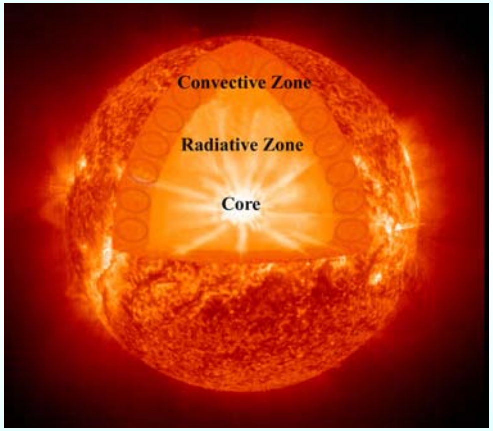
Hungarian Academy of Science
KFKI Institute for Particle and Nuclear Physics
Theoretical Physics Department
April, 2010

Plasma Polarization in Massive Astrophysical Objects



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Moscow Institute of Physics and Technology (State University)

astro-ph:0901.2547

astro-ph:0902.2386

Basic Idea

Gravitation attracts (heavy) ions and does not attracts electrons.
It leads to a small violation of electroneutrality and polarizes plasma in MAO
(*Sutherland, 1903*)

Polarization field compensates (totally or partially) gravitational (and any other mass-acting) force in thermodynamically equilibrium state
(*macroscopic screening*)

Comment: Ions in thermodynamic equilibrium are suspended, figuratively speaking, in electrostatic field of strongly degenerated and weakly compressed electrons

Expected consequences

Polarization **always** accompanies gravitation

Polarization field must be of the **same order** as gravitation field (*per one proton*)

Polarization field must be **congruent** to gravitation field

Any mass-acting force must be accompanied by polarization

- Rotation – centrifugal force F_c ⇔ ($F_E \sim -\alpha F_c$)
- Expansion *or* compression – inertial force F_a ⇔ ($F_E \sim -\alpha F_a$)
- Vibration ⇔ no pure acoustic oscillations ⇔ *(+ electromagnetic oscillations)*

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Basic statement

(*J. Phys. A: Math. & Theor. 2009*)

New "**Coulomb non-ideality force**" is third "**participant**" in competition between gravitation and polarization forces in equilibrium MAO.

In most cases this new "force" **increases** final **electrostatic field** in comparison with that of ideal-gas solution.

astro-ph:0901.2547 arXiv:0902.2386v1

Iosilevskiy I. / Int. Conf. "***Physics of Neutron Stars***", St.-Pb. Russia, 2008

Iosilevskiy I. / Int. Conf. "***Physics of Non-Ideal Plasmas***", Moscow, Russia, 2009

Plasma Screening

(*historical comments*)

Gouy G. *J. Phys. Radium* **9** 457 (**1910**)

Chapman D. *Phil. Mag*. **25** 475 (**1913**)

Micro- & Macro- Screening

Peter Debye

Erich Hückel

Microscopic screening (*ideal plasma*)

Debye - Hückel screening $(n\lambda^3 << 1)$

Thomas - Fermi screening $(n\lambda^3 >> 1)$

$$F_{av}(\mathbf{r}) = F_{ext}(\mathbf{r}) + F_{scr}(\mathbf{r}) \approx F_{ext}(\mathbf{r})\exp\{-r/r_{scr}\} \to 0$$

$$r \to \infty$$

Macroscopic screening (*ideal plasma*)

Pannekoek - Rosseland screening $(n\lambda^3 << 1)$

Bildsten *et al* screening $(n\lambda^3 >> 1)$

$$F_{av}^{(Z)}(\mathbf{r}) = F_{grav}^{(Z)}(\mathbf{r}) - F_{scr}^{(Z)}(\mathbf{r}) \approx 0$$

What is the problem ?

Micro-scopic screening: **-** *Correct screening for* ***non-ideal*** *plasma at* ***micro-*** *level*

Macro-scopic screening: **-** *Correct screening for* ***non-ideal*** *plasma at* ***macro-*** *level*

$$e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{M}\rangle}{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{Z}\rangle} \longleftrightarrow \mathbf{D}_{\mu}^{n} = \left(\mathbf{D}_{\mu}^{n}\right)^{id} + \Delta\mathbf{D}_{\mu}^{n}$$

$\mathbf{D}_{\mu}^{n}$ **- Jacobi matrix** $[\![\delta n_j / \delta\mu_k]\!]_{T,\mu_i(i\neq k)}$ $(j,k = 1,2,3,\ldots)$

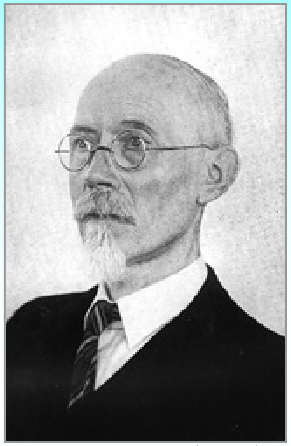
Historical comments

Plasma polarization at micro-level – Debye and Hückel, *Phys. Zeitschr.*, **24**, 8, 1923.

Plasma polarization at macro-level – Pannekoek A. *Bull. Astron. Inst. Neth.*, 1 (1922)

= = «» = = – Rosseland S. *Mon. Roy. Astron. Soc.*, **84**, (1924)

Pannekoek - Rosseland electrostatic field


A. Pannekoek

Application to plasma:

1) - ideal
2) - non-degenerate
3) - equilibrium
4) - isothermal ($T = const$)
5) - electroneutral
{ $n_+(r) = n_-(r)$ }

$$dP_e/dr = -GMm_e n_e/r^2 - n_e eE$$
$$dP_i/dr = -GMm_i n_i/r^2 + n_i qE$$

M – mass of the Sun,
G – gravitational constant,
m_e, m_p – electron & proton masses

$$F_E^{(p)} = -(1/2)F_G^{(p)}$$

$$F_E^{(e)} = +(1/2)F_G^{(p)}$$


N. Bohr & S. Rosseland

Generalization to ideal plasma of ions (A,Z) and electrons

$$F_E^{(p)} = -\frac{A}{(Z+1)}F_G^{(p)}$$

$$F_E^{(Z)} = -\frac{Z}{(Z+1)}F_G^{(Z)}$$

(*) $F_E^{(p)}$, $F_G^{(p)}$, $F_E^{(Z)}$, $F_G^{(Z)}$, - electrostatic and gravitational forces acting on one proton (p) and ion (A,Z)

Extension to the strongly degenerated plasma

The model of **L. Bildsten** *et al.* (2001 – 2007)

L. Bildsten & D. Hall //*Ap.J.*, 549: (2001) *Gravitational settling of ^{22}Ne in liquid white dwarf interior*
P. Chang & L. Bildsten // *Ap.J.*, 585 (2003) *Diffusive nuclear burning in neutron star envelopes*

$$\frac{dP_e}{dr} = -n_e(r)\{m_e g(r) + eE\}$$

$$\frac{dP_i}{dr} = -n_i(r)\{A_i m_p g(r) - Z_i eE\}$$

1) - **ideal**
2) - **strongly degenerated**
3) - isothermal $(T = const)$
4) - electroneutral
$\{ n_+(r) = n_-(r) \}$
- - - - - - - - - - - - - - - - -
5) - equilibrium

The SUN

$(p^+ + e^-)$

$$F_E^{(p)} \approx -(1/2)F_G^{(p)}$$

White Dwarf

$({}_{16}O^{8+},\ {}_{12}C^{6+},\ {}_{4}He^{2+})$

$$F_E^{(p)} \approx -2F_G^{(p)}$$

$$\boldsymbol{F}_E^{(Z)} \approx -\boldsymbol{F}_G^{(Z)}$$

Accuracy ~ small parameter x_c

$$x_c \equiv \left(\frac{\partial n_e}{\partial p_e}\right)_T \bigg/ \left(\frac{\partial n_i}{\partial p_i}\right)_T$$

NB !

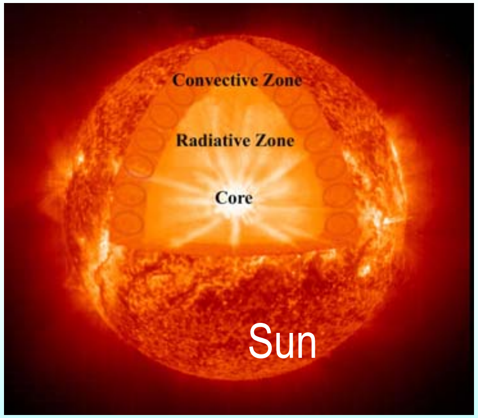
- Average electrostatic field must be of the *same order* as gravitational one*

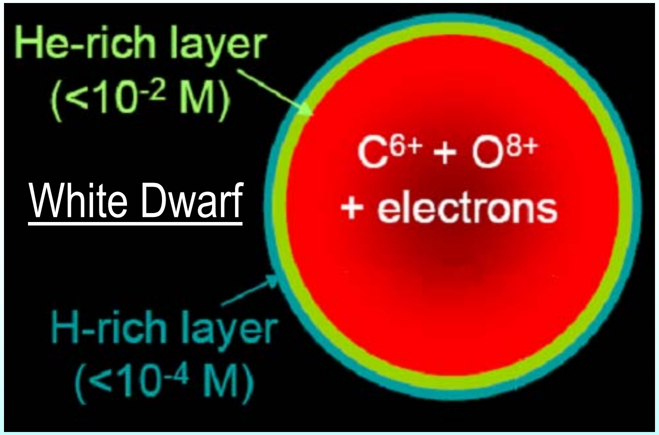
(* - counting per one proton)

Question: *(Bally & Harrison, 1978)*

? - Do both limiting cases (*ideal non-degenerate and degenerate electrons*) **restrict interval** of **possible *ratio*** of ***gravitational*** and ***electrostatic*** **forces - ?**

$$F_E^{(p)} = -(1/2)F_G^{(p)}$$ ← **?** → $$F_E^{(p)} = -2F_G^{(p)}$$





Answer:

! Yes : - if one takes into account the **electron degeneracy** only **!**

! No : - if one takes into account **non-ideality** effects additionally **!**

(*see below*)

It may be $| F_E^{(p)} / F_G^{(p)} | \geq 2$ i.e. $| F_E^{(Z)} / F_G^{(Z)} | \geq 1$

(“*Overcompensation*”)

Iosilevskiy I. “***Physics of NS***”, S-Pb. Russia, 2008

***J. Phys. A*, 42**, 2009 // astro-ph:0901.2547

Historical comments

Plasma polarization at micro-level – Debye and Hückel, *Phys. Zeitschr.*, **24**, 8, 1923.

Plasma polarization at macro-level – Pannekoek A., *Bull. Astron. Inst. Neth.*, 1 (1922)

== « » == Rosseland S. *Mon. Roy. Astron. Soc.*, **84**, (1924)

Mr. S. Rosseland, Electrical State of a Star.

Application to plasma:
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$\{ n_+(r) = n_-(r) \}$

5) - equilibrium

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M – mass of the Sun,
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Macroscopic screening *in MAO*

J.Bally & E.Harrison, *Astrophys. Journal, 220, 1978*

The Electrically Polarized Universe

THE ELECTRICALLY POLARIZED UNIVERSE

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Department of Physics and Astronomy, University of Massachusetts

Received 1977 September 8; accepted 1977 September 22

ABSTRACT

It is shown that all gravitationally bound systems—stars, galaxies, and clusters of galaxies—are positively charged and have a charge-to-mass ratio of ~100 coulombs per solar mass. The freely expanding intergalactic medium has a compensating negative charge. The immediate physical consequences of an electrically polarized universe are found to be extremely small.

Subject headings: cosmology — galaxies: intergalactic medium — hydromagnetics

Eddington (1926; see also Rossland 1924) showed in *The Internal Constitution of the Stars* that a star has an internal electric field

$$-\nabla\phi = \alpha(m_p/e)\nabla\psi\,, \qquad (1)$$

where ϕ is the electrical potential, ψ is the gravitational potential, m_p is the mass, and e is the charge of a proton. For a nondegenerate electron gas

$$\alpha = \sum n_i A_i \Big/ \sum n_i(1 + Z_i)$$

where the summations are over ion spec n_i, atomic weight A_i, and effective char fully ionized gas of arbitrary compositi $\frac{1}{2} \leq \alpha \leq 2$. When radiation pressure degeneracy are included, α has similar general $\alpha \sim 1$.

From the divergence of equation (1) i

$$\sigma/\rho = G\alpha m_p/e\,,$$

where σ is the positive gravitationally i density and ρ is the mass density. For a star of total charge Q and mass M the charge-to-mass ratio is

$$Q/M = G\alpha m_p/e\,, \qquad (4)$$

and with $\alpha \sim 1$, is of order 100 coulombs per solar mass. This positive charge exists because electrons, despite their low mass, contribute substantially to the pressure, and an electric field is therefore needed to hold in the electron gas. In effect, some electrons escape (most electrons have velocities exceeding the escape velocity), and the remaining electrons are retained by the positively charged star.

It has previously seemed reasonable to suppose that the positive charge within a star is screened by a negatively charged atmosphere containing the expelled electrons. It can be shown, however, that screening occurs in the atmosphere only when the scale height is less than a Debye length.

By allowing for the difference in charge densities in the hydrostatic equations, we find

$$\nabla^2\sigma = -\lambda_D^{-2}(\sigma - G\alpha m_p/e)\,, \qquad (5)$$

in place of equation (3), where

$$\lambda_D = (kT/4\pi n_e e^2)^{1/2} \sim 10(T/n_e)^{1/2}\ \text{cm}\,, \qquad (6)$$

is the Debye length and n_e is the electron density in a gas of temperature T. Thus, if L is a scale height, and $\nabla^2 \sim L^{-2}$, then equation (3) is recovered whenever $\lambda_D \ll L$. The charge density σ can only become negative in tenuous outer regions of a stellar atmosphere

pared with the Debye length of their interstellar media.

Our equations neglect—among other things—rotational inertial forces and are therefore not correct for rotationally supported gaseous systems. The charge-to-mass ratio of equation (4) does apply, however, to spiral galaxies in which the interstellar gas accounts for only a small fraction of their total mass.

Possibly most galaxies are me tionally bound clusters. Since the galaxies is larger than the Debye l cluster medium (for all conceivable and temperatures), it follows that a also a charge-to-mass ratio given b

All gravitationally bound system and clusters of galaxies—are posit the freely expanding intergalactic clusters of galaxies contains the exp is therefore negatively charged. S Sun have center-to-surface poten $\sim 10^3$ V, giant galaxies have poten $\sim 10^9$ V, and rich clusters such as have potential differences of $\sim 10^{1}$

All gravitationally bound systems—stars, galaxies, and clusters of galaxies—are positively charged, and the freely expanding intergalactic medium between clusters of galaxies contains the expelled electrons and is therefore negatively charged.

two examples illustrate how small are the physical consequences of an electrically polarized universe.

Blackett (1947) advanced the hypothesis that all massive rotating bodies have magnetic moments of

$$P = \beta G^{1/2} J/c\,, \qquad (7)$$

where J denotes angular momentum, c is the speed of light, and β is a dimensionless constant of order unity. In Blackett's words: "It is suggested tentatively that the balance of evidence is that the above equation represents some new and fundamental property of rotating matter." It is now known that numerous astronomical objects (planets, magnetic variable stars, pulsars, etc.) do not obey equation (7) with $\beta \sim 1$. All gravitationally bound systems, however, having the

generating seed magnetic fields (Harrison 1970, 1973).

Two charged stars in orbit about each other emit electromagnetic radiation; and if they have different charge-to-mass ratios denoted by α_1, and α_2, then

$$L_{EM}/L_G \sim (\alpha_1 + \alpha_2)^2\beta^2 \sim 10^{-36}\,, \qquad (9)$$

where L_{EM} is the magnetic dipole radiation luminosity and L_G is the gravitational radiation luminosity. In the case of electric dipole radiation

$$L_{EM}/L_G \sim (\alpha_1 - \alpha_2)^2\beta^2(cP/a)^2\,, \qquad (10)$$

where P is the orbital period and a is the separating distance of the two stars. It is again apparent that the results derived are of no astrophysical importance.

The picture presented consists of positively charged stronomical systems embedded in an intergalactic sea f negative charge. It provides a theoretical basis for lackett's hypothesis, although the magnetic fields re much weaker than Blackett anticipated. We find he picture of an electrically polarized universe ntriguing, and yet, rather surprisingly, we have so far iled to discover any physically significant effects of mmediate consequence.

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arrison, E. R. 1970, *M.N.R.A.S.*, **147**, 279.
——. 1973, *M.N.R.A.S.*, **165**, 185.
ossland, S. 1924, *M.N.R.A.S.*, **84**, 308.

JOHN BALLY and E. R. HARRISON: University of Massachusetts, Department of Physics and Astronomy, GR Tower B, Amherst, MA 01002

. . . . We find the picture of an electrically polarized universe intriguing, and yet, rather surprisingly, we have so far failed to discover any physically significant effects of immediate consequence.

Application - I

Electrostatics of a star

Proportionality (congruence) of average electrostatic and gravitational potentials

Excess charge profile in a star is similar (*proportional*) to their density profile

$$Q(\mathbf{r}) \sim \rho(\mathbf{r})$$

NB ! $Q(\mathbf{r}) <<< \rho(\mathbf{r})$

Primitive estimation:

– Maximal value of electrostatic field (*at the surface*) – $E_{max}(r = R)$

– Maximal value of electrostatic potential (*in the centre*) – $U_{max}(r = 0)$

$$E_{max} \cong gm_p/e = (GMm_p/R^2e) \approx 2.85 \cdot 10^{-8} \cdot [M^*/(R^*)^2] \text{ V/cm}$$

$$U_{max} \cong gR/2 = (GMm_p/2R) \approx 1 \cdot 10^3 (M^*/R^*) \text{ eV}$$

$M^* \equiv M/M_☼$; $R^* \equiv R/R_☼$

$M_☼ \cong 1.99 \cdot 10^{33}$ g. $R_☼ \cong 6.96 \cdot 10^{10}$ cm

-- mass and radius of the Sun

Electrostatic potential parameters:

	SUN $M \equiv M_☼$ $R \equiv R_☼$	**White Dwarf** $M_{WD} = M_☼$ $R_{WD} = R_{Earth}$	**Neutron Star** $M_{NS} = M_☼$ $R_{NS} = 10$ km
U_{max} [eV]	1 keV	1 MeV	70 MeV
E_{max} [V/cm]	$3 \cdot 10^{-8}$	0.03	150

Widely used approach *(standard)*

From unique equation of hydrostatic (i.e. mechanical) **equilibrium of electro-neutral matter in gravitational field . . .**

$$\frac{dP_\Sigma}{dr} = -\{n_e(r)m_e + n_i(r)m_i\}g(r) = -\rho(r)g(r)$$

. . . to the set of separate equations of hydrostatic equilibrium for each charged specie *(in terms of partial pressures)*

?

$$\frac{dP_e}{dr} = -n_e\{m_e g(r) + eE\}$$

$$\frac{dP_i}{dr} = -n_i\{A_i m_p g(r) - Z_i eE\}$$

What is non-correct ?

NB !

– ***partial pressures*** and ***separate equations of "hydrostatic" equilibrium*** are ***not well-defined*** quantities in ***non-ideal*** plasmas of compact stars

What should be done instead ?

Quasi-stationary state in non-ideal self-gravitating body

(*the problem in general*)

Joint self-consistent *description of* ***thermodynamics*** *and* ***kinetics*** *for* **heat, mass** *and* **impulse transfer** (*diffusion, thermo-conductivity and equation of state*)

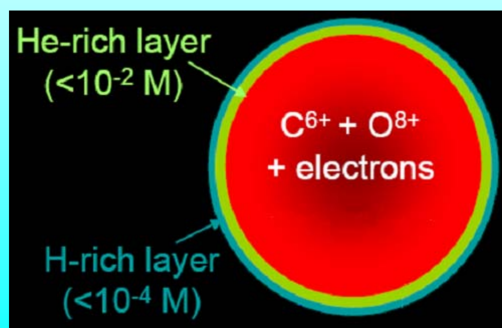
$$e\mathbf{E} + m_e\mathbf{G} + \nabla\mu_e + c_T\nabla T = 0$$

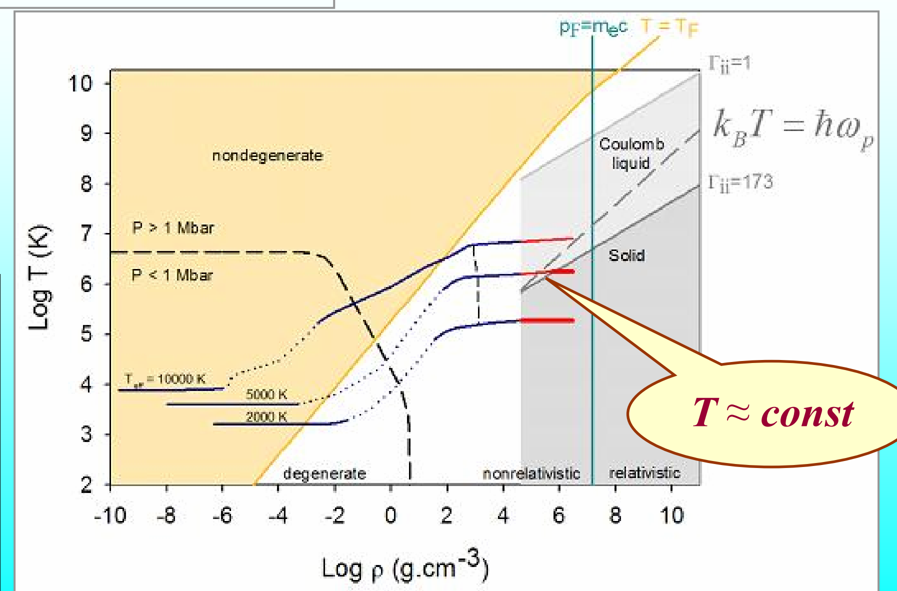
Simplified case

- **Total thermodynamic equilibrium** (T = const)

- **No influence of magnetic field**
- **No relativistic effects**
- **No energy loss or deposition**

for example:
White Dwarfs





General approach

Variational formulation of equilibrium statistical mechanics

C. De Dominicis,1962 // Hohenberg & Kohn,1964 // R. Evans,1979 etc..

NB !
- ***three small parameters***

$$x_m \equiv (m_e / m_i)$$

$$x_c \equiv \left(\frac{\partial n_e}{\partial p_e}\right)_T^{id} \Big/ \left(\frac{\partial n_i}{\partial p_i}\right)_T^{id}$$

$$\alpha \equiv \frac{Gm^2}{e^2} \sim 10^{-36}$$

NB !
- ***two large parameters***
  - Range of Coulomb forces
  - Range of gravitational forces

Standard trick to avoid Coulomb singularities:
- Start with Yukawa: $V(r) = (A/r)\exp\{-r/\lambda\}$
- Solution of many-body problem
- Range of interaction is tended to infinity ($\lambda \to \infty$)

$$V(r) = \frac{A}{r}\exp(-r/\lambda)$$

Integral form of thermodynamic equilibrium conditions

Variational formulation (*multi-component version*)

$$F = \min \mathbf{F}\left[T, V, \{N\} \,|\, \{n_j(\mathbf{r})\} : \{n_{jk}(\mathbf{r},\mathbf{r}')\} \ldots\right]^{\{T=const,\, N_k=const\}}_{V_1(\mathbf{r}), V_{1,2}(\mathbf{r},\mathbf{r}'), V_{1,2,3}(\mathbf{r},\mathbf{r}',\mathbf{r}'')\ldots=const}$$

The main problem – ***strong non-locality*** of the **free energy functional** due to **long-range nature** of **Coulomb** and **gravitational interaction**

Standard: separation of main non-local parts.

$$F\{T, V(r) / \left[\{n_i(\cdot)\} / \{n_{ij}(\cdot,\cdot)\}\right] \equiv$$

$$\equiv -\sum_{jk} \frac{G m_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + F^*\left[\{n_i(\cdot)\} // \{n_{ij}(\cdot,\cdot)\}\right]$$

NB! The rest $F^*\{\ldots\}$ is the *free energy* of *new* system on *compensating background(s)*

It's assumed that the rest free energy functional $F^*[n_i // n_{ij}]$ is *weakly non-local*

Hence *weakly non-local chemical potentials*: $\mu_j^{(chem)}$ - could be introduced

$$\mu_j^{(chem)} \equiv \left(\delta F^*[\cdots] / \delta n_j(\cdot)\right)_{T, n_{k \neq j}}$$

Local forms of thermodynamic equilibrium conditions

Heat exchange:

$$T(\mathbf{r}) = \text{const}$$

Impulse exchange:

$$\nabla P_\Sigma = -\rho(\mathbf{r})\nabla\varphi_G(\mathbf{r})$$

Particle exchange:

In terms of potentials

Constance of total (generalized) *electro-chemical* potential

$$m_j\varphi_{\mathrm{G}}(\mathbf{r}) + q_j\varphi_{\mathrm{E}}(\mathbf{r}) + \mu_j^{(\mathrm{chem})}\{n_{\mathrm{i}}(\mathbf{r}),\, n_{\mathrm{e}}(\mathbf{r}),\, \{n_{jk}(\mathbf{x},\mathbf{y})\}\ T\} = \text{const}$$

(j,k = electrons, ions)

In terms of forces

Balance of forces including generalized "*non-ideality*" force

$$m_j\nabla\varphi_{\mathrm{G}}(\mathbf{r}) + q_j\nabla\varphi_{\mathrm{E}}(\mathbf{r}) + \nabla\mu_j^{(\mathrm{chem})}\{n_{\mathrm{i}}(\mathbf{r}),\, n_{\mathrm{e}}(\mathbf{r}),\, \{n_{\mathrm{jk}}(\mathbf{x},\mathbf{y})\}\ T\} = 0$$

(j,k = electrons, ions)

$\varphi_{\mathrm{G}}(\mathbf{r})$ и $\varphi_{\mathrm{E}}(\mathbf{r})$ – *gravitational and electrostatic potentials*

NB !

The set of equations for *electro-chemical potentials* instead of the set of separate equations of "hydrostatic" equilibrium for partial pressures !

Quickly rotating star

(*centrifugal force addition*)

Constance of total (**generalized**) ***electro-chemical*** **potential**

$$m_j \{\varphi_G(\mathbf{r}) + \varphi_C(\mathbf{r})\} + q_j \varphi_E(\mathbf{r}) + \mu_j^{(chem)}\{n_i(\mathbf{r}), n_e(\mathbf{r}), \{n_{jk}(\mathbf{x},\mathbf{y})\}\ T\} = \mathrm{const}$$

(j,k = electrons, ions)

Balance of **forces** including **generalized** "***non-ideality***" **force**

$$m_j\{\nabla\varphi_G(\mathbf{r}) + \nabla\varphi_C(\mathbf{r})\} + q_j\nabla\varphi_E(\mathbf{r}) + \nabla\mu_j^{(chem)}\{n_i(\mathbf{r}), n_e(\mathbf{r}), \{n_{jk}(\mathbf{x},\mathbf{y})\}\ T\} = 0$$

(j,k = electrons, ions)

$\varphi_G(\mathbf{r})$, $\varphi_C(\mathbf{r})$ *and* $\varphi_E(\mathbf{r})$ – *gravitational, centrifugal and electrostatic potentials*

Polarization field should be equal to zero in the case of the rotation limit when the centrifugal force is equal to the gravitational one.

$$F = \min F\left(T, V, \{N_k\} / \{n_i(\cdot)\} / \{n_{ij}(\cdot,\cdot)\}\right) \equiv$$

$$\equiv - \sum_{jk} \frac{G m_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + F^*\left[\{n_i(\cdot)\} / \{n_{ij}(\cdot,\cdot)\}\right]$$

NB! *Extremely low* strength of *gravitational* interaction in comparison with *Coulomb* one

small parameter !

$$\alpha \equiv \frac{G m^2}{e^2} \sim 10^{-36}$$

Even *extremely small* deviation from electroneutrality in *Coulomb term* leads to significant *energy variation* in free energy functional

Thermodynamically equilibrium star is electroneutral *almost everywhere*

Extremely small but *non-zero* violation of global electroneutrality !

Total charge disbalance - ΔQ

$\Delta Q \sim \alpha N_{\Sigma}^{barion}$ $\quad N_{\Sigma}^{barion} \approx 10^{57}$ $\quad \Delta Q \sim \alpha \cdot 10^{57} \approx \left(10^{21} - 10^{22}\right) e$ $\quad \approx 100$ Q

NB! Deviation from electroneutrality *must not* be uniform *totally everywhere*

Exceptions: - discontinuity surfaces

(*phase boundaries, jump-like change in ionic composition etc.*)

Macroscopic Screening in Non-Ideal Plasma

In electroneutrality regions one obtains:

$$e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{M}\rangle}{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{Z}\rangle}$$

Here:

$\mathbf{D}_{\mu}^{n}(\mathbf{r})$ matrix $\Leftrightarrow$ $\{\delta\mathbf{n}(\mathbf{r})/\delta\boldsymbol{\mu}(\mathbf{r}')\}_T \equiv [\![\delta n_j(\mathbf{r})/\delta\mu_k(\mathbf{r}')]\!]_{T,\mu_i(i\neq k)} \equiv [\![\delta^2 F^*/\delta\mu_j(\mathbf{r})\delta\mu_k(\mathbf{r}')]\!]_{T,\mu_i(i\neq k)}$

$\langle\mathbf{Z}| \equiv \{Z_j\}$

$|\mathbf{M}\rangle \equiv \{M_j\}$

$\mathbf{D}_{\mu}^{n}$ **is inverse matrix to:** $\mathbf{D}_{n}^{\mu} \equiv [\![\delta^2 F^*/\delta n_j(\mathbf{r})\delta n_k(\mathbf{r}')]\!]_{T,n_i(i\neq k)}$

$$\mathbf{D}_{\mu}^{n} * \mathbf{D}_{n}^{\mu} = \mathbf{E}$$

Non-ideality effects $\Leftrightarrow$ $\mathbf{D}_{\mu}^{n} = \left(\mathbf{D}_{\mu}^{n}\right)^{id} + \Delta\mathbf{D}_{\mu}^{n}$

$$e\nabla \varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla \varphi_{\mathrm{G}}(\mathbf{r}) \frac{\langle \mathrm{Z} | \mathbf{D}_{\mu}^{n} | \mathrm{M} \rangle}{\langle \mathrm{Z} | \mathbf{D}_{\mu}^{n} | \mathrm{Z} \rangle}$$

Does not restricted by:

Spherical symmetry condition
Nomenclature of ions
Degree of ionization
Degree of Coulomb non-ideality
Degree of electronic degeneracy
....

NB! *Matrix* $\mathbf{D}_{\mu}^{n}$ *is still non-local*

$$F = \min F\left(T, V, \{N_k\}/\{n_i(\cdot)\}/\{n_{ij}(\cdot,\cdot)\}\right) \equiv$$

$$\equiv -\sum_{jk} \frac{G m_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + F^*\left[\{n_i(\cdot)\}/\{n_{ij}(\cdot,\cdot)\}\right]$$

"Quasi-uniformity" approximation

$$F = \min F\left(T, V, \{N_k\}/\{n_i(\cdot)\}\right) \equiv$$

$$\equiv -\sum_{jk} \frac{G m_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \int f^*\left(\{n_i(\mathbf{r}) \ldots n_k(\mathbf{r})\}\right) d\mathbf{r}$$

μ *is a function, not functional !*

$$\mu_j^{(chem)}(\mathbf{r}) \equiv \left(\partial f^*[T, \{n_k(\mathbf{r})\}]/\partial n_j\right)_{T, n_{k \neq j}}$$

$$f^*(\{n\}) \equiv \lim \left\{ \frac{F(N_i \ldots N_k, V, T)}{V} \right\}_{\{N_k\}, V \to \infty}^{N_k/V \to n_k}$$

In terms of potentials

$$m_j \varphi_G(\mathbf{r}) + q_j \varphi_E(\mathbf{r}) + \mu_j^{(chem)}[\{n_k(\mathbf{r})\}, T] = \text{const} \qquad (j,k = \text{electrons, ions})$$

In terms of forces

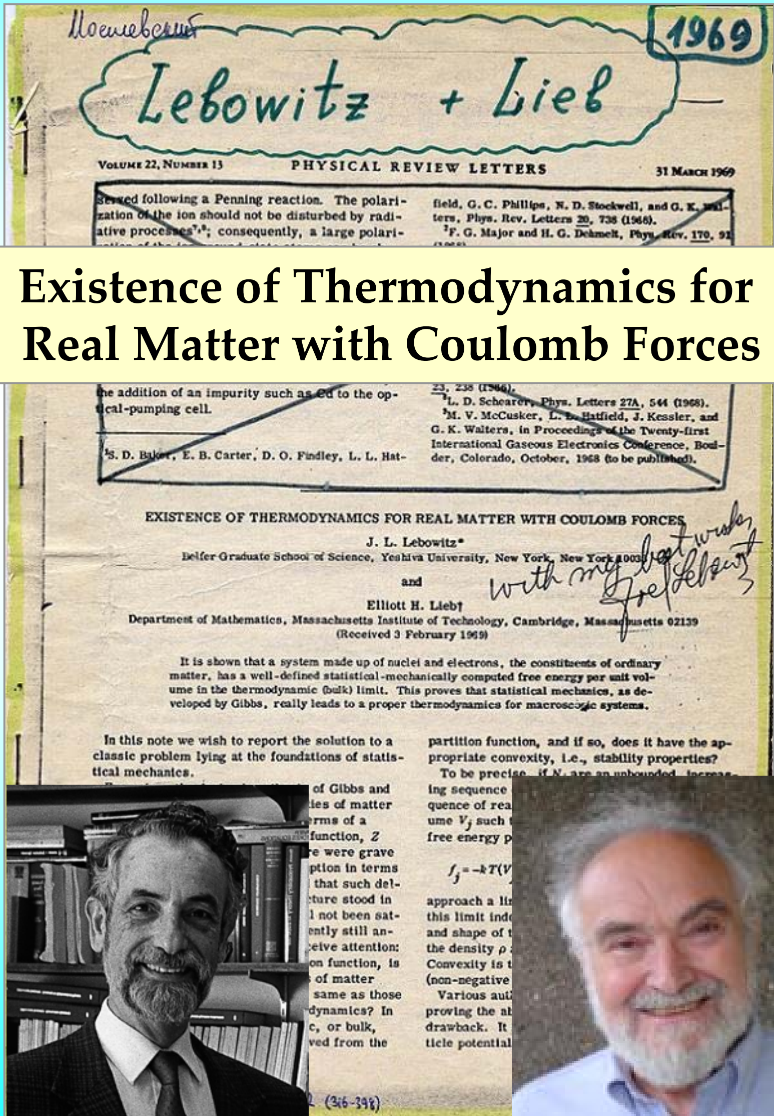
$$m_j \nabla\varphi_G(\mathbf{r}) + q_j \nabla\varphi_E(\mathbf{r}) + \nabla\mu_j^{(chem)}[\{n_k(\mathbf{r})\}, T] = 0 \qquad (j,k = \text{electrons, ions})$$

NB! The *local* free energy density $f^*(\{n\})$ must be defined for *non-electroneutral* densities $\{n_k\}$

The problem of thermodynamic limit in Coulomb system

Lebowitz J.L. & Lieb E.H. *PRL,* **22** 631 (1969)

$$f^*(\{n\},T) \equiv \lim \left\{ \frac{F(N_i \ldots N_k, V, T)}{V} \right\}_{\{N_k\}, V \to \infty}^{N_k/V \to n_k}$$



Existence of Thermodynamics for Real Matter with Coulomb Forces

Elliot Lieb

Joel Lebowitz

Thermodynamic limit strongly depends on disbalance of net electric charge

$Q \to 0$ | $Q \sim N^{\varepsilon\,(<2/3)}$ | $Q \sim N^{\varepsilon\,(>2/3)}$

Could be avoided in

Electroneutral

Grand Canonical Ensemble

Macroscopic Screening in Non-Ideal Plasma

$$e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle \mathrm{Z}|\mathbf{D}^{n}_{\mu}|\mathrm{M}\rangle}{\langle \mathrm{Z}|\mathbf{D}^{n}_{\mu}|\mathrm{Z}\rangle}$$

In "quasi-uniformity" approximation

Here:

$\mathbf{D}^{n}_{\mu}(\mathbf{r})$ matrix $\Leftrightarrow$ $\{\partial\mathbf{n}(\mathbf{r})/\partial\boldsymbol{\mu}(\mathbf{r}')\}_T \equiv [\![\partial n_j(\mathbf{r})/\partial\mu_k(\mathbf{r}')]\!]_{T,\mu_i(i\neq k)}$

$\langle\mathbf{Z}| \equiv \{Z_j\}$

$|\mathbf{M}\rangle \equiv \{M_j\}$

$\mathbf{D}^{n}_{\mu}$ **is inverse matrix to:** $\mathbf{D}^{\mu}_{n} \equiv [\![\partial^2 F^*/\partial n_j(\mathbf{r})\partial n_k(\mathbf{r}')]\!]_{T,n_i(i\neq k)}$

$$\mathbf{D}^{n}_{\mu} * \mathbf{D}^{\mu}_{n} = \mathbf{E}$$

Non-ideality effects $\Leftrightarrow$ $\mathbf{D}^{n}_{\mu} = \left(\mathbf{D}^{n}_{\mu}\right)^{id} + \Delta\mathbf{D}^{n}_{\mu}$

Details of Variational Procedure

$$F = \min F\left(T, V, \{N_j\}/\{n_i(\cdot)\}/\{n_{ij}(\cdot,\cdot)\}\right) \equiv$$

$$\equiv -\sum_{jk} \frac{G m_j m_k}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\mathbf{r}) \cdot n_k(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + F^*\left[\{n_i(\cdot)\}/\{n_{ij}(\cdot,\cdot)\}\right]$$

Dilemma: ***Physical*** *or* ***Chemical*** representation **?**

Physical *picture* ⇔ ***Chemical*** *picture*

Basic Units →

Nuclei *and* **electrons**

Atoms, molecules . . .
free ions *and* **free electrons**

Planets, BD, ...

$H^+ + He^{++} + e^{(-)}$

$H + H_2 + H^{(-)} + H_2^+ + H^+ +$
$He + He^+ + He^{++} + \ldots + e^{(-)}$

NB !

In each point Saha-like equations are valid !

$AB \Leftrightarrow A + B$ → $\mu_{AB}(\mathbf{r}) = \mu_A(\mathbf{r}) + \mu_B(\mathbf{r})$

Saha-like equations for local parameters

Dilemma: *Physical* or *Chemical* representation ?

Physical *picture* ⇔ ***Chemical*** *picture*

Nuclear Plasma

Basic Units → ***n*, *p*,** *and* **electrons** | ***n**, *p**, *N(A,Z)*** *and* **electrons***

"Free" neutrons, protons
and their "**clusters**"

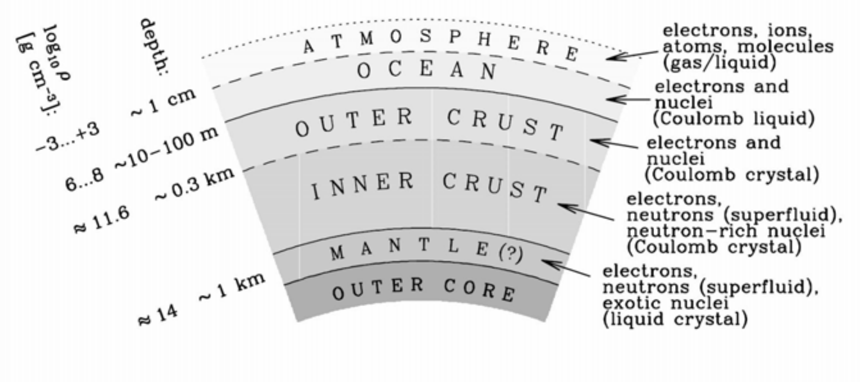


Figure 2.1. Schematic structure of an envelope of a neutron star with the internal temperature $\sim 10^8$ K (see text for more details).

Haensel P., Potekhin A., Yakovlev D.
Neutron Stars, Springer, New York, 2007

Typel S., Roepke G., Klahn T., Blaschke D., and Wolter H. arXiv:0908.2344v1

$$N(A,Z) \Leftrightarrow Zp + (A-Z)n$$

↓

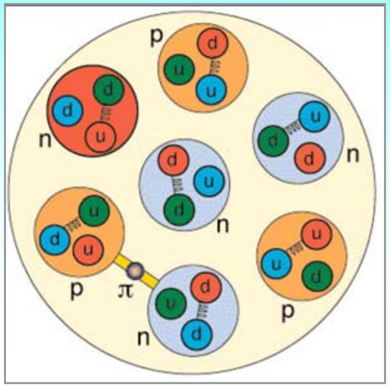
Saha-like equations are valid !

$$\mu_{N(A,Z)}(\mathbf{r}) = Z\mu_p(\mathbf{r}) + (A-Z)\mu_n(\mathbf{r})$$

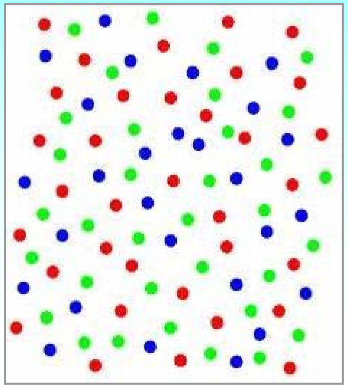
Dilemma: *Physical* or *Chemical* representation ?

Physical picture ?

Strange (hybrid) stars



⇔



u, d, s, p, n, e	$\mu_u, \mu_d, \mu_s, \mu_p, \mu_n, \mu_e$

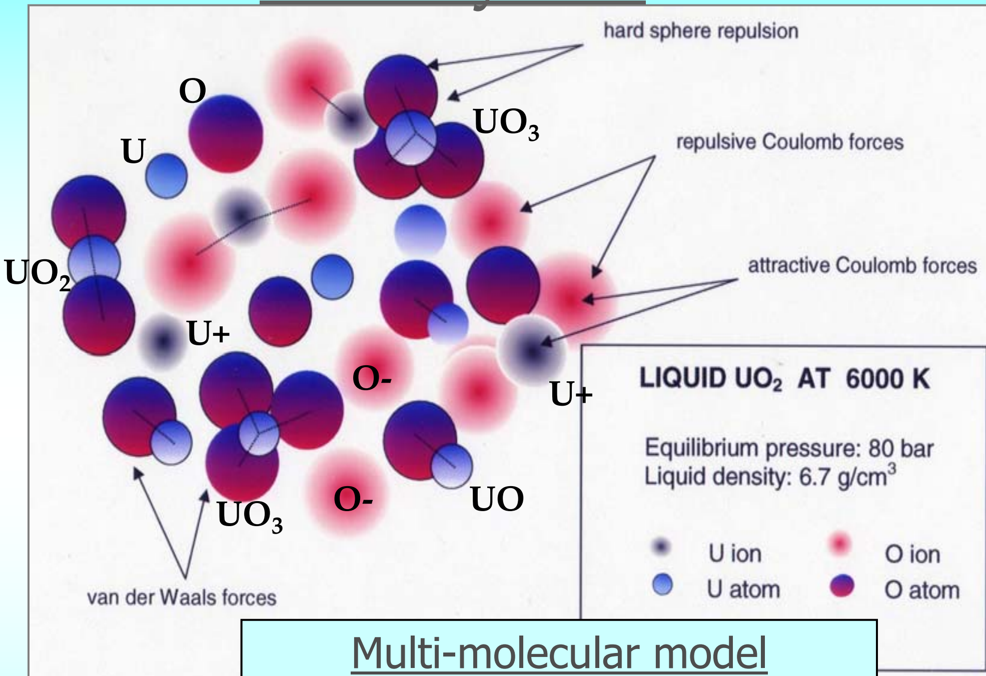
u + e ⇔ d
d ⇔ s
p + e ⇔ n
n ⇔ u + 2d
(p ⇔ 2u + d)

$$\mu_u + \mu_e = \mu_d,$$
$$\mu_d = \mu_s,$$
$$\mu_p + \mu_e = \mu_n \equiv \mu_B,$$
$$\mu_n = \mu_u + 2\mu_d,$$
$$(\mu_p = 2\mu_u + \mu_d).$$

Endo T., Maruyama T., Chiba S., Tatsumi T.
astro-ph/0601017v1/ 2006 /

Chemical picture

U – O system



Multi-molecular model
(*Liquid & Gas*)

$U + O + O_2 + UO + UO_2 + UO_3$

$U^+ + UO^+ + UO_2^+ + O^- + UO_3^- + e^-$

$U + 2O \Leftrightarrow UO_2$
$2O \Leftrightarrow O_2$
$U^+ + e \Leftrightarrow U$
$UO_3 + e \Leftrightarrow UO_3^-$
. . . . . .

$\mu_U + 2\mu_U = \mu_{UO2}$
$2\mu_O = \mu_{O2}$
$\mu_{U+} + \mu_e = \mu_U$
$\mu_{UO3} + \mu_e = \mu_{UO3-}$
. . . .

Iosilevskiy I. / "*Physics of Neutron Stars*", S-Pb. Russia, 2008

$$e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{M}\rangle}{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{Z}\rangle}$$

Simplified cases:

- Ideal-mixture approximation

(*multi-component "chemical picture*")

- Classical weakly non-ideal plasma

(*Debye approximation in Grand Canonical Ensemble*)

- Strongly non-ideal ionic mixture on strongly degenerated weakly non-ideal electrons

(*switching-off the electron-ionic correlations*)

- Two-component electron-ionic system with arbitrary degree of degeneracy and non-ideality

(*strongly correlated system*)

Ideal-mixture approximation

$$\mathbf{D}^n_\mu = \left(\mathbf{D}^n_\mu\right)^{id}$$

(*chemical picture: - a, b, ab, ab$_2$, a$_2$b, . . . a$_n$b$_m$*)

$$e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle \mathrm{Z}|\mathbf{D}^n_\mu|\mathrm{M}\rangle}{\langle \mathrm{Z}|\mathbf{D}^n_\mu|\mathrm{Z}\rangle} \Leftrightarrow e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{M}}(\mathbf{r})\frac{\left(\sum_j \tilde{n}_j M_j Z_j\right)}{\left(\sum_j \tilde{n}_j Z_j^2\right)}$$

$$\langle\mathbf{Z}| \equiv \{Z_j\} \qquad |\mathbf{M}\rangle \equiv \{M_j\}$$

$$\tilde{n}_j \equiv kT\left(\partial n_j/\partial\mu_j\right)^{id.gas}_{T,n_{k\neq j}} \quad (j = 1,2,3\ldots)$$

$$\tilde{n}_e \rightarrow 0 \qquad \left(n_e\lambda_e^3 \gg 1\right)$$

NB ! Electronic contribution falls out from $e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{M}}(\mathbf{r})\frac{\left(\sum_j \tilde{n}_j M_j Z_j\right)}{\left(\sum_j \tilde{n}_j Z_j^2\right)}$ in the limit of strong electron degeneracy due to diminishing of ideal-gas electronic compressibility:

Here:

$$\mathbf{D}^n_\mu(\mathbf{r}) \Leftrightarrow \{\partial\mathbf{n}(\mathbf{r})/\partial\boldsymbol{\mu}(\mathbf{r}')\}_T \equiv \left[\!\left[\partial n_j(\mathbf{r})/\partial\mu_k(\mathbf{r}')\right]\!\right]_{T,\mu_i(i\neq k)}$$

$\mathbf{D}^n_\mu$ is inverse matrix to: $\mathbf{D}^\mu_n \equiv \left[\!\left[\partial^2 F^*/\partial n_j(\mathbf{r})\partial n_k(\mathbf{r}')\right]\!\right]_{T,n_i(i\neq k)}$ $\quad \mathbf{D}^n_\mu * \mathbf{D}^\mu_n = \mathbf{E}$

Non-ideality effects in two-component plasma

{+Z, e}

Equilibrium condition with "*non-ideality force*"

$$m_k\nabla\varphi_{\mathrm{G}}(\mathbf{r}) + Z_k e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) + \nabla\mu_k^{(\mathrm{chem})}\{n_{\mathrm{i}}(\mathbf{r}),\ n_{\mathrm{e}}(\mathbf{r}),\ T\} = 0 \qquad (k = \text{electrons, ions})$$

Final equation for average electrostatic field

(*with taking into account non-ideality and degeneracy effects*)

$$m_i\nabla\varphi_G(\mathbf{r}) + Z_i e\nabla\varphi_E(\mathbf{r})\left[1+\frac{(\mu_{ii}^0+\Delta_i^i+Z\Delta_e^i)}{Z(Z\mu_{ee}^0+Z\Delta_e^e+\Delta_i^e)}\right] = 0$$

Here:

$\mu_j^0(n_j,T)$ – ideal-gas part of (*local*) chemical potential of specie ***j***

$\Delta\mu_j^{(chem)}(n_j,n_i\ldots.n_k,T)$ – non-ideal-gas part of (*local*) chemical potential of specie ***j***

$$\mu_{jj}^0 \equiv \left(\frac{\partial\mu_j^0}{\partial n_j}\right) \qquad \Delta_k^j \equiv \left(\frac{\partial\Delta\mu_j}{\partial n_k}\right)$$

Non-ideality effects in local density approximation

(continued)

1) **Ideal** and **non-degenerate gas** $(n\lambda_e^3 << 1)$

$$F_G^{(Z)} + 2F_E^{(Z)} = 0$$

Polarization compensates just *one half* of gravitational attraction (*for symmetric ion A=2Z*)

2) **Non-ideal** and **non-degenerate gas** $(n\lambda_e^3 << \mathbf{1})$

Polarization compensates *more* than *one half* of gravitational attraction (*for symmetric ion*)

$$F_G^{(Z)} + F_E^{(Z)}[2 - \varepsilon(\Gamma)] = 0 \qquad 0 < \varepsilon(\Gamma) < 1$$

3) **Ideal** and **highly-degenerate gas** $(n\lambda_e^3 >> 1)$

$$F_G^{(Z)} + F_E^{(Z)} \cong 0$$

Polarization compensates gravitational attraction of ions *almost totally*

4) **Non-ideal** and **highly-degenerate gas** $(n\lambda_e^3 >> \mathbf{1})$

$$F_{\mathrm{E}}^{(Z)} + F_{\mathrm{G}}^{(Z)}[1 + \varepsilon(\Gamma, n\lambda^3)] = 0$$

Polarization compensates *not only* gravitational attraction
but additional "*non-ideality force*" directed towards the center of a star **!**

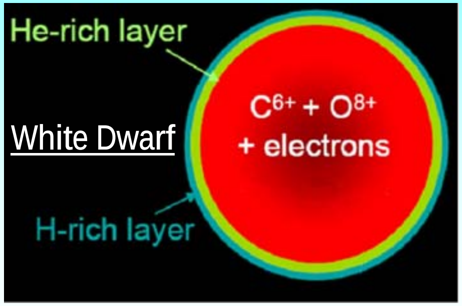
«Global» non-ideality effect !

“…Что касается электростатического потенциала звезд, то трудно себе вообразить какие-либо особенные его проявления. …” ***NN****

Observable consequences *for* plasma polarization

Two well-known examples

Accretion ➔ diffusion ➔ burning *of* hydrogen *in* outer layer *of* compact stars



Chang & Bildsten (2003) *Diffusive nuclear burning in neutron star envelopes*

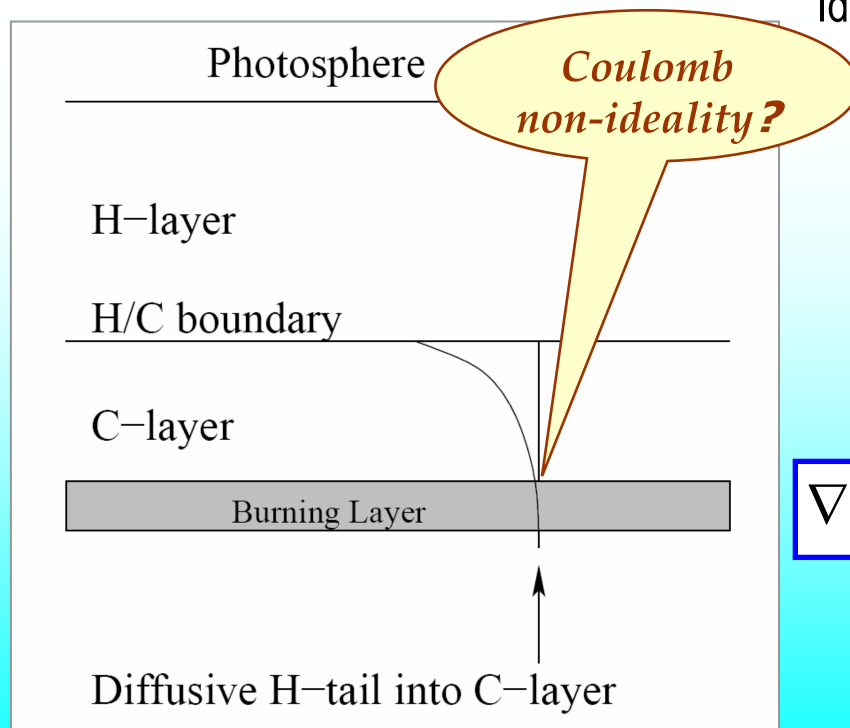
Mixture ${}_{12}C^{6+}$, ${}_{16}O^{8+}$, ${}_{4}He^{2+}$

$$F_E^{(p)} = -\frac{A}{Z} F_G^{(p)} \approx -2F_G^{(p)}$$

Ideal ions – degenerated electrons

$$F_E^{(p)} = -\frac{A}{(Z+1)} F_G^{(p)} \approx -(1.33-1.8)F_G^{(p)}$$

Ideal ions – non-degenerated electrons



$\nabla \varphi_G(\mathbf{r})$ $\nabla \varphi_E(\mathbf{r})$

$\nabla \varphi_G(\mathbf{r})$ $\nabla \varphi_E(\mathbf{r})$

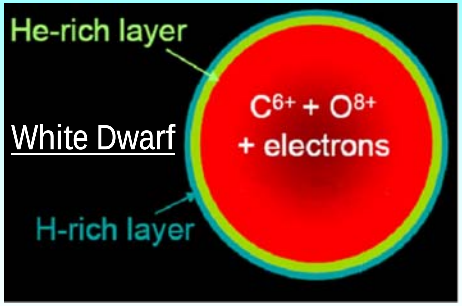
Pure hydrogen

$$F_E^{(p)} \approx -(1/2)F_G^{(p)}$$

Two well-known examples

Diffusion *and* sedimentation *of* Ne *in* interior *of* WD

Bildsten & Hall (2001) *Gravitational settling of* ^{22}*Ne in liquid white dwarf interior*



Mixture $_{12}C^{6+}$, $_{16}O^{8+}$, $_{4}He^{2+}$ → $F_E^{(p)} = -\frac{A}{Z} F_G^{(p)} \approx -2F_G^{(p)}$

The net force on ^{22}Ne

$$\boldsymbol{F} = -22 m_p g \hat{\boldsymbol{r}} + 10 e E \hat{\boldsymbol{r}} = -2 m_p g \hat{\boldsymbol{r}}$$

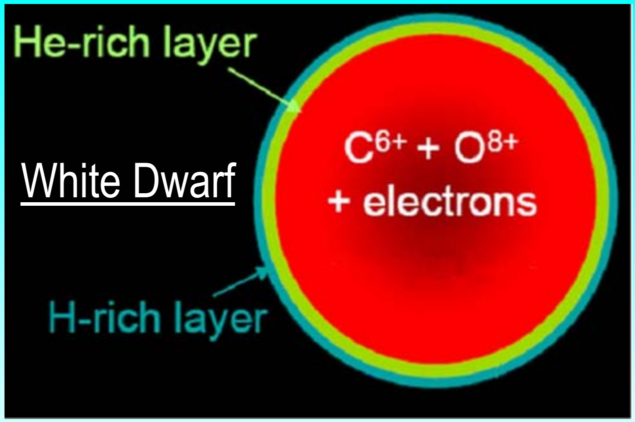
.... The total increase in cooling age by the time the WD completely crystallizes ranges from 0.25-1.6 Gyr, depending on the value of D and the WD mass.

NB !

Coulomb non-ideality at ***micro-level*** discriminates $_{16}O^{8+}$ *in* $_{12}C^{6+}$, *and* $_{12}C^{6+}$ *in* $_{4}He^{2+}$... and accelerates Rayleigh–Taylor hydrodynamic instability

Coulomb non-ideality effect at ***macro-level*** (plasma polarization) ***suppresses*** Rayleigh–Taylor hydrodynamic instability

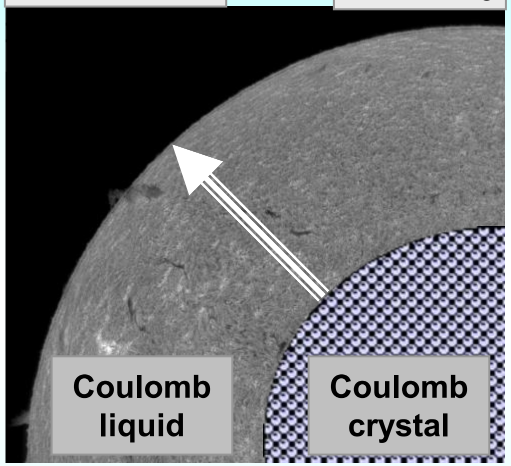
Plasma polarization *and* hydrodynamics in compact stars



White Dwarf

Typical WD ⇔ mixture ${}_{12}C^{6+}$, ${}_{16}O^{8+}$, ${}_{4}He^{2+}$ +
+ electronic background (*strongly degenerated*)

$T \sim const$ $M \sim M_{\odot}$



WD – is **strongly non-ideal** ($\Gamma \sim 10^2 – 10^3 >> 1$)

$$e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle Z|\mathbf{D}^n_\mu|M\rangle}{\langle Z|\mathbf{D}^n_\mu|Z\rangle} \rightarrow F_{\mathrm{E}}^{(Z)} \approx -F_{\mathrm{G}}^{(Z)}\left[1-\frac{a_{\mathrm{M}}\Gamma_Z}{Z}x_c(\zeta_e)\right]^{-1} \approx -F_{\mathrm{G}}^{(Z)}$$

$$F_E^{(Z)} + F_G^{(Z)} \approx 0$$

Total force acting on every ion
(nuclei: ${}_{12}C^{6+}$, ${}_{16}O^{8+}$, ${}_{4}He^{2+}$)
is ***equal to zero*** **!**

$T \sim 10^6 \div 10^7\,\mathrm{K}$ $\rho \sim 10^6\, g/cm^3$

$n_c \sim 3\cdot10^{29} \div 3\cdot10^{32}\, cm^{-3}$

$\zeta_e \equiv n_e \lambda\!\!\!^{-}{}_e^3 \sim 10^5$

$$x_c(\zeta_e) \equiv \left(\frac{\mu^0_{ii}}{Z\mu^0_{ee}}\right) \sim 10^{-3} \div 3\cdot10^{-4}$$

NB !
White Dwarf is in **weightless state** in fact **!**

What does it mean – hydrodynamics of a star in weightless state ?

Hydrodynamics of a star in weightless state ?

$$F_E^{(Z)} + F_G^{(Z)} \approx 0$$

Carbon, oxygen and helium does **not sink** or **float** in each other !

Any hypothetical **layered structure** from ${}_{12}C^{6+}$, ${}_{16}O^{8+}$, ${}_{4}He^{2+}$ is **hydrodynamically stable** as well as homogeneous mixture

Rayleigh-Taylor **hydrodynamic instability «does not work»** in WD !

R-T instability comes out of sources, which induce convection in WD !

NB !

Plasma polarization due to gravitation and non-ideality can **suppress hydrodynamic instability** in interiors of compact stars **!**

Given:

Total force acting on every ion (nuclei: $_{12}C^{6+}$, $_{16}O^{8+}$, $_{4}He^{2+}$) is ***equal to zero*** !

$$F_E^{(Z)} + F_G^{(Z)} \approx 0$$

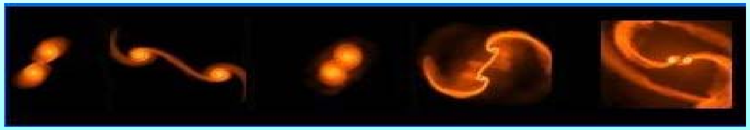
Naive questions *

Why compact star is spherical ?

Why rotating star is spherical ? (*pancake ? roll ? more complicated ?)*

Why rotating binaries are spherical ?

What is the form of mergers (*if polarization field is taken into account*) **?**



Are all these questions meaningful ?

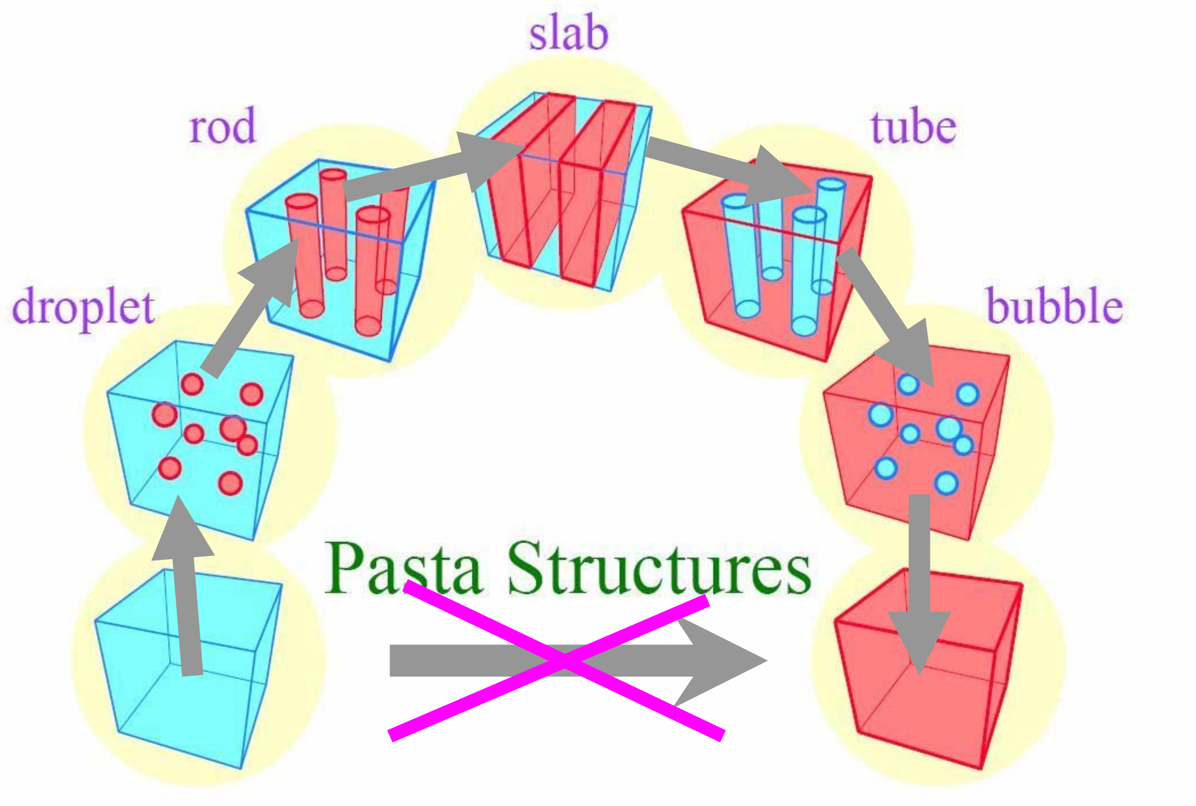
* *(в помощь лектору по астрофизике)*

"…*Что касается* *электростатического потенциала*, *то . . . трудно себе вообразить какие-либо особенные его проявления*. …" *NN**

Naive questions II

Structured Mixed Phase ⇔ "Pasta" plasma

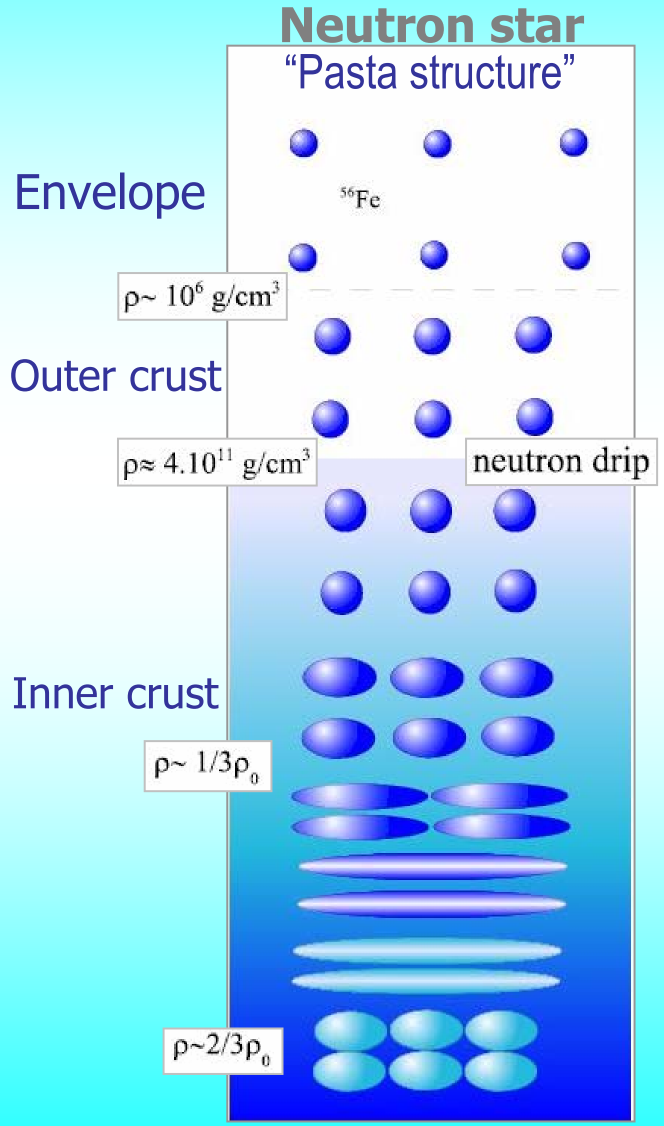
Structured Mixed Phase Concept ⇔ "Pasta"



Schematic picture of pasta structures. Phase transition from blue phase (left-bottom) to red phase (right-bottom) is considered.

Pasta structures in compact stars
/arXiv:nucl-th/0605075v2 /2006/

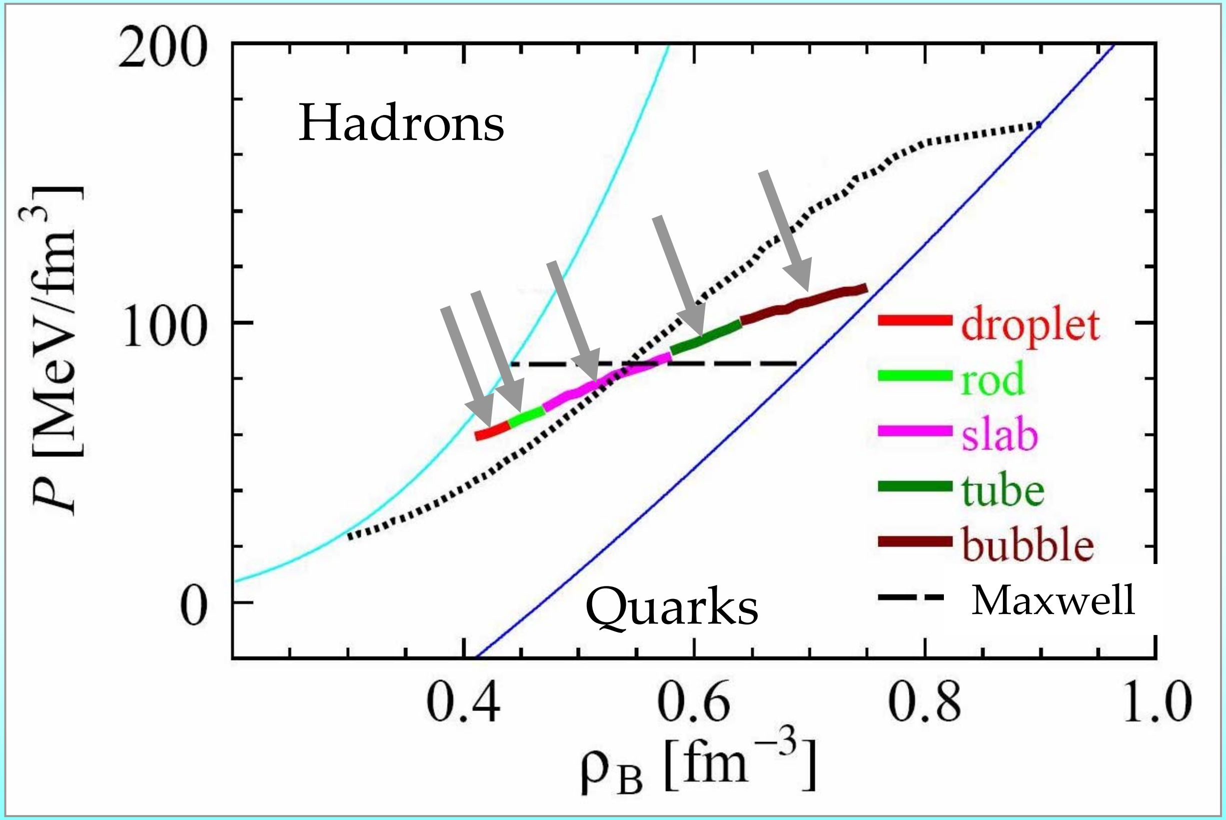
Maruyama T., Tatsumi T., Endo T., Chiba S.

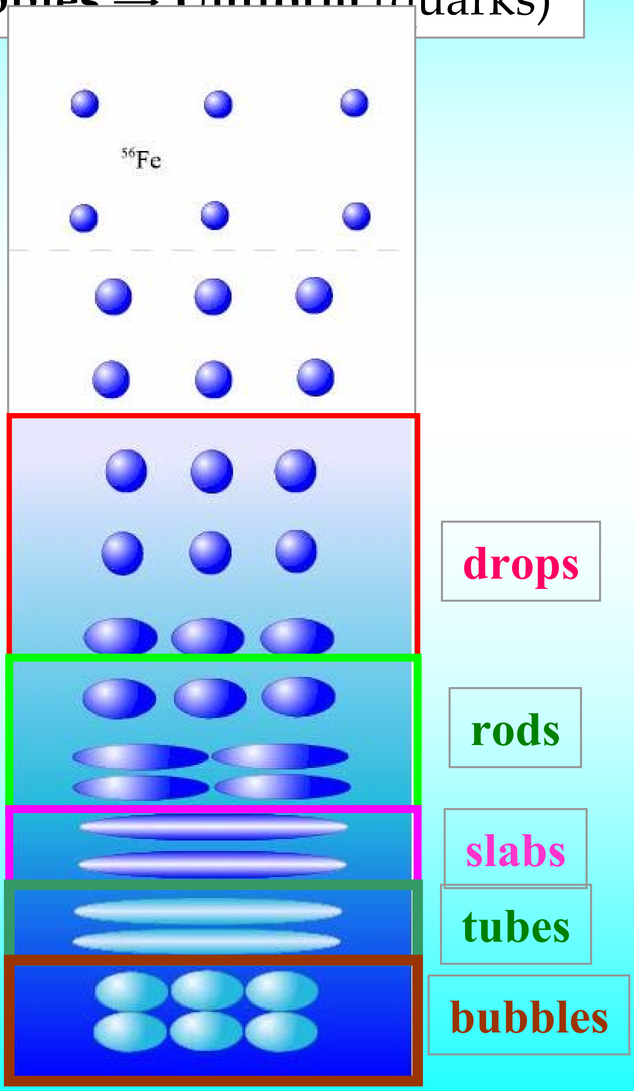


Structured Mixed Phase Concept ⇔ "Pasta"

The sequence of five (or more ?) phase transitions !

Uniform (nucleons) → **Drops** → **Rods** → **Slabs** → **Bubbles** → **Uniform** (quarks)

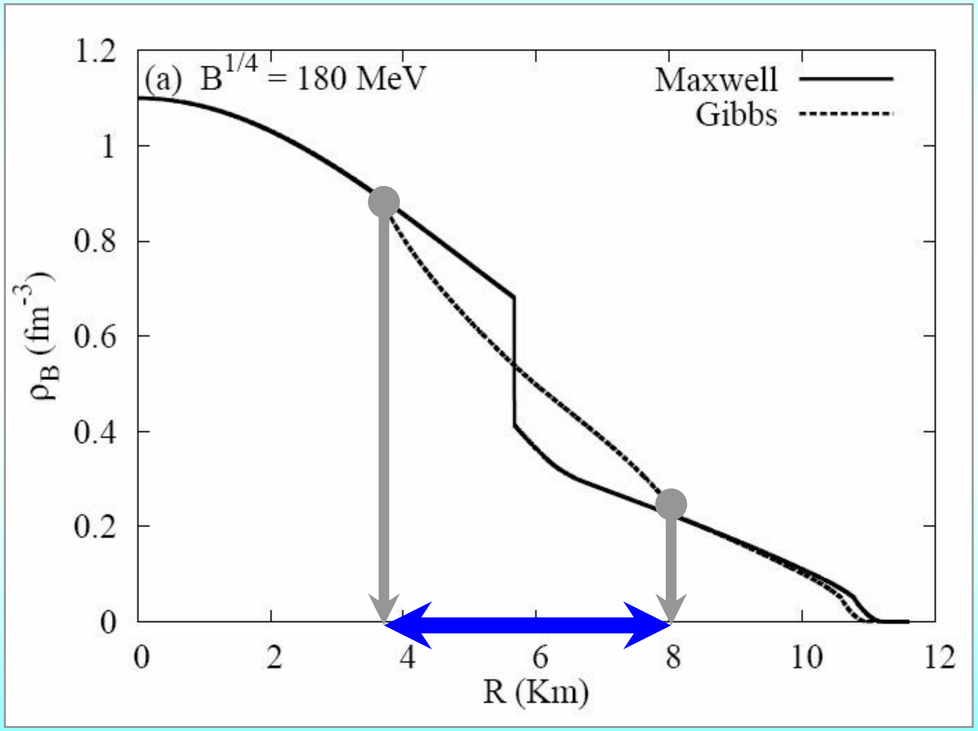




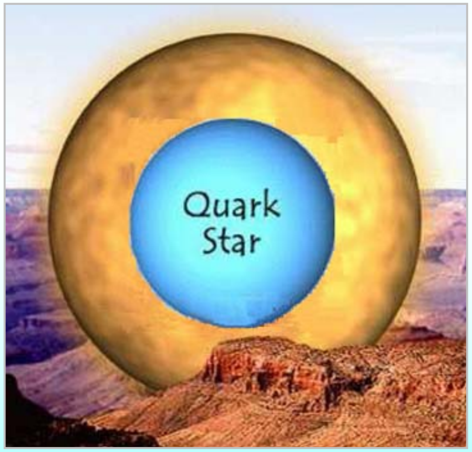
Maryuama T., Tatsumi T., Endo T., Chiba S.
arXiv/0605075v2

Mixed Phase Layer in Hybrid Star

may be about 40% !



Hybrid Stars
Quark core + Hadron crust

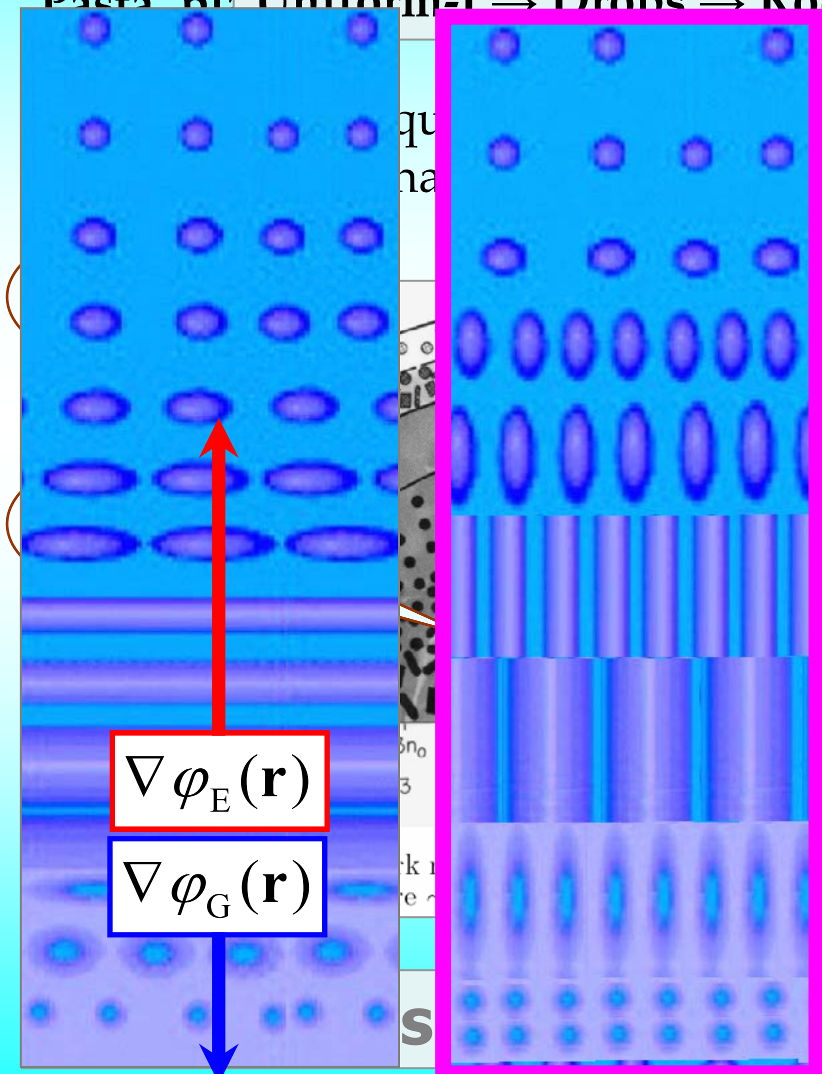


|← R ~ 10 km →|

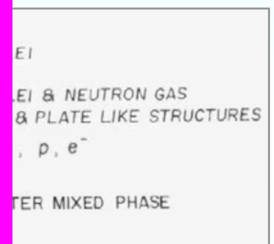
Bhattacharyya A., Mishustin I., Greiner W. <arXiv0905.0352b> (2009)

Structured Mixed Phase ⇔ "Pasta" plasma

'Pasta' pl Uniform-I → Drops → Rods → Slabs → Bubbles → Uniform-II



What *is the* **orientation** *of* **spaghetti** *and* **lasagne ?**



Heise … Hjorth-Jensen

Phase T… s in Neutron Stars

a… 028v1 (1998)

?

What *is the* **topology** *(connectivity)* *of* **spaghetti** *and* **lasagne ?**

Honeycomb ?

What *are the* **transport properties** *of such* **mist-net-foam** *structure* **?**

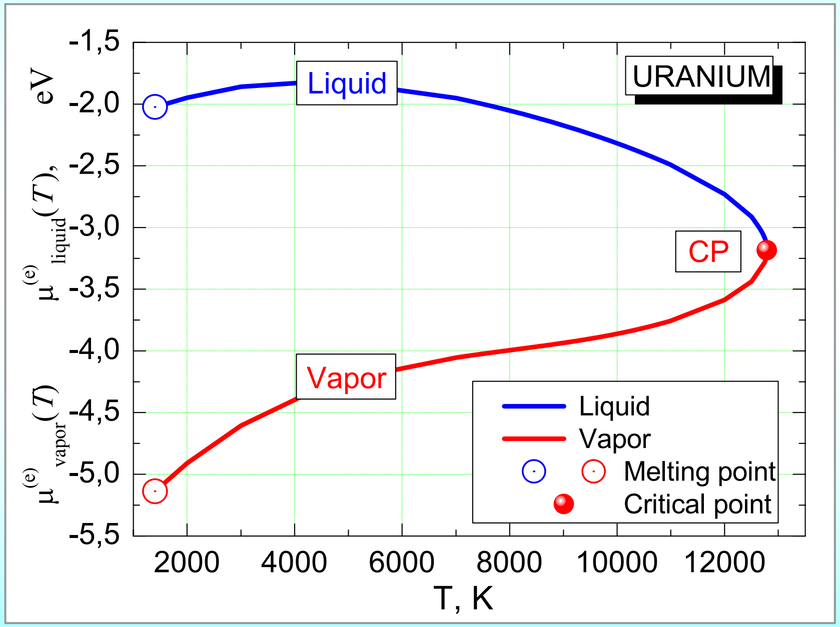
Electrostatics *of* **Phase Boundaries** *in* **Coulomb Systems**

Iosilevskiy I. / Int. Conference “***Physics of Neutron Stars***”, St.-Pb. Russia, 2008

Iosilevskiy I. / Int. Conference “***Physics of Non-ideal Plasmas***”, Moscow, Russia, 2009

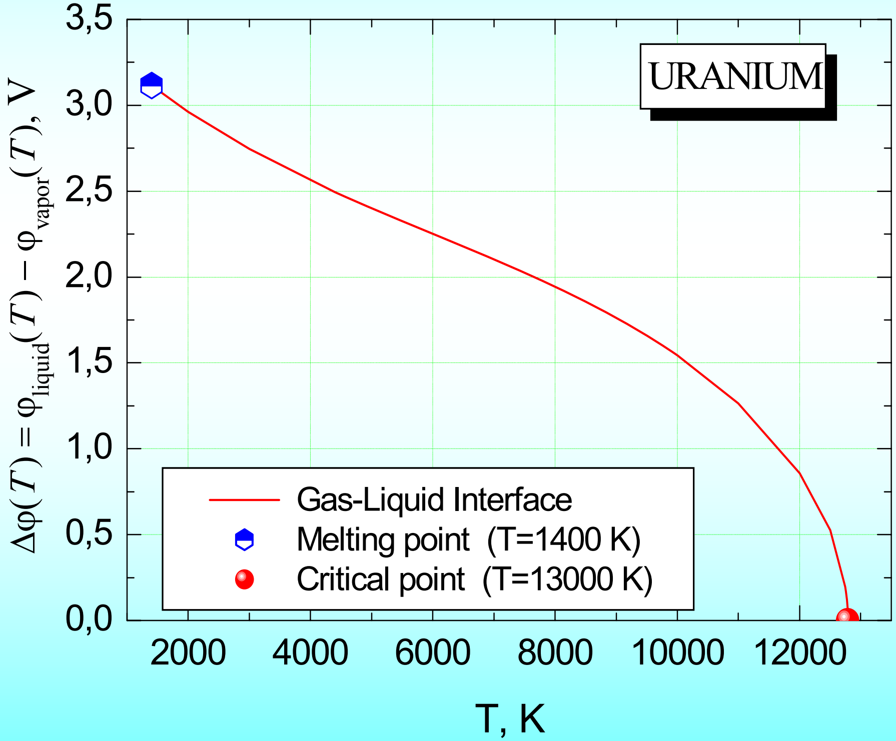
Potential of gas-liquid interface in Uranium

Chemical potential vs temperature



Electrochemical Phase Diagram

$$e\Delta\varphi = (\mu_e)_{liquid} - (\mu_e)_{vapor}$$



Calculation of gas-liquid equilibrium via plasma model (code "SAHA-IV")

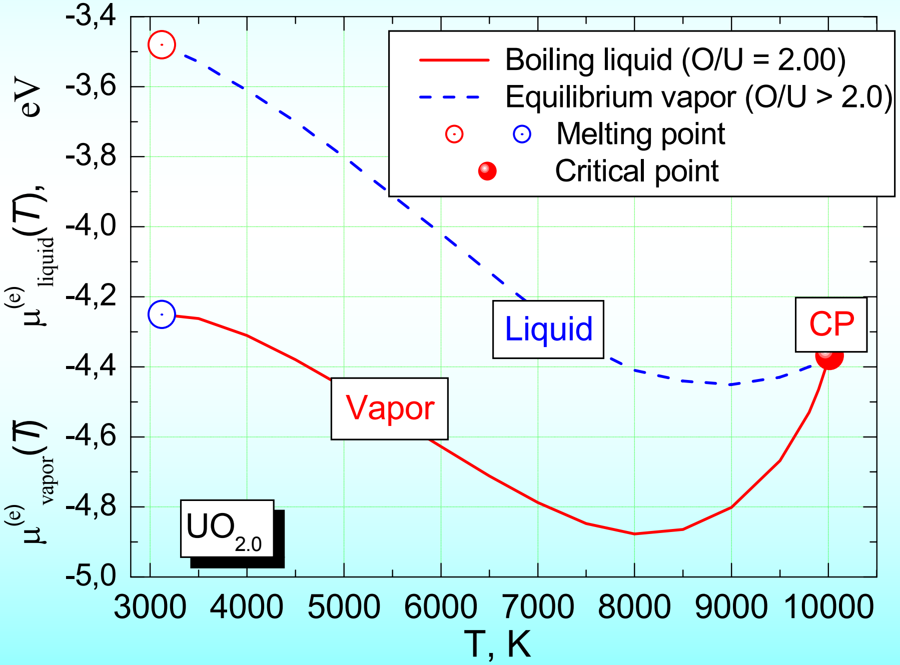
(*Gryaznov & Iosilevskiy, 2005*)

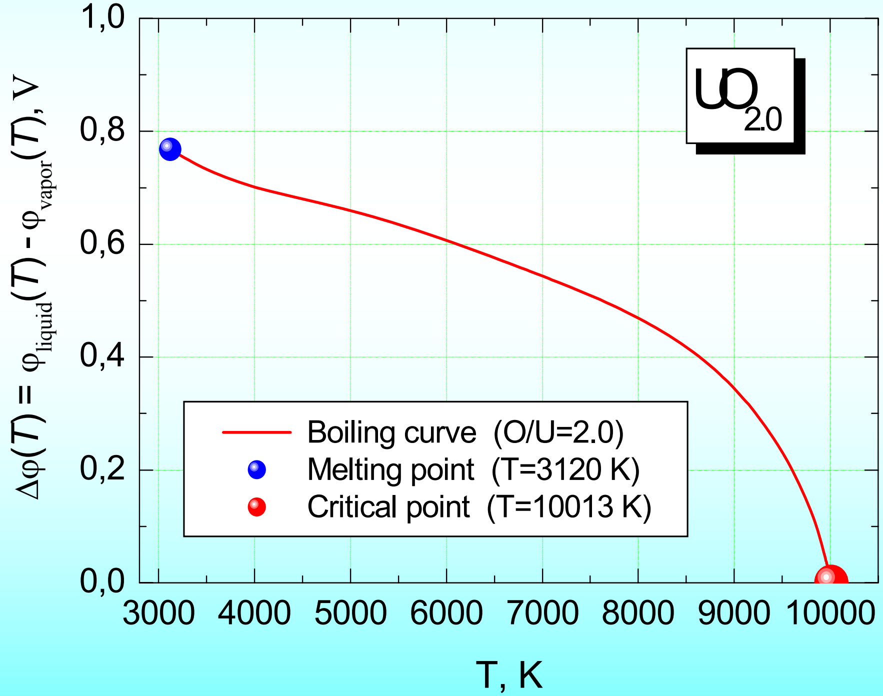
Iosilevskiy & Chigvintsev, *J. de Physique IV*, (2000)

Potential of non-congruent phase boundaries in U-O system

$$e\Delta\varphi = (\mu_e)_{liquid} - (\mu_e)_{vapor}$$

Electrochemical Phase Diagram





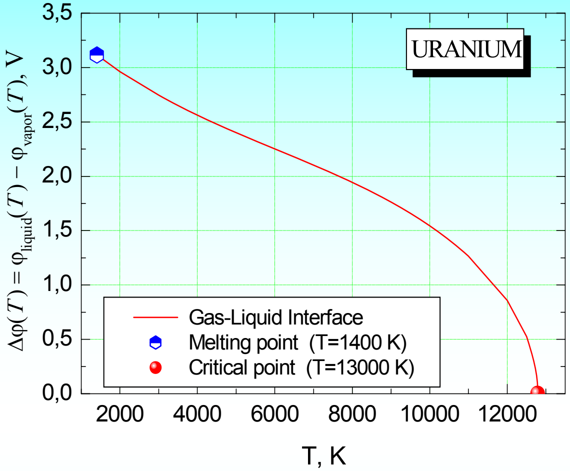
Calculation of non-congruent gas-liquid equilibrium (code "SAHA-VI")

Iosilevskiy, Gryaznov et al., *Contrib. Plasma Phys.* (2003)

Electrostatics of phase boundaries in Coulomb systems

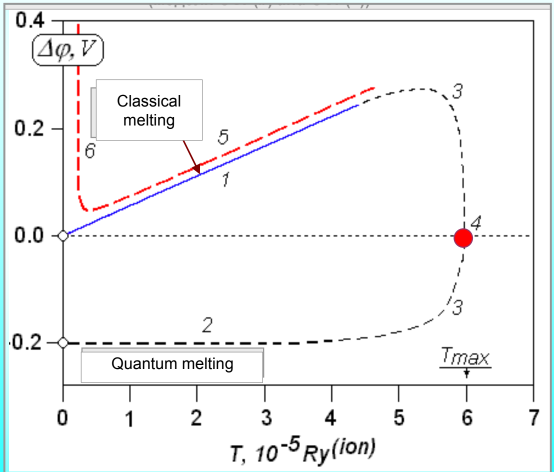
Terrestrial applications

Electrostatic (Galvani) potential



Iosilevskiy & Gryaznov, *J.Nucl.Mat.* (2005)

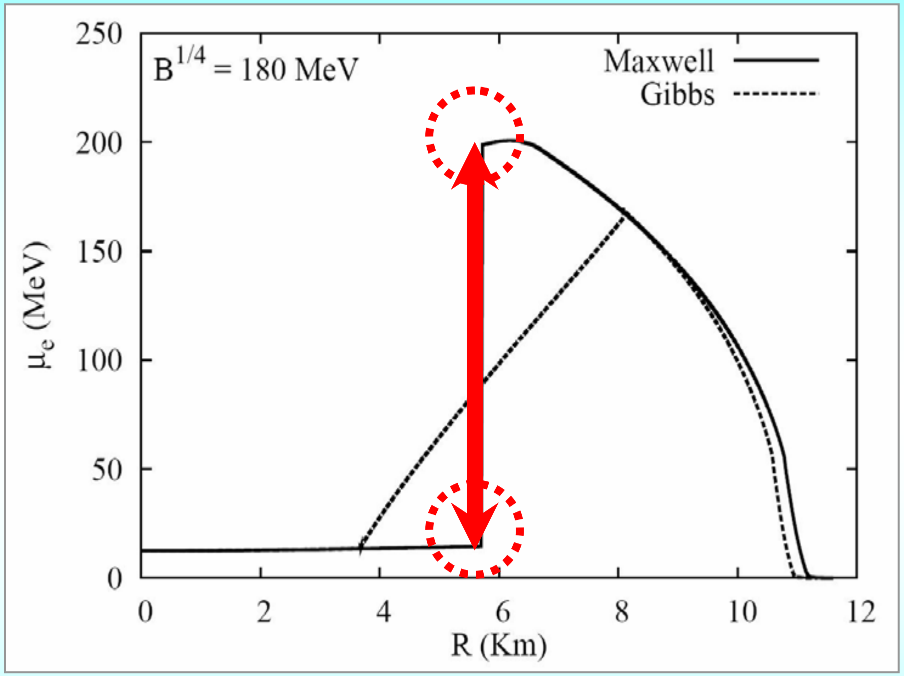
Electrostatic "portrait" of Wigner crystal in OCP



Iosilevskiy & Chigvintsev, *J. Physique* (2000)

Quark-Hadron phase transition in Hybrid Star

Bhattacharyya A., Mishustin I., Greiner W., arXiv:0905.0352v1 (2009)



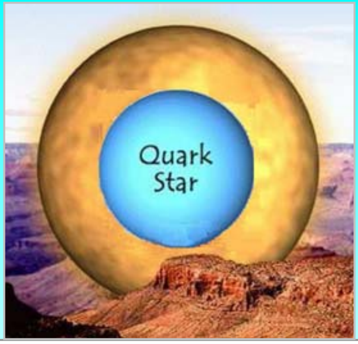
$$e\Delta\varphi_{HQ} = (\mu_e)_{\text{Hardron phase}} - (\mu_e)_{\text{Quark phase}}$$

$$e\Delta\varphi_{HQ} \approx 200\ \text{MeV}\ !$$

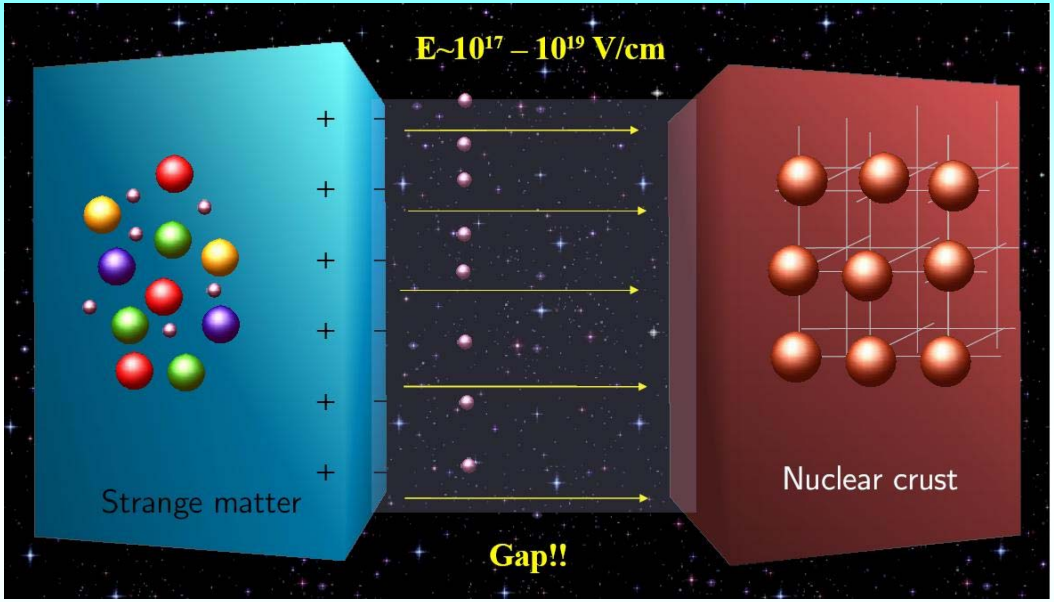
$$\delta_{HQ} \approx 10^3\ \text{fm} \rightarrow E \sim 10^{18}\ \text{V/cm}$$

For comparison: Alcock *et al.*, 1986: → $E \sim \mathbf{10^{17}}$ V/cm

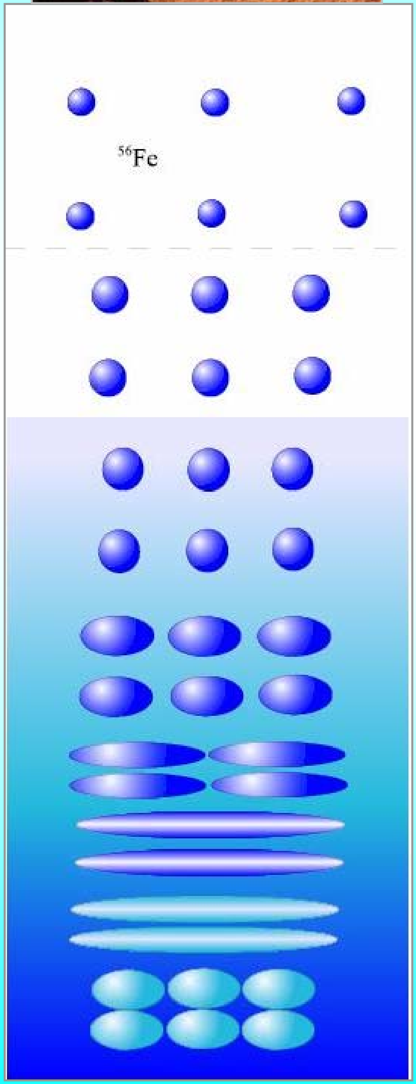
Electrostatics of Quark-Hadron Interface



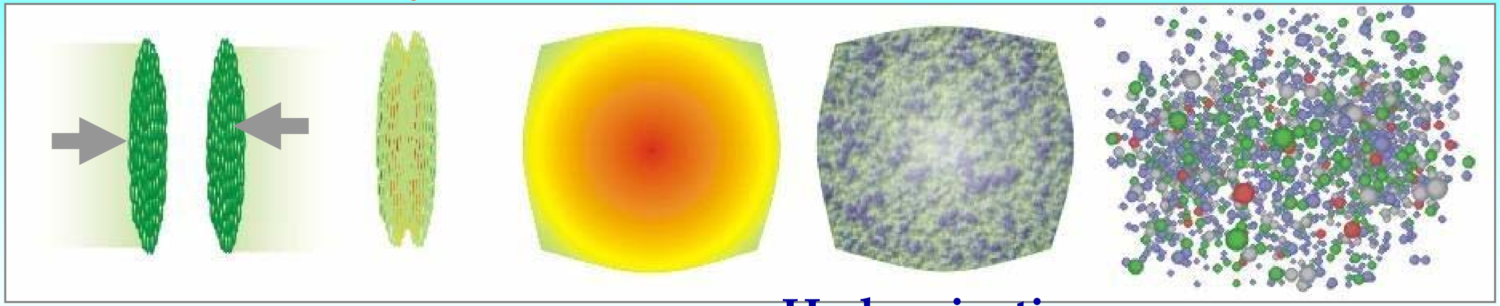
Nuclear Crust on Strange Core

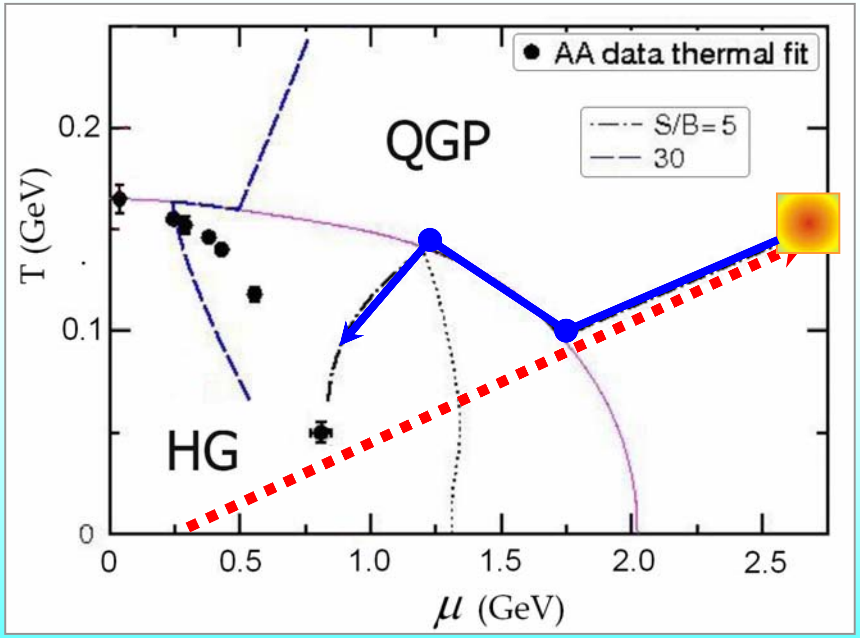


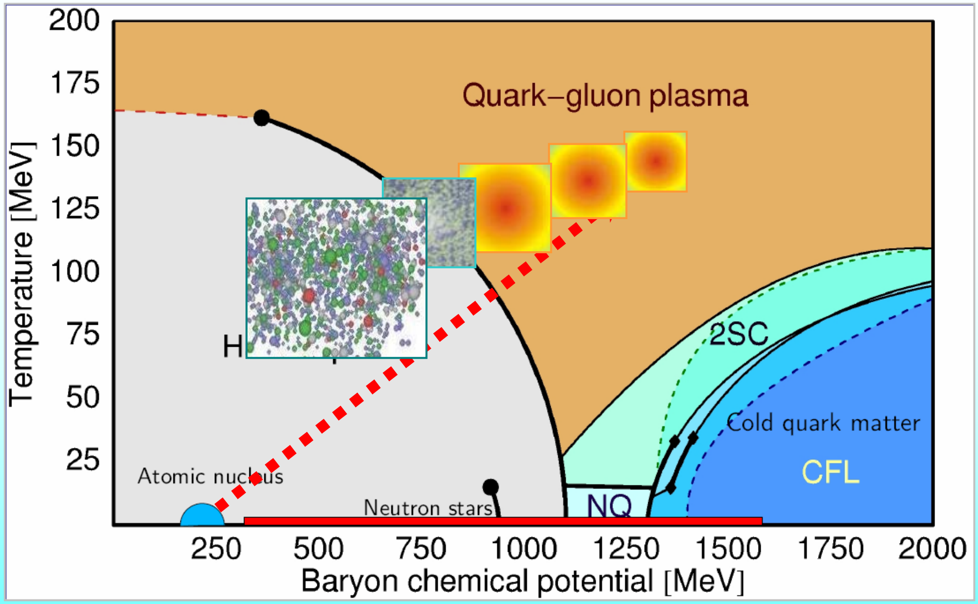
After Fridolin Weber, WEH Seminar, Bad Honnef, 2006



Impact *and* hydrodynamics *of* fireball







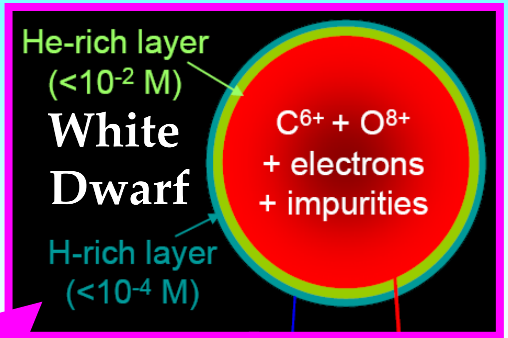
L.Satarov, M.Dmitriev, I.Mishustin //arXiv: 0901.1430v1

After David Blaschke, WEHS Seminar, Bad Honnef, 2007

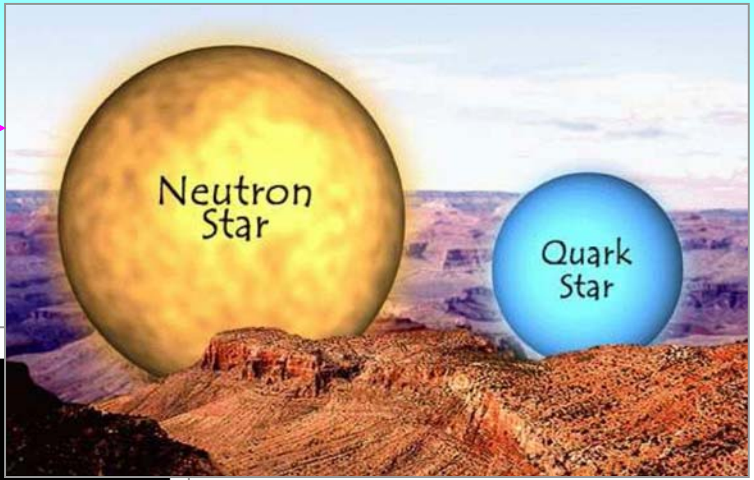
Macroscopic charge *on* phase boundaries *in* Compact Stars

Compact stars

White dwarfs, Neutron stars, "Strange" (quark) stars, Hybrid stars



Neutron and "Strange" Stars



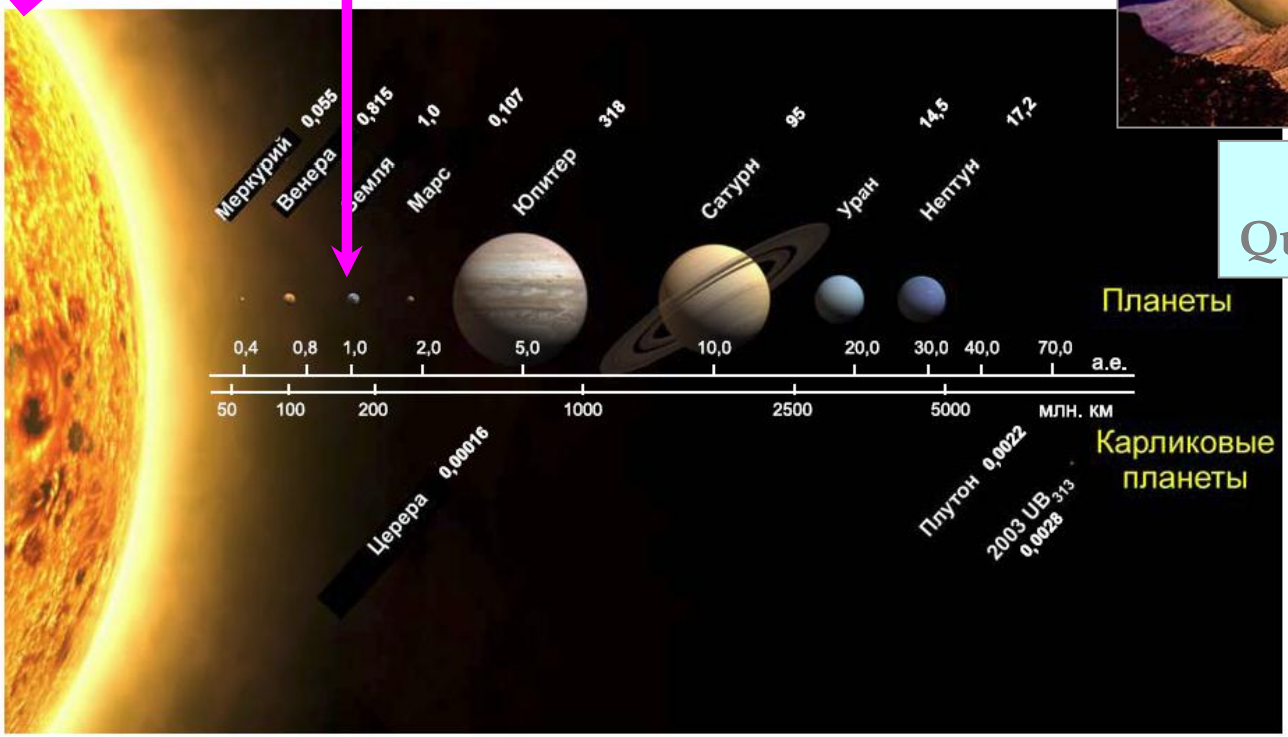
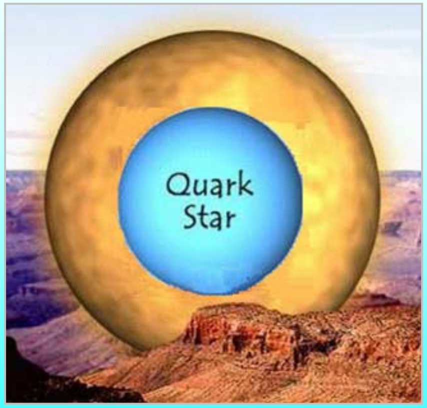


Рис. 65. Массы планет (в единицах массы Земли) и их среднее расстояние от Солнца [371]

Hybrid Stars

Quark core + Hadron Crust



|← R ~ 10 km →|

Macroscopic charge *on* phase boundaries *in MAO*

$$e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\left\langle \mathrm{Z}\middle|\mathbf{D}_{\mu}^{n}\middle|\mathrm{M}\right\rangle}{\left\langle \mathrm{Z}\middle|\mathbf{D}_{\mu}^{n}\middle|\mathrm{Z}\right\rangle}$$

Any **jump-like discontinuity** in thermodynamic parameters (**phase boundary**, **jump** in ionic **composition** *etc*) must be accompanied with existing of **macroscopic charge** localized at this interface.

astro-ph:0901.2547 / astro-ph:0902.2386

Iosilevskiy I. / Int. Conference "*Physics of Neutron Stars*", St.-Pb. Russia, 2008

Iosilevskiy I. / Int. Conference "*Physics of Non-ideal Plasmas*", Moscow, Russia, 2009

Plasma polarization in thermodynamics of neutron stars

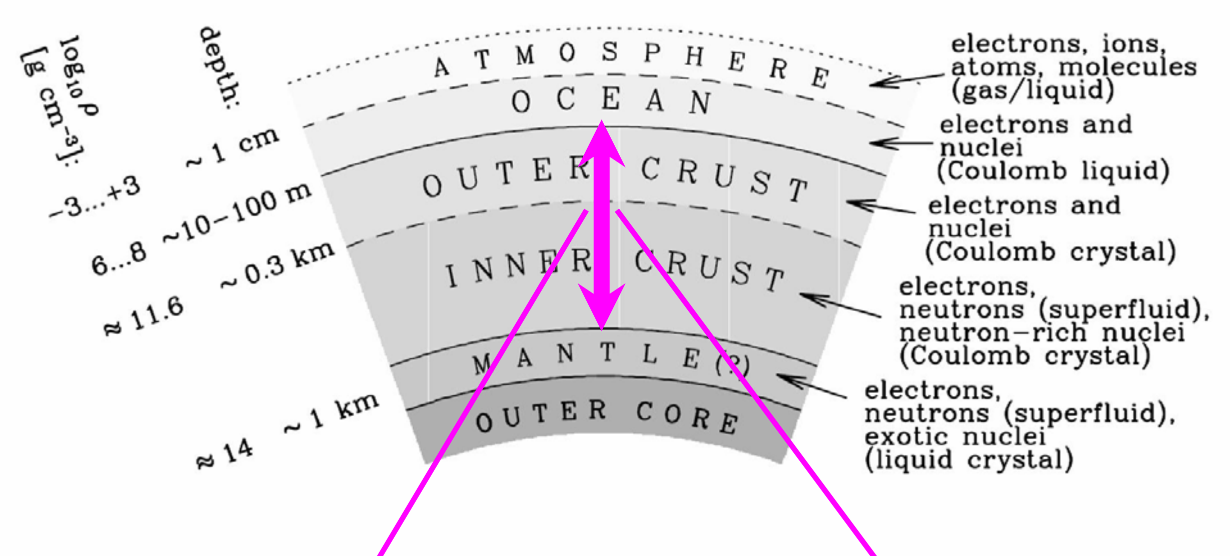
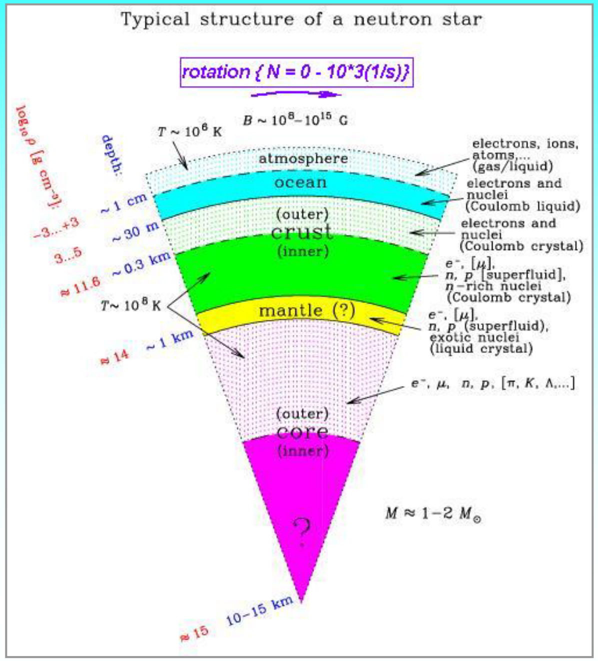
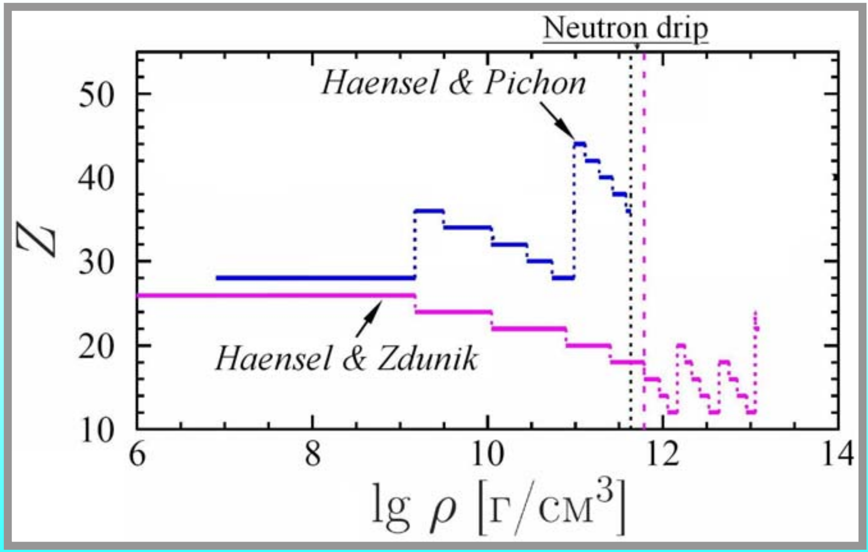


Figure 2.1. Schematic structure of an envelope of a neutron star with the internal temperature $\sim 10^8$ K (see text for more details).



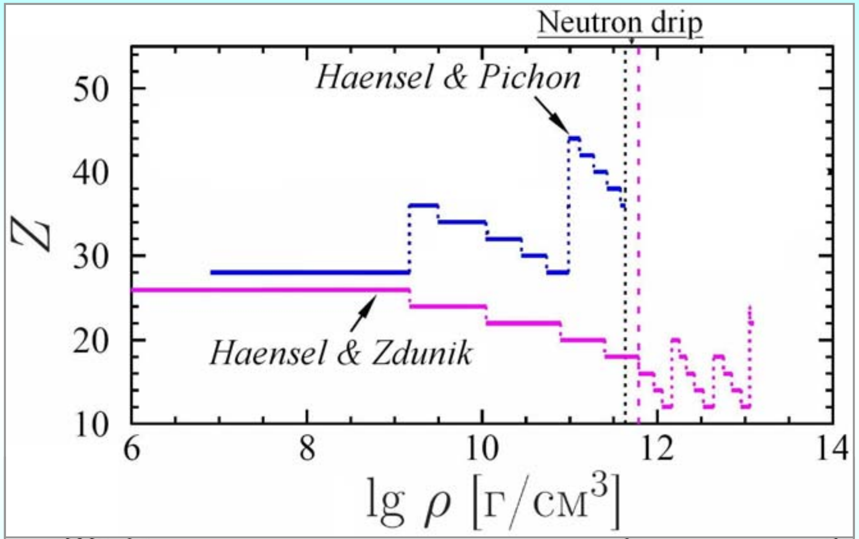


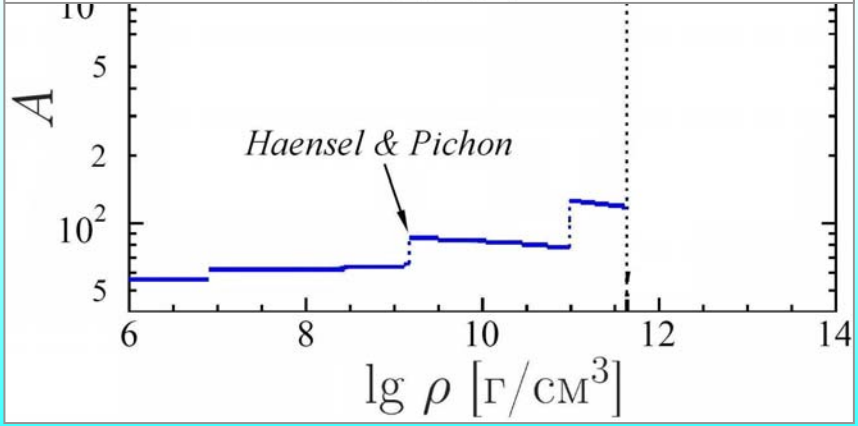
After / Haensel P, Potekhin A, Yakovlev D, *Neutron Stars* // Springer, 2007 /

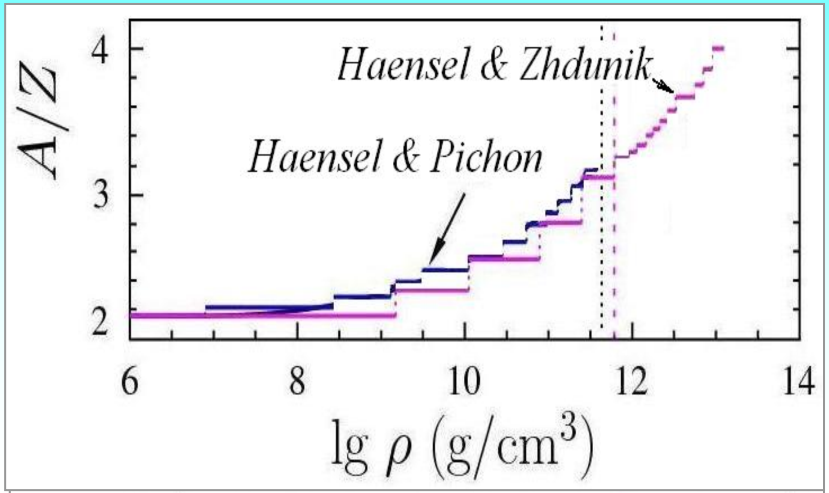
Macroscopic charge on phase boundaries in MAO

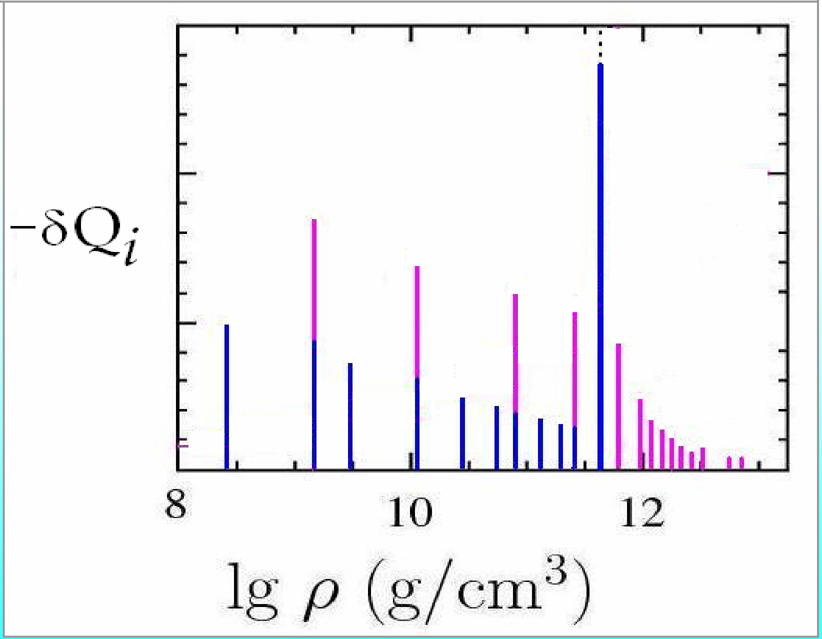
Typically – ratio *A/Z increases* when we cross the interface toward the inner layer.
It means *decreasing* of electrostatic field, i.e. macroscopic *negative charge* localized on two-layer interface.

$$e\nabla\varphi_{\mathrm{E}}(\mathbf{r}) = -\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{M}\rangle}{\langle \mathrm{Z}|\mathbf{D}_{\mu}^{n}|\mathrm{Z}\rangle} \cong -m_{p}\nabla\varphi_{\mathrm{G}}(\mathbf{r})\frac{A}{Z}$$









Iosilevskiy I. / Int. Conference *"Physics of Neutron Stars"*, St.-Pb. Russia, 2008

Conclusions and perspectives

- **Plasma polarization** in massive astrophysical bodies is **general** phenomenon

- **Plasma polarization** in massive astrophysical bodies is **universal** phenomenon

- **Plasma polarization** in massive astrophysical bodies is **interesting** phenomenon

- **Plasma polarization** in massive astrophysical bodies manifests itself in great number of **observable consequences** in **thermodynamics** of MAO

- **Plasma polarization** in massive astrophysical bodies manifests itself in great number of **observable consequences** in **hydrodynamics** of MAO

- **Coulomb non-ideality** effects at **micro**-level could **amplify hydrodynamic instability** in MAO, while **Coulomb non-ideality** at **macro**-level could **suppress hydrodynamic instability**

Outlook

- Local and global thermodynamic stability of (*strongly non-ideal*) matter in MAO ?
- Electrostatic potential (*micro* *and* *macro*) in "pasta plasma" inside compact star ?
- Gravitational polarization inside QGP-plasma of Strange Star ?
- Electrostatics of plasma oscillations (vibrations) in compact stars ?
- Inertial polarization of rotating stars and binaries ?
- Electrostatics of Supernova explosions ?
- Electrostatics of Black Holes ?
- Electrostatics of expanding "fairball" ?

Questions out of my understanding

- Gravitational polarization with relativistic effects ?
- What does it mean: gravitational polarization in media, where mass is not constant ?
- Polarization in compact star with strong magnetic field ?

"Nothing is secret which shall not be manifested…"

Luke 8:17

Thank you!

Support: **ISTC** 3755 // **CRDF** MO-011-0, and by **RAS Scientific Programs**
"Physics and Chemistry of Extreme States of Matter" and "Physics of Compressed Matter and Interiors of Planets"
MIPT Education Center "Physics of High Energy Density Matter"

There will be enough challenges

to keep us all happily occupied for years to come…

Hugh Van Horn (1990)

(Phase Transitions in Dense Astrophysical Plasmas)

The end

Milestones

1903 // W. Sutherland – Discussed basic idea of gravitational polarization in MAO

1922 // A. Pannekoek 1924 // S. Rosseland	Obtained the key relation for proportionality of average gravitational and electrostatic fields (counting per proton) for the case of ideal non-degenerated plasma of the Sun { $F_E = ½ F_G$}

1924 // E. Milne – Net charge on the star // Discussed basic idea of non-electroneutrality of stars
1926 // A. Eddington – Respected these ideas in his book
1968 // L. Rosen – Discussed gravitational polarization in the stars as a standard

1976 // T.Montmerle & A.Mishaud 1979 // A.Mishaud & G.Fontain	*Idea*:- protons are “repelled out” by electrostatic field from helium star envelope due to the gravitational polarization

1978 // J. Bally & E. Harrison – *The Electrically Polarized Universe* // Idea of non-electronuetrality of all gravitational objects in the Universe, including stars, galaxies and their clusters

1980 // C.Alkock – *Electric field of a chemically inhomogeneous star* /Electrostatic pollution of hydrogen from helium envelope of white dwarfs
1986 // C.Alkock, Fachri, Olinto – *Electric field on the Strange Star Surface* / Idea of huge local charge densities and average electrostatic field at the surface of the “strange” star
1992 // N.Glendenning / Introduced concept of «Structured Mixed Phase» for quark-hadron phase transition / *Compact Stars: Springer, 2000*.
1996 // D. Kirzhnits – Gravitational polarization give no noticeable observable effects
2001-2005 // L.Bildsten *et al* – Extended the idea of influence of gravitational polarization on diffusion of heavy ions in interiors of white dwarfs. Influence on star cooling and evolution

2003-2005 // S.Ray et al. 2005 // A.Mattei 2007 // A.Di Prisco et al.	*Exotics*: Ideas of ultra high charges and fields, charged black holes, charged gravitational collapse . . . *etc.*

And many other papers probably missed by this list . . .

Crystallization *in* C/O mixture *of* White Dwarfs

Phase diagram in C/O mixture

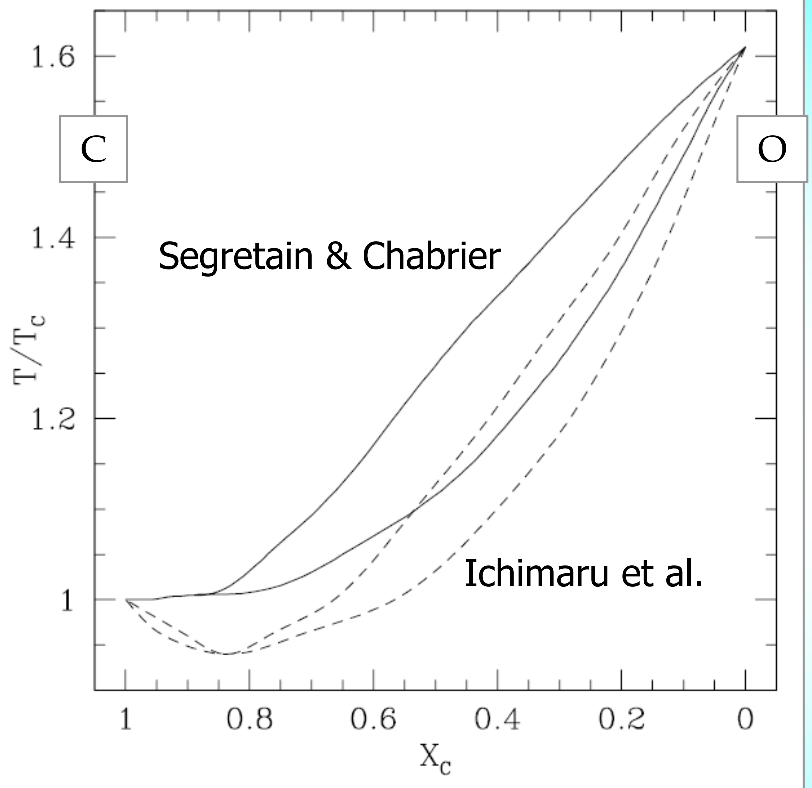


FIG. 1.—Phase diagrams for a C/O mixture as computed by Ichimaru et al. (1988, *dashed line*) and Segretain & Chabrier (1993, *solid line*), where

Phase diagram in C/O mixture

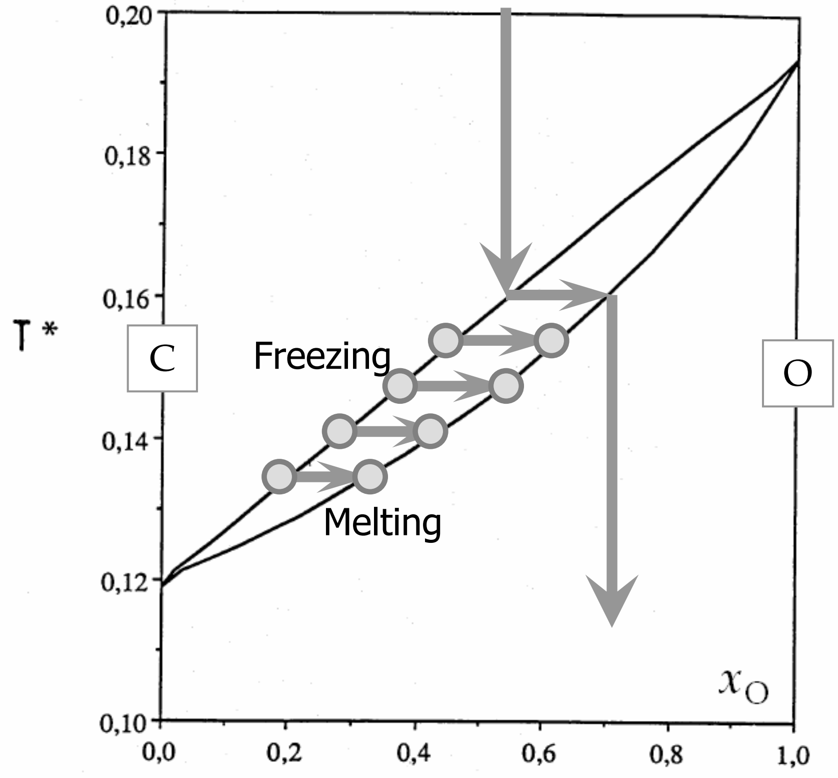
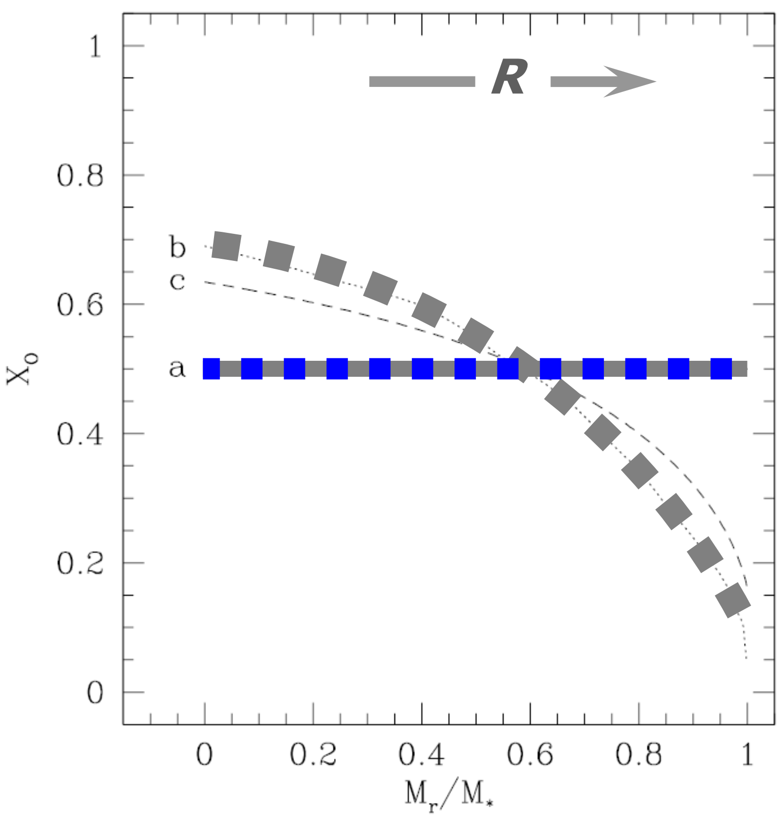


Fig. 1. Phase diagram of the carbon-oxygen mixture at constant electronic pressure. $T^* = 1/\Gamma$ is the reduced temperature,

J.Barrat, J.P.Hansen, R.Mochkovich (1988)

Crystallization *in* C/O mixture *of* White Dwarfs

Oxygen profile in WD



a) – initial
b) – final (Ichimaru)
c) – final (Segretain & Chabrier)

Phase diagram in C/O mixture

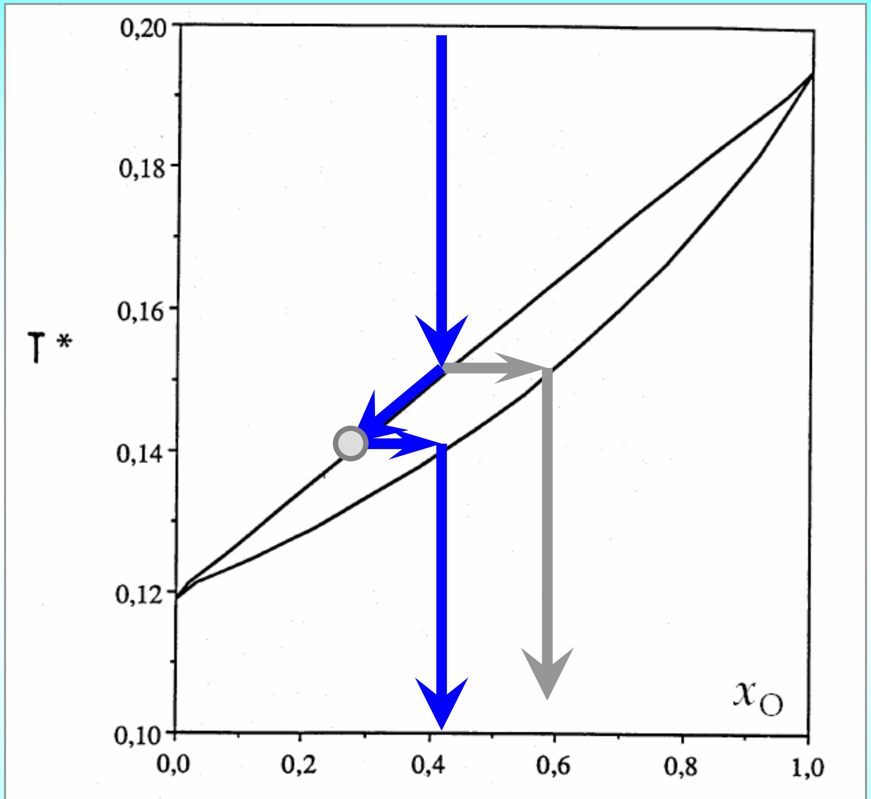


Fig. 1. Phase diagram of the carbon-oxygen mixture at constant electronic pressure. $T^* = 1/\Gamma$ is the reduced temperature,

J.Barrat, J.P.Hansen, R.Mochkovich (1988)