SPH for simulating early planetary systems

Impacts and collisions

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Agenda

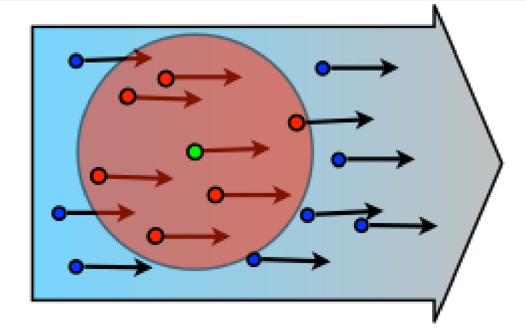
- SPH quick overview
- First results
- Outlook
- References

Smoothed Particle Hydrodynamics (SPH)

- SPH origin: simulating hydrodynamic problems in astrophysics
  - Extended to elasto-plastic dynamics & self gravity
- Application examples include
  - Cosmology
  - Star and planet formation
  - Interactions of stars, black holes,...
  - Accretion discs
  - Material science
  - …

SPH is a mesh-free Lagrangian particle method

- Completely different from finite difference and finite volume methods → well suited for comparisons
- The simulated system is represented as a set of interacting “SPH particles” which
  - carry all physical properties of their “fluid part”
  - determine the density in their region (= number of SPH particles in a specific region)
  - move like point masses governed by the Lagrangian form of the equation of motion
  - are to be interpreted as a numerical vehicle rater than physical particles



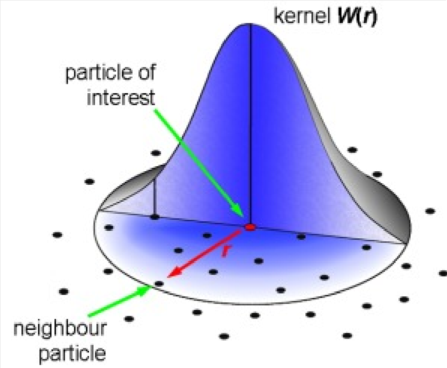
Speith (2012)

SPH principle

- System of coupled PDEs → system of ODEs
  1. Smooth quantities via kernel convolution

$$f(\boldsymbol{r}) \longrightarrow \int f(\boldsymbol{r}')\, W(|\boldsymbol{r}-\boldsymbol{r}'|)\, \mathrm{d}\boldsymbol{V}' = f(\boldsymbol{r}) + \mathcal{O}(h^2)$$

h... smoothing length, “radius” of kernel, determines spatial resolution



NUI Galway (2012)

  2. Remove spatial derivatives

$$\nabla f(\boldsymbol{r}) \longrightarrow \int f(\boldsymbol{r}')\, \nabla W(|\boldsymbol{r}-\boldsymbol{r}'|)\, \mathrm{d}\boldsymbol{V}'$$

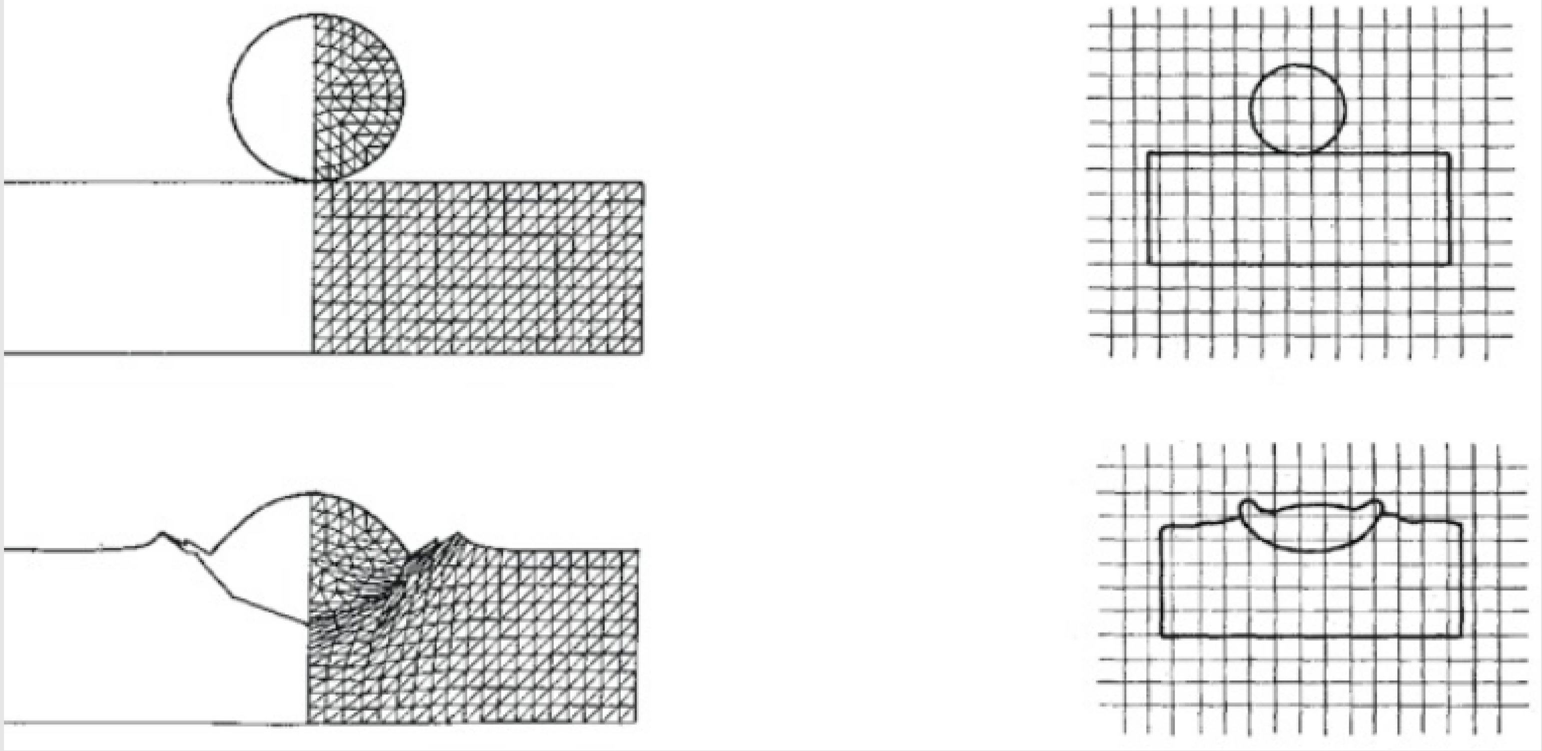
  3. Discretize

$$\nabla f(\boldsymbol{r}^i) \approx \sum_j \frac{m^j}{\rho^j}\, f(\boldsymbol{r}^j)\, \nabla W(|\boldsymbol{r}^i-\boldsymbol{r}^j|, h)$$

SPH for impact simulations

Mesh-free Lagrangian method provides natural reference frame for treating deformations and fragmentation

Lagrange scheme Euler scheme



Speith (2012)

Impacts: solid body mechanics

Continuity equation:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\,\frac{\partial v_\gamma}{\partial x_\gamma} = 0$$

EOM:

$$\frac{\mathrm{d}v_\alpha}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial p}{\partial x_\alpha} + \frac{1}{\rho}\frac{\partial S_{\alpha\beta}}{\partial x_\beta} - \frac{\partial \Phi}{\partial x_\alpha}$$

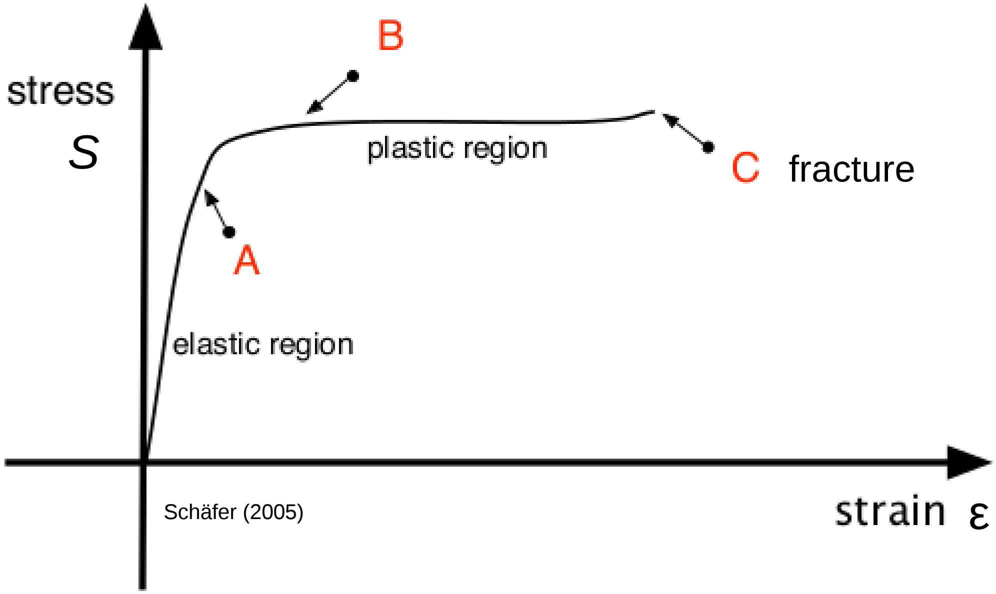
Energy:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{p}{\rho}\frac{\partial v_\alpha}{\partial x_\alpha} + \frac{1}{\rho}S_{\alpha\beta}\dot{\varepsilon}_{\alpha\beta}$$

Constitutive equation:

$$\frac{\mathrm{d}S_{\alpha\beta}}{\mathrm{d}t} = 2\mu\left(\dot{\varepsilon}_{\alpha\beta} - \frac{1}{3}\delta_{\alpha\beta}\dot{\varepsilon}_{\gamma\gamma}\right) + S_{\gamma\beta}R_{\gamma\beta} - R_{\alpha\gamma}S_{\gamma\beta}$$

$$R_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v_\alpha}{\partial x_\beta} - \frac{\partial v_\beta}{\partial x_\alpha}\right)$$



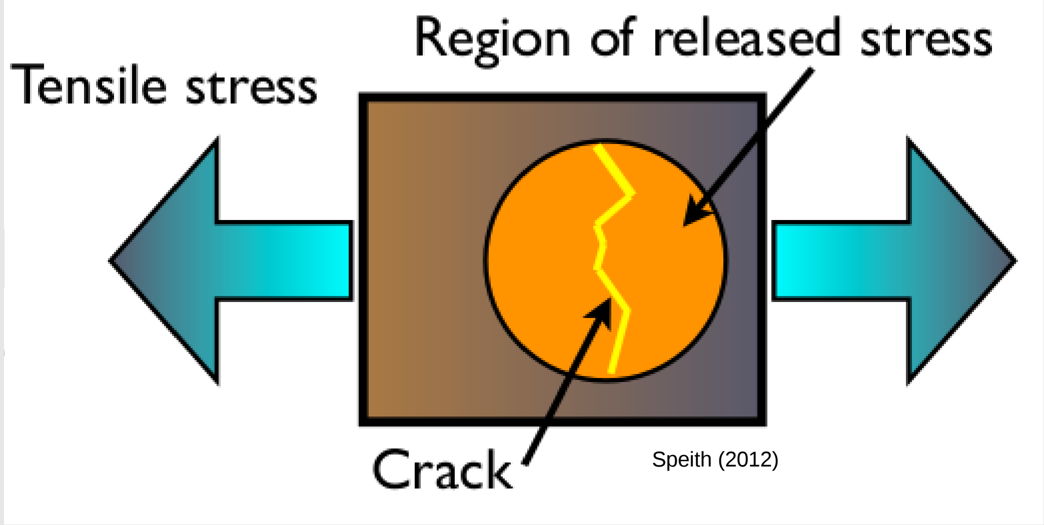
Schäfer (2005)

Equations of state relate density ρ, pressure p, and (internal) energy u, e.g.

$$\text{Tillotson (1962):}\quad p = \left(a + \frac{b}{\frac{u}{u_0}\frac{\rho_0^2}{\rho^2} + 1}\right)\rho u + A\left(\frac{\rho}{\rho_0} - 1\right) + B\left(\frac{\rho}{\rho_0} - 1\right)^2$$

$$\text{Murnaghan:}\quad p = \frac{K}{n}\left(\frac{\rho}{\rho_0} - 1\right)^n$$

Damage and brittle fracture



Speith (2012)

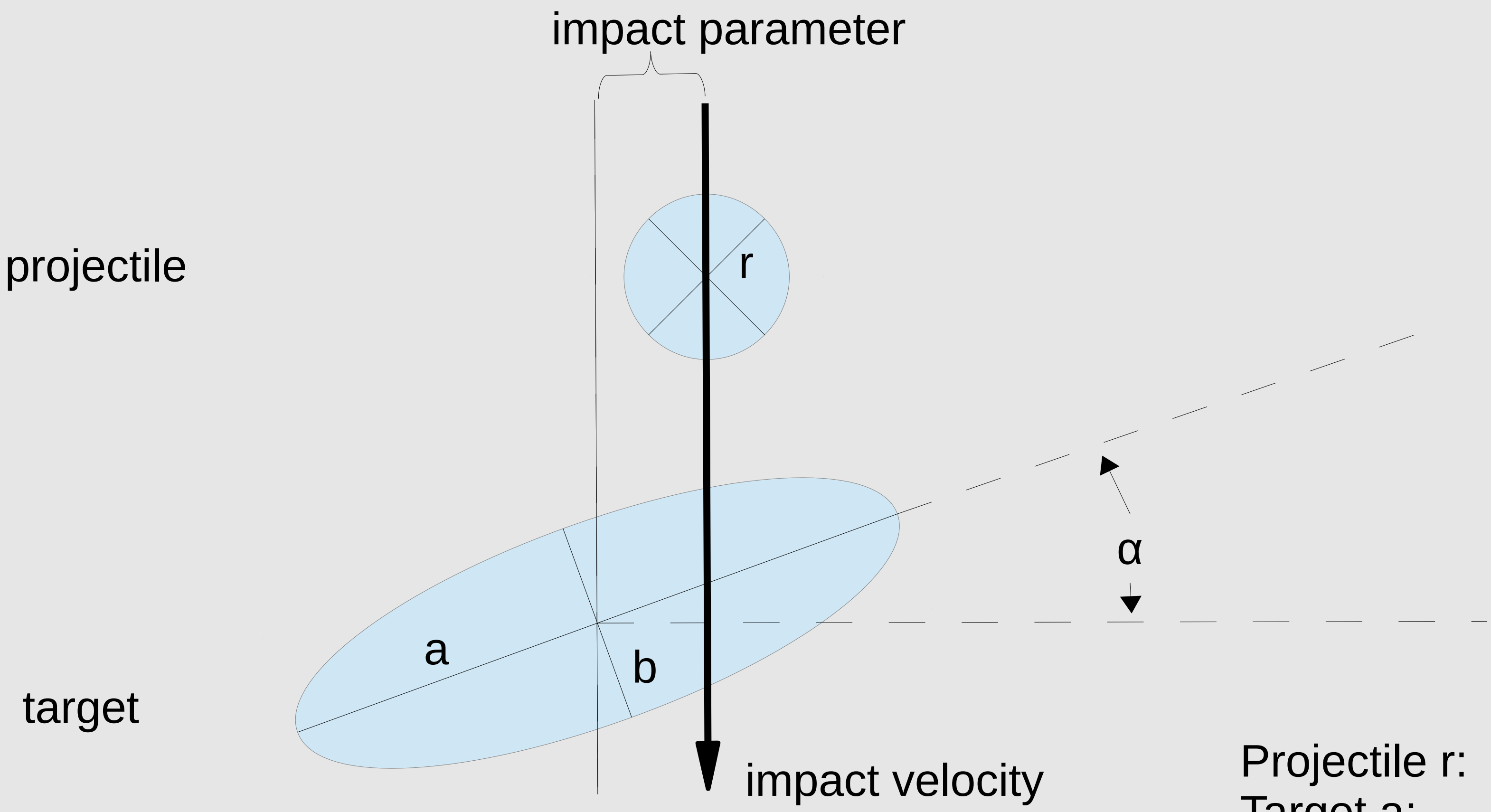
- Large enough strain causes flaws in the solids to develop into cracks
- Cracks grow until the local stress is relieved
- Grady & Kipp (1980) damage model:
  - damage D with $0 \leq D \leq 1$
  - stress ~ (1-D)
- Flaws distributed according to Weibull probability distribution (Weibull, 1939)
  - Number of flaws *n* per unit volume that are activated by strains < ε

$$n(\varepsilon) = k\varepsilon^m$$

Numerical tests (first results)

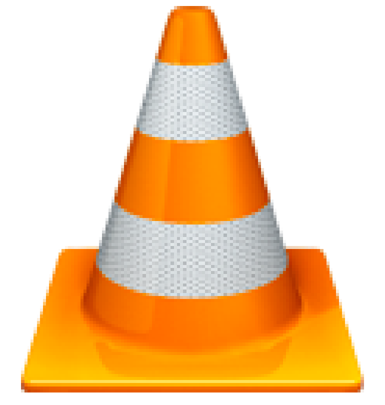
- Collisions of brittle bodies
- Murnaghan EOS with test values for K, n, and ρ_0
- Projectile:
  - Spherical, e.g. small asteroid
  - No flaws
- Target:
  - Spheroidal, e.g., irregular shaped small asteroid
  - Bigger than projectile
  - Half of the projectile density
  - Flaws according to Weibull constants
    $m = 8.5$, $k = 1.4\text{e}23 / \text{m}^3$

Numerical test geometry



Projectile r:	0.015, 0.5
Target a:	0.1
Target b:	0.05
Impact par.:	0.0, 0.05
α:	**0°**, 30°, 60°, **90°**
Impact vel.:	-0.1
(all SI units)	

Videos...



Outlook

- Immediate implementations
  - Measure fragmentation and merging
  - Realistic equation of state coefficients (Melosh, 1989)
- Goal: water in early planetary systems
  - What influence does water content have?
    - Fragmentation
    - Merging
  - Water in/on protoplanets
  - Influence of different water/ice distributions
  - Thermal effects beyond Murnaghan EOS

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Thank you!