Gravitational waves of spinning and eccentric binary sources

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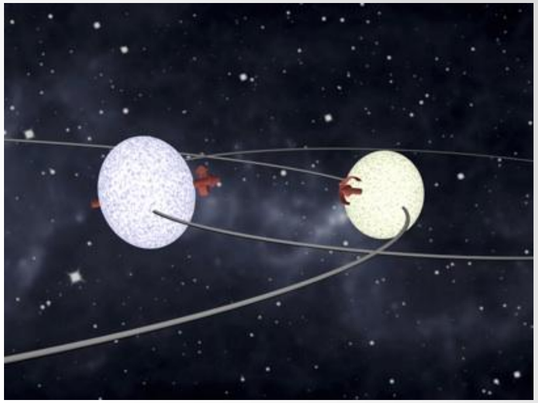
Introduction

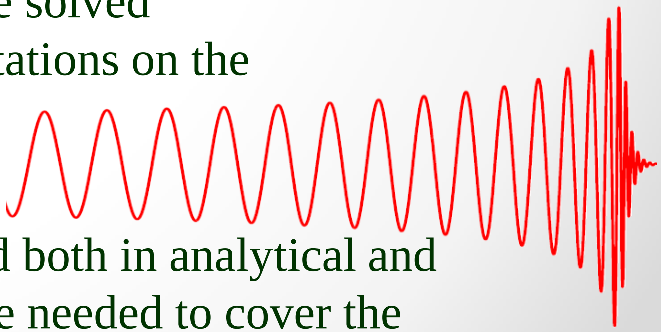


- GR is non-linear, no hope for analytic solutions describing GW emission.

- In the post-Newtonian approximation the velocities of the source and the gravitational potential are treated perturbatively,
    - the PN field equations are solved in the *near* and *wave zones*
    - in the overlap of these regions the solutions are matched to each other

  ➢ the radiation field far from the source is expressed in terms of integrals over the source

- In numerical relativity the nonlinear field equations are solved without limitations on the speed of the source but limitations on the computational power.



- GWs from the three stages of inspiral can be computed both in analytical and numerical relativity. Sufficiently accurate templates are needed to cover the largest possible parameter domain.

CBwaves – motivations

- Our main purpose to develop was
    - to provide a fast and accurate computational tool to determine the gravitational waveforms yielded by generic spinning binaries of neutron stars and/or black holes on eccentric orbits.
    - The current version of CBwaves involves all the possible terms – in the acceleration, radiation field and energy – within the applied PN approximation for an accurate description of these type of generic binary configurations.

CBwaves – basic features

- Analytic setup within the PN framework, synthesis of all the recent developments.

- The 3.5PN accurate equations of motion are integrated, 4th order Runge-Kutta scheme

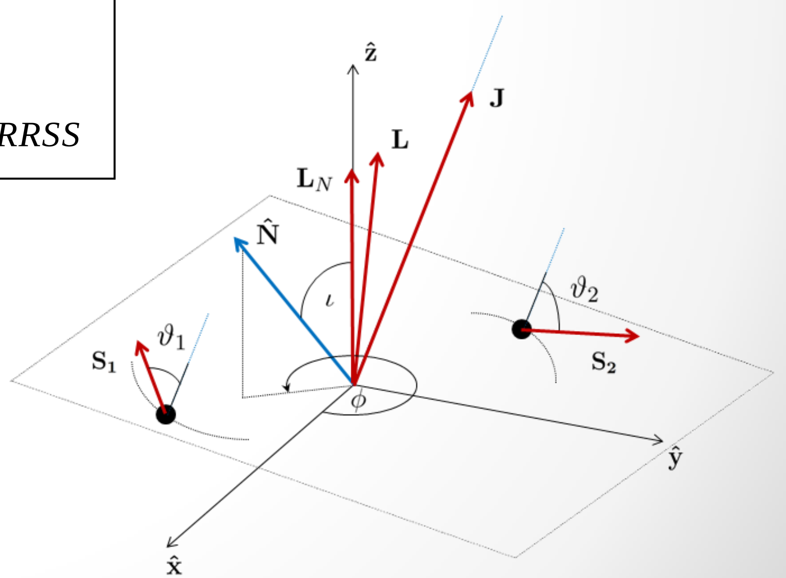
$$\mathbf{a} = \mathbf{a}_N + \mathbf{a}_{PN} + \mathbf{a}_{SO} + \mathbf{a}_{2PN} + \mathbf{a}_{SS} + \mathbf{a}_{RR} + \mathbf{a}_{PNSO} + \mathbf{a}_{3PN} + \mathbf{a}_{RR1PN} + \mathbf{a}_{RRSO} + \mathbf{a}_{RRSS}$$

- Spin precession

$$\begin{aligned}\dot{\mathbf{S}}_i &= \frac{G}{c^2 r^3}\Big\{\frac{4+3\zeta_i}{2}\mathbf{L}_N - \mathbf{S}_j + 3\,(\hat{\mathbf{n}}\cdot\mathbf{S}_j)\,\hat{\mathbf{n}} \\ &+ \frac{m_i^2}{8c^2m^2}\Big[\left(16+22\zeta_i+10\zeta_i^2+\zeta_i^3\right)v^2 \\ &-2\left(4+7\zeta_i+7\zeta_i^2+2\zeta_i^3\right)\frac{Gm}{r} - 6\,(4+3\zeta_i)\,\dot{r}^2\Big]\mathbf{L}_N \\ &+ \frac{G^2\mu m}{c^5 r^2}\left[\frac{2}{3}\,(\mathbf{v}\cdot\mathbf{S}_j) + 30\dot{r}\,(\hat{\mathbf{n}}\cdot\mathbf{S}_j)\right]\hat{\mathbf{n}}\Big\}\times\mathbf{S}_i\end{aligned}$$

Contributions to the acceleration

- L. Kidder (95)
  2PN with spins, radiation reaction at 2.5PN
- H. Tagoshi et.al. (01), G Faye et.al. (06)
  1PN correction to SO
- T. Mora and C. M. Will (04) 3PN
- B. R. Iyer and C. M. Will (95)
  3.5PN radiation reaction
- C. M. Will (05) , H. Wang and C. M. Will (07)
  SO and SS contributions to radiation reaction



CBwaves – basic features

- The radiation field is determined by the simultaneous evolution of the analytic waveforms – all the contributions are included that have been worked out for generic eccentric orbits up to 2PN order

Contributions to the waveform

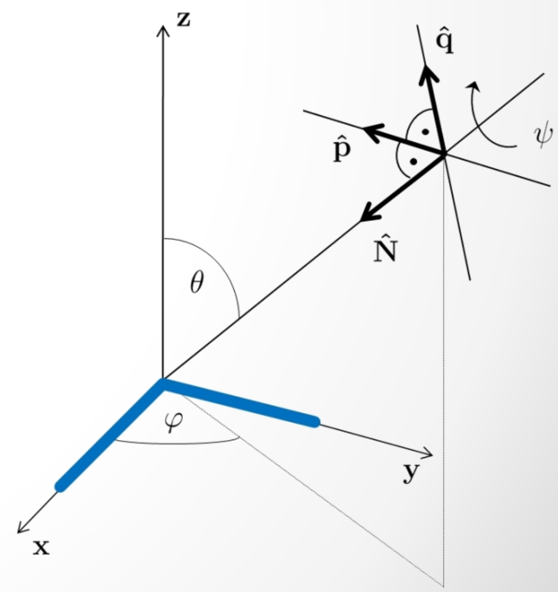
- R. V Wagoner and C. M. Will (76,77)
  0.5 and 1PN correctons to Q_{ij}
- A. G. Wiseman (92) 1.5PN Q_{ij} term
- L. Kidder (95) spin terms
- C. M. Will and A. G. Wiseman (96)
  2PN contribution to Q_{ij}

$$h_{ij} = \frac{2G\mu}{c^4 D}\left(Q_{ij} + P^{0.5}Q_{ij} + PQ_{ij} + PQ_{ij}^{SO} + P^{1.5}Q_{ij} + P^{1.5}Q_{ij}^{SO} + P^2 Q_{ij} + P^2 Q_{ij}^{SS}\right)$$

- The strain is calculated from the polarization states with respect to the orthonormal triad (radiation frame) $(\hat{\mathbf{N}}, \hat{\mathbf{p}}, \hat{\mathbf{q}})$

$$h_+ = \tfrac{1}{2}(\hat{p}_i\hat{p}_j - \hat{q}_i\hat{q}_j)h_{ij}^{TT}, \quad h_\times = \tfrac{1}{2}(\hat{p}_i\hat{q}_j + \hat{q}_i\hat{p}_j)h_{ij}^{TT}$$
$$h(t) = F_+ h_+(t) + F_\times h_\times(t)$$

- Waveforms are calculated in time and frequency domain (implemented FFT).
- Provides expansion of the radiation field in s = -2 spin weighted spherical harmonics.



CBwaves – basic features

- The most important input parameters
  - separation, masses, spins and their orientation, initial eccentricity
- In order to make the submission of the software to research clusters a Condor job description file generator script is provided along with man pages, rpm package, etc.
- The source code of the current version of CBwaves can be downloaded

http://grid.kfki.hu/project/virgo/cbwaves/

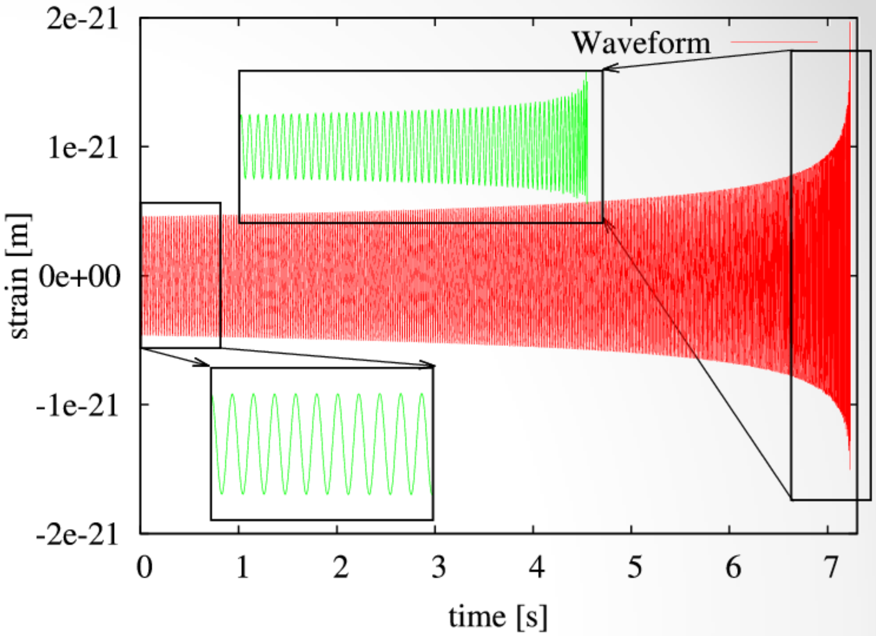
- For a similar GW generating software – for generic spinning binaries moving possibly on eccentric orbits – see Levin et.al.
  - equations of motion up to 3.5PN
  - some contributions related to the generic motion of the binary is missing
  - waveforms to quadrupole order

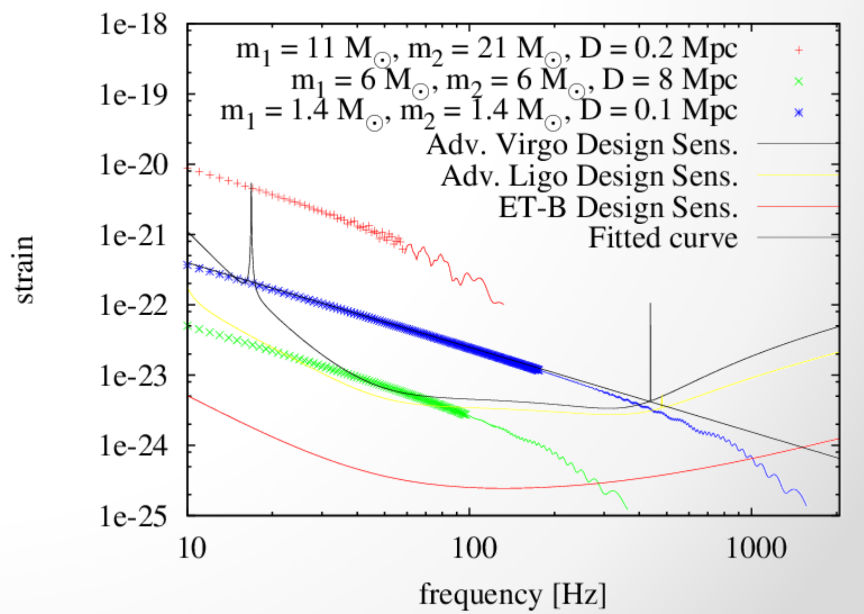
J. Levin et.al.
CQG **28**, 175001 (2011)

Circular orbits

- Circular non-spinning waveforms
  - slow rise in the amplitude and the frequency
  - Comparison with phenomenological waveforms

- The interval while the source is visible by the advanced detectors is significantly larger than for the current ones.
  - The 40 Hz lower frequency cutoff of the current detectors limits the signal duration of a 1.4 - 1.4 $M_\odot$ binary to ≈ 26s, while from 10 Hz relevant for the advanced detectors the signal from the same binary spends ≈ 950s.
  - Validity range of the PN approximation





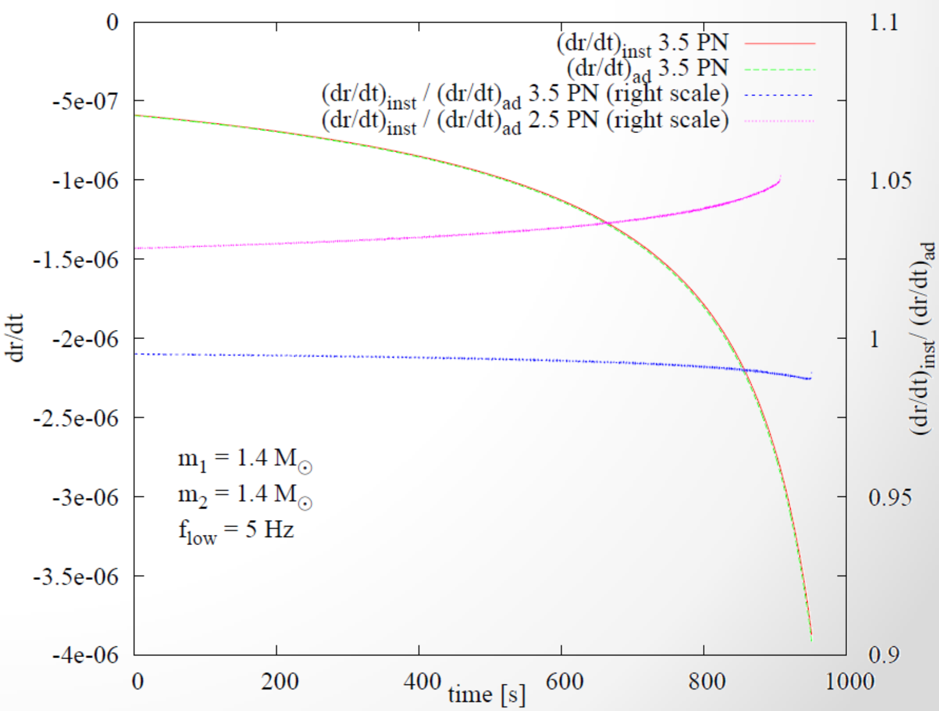
Circular orbits

- The adiabatic approximation is expected to hold whenever the time scales of precession and shrinkage of the orbit are long compared to the orbital period,
  - quasi-circular inspiral orbits

$$\frac{dr_{ad}}{dt} = \frac{dE/dt}{dE/dr} \qquad \frac{dr}{dt}$$

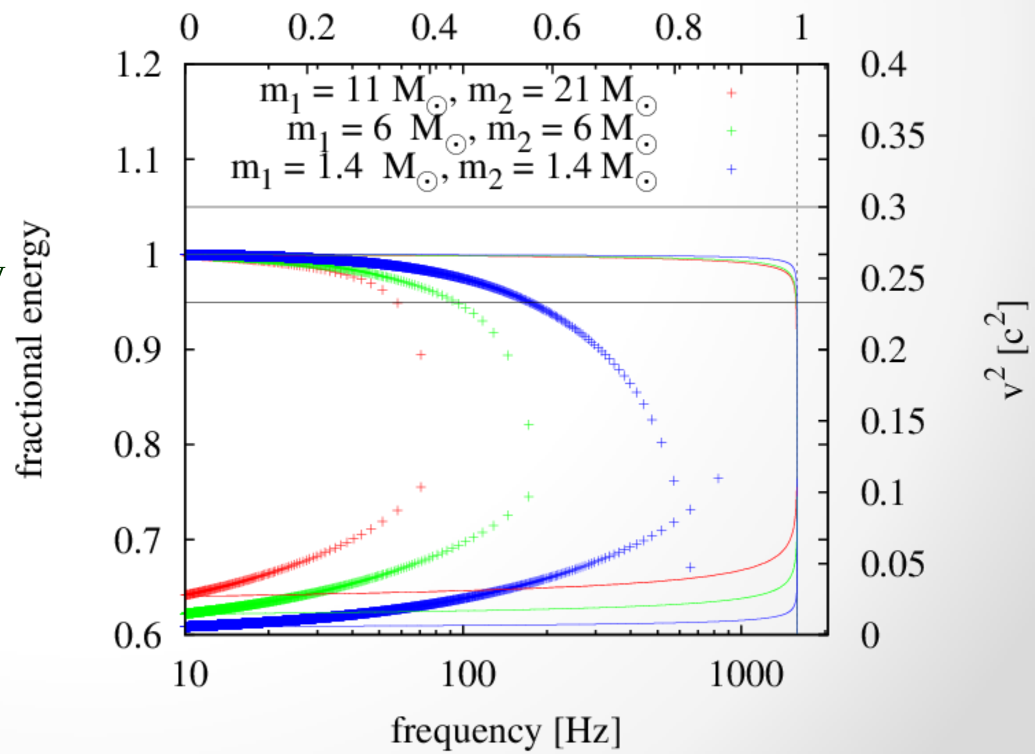
- To check this approximation the ratio of the rate of inspiral of the adiabatic approximation and the corresponding time evolution is monitored.

- It was found that the difference between the adiabatic approximation and the instantaneous change is less then 5%
  - need for a revision of the assumptions applied e.g. in DA
  - use of physically realistic waveforms

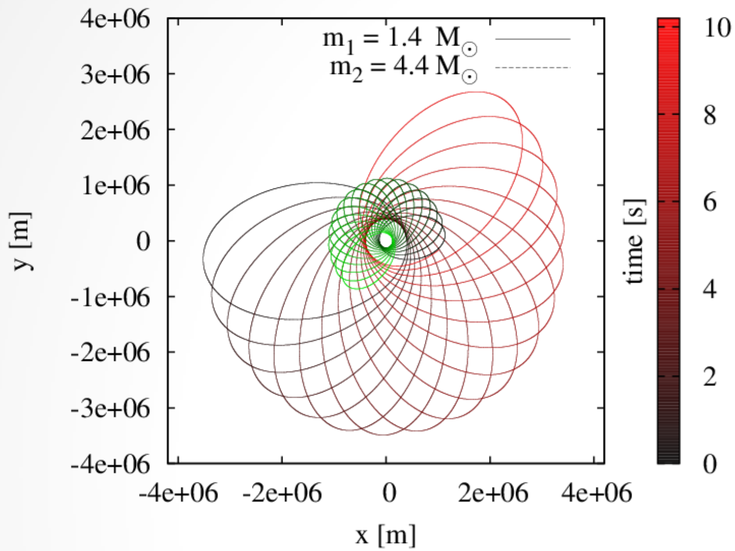


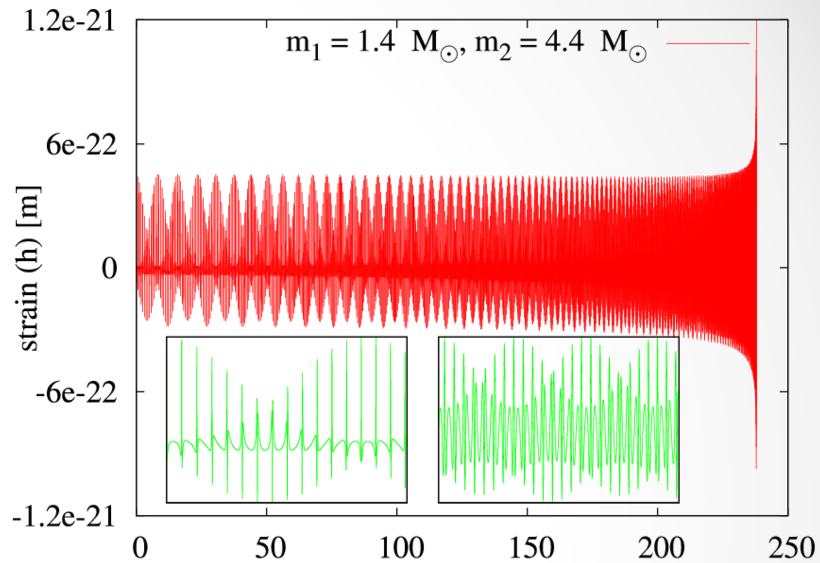
Validity range of PN

- Validity of the PN approximation: until the ISCO ?
- By investigating the validity of the energy balance relation, e.g. the fractional energy, which is the ratio of the actual and initial energies, we have found that the PN approximation should not be applied when the PN parameter reaches the domain $\varepsilon \sim 0.08\text{-}0.1$.
- The loss of accuracy is insensitive, i.e. to the
  - variations of the binary parameters
  - size of the applied time-step
- The loss of accuracy is more transparent in frequency domain and is getting more significant before reaching the frequency ranges of the upgraded ground based detectors.
- In accordance with the results of Levin et. al., where the relative magnitude of the higher order contributions to the equations of motion were monitored.



Effect of eccentricity





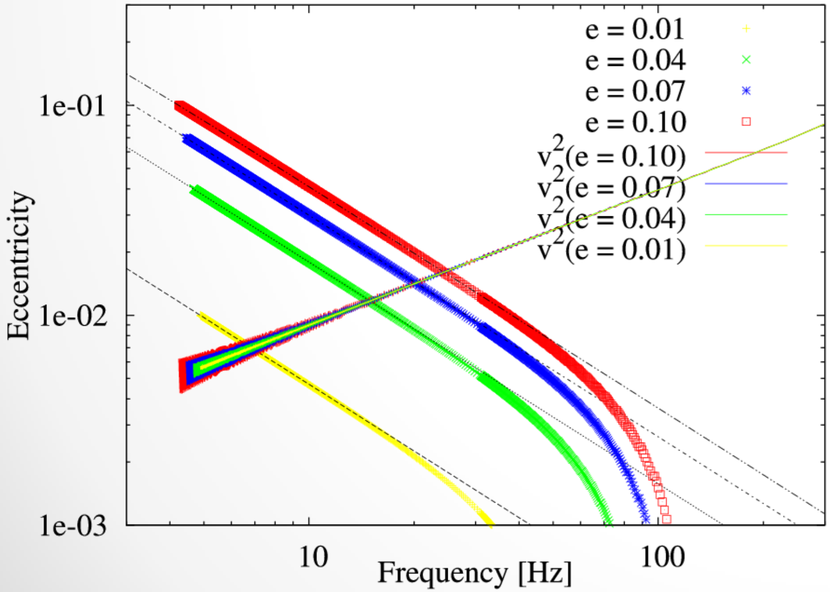
- Radiation reaction drives the binary towards circularization.

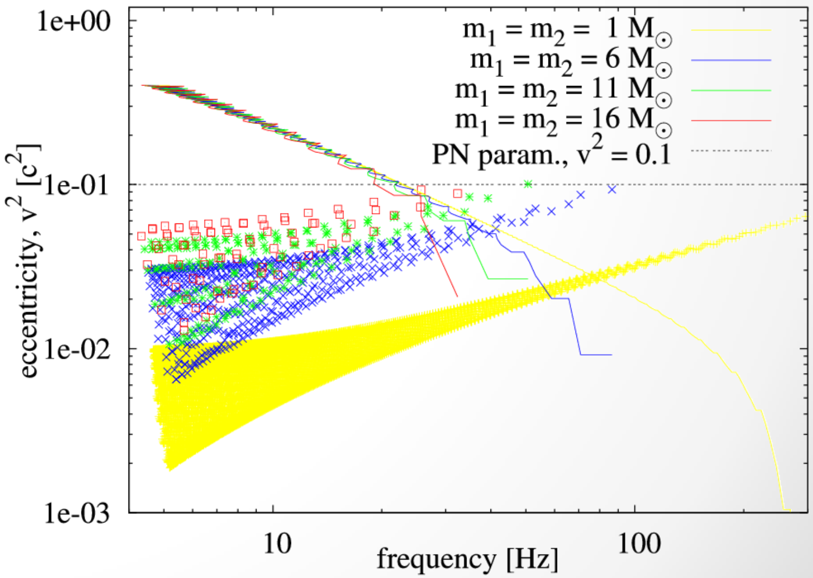
$$e = \frac{r_{max} - r_{min}}{r_{max} + r_{min}}$$

- Use of the geometric, not instantaneous eccentricity
- Detection pipelines are mainly using circular waveform templates
  - the binary is circularized by the time the GW enters the frequency band of the detectors.

Effect of eccentricity

- In the early phase of inspiral eccentric evolution is perfectly explained by analytic estimates. Toward the end of the evolution there is a difference, although the PN parameter is still below its critical value ε ~ 0.08-0.1.
- Universality: the frequency dependence of the eccentricity is insensitive to the total mass or the mass ratio.



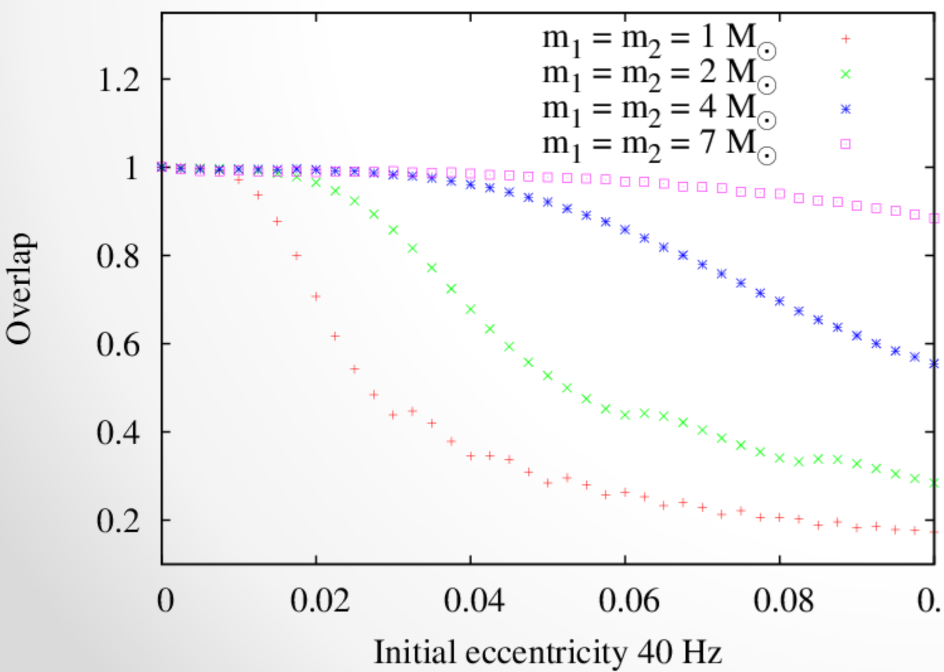


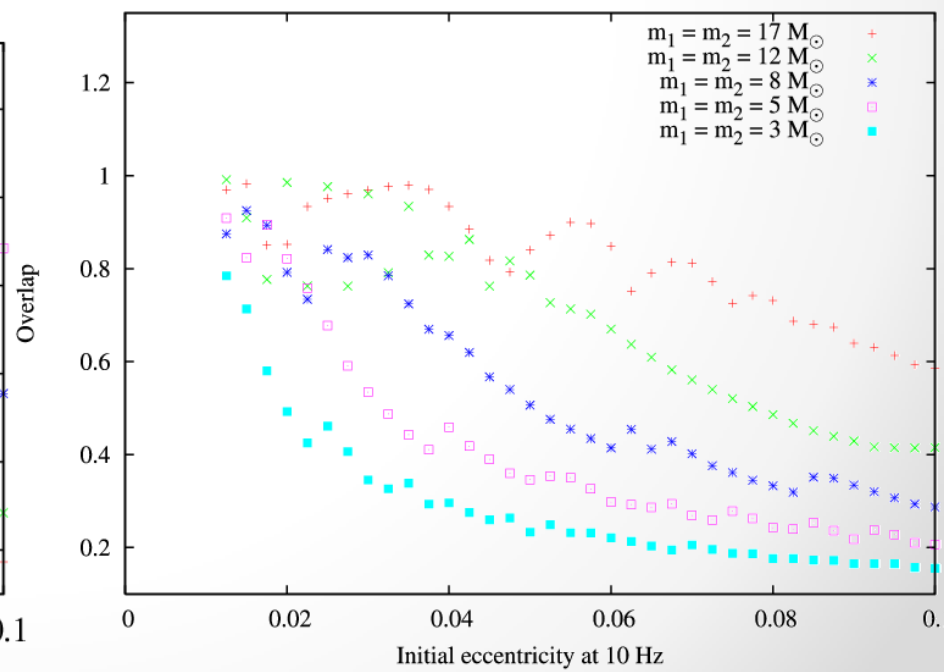
Effect of eccentricity

$$\mathcal{O}_{s,t} = \frac{(s|t)}{\sqrt{(s|s)(t|t)}}$$

$$(a|b) = 2\int_{f_{min}}^{f_{max}} \frac{\tilde{a}^*(f)\tilde{b}(f)+\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)} df$$

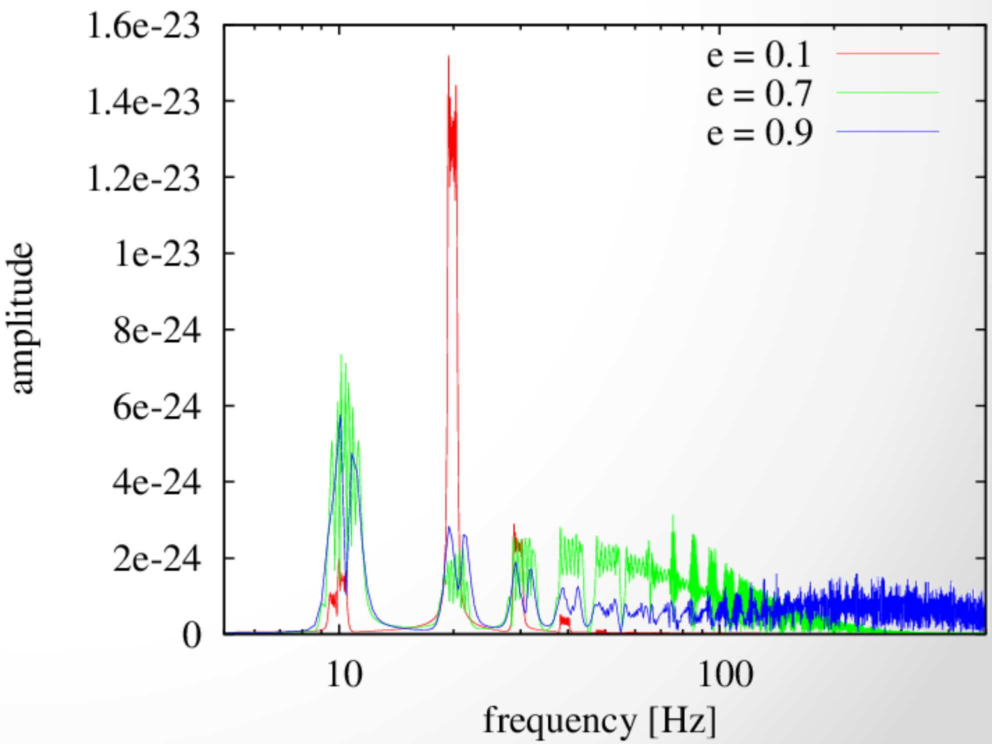
- Besides circularization tiny eccentricity may remain. Non-negligible orbital eccentricity, i.e. from BH binaries formed by tidal capture in globular clusters or galactic nuclei.
- Using circular templates for systems with even tiny residual eccentricities lead to a large Overlap/SNR loss which is studied with CBwaves for AdV and ET.
- The drop of overlap is even more significant than it was for initial LIGO basically because the signals and templates spend longer period within the sensitivity ranges of these more advanced detectors.





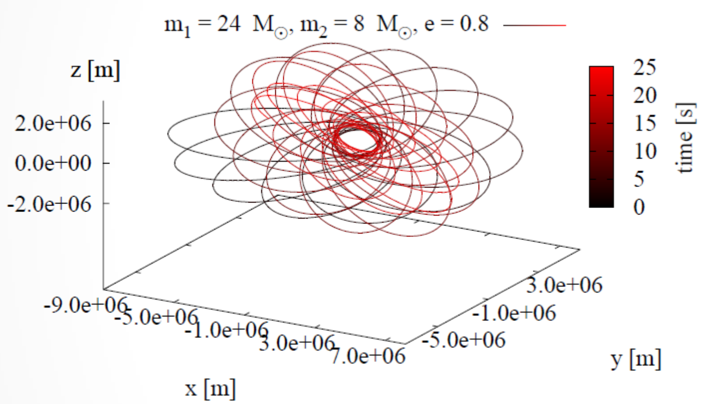
Effect of eccentricity

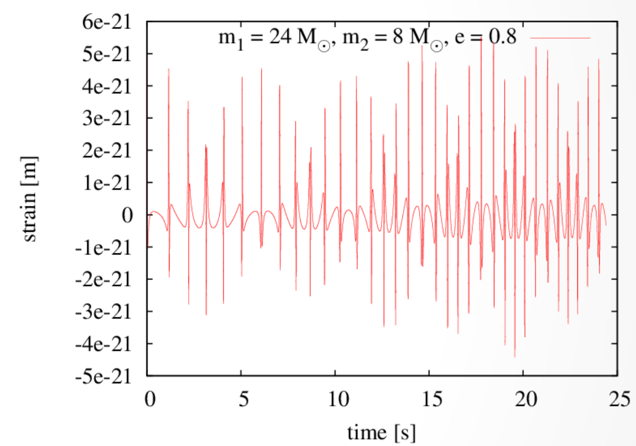
- The early phase of GWs emitted by strongly eccentric sources – formed e.g. in various many-body interactions in the galactic halo – possess specific characteristics
  - burst type character, which may be used to determine the physical parameters of the system
  - from the fractional power present at various frequency bands the eccentricity of the system may be deduced
  - The waveforms spread over several – although not adjacent – frequency bands. This provides a unique imprint to them which may help in constructing sensible detection pipelines robust against transient noises.

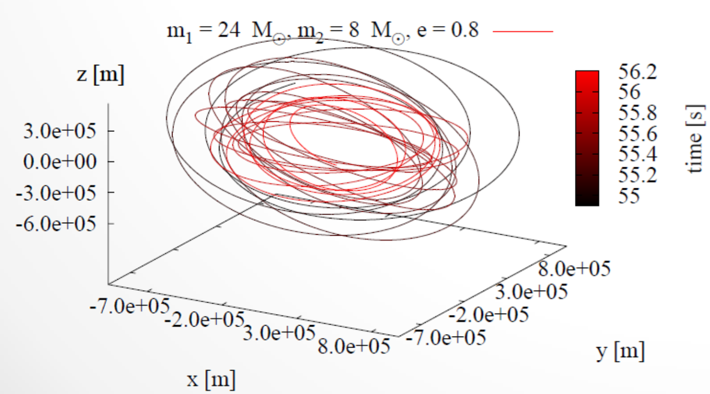


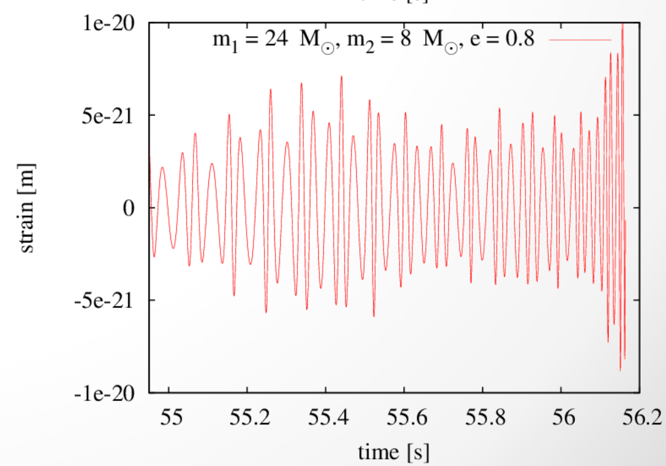
Spinning sources

- In the presence of spins the waveform acquires amplitude and frequency modulation even with zero orbital eccentricity.
- The early phase has burst type character.









Main results

- Investigating the validity of the adiabatic approximation it was found that the adiabatic approximation yields about 5% difference compared to the time evolution which invokes a need for a revision of those data analyzing processes based on the assumption of the equality of these rates.
- Using the designed sensitivity of the advanced and 3rd generation detectors it is demonstrated that circular template banks are even less effective than the initial detectors were in identifying binaries possessing only a tiny residual orbital eccentricity.
- The ranges of correspondences, along with some of the discrepancies, between the analytic and numerical description of the evolution of eccentric binaries, along with some universal properties characterizing their evolution, were pointed out.
- By inspecting the energy balance relation it was shown that, on contrary to the general expectations, the post-Newtonian approximation should not be applied once the post-Newtonian parameter gets beyond its critical value $\varepsilon \sim 0.08$-0.1, which was found to precede considerably the phase of reaching the ISCO.
- By studying gravitational waves emitted during the early phase of the evolution of strongly eccentric binary systems it was found that they possess very specific characteristics which may make the detection of these type of binary systems to be feasible.

Final remarks

Gravitational waves from spinning eccentric binaries*

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- arXiv:1207.0001
  Submitted to ***Classical and Quantum Gravity***

- **RMKI Virgo Group** The CBwaves software is available for public use

http://grid.kfki.hu/project/virgo/cbwaves/

- Easy to use, suited for DA purposes

- Future plans: porting to GPU, open orbits, spinflip, SSC ….