Stability of the spatial restricted three-body problem & secondary resonances

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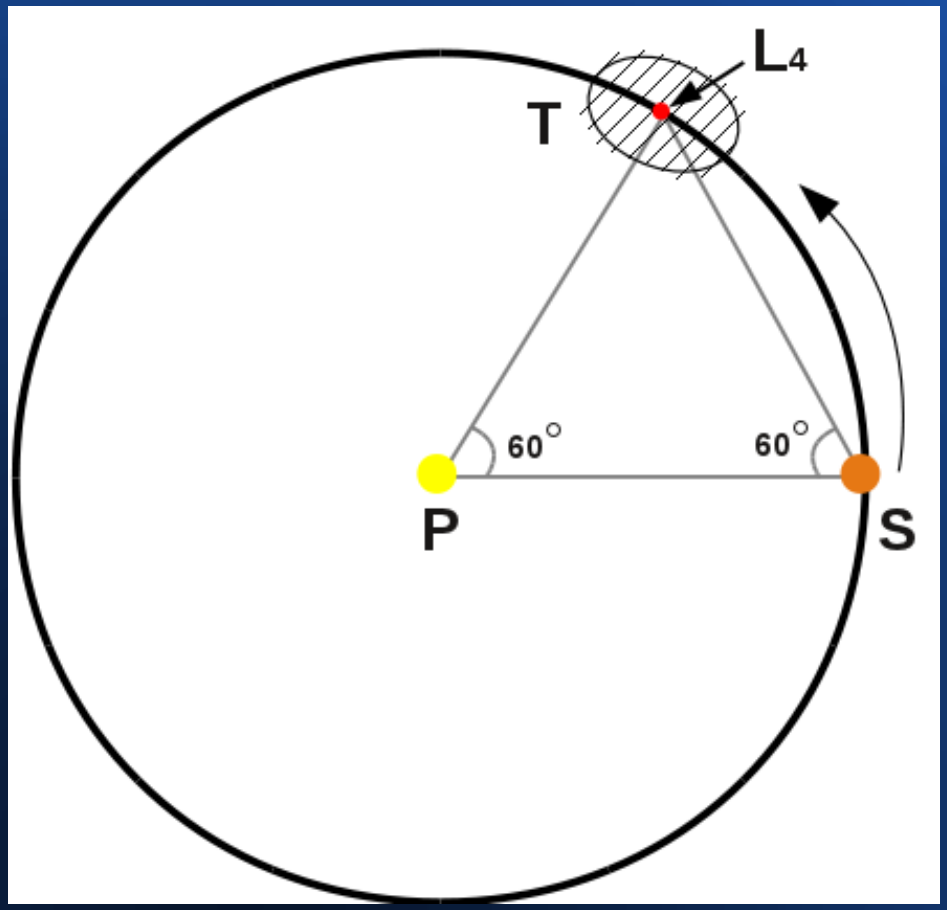
Schwarz et al., 2012, MNRAS (in press)

Outline

- Introduction
  - Restricted 3-Body-Problem (R3BP)
  - Applications
- Stability
  - CR3BP
  - ER3BP
- Resonances
- Summary

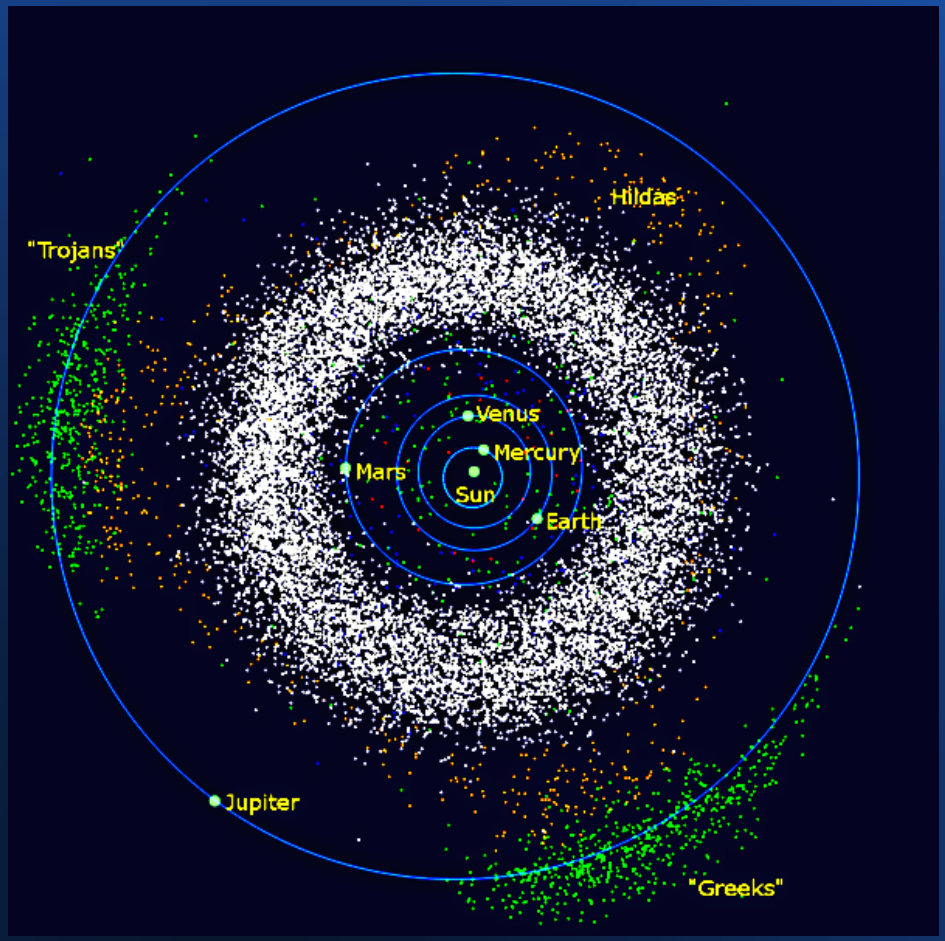
Configuration of R3BP



- 2 massive objects
  - Primary P
  - Secondary S
  - massless Trojan T
- circular (CR3BP) or elliptic (ER3BP) motion of S
- libration of T around Lagrange point L_4/L_5

Szebehely, 1967

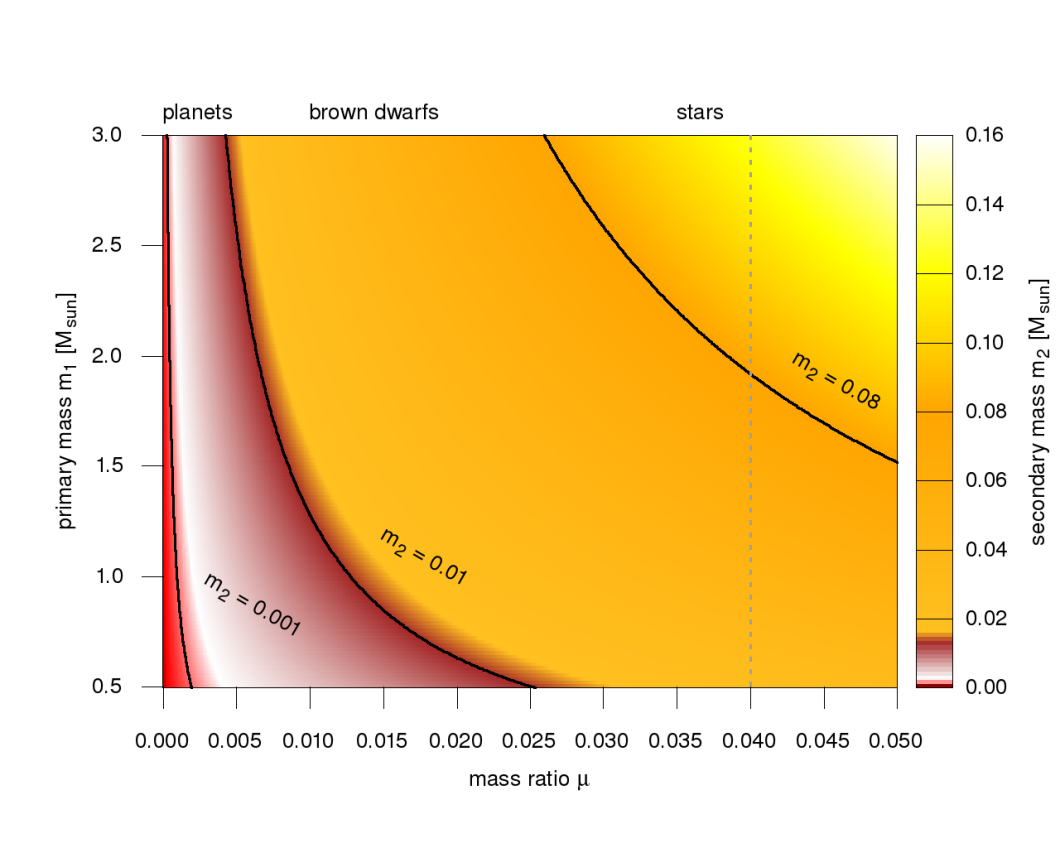
Applications of R3BP



- Trojans in the Solar System
  - Earth: 1 (2010TK7)
  - Mars: 3
  - Jupiter: > 5200
  - Neptune: 8

minorplanetcenter.net, Aug 2012

Applications of R3BP



- Extrasolar planetary systems
- ~70% of stars in solar neighborhood in binary/multiple star systems
- wide range of masses
- T-type motion

Stability

Stability depending on 3 parameters

- mass ratio S/(P+S), $\mu \lesssim 1/25$ (Gascheau, 1843)

$$\mu = \frac{m_2}{m_1 + m_2}$$

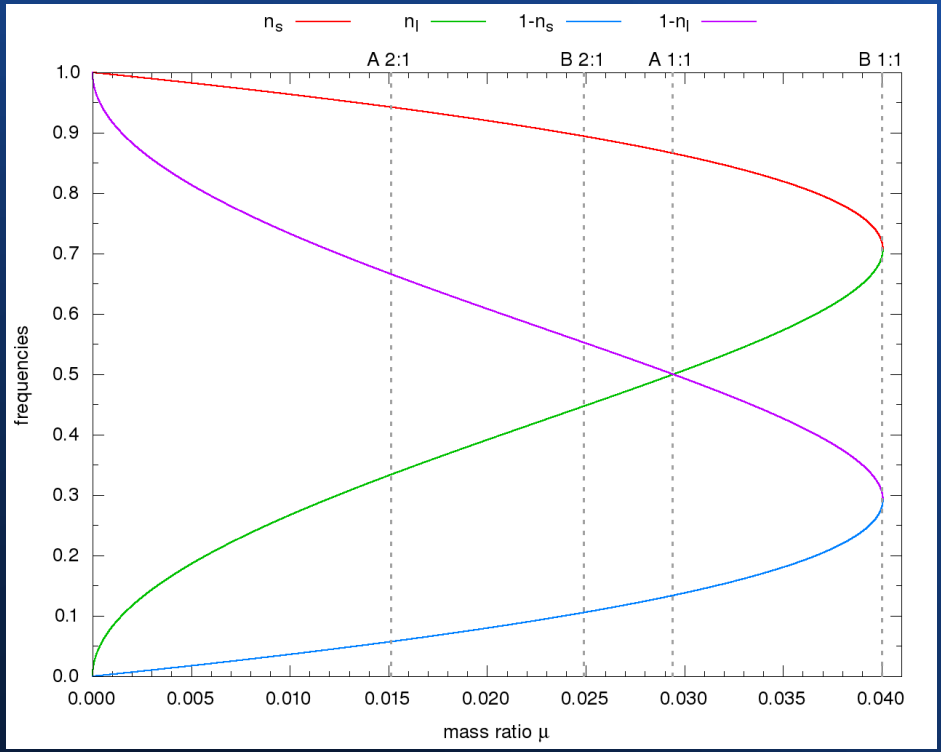
- inclination *i* of T, $i < 61.5°$ (Brasser et al., 2004)
- eccentricity *e* of S

Methods

- Numerical integration of equations of motion
  - mass ratio $0.0005 \leq \mu \leq 0.05$
  - inclination $0 \leq i \leq 90°$
  - eccentricity $0 \leq e \leq 0.99$
  - integration time 10^6 periods
- DFT / FFT analysis – Laplace-Lagrange variables

$$h = e\cos(\omega + \Omega) \qquad p = \sin i \cos \Omega$$
$$k = e\sin(\omega + \Omega) \qquad q = \sin i \sin \Omega$$

Secondary resonances

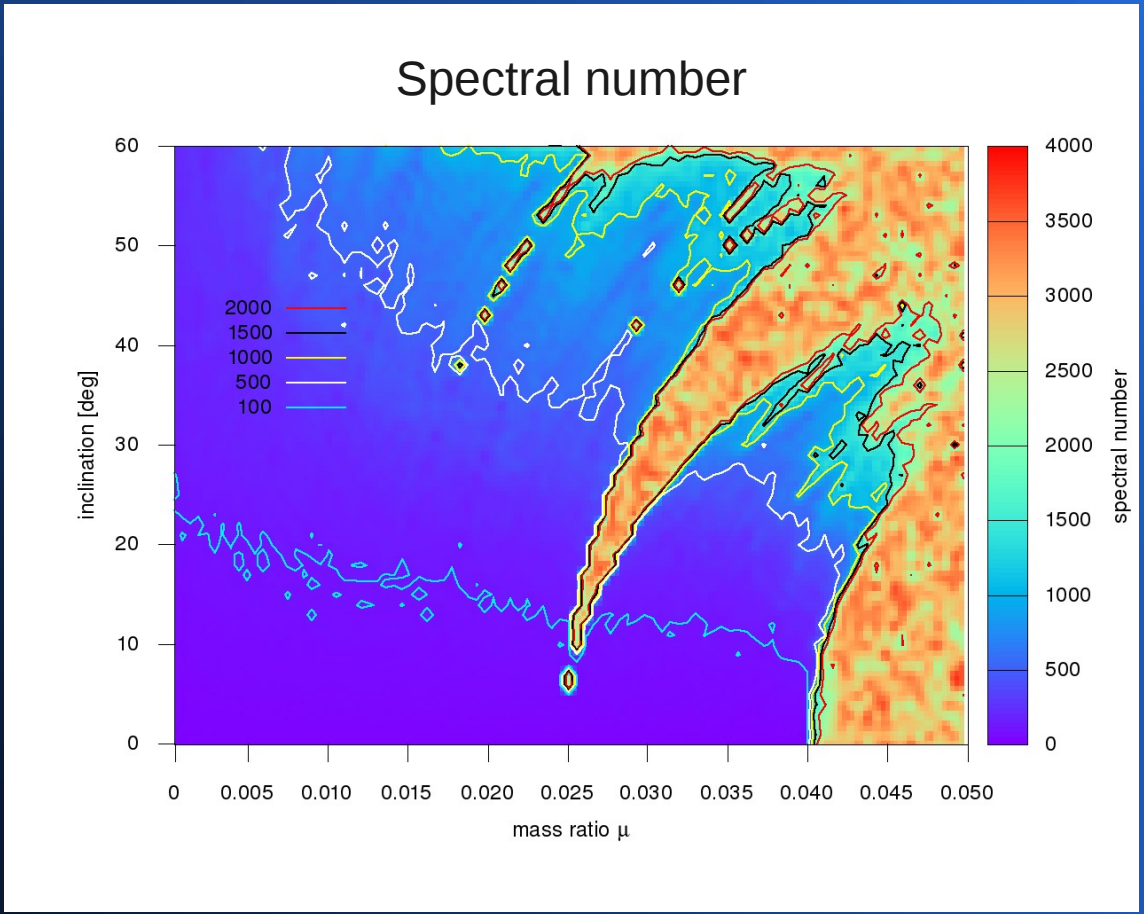


$$A = (1 - n_l) : n_l \qquad B = n_s : n_l$$
$$C = (1 - n_l) : (1 - n_s) \qquad D = n_s : (1 - n_l)$$
$$E = n_s : (1 - n_s) \qquad F = n_l : (1 - n_s)$$

- librational frequencies:
  - short period comp. n_s
  - long period comp. n_l
- orbital frequency of S: $n = 1$
- resonances for integer combinations of freq.

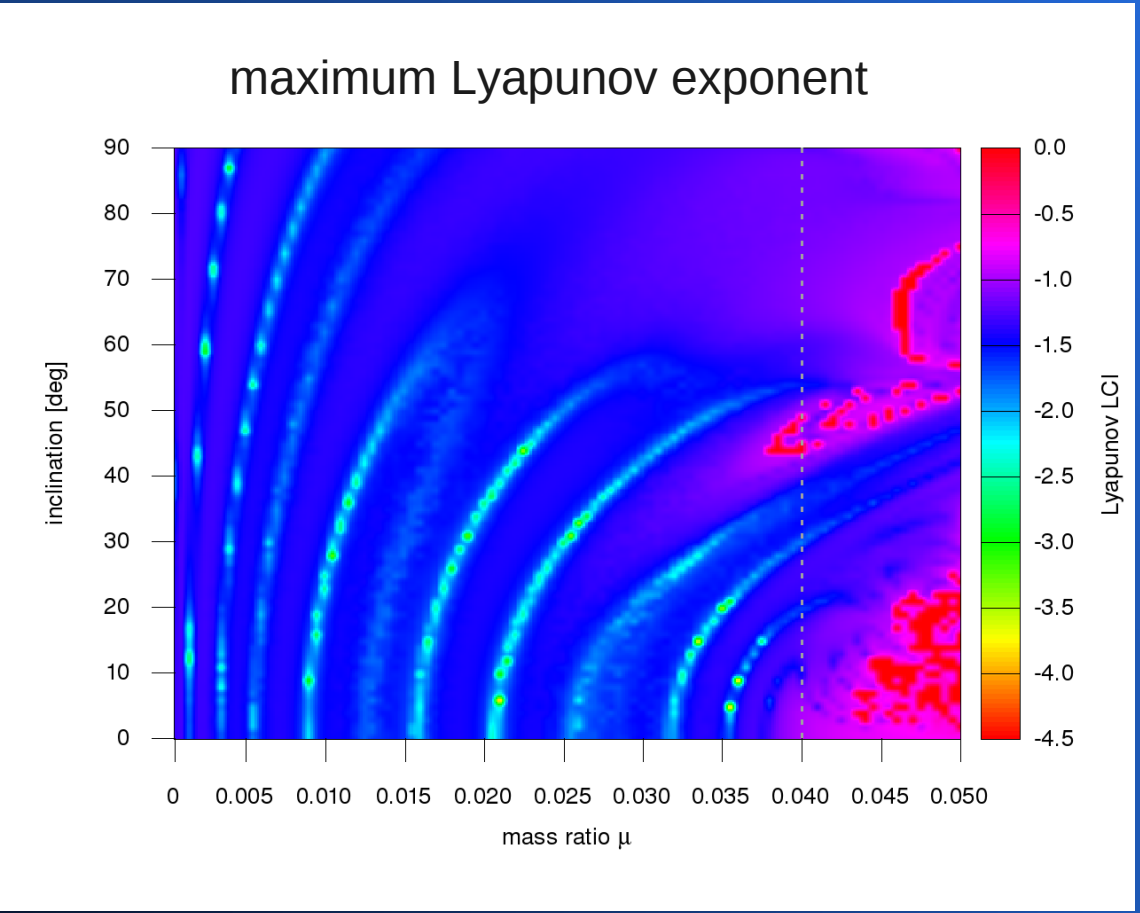
Érdi et al., 2007

Stability maps for CR3BP

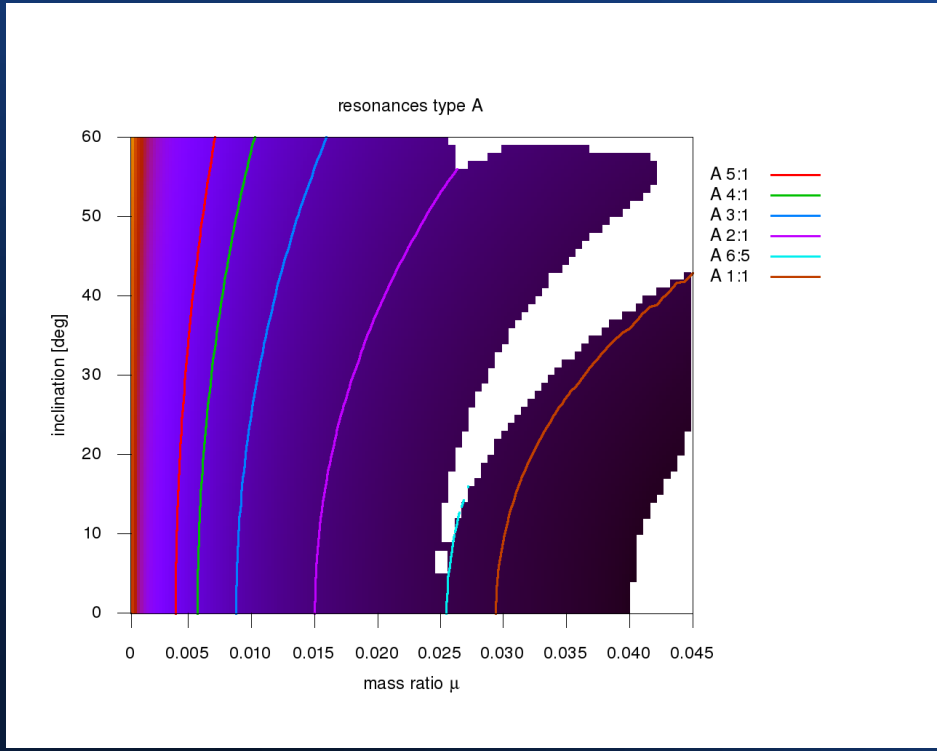


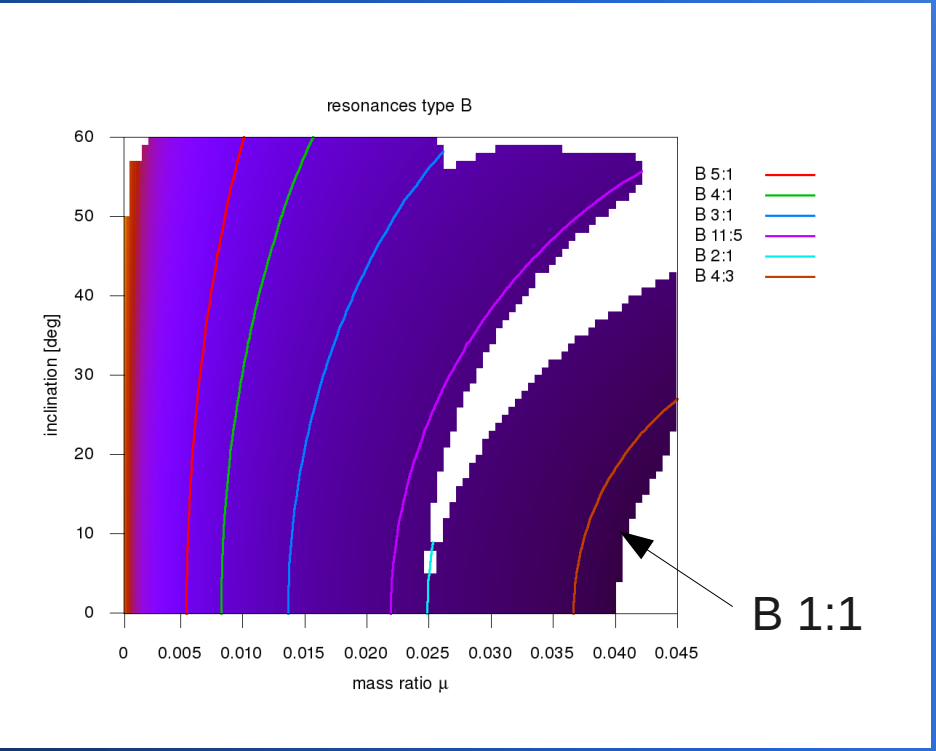
Zhou et al., 2009

Stability maps for CR3BP



Location of resonances





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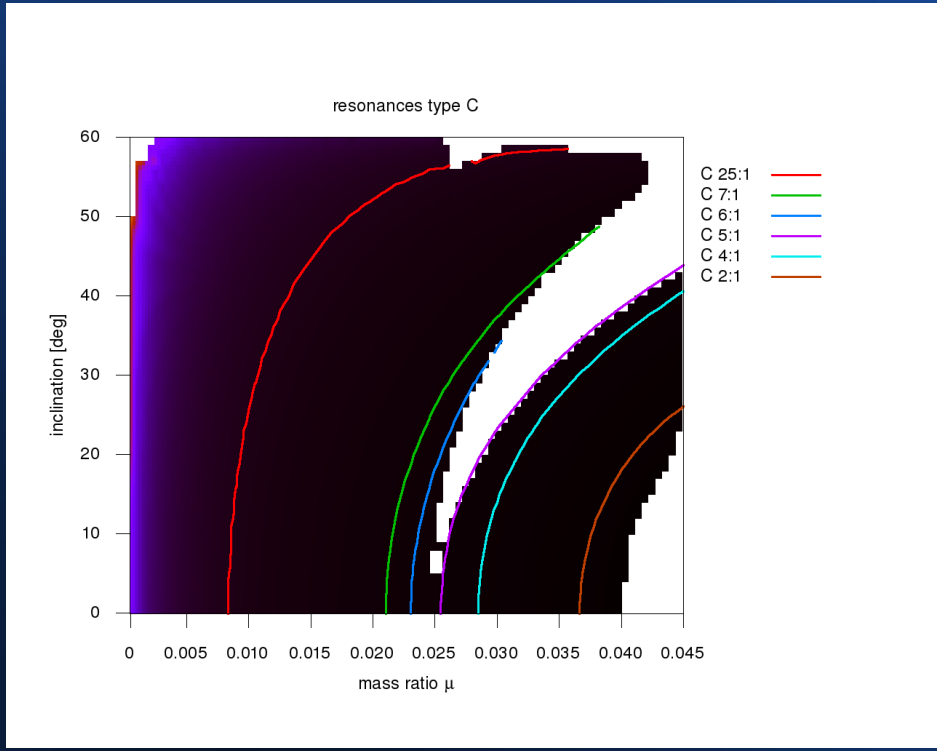
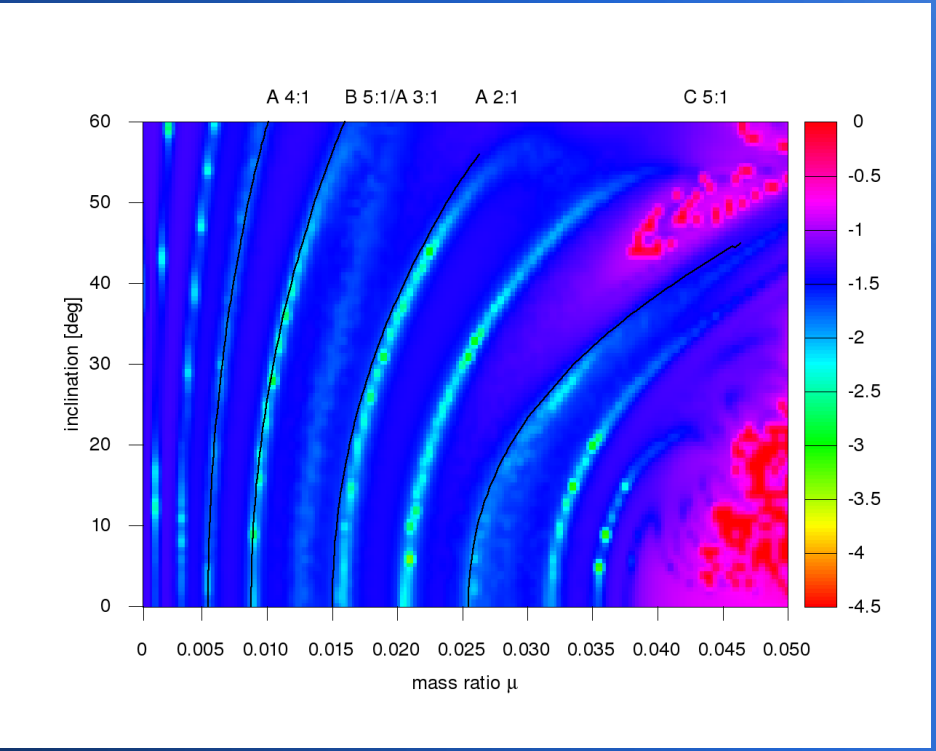
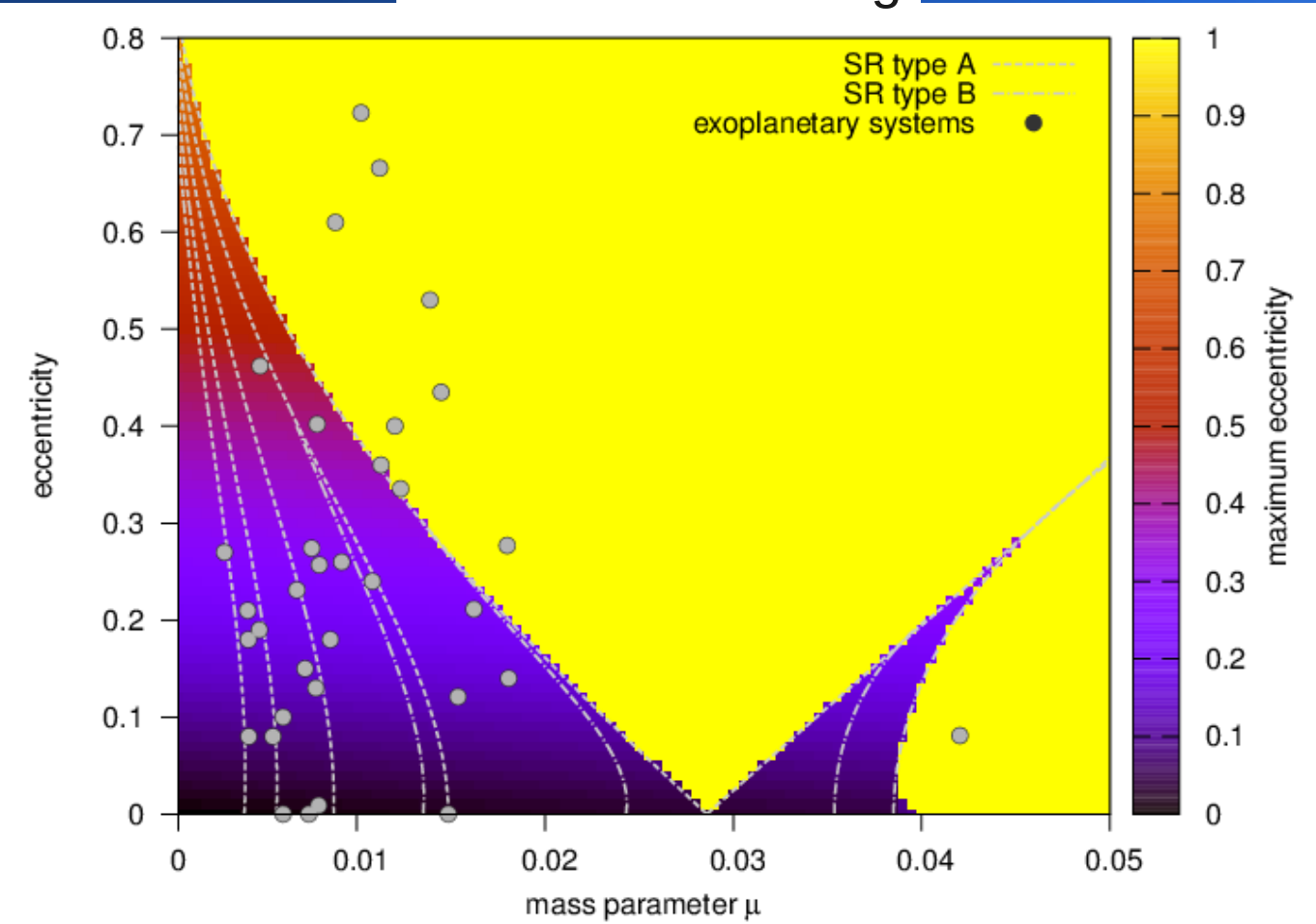




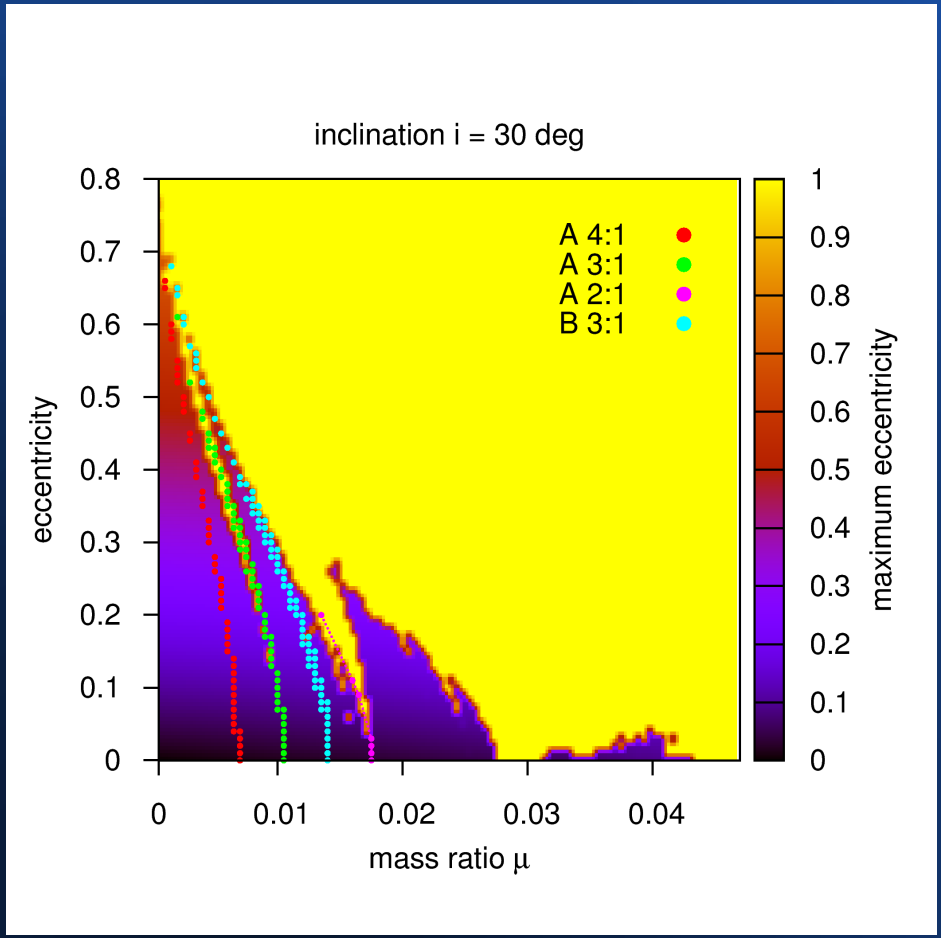
Stability maps for ER3BP

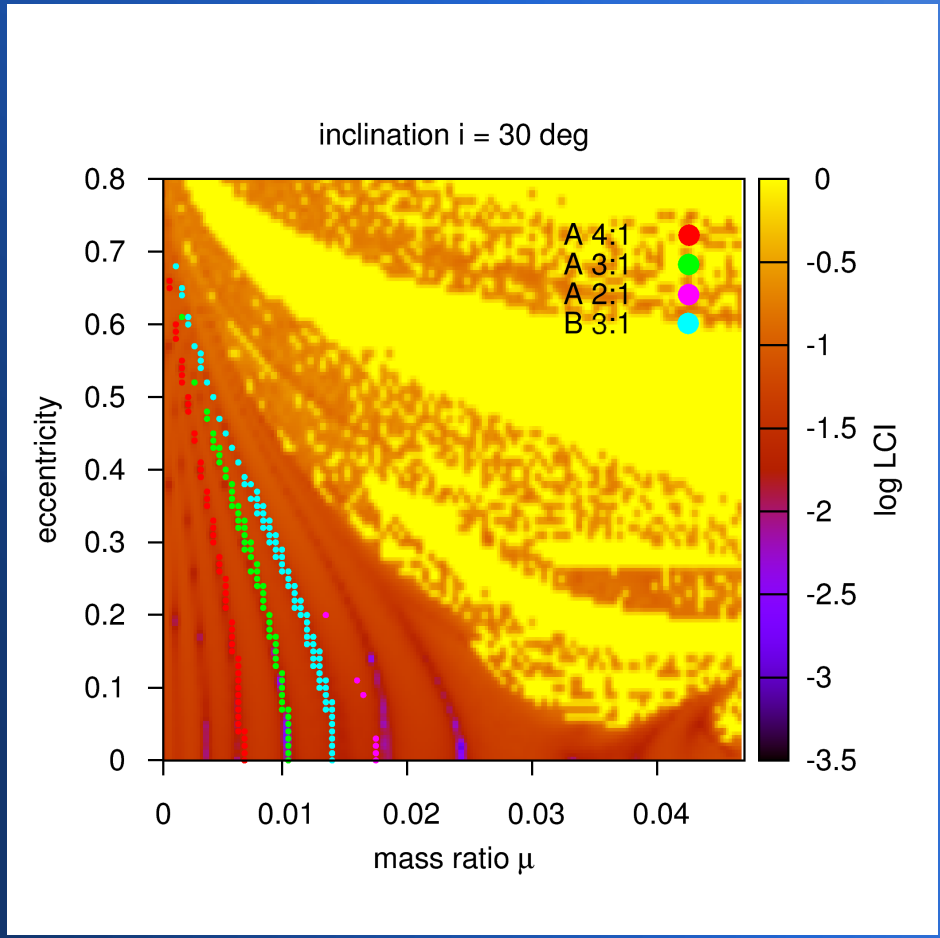
inclination i = 0 deg



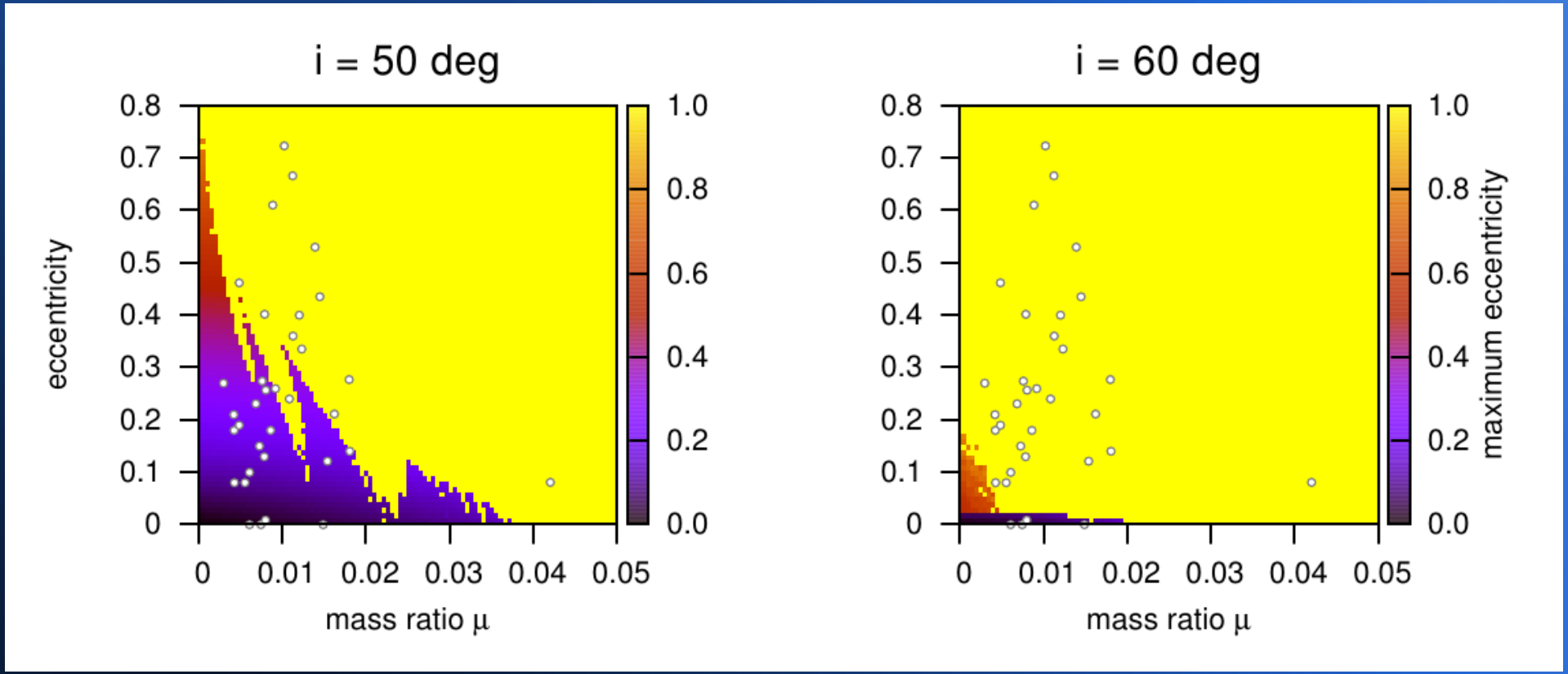
Érdi et al., 2009, Schwarz et al., 2012

Stability maps for ER3BP

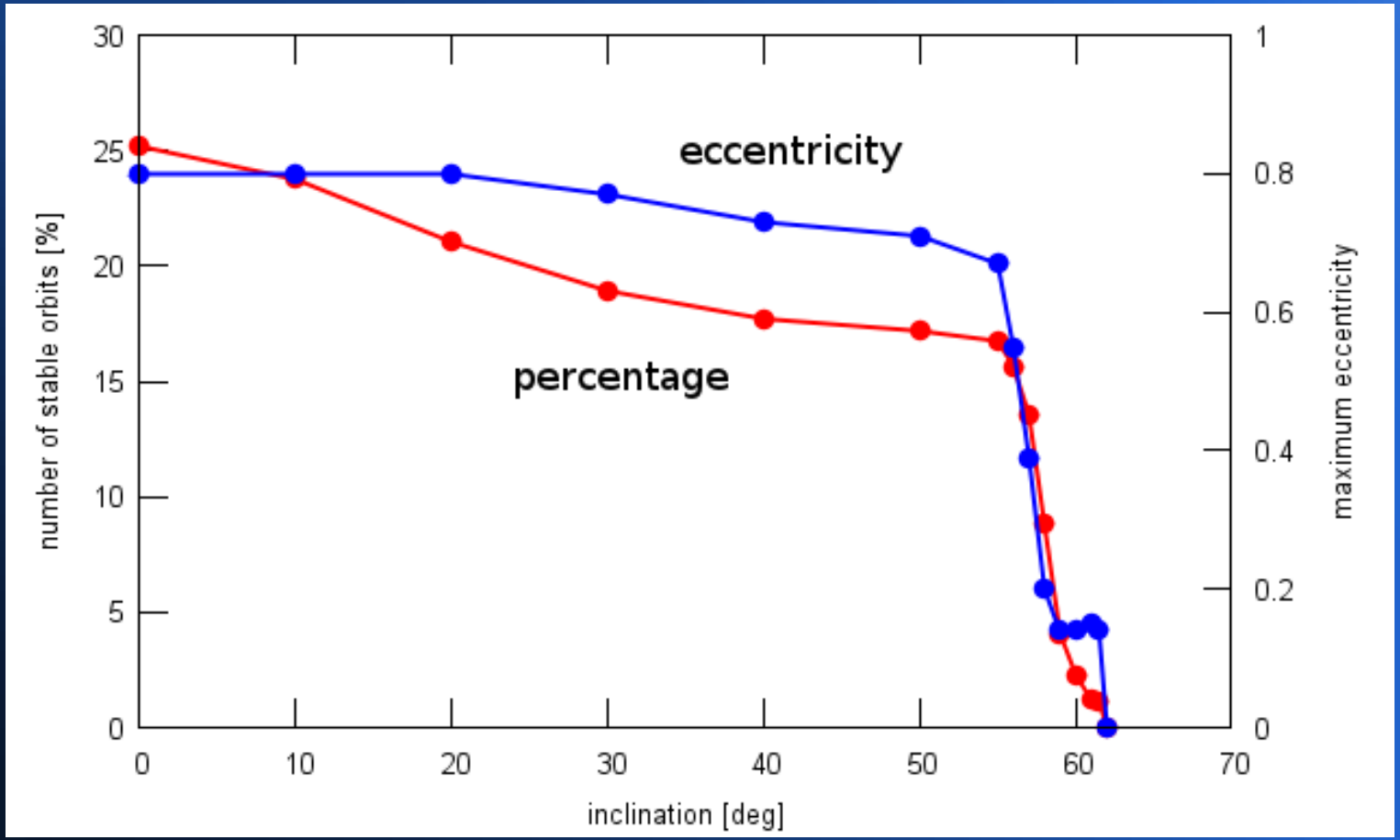




ER3BP at high inclination



Stability summary



Summary

- Circular R3BP
  - stable orbits at high inclinations
  - long-time stability for > 10^6 periods
  - secondary resonances related to chaotic regions
- Elliptic R3BP
  - stability region shrinks strongly with increasing inclination
  - model for possible exoplanetary systems