Some of the recent developments in general relativity

István Rácz
Wigner RCP, Budapest

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Outline

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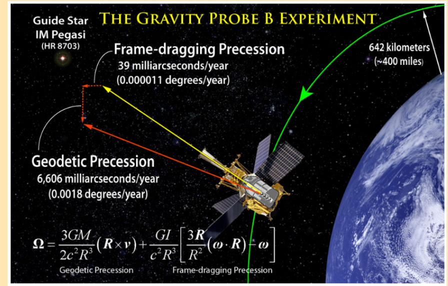
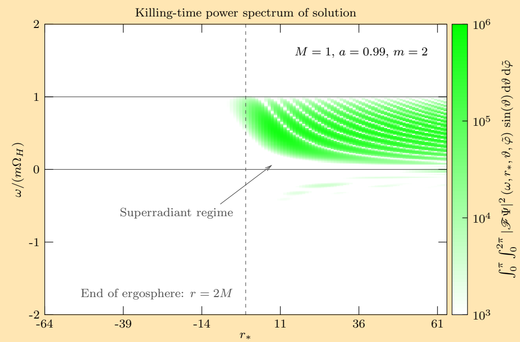
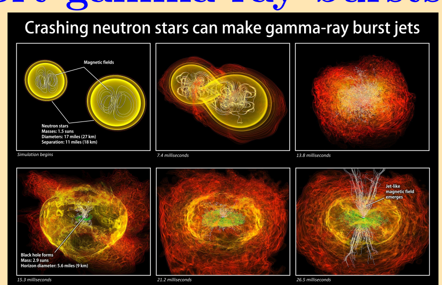


- **Superradiance or total reflection?**

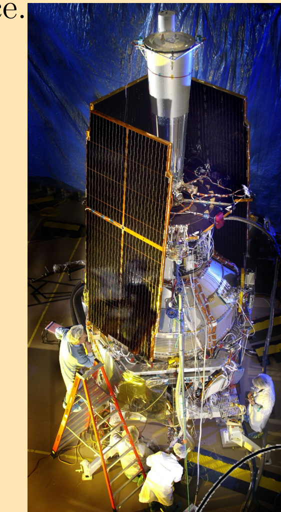


- **The engine for short gamma ray bursts**



Gravity Probe B

- To test Einstein's theory of general relativity, Gravity Probe B attempt to measure two minuscule angles with spinning gyroscopes, floating in space.

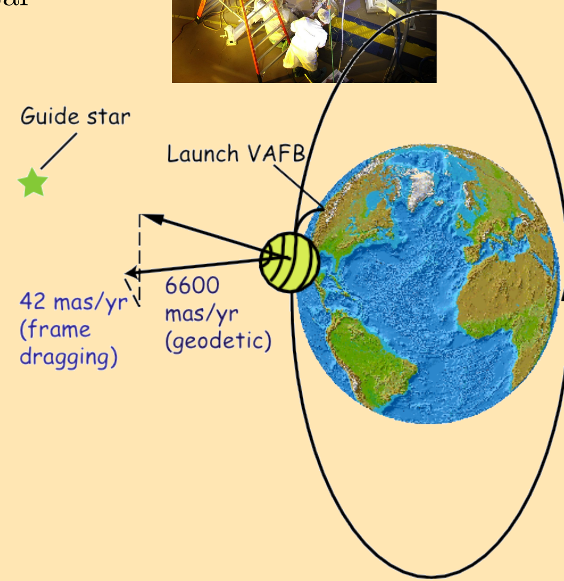


  - After 31 years of research and development,
  - 10 years of flight preparation,
  - a 1.5 year flight mission (Apr. 2004 - Sept. 2005)
  - 5 years of data analysis (!)

  the GP-B team announced the final experimental results May 4, 2011.

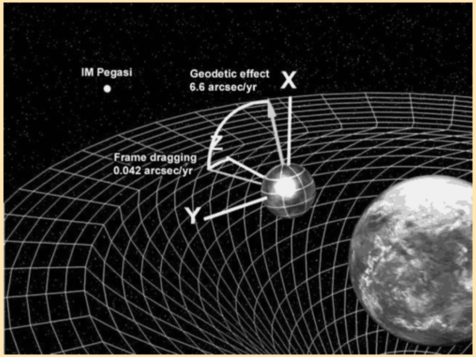
  - GP-B was designed to measure two key predictions
    - the geodetic precession
    - the frame dragging

    of Einstein's general theory of relativity by monitoring the orientations of ultra-sensitive gyroscopes relative to a distant guide star.



Gravity Probe B

- The warping of spacetime near the rotating Earth exerts a torque on the gyroscope so that its axis slowly precesses—by about 6.6 arcseconds (or 1.8 thousandths of a degree) per year—in the plane of the satellite's orbit.
  - To picture this precession, or "geodetic effect," imagine a stick moving parallel to its length on a closed path along the curved surface of the Earth, returning to its origin pointing in a slightly different direction than when it started.
- The rotation of the Earth also exerts a "frame-dragging" effect on the gyro. In this case, the precession is perpendicular to the orbital plane and advances by 40 milliarcseconds per year.
- Josef Lense and Hans Thirring first pointed out the existence of the frame-dragging phenomenon in 1918. The experiment with gyroscopes was first suggested in late 1959 by George Pugh and Leonard Schiff .

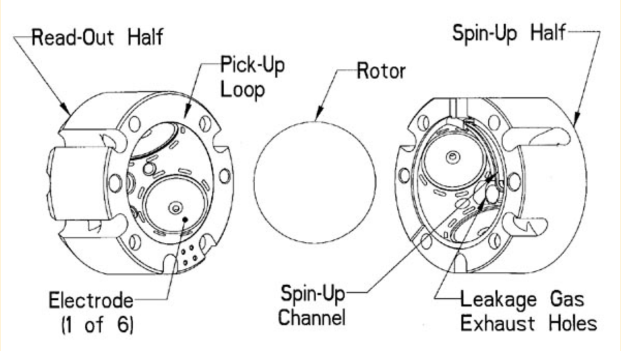


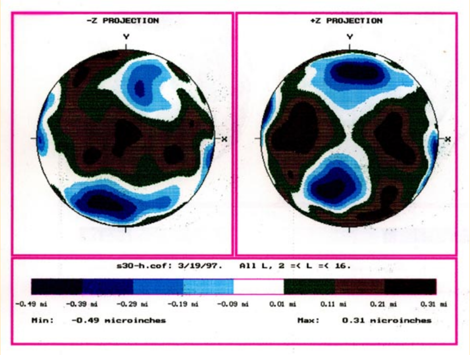
- Disclosure: Clifford M. Will chaired NASA's Science Advisory Committee for GP-B (1998-2011)

The gyroscopes

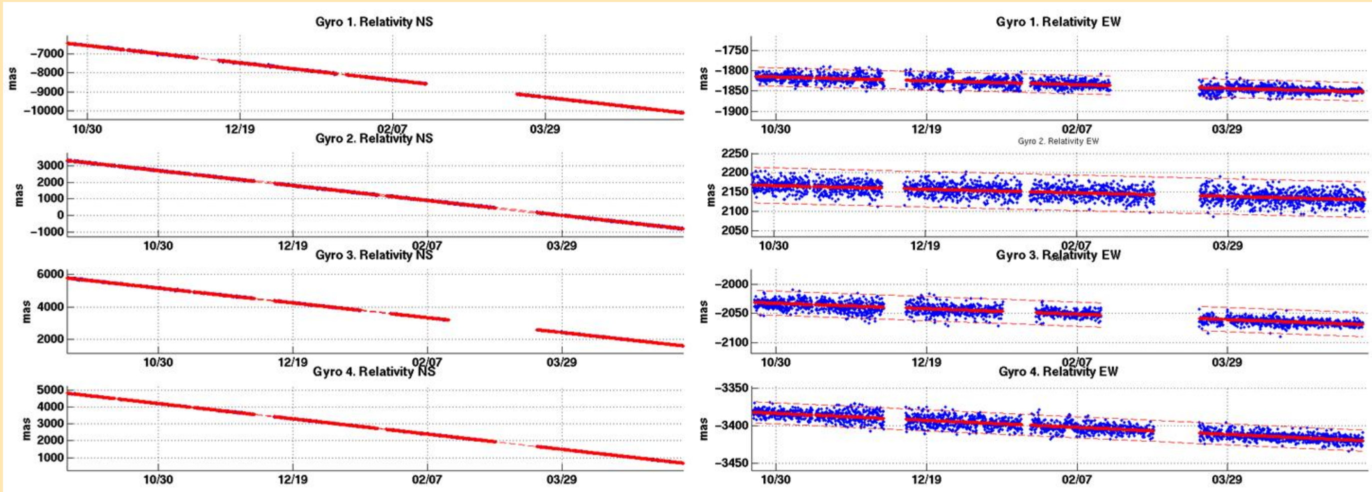
- To measure the minuscule angles predicted by Einstein's theory, the GP-B team needed to build a near-perfect gyroscope—one whose spin axis would not drift away from its starting point by more than one hundred-billionth of a degree each hour that it was spinning.
- By comparison, the spin-axis drift in the most sophisticated Earth-based gyroscopes, found in high-tech aircraft and nuclear submarines, is seven orders of magnitude (more than ten million times) greater than GP-B could allow.





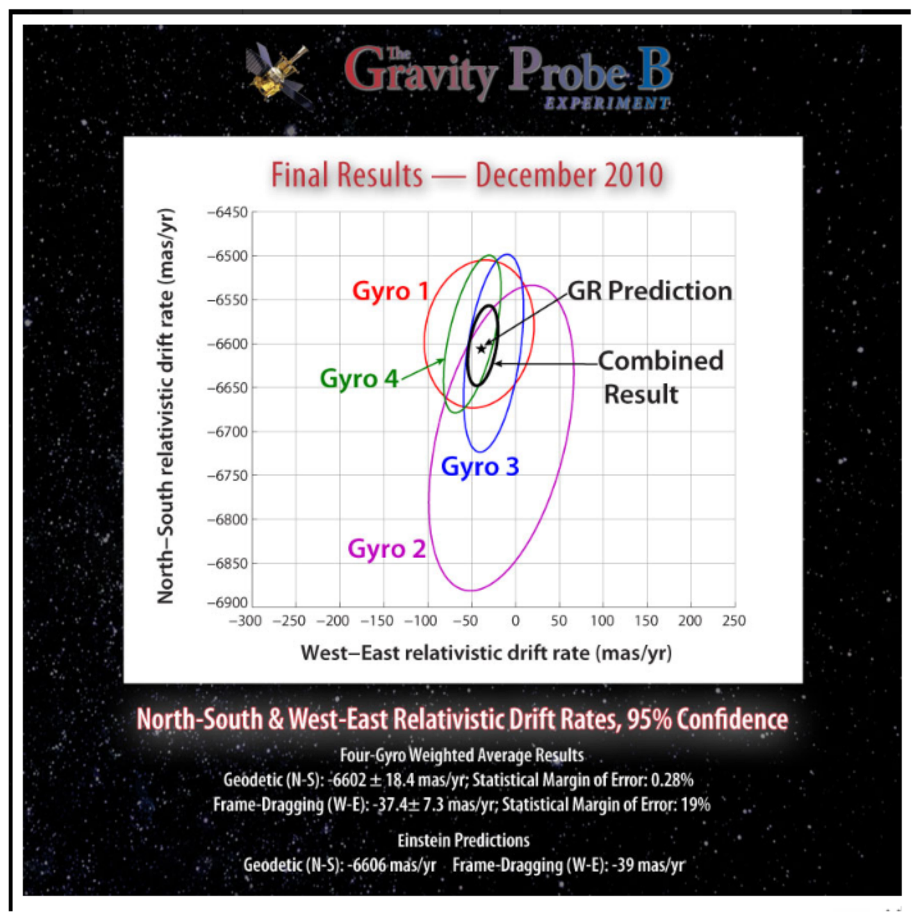


Gravity Probe B: The Experiment – Gyroscopes, May 4, 2011



- The story: Clifford M. Will: Finally, results from Gravity Probe B, Physics 4, 43 (2011)
- Three important, but unexpected, phenomena were discovered during the experiment that affected the accuracy of the results:
  - First, because each rotor is not exactly spherical, its principal axis rotates around its spin axis with a period of several hours, with a fixed angle between the two axes.
  - In addition, over the course of a day, each rotor was found to make occasional, seemingly random "jumps" in its orientation—some as large as 100 milliarcseconds. Some rotors displayed more frequent jumps than others. Without being able to continuously monitor the rotors' orientation, Everitt and his team couldn't fully exploit the calibrating effect of the stellar aberration in their analysis.
  - Finally, during a planned 40-day, end-of-mission calibration phase, the team discovered that when the spacecraft was deliberately pointed away from the guide star by a large angle, the misalignment induced much larger torques on the rotors than expected. From this, they inferred that even the very small misalignment that occurred during the science phase of the mission induced torques that were probably several hundred times larger than the designers had estimated.
  - What ensued during the data analysis phase was worthy of a detective novel. The team was able to account for all these effects in a parametrized model.

Gravity Probe B



N-S geodetic precession, W-E frame dragging

Superradiance or total reflection?

Joint work with András László

The stability of the Kerr black hole

- The stability problem for the Kerr family of black hole solutions to the vacuum Einstein equations is one of the most important unresolved issues in GR.
  - The ultimate goal is to understand the dynamical stability of Kerr, as a family of solutions, to the Cauchy problem for the system of nonlinear hyperbolic equations

$$\boxed{R_{ab}(g) = 0}$$

- Essentially all work in the black hole case has been confined to the linearized setting
  - The simplest problem: scalar perturbations on a fixed Kerr background

$$\boxed{\Box_g \Phi = 0}$$

  - which is a poor man's substitute for the more complicated problem of gravitational perturbations, obtained by linearizing $R_{ab}(g) = 0$ around a Kerr BH.
- A complete proof, covering the general subextremal case, of linear stability for scalar perturbations was given recently by M. Dafermos & I. Rodnianski [arXiv:1010.5137].

Superradiance

- The wave analog of the Penrose process: *allows energy to be extracted from black holes*.
  - "...if scalar, electromagnetic or gravitational wave is incident upon a black hole, part of the wave (the "transmitted wave") will be absorbed by the black hole and part of the wave (the "reflected wave") will escape to infinity."
- Superradiance, discovered at the early 70's as a new phenomenon, may be related to the names of Misner, Zel'dovich and Starobinskii
- By using the Teukolsky equation scalar, electromagnetic and gravitational perturbations can be investigated within the same setting.
- The conventional arguments ending up with superradiance, including the ones based on Teukolsky's equation, all refer to properties of individual modes.
- As it was shown first by Bekenstein whenever superradiance happens it can be seen to be completely consistent with the laws of BH thermodynamics.
- The aforementioned proof of linear stability by M. Dafermos & I. Rodnianski does not include a detailed investigation of superradiance.
  - Their main concern was to provide boundedness and decay statements for solutions of $\Box_g\Phi = 0$ arising from arbitrary finite-energy initial data.

Superradiance (mode analysis)

- It was realized first by Carter the d'Alembert operator separates for the t-Fourier transformed field.
  - the temporal Fourier transform, $\mathscr{F}\Phi$, of a solution to $\Box_g \Phi = 0$, in coordinates $t, r_*, \vartheta, \varphi$, may be decomposed as

$$\boxed{\mathscr{F}\Phi(\omega, r_*, \vartheta, \varphi) = \frac{1}{\sqrt{r^2+a^2}} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} R^m_{\ell,\omega}(r_*) S^m_{\ell,a\omega}(\vartheta, \varphi)\,,} \tag{1}$$

  - ω is the frequency in the time translation direction
  - $S^m_{\ell,a\omega}$ denotes the *oblate spheroidal harmonic functions* with oblateness parameter $a\omega$ and with angular momentum quantum numbers ℓ, m ($S^m_{\ell,a\omega}$ eigenfunctions of a self-adjoint op.)
  - for the radial functions $R^m_{\ell,\omega}$ a *one-dimensional Schrödinger equation* of the form

$$\boxed{\frac{d^2 R^m_{\ell,\omega}}{dr_*^2} + \left[\left(\omega - \frac{ma}{r^2+a^2}\right)^2 + \Delta \cdot V^m_{\ell,\omega}(r_*)\right] R^m_{\ell,\omega} = 0\,,} \tag{2}$$

  with suitable real potentials $V^m_{\ell,\omega}(r_*)$, can be derived from $\Box_g \Phi = 0$.

Superradiance (mode analysis)

- The "physical solutions" to (2) are supposed to possess the asymptotic behavior

$$\boxed{R^m_{\ell,\omega} \sim \begin{cases} e^{-i\omega r_*} + \mathcal{R}\, e^{+i\omega r_*} & \text{as } \; r \to \infty \\ \mathcal{T}\, e^{-i(\omega - m\Omega_H) r_*} & \text{as } \; r \to r_+ \end{cases}} \tag{3}$$

  - Ω_H: the angular velocity of the BH w.r.t the asymptotically stationary observers
  - with reflection and transmission coefficients, $\mathcal{R}$ and $\mathcal{T}$, respectively.

- **!** (3) presumes the existence of a transmitted wave submerging into the ergoregion.

- By evaluating the Wronskian of the corresponding fundamental solutions, "close" to infinity and "close" to the horizon, it can be shown that

$$\boxed{(\omega - m\Omega_H)\,\mathcal{T} = (1 - \mathcal{R})\,\omega.} \tag{4}$$

- Whenever $\mathcal{R} > 1$—or equivalently whenever the inequality

$$\boxed{0 < \omega < m\,\Omega_H} \tag{5}$$

holds—positive energy is supposed to be acquired by the backscattered scalar mode due to its interaction with the Kerr black hole.

Superradiance (mode analysis)

- It is precisely in the frequency range $0 < \omega < m\,\Omega_H$ where for individual modes the sign of the energy flux through the event horizon is negative.
- An analogous conclusion can be drawn by looking at the "particle number current" in the scalar and electromagnetic wave cases.
- The linear stability problem solved first by Kay and Wald taught us important lessons:
  - **statements at the level of individual modes typically do not imply statements for the superposition of infinitely many modes**

Numerical studies

- We studied the evolution of complex scalar fields on Kerr background
  - **GridRipper (3+1)** is fully spectral in the angular directions while the dynamics in the complementary 1+1 Lorentzian spacetime is followed by making use of a fourth order finite differencing scheme with adaptive mesh refinement (AMR).

The initial data

- To investigate the way an incident scalar wave acquires extra energy by submerging into the ergoregion the solution, in the asymptotic region, was expected to posses the form

$$\boxed{\Phi(t, r_*, \vartheta, \tilde{\varphi}) \approx e^{-\mathrm{i}\,\omega_0\,(r_* - r_{*0} + t)} f(r_* - r_{*0} + t)\, Y_\ell^m(\vartheta, \tilde{\varphi}).} \tag{6}$$

  where $f : \mathbb{R} \to \mathbb{C}$ is a smooth function of compact support and ω_0, r_{*0} are real parameters.

- This, in a sufficiently small neighborhood of the initial data surface in the asymptotic region, may be generated by choosing the initial data as

$$\boxed{\begin{aligned} \phi(r_*, \vartheta, \tilde{\varphi}) &= e^{-\mathrm{i}\,\omega_0\,(r_* - r_{*0})} f(r_* - r_{*0})\, Y_\ell^m(\vartheta, \tilde{\varphi})\,, \\ \phi_t(r_*, \vartheta, \tilde{\varphi}) &= -\mathrm{i}\,\omega_0\, \phi(r_*, \vartheta, \tilde{\varphi}) + e^{-\mathrm{i}\,\omega_0\,(r_* - r_{*0})}\, f'(r_* - r_{*0})\, Y_\ell^m(\vartheta, \tilde{\varphi})\,, \end{aligned}}$$

  where f' denotes the first derivative of $f : \mathbb{R} \to \mathbb{C}$.

- the Fourier transform, $\mathscr{F}\Phi$, of the approximate solution (6) reads as

$$\boxed{\mathscr{F}\Phi(\omega, r_*, \vartheta, \tilde{\varphi}) \approx e^{-\mathrm{i}\,\omega\,(r_* - r_{*0})} \mathscr{F}f(\omega - \omega_0)\, Y_\ell^m(\vartheta, \tilde{\varphi}),} \tag{7}$$

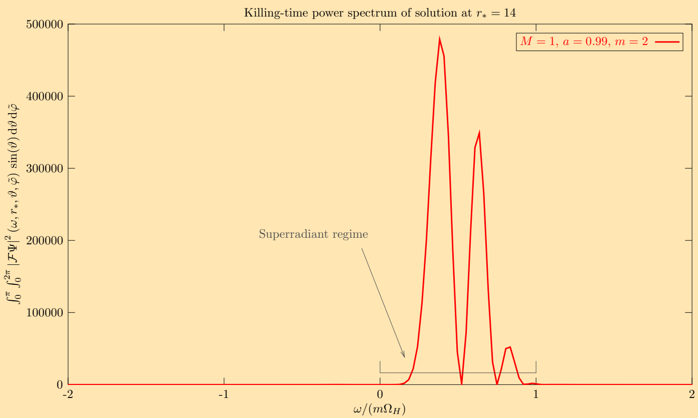
  - ω is the temporal frequency
  - $\mathscr{F}f$ stands for the Fourier-transform of f, playing the role of a frequency profile.

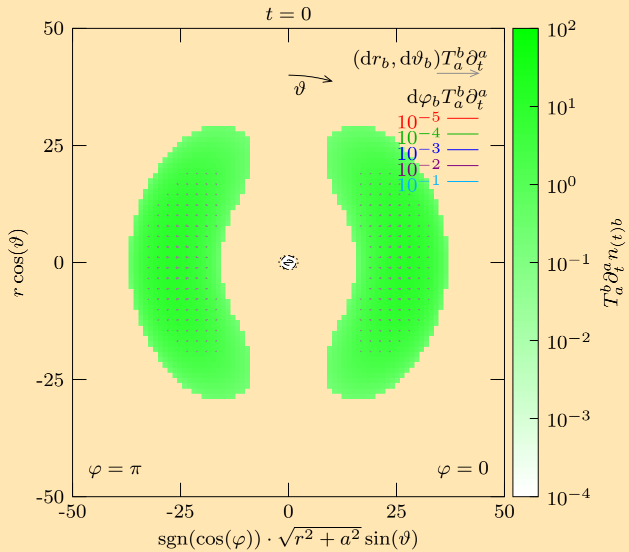
- Assuming that $\mathscr{F}f$ is sufficiently narrow the approximate solution (6) should be close to a monochromatic wave packet.

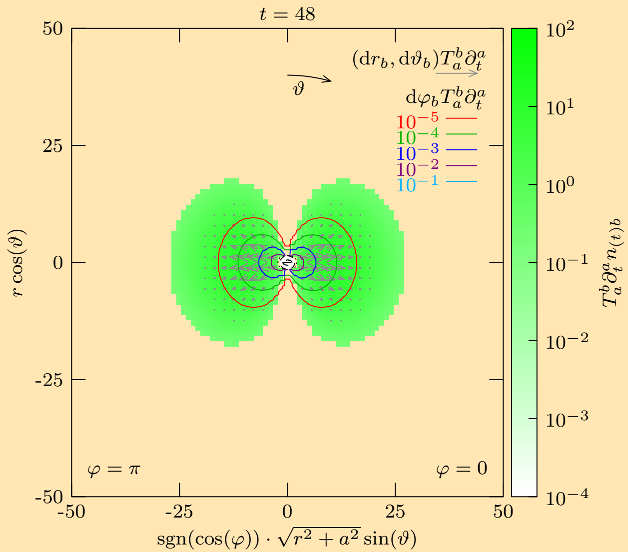
The chosen type of initial data is to be superradiant

- the frequency spectrum of a to be superradiant solution at $r_* = 14$ located towards the black hole with respect to the compact support of the initial data

$M = 1,\ a = 0.99,\ \ell = m = 2,$
$\omega_0 = \frac{1}{2} m \Omega_H,\ r_{*0} = 31.823$
(!pure quadrupole type initial data!)







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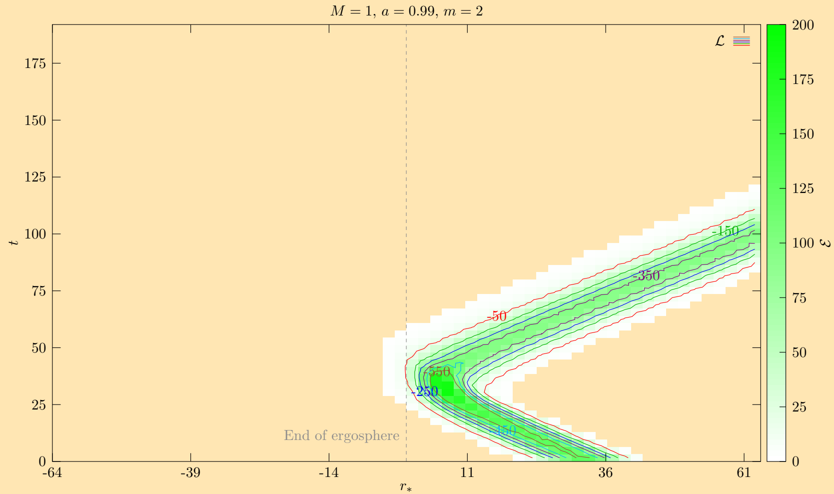
The time dependence of the radial energy and angular momentum distributions & and the power spectrum

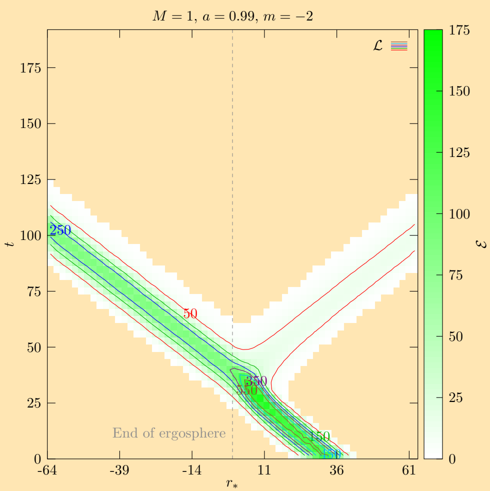
the energy and angular momentum, E and L, on a $t = const$ time level surface can be given as

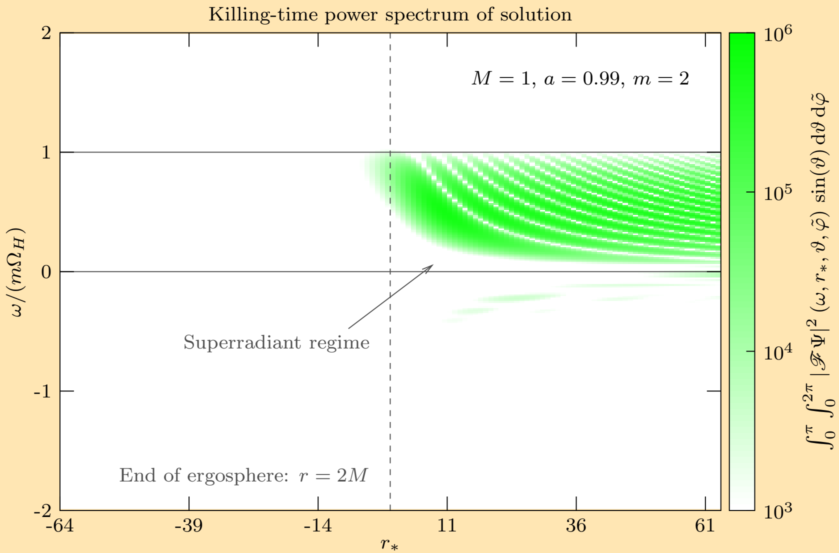
$$\boxed{E = \int_{t=const} \mathscr{E} \mathrm{d}r_*}$$

and

$$\boxed{L = \int_{t=const} \mathscr{L} \mathrm{d}r_*}$$

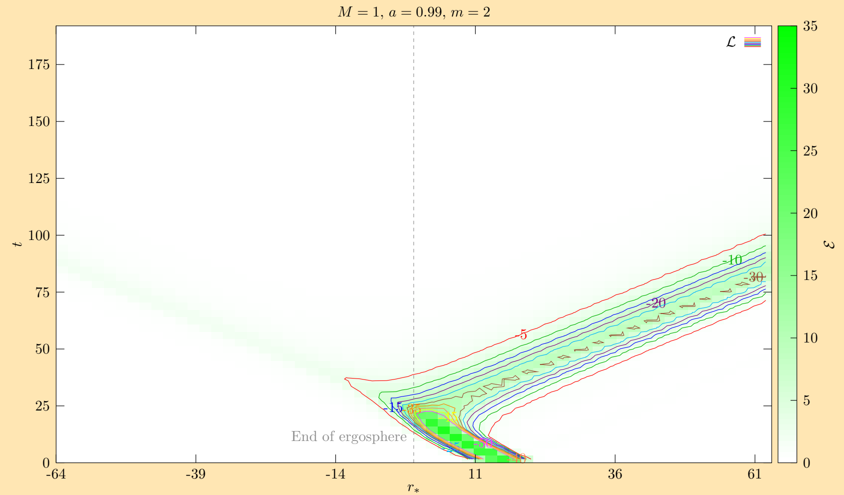


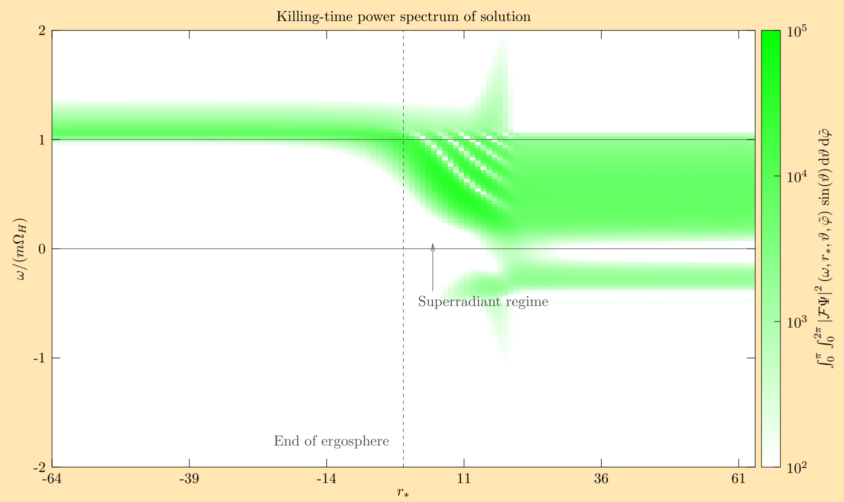




The time dependence of the radial energy and angular momentum distributions & and the power spectrum (2)

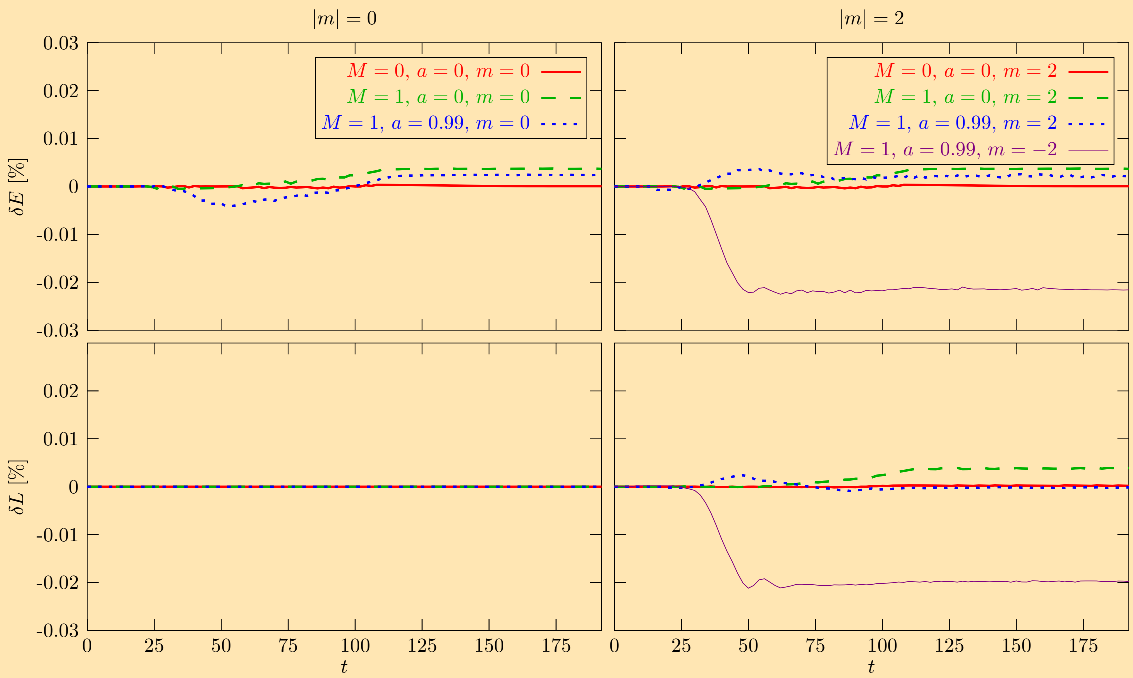
For an almost to be superradiant solution





"statements at the level of individual modes typically do not imply statements for the superposition of infinitely many modes"

The accuracy



- Time dependence of the relative variation of the energy and angular momentum balances

$$\delta E = \frac{[E(t)+E_{rad}(t)]-E_0}{E_0} \quad \text{and} \quad \delta L = \frac{[L(t)+L_{rad}(t)]-L_0}{L_0}$$

  – E_0 and L_0 are the initial energy and angular momentum, respectively.

Summary

- The evolution of massless scalar field on Kerr background, arising from initial data with compact support in the distant region, was considered.
- The incident wave packet was tuned to maximize the effect of superradiance.
- For perfectly tuned data instead of the occurrence of energy extraction from black hole the inward sent radiation fail to reach the ergoregion rather it suffers total reflection.
- By examining the energy to angular momentum content of the to be superradiant wave packets it is clear that far too much angular momentum is stored by them, $E < \Omega_H L$, which does not allow them to reach the horizon of the black hole.
- This new phenomenon may be considered as the field theoretical analog of the one in Wald's thought experiments demonstrating, in the early 70', that a Kerr black hole does not capture a particle that would cause a violation of the relation $m^2 \geq a^2 + e^2$.
- Our findings do also have implications related to the concept of BH bomb. If superradiance does not occur the solutions to the massive Klein-Gordon equation

  $$\boxed{\Box_g \Phi = \mu^2 \Phi}$$

  on Kerr background, as opposed to some fashionable speculations, should remain bounded and, in turn, our results support the stability of Kerr black holes.

The engine for short gamma ray bursts

The goals of numerical relativity

- Using NR to explore fundamental physics and astrophysics
- Numerical relativity solves Einstein equations in those regimes in which no approximation is expected to hold. The codes developed may be used as "theoretical laboratories".

The problem of time evolution in GR

$\boxed{R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}}$ field equations

$\boxed{\nabla^a T_{ab} = 0}$ conservation of en. & imp.

$\boxed{\nabla_a (\rho u^a) = 0}$ conservation of baryon no.

$\boxed{p = p(\rho, \epsilon, ...)u}$ EOS ... thermodynamics

$\boxed{\nabla^a F^*_{ab} = 0}$ Maxwell eqs., induction, zero div.

$\boxed{T_{ab} = T^{fluid}_{ab} + T^{em}_{ab} + ...}$

- In non-vacuum spacetimes the truncation error is the only measurable error: "SIMULATION"
- It's our approximation to "reality" and it can be continuously improved: microphysics, magnetic fields, viscosity, radiation support, ...

The two-body problem

- For BHs we know what to expect:

$$\boxed{\mathbf{BH + BH \Longrightarrow BH + gravitational\ waves\ (GWs)}}$$

- For NSs the question is more subtle: the merger leads to an hyper-massive neutron star (HMNS), i.e. a metastable equilibrium:

$$\boxed{\mathbf{NS + NS \Longrightarrow HMNS + \ldots\ ? \Longrightarrow BH + torus + \ldots\ ? \Longrightarrow BH}}$$

- All the physics and complications are in the intermediate stages;

  the rewards are however high:

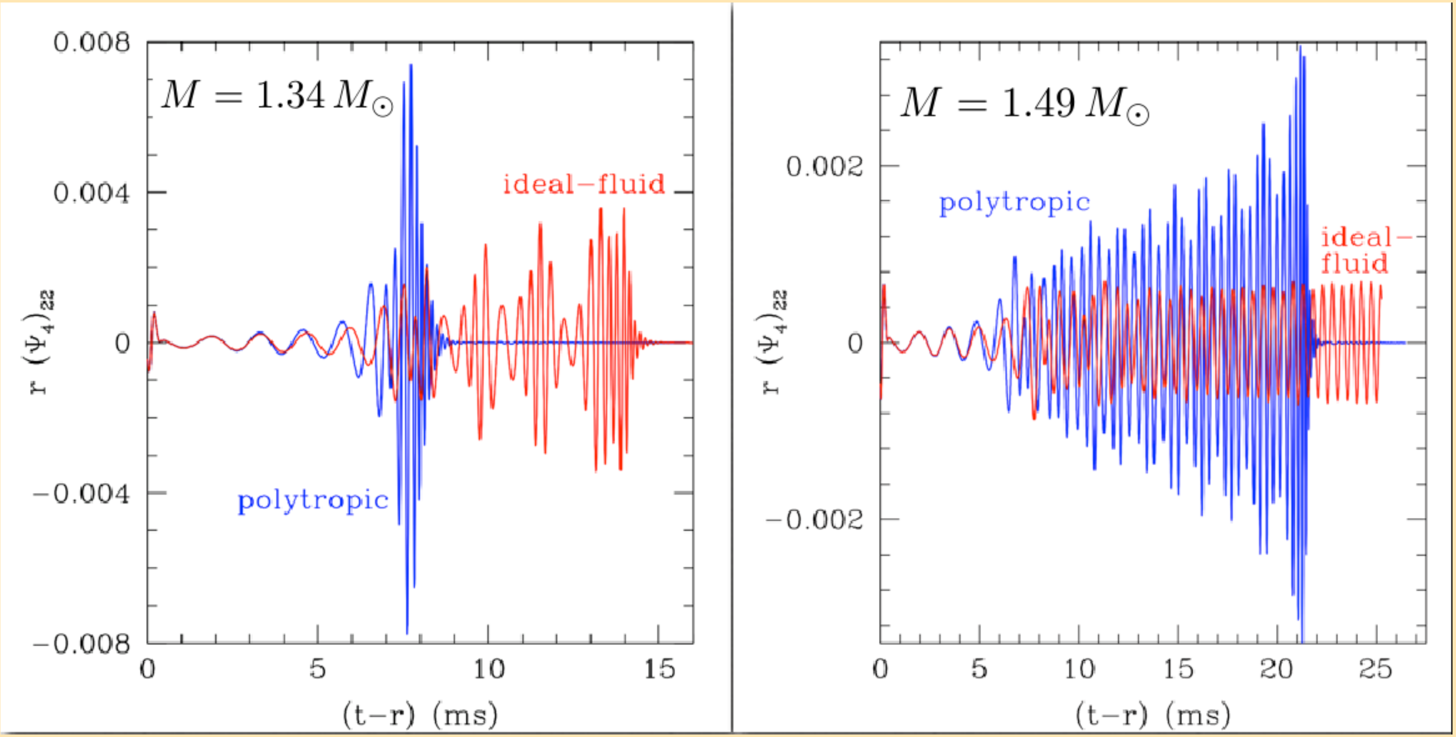
  – studying the HMNS will give an imprint on the EOS

  – studying the BH+torus will tell us on the central engine of GRBs

Don't forget: with advanced detectors we expect to have a realistic rate of 40 BNSs inspirals a year, i.e. ~ 1 a week (range is 0.4/400 for pessimistic/optimistic estimates; Abadie+ 2010).

The role of EOS in binary NSs problem

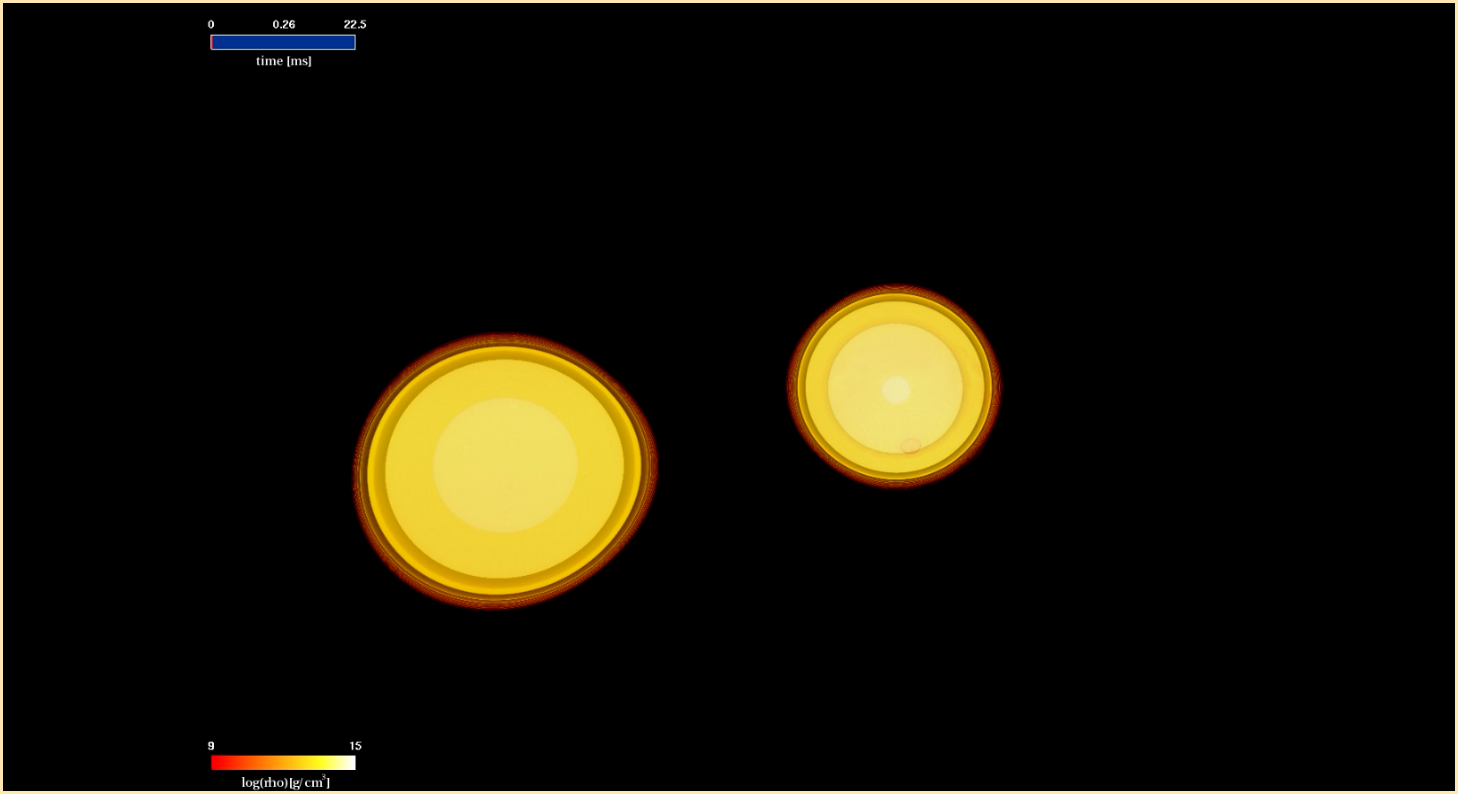
There are clear differences for the same mass and for the same EOS: multidimensional parameter space



The evolution of a binary neutron star (BNS)

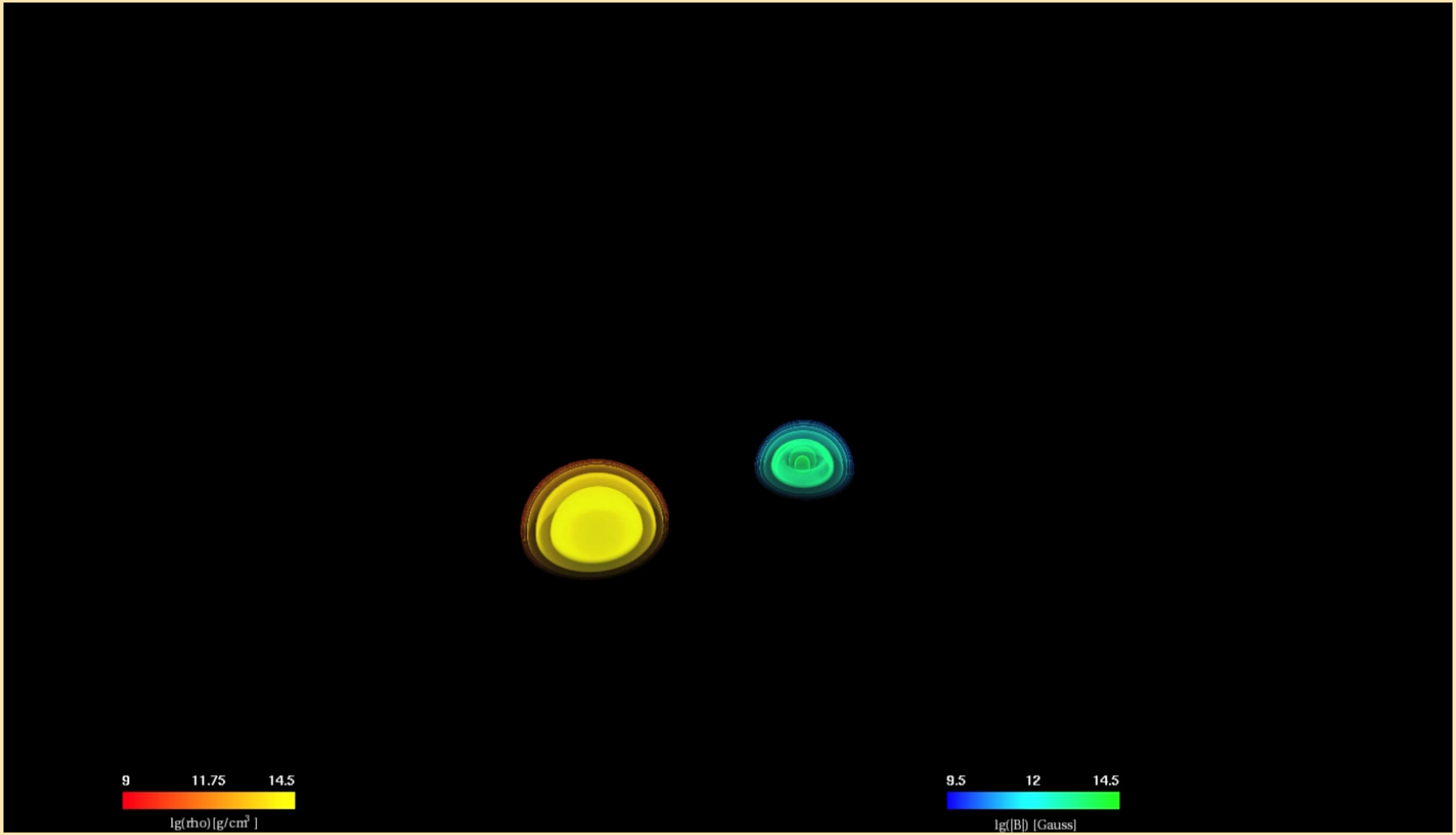
[as viewed by Luciano Rezzolla's group at AEI]

- After a few orbits the two neutron stars merge and following a short intermediate period with a highly unstable HMNS a black hole is formed at the center.
- Notice the formation of a thick accretion disk with an oblate toroidal shape.



The evolution of a highly magnetized binary neutron star (BNS)

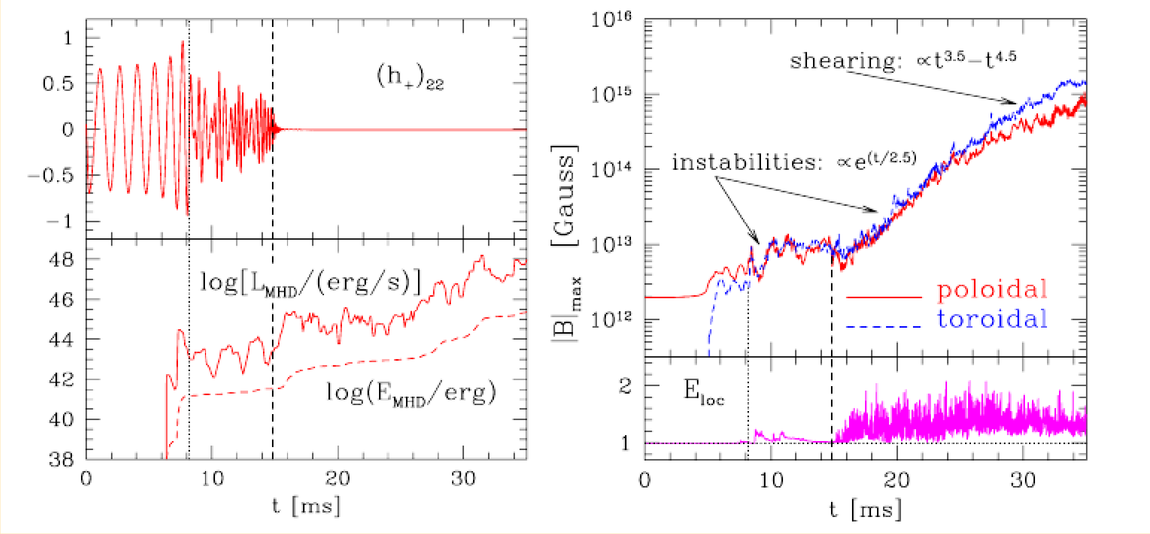
- After a few orbits the two neutron stars merge and following a short intermediate period with a highly unstable HMNS a black hole is formed at the center.
- Notice the variation and the redistribution of the magnetic field by the end of the simulation.



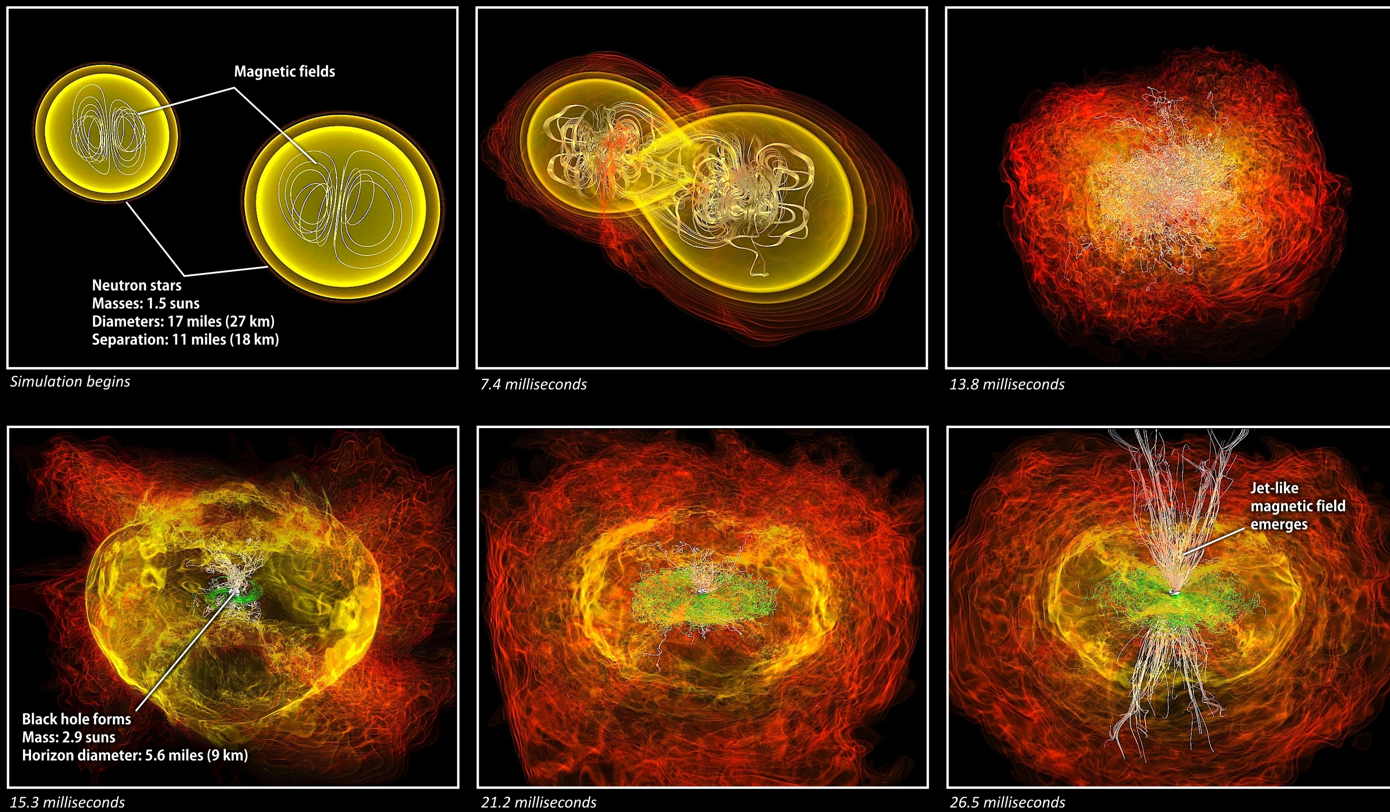
The two-body problem: GR

- Left panel: GW signal shown through the $\ell = 2$, $m = 2$ mode of the + polarization, $(h_+)_{22}$ (top part), and the MHD luminosity, LMHD (bottom part) as computed from the integrated Pointing flux and shown with a solid line. The corresponding energy, EMHD , is shown with a dashed line. The dotted and dashed vertical lines show the times of merger (as deduced from the first peak in the evolution of the GW amplitude) and BH formation, respectively.

- Right panel: evolution of the maximum of the magnetic field in its poloidal (red solid line) and toroidal (blue dashed line) components. The bottom panel shows the maximum local fluid energy, indicating that an unbound outflow (i.e., $E_{loc} > 1$) develops and is sustained after BH formation.

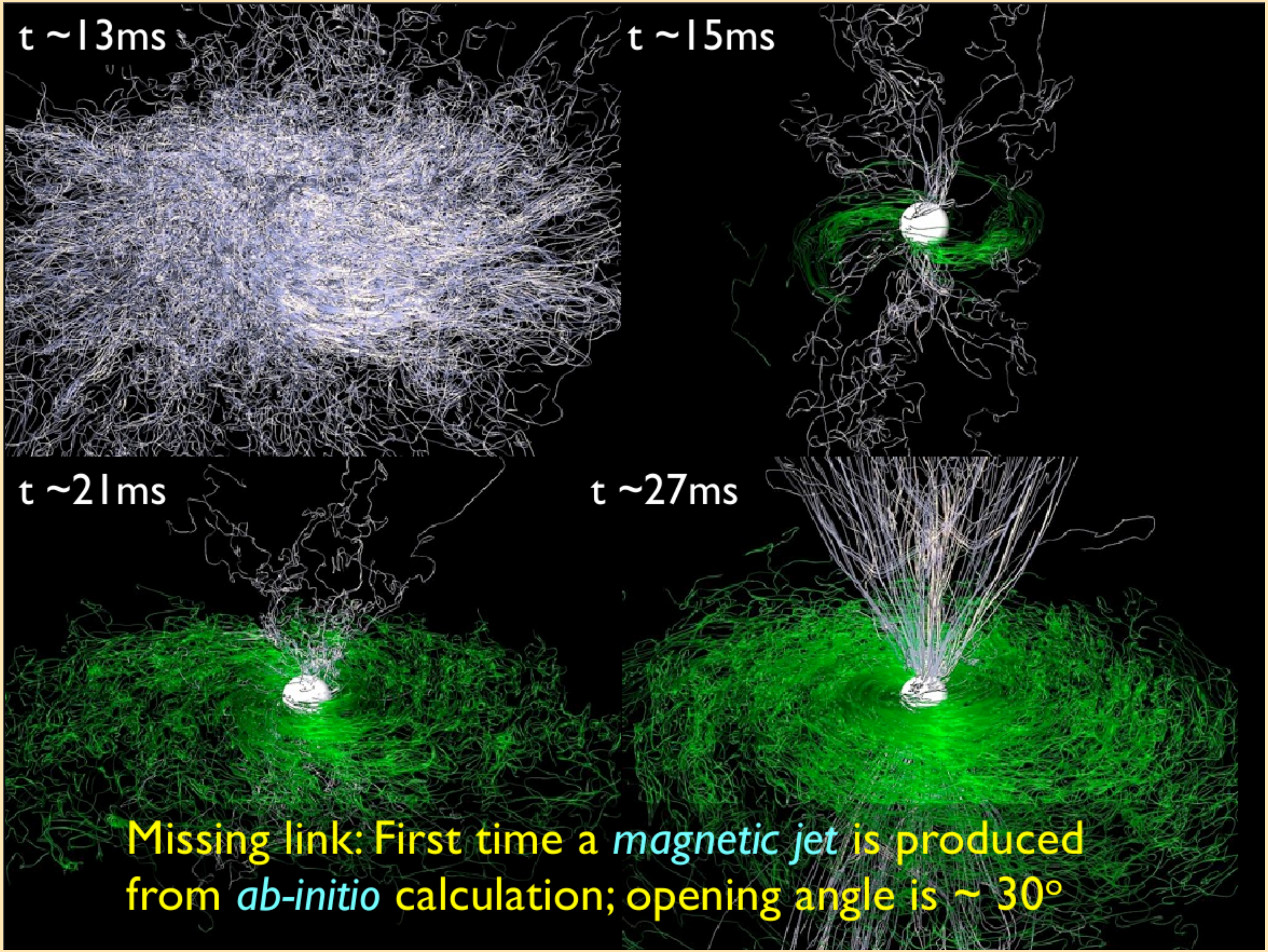


Crashing neutron stars can make gamma-ray burst jets



Credit: NASA/AEI/ZIB/M. Koppitz and L. Rezzolla

$J/M^2 = 0.83 \qquad \mathrm{M_{tor}} = 0.063 M_\odot \qquad \mathrm{t_{accr}} \simeq M_{\rm tor}/\dot{M} \simeq 0.3\ \mathrm{s}$



Thanks for your attention