculations are shown also in Fig.7. One can see this more than satisfactory accordance of these results with the data obtained from the procedure of helioseismic inversion [1], but to our opinion, it has to be appreciated as occasional at the moment. In Fig 8 the distribution of mean ion charges of each chemical elements versus temperature is represented. We must note that ionization degree for H and He reaches its maximum in the convection zone, for C it occurs in the core only, whereas for other chemical elements it does not reach the maxima even in the center of the Sun.

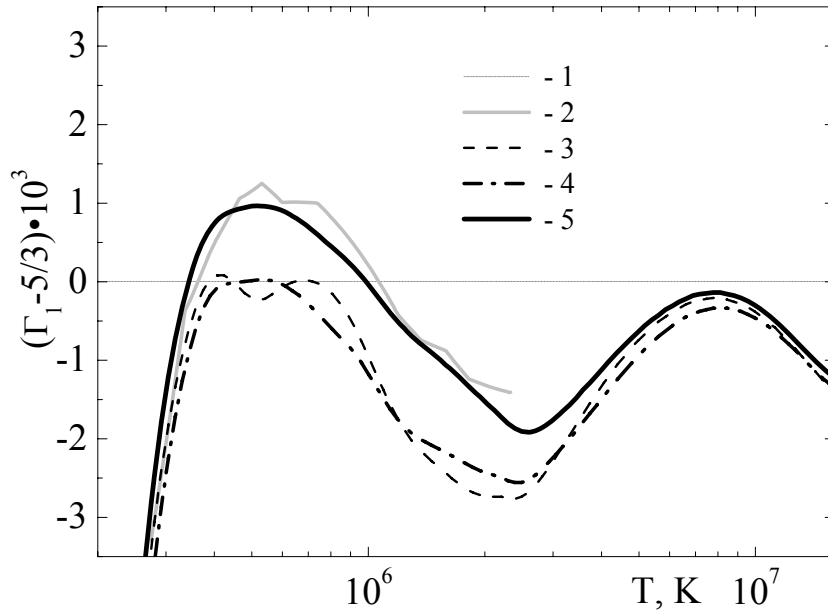


FIGURE 7. Influence of extension (4) of heavy element composition on the behavior of AE; 1, 2, 3 are the same as 1, 2, 6 in Fig.6, 5 – calculation with extended composition and double diminished Z.

It points on a significant role of bound states of atoms and ions in the solar plasma and the knowledge of their energy level structure as an adequate description of contribution to thermodynamic functions and equation of state.

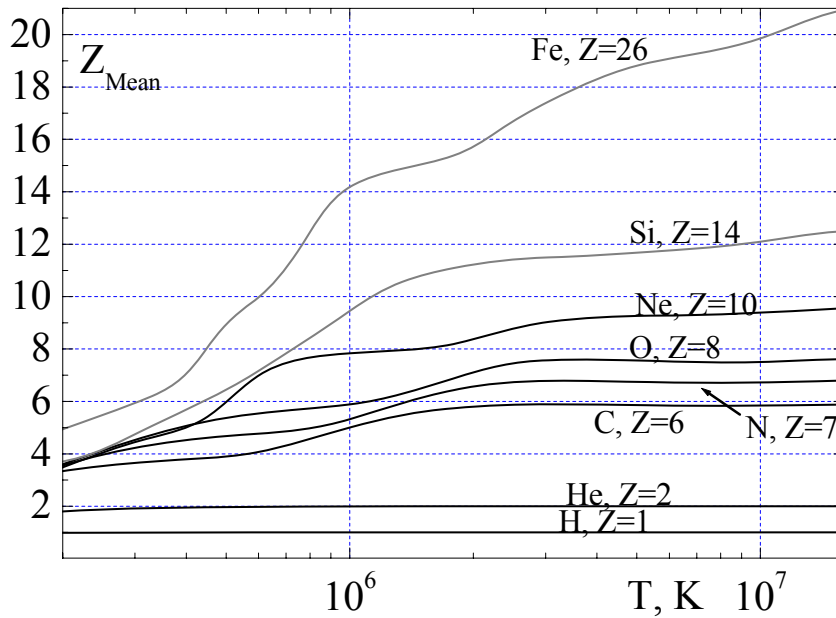


FIGURE 8. Mean charge of ions of chemical elements in the solar plasma. Names of chemical elements and charges of their nuclei are pointed out.

CONCLUSIONS

In this work we have proposed the thermodynamic model SAHA-S of the solar plasma. The model is based on the chemical picture of plasma and includes the Coulomb correlations in the frame of the Debye approximation in the grand canonical ensemble with the exchange and diffraction corrections, free electron degeneracy effects, relativistic effects, new approach for the calculation of atomic and ionic partition functions, the radiation contribution to the plasma pressure and energy taken into account. The results of this model are in good agreement with the solar thermodynamic models now in use. To isolate discrepancies in I_l between the theoretical data and the data of the helioseismic inversion contribution of various plasma effects has been analyzed. The most important factors which affect the behavior of the adiabatic exponent in the convection zone are the Coulomb correlations and the bound states of atoms and ions. In the region of the solar radiation zone and core the relativistic and radiation effects are of great importance. Extending the set of heavy elements with Fe and Si leads to the removal of additional peaks of the adiabatic exponent because of the ionization of Ne inner shells. Double decrease of extended heavy element abundance makes it possible to reach improved agreement between the helioseismic and calculated data.

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