Elimination of long term variation from chaotic light curves

What type of stars can have chaotic light curve?

How to detect chaos?

Why do long term variations disturb us?

How to eliminate long term variations?

Emese Plachy (ELTE, Department of Astronomy) *FIKUT6*

Theory: chaos can be caused by interacting pulsating modes that are close to a resonance

Chaotic pulsators:

- RV Tauri Type stars: R Sct, AC Her
- Mira type star: R Cyg
- Semiregular variable stars: R UMi, RS Cyg, V CVn, UX Dra, SX Her
- W Vir stellar models
- RR Lyrae stellar models ***new!***

Sign of period doubling bifurcation:

- RR Lyrae stars (Kepler Space Telescope) ***new!***
- BL Her stellar models and stars ***new!***
- White dwarf stars

How do we detect chaos in light curves?

Global flow reconstruction:

$g(t_n)$ time series: the luminosity of the star = **input**

$\boldsymbol{X}^n = \{ g(t_n), g(t_n-\Delta), \ldots, g(t_n-(d_e-1)\Delta)\}$ a „delay vector” that represents the flow in the phase space: trajectory

d_e :embedding dimension (4, 5, 6)

Δ: delay

$\boldsymbol{F}=\{F_1, F_2, \ldots, F_{de}\}$ operator (map)

connects the neighbouring points: $\boldsymbol{X}^{n+1}=\boldsymbol{F}(\boldsymbol{X}^n)$

--> **synthetic light curve = output**

--> quantitative dynamical properties:

$$d_l = K + \frac{1}{|\lambda_{K+1}|}\sum_{i=1}^{K} \lambda_i$$

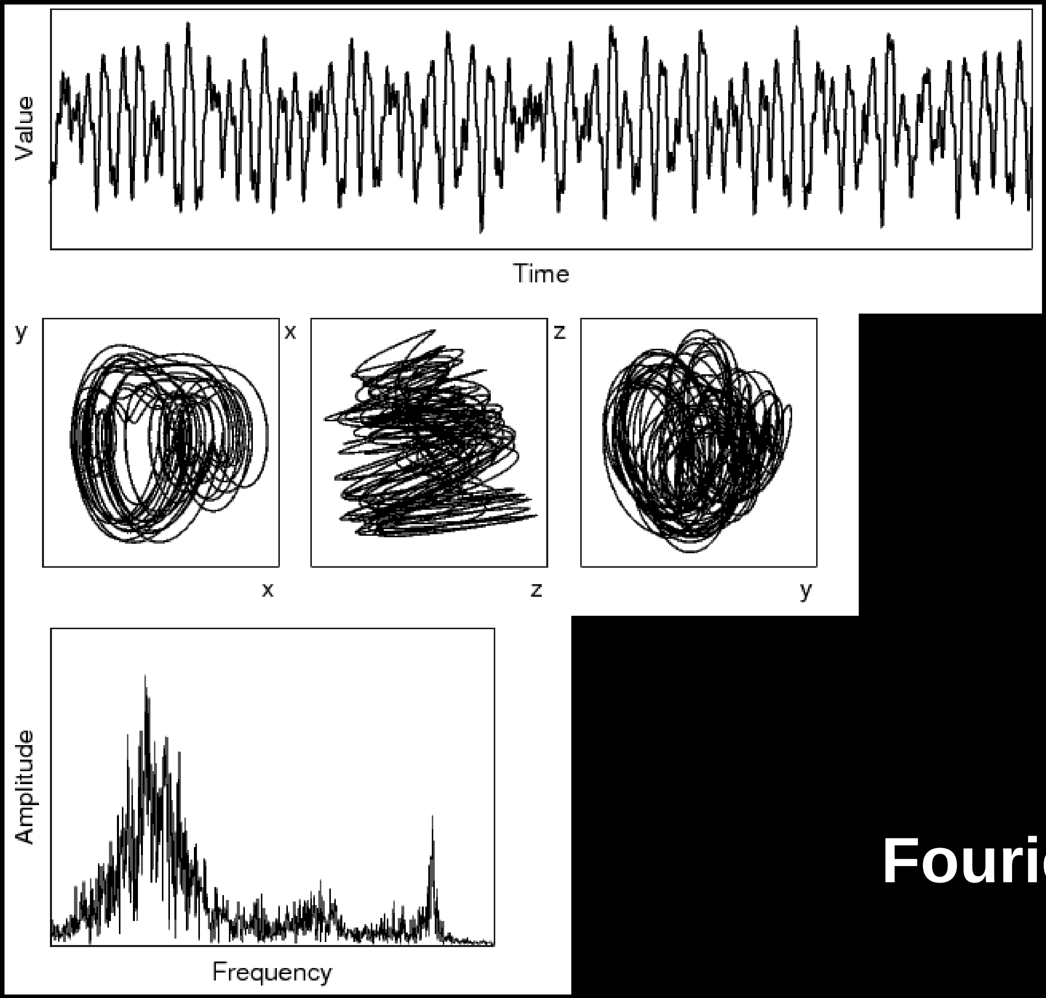
d_l : Lyapunov dimension,

λ_i : Lyapunov exponents, the divergence of neighbouring trajectories

Comparison of input and output data

The reconstruction is successful if the output synthetic light curve is similar to the original input light curve.

Chaotic example: (coupled Rössler oscillators)



Shape of the light curve

Broomhead-King Projection

Fourier Spectrum

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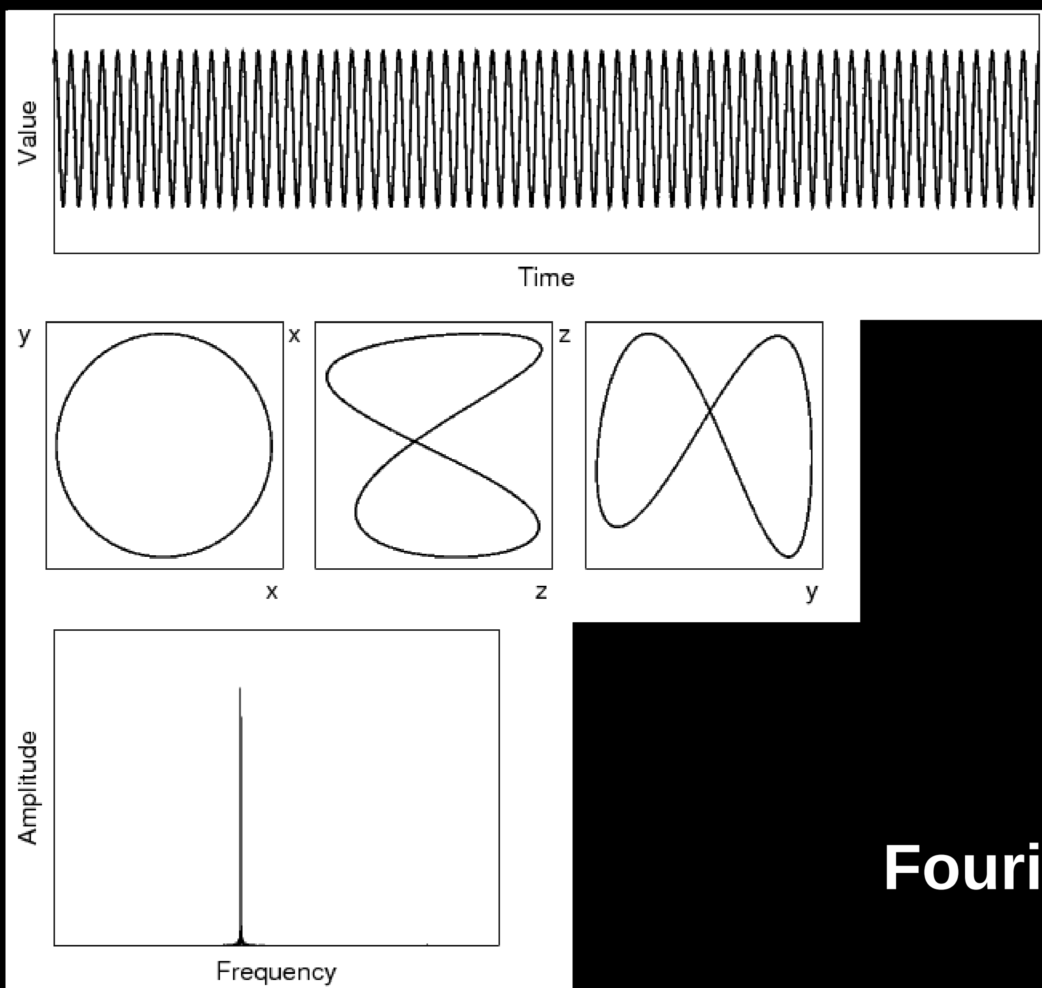

Shape of the light curve

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Fourier Spectrum

Requirements for input time series

Long (i. e. dozens of cycles) and equally spaced noiseless and continous data with optimal sampling (~100 point/cycle)

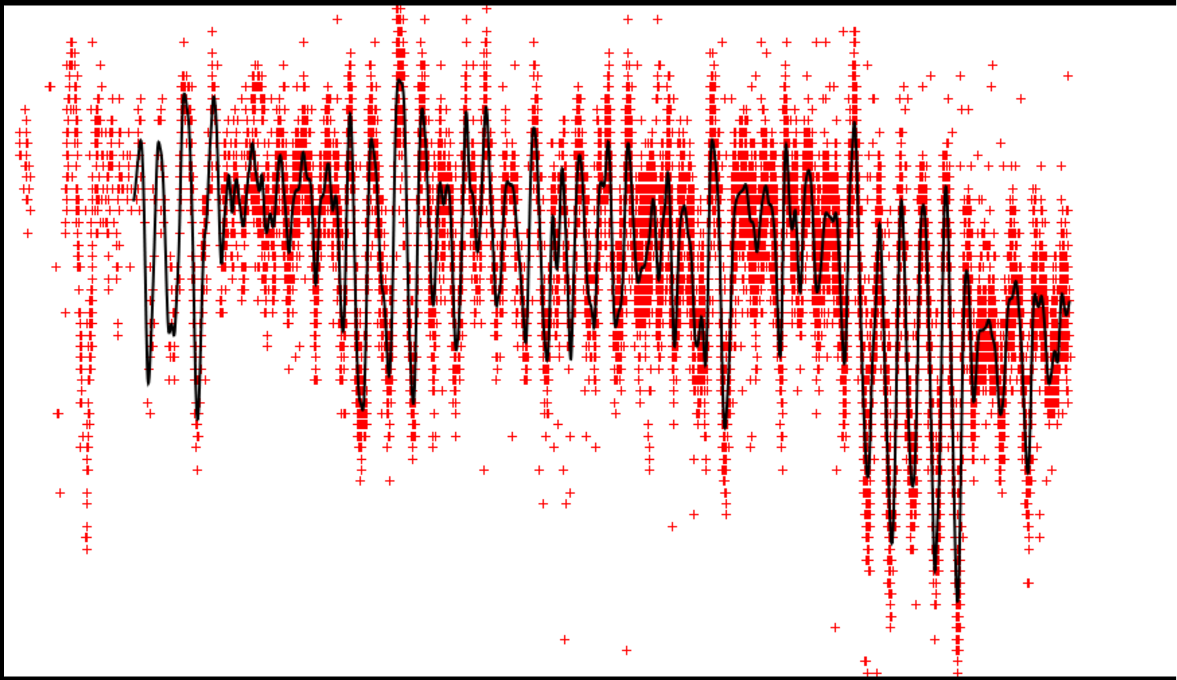
Visual light curves of semiregulars of the AAVSO

(Americanc Association of Variable Star Observers)

100 years of data

(tipical period month -years)

noisy, uneven



--> Resample:

Averaging, Spline smoothing

Gaussian smoothing

The main characteristics should not change!

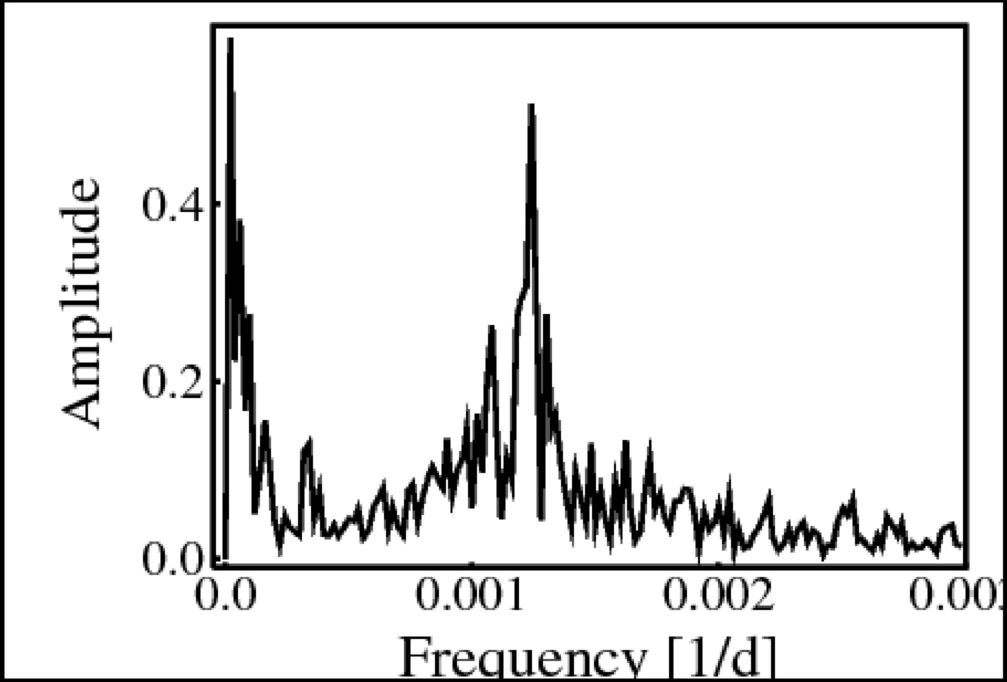
Trends & long term variations

Most probably not connected to the oscillation:

evolutionary changes or surroundings changes (mass loss, stellar wind, dust clumps)

Effects:

- on Fourier Spectra: low frequency peaks



- on Global Flow Reconstruction:

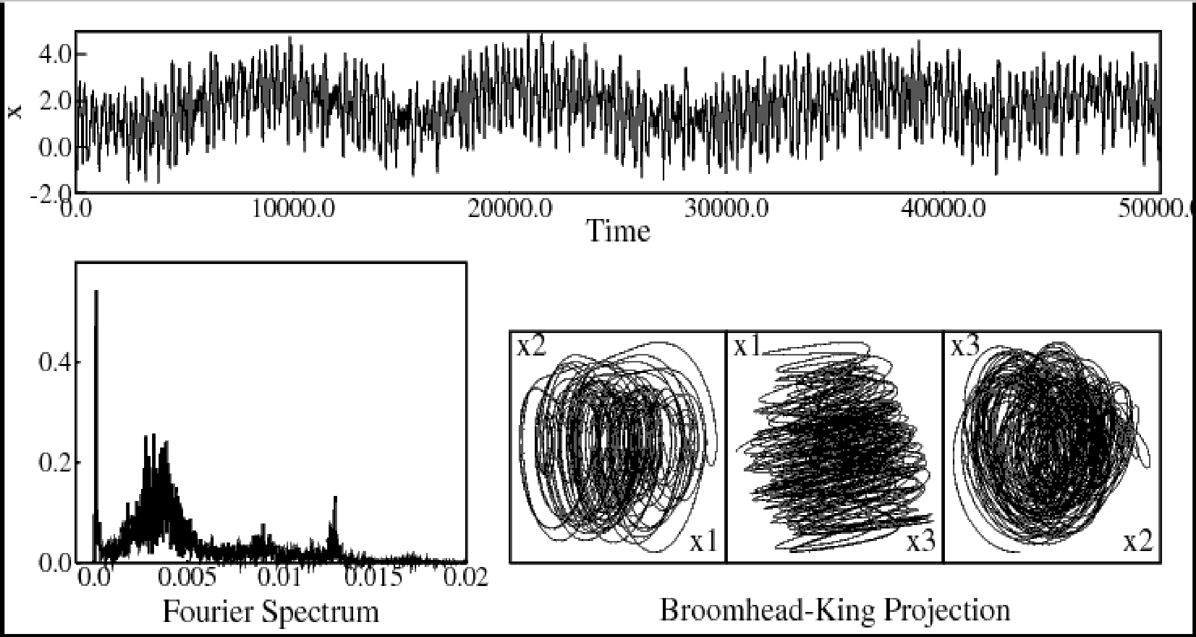
  extra dimensions

Reconstruction of test data with long term variation

Input:

coupled Rössler oscillator

+ extra variation



Tipical output:

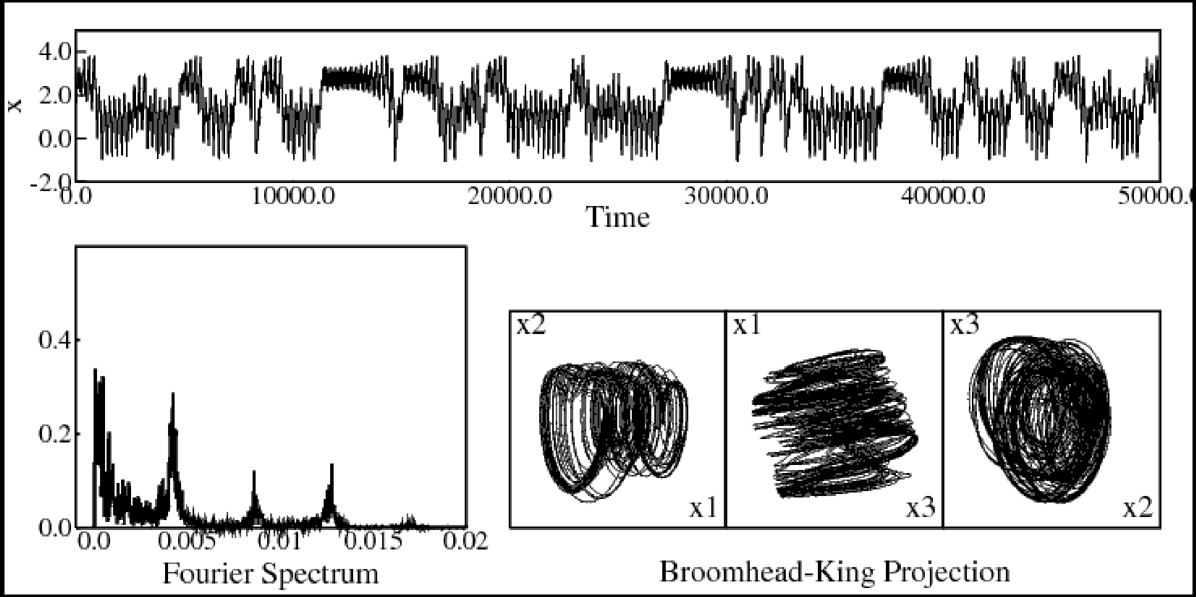
chaotic signal but <u>not similar</u>

Statistics:

(varying d_e ,Δ,

smoothing parameters)

27 % of synthetic signals are useful



Eliminating trends & long term variations

- Polinomial fitting method ~ eliminating atmospheric extinction
- Fourier filtering ~ high-pass frequency filter
- Empirical Mode Decomposition (EMD) method

~ decompose the signal into so-called Intrinsic Mode Functions (IMF)

works well for nonstationary and nonlinear data

Some examples for applications:

- image processing
- ocean engineering
- meteorological and atmospheric applications
- seismic studies
- neuroscience experiments
- solar physics: long-term variations of the coronal rotation and solar activity

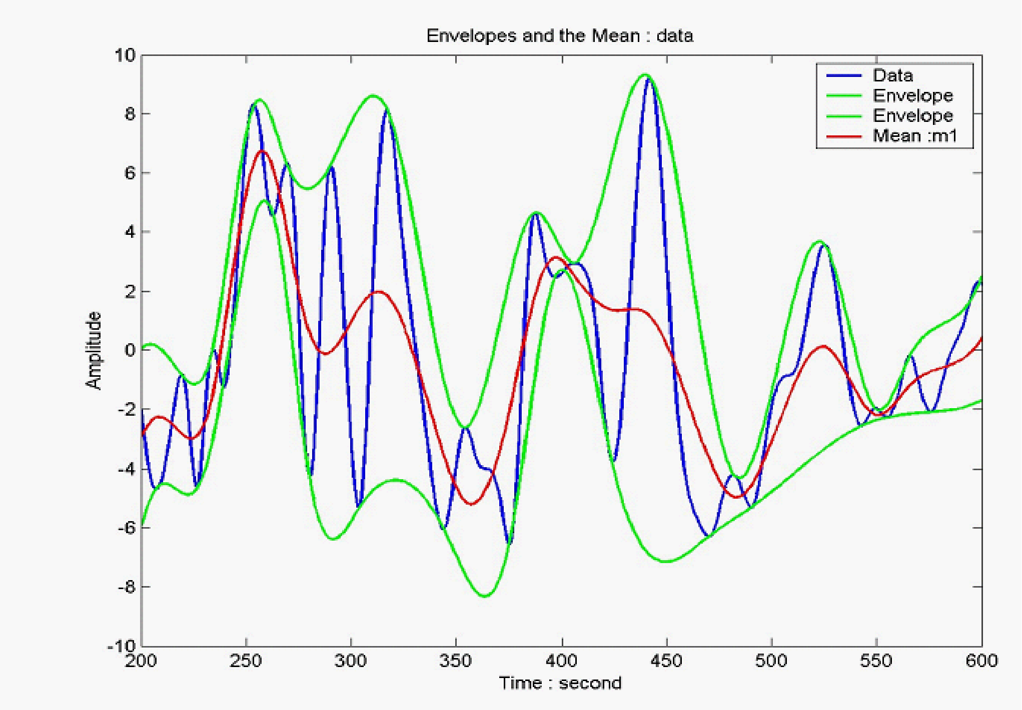
Empirical Mode Decomposition (EMD)

Procedure:

1. Identify local extrema
2. Connect all the local maxima by a cubic spline line: upper envelope.
3. Repeat it for the local minima: lower envelope.
4. The mean of the envelopes is m_1.
5. We extract the mean envelope

   from the original data:

   $$X(t) - m_1 = h_1$$



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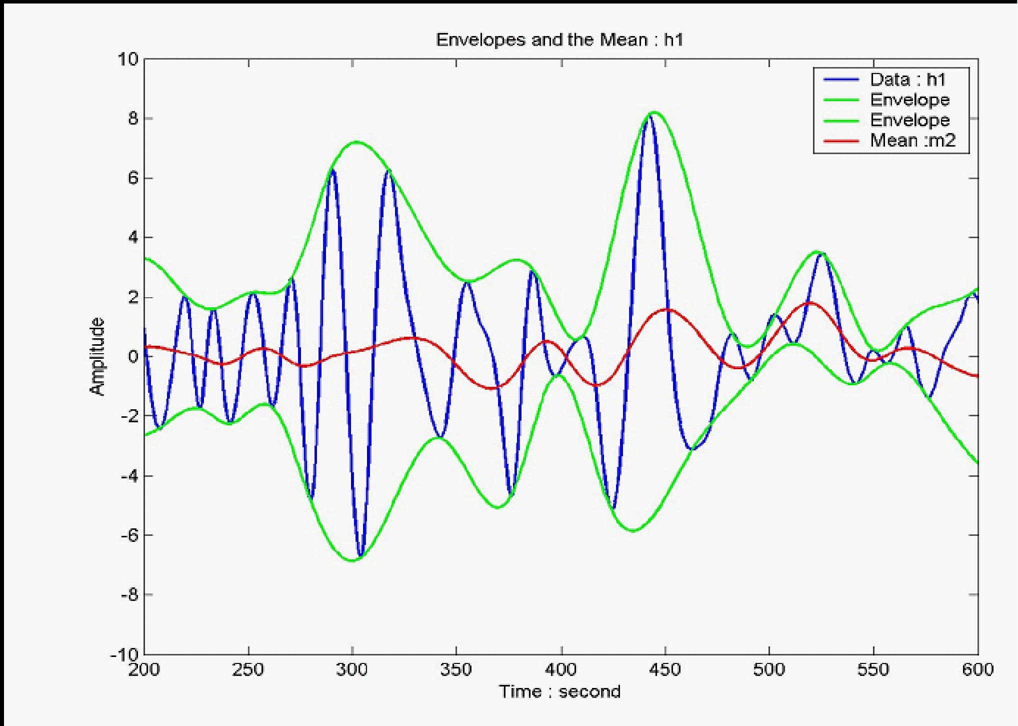
Repeat 1.-5.

$h_1 - m_{11} = h_{11}$ .... $h_{1(k-1)} - m_{1k} = h_{1k}$

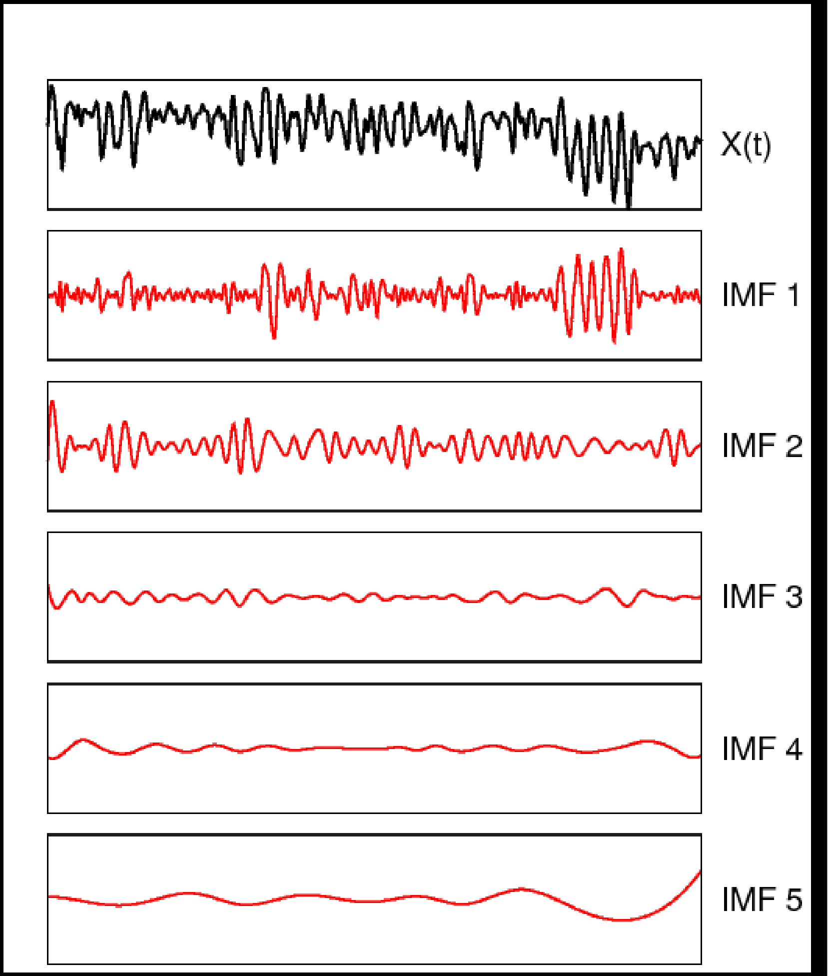
repeat till h_{1k} satisfy the definition of an IMF



Intrinsic Mode Function (IMF)

- The number of extrema and the number of zero-crossings must either be equal or differ at most by one.
- At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

$IMF_1 = h_{1k}$ first IMF component



We separate IMF 1 from the data:

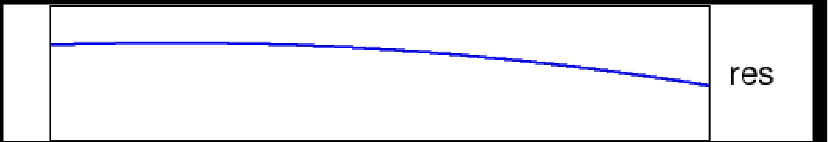
$X(t) – IMF_1 = r_1$

Repeat procedure with the rest of the data:

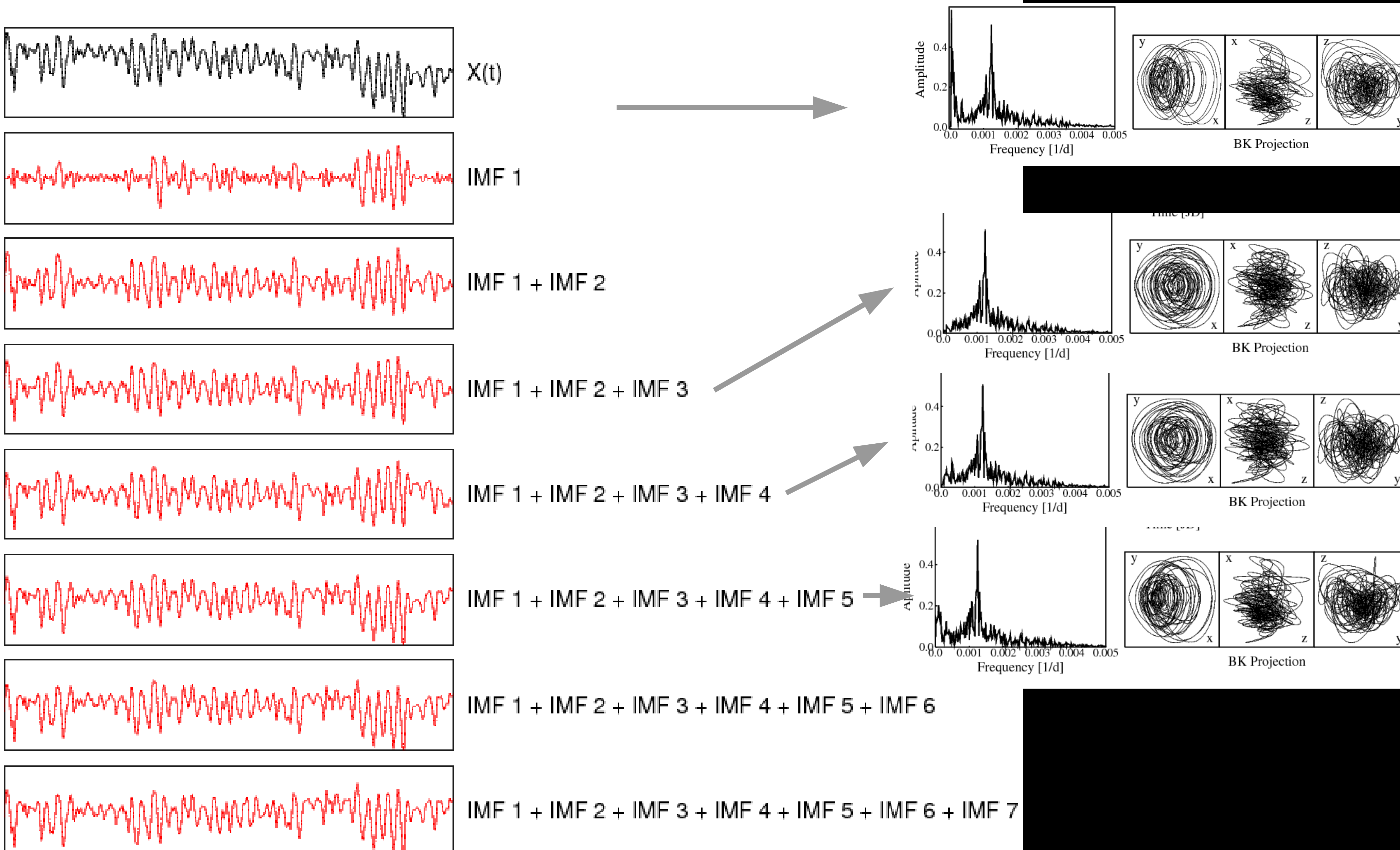
$r_{n-1}-IMF_n = r_n$

Process stops if r_n becomes monotonic

---> $X(t) = \sum IMF_j + r_n$



Adding IMFs together

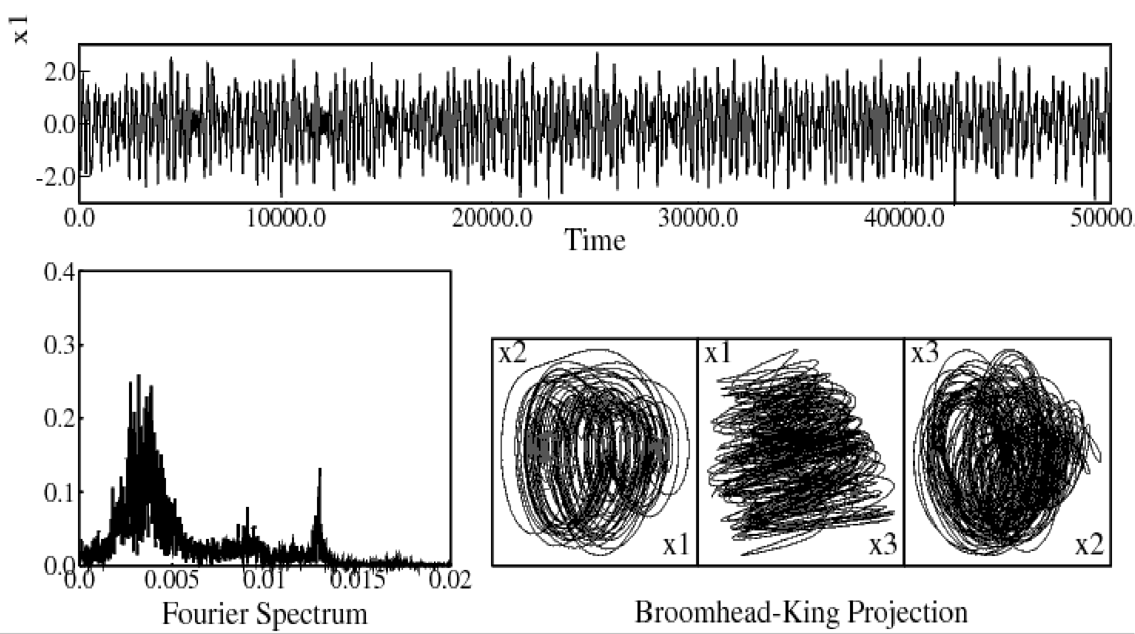


Application of EMD for reconstructions

$IMF_1+IMF_2+IMF_3$

76% of synthetic signals

are „good”



Conclusion:

Elimination of long term variation with EMD method can help to detect chaos with the global flow reconstruction.

Future work

- Further tests (with different long term variations)
- Compare EMD to other methods (Fourier filtering)
- Apply on real light curves