

# Plasma polarization in massive astrophysical objects

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## Abstract

Macroscopic plasma polarization, which is created by gravitation and other mass-acting (inertial) forces in massive astrophysical objects (MAOs), is under discussion. A non-ideality effect due to strong Coulomb interaction of charged particles is introduced into consideration as a new source of such polarization. A simplified situation of a totally equilibrium isothermal star without relativistic effects and influence of magnetic field is considered. The study is based on a density functional approach combined with a 'local density approximation'. It leads to conditions of constancy for generalized (electro) chemical potentials and/or conditions of equilibrium for the forces acting on each charged species. A new 'non-ideality force' appears in this consideration. Hypothetical sequences of gravitational, inertial and non-ideality polarization on thermo- and hydrodynamics of MAO are under discussion.

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## 1. Introduction

The long-range nature of Coulomb and gravitational interactions leads to specific manifestation of their joint action in massive astrophysical objects (MAO). The main one of them is polarization of plasmas under gravitational attraction of ions. Extraordinary smallness of the gravitational field in comparison with the electric one (the ratio of gravitational to electric forces for two protons is  $\sim 10^{-36}$ ) leads to the fact that extremely small and thermodynamically (energetically) negligible deviation from electroneutrality can provide thermodynamically noticeable (even significant) consequences at the level of first (thermodynamic) derivatives. This is the main topic of the present paper.

## 2. Electrostatics of massive astrophysical objects

Gravitational attraction polarizes plasma of massive astrophysical bodies due to two factors: (i) smallness of the electronic mass in comparison with the ionic one and (ii) general

non-uniformity of MAO due to the long-range nature of gravitational forces. The first, mass-dependent type of gravitational polarization is part of the more general phenomenon: (A) any inertial (mass-acting) force (due to rotation, vibration, inertial expansion, compression, etc) polarizes an ion–electron plasma due to the same reason: low mass of electron in comparison with that of ions. The second type of discussed polarization is also part of the more general phenomenon (see, for example [1]): (B) any non-uniformity in the equilibrium Coulomb system is accompanied by its polarization and existence of the stationary profile of the average electrostatic potential. This potential is a thermodynamic quantity because it depends on thermodynamic parameters. The important particular case is the existence of stationary drop of average electrostatic potential at any two-phase interface in the equilibrium Coulomb system [2]. It is valid for terrestrial applications: two-phase interfaces in ordinary and dusty plasmas, as well as in ionic liquids and molten salts (*Galvani potential*) [3, 4]. It is also valid for astrophysical applications: phase boundaries in planets and compact stars [5, 6]. It is also valid for simplified Coulomb models [7–9]. The equilibrium potential of a two-phase interface is a thermodynamic quantity: it depends on thermodynamic parameters (bulk properties) of coexisting phases only. In contrast to the electron work function, the Galvani potential does not depend on the properties of the two-phase interface itself: i.e. its form, purity, etc [10].

A remarkable feature of gravitational polarization is that the resulting average electrostatic field *must be* of the *same order* as the gravitational field (counting per one proton). The average electrostatic force must be equal to *one half* of the gravitational force in ideal and non-degenerated isothermal electron–proton plasma of outer layers of a star [11, 12]. The average electrostatic force is supposed to be equal to just *twice* the gravitational force (counting per one proton) in the opposite case of ionic plasma on a strongly degenerated electronic background in compact stars (white dwarfs, neutron stars, etc) [5, 13], etc. Ions in thermodynamically equilibrium MAO are suspended, figuratively speaking, in the electrostatic field of strongly degenerated and weakly compressible electrons. Exact equality ( $F_E^{(p)} = 2F_G^{(p)}$ ) corresponds to zero-order approximation in expansion by a small parameter  $x_m \equiv m_e/m_i$ . This proportionality (*congruence*) of gravitational and average electrostatic fields is not restricted by the condition of strong ionization. The same rule is valid for weakly ionized plasmas. The key (dominating) factors for this ratio are (i) Coulomb non-ideality and (ii) degree of electronic degeneracy.

Real plasmas of compact stars (white dwarfs and neutron stars) are close to isothermal conditions due to high thermal conductivity of degenerated electrons. At the same time, the plasma of ordinary stars, for example, of the Sun, is not isothermal. Temperature profile, heat transfer and thermo-diffusion exist in such plasmas. This should be taken into account self-consistently in the calculation of average electrostatic field.

### 3. Ideal-gas approximation

Plasma polarization at *micro*-level is well known in the classical case as Debye–Hückel screening [15] and in the case of degenerated electrons as Thomas–Fermi screening [16]. Plasma polarization under gravitational forces at *macro*-level is less known although it was claimed [17] and proved during the same years [11, 12]. The average electrostatic potential (*Pannekoek–Rosseland electrostatic field*) was calculated for idealized thermodynamically equilibrium, ideal and non-degenerate, isothermal and electroneutral plasma in the outer layers of normal stars. The exact relation between electrostatic and gravitational forces was obtained for a two-component plasma of electrons and ions of charge  $Z$  and atomic

number  $A$ :

$$F_E^{(p)} = -F_E^{(e)} = -\frac{A}{(Z+1)} F_G^{(p)}, \quad F_E^{(Z)} = -\frac{Z}{(Z+1)} F_G^{(Z)}. \quad (1)$$

Here  $F_E^{(p)}$ ,  $F_G^{(p)}$ ,  $F_E^{(e)}$ ,  $F_E^{(Z)}$ ,  $F_G^{(Z)}$  are the electrostatic and gravitational forces acting on one proton (p), electron (e) and ion (Z). The ideal-gas formula (1) was extended approximately under conditions of dense non-ideal plasma with highly degenerated electrons in interiors of compact stars by Bilsten *et al* ([13, 14], etc). A new small parameter was introduced into consideration:  $x_C$  the ratio of ideal-gas compressibility of electrons and ions at given density. The screening effect proved to be higher under these conditions so that the polarization force compensates (screens) almost totally the gravitation force acting on each ion at  $x_C \ll 1$ . For example, the average electric force acting on an impurity proton is twice higher than the gravitation force under conditions of dense mixture of nuclei  $\{_{16}\text{O}^{8+}, _{12}\text{C}^{6+}, _4\text{He}^{2+}\}$  on a highly degenerate electronic background in the interior of typical white dwarfs (WD). It means that such protons are repelled out from the degenerate WD interior until they reach non-degenerate conditions near the surface (for example [14]),

$$F_E^{(p)} = -F_E^{(e)} \cong -\frac{A}{Z} F_G^{(p)}, \quad F_E^{(Z)} \cong -F_G^{(Z)}. \quad (2)$$

It seems natural to suppose that in the general case the value of the discussed compensation lies between two limits (1) and (2). The point of the present paper is that it is valid for the ideal-gas assumption only (with arbitrary degree of electron degeneracy). In fact, it is not correct if one takes into account non-ideality effects. There may be conditions when polarization force *overcompensates* gravitational force due to an additional non-ideality effect, i.e.  $|F_E^{(Z)}| \geq |F_G^{(Z)}|$  (see below).

#### 4. Gravitational polarization with non-ideality effects

Our goal is to improve the presently existing approach to describing gravitational polarization in a strongly non-ideal plasma, which is typical for planets and compact stars' interior. The approach accepted by [5, 13, 18], etc operates with the idea of individual *partial pressures* for ions and electrons,  $P_i(n_i, T)$  and  $P_e(n_e, T)$ , and based on the solution of several separate ('partial') *hydrostatic equilibrium equations* for each species of particles instead of *unique* hydrostatic equilibrium equation for *total* pressure and total mass density in the standard approach [19, 20]. The point of the present work is that partial pressures and partial hydrostatic equilibrium equations are not well-defined quantities in the general case of an equilibrium non-ideal system.

Let us consider the simplified case of a hypothetical non-uniform self-gravitating body in total thermodynamic equilibrium without relativistic effects and influence of magnetic field. A general approach for the description of thermodynamic equilibrium in this case is multi-component variational formulation of statistical mechanics [21–23]. Thermodynamic equilibrium conditions may be written in three forms: (i) extremum condition for the thermodynamic potential of the total system (free energy functional) regarding variations of one-, two-, three-particle, etc correlations in the system; (ii) constancy conditions for generalized 'electro-chemical' potentials [2] for all species (electrons, ions, etc), and (iii) zero conditions for the sum of (generalized) average forces acting on each species of particles in the system. The problem is that all three values: (total) thermodynamic potential, (partial) electrochemical potentials and (partial) average forces are essentially non-local functionals on

mean-particle correlations. The next (standard) technique is the separation of the main non-local parts of the free energy functional—electrostatic and gravitational energies in mean-field approximation:

$$\begin{aligned}
 F\{T, V, N_j, N_k, \dots\} &= \min F(T, V | [\{n_i(\cdot)\}, \{n_{ij}(\cdot, \cdot)\}]) \\
 &\equiv \min \left( - \sum_{j,k} \frac{G m_j m_k}{2} \int \frac{n_j(\mathbf{x}) \cdot n_k(\mathbf{y})}{|\mathbf{x} - \mathbf{y}|} d\mathbf{x} d\mathbf{y} \right. \\
 &\quad \left. + \sum_{j,k} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{x}) \cdot n_k(\mathbf{y})}{|\mathbf{x} - \mathbf{y}|} d\mathbf{x} d\mathbf{y} + F^*[T, V | \{n_j(\cdot)\}, \{n_{jk}(\cdot, \cdot)\}] \right).
 \end{aligned} \tag{3}$$

Here  $n(\mathbf{x})$  and  $n(\mathbf{x}, \mathbf{y})$  are one- and two-particle densities. It is assumed that all non-local effects are exhausted by the first two terms of the right-hand side of (3). Consequently, the next widely used technique is the ‘local-density’ approximation (4) for the free energy term  $F^*[\dots]$  of the hypothetical non-ideal charge system with extracted electrostatic and gravitational energies in mean-field approximation:

$$\begin{aligned}
 F^*[T, V | \{n(\cdot)\}] &\approx \int_V f^*(n_j(\mathbf{r}), n_k(\mathbf{r}), \dots, T) d\mathbf{r}, \\
 f^*(n_j, n_k, \dots, T) &\equiv \lim_{\substack{(N_j/V) \rightarrow n_j, (N_k/V) \rightarrow n_k, \dots \\ \{N_j, N_k, \dots\} \rightarrow \infty, V \rightarrow \infty}} \left\{ \frac{F^*(N_j, N_k, \dots, V, T)}{V} \right\}.
 \end{aligned} \tag{4}$$

It should be stressed [24, 25] that (local) free energy density  $f^*(n_i, n_k, \dots, T)$  in (4) must be defined as the thermodynamic limit of specific free energy of the (new) *uniform* macroscopic *non-electroneutral* multi-component charge system with charge particle densities  $(n_i, n_k, \dots)$  *on compensating Coulomb (and strictly speaking gravitational) background(s)* [1, 26]. It means that we deal with the free energy  $F^*(\mathbf{N}, T)$  of the artificial system with additional attraction, which could be, formally speaking, thermodynamically unstable under conditions of strong Coulomb non ideality [1], i.e. matrix  $\|\partial^2 F^* / \partial \mathbf{N}^2\| \equiv \|\partial \boldsymbol{\mu}^{(\text{chem})} / \partial \mathbf{n}\|$  could lose its positiveness under strong non-ideality conditions ( $\Gamma \gg 1$ , see below). It should be stressed that it does not mean thermodynamic instability of the *whole* non-uniform Coulomb system (star), which is stabilized in a long-wave limit via the mean-field Coulomb term in (3). Nevertheless artificial short-wave instability still remains in the equilibrium ionic profile  $n_i(\mathbf{r})$  due to the local density approximation (4). It should be suppressed by the addition of the corresponding gradient terms in the free energy density functional  $F^*$  in (3). The discussed artificial short-wave instability could be avoided also in the framework of the local density approximation (4) via a special choice of long-range potential in the mean-field Coulomb term in (3) [27]. Both these tricks are not necessary in the context of the present paper for general illustration of non-ideality influence on plasma polarization in high-gravity astrophysical objects.

A thermodynamic equilibrium condition in the integral form (4) leads to two sets of corresponding local forms in terms of electrochemical potentials (5) and in terms of generalized thermodynamic forces (6):

$$m_j \varphi_G(\mathbf{r}) + q_j \varphi_E(\mathbf{r}) + \mu_j^{(\text{chem})} \{n_i(\mathbf{r}), n_e(\mathbf{r}); T\} = \mu_j^{(\text{el.chem})} = \text{const.} \tag{5}$$

( $j$  = electrons, ions)

$$m_j \nabla \varphi_G(\mathbf{r}) + q_j \nabla \varphi_E(\mathbf{r}) + \nabla \mu_j^{(\text{chem})} \{n_i(\mathbf{r}), n_e(\mathbf{r}); T\} = \nabla \mu_j^{(\text{el.chem})} = 0 \tag{6}$$

( $j$  = electrons, ions).

Here  $\varphi_G(\mathbf{r})$  и  $\varphi_E(\mathbf{r})$  are the gravitational and electrostatic potentials,  $\mu_j^{(\text{chem})}$  and  $\mu_j^{(\text{el.chem})}$  are the local chemical and non-local electrochemical potentials,  $m_j$  and  $q_j$  are the mass and charge of species  $j$  ( $j = i, e$ ),  $\nabla\varphi(\mathbf{r})$ ,  $\nabla\mu(\mathbf{r})$  are spatial gradients. It should be stressed that the set of equations (5) and/or (6) is well-defined equivalents (substitute) for the set of separate equations of hydrostatic equilibrium for the above-mentioned partial pressures and densities of charged species under ideal-gas conditions [11, 12].

Thermodynamic equilibrium conditions in the form (6) with electroneutrality condition lead to the final equation for average electrostatic field in a simplified case of two-component non-ideal electron–ionic plasma with an arbitrary degree of non-ideality and electron degeneracy [26]:

$$Ze\nabla\varphi_E(\mathbf{r}) \left[ 1 + \frac{\Theta}{Z} \right] = -M\nabla\varphi_G(\mathbf{r}) \left[ 1 - m \frac{\Theta}{M} \right], \quad \Theta \equiv \frac{(\mu_{ii}^0 + \Delta_i^i + Z_i \Delta_e^i)}{(Z\mu_{ee}^0 + Z\Delta_e^e + \Delta_i^e)}, \quad (7)$$

$$\Delta_k^j \equiv \left( \frac{\partial \Delta\mu_j^{(\text{chem})}}{\partial n_k} \right)_{T, n_{i \neq k}}, \quad \mu_{jj}^0 \equiv \left( \frac{\partial \mu_j^0}{\partial n_j} \right)_{T, n_{k \neq j}}.$$

Here  $m$ ,  $M$ ,  $Z$  are the masses and charge of electrons and ions.  $\mu_j^0(n_j, T)$  and  $\Delta\mu_j^{(\text{chem})}(n_i, n_e, T)$  are the ideal and non-ideal parts of the *local* chemical potential of species  $j$  ( $j = i, e$ ). Note that for the Coulomb interaction, non-ideal corrections  $\Delta\mu^{(\text{chem})}$  and their derivatives  $\Delta_k^j$  in (7) are negative.

*Comments: thermodynamics.*

- In the ideal-gas approximation equation (7) reproduces both known limits, (1) and (2), with non-degenerated or highly degenerated electrons correspondingly, and gives *monotonic* growth of screening effect from (1) to (2) at intermediate degree of electronic degeneracy:

$$F_E^{(Z)} = -\frac{(1 - x_c m M^{-1})}{(1 + x_c Z^{-1})} F_G^{(Z)} \approx -\left(1 + \frac{x_c}{Z}\right)^{-1} F_G^{(Z)}, \quad (7a)$$

$$x_c(\zeta_e) \equiv \left( \frac{\mu_{ii}^0}{Z\mu_{ee}^0} \right), \quad \zeta_e \equiv n_e \lambda_e^3$$

( $1 \geq x_c \geq 0$  when  $0 \leq \zeta_e \leq \infty$ ). A simplified approximation for  $x_c(\zeta_e)$  could be used for estimations:  $x_c(\zeta_e) \approx (1 + a_1 \zeta + a_2 \zeta^2)^{2/3} (1 + b_1 \zeta + b_2 \zeta^2)^{-1}$  (here  $a_1, a_2, b_1, b_2 \approx 0.213\,14, 0.028\,27, 0.284\,18, 0.047\,12$ ).

In ultrahigh densities ( $\rho \geq 10^6$  g cc<sup>-1</sup>), when the electronic subsystem became relativistic [19, 20], a useful approximation for  $x_c(\zeta_e)$  could be found elsewhere [28].

- In contrast to the ideal-gas approximation (7a) equation (7) in the general case describes equilibrium conditions as competition between not *two*, but *three* sources of influence: gravitation field, polarization field and generalized ‘*non-ideality force*’.
- Coulomb ‘non-ideality force’, when it is taken into account in (7), moves positive ions *inside* the star in addition to gravitation. Hence a ‘non-ideality force’ *increases* the compensating electrostatic field  $\Delta\varphi_E(\mathbf{r})$  in comparison with the ideal-gas approximation (7a).
- In the case of classical (non-degenerated) plasma the function  $\Theta$  in (7) depends on Coulomb non-ideality. In the weak non-ideality limit for two-component electron–ionic ( $Z$ ) plasma it could be described in the Debye–Hückel approximation

$$F_G^{(Z)} \approx -F_E^{(Z)} \left[ 1 + \frac{(1 - Z^2 \Gamma_D/4)}{Z(1 - \Gamma_D/4)} \right], \quad \Gamma_D \equiv (e^2/kT r_D) \ll 1, \quad \zeta_e \equiv n_e \lambda_e^3 \ll 1,$$

$$\{r_D^{-2} \equiv (4\pi e^2(1 + Z^2)/kT)\}. \quad (7b)$$

- It is non-symmetry in thermodynamic properties of electrons and ions which manifests itself in discussing the ‘non-ideality force’. For example, in a *symmetrical* classical (non-degenerated) electron–protonic system Coulomb non-ideality corrections in the numerator and denominator of the non-ideality function  $\Theta$  in (7) cancel each other totally, so that  $\Theta = 1$  and the resulting polarization field is equal to its ideal-gas limit (1) at *any* degree of Coulomb non-ideality [ $F_E^{(p)} = -\frac{1}{2}F_G^{(p)}$ ].
- The non-ideality function  $\Theta$  may be negative and the bracket term  $[1 + (\Theta/Z)]$  in the right-hand side of (7) may be *less* than *unity* in the case when a strongly non-ideal ionic subsystem is combined with highly degenerated and almost ideal electrons (for example, in white dwarfs). In this case one meets ‘*overcompensation*’ when the polarization field could be *higher* (by absolute value) than the gravitation field: i.e.  $|F_E^{(Z)}| > |F_G^{(Z)}|$ . The rough approximation (7c) could be useful. A more accurate approximation for EOS of OCP could be found elsewhere (see for example [29])

$$F_G^{(Z)} \approx -F_E^{(Z)} \left[ 1 - \frac{a_M \Gamma_Z}{Z} x_c(\zeta_e) \right],$$

$$\{\Gamma_Z \equiv Z^2 e^2 (4\pi n_i/3)^{1/3} / kT \gg 1, \zeta_e \equiv n_e \lambda_e^3 \gg 1, a_M \approx 0.4\}. \quad (7c)$$

- Any jump-like discontinuity in a local thermodynamic state, in particular phase transition interface or the set of interfaces between mono-ionic layers with different  $M_i$ ,  $Z_i$  and  $\Gamma_Z$  in neutron star crust [30], leads in the general case to the corresponding jump-like discontinuity in ‘non-ideality force’ in (7) and consequently, to a jump-like discontinuity in the final polarization field  $\Delta\varphi_E(\mathbf{r})$ . It means in its turn appearance of *macroscopic charge* at all discussed mean-phase and mean-layer *interfaces* in addition to electrostatic potential drop (Galvani potential) mentioned at the beginning of the paper.
- Equation (7), which connects polarization field with gravitation and non-ideality forces, could be generalized for the case of ionic mixture ( $Z_1, Z_2, \dots$ ) [27]. In this case average polarization field repels out ions with a smaller ratio  $A/Z$  and pulls inside the ions with a higher ratio  $A/Z$ . In addition, the polarization field pulls inside the ions with higher charge  $Z$  due to non-ideality effects. For example, such repulsion of minor proton impurity from fully ionized helium plasma in the outer layer of white dwarfs and neutron stars prevents proton diffusion and subsequent burning in the deeper layers of a star (see for example [14]).
- Equation (7) is not restricted by spherical symmetry conditions. It is valid for rotating stars and stars in binary systems, etc. It is valid for any self-gravitating system in *total thermodynamic equilibrium* (see the above comment about the non-isothermal state).
- Strong correlation between gravitation and polarization fields (7) is not restricted by a condition of high degree of plasma ionization. Weakly ionized but non-ideal plasmas of outer layers of self-gravitating bodies (planets, stars, etc) must obey the same equations (3)–(7). A multi-component variant of equations (4)–(7) should be used (so-called ‘chemical picture’) in this case and equation (7) will include contributions from neutral-charge interactions, i.e. not just the Coulomb non-ideal terms.

*Comments: hydrodynamics.*

- Plasma polarization in MAO leads to noticeable hydrodynamic consequences.
- Plasma polarization could suppress hydrodynamic instabilities in MAO. For example, plasma polarization could suppress the hypothetical Rayleigh–Taylor instability in the liquid mixture of nuclei  $\{_{16}\text{O}^{8+} + _{12}\text{C}^{6+} + _4\text{He}^{2+}\}$  in the interior of a typical white dwarf in the vicinity of its freezing boundary [31]. Accordingly (7), the polarization field compensates almost totally the gravitation field acting on any nucleus, O, C and

**Table 1.** Estimated parameters of average electrostatic potential of stars.

	Sun $M \equiv M_{\odot}, R \equiv R_{\odot}$	White dwarf $M_{\text{WD}} = M_{\odot}, R_{\text{WD}} = R_{\text{Earth}}$	Neutron star $M_{\text{NS}} = M_{\odot}, R_{\text{NS}} = 10 \text{ km}$
$U_{\text{max}}$ (eV)	1 keV	1 MeV	70 MeV
$E_{\text{max}}$ (V cm <sup>-1</sup> )	$3 \times 10^{-8}$	0.03	150

He, due to their symmetry ( $A/Z = 2$ ) so that the total force is roughly equal to zero: ( $F_{\text{E}}^{(Z)} + F_{\text{G}}^{(Z)} \approx 0$ ). The final weak discrimination in the total force acting on each ion in the mixture  $\{_{16}\text{O}^{8+} + _{12}\text{C}^{6+} + _4\text{He}^{2+}\}$  depends on interplay between Coulomb non-ideality effects and electron degeneracy.

- Besides gravitation any inertial (mass-acting) force (rotation, vibration, inertial expansion (explosion) and compression (collapse), etc) could polarize electron–ionic plasma due to the same reason—the low mass ratio of electrons and ions.
- The effect of the rotation of MAO (including the differential one) could be taken into account naturally in the discussed form of equilibrium conditions (3)–(7). For this purpose rotation energy functional, local centrifugal potential and local centrifugal force should be added to the total free energy functional (3) and should be included in the electrochemical potential (5) and the dynamic equilibrium equation (6) correspondingly. For example, the polarization field should be equal to zero in the case of the rotation limit when the centrifugal force is equal to the gravitational one.
- Any acoustic oscillation slow enough relatively to electronic and ionic relaxation must be accompanied with electron–ionic polarization and consequent electromagnetic oscillation. Hence acoustic properties of a star must include in the general case dependence on the parameters of Coulomb interaction: ionic charge  $Z$ , ratio  $A/Z$ , Coulomb non-ideality parameter, etc.

*Comments: Polarization parameters.*

Proportionality (congruence) of average electrostatic and gravitational fields in a star means that excess charge profile, while being very small [ $Q(\mathbf{r}) \ll \rho(\mathbf{r})$ ], is proportional (congruent) to the density profile of the star [ $Q(\mathbf{r}) \sim \rho(\mathbf{r})$ ]. It gives a simple estimation of contour parameters of the discussed polarization (table 1): the maximal value of electrostatic field  $E_{\text{max}}$  (at the surface) and the maximal value of electrostatic potential  $U_{\text{max}}$  (in the center):

$$E_{\text{max}}(r = R) \cong gm_{\text{p}}/e = (GMm_{\text{p}}/R^2e) \approx 2.85 \times 10^{-8} [M^*/(R^*)^2] \text{ V cm}^{-1}$$

$$U_{\text{max}}(r = 0) \cong gm_{\text{p}}R/2 = (GMm_{\text{p}}/2R) \approx 1 \times 10^3 (M^*/R^*) \text{ eV}.$$

Here  $M^* \equiv M/M_{\odot}$ ;  $R^* \equiv R/R_{\odot}$  ( $M_{\odot} \cong 1.99 \times 10^{33} \text{ g}$ ,  $R_{\odot} \cong 6.96 \times 10^{10} \text{ cm}$  are the mass and radius of the Sun).

One can conclude that contour parameters of the discussed polarization are extremely small for non-exotic MAO (stars, planets, etc) [32]. At the same time they are noticeable and even significant in exotic situation in neutron and combine (*strange*) stars [5, 18], combine (strange) white dwarfs [33] etc.

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